Computer Algebra
Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

4-Trig-functions/4.7-Miscellaneous/136-4.7.2-trig^m-a-trig+b-trig-^n

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [294]. This is test number [136].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	100.00 (294)	0.00 (0)
Rubi	98.98 (291)	1.02 (3)
Maple	98.64 (290)	1.36 (4)
Fricas	98.64 (290)	1.36 (4)
Mupad	98.64 (290)	1.36 (4)
Giac	95.58 (281)	4.42 (13)
Maxima	92.18 (271)	7.82 (23)
Sympy	23.13 (68)	76.87 (226)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

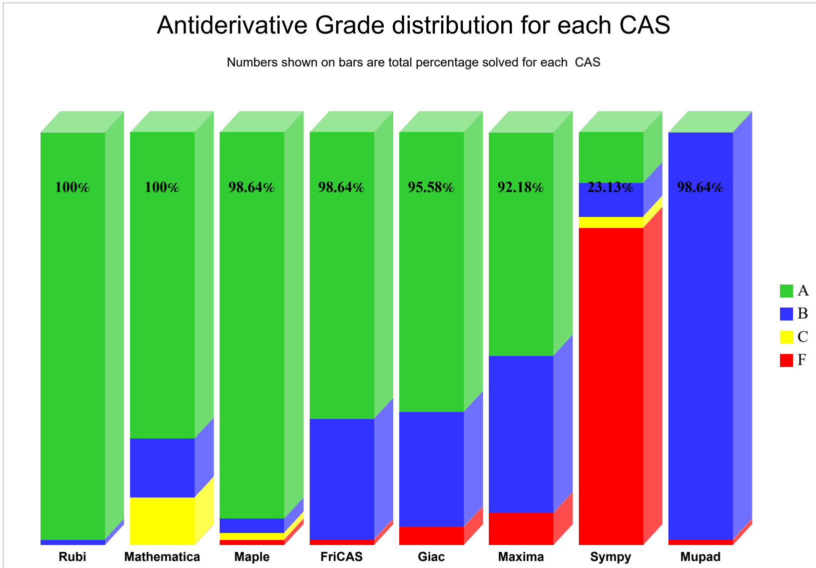
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

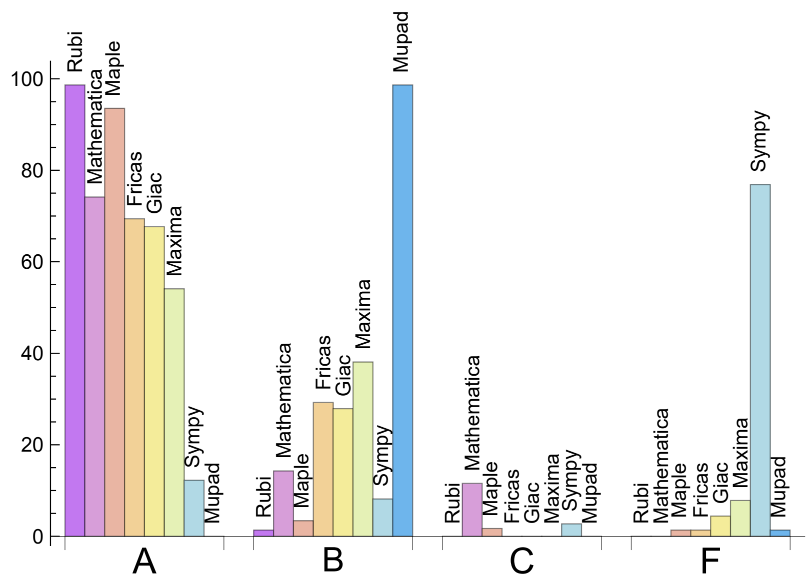
System	% A grade	% B grade	% C grade	% F grade
Rubi	95.918	3.061	0.000	1.020
Maple	93.537	3.401	1.701	1.361
Mathematica	74.150	14.286	11.565	0.000
Fricas	69.388	29.252	0.000	1.361
Giac	67.687	27.891	0.000	4.422
Maxima	54.082	38.095	0.000	7.823
Sympy	12.245	8.163	2.721	76.871
Mupad	0.000	98.639	0.000	1.361

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Mathematica	0	0.00	0.00	0.00
Rubi	3	100.00	0.00	0.00
Fricas	4	100.00	0.00	0.00
Maple	4	100.00	0.00	0.00
Mupad	4	0.00	100.00	0.00
Giac	13	38.46	7.69	53.85
Maxima	23	17.39	0.00	82.61
Sympy	226	65.04	31.86	3.10

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.26
Fricas	0.27
Rubi	0.48
Giac	0.94
Mathematica	1.53
Maple	1.84
Sympy	6.58
Mupad	24.24

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maple	109.18	1.09	91.00	0.97
Rubi	116.31	1.07	88.00	1.00
Fricas	172.00	1.66	115.50	1.20
Maxima	177.42	2.02	118.00	1.50
Mathematica	178.11	1.60	91.00	1.01
Sympy	258.71	3.14	164.00	1.88
Giac	409.44	4.81	118.00	1.47
Mupad	610.58	4.77	149.50	1.79

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

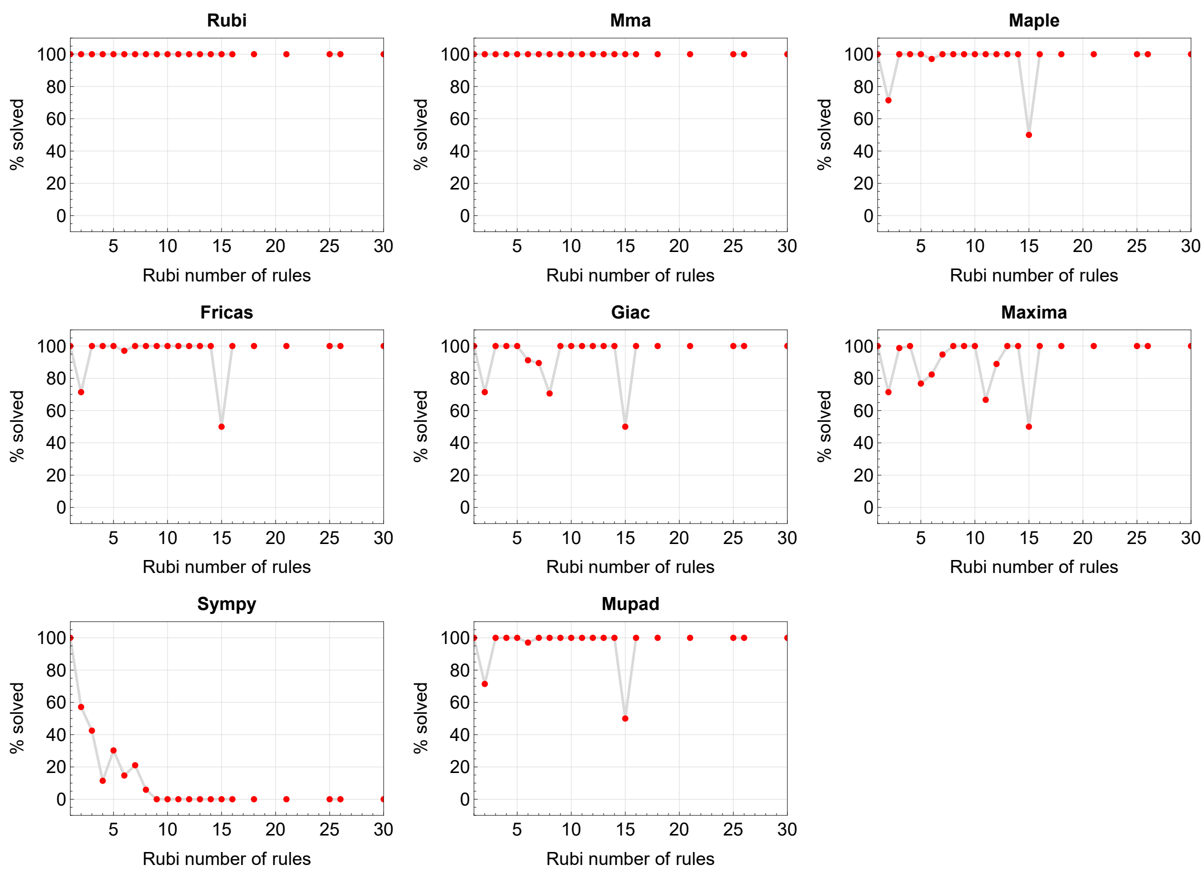


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

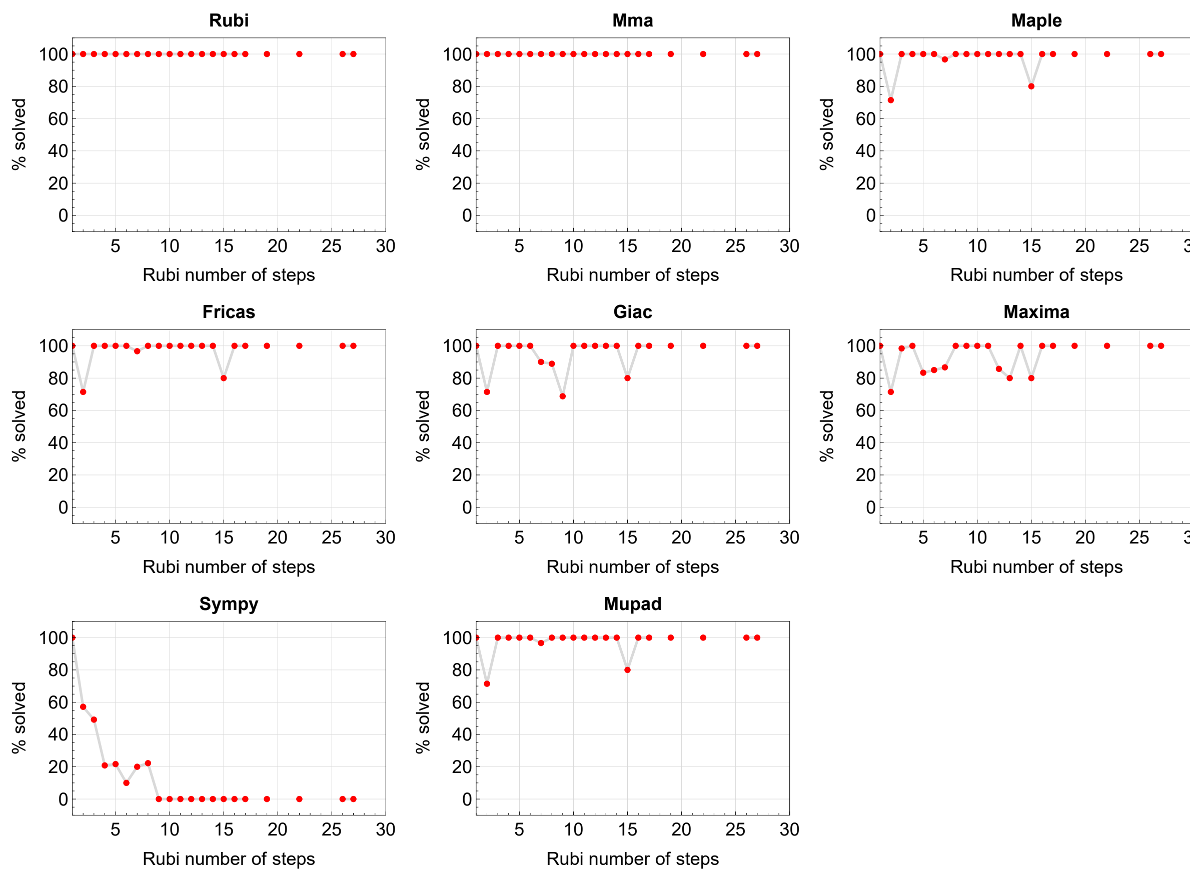


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

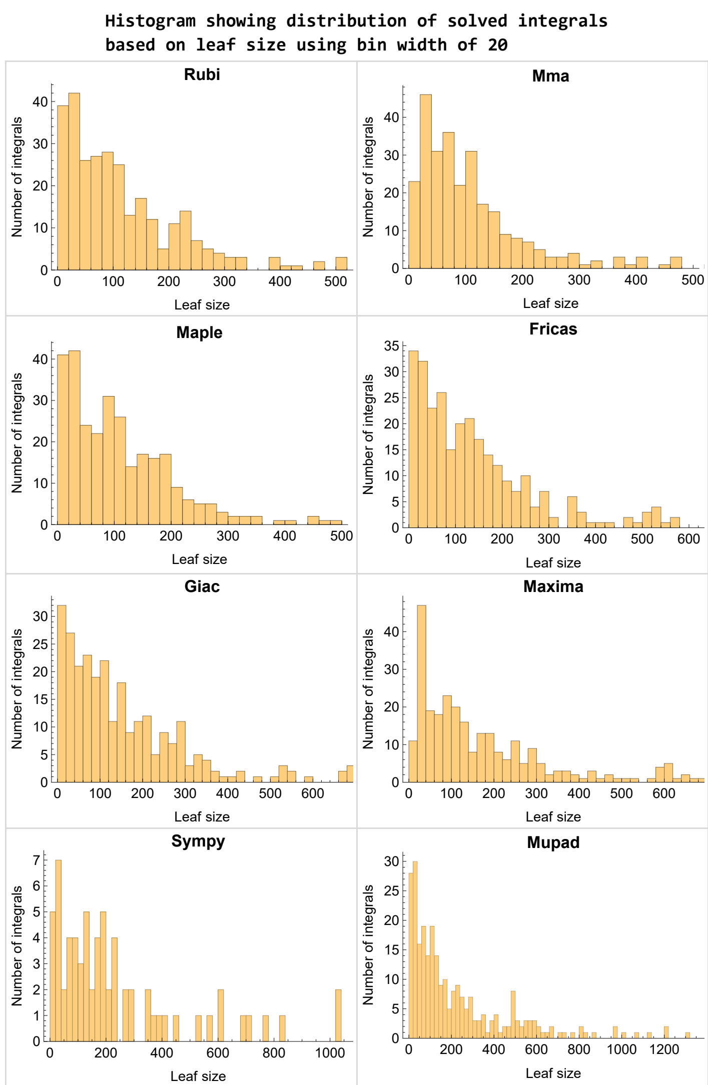


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

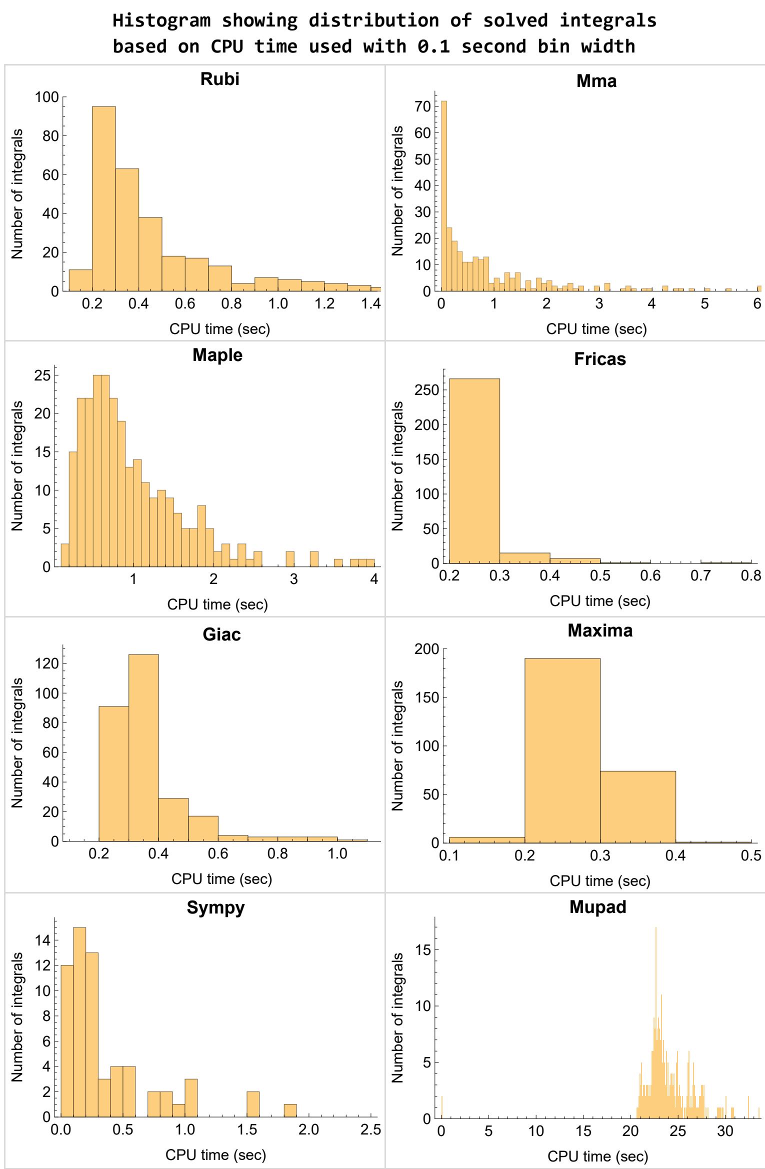


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

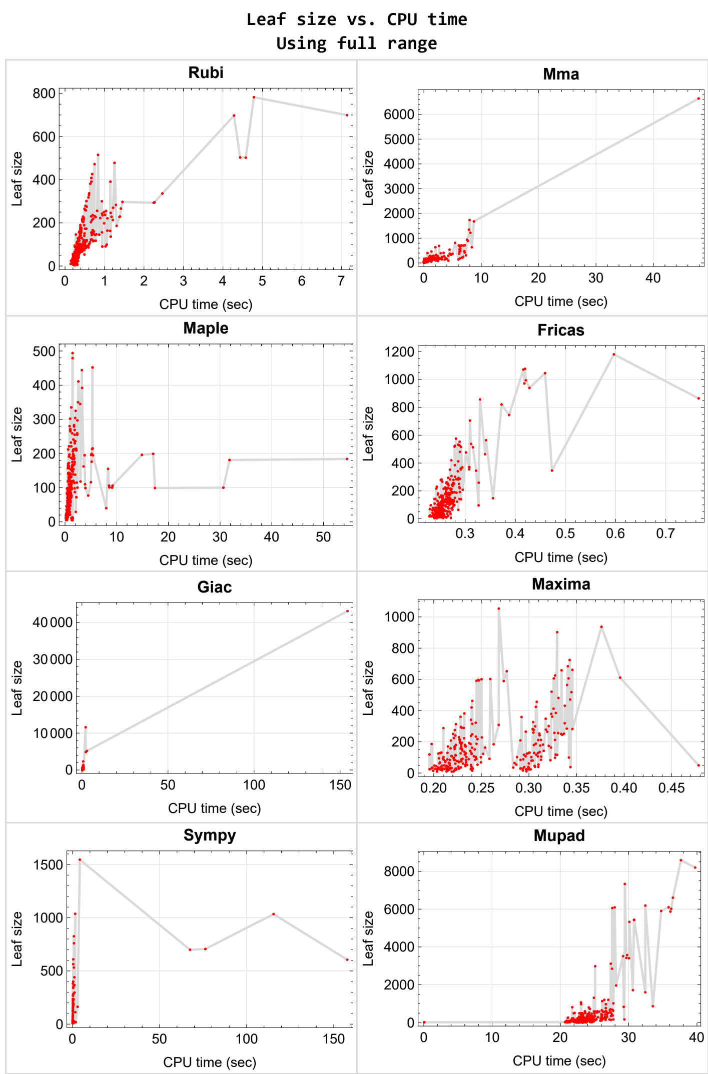


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica $\{29, 272\}$

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

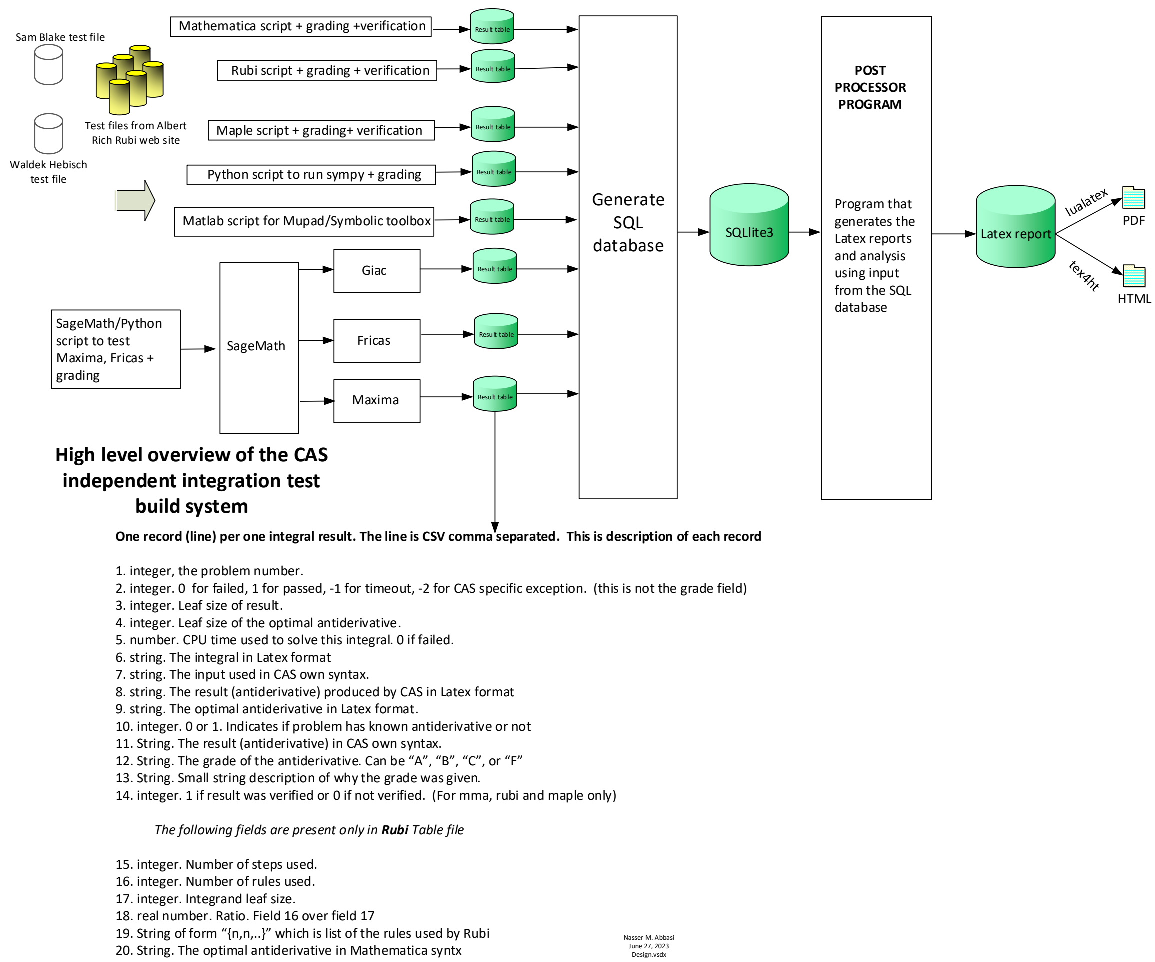
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 293, 294 }

B grade { 15, 23, 131, 142, 285, 286, 287, 288, 290 }

C grade { }

F normal fail { 289, 291, 292 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 7, 9, 11, 12, 13, 14, 15, 17, 18, 19, 20, 23, 26, 27, 28, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 64, 67, 69, 71, 73, 74, 75, 76, 78, 79, 80, 82, 87, 89, 90, 91, 92, 93, 95, 96, 97, 106, 107, 108, 109, 110, 111, 113, 114, 115, 116, 117, 118, 120, 122, 123, 125, 126, 127, 128, 130, 133, 136, 137, 138, 140, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 170, 172, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 185, 186, 187, 189, 194, 195, 196, 198, 201, 202, 204, 206, 208, 210, 212, 213, 214, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 266, 269, 270, 271, 273, 274, 275, 277, 279, 281, 283, 285, 287, 288, 289, 291, 293, 294 }

B grade { 6, 21, 24, 63, 66, 68, 70, 72, 81, 84, 85, 86, 88, 98, 99, 100, 101, 103, 104, 105, 119, 121, 134, 143, 169, 171, 173, 179, 188, 190, 191, 192, 193, 197, 199, 200, 203, 205, 207, 209, 211, 265 }

C grade { 8, 10, 16, 22, 25, 29, 50, 65, 77, 83, 94, 102, 112, 124, 129, 131, 132, 135, 139, 141, 142, 215, 264, 267, 268, 272, 276, 278, 280, 282, 284, 286, 290, 292 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 26, 27, 28, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 143, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 188, 189, 190, 192, 193, 194, 195, 196, 197, 199, 200, 201, 203, 204, 206, 207, 208, 209, 210, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 273, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294 }

B grade { 23, 24, 25, 67, 85, 104, 133, 142, 157, 205 }

C grade { 144, 191, 198, 202, 211 }

F normal fail { 29, 187, 272, 274 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 2, 3, 4, 5, 7, 8, 10, 12, 30, 31, 32, 33, 34, 35, 36, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 105, 106, 107, 108, 109, 110, 111, 112, 114, 116, 118, 119, 120, 121, 122, 126, 139, 150, 151, 152, 153, 154, 155, 156, 158, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 188, 189, 190, 193, 194, 195, 196, 197, 198, 200, 201, 202, 203, 204, 206, 207, 208, 209, 210, 212, 213, 214, 216, 217, 218, 219, 221, 222, 224, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 273, 276, 277, 278, 280, 281, 282, 283, 284, 286, 288, 290, 292 }

B grade { 6, 9, 11, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 37, 67, 85, 104, 113, 115, 117, 123, 124, 125, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 157, 159, 161, 171, 172, 173, 184, 185, 186, 191, 192, 199, 205, 211, 215, 220, 223, 225, 226, 264, 265, 266, 267, 268, 269, 270, 271, 275, 279, 285, 287, 289, 291, 293, 294 }

C grade { }

F normal fail { 29, 187, 272, 274 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.5 Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 9, 11, 16, 18, 20, 27, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 105, 106, 107, 108, 109, 113, 115, 122, 124, 126, 128, 130, 143, 145, 147, 149, 155, 167, 168, 170, 172, 174, 178, 179, 180, 181, 193, 198, 200, 201, 202, 206, 208, 212, 214, 217, 218, 219, 224, 225, 227, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 271, 273, 275, 284, 294 }

B grade { 8, 10, 12, 13, 14, 15, 17, 19, 21, 22, 23, 24, 25, 26, 28, 67, 85, 104, 110, 111, 112, 114, 116, 117, 118, 119, 120, 121, 123, 125, 127, 129, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 144, 146, 148, 156, 157, 158, 159, 160, 161, 162, 169, 171, 173, 182, 183, 184, 185, 186, 188, 189, 190, 191, 192, 194, 195, 196, 197, 199, 203, 204, 205, 207, 209, 210, 211, 213, 215, 216, 220, 221, 222, 223, 226, 228, 264, 265, 266, 267, 268, 269, 270, 276, 277, 278, 279, 280, 281, 282, 283, 285, 286, 287, 288, 289, 290, 291, 292, 293 }

C grade { }

F normal fail { 29, 187, 272, 274 }

F(-1) timedout fail { }

F(-2) exception fail { 150, 151, 152, 153, 154, 163, 164, 165, 166, 175, 176, 177, 257, 258, 259, 260, 261, 262, 263 }

2.1.6 Giac

A grade { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 14, 15, 17, 18, 19, 20, 21, 24, 26, 27, 28, 30, 31, 32, 33, 34, 35, 36, 38, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 69, 71, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 87, 89, 91, 92, 93, 94, 95, 96, 98, 99, 100, 101, 102, 103, 106, 108, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 122, 124, 125, 126, 127, 128, 129, 130, 131, 134, 136, 137, 138, 139, 140, 143, 145, 147, 148, 149, 150, 151, 152, 153, 154, 155, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 185, 186, 189, 191, 192, 193, 194, 196, 198, 199, 200, 201, 202, 204, 206, 207, 208, 210, 212, 213, 214, 216, 217, 218, 219, 221, 223, 225, 228, 232, 241, 242, 253, 254, 256, 258, 259, 260, 263, 275, 276, 277, 278, 279, 280, 281, 282, 285, 286, 287, 288, 290, 291, 294 }

B grade { 5, 6, 13, 16, 22, 23, 25, 37, 39, 41, 53, 55, 67, 68, 70, 72, 84, 85, 86, 88, 90, 97, 104, 105, 107, 109, 121, 123, 132, 133, 135, 141, 142, 144, 146, 156, 173, 179, 188, 190, 195, 197, 203, 205, 209, 211, 215, 220, 222, 224, 226, 227, 229, 230, 231, 233, 234, 235, 236, 237, 238, 239, 240, 250, 251, 252, 255, 257, 261, 262, 264, 265, 266, 267, 268, 269, 270, 283, 284, 289, 292, 293 }

C grade { }

F normal fail { 29, 187, 272, 273, 274 }

F(-1) timedout fail { 243 }

F(-2) exception fail { 244, 245, 246, 247, 248, 249, 271 }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 273, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294 }

C grade { }

F normal fail { }

F(-1) timedout fail { 29, 187, 272, 274 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 2, 3, 4, 5, 6, 7, 31, 33, 35, 43, 45, 47, 58, 59, 60, 74, 76, 77, 78, 93, 94, 95, 150, 152, 154, 155, 164, 166, 167, 168, 175, 177, 178, 180, 202, 232 }

B grade { 1, 18, 30, 32, 34, 44, 46, 48, 57, 61, 62, 75, 79, 92, 96, 97, 151, 153, 163, 165, 176, 179, 188, 195 }

C grade { 9, 10, 11, 113, 114, 115, 124, 275 }

F normal fail { 12, 13, 14, 19, 20, 21, 26, 27, 28, 29, 36, 37, 38, 39, 40, 49, 50, 51, 52, 63, 64, 65, 80, 81, 98, 116, 117, 118, 119, 120, 121, 126, 127, 128, 129, 130, 136, 137, 138, 139, 140, 146, 147, 148, 149, 156, 157, 158, 159, 160, 161, 162, 169, 170, 171, 172, 173, 174, 181, 182, 183, 184, 185, 186, 187, 189, 190, 191, 192, 193, 194, 196, 197, 198, 199, 200, 201, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 251, 252, 253, 254, 255, 256, 258, 259, 260, 261, 262, 263, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 293, 294 }

F(-1) timedout fail { 8, 15, 23, 24, 41, 42, 53, 54, 55, 56, 66, 67, 68, 69, 70, 71, 72, 73, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 122, 123, 131, 133, 134, 135, 141, 142, 143, 144, 145, 250, 257, 264, 276, 277, 278, 279, 280, 281, 282, 283, 285, 286, 287, 288, 289, 290, 291, 292 }

F(-2) exception fail { 16, 17, 22, 25, 125, 132, 284 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	34	28	25	36	75	33	35
N.S.	1	1.00	0.94	0.78	0.69	1.00	2.08	0.92	0.97
time (sec)	N/A	0.208	0.043	0.420	0.217	0.242	0.148	0.271	21.216

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	26	20	20	27	27	25	32
N.S.	1	1.00	1.08	0.83	0.83	1.12	1.12	1.04	1.33
time (sec)	N/A	0.205	0.041	0.434	0.214	0.258	0.100	0.274	22.554

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	21	19	37	19	19
N.S.	1	1.00	1.00	0.80	0.84	0.76	1.48	0.76	0.76
time (sec)	N/A	0.199	0.008	0.185	0.207	0.236	0.079	0.273	21.222

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	10	8	10	10
N.S.	1	1.00	1.00	1.10	1.00	1.00	0.80	1.00	1.00
time (sec)	N/A	0.138	0.049	0.155	0.213	0.253	0.028	0.272	20.984

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	14	10	9	11	8	24	54
N.S.	1	1.00	1.56	1.11	1.00	1.22	0.89	2.67	6.00
time (sec)	N/A	0.180	0.018	0.359	0.203	0.263	0.514	0.278	21.158

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	25	17	24	33	24	33	24
N.S.	1	1.00	2.08	1.42	2.00	2.75	2.00	2.75	2.00
time (sec)	N/A	0.198	0.045	0.409	0.207	0.246	0.951	0.276	20.675

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	15	19	17	13	14
N.S.	1	1.00	1.00	0.93	1.00	1.27	1.13	0.87	0.93
time (sec)	N/A	0.211	0.049	0.569	0.222	0.236	1.840	0.272	20.816

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	92	94	97	209	93	0	148	3512
N.S.	1	1.01	1.03	1.07	2.30	1.02	0.00	1.63	38.59
time (sec)	N/A	0.484	0.346	0.551	0.291	0.253	0.000	0.289	29.176

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	62	84	106	144	706	94	94
N.S.	1	1.00	0.91	1.24	1.56	2.12	10.38	1.38	1.38
time (sec)	N/A	0.343	0.250	0.369	0.309	0.271	76.317	0.306	20.937

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	47	46	88	46	165	55	970
N.S.	1	1.00	1.34	1.31	2.51	1.31	4.71	1.57	27.71
time (sec)	N/A	0.280	0.173	0.280	0.294	0.243	0.321	0.273	23.048

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	38	35	61	96	112	61	31
N.S.	1	1.00	1.06	0.97	1.69	2.67	3.11	1.69	0.86
time (sec)	N/A	0.192	0.065	0.297	0.300	0.260	1.567	0.307	21.374

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	20	21	48	44	0	22	32
N.S.	1	1.00	0.87	0.91	2.09	1.91	0.00	0.96	1.39
time (sec)	N/A	0.302	0.369	0.442	0.217	0.255	0.000	0.293	21.418

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	67	81	107	133	0	108	170
N.S.	1	1.00	1.22	1.47	1.95	2.42	0.00	1.96	3.09
time (sec)	N/A	0.339	0.341	0.435	0.297	0.277	0.000	0.322	22.965

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	53	48	53	119	117	0	78	91
N.S.	1	0.96	0.87	0.96	2.16	2.13	0.00	1.42	1.65
time (sec)	N/A	0.485	1.297	0.559	0.196	0.267	0.000	0.277	21.322

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	283	107	140	253	240	0	186	224
N.S.	1	2.64	1.00	1.31	2.36	2.24	0.00	1.74	2.09
time (sec)	N/A	1.232	0.599	0.612	0.311	0.267	0.000	0.307	21.871

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	75	121	81	117	132	0	139	626
N.S.	1	1.17	1.89	1.27	1.83	2.06	0.00	2.17	9.78
time (sec)	N/A	0.481	0.420	0.541	0.295	0.263	0.000	0.277	27.715

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-2)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	62	97	128	164	0	103	86
N.S.	1	1.00	1.03	1.62	2.13	2.73	0.00	1.72	1.43
time (sec)	N/A	0.285	0.265	0.388	0.309	0.256	0.000	0.298	21.686

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	14	39	605	13	29
N.S.	1	1.00	1.00	0.82	0.82	2.29	35.59	0.76	1.71
time (sec)	N/A	0.172	0.034	0.367	0.199	0.241	157.808	0.267	21.078

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	72	85	128	220	0	109	492
N.S.	1	1.00	1.14	1.35	2.03	3.49	0.00	1.73	7.81
time (sec)	N/A	0.324	0.612	0.563	0.300	0.289	0.000	0.306	21.951

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	76	56	62	134	0	63	114
N.S.	1	1.00	1.55	1.14	1.27	2.73	0.00	1.29	2.33
time (sec)	N/A	0.266	2.433	0.642	0.217	0.274	0.000	0.277	21.710

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	156	270	158	242	345	0	215	511
N.S.	1	1.32	2.29	1.34	2.05	2.92	0.00	1.82	4.33
time (sec)	N/A	0.939	2.421	0.714	0.311	0.322	0.000	0.313	22.085

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	121	114	134	359	282	0	242	5324
N.S.	1	1.23	1.16	1.37	3.66	2.88	0.00	2.47	54.33
time (sec)	N/A	0.655	1.054	0.750	0.292	0.270	0.000	0.282	30.100

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	300	92	212	299	282	0	197	263
N.S.	1	3.26	1.00	2.30	3.25	3.07	0.00	2.14	2.86
time (sec)	N/A	0.917	0.606	0.602	0.307	0.253	0.000	0.307	21.145

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	19	47	29	84	116	0	20	48
N.S.	1	1.27	3.13	1.93	5.60	7.73	0.00	1.33	3.20
time (sec)	N/A	0.185	0.169	0.444	0.244	0.270	0.000	0.282	21.077

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	101	157	250	225	0	166	216
N.S.	1	1.00	1.38	2.15	3.42	3.08	0.00	2.27	2.96
time (sec)	N/A	0.277	0.205	0.530	0.334	0.260	0.000	0.299	21.692

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	96	73	172	220	0	77	131
N.S.	1	1.00	1.63	1.24	2.92	3.73	0.00	1.31	2.22
time (sec)	N/A	0.271	0.653	0.698	0.233	0.266	0.000	0.285	21.556

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	213	193	163	276	463	0	212	813
N.S.	1	1.16	1.05	0.89	1.50	2.52	0.00	1.15	4.42
time (sec)	N/A	1.178	1.288	0.759	0.305	0.340	0.000	0.331	21.745

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	208	128	308	385	0	146	253
N.S.	1	1.00	1.78	1.09	2.63	3.29	0.00	1.25	2.16
time (sec)	N/A	0.344	2.024	0.905	0.268	0.279	0.000	0.298	21.082

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	66	66	367	0	0	0	0	0	0
N.S.	1	1.00	5.56	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.243	5.003	0.000	0.000	0.000	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	57	62	62	62	175	95	149
N.S.	1	1.00	0.66	0.71	0.71	0.71	2.01	1.09	1.71
time (sec)	N/A	0.288	0.138	0.732	0.206	0.258	0.357	0.309	24.580

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	46	49	51	87	85	67
N.S.	1	1.00	1.00	0.77	0.82	0.85	1.45	1.42	1.12
time (sec)	N/A	0.262	0.010	0.688	0.211	0.256	0.225	0.288	22.662

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	62	52	48	51	128	65	107
N.S.	1	1.00	0.95	0.80	0.74	0.78	1.97	1.00	1.65
time (sec)	N/A	0.261	0.108	0.649	0.201	0.269	0.184	0.285	24.891

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	36	35	38	63	55	47
N.S.	1	1.00	1.00	0.82	0.80	0.86	1.43	1.25	1.07
time (sec)	N/A	0.246	0.012	0.611	0.199	0.248	0.114	0.284	21.311

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	46	36	37	35	73	35	35
N.S.	1	1.00	1.07	0.84	0.86	0.81	1.70	0.81	0.81
time (sec)	N/A	0.226	0.056	0.398	0.207	0.264	0.096	0.273	21.439

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	46	23	24	23	31	24	38
N.S.	1	1.00	1.92	0.96	1.00	0.96	1.29	1.00	1.58
time (sec)	N/A	0.148	0.008	0.239	0.203	0.256	0.081	0.261	20.792

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	23	30	21	0	27	70
N.S.	1	1.00	1.00	1.35	1.76	1.24	0.00	1.59	4.12
time (sec)	N/A	0.195	0.019	0.664	0.198	0.255	0.000	0.272	20.922

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	32	40	54	0	54	38
N.S.	1	1.00	1.00	1.33	1.67	2.25	0.00	2.25	1.58
time (sec)	N/A	0.221	0.016	0.595	0.240	0.263	0.000	0.291	21.124

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	30	30	0	25	23
N.S.	1	1.00	1.00	0.89	1.07	1.07	0.00	0.89	0.82
time (sec)	N/A	0.236	0.016	0.828	0.214	0.237	0.000	0.295	20.954

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	52	50	61	74	0	99	105
N.S.	1	1.00	1.00	0.96	1.17	1.42	0.00	1.90	2.02
time (sec)	N/A	0.250	0.011	0.855	0.216	0.267	0.000	0.300	22.908

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	41	38	41	45	0	48	40
N.S.	1	1.00	0.93	0.86	0.93	1.02	0.00	1.09	0.91
time (sec)	N/A	0.243	0.104	0.874	0.215	0.249	0.000	0.311	21.957

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	74	63	86	88	0	141	175
N.S.	1	1.00	1.00	0.85	1.16	1.19	0.00	1.91	2.36
time (sec)	N/A	0.270	0.013	0.987	0.210	0.276	0.000	0.321	26.168

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	53	48	53	57	0	70	65
N.S.	1	1.00	0.88	0.80	0.88	0.95	0.00	1.17	1.08
time (sec)	N/A	0.260	0.174	1.026	0.246	0.244	0.000	0.329	21.170

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	92	99	98	94	187	155	176
N.S.	1	1.00	0.67	0.72	0.72	0.69	1.36	1.13	1.28
time (sec)	N/A	0.354	0.329	1.096	0.221	0.254	0.496	0.345	21.809

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	147	118	102	95	340	132	156
N.S.	1	1.00	0.84	0.68	0.59	0.55	1.95	0.76	0.90
time (sec)	N/A	0.383	0.862	1.023	0.210	0.258	0.368	0.355	21.947

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	69	79	77	74	138	114	115
N.S.	1	1.00	0.67	0.77	0.75	0.72	1.34	1.11	1.12
time (sec)	N/A	0.333	0.179	0.967	0.224	0.253	0.240	0.327	21.691

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	98	84	75	75	238	85	89
N.S.	1	1.00	0.78	0.67	0.60	0.60	1.89	0.67	0.71
time (sec)	N/A	0.335	0.957	0.704	0.229	0.255	0.195	0.309	20.895

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	67	52	52	53	85	73	77
N.S.	1	1.00	1.00	0.78	0.78	0.79	1.27	1.09	1.15
time (sec)	N/A	0.271	0.050	0.642	0.235	0.240	0.120	0.310	21.750

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	52	57	68	52	128	50	63
N.S.	1	1.00	0.95	1.04	1.24	0.95	2.33	0.91	1.15
time (sec)	N/A	0.194	0.075	0.471	0.250	0.256	0.095	0.293	21.161

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	84	53	60	62	0	89	66
N.S.	1	1.00	1.53	0.96	1.09	1.13	0.00	1.62	1.20
time (sec)	N/A	0.255	0.119	0.717	0.231	0.259	0.000	0.311	21.410

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	69	44	49	60	0	44	118
N.S.	1	1.00	1.77	1.13	1.26	1.54	0.00	1.13	3.03
time (sec)	N/A	0.305	0.115	0.779	0.478	0.254	0.000	0.302	23.216

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	67	83	89	96	0	122	106
N.S.	1	1.00	1.00	1.24	1.33	1.43	0.00	1.82	1.58
time (sec)	N/A	0.283	0.015	0.773	0.237	0.250	0.000	0.317	22.225

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	46	48	45	55	0	41	68
N.S.	1	1.00	1.53	1.60	1.50	1.83	0.00	1.37	2.27
time (sec)	N/A	0.211	0.022	0.986	0.213	0.246	0.000	0.310	21.592

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	120	118	129	120	0	249	216
N.S.	1	1.00	1.00	0.98	1.08	1.00	0.00	2.08	1.80
time (sec)	N/A	0.337	0.019	1.149	0.239	0.264	0.000	0.329	24.788

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	77	54	82	70	79	0	80	98
N.S.	1	0.91	0.64	0.96	0.82	0.93	0.00	0.94	1.15
time (sec)	N/A	0.266	0.190	1.144	0.227	0.241	0.000	0.323	22.483

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	168	149	180	142	0	343	328
N.S.	1	1.00	1.00	0.89	1.07	0.85	0.00	2.04	1.95
time (sec)	N/A	0.381	0.018	1.335	0.233	0.262	0.000	0.343	25.041

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	112	104	110	91	100	0	118	130
N.S.	1	0.90	0.83	0.88	0.73	0.80	0.00	0.94	1.04
time (sec)	N/A	0.292	0.717	1.243	0.245	0.246	0.000	0.344	22.703

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	265	265	235	175	163	150	532	218	523
N.S.	1	1.00	0.89	0.66	0.62	0.57	2.01	0.82	1.97
time (sec)	N/A	0.497	1.351	1.411	0.254	0.267	0.781	0.417	24.216

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	141	131	126	123	233	197	214
N.S.	1	1.00	0.81	0.75	0.72	0.70	1.33	1.13	1.22
time (sec)	N/A	0.418	3.162	1.197	0.228	0.257	0.594	0.410	24.043

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	171	154	131	128	400	157	407
N.S.	1	1.00	0.79	0.71	0.61	0.59	1.85	0.73	1.88
time (sec)	N/A	0.443	1.895	1.097	0.208	0.257	0.404	0.381	24.120

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	121	111	107	102	182	145	147
N.S.	1	1.00	0.86	0.79	0.76	0.73	1.30	1.04	1.05
time (sec)	N/A	0.384	1.223	1.108	0.242	0.246	0.283	0.373	22.843

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	95	94	109	91	100	272	104	281
N.S.	1	1.22	1.21	1.40	1.17	1.28	3.49	1.33	3.60
time (sec)	N/A	0.241	1.009	0.743	0.230	0.269	0.210	0.340	23.855

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	56	81	75	84	77	117	91	104
N.S.	1	0.97	1.40	1.29	1.45	1.33	2.02	1.57	1.79
time (sec)	N/A	0.212	0.322	0.955	0.234	0.250	0.133	0.297	22.400

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	104	401	98	91	79	0	93	156
N.S.	1	1.14	4.41	1.08	1.00	0.87	0.00	1.02	1.71
time (sec)	N/A	0.312	0.633	0.974	0.259	0.253	0.000	0.349	23.259

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	131	96	84	109	0	150	116
N.S.	1	1.00	1.52	1.12	0.98	1.27	0.00	1.74	1.35
time (sec)	N/A	0.315	1.103	0.978	0.217	0.264	0.000	0.347	22.859

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	79	70	85	88	0	71	183
N.S.	1	1.00	1.10	0.97	1.18	1.22	0.00	0.99	2.54
time (sec)	N/A	0.439	0.244	1.075	0.313	0.262	0.000	0.355	23.584

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	293	116	118	123	0	171	160
N.S.	1	1.00	2.84	1.13	1.15	1.19	0.00	1.66	1.55
time (sec)	N/A	0.320	1.686	1.180	0.233	0.274	0.000	0.378	25.262

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	57	72	87	78	0	57	88
N.S.	1	1.00	1.90	2.40	2.90	2.60	0.00	1.90	2.93
time (sec)	N/A	0.216	0.179	1.207	0.229	0.236	0.000	0.368	22.631

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	464	154	157	147	0	333	293
N.S.	1	1.00	2.94	0.97	0.99	0.93	0.00	2.11	1.85
time (sec)	N/A	0.409	1.315	1.625	0.228	0.266	0.000	0.384	26.530

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	108	54	119	122	105	0	112	123
N.S.	1	0.90	0.45	0.99	1.02	0.88	0.00	0.93	1.02
time (sec)	N/A	0.297	0.374	1.289	0.240	0.253	0.000	0.390	22.478

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	637	186	208	170	0	465	423
N.S.	1	1.00	3.03	0.89	0.99	0.81	0.00	2.21	2.01
time (sec)	N/A	0.433	2.007	1.794	0.218	0.268	0.000	0.411	26.803

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	156	115	147	154	128	0	166	156
N.S.	1	0.90	0.66	0.84	0.89	0.74	0.00	0.95	0.90
time (sec)	N/A	0.336	0.592	1.510	0.229	0.245	0.000	0.402	23.091

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	259	259	810	214	248	192	0	597	547
N.S.	1	1.00	3.13	0.83	0.96	0.74	0.00	2.31	2.11
time (sec)	N/A	0.496	5.455	2.144	0.241	0.289	0.000	0.422	26.784

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	190	177	175	184	150	0	220	189
N.S.	1	0.89	0.83	0.82	0.86	0.70	0.00	1.03	0.89
time (sec)	N/A	0.377	2.170	1.851	0.263	0.280	0.000	0.421	23.921

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	279	370	193	186	177	367	269	334
N.S.	1	1.00	1.33	0.69	0.67	0.63	1.32	0.96	1.20
time (sec)	N/A	0.518	6.165	1.730	0.239	0.260	1.067	0.529	24.666

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	381	222	241	199	184	760	245	343
N.S.	1	1.00	0.58	0.63	0.52	0.48	1.99	0.64	0.90
time (sec)	N/A	0.614	3.118	1.616	0.228	0.263	0.808	0.489	23.874

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	165	163	154	149	286	229	291
N.S.	1	1.00	0.75	0.74	0.70	0.68	1.30	1.04	1.32
time (sec)	N/A	0.458	2.947	1.468	0.222	0.254	0.539	0.445	23.255

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	301	178	187	170	151	563	187	471
N.S.	1	1.00	0.59	0.62	0.56	0.50	1.87	0.62	1.56
time (sec)	N/A	0.534	1.866	1.175	0.233	0.254	0.419	0.427	24.589

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	128	131	123	123	206	165	204
N.S.	1	1.00	0.78	0.79	0.75	0.75	1.25	1.00	1.24
time (sec)	N/A	0.388	1.458	1.066	0.252	0.252	0.263	0.400	22.837

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	111	107	135	136	121	381	122	320
N.S.	1	1.03	0.99	1.25	1.26	1.12	3.53	1.13	2.96
time (sec)	N/A	0.303	1.110	1.049	0.232	0.253	0.209	0.311	24.138

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	181	116	126	121	0	217	190
N.S.	1	1.00	1.21	0.77	0.84	0.81	0.00	1.45	1.27
time (sec)	N/A	0.367	2.421	1.311	0.234	0.270	0.000	0.385	26.105

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	129	477	155	135	136	0	128	255
N.S.	1	1.08	4.01	1.30	1.13	1.14	0.00	1.08	2.14
time (sec)	N/A	0.411	6.352	1.305	0.311	0.269	0.000	0.389	22.908

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	268	157	142	153	0	206	221
N.S.	1	1.00	1.77	1.04	0.94	1.01	0.00	1.36	1.46
time (sec)	N/A	0.380	3.549	1.330	0.238	0.268	0.000	0.410	24.833

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	105	101	116	119	0	104	546
N.S.	1	1.00	1.02	0.98	1.13	1.16	0.00	1.01	5.30
time (sec)	N/A	0.593	0.389	1.269	0.330	0.265	0.000	0.415	23.648

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	936	198	192	163	0	325	278
N.S.	1	1.00	5.57	1.18	1.14	0.97	0.00	1.93	1.65
time (sec)	N/A	0.397	7.520	1.573	0.237	0.252	0.000	0.450	25.486

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	73	96	103	109	0	73	139
N.S.	1	1.00	2.43	3.20	3.43	3.63	0.00	2.43	4.63
time (sec)	N/A	0.217	0.351	1.306	0.244	0.248	0.000	0.436	22.349

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	258	1342	255	251	187	0	536	419
N.S.	1	1.00	5.20	0.99	0.97	0.72	0.00	2.08	1.62
time (sec)	N/A	0.516	7.849	1.859	0.220	0.275	0.000	0.452	26.550

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	129	54	166	151	142	0	144	186
N.S.	1	0.90	0.38	1.16	1.06	0.99	0.00	1.01	1.30
time (sec)	N/A	0.315	0.621	1.642	0.243	0.239	0.000	0.453	24.207

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	330	330	1732	304	322	214	0	706	566
N.S.	1	1.00	5.25	0.92	0.98	0.65	0.00	2.14	1.72
time (sec)	N/A	0.585	7.976	1.951	0.244	0.273	0.000	0.473	26.244

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	181	115	213	193	167	0	214	447
N.S.	1	0.90	0.57	1.06	0.96	0.83	0.00	1.06	2.22
time (sec)	N/A	0.371	0.897	1.902	0.228	0.252	0.000	0.477	26.003

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	408	408	242	350	382	251	0	880	703
N.S.	1	1.00	0.59	0.86	0.94	0.62	0.00	2.16	1.72
time (sec)	N/A	0.672	3.158	2.509	0.232	0.278	0.000	0.501	27.359

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	229	175	259	233	194	0	284	560
N.S.	1	0.90	0.69	1.02	0.92	0.76	0.00	1.12	2.20
time (sec)	N/A	0.428	1.844	2.357	0.244	0.273	0.000	0.481	26.398

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	515	515	307	325	290	250	1037	342	801
N.S.	1	1.00	0.60	0.63	0.56	0.49	2.01	0.66	1.56
time (sec)	N/A	0.778	7.337	2.049	0.229	0.288	1.508	0.492	24.163

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	337	337	542	236	224	217	440	313	495
N.S.	1	1.00	1.61	0.70	0.66	0.64	1.31	0.93	1.47
time (sec)	N/A	0.595	6.210	1.869	0.221	0.271	1.033	0.532	26.800

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	426	426	259	279	228	220	826	278	650
N.S.	1	1.00	0.61	0.65	0.54	0.52	1.94	0.65	1.53
time (sec)	N/A	0.691	3.568	1.740	0.251	0.264	0.769	0.581	26.059

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	450	206	194	186	357	259	372
N.S.	1	1.00	1.64	0.75	0.71	0.68	1.30	0.94	1.35
time (sec)	N/A	0.536	6.210	1.517	0.230	0.267	0.545	0.537	27.647

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	146	188	221	187	182	609	211	472
N.S.	1	1.16	1.49	1.75	1.48	1.44	4.83	1.67	3.75
time (sec)	N/A	0.273	3.821	1.197	0.245	0.261	0.420	0.489	24.994

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	89	156	167	172	155	267	187	248
N.S.	1	0.95	1.66	1.78	1.83	1.65	2.84	1.99	2.64
time (sec)	N/A	0.237	1.737	1.449	0.234	0.249	0.288	0.364	23.199

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	189	711	192	170	160	0	199	297
N.S.	1	1.11	4.18	1.13	1.00	0.94	0.00	1.17	1.75
time (sec)	N/A	0.471	6.512	1.554	0.215	0.263	0.000	0.508	24.929

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	632	169	162	177	0	283	277
N.S.	1	1.00	3.08	0.82	0.79	0.86	0.00	1.38	1.35
time (sec)	N/A	0.450	8.365	1.820	0.240	0.264	0.000	0.512	26.784

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	173	571	209	179	177	0	173	354
N.S.	1	1.02	3.38	1.24	1.06	1.05	0.00	1.02	2.09
time (sec)	N/A	0.481	6.356	1.582	0.305	0.264	0.000	0.527	25.009

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	892	230	181	190	0	281	302
N.S.	1	1.00	4.37	1.13	0.89	0.93	0.00	1.38	1.48
time (sec)	N/A	0.445	7.691	1.878	0.229	0.278	0.000	0.561	26.853

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	126	139	174	155	0	144	971
N.S.	1	1.00	0.86	0.95	1.18	1.05	0.00	0.98	6.61
time (sec)	N/A	0.786	0.843	1.558	0.321	0.256	0.000	0.544	26.658

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	1219	236	230	196	0	410	345
N.S.	1	1.00	5.44	1.05	1.03	0.88	0.00	1.83	1.54
time (sec)	N/A	0.450	8.115	1.980	0.227	0.262	0.000	0.563	27.407

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	89	120	166	144	0	89	169
N.S.	1	1.00	2.97	4.00	5.53	4.80	0.00	2.97	5.63
time (sec)	N/A	0.216	0.544	1.600	0.222	0.255	0.000	0.578	23.901

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	318	318	1677	297	289	227	0	680	514
N.S.	1	1.00	5.27	0.93	0.91	0.71	0.00	2.14	1.62
time (sec)	N/A	0.572	8.721	2.387	0.232	0.288	0.000	0.574	27.405

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	160	54	209	223	176	0	176	419
N.S.	1	0.90	0.31	1.18	1.26	0.99	0.00	0.99	2.37
time (sec)	N/A	0.344	0.488	2.191	0.232	0.263	0.000	0.580	27.565

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	391	391	331	345	360	257	0	888	675
N.S.	1	1.00	0.85	0.88	0.92	0.66	0.00	2.27	1.73
time (sec)	N/A	0.661	4.740	2.934	0.228	0.276	0.000	0.616	27.631

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	218	115	255	275	207	0	262	548
N.S.	1	0.90	0.48	1.05	1.14	0.86	0.00	1.08	2.26
time (sec)	N/A	0.410	1.264	2.355	0.222	0.271	0.000	0.620	27.273

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	472	472	374	392	420	287	0	1096	831
N.S.	1	1.00	0.79	0.83	0.89	0.61	0.00	2.32	1.76
time (sec)	N/A	0.731	4.518	3.296	0.240	0.289	0.000	0.647	29.272

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	221	218	197	564	208	0	322	6099
N.S.	1	0.97	0.96	0.87	2.48	0.92	0.00	1.42	26.87
time (sec)	N/A	1.044	0.876	0.854	0.340	0.295	0.000	0.309	35.841

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	137	221	379	262	0	286	342
N.S.	1	1.00	0.83	1.33	2.28	1.58	0.00	1.72	2.06
time (sec)	N/A	0.682	1.467	0.768	0.324	0.263	0.000	0.344	26.025

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	120	143	120	284	119	0	182	3572
N.S.	1	1.01	1.20	1.01	2.39	1.00	0.00	1.53	30.02
time (sec)	N/A	0.557	0.598	0.638	0.340	0.259	0.000	0.305	29.780

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	79	90	142	187	1034	118	110
N.S.	1	1.00	0.87	0.99	1.56	2.05	11.36	1.30	1.21
time (sec)	N/A	0.395	0.513	0.516	0.315	0.283	115.445	0.328	22.618

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	41	62	124	61	296	74	1069
N.S.	1	1.00	0.91	1.38	2.76	1.36	6.58	1.64	23.76
time (sec)	N/A	0.313	0.229	0.461	0.298	0.259	1.070	0.303	23.006

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	45	43	80	131	163	74	39
N.S.	1	1.00	0.96	0.91	1.70	2.79	3.47	1.57	0.83
time (sec)	N/A	0.204	0.026	0.408	0.304	0.254	2.901	0.319	22.557

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	18	19	103	59	0	19	62
N.S.	1	1.00	0.44	0.46	2.51	1.44	0.00	0.46	1.51
time (sec)	N/A	0.341	0.017	0.606	0.234	0.250	0.000	0.295	23.456

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	109	129	163	191	0	136	310
N.S.	1	1.00	1.36	1.61	2.04	2.39	0.00	1.70	3.88
time (sec)	N/A	0.387	0.113	0.708	0.323	0.278	0.000	0.347	22.735

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	86	52	53	238	117	0	54	300
N.S.	1	0.98	0.59	0.60	2.70	1.33	0.00	0.61	3.41
time (sec)	N/A	0.566	0.151	0.726	0.239	0.266	0.000	0.308	23.128

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	150	321	269	361	259	0	278	724
N.S.	1	0.98	2.10	1.76	2.36	1.69	0.00	1.82	4.73
time (sec)	N/A	0.695	2.151	1.000	0.324	0.327	0.000	0.348	24.304

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	145	99	106	462	183	0	120	575
N.S.	1	0.92	0.63	0.67	2.92	1.16	0.00	0.76	3.64
time (sec)	N/A	0.839	1.240	1.130	0.241	0.271	0.000	0.340	25.692

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	246	661	479	625	346	0	554	2979
N.S.	1	0.94	2.52	1.83	2.39	1.32	0.00	2.11	11.37
time (sec)	N/A	1.142	6.273	1.461	0.328	0.473	0.000	0.360	25.105

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	173	149	154	282	279	0	250	6604
N.S.	1	1.19	1.03	1.06	1.94	1.92	0.00	1.72	45.54
time (sec)	N/A	0.541	1.603	1.042	0.346	0.277	0.000	0.317	36.476

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	222	130	172	348	302	0	286	286
N.S.	1	1.61	0.94	1.25	2.52	2.19	0.00	2.07	2.07
time (sec)	N/A	0.857	1.306	0.757	0.318	0.280	0.000	0.357	26.055

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	93	192	97	131	173	1545	159	3114
N.S.	1	1.13	2.34	1.18	1.60	2.11	18.84	1.94	37.98
time (sec)	N/A	0.532	0.884	0.593	0.301	0.268	4.202	0.313	27.382

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-2)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	79	118	182	215	0	138	136
N.S.	1	1.00	0.95	1.42	2.19	2.59	0.00	1.66	1.64
time (sec)	N/A	0.329	0.524	0.530	0.315	0.257	0.000	0.327	22.657

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	21	21	57	0	20	47
N.S.	1	1.00	1.00	0.66	0.66	1.78	0.00	0.62	1.47
time (sec)	N/A	0.191	0.028	0.474	0.222	0.258	0.000	0.292	22.383

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	120	135	212	293	0	166	383
N.S.	1	1.00	1.30	1.47	2.30	3.18	0.00	1.80	4.16
time (sec)	N/A	0.379	0.826	0.819	0.313	0.286	0.000	0.357	23.042

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	70	51	57	60	178	0	71	382
N.S.	1	0.93	0.68	0.76	0.80	2.37	0.00	0.95	5.09
time (sec)	N/A	0.283	0.269	1.079	0.223	0.266	0.000	0.296	24.403

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	228	709	259	471	355	0	280	585
N.S.	1	1.27	3.96	1.45	2.63	1.98	0.00	1.56	3.27
time (sec)	N/A	1.146	6.082	1.386	0.343	0.308	0.000	0.368	25.120

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	129	140	114	115	281	0	149	1132
N.S.	1	0.91	0.99	0.81	0.82	1.99	0.00	1.06	8.03
time (sec)	N/A	0.332	6.019	1.816	0.223	0.287	0.000	0.322	26.116

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	C	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	478	211	283	658	480	0	399	610
N.S.	1	2.21	0.98	1.31	3.05	2.22	0.00	1.85	2.82
time (sec)	N/A	1.187	1.971	1.742	0.334	0.290	0.000	0.422	26.657

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	144	154	140	481	341	0	265	6190
N.S.	1	1.18	1.26	1.15	3.94	2.80	0.00	2.17	50.74
time (sec)	N/A	0.739	1.966	1.125	0.331	0.285	0.000	0.385	32.439

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	236	119	280	412	352	0	293	443
N.S.	1	1.98	1.00	2.35	3.46	2.96	0.00	2.46	3.72
time (sec)	N/A	0.683	1.284	0.879	0.326	0.270	0.000	0.405	23.984

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	30	57	21	171	142	0	20	85
N.S.	1	1.36	2.59	0.95	7.77	6.45	0.00	0.91	3.86
time (sec)	N/A	0.206	0.491	0.654	0.222	0.241	0.000	0.367	22.241

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	132	191	326	294	0	221	260
N.S.	1	1.00	1.28	1.85	3.17	2.85	0.00	2.15	2.52
time (sec)	N/A	0.330	0.298	0.773	0.304	0.266	0.000	0.322	24.647

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	81	57	63	315	284	0	62	396
N.S.	1	0.94	0.66	0.73	3.66	3.30	0.00	0.72	4.60
time (sec)	N/A	0.295	0.602	1.035	0.240	0.282	0.000	0.352	26.108

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	297	396	269	518	513	0	314	1311
N.S.	1	1.14	1.52	1.03	1.99	1.97	0.00	1.21	5.04
time (sec)	N/A	1.460	2.940	1.864	0.345	0.315	0.000	0.427	24.903

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	147	140	115	652	354	0	140	1204
N.S.	1	0.91	0.87	0.71	4.05	2.20	0.00	0.87	7.48
time (sec)	N/A	0.362	3.645	2.273	0.277	0.291	0.000	0.386	27.019

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	383	697	688	444	902	564	0	510	1203
N.S.	1	1.82	1.80	1.16	2.36	1.47	0.00	1.33	3.14
time (sec)	N/A	4.041	2.655	3.254	0.330	0.342	0.000	0.444	26.428

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	210	272	195	1053	476	0	243	1712
N.S.	1	0.91	1.17	0.84	4.54	2.05	0.00	1.05	7.38
time (sec)	N/A	0.443	1.427	3.794	0.268	0.302	0.000	0.408	30.616

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	198	419	183	385	575	0	370	8586
N.S.	1	1.20	2.54	1.11	2.33	3.48	0.00	2.24	52.04
time (sec)	N/A	0.990	7.104	1.706	0.328	0.282	0.000	0.381	37.653

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	C	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	391	165	494	724	524	0	524	764
N.S.	1	2.49	1.05	3.15	4.61	3.34	0.00	3.34	4.87
time (sec)	N/A	1.117	2.060	1.463	0.343	0.280	0.000	0.412	26.646

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	124	21	53	255	0	20	224
N.S.	1	1.00	4.13	0.70	1.77	8.50	0.00	0.67	7.47
time (sec)	N/A	0.219	1.516	0.974	0.233	0.263	0.000	0.369	24.263

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	151	128	335	606	420	0	426	505
N.S.	1	1.07	0.91	2.38	4.30	2.98	0.00	3.02	3.58
time (sec)	N/A	0.476	1.323	1.214	0.342	0.272	0.000	0.391	26.965

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	85	64	85	217	0	50	222
N.S.	1	1.00	0.87	0.65	0.87	2.21	0.00	0.51	2.27
time (sec)	N/A	0.311	0.407	0.836	0.221	0.242	0.000	0.303	23.674

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	235	290	411	661	745	0	527	2848
N.S.	1	1.02	1.26	1.78	2.86	3.23	0.00	2.28	12.33
time (sec)	N/A	0.956	4.257	2.585	0.346	0.388	0.000	0.395	27.550

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	128	133	118	144	537	0	138	666
N.S.	1	0.93	0.96	0.86	1.04	3.89	0.00	1.00	4.83
time (sec)	N/A	0.352	2.377	2.990	0.230	0.312	0.000	0.347	27.241

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	400	782	538	452	936	820	0	548	1961
N.S.	1	1.96	1.34	1.13	2.34	2.05	0.00	1.37	4.90
time (sec)	N/A	4.270	4.477	5.318	0.376	0.373	0.000	0.409	28.161

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	216	295	195	217	553	0	249	1599
N.S.	1	0.93	1.27	0.84	0.94	2.38	0.00	1.07	6.89
time (sec)	N/A	0.455	2.270	5.415	0.246	0.289	0.000	0.370	32.433

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	102	82	96	0	76	219	116	164
N.S.	1	1.03	0.83	0.97	0.00	0.77	2.21	1.17	1.66
time (sec)	N/A	0.386	0.863	1.224	0.000	0.249	0.220	0.299	29.372

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	74	111	84	0	63	196	119	134
N.S.	1	1.06	1.59	1.20	0.00	0.90	2.80	1.70	1.91
time (sec)	N/A	0.358	0.774	0.963	0.000	0.238	0.246	0.292	24.953

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	78	60	60	0	54	151	95	111
N.S.	1	1.04	0.80	0.80	0.00	0.72	2.01	1.27	1.48
time (sec)	N/A	0.352	0.773	0.835	0.000	0.252	0.176	0.297	26.042

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	56	73	49	0	41	126	67	78
N.S.	1	1.08	1.40	0.94	0.00	0.79	2.42	1.29	1.50
time (sec)	N/A	0.338	0.743	0.668	0.000	0.243	0.170	0.296	23.329

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	38	26	0	32	60	58	39
N.S.	1	1.00	0.83	0.57	0.00	0.70	1.30	1.26	0.85
time (sec)	N/A	0.203	0.397	0.490	0.000	0.245	0.093	0.287	22.657

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	19	29	17	31	21	25
N.S.	1	1.00	1.00	0.66	1.00	0.59	1.07	0.72	0.86
time (sec)	N/A	0.184	0.246	0.405	0.223	0.240	0.074	0.274	22.592

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	27	23	22	101	26	0	57	41
N.S.	1	1.17	1.00	0.96	4.39	1.13	0.00	2.48	1.78
time (sec)	N/A	0.283	0.152	0.511	0.249	0.251	0.000	0.289	22.626

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	34	35	65	83	80	0	58	43
N.S.	1	1.10	1.13	2.10	2.68	2.58	0.00	1.87	1.39
time (sec)	N/A	0.312	0.403	0.588	0.222	0.242	0.000	0.292	23.156

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	38	28	23	108	33	0	27	25
N.S.	1	1.12	0.82	0.68	3.18	0.97	0.00	0.79	0.74
time (sec)	N/A	0.325	0.105	0.586	0.219	0.237	0.000	0.306	24.352

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	63	54	100	186	174	0	99	116
N.S.	1	1.05	0.90	1.67	3.10	2.90	0.00	1.65	1.93
time (sec)	N/A	0.338	0.317	0.655	0.198	0.246	0.000	0.301	24.856

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	56	52	36	211	72	0	47	99
N.S.	1	1.08	1.00	0.69	4.06	1.38	0.00	0.90	1.90
time (sec)	N/A	0.345	0.138	0.615	0.227	0.241	0.000	0.308	23.560

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	87	66	122	288	266	0	138	193
N.S.	1	1.04	0.79	1.45	3.43	3.17	0.00	1.64	2.30
time (sec)	N/A	0.376	0.620	0.700	0.210	0.254	0.000	0.310	26.610

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	74	74	47	313	109	0	67	139
N.S.	1	1.06	1.06	0.67	4.47	1.56	0.00	0.96	1.99
time (sec)	N/A	0.359	0.292	0.707	0.223	0.239	0.000	0.311	24.335

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	92	149	102	0	74	231	145	161
N.S.	1	1.08	1.75	1.20	0.00	0.87	2.72	1.71	1.89
time (sec)	N/A	0.423	0.845	0.894	0.000	0.243	0.286	0.317	27.171

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	88	82	79	0	65	189	103	138
N.S.	1	0.87	0.81	0.78	0.00	0.64	1.87	1.02	1.37
time (sec)	N/A	0.273	0.772	0.818	0.000	0.250	0.197	0.308	27.413

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	73	111	67	0	52	163	93	90
N.S.	1	1.07	1.63	0.99	0.00	0.76	2.40	1.37	1.32
time (sec)	N/A	0.399	0.774	0.853	0.000	0.252	0.203	0.311	24.291

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	93	60	44	0	43	117	68	69
N.S.	1	1.04	0.67	0.49	0.00	0.48	1.31	0.76	0.78
time (sec)	N/A	0.332	0.731	0.573	0.000	0.235	0.136	0.321	24.894

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	58	73	38	45	30	92	47	79
N.S.	1	1.12	1.40	0.73	0.87	0.58	1.77	0.90	1.52
time (sec)	N/A	0.342	0.449	0.527	0.233	0.243	0.134	0.299	22.725

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	19	22	17	44	30	31
N.S.	1	1.00	1.00	0.61	0.71	0.55	1.42	0.97	1.00
time (sec)	N/A	0.183	0.226	0.448	0.224	0.233	0.077	0.298	22.439

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	50	184	54	117	64	0	57	44
N.S.	1	1.09	4.00	1.17	2.54	1.39	0.00	1.24	0.96
time (sec)	N/A	0.336	0.357	0.598	0.308	0.258	0.000	0.313	22.948

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	41	36	30	30	70	0	100	83
N.S.	1	0.75	0.65	0.55	0.55	1.27	0.00	1.82	1.51
time (sec)	N/A	0.253	0.076	0.632	0.212	0.249	0.000	0.307	22.695

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	61	146	89	167	134	0	95	104
N.S.	1	1.09	2.61	1.59	2.98	2.39	0.00	1.70	1.86
time (sec)	N/A	0.366	0.552	0.704	0.229	0.266	0.000	0.317	23.163

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	36	20	35	54	0	35	49
N.S.	1	1.00	1.06	0.59	1.03	1.59	0.00	1.03	1.44
time (sec)	N/A	0.230	0.099	0.649	0.205	0.247	0.000	0.312	23.712

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	89	215	111	295	230	0	151	136
N.S.	1	1.06	2.56	1.32	3.51	2.74	0.00	1.80	1.62
time (sec)	N/A	0.411	1.230	0.767	0.229	0.244	0.000	0.328	26.619

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	55	50	36	47	97	0	47	76
N.S.	1	0.79	0.71	0.51	0.67	1.39	0.00	0.67	1.09
time (sec)	N/A	0.255	0.230	0.684	0.209	0.231	0.000	0.316	23.254

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	106	106	97	0	76	224	115	164
N.S.	1	0.85	0.85	0.78	0.00	0.61	1.79	0.92	1.31
time (sec)	N/A	0.290	0.968	0.956	0.000	0.250	0.229	0.354	27.763

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	118	149	85	0	63	197	119	134
N.S.	1	1.11	1.41	0.80	0.00	0.59	1.86	1.12	1.26
time (sec)	N/A	0.463	0.841	0.822	0.000	0.235	0.259	0.340	25.107

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	140	84	62	0	54	155	78	96
N.S.	1	1.07	0.64	0.47	0.00	0.41	1.18	0.60	0.73
time (sec)	N/A	0.475	0.795	0.691	0.000	0.252	0.168	0.341	25.839

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	100	111	56	69	41	131	73	133
N.S.	1	1.11	1.23	0.62	0.77	0.46	1.46	0.81	1.48
time (sec)	N/A	0.428	0.750	0.659	0.234	0.249	0.167	0.348	23.001

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	77	23	51	30	95	57	100
N.S.	1	1.00	2.41	0.72	1.59	0.94	2.97	1.78	3.12
time (sec)	N/A	0.199	0.444	0.611	0.239	0.240	0.118	0.325	22.952

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	19	29	17	44	36	68
N.S.	1	1.00	1.00	0.61	0.94	0.55	1.42	1.16	2.19
time (sec)	N/A	0.182	0.218	0.555	0.227	0.244	0.083	0.294	24.444

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	49	40	35	99	55	0	100	101
N.S.	1	0.80	0.66	0.57	1.62	0.90	0.00	1.64	1.66
time (sec)	N/A	0.238	0.152	0.741	0.342	0.240	0.000	0.315	23.364

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	68	109	86	319	112	0	110	105
N.S.	1	1.10	1.76	1.39	5.15	1.81	0.00	1.77	1.69
time (sec)	N/A	0.389	0.453	0.773	0.338	0.252	0.000	0.321	23.082

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	57	48	41	301	113	0	128	104
N.S.	1	0.76	0.64	0.55	4.01	1.51	0.00	1.71	1.39
time (sec)	N/A	0.259	0.121	0.762	0.321	0.247	0.000	0.341	23.215

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	83	64	100	215	182	0	112	135
N.S.	1	1.09	0.84	1.32	2.83	2.39	0.00	1.47	1.78
time (sec)	N/A	0.403	0.630	0.803	0.227	0.246	0.000	0.347	24.931

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	50	21	240	69	0	47	55
N.S.	1	1.00	1.47	0.62	7.06	2.03	0.00	1.38	1.62
time (sec)	N/A	0.228	0.151	0.819	0.223	0.229	0.000	0.341	22.625

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	113	115	122	341	278	0	164	150
N.S.	1	1.09	1.11	1.17	3.28	2.67	0.00	1.58	1.44
time (sec)	N/A	0.463	0.617	0.902	0.239	0.251	0.000	0.363	25.988

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	66	66	90	0	0	0	0	0	0
N.S.	1	1.00	1.36	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.243	4.268	0.000	0.000	0.000	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	16	6	31	5	17	22	21
N.S.	1	1.00	3.20	1.20	6.20	1.00	3.40	4.40	4.20
time (sec)	N/A	0.200	0.033	0.246	0.217	0.256	0.073	0.294	24.594

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	19	11	54	10	0	10	21
N.S.	1	1.00	1.90	1.10	5.40	1.00	0.00	1.00	2.10
time (sec)	N/A	0.264	0.043	1.513	0.287	0.241	0.000	0.281	22.559

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	63	5	30	4	0	14	4
N.S.	1	1.00	15.75	1.25	7.50	1.00	0.00	3.50	1.00
time (sec)	N/A	0.240	0.087	1.436	0.294	0.244	0.000	0.284	22.645

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	25	15	28	24	0	12	12
N.S.	1	1.00	2.27	1.36	2.55	2.18	0.00	1.09	1.09
time (sec)	N/A	0.270	0.068	0.228	0.291	0.249	0.000	0.286	22.606

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	20	14	23	22	0	10	23
N.S.	1	1.00	2.22	1.56	2.56	2.44	0.00	1.11	2.56
time (sec)	N/A	0.296	0.082	0.365	0.297	0.253	0.000	0.282	22.429

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	23	11	15	18	0	10	10
N.S.	1	1.00	2.30	1.10	1.50	1.80	0.00	1.00	1.00
time (sec)	N/A	0.199	0.029	0.210	0.241	0.233	0.000	0.277	22.473

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	20	8	25	13	0	12	15
N.S.	1	1.00	1.82	0.73	2.27	1.18	0.00	1.09	1.36
time (sec)	N/A	0.241	0.042	0.349	0.216	0.260	0.000	0.294	22.409

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	18	8	29	9	17	20	19
N.S.	1	1.00	2.00	0.89	3.22	1.00	1.89	2.22	2.11
time (sec)	N/A	0.210	0.037	0.239	0.241	0.262	0.071	0.284	22.955

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	23	13	54	14	0	14	23
N.S.	1	1.00	1.64	0.93	3.86	1.00	0.00	1.00	1.64
time (sec)	N/A	0.277	0.045	1.420	0.310	0.256	0.000	0.297	23.406

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	34	7	30	6	0	14	6
N.S.	1	1.00	5.67	1.17	5.00	1.00	0.00	2.33	1.00
time (sec)	N/A	0.246	0.076	1.412	0.292	0.239	0.000	0.286	22.337

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	29	17	28	28	0	14	14
N.S.	1	1.00	1.93	1.13	1.87	1.87	0.00	0.93	0.93
time (sec)	N/A	0.282	0.079	0.240	0.300	0.249	0.000	0.287	22.803

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	18	14	23	20	0	8	23
N.S.	1	1.00	2.57	2.00	3.29	2.86	0.00	1.14	3.29
time (sec)	N/A	0.299	0.077	0.355	0.300	0.250	0.000	0.286	22.692

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	25	11	15	17	0	10	10
N.S.	1	1.00	2.27	1.00	1.36	1.55	0.00	0.91	0.91
time (sec)	N/A	0.213	0.053	0.214	0.215	0.229	0.000	0.289	22.354

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	15	22	8	25	15	0	14	15
N.S.	1	1.15	1.69	0.62	1.92	1.15	0.00	1.08	1.15
time (sec)	N/A	0.247	0.040	0.318	0.205	0.240	0.000	0.281	22.123

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	15	20	22	21	27	16	14
N.S.	1	1.00	0.65	0.87	0.96	0.91	1.17	0.70	0.61
time (sec)	N/A	0.324	0.117	0.530	0.216	0.244	0.874	0.287	22.544

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	14	7	38	6	0	18	6
N.S.	1	1.00	2.33	1.17	6.33	1.00	0.00	3.00	1.00
time (sec)	N/A	0.247	0.016	1.353	0.344	0.272	0.000	0.285	23.780

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	20	11	34	12	0	10	24
N.S.	1	1.00	2.00	1.10	3.40	1.20	0.00	1.00	2.40
time (sec)	N/A	0.277	0.020	1.244	0.300	0.249	0.000	0.277	23.596

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	36	25	39	20	0	22	11
N.S.	1	1.00	5.14	3.57	5.57	2.86	0.00	3.14	1.57
time (sec)	N/A	0.309	0.058	0.322	0.299	0.266	0.000	0.289	23.402

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	10	15	23	17	0	8	8
N.S.	1	1.00	0.83	1.25	1.92	1.42	0.00	0.67	0.67
time (sec)	N/A	0.275	0.033	0.230	0.302	0.251	0.000	0.277	23.242

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	25	6	29	15	0	12	11
N.S.	1	1.00	2.27	0.55	2.64	1.36	0.00	1.09	1.00
time (sec)	N/A	0.257	0.021	0.395	0.209	0.246	0.000	0.271	22.988

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	6	5	9	9	0	4	4
N.S.	1	1.00	0.67	0.56	1.00	1.00	0.00	0.44	0.44
time (sec)	N/A	0.209	0.060	0.223	0.219	0.240	0.000	0.279	23.160

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	14	5	38	4	0	18	4
N.S.	1	1.00	3.50	1.25	9.50	1.00	0.00	4.50	1.00
time (sec)	N/A	0.250	0.043	1.318	0.295	0.239	0.000	0.281	23.285

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	20	9	46	10	0	10	31
N.S.	1	1.00	2.00	0.90	4.60	1.00	0.00	1.00	3.10
time (sec)	N/A	0.278	0.024	1.276	0.306	0.248	0.000	0.281	22.588

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	46	23	39	18	0	20	9
N.S.	1	1.00	9.20	4.60	7.80	3.60	0.00	4.00	1.80
time (sec)	N/A	0.309	0.049	0.339	0.306	0.254	0.000	0.288	23.356

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	23	14	0	12	10
N.S.	1	1.00	1.00	1.06	1.44	0.88	0.00	0.75	0.62
time (sec)	N/A	0.286	0.070	0.223	0.298	0.247	0.000	0.290	22.765

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	15	25	6	41	15	0	14	19
N.S.	1	1.15	1.92	0.46	3.15	1.15	0.00	1.08	1.46
time (sec)	N/A	0.260	0.022	0.368	0.226	0.245	0.000	0.288	22.652

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	8	9	10	10	0	8	6
N.S.	1	1.00	0.67	0.75	0.83	0.83	0.00	0.67	0.50
time (sec)	N/A	0.211	0.036	0.197	0.221	0.240	0.000	0.295	22.678

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	61	21	176	44	0	68	42
N.S.	1	1.00	2.65	0.91	7.65	1.91	0.00	2.96	1.83
time (sec)	N/A	0.244	0.631	0.380	0.312	0.250	0.000	0.300	23.380

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	32	30	29	252	52	0	82	62
N.S.	1	0.63	0.59	0.57	4.94	1.02	0.00	1.61	1.22
time (sec)	N/A	0.303	0.597	0.402	0.337	0.266	0.000	0.293	23.797

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	20	17	16	18	0	16	16
N.S.	1	1.00	1.11	0.94	0.89	1.00	0.00	0.89	0.89
time (sec)	N/A	0.208	0.126	0.283	0.294	0.243	0.000	0.299	0.080

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	27	24	37	35	37	0	37	61
N.S.	1	0.93	0.83	1.28	1.21	1.28	0.00	1.28	2.10
time (sec)	N/A	0.215	0.052	0.572	0.284	0.257	0.000	0.336	23.923

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	11	0	11	45
N.S.	1	1.00	1.00	1.09	1.00	1.00	0.00	1.00	4.09
time (sec)	N/A	0.195	0.036	0.331	0.297	0.243	0.000	0.310	23.942

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	30	19	39	30	0	79	14
N.S.	1	1.00	1.88	1.19	2.44	1.88	0.00	4.94	0.88
time (sec)	N/A	0.209	0.052	0.269	0.310	0.252	0.000	0.320	23.741

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	22	22	20	245	46	0	72	56
N.S.	1	0.46	0.46	0.42	5.10	0.96	0.00	1.50	1.17
time (sec)	N/A	0.257	0.100	0.381	0.335	0.254	0.000	0.300	24.866

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	13	28	12	0	28	20
N.S.	1	1.00	1.00	1.30	2.80	1.20	0.00	2.80	2.00
time (sec)	N/A	0.222	0.008	0.336	0.223	0.238	0.000	0.280	24.587

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	20	23	21	64	31	0	18	33
N.S.	1	1.43	1.64	1.50	4.57	2.21	0.00	1.29	2.36
time (sec)	N/A	0.293	0.005	0.385	0.285	0.245	0.000	0.282	23.704

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	20	12	13	24	14	0	26	14
N.S.	1	1.67	1.00	1.08	2.00	1.17	0.00	2.17	1.17
time (sec)	N/A	0.208	0.007	0.444	0.196	0.254	0.000	0.295	22.686

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	40	34	52	46	61	0	48	69
N.S.	1	1.18	1.00	1.53	1.35	1.79	0.00	1.41	2.03
time (sec)	N/A	0.218	0.019	0.497	0.213	0.243	0.000	0.350	23.040

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	26	28	0	28	15
N.S.	1	1.00	1.00	1.09	2.36	2.55	0.00	2.55	1.36
time (sec)	N/A	0.201	0.005	0.458	0.211	0.249	0.000	0.286	22.117

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	21	15	14	17	13	0	46	13
N.S.	1	1.40	1.00	0.93	1.13	0.87	0.00	3.07	0.87
time (sec)	N/A	0.207	0.016	0.217	0.209	0.252	0.000	0.309	0.051

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	44	18	0	10	29
N.S.	1	1.00	1.00	1.10	4.40	1.80	0.00	1.00	2.90
time (sec)	N/A	0.233	0.004	0.325	0.203	0.248	0.000	0.288	22.282

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	29	28	28	0	11588	29
N.S.	1	1.00	1.00	0.88	0.85	0.85	0.00	351.15	0.88
time (sec)	N/A	0.285	0.019	2.077	0.217	0.245	0.000	2.141	23.389

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	38	29	28	28	0	99	49
N.S.	1	1.00	1.15	0.88	0.85	0.85	0.00	3.00	1.48
time (sec)	N/A	0.304	0.162	0.941	0.226	0.242	0.000	0.356	22.959

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	29	40	26	25	25	0	102	28
N.S.	1	1.32	1.82	1.18	1.14	1.14	0.00	4.64	1.27
time (sec)	N/A	0.260	0.009	0.428	0.215	0.237	0.000	0.337	22.138

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	37	25	25	25	37	27	40
N.S.	1	1.00	1.42	0.96	0.96	0.96	1.42	1.04	1.54
time (sec)	N/A	0.154	0.019	0.309	0.239	0.252	0.093	0.275	22.072

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	23	32	34	0	107	40
N.S.	1	1.00	1.00	0.92	1.28	1.36	0.00	4.28	1.60
time (sec)	N/A	0.268	0.014	0.303	0.214	0.245	0.000	0.357	22.640

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	27	24	0	71	24
N.S.	1	1.00	1.00	0.89	0.96	0.86	0.00	2.54	0.86
time (sec)	N/A	0.270	0.015	0.423	0.201	0.230	0.000	0.348	22.605

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	32	26	0	97	29
N.S.	1	1.00	1.00	0.85	0.97	0.79	0.00	2.94	0.88
time (sec)	N/A	0.277	0.024	0.858	0.219	0.239	0.000	0.369	22.442

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	117	77	100	68	85	0	43089	101
N.S.	1	1.10	0.73	0.94	0.64	0.80	0.00	406.50	0.95
time (sec)	N/A	0.747	0.375	9.111	0.207	0.247	0.000	154.270	24.060

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	84	82	77	66	74	0	5161	76
N.S.	1	0.98	0.95	0.90	0.77	0.86	0.00	60.01	0.88
time (sec)	N/A	0.498	0.721	4.479	0.216	0.251	0.000	2.935	22.332

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	90	117	72	76	83	0	4849	121
N.S.	1	1.03	1.34	0.83	0.87	0.95	0.00	55.74	1.39
time (sec)	N/A	0.903	0.521	2.197	0.212	0.266	0.000	2.206	22.845

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	116	77	84	108	0	2320	143
N.S.	1	1.00	1.51	1.00	1.09	1.40	0.00	30.13	1.86
time (sec)	N/A	0.343	0.798	1.230	0.291	0.264	0.000	0.752	22.875

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	91	75	98	102	126	0	171	147
N.S.	1	1.01	0.83	1.09	1.13	1.40	0.00	1.90	1.63
time (sec)	N/A	0.991	0.224	1.845	0.288	0.259	0.000	0.935	22.201

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	66	92	82	115	0	158	227
N.S.	1	1.00	0.67	0.93	0.83	1.16	0.00	1.60	2.29
time (sec)	N/A	1.023	0.267	1.027	0.323	0.257	0.000	0.987	22.510

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	136	78	138	129	129	0	226	177
N.S.	1	1.09	0.62	1.10	1.03	1.03	0.00	1.81	1.42
time (sec)	N/A	1.137	0.363	1.967	0.214	0.275	0.000	1.002	25.255

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	67	114	100	95	100	0	0	149
N.S.	1	0.87	1.48	1.30	1.23	1.30	0.00	0.00	1.94
time (sec)	N/A	0.399	0.972	30.633	0.210	0.245	0.000	0.000	22.713

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	121	106	99	94	101	0	0	237
N.S.	1	1.01	0.88	0.82	0.78	0.84	0.00	0.00	1.98
time (sec)	N/A	0.438	0.215	17.398	0.206	0.255	0.000	0.000	23.726

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	111	98	106	87	128	0	0	225
N.S.	1	0.99	0.88	0.95	0.78	1.14	0.00	0.00	2.01
time (sec)	N/A	0.422	1.090	9.198	0.214	0.265	0.000	0.000	25.904

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	103	102	116	113	123	0	0	219
N.S.	1	0.89	0.88	1.00	0.97	1.06	0.00	0.00	1.89
time (sec)	N/A	0.347	0.420	5.011	0.223	0.277	0.000	0.000	26.128

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	107	100	101	109	122	0	0	219
N.S.	1	0.93	0.87	0.88	0.95	1.06	0.00	0.00	1.90
time (sec)	N/A	0.475	1.404	8.564	0.218	0.262	0.000	0.000	26.183

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	97	106	96	107	0	0	223
N.S.	1	1.00	0.87	0.95	0.86	0.96	0.00	0.00	2.01
time (sec)	N/A	0.455	2.063	8.522	0.221	0.275	0.000	0.000	26.506

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	99	110	128	109	0	0	220
N.S.	1	1.00	0.83	0.92	1.08	0.92	0.00	0.00	1.85
time (sec)	N/A	0.445	0.498	3.871	0.207	0.272	0.000	0.000	26.301

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	110	100	100	188	123	0	303	161
N.S.	1	0.97	0.88	0.88	1.66	1.09	0.00	2.68	1.42
time (sec)	N/A	0.616	0.778	2.409	0.307	0.284	0.000	0.370	24.399

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	97	80	85	129	98	0	190	117
N.S.	1	1.05	0.87	0.92	1.40	1.07	0.00	2.07	1.27
time (sec)	N/A	0.564	0.542	1.126	0.305	0.278	0.000	0.372	23.645

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	70	75	102	75	0	257	93
N.S.	1	1.00	0.88	0.94	1.28	0.94	0.00	3.21	1.16
time (sec)	N/A	0.444	0.319	0.532	0.303	0.273	0.000	0.365	22.354

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	76	63	70	71	64	0	100	67
N.S.	1	1.03	0.85	0.95	0.96	0.86	0.00	1.35	0.91
time (sec)	N/A	0.319	0.080	0.461	0.212	0.286	0.000	0.298	22.625

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	80	64	70	73	65	0	101	68
N.S.	1	1.07	0.85	0.93	0.97	0.87	0.00	1.35	0.91
time (sec)	N/A	0.416	0.204	0.879	0.226	0.274	0.000	0.338	22.286

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	101	103	87	125	96	0	253	93
N.S.	1	1.07	1.10	0.93	1.33	1.02	0.00	2.69	0.99
time (sec)	N/A	0.483	0.329	1.908	0.205	0.327	0.000	0.360	22.517

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	115	92	99	158	147	0	190	118
N.S.	1	1.06	0.85	0.92	1.46	1.36	0.00	1.76	1.09
time (sec)	N/A	0.507	0.612	3.943	0.218	0.355	0.000	0.388	22.255

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	164	215	0	857	0	1362	7329
N.S.	1	1.00	0.67	0.88	0.00	3.53	0.00	5.60	30.16
time (sec)	N/A	0.874	3.989	5.342	0.000	0.330	0.000	0.769	29.441

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	151	184	0	705	0	331	6093
N.S.	1	1.00	0.67	0.81	0.00	3.11	0.00	1.46	26.84
time (sec)	N/A	0.758	2.589	1.433	0.000	0.309	0.000	0.542	27.944

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	131	155	0	518	0	282	213
N.S.	1	1.00	0.60	0.71	0.00	2.37	0.00	1.29	0.97
time (sec)	N/A	0.679	1.832	1.369	0.000	0.279	0.000	0.536	23.403

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	128	162	0	526	0	289	245
N.S.	1	1.00	0.63	0.80	0.00	2.59	0.00	1.42	1.21
time (sec)	N/A	0.623	1.463	1.394	0.000	0.289	0.000	0.326	23.428

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	160	127	162	0	532	0	288	245
N.S.	1	1.18	0.93	1.19	0.00	3.91	0.00	2.12	1.80
time (sec)	N/A	0.720	1.453	3.588	0.000	0.286	0.000	0.454	23.146

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	157	121	155	0	516	0	284	215
N.S.	1	1.20	0.92	1.18	0.00	3.94	0.00	2.17	1.64
time (sec)	N/A	0.682	1.055	8.324	0.000	0.292	0.000	0.498	22.742

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	196	199	0	864	0	354	6056
N.S.	1	1.00	0.85	0.86	0.00	3.74	0.00	1.53	26.22
time (sec)	N/A	0.715	2.676	17.052	0.000	0.765	0.000	0.553	27.597

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	270	713	213	684	1180	0	848	527
N.S.	1	1.09	2.88	0.86	2.76	4.76	0.00	3.42	2.12
time (sec)	N/A	1.193	7.137	5.241	0.341	0.596	0.000	0.951	25.342

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	255	477	199	589	1045	0	676	491
N.S.	1	1.10	2.06	0.86	2.54	4.50	0.00	2.91	2.12
time (sec)	N/A	1.024	7.051	5.151	0.274	0.459	0.000	0.877	22.887

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	237	184	176	593	994	0	690	490
N.S.	1	1.12	0.87	0.83	2.81	4.71	0.00	3.27	2.32
time (sec)	N/A	0.983	6.294	5.151	0.247	0.421	0.000	0.872	22.774

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	256	696	196	601	1071	0	800	494
N.S.	1	1.12	3.04	0.86	2.62	4.68	0.00	3.49	2.16
time (sec)	N/A	0.802	6.512	5.072	0.250	0.416	0.000	0.387	23.240

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	250	703	196	602	1076	0	801	496
N.S.	1	1.08	3.04	0.85	2.61	4.66	0.00	3.47	2.15
time (sec)	N/A	0.958	6.878	14.865	0.260	0.420	0.000	0.672	23.578

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	230	217	181	596	939	0	689	492
N.S.	1	1.08	1.02	0.85	2.81	4.43	0.00	3.25	2.32
time (sec)	N/A	0.642	6.639	31.823	0.247	0.428	0.000	0.735	23.149

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	212	217	184	591	971	0	675	490
N.S.	1	0.93	0.95	0.81	2.59	4.26	0.00	2.96	2.15
time (sec)	N/A	0.622	6.745	54.578	0.245	0.418	0.000	0.857	22.808

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	246	212	180	412	0	0	861
N.S.	1	1.00	1.59	1.37	1.16	2.66	0.00	0.00	5.55
time (sec)	N/A	0.652	6.861	1.639	0.223	0.286	0.000	0.000	33.537

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	264	266	6639	0	0	0	0	0	0
N.S.	1	1.01	25.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.375	47.880	0.000	0.000	0.000	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	35	40	36	35	0	0	35
N.S.	1	1.00	0.90	1.03	0.92	0.90	0.00	0.00	0.90
time (sec)	N/A	0.301	0.442	7.969	0.229	0.271	0.000	0.000	23.040

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	144	144	106	0	0	0	0	0	0
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.636	2.315	0.000	0.000	0.000	0.000	0.000	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	61	82	105	142	699	94	93
N.S.	1	1.00	0.94	1.26	1.62	2.18	10.75	1.45	1.43
time (sec)	N/A	0.386	0.234	0.374	0.313	0.265	67.472	0.313	22.410

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	153	98	211	94	0	152	3401
N.S.	1	1.00	1.66	1.07	2.29	1.02	0.00	1.65	36.97
time (sec)	N/A	0.557	0.634	0.430	0.310	0.253	0.000	0.288	30.048

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	123	113	163	278	210	0	190	286
N.S.	1	1.01	0.93	1.34	2.28	1.72	0.00	1.56	2.34
time (sec)	N/A	0.663	1.159	0.629	0.318	0.262	0.000	0.344	23.403

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	91	82	98	212	94	0	156	3419
N.S.	1	0.98	0.88	1.05	2.28	1.01	0.00	1.68	36.76
time (sec)	N/A	0.525	0.589	0.434	0.327	0.255	0.000	0.289	29.670

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	115	115	165	281	215	0	192	277
N.S.	1	1.03	1.03	1.47	2.51	1.92	0.00	1.71	2.47
time (sec)	N/A	0.708	0.879	0.592	0.304	0.264	0.000	0.319	23.259

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	164	178	163	431	174	0	275	5902
N.S.	1	0.93	1.01	0.93	2.45	0.99	0.00	1.56	33.53
time (sec)	N/A	1.055	0.802	0.767	0.338	0.265	0.000	0.294	34.756

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	112	170	281	213	0	201	291
N.S.	1	1.00	0.91	1.38	2.28	1.73	0.00	1.63	2.37
time (sec)	N/A	0.663	0.670	0.599	0.304	0.263	0.000	0.304	22.967

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	163	287	163	424	175	0	273	5870
N.S.	1	0.93	1.64	0.93	2.42	1.00	0.00	1.56	33.54
time (sec)	N/A	1.064	1.038	0.602	0.307	0.270	0.000	0.290	36.112

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	186	223	302	521	307	0	361	600
N.S.	1	0.96	1.16	1.56	2.70	1.59	0.00	1.87	3.11
time (sec)	N/A	1.258	1.637	0.987	0.324	0.273	0.000	0.323	23.582

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	117	144	84	118	138	0	144	1017
N.S.	1	1.67	2.06	1.20	1.69	1.97	0.00	2.06	14.53
time (sec)	N/A	0.696	0.294	0.592	0.328	0.257	0.000	0.286	27.764

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	229	111	142	265	252	0	209	249
N.S.	1	2.08	1.01	1.29	2.41	2.29	0.00	1.90	2.26
time (sec)	N/A	1.344	0.812	0.637	0.297	0.264	0.000	0.326	23.440

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	C	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	295	226	138	259	236	0	223	5431
N.S.	1	2.29	1.75	1.07	2.01	1.83	0.00	1.73	42.10
time (sec)	N/A	2.243	1.800	0.816	0.305	0.275	0.000	0.280	30.774

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	228	110	143	264	252	0	204	253
N.S.	1	2.09	1.01	1.31	2.42	2.31	0.00	1.87	2.32
time (sec)	N/A	1.311	0.808	0.642	0.319	0.266	0.000	0.314	24.447

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	336	145	143	257	244	0	219	6012
N.S.	1	2.56	1.11	1.09	1.96	1.86	0.00	1.67	45.89
time (sec)	N/A	2.429	1.968	0.842	0.327	0.274	0.000	0.281	36.239

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	B	B	F(-1)	B	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	0	200	271	611	360	0	342	594
N.S.	1	0.00	1.16	1.58	3.55	2.09	0.00	1.99	3.45
time (sec)	N/A	0.000	1.332	1.029	0.396	0.291	0.000	0.330	25.436

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	C	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	293	221	141	256	252	0	214	5428
N.S.	1	2.29	1.73	1.10	2.00	1.97	0.00	1.67	42.41
time (sec)	N/A	2.187	1.628	0.863	0.331	0.277	0.000	0.282	30.827

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	B	B	F(-1)	A	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	0	198	265	606	369	0	335	586
N.S.	1	0.00	1.12	1.51	3.44	2.10	0.00	1.90	3.33
time (sec)	N/A	0.000	1.488	1.021	0.326	0.294	0.000	0.314	24.906

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	C	A	B	A	F(-1)	B	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	0	409	232	456	371	0	435	8198
N.S.	1	0.00	1.95	1.10	2.17	1.77	0.00	2.07	39.04
time (sec)	N/A	0.000	3.438	1.134	0.308	0.308	0.000	0.289	39.734

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	76	63	98	140	0	90	408
N.S.	1	1.00	1.62	1.34	2.09	2.98	0.00	1.91	8.68
time (sec)	N/A	0.268	0.207	0.536	0.327	0.292	0.000	0.317	23.288

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	60	49	79	142	0	75	123
N.S.	1	1.00	1.25	1.02	1.65	2.96	0.00	1.56	2.56
time (sec)	N/A	0.273	0.136	0.523	0.303	0.289	0.000	0.309	23.207

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [286] had the largest ratio of [1.44443999999999995]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	3	1.00	14	0.214
2	A	3	3	1.00	14	0.214
3	A	3	3	1.00	12	0.250
4	A	1	1	1.00	9	0.111
5	A	3	3	1.00	12	0.250
6	A	3	3	1.00	14	0.214
7	A	3	3	1.00	14	0.214
8	A	8	8	1.01	16	0.500
9	A	7	6	1.00	16	0.375
10	A	4	4	1.00	14	0.286
11	A	4	3	1.00	11	0.273
12	A	6	6	1.00	14	0.429
13	A	7	6	1.00	16	0.375
14	A	11	10	0.96	16	0.625
15	B	3	3	2.64	16	0.188
16	A	9	9	1.17	16	0.562
17	A	6	5	1.00	14	0.357
18	A	2	2	1.00	11	0.182
19	A	7	6	1.00	14	0.429
20	A	5	4	1.00	16	0.250
21	A	15	14	1.32	16	0.875
22	A	12	12	1.23	16	0.750

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	B	3	3	3.26	16	0.188
24	A	4	3	1.27	14	0.214
25	A	6	5	1.00	11	0.455
26	A	5	4	1.00	14	0.286
27	A	17	16	1.16	16	1.000
28	A	5	4	1.00	16	0.250
29	A	2	2	1.00	33	0.061
30	A	3	3	1.00	26	0.115
31	A	3	3	1.00	26	0.115
32	A	3	3	1.00	26	0.115
33	A	3	3	1.00	26	0.115
34	A	3	3	1.00	24	0.125
35	A	1	1	1.00	17	0.059
36	A	3	3	1.00	24	0.125
37	A	3	3	1.00	26	0.115
38	A	3	3	1.00	26	0.115
39	A	3	3	1.00	26	0.115
40	A	3	3	1.00	26	0.115
41	A	3	3	1.00	26	0.115
42	A	3	3	1.00	26	0.115
43	A	3	3	1.00	28	0.107
44	A	3	3	1.00	28	0.107
45	A	3	3	1.00	28	0.107
46	A	3	3	1.00	28	0.107
47	A	3	3	1.00	26	0.115
48	A	3	3	1.00	19	0.158
49	A	3	3	1.00	26	0.115
50	A	6	6	1.00	28	0.214
51	A	3	3	1.00	28	0.107
52	A	4	3	1.00	28	0.107
53	A	3	3	1.00	28	0.107
54	A	5	4	0.91	28	0.143
55	A	3	3	1.00	28	0.107
56	A	5	4	0.90	28	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	3	3	1.00	28	0.107
58	A	3	3	1.00	28	0.107
59	A	3	3	1.00	28	0.107
60	A	3	3	1.00	28	0.107
61	A	7	6	1.22	26	0.231
62	A	4	3	0.97	19	0.158
63	A	7	6	1.14	26	0.231
64	A	3	3	1.00	28	0.107
65	A	8	8	1.00	28	0.286
66	A	3	3	1.00	28	0.107
67	A	4	3	1.00	28	0.107
68	A	3	3	1.00	28	0.107
69	A	5	4	0.90	28	0.143
70	A	3	3	1.00	28	0.107
71	A	5	4	0.90	28	0.143
72	A	3	3	1.00	28	0.107
73	A	5	4	0.89	28	0.143
74	A	3	3	1.00	28	0.107
75	A	3	3	1.00	28	0.107
76	A	3	3	1.00	28	0.107
77	A	3	3	1.00	28	0.107
78	A	3	3	1.00	26	0.115
79	A	5	5	1.03	19	0.263
80	A	3	3	1.00	26	0.115
81	A	7	6	1.08	28	0.214
82	A	3	3	1.00	28	0.107
83	A	10	10	1.00	28	0.357
84	A	3	3	1.00	28	0.107
85	A	4	3	1.00	28	0.107
86	A	3	3	1.00	28	0.107
87	A	5	4	0.90	28	0.143
88	A	3	3	1.00	28	0.107
89	A	5	4	0.90	28	0.143
90	A	3	3	1.00	28	0.107

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
91	A	5	4	0.90	28	0.143
92	A	3	3	1.00	28	0.107
93	A	3	3	1.00	28	0.107
94	A	3	3	1.00	28	0.107
95	A	3	3	1.00	28	0.107
96	A	8	7	1.16	26	0.269
97	A	5	4	0.95	19	0.211
98	A	9	8	1.11	26	0.308
99	A	3	3	1.00	28	0.107
100	A	7	6	1.02	28	0.214
101	A	3	3	1.00	28	0.107
102	A	12	12	1.00	28	0.429
103	A	3	3	1.00	28	0.107
104	A	4	3	1.00	28	0.107
105	A	3	3	1.00	28	0.107
106	A	5	4	0.90	28	0.143
107	A	3	3	1.00	28	0.107
108	A	5	4	0.90	28	0.143
109	A	3	3	1.00	28	0.107
110	A	14	14	0.97	28	0.500
111	A	11	10	1.00	28	0.357
112	A	8	8	1.01	28	0.286
113	A	7	6	1.00	28	0.214
114	A	4	4	1.00	26	0.154
115	A	4	3	1.00	19	0.158
116	A	5	5	1.00	26	0.192
117	A	7	6	1.00	28	0.214
118	A	10	9	0.98	28	0.321
119	A	11	10	0.98	28	0.357
120	A	13	12	0.92	28	0.429
121	A	15	14	0.94	28	0.500
122	A	7	6	1.19	28	0.214
123	A	5	4	1.61	28	0.143
124	A	8	8	1.13	28	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
125	A	6	5	1.00	26	0.192
126	A	2	2	1.00	19	0.105
127	A	7	6	1.00	26	0.231
128	A	5	4	0.93	28	0.143
129	A	15	14	1.27	28	0.500
130	A	5	4	0.91	28	0.143
131	B	5	4	2.21	28	0.143
132	A	10	10	1.18	28	0.357
133	A	9	8	1.98	28	0.286
134	A	4	3	1.36	26	0.115
135	A	6	5	1.00	19	0.263
136	A	5	4	0.94	26	0.154
137	A	17	16	1.14	28	0.571
138	A	5	4	0.91	28	0.143
139	A	27	26	1.82	28	0.929
140	A	5	4	0.91	28	0.143
141	A	12	12	1.20	28	0.429
142	B	11	10	2.49	28	0.357
143	A	4	3	1.00	28	0.107
144	A	9	8	1.07	26	0.308
145	A	4	4	1.00	19	0.211
146	A	13	12	1.02	26	0.462
147	A	5	4	0.93	28	0.143
148	A	31	30	1.96	28	1.071
149	A	5	4	0.93	28	0.143
150	A	5	5	1.03	31	0.161
151	A	5	5	1.06	31	0.161
152	A	5	5	1.04	31	0.161
153	A	5	5	1.08	31	0.161
154	A	3	3	1.00	29	0.103
155	A	2	2	1.00	22	0.091
156	A	5	5	1.17	29	0.172
157	A	5	5	1.10	31	0.161
158	A	5	5	1.12	31	0.161

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
159	A	5	5	1.05	31	0.161
160	A	5	5	1.08	31	0.161
161	A	5	5	1.04	31	0.161
162	A	5	5	1.06	31	0.161
163	A	5	5	1.08	31	0.161
164	A	7	6	0.87	31	0.194
165	A	5	5	1.07	31	0.161
166	A	5	5	1.04	31	0.161
167	A	5	5	1.12	29	0.172
168	A	2	2	1.00	22	0.091
169	A	5	5	1.09	29	0.172
170	A	7	6	0.75	31	0.194
171	A	5	5	1.09	31	0.161
172	A	6	5	1.00	31	0.161
173	A	5	5	1.06	31	0.161
174	A	7	6	0.79	31	0.194
175	A	7	6	0.85	31	0.194
176	A	5	5	1.11	31	0.161
177	A	7	7	1.07	31	0.226
178	A	5	5	1.11	31	0.161
179	A	5	4	1.00	29	0.138
180	A	2	2	1.00	22	0.091
181	A	7	6	0.80	29	0.207
182	A	5	5	1.10	31	0.161
183	A	7	6	0.76	31	0.194
184	A	5	5	1.09	31	0.161
185	A	6	5	1.00	31	0.161
186	A	5	5	1.09	31	0.161
187	A	2	2	1.00	33	0.061
188	A	6	5	1.00	7	0.714
189	A	7	6	1.00	10	0.600
190	A	5	5	1.00	10	0.500
191	A	6	6	1.00	10	0.600
192	A	7	7	1.00	10	0.700

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
193	A	4	4	1.00	10	0.400
194	A	8	7	1.00	10	0.700
195	A	6	5	1.00	9	0.556
196	A	8	7	1.00	12	0.583
197	A	5	5	1.00	12	0.417
198	A	6	6	1.00	12	0.500
199	A	7	7	1.00	12	0.583
200	A	4	4	1.00	12	0.333
201	A	8	7	1.15	12	0.583
202	A	8	7	1.00	20	0.350
203	A	5	5	1.00	10	0.500
204	A	8	7	1.00	10	0.700
205	A	7	7	1.00	10	0.700
206	A	6	6	1.00	10	0.600
207	A	9	8	1.00	10	0.800
208	A	4	4	1.00	10	0.400
209	A	5	5	1.00	12	0.417
210	A	9	8	1.00	12	0.667
211	A	7	7	1.00	12	0.583
212	A	6	6	1.00	12	0.500
213	A	9	8	1.15	12	0.667
214	A	4	4	1.00	12	0.333
215	A	6	5	1.00	15	0.333
216	A	5	4	0.63	22	0.182
217	A	4	3	1.00	22	0.136
218	A	6	5	0.93	22	0.227
219	A	4	3	1.00	22	0.136
220	A	5	4	1.00	22	0.182
221	A	4	3	0.46	22	0.136
222	A	6	5	1.00	17	0.294
223	A	5	4	1.43	24	0.167
224	A	4	3	1.67	24	0.125
225	A	5	4	1.18	24	0.167
226	A	4	3	1.00	24	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
227	A	4	3	1.40	24	0.125
228	A	4	3	1.00	24	0.125
229	A	7	6	1.00	26	0.231
230	A	9	8	1.00	26	0.308
231	A	8	7	1.32	24	0.292
232	A	1	1	1.00	17	0.059
233	A	8	7	1.00	24	0.292
234	A	8	7	1.00	26	0.269
235	A	7	6	1.00	26	0.231
236	A	12	12	1.10	28	0.429
237	A	9	9	0.98	28	0.321
238	A	16	16	1.03	26	0.615
239	A	5	5	1.00	19	0.263
240	A	15	15	1.01	26	0.577
241	A	16	16	1.00	28	0.571
242	A	19	18	1.09	28	0.643
243	A	7	6	0.87	28	0.214
244	A	9	8	1.01	28	0.286
245	A	9	8	0.99	26	0.308
246	A	8	7	0.89	19	0.368
247	A	9	8	0.93	26	0.308
248	A	9	8	1.00	28	0.286
249	A	9	8	1.00	28	0.286
250	A	10	9	0.97	28	0.321
251	A	12	11	1.05	28	0.393
252	A	8	7	1.00	26	0.269
253	A	11	10	1.03	19	0.526
254	A	7	6	1.07	26	0.231
255	A	10	9	1.07	28	0.321
256	A	8	7	1.06	28	0.250
257	A	6	6	1.00	28	0.214
258	A	5	5	1.00	28	0.179
259	A	6	6	1.00	26	0.231
260	A	5	5	1.00	19	0.263

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
261	A	13	12	1.18	26	0.462
262	A	12	11	1.20	28	0.393
263	A	6	6	1.00	28	0.214
264	A	10	9	1.09	28	0.321
265	A	11	10	1.10	28	0.357
266	A	10	9	1.12	26	0.346
267	A	9	8	1.12	19	0.421
268	A	10	9	1.08	26	0.346
269	A	12	11	1.08	28	0.393
270	A	7	6	0.93	28	0.214
271	A	8	7	1.00	28	0.250
272	A	15	15	1.01	28	0.536
273	A	7	6	1.00	26	0.231
274	A	7	6	1.00	28	0.214
275	A	8	7	1.00	16	0.438
276	A	11	10	1.00	18	0.556
277	A	13	12	1.01	18	0.667
278	A	11	10	0.98	18	0.556
279	A	14	13	1.03	20	0.650
280	A	19	18	0.93	20	0.900
281	A	13	12	1.00	18	0.667
282	A	19	18	0.93	20	0.900
283	A	22	21	0.96	20	1.050
284	A	9	9	1.67	16	0.562
285	B	19	18	2.08	18	1.000
286	B	27	26	2.29	18	1.444
287	B	19	18	2.09	18	1.000
288	B	22	21	2.56	20	1.050
289	F	0	0	N/A	0.000	N/A
290	B	26	25	2.29	18	1.389
291	F	0	0	N/A	0.000	N/A
292	F	0	0	N/A	0.000	N/A
293	A	3	3	1.00	14	0.214

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
294	A	3	3	1.00	14	0.214

2.3. Detailed conclusion table specific for Rubi results

CHAPTER 3

Listing of integrals

3.1 $\int \sin^3(x)(a\cos(x)+b\sin(x))\,dx$

3.1.1 Optimal result

Integrand size = 14, antiderivative size = 36

$$\int \sin^3(x)(a\cos(x)+b\sin(x))\,dx=\frac{3bx}{8}-\frac{3}{8}b\cos(x)\sin(x)-\frac{1}{4}b\cos(x)\sin^3(x)+\frac{1}{4}a\sin^4(x)$$

output
```
3/8*b*x-3/8*b*cos(x)*sin(x)-1/4*b*cos(x)*sin(x)^3+1/4*a*sin(x)^4
```

3.1.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \sin^3(x)(a\cos(x)+b\sin(x))\,dx=\frac{3bx}{8}+\frac{1}{4}a\sin^4(x)-\frac{1}{4}b\sin(2x)+\frac{1}{32}b\sin(4x)$$

input
```
Integrate[Sin[x]^3*(a*Cos[x] + b*Sin[x]),x]
```

output
```
(3*b*x)/8 + (a*Sin[x]^4)/4 - (b*Sin[2*x])/4 + (b*Sin[4*x])/32
```

3.1.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 3568, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(x)(a\cos(x)+b\sin(x))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(x)^3(a\cos(x)+b\sin(x))dx$$

$$\Big\downarrow \text{ 3568}$$

$$\int \left(a\sin^3(x)\cos(x)+b\sin^4(x)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}a\sin^4(x)+\frac{3bx}{8}-\frac{1}{4}b\sin^3(x)\cos(x)-\frac{3}{8}b\sin(x)\cos(x)$$

input
```
Int[Sin[x]^3*(a*Cos[x] + b*Sin[x]),x]
```

output
```
(3*b*x)/8 - (3*b*Cos[x]*Sin[x])/8 - (b*Cos[x]*Sin[x]^3)/4 + (a*Sin[x]^4)/4
```

3.1.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3568
```
Int[sin[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[sin[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

3.1.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

method	result
default	$b\left(-\frac{\left(\sin(x)^3+\frac{3\sin(x)}{2}\right)\cos(x)}{4}+\frac{3x}{8}\right)+\frac{a\sin(x)^4}{4}$
parts	$b\left(-\frac{\left(\sin(x)^3+\frac{3\sin(x)}{2}\right)\cos(x)}{4}+\frac{3x}{8}\right)+\frac{a\sin(x)^4}{4}$
risch	$\frac{3xb}{8}+\frac{a\cos(4x)}{32}+\frac{b\sin(4x)}{32}-\frac{a\cos(2x)}{8}-\frac{b\sin(2x)}{4}$
parallelrisch	$\frac{3xb}{8}+\frac{3a}{32}-\frac{a\cos(2x)}{8}+\frac{a\cos(4x)}{32}+\frac{b\sin(4x)}{32}-\frac{b\sin(2x)}{4}$
norman	$\frac{4a\tan(\frac{x}{2})^4-\frac{3b\tan(\frac{x}{2})}{4}-\frac{11b\tan(\frac{x}{2})^3}{4}+\frac{11b\tan(\frac{x}{2})^5}{4}+\frac{3b\tan(\frac{x}{2})^7}{4}+\frac{3xb}{8}+\frac{3xb\tan(\frac{x}{2})^2}{2}+\frac{9xb\tan(\frac{x}{2})^4}{4}+\frac{3xb\tan(\frac{x}{2})^6}{2}+\frac{3xb\tan(\frac{x}{2})^8}{8}}{\left(1+\tan(\frac{x}{2})^2\right)^4}$

input
```
int(sin(x)^3*(a*cos(x)+b*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
b*(-1/4*(sin(x)^3+3/2*sin(x))*cos(x)+3/8*x)+1/4*a*sin(x)^4
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \sin^3(x)(a\cos(x)+b\sin(x))\,dx = \frac{1}{4}\,a\cos{(x)}^4-\frac{1}{2}\,a\cos{(x)}^2+\frac{3}{8}\,bx+\frac{1}{8}\left(2\,b\cos{(x)}^3-5\,b\cos{(x)}\right)\sin{(x)}$$

input
```
integrate(sin(x)^3*(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

output
```
1/4*a*cos(x)^4 - 1/2*a*cos(x)^2 + 3/8*b*x + 1/8*(2*b*cos(x)^3 - 5*b*cos(x)
)*sin(x)
```

3.1.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(37) = 74.

Time = 0.15 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.08

$$\int \sin^3(x)(a\cos(x)+b\sin(x))\,dx = \frac{a\sin^4(x)}{4}+\frac{3bx\sin^4(x)}{8}+\frac{3bx\sin^2(x)\cos^2(x)}{4}+\frac{3bx\cos^4(x)}{8}-\frac{5b\sin^3(x)\cos(x)}{8}-\frac{3b\sin(x)\cos^3(x)}{8}$$

input
```
integrate(sin(x)**3*(a*cos(x)+b*sin(x)),x)
```

output
```
a*sin(x)**4/4 + 3*b*x*sin(x)**4/8 + 3*b*x*sin(x)**2*cos(x)**2/4 + 3*b*x*co
s(x)**4/8 - 5*b*sin(x)**3*cos(x)/8 - 3*b*sin(x)*cos(x)**3/8
```

3.1.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

$$\int \sin^3(x)(a\cos(x)+b\sin(x))\,dx = \frac{1}{4}\,a\sin(x)^4+\frac{1}{32}\,b(12\,x+\sin(4\,x)-8\,\sin(2\,x))$$

input
```
integrate(sin(x)^3*(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

output
```
1/4*a*sin(x)^4 + 1/32*b*(12*x + sin(4*x) - 8*sin(2*x))
```

3.1.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int \sin^3(x)(a\cos(x)+b\sin(x))\,dx = \frac{3}{8}\,bx+\frac{1}{32}\,a\cos(4\,x)-\frac{1}{8}\,a\cos(2\,x)+\frac{1}{32}\,b\sin(4\,x)-\frac{1}{4}\,b\sin(2\,x)$$

input
```
integrate(sin(x)^3*(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

output
```
3/8*b*x + 1/32*a*cos(4*x) - 1/8*a*cos(2*x) + 1/32*b*sin(4*x) - 1/4*b*sin(2
*x)
```

3.1.9 Mupad [B] (verification not implemented)

Time = 21.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int \sin^3(x)(a\cos(x)+b\sin(x))\,dx = \frac{a\cos(x)^4}{4}+\frac{b\,\sin(x)\,\cos(x)^3}{4} - \frac{a\cos(x)^2}{2}-\frac{5\,b\,\sin(x)\,\cos(x)}{8}+\frac{3\,b\,x}{8}$$

input
```
int(sin(x)^3*(a*cos(x) + b*sin(x)),x)
```

output
```
(3*b*x)/8 - (a*cos(x)^2)/2 + (a*cos(x)^4)/4 - (5*b*cos(x)*sin(x))/8 + (b*c
os(x)^3*sin(x))/4
```

3.2 $\int \sin^2(x)(a\cos(x)+b\sin(x))\,dx$

3.2.1 Optimal result

Integrand size = 14, antiderivative size = 24

$$\int \sin^2(x)(a\cos(x)+b\sin(x))\,dx=-b\cos(x)+\frac{1}{3}b\cos^3(x)+\frac{1}{3}a\sin^3(x)$$

output
```
-b*cos(x)+1/3*b*cos(x)^3+1/3*a*sin(x)^3
```

3.2.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \sin^2(x)(a\cos(x)+b\sin(x))\,dx=-\frac{3}{4}b\cos(x)+\frac{1}{12}b\cos(3x)+\frac{1}{3}a\sin^3(x)$$

input
```
Integrate[Sin[x]^2*(a*Cos[x] + b*Sin[x]),x]
```

output
```
(-3*b*Cos[x])/4 + (b*Cos[3*x])/12 + (a*Sin[x]^3)/3
```

3.2.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 3568, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(x)(a\cos(x)+b\sin(x))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x)^2(a\cos(x)+b\sin(x))dx$$

$$\Big\downarrow \text{3568}$$

$$\int \left(a\sin^2(x)\cos(x)+b\sin^3(x)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}a\sin^3(x)+\frac{1}{3}b\cos^3(x)-b\cos(x)$$

input
```
Int[Sin[x]^2*(a*Cos[x] + b*Sin[x]),x]
```

output
```
-(b*Cos[x]) + (b*Cos[x]^3)/3 + (a*Sin[x]^3)/3
```

3.2.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3568
```
Int[sin[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[sin[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

3.2.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{b\left(2+\sin(x)^2\right)\cos(x)}{3}+\frac{a\sin(x)^3}{3}$	20
parts	$-\frac{b\left(2+\sin(x)^2\right)\cos(x)}{3}+\frac{a\sin(x)^3}{3}$	20
risch	$-\frac{3b\cos(x)}{4}+\frac{a\sin(x)}{4}+\frac{b\cos(3x)}{12}-\frac{a\sin(3x)}{12}$	26
norman	$\frac{-4\tan\left(\frac{x}{2}\right)^2b+\frac{8\tan\left(\frac{x}{2}\right)^3a}{3}-\frac{4b}{3}}{\left(1+\tan\left(\frac{x}{2}\right)^2\right)^3}$	34
parallelrisch	$\frac{-4\tan\left(\frac{x}{2}\right)^2b+\frac{8\tan\left(\frac{x}{2}\right)^3a}{3}-\frac{4b}{3}}{\left(1+\tan\left(\frac{x}{2}\right)^2\right)^3}$	35

input
```
int(sin(x)^2*(a*cos(x)+b*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/3*b*(2+sin(x)^2)*cos(x)+1/3*a*sin(x)^3
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.12

$$\int \sin^2(x)(a\cos(x)+b\sin(x))\,dx=\frac{1}{3}\,b\cos\left(x\right)^3-b\cos\left(x\right)-\frac{1}{3}\left(a\cos\left(x\right)^2-a\right)\sin\left(x\right)$$

input
```
integrate(sin(x)^2*(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

output
```
1/3*b*cos(x)^3 - b*cos(x) - 1/3*(a*cos(x)^2 - a)*sin(x)
```

3.2.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.12

$$\int \sin^2(x)(a\cos(x)+b\sin(x))\,dx=\frac{a\sin^3{(x)}}{3}-b\sin^2{(x)}\cos{(x)}-\frac{2b\cos^3{(x)}}{3}$$

input
```
integrate(sin(x)**2*(a*cos(x)+b*sin(x)),x)
```

output
```
a*sin(x)**3/3 - b*sin(x)**2*cos(x) - 2*b*cos(x)**3/3
```

3.2.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \sin^2(x)(a\cos(x)+b\sin(x))\,dx=\frac{1}{3}\,a\sin{(x)}^3+\frac{1}{3}\,\left(\cos{(x)}^3-3\,\cos{(x)}\right)b$$

input
```
integrate(sin(x)^2*(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

output
```
1/3*a*sin(x)^3 + 1/3*(cos(x)^3 - 3*cos(x))*b
```

3.2.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \sin^2(x)(a\cos(x)+b\sin(x))\,dx=\frac{1}{12}\,b\cos{(3\,x)}-\frac{3}{4}\,b\cos{(x)}-\frac{1}{12}\,a\sin{(3\,x)}+\frac{1}{4}\,a\sin{(x)}$$

input
```
integrate(sin(x)^2*(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

output
```
1/12*b*cos(3*x) - 3/4*b*cos(x) - 1/12*a*sin(3*x) + 1/4*a*sin(x)
```

3.2.9 Mupad [B] (verification not implemented)

Time = 22.55 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

$$\int \sin^2(x)(a\cos(x)+b\sin(x))\,dx=-\frac{4\left(-2\,a\tan\left(\frac{x}{2}\right)^3+3\,b\tan\left(\frac{x}{2}\right)^2+b\right)}{3\left(\tan\left(\frac{x}{2}\right)^2+1\right)^3}$$

input
```
int(sin(x)^2*(a*cos(x) + b*sin(x)),x)
```

output
```
-(4*(b - 2*a*tan(x/2)^3 + 3*b*tan(x/2)^2))/(3*(tan(x/2)^2 + 1)^3)
```

3.3 $\int \sin(x)(a\cos(x)+b\sin(x))\,dx$

3.3.1 Optimal result

Integrand size = 12, antiderivative size = 25

$$\int \sin(x)(a\cos(x)+b\sin(x))\,dx=\frac{bx}{2}-\frac{1}{2}b\cos(x)\sin(x)+\frac{1}{2}a\sin^2(x)$$

output
```
1/2*b*x-1/2*b*cos(x)*sin(x)+1/2*a*sin(x)^2
```

3.3.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \sin(x)(a\cos(x)+b\sin(x))\,dx=\frac{bx}{2}-\frac{1}{2}a\cos^2(x)-\frac{1}{4}b\sin(2x)$$

input
```
Integrate[Sin[x]*(a*Cos[x] + b*Sin[x]),x]
```

output
```
(b*x)/2 - (a*Cos[x]^2)/2 - (b*Sin[2*x])/4
```

3.3.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3568, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)(a\cos(x)+b\sin(x))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x)(a\cos(x)+b\sin(x))dx$$

$$\Big\downarrow \text{3568}$$

$$\int \left(a\sin(x)\cos(x)+b\sin^2(x)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}a\sin^2(x)+\frac{bx}{2}-\frac{1}{2}b\sin(x)\cos(x)$$

input
```
Int[Sin[x]*(a*Cos[x] + b*Sin[x]),x]
```

output
```
(b*x)/2 - (b*Cos[x]*Sin[x])/2 + (a*Sin[x]^2)/2
```

3.3.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3568
```
Int[sin[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[sin[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

3.3.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
risch	$\frac{xb}{2}-\frac{a\cos(2x)}{4}-\frac{b\sin(2x)}{4}$	20
default	$b\left(-\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)-\frac{\cos(x)^2 a}{2}$	21
parts	$b\left(-\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)+\frac{a\sin(x)^2}{2}$	21
parallelrisch	$\frac{xb}{2}+\frac{a}{4}-\frac{b\sin(2x)}{4}-\frac{a\cos(2x)}{4}$	23
meijerg	$\frac{a\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(2x)}{\sqrt{\pi}}\right)}{4}+\frac{b\sqrt{\pi}\left(\frac{2x}{\sqrt{\pi}}-\frac{\sin(2x)}{\sqrt{\pi}}\right)}{4}$	43
norman	$\frac{b\tan\left(\frac{x}{2}\right)^3+2\tan\left(\frac{x}{2}\right)^2 a+xb\tan\left(\frac{x}{2}\right)^2-b\tan\left(\frac{x}{2}\right)+\frac{xb}{2}+\frac{xb\tan\left(\frac{x}{2}\right)^4}{2}}{\left(1+\tan\left(\frac{x}{2}\right)^2\right)^2}$	60

input
```
int(sin(x)*(a*cos(x)+b*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*b-1/4*a*cos(2*x)-1/4*b*sin(2*x)
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \sin(x)(a\cos(x)+b\sin(x))\,dx=-\frac{1}{2}\,a\cos\left(x\right)^2-\frac{1}{2}\,b\cos\left(x\right)\sin\left(x\right)+\frac{1}{2}\,bx$$

input
```
integrate(sin(x)*(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

output
```
-1/2*a*cos(x)^2 - 1/2*b*cos(x)*sin(x) + 1/2*b*x
```

3.3.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.48

$$\int \sin(x)(a\cos(x)+b\sin(x))\,dx = \frac{a\sin^2(x)}{2}+\frac{bx\sin^2(x)}{2}+\frac{bx\cos^2(x)}{2}-\frac{b\sin(x)\cos(x)}{2}$$

input
```
integrate(sin(x)*(a*cos(x)+b*sin(x)),x)
```

output
```
a*sin(x)**2/2 + b*x*sin(x)**2/2 + b*x*cos(x)**2/2 - b*sin(x)*cos(x)/2
```

3.3.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \sin(x)(a\cos(x)+b\sin(x))\,dx = -\frac{1}{2}\,a\cos(x)^2+\frac{1}{4}\,b(2\,x-\sin(2\,x))$$

input
```
integrate(sin(x)*(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

output
```
-1/2*a*cos(x)^2 + 1/4*b*(2*x - sin(2*x))
```

3.3.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \sin(x)(a\cos(x)+b\sin(x))\,dx = \frac{1}{2}\,bx-\frac{1}{4}\,a\cos(2\,x)-\frac{1}{4}\,b\sin(2\,x)$$

input
```
integrate(sin(x)*(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

output
```
1/2*b*x - 1/4*a*cos(2*x) - 1/4*b*sin(2*x)
```

3.3.9 Mupad [B] (verification not implemented)

Time = 21.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \sin(x)(a\cos(x)+b\sin(x))\,dx=\frac{a\sin\left(x\right)^2}{2}-\frac{b\,\cos\left(x\right)\,\sin\left(x\right)}{2}+\frac{b\,x}{2}$$

input
```
int(sin(x)*(a*cos(x) + b*sin(x)),x)
```

output
```
(a*sin(x)^2)/2 + (b*x)/2 - (b*cos(x)*sin(x))/2
```

3.4 $\int (a\cos(x)+b\sin(x))\,dx$

3.4.1 Optimal result

Integrand size = 9, antiderivative size = 10

$$\int (a\cos(x)+b\sin(x))\,dx=-b\cos(x)+a\sin(x)$$

output
```
-b*cos(x)+a*sin(x)
```

3.4.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int (a\cos(x)+b\sin(x))\,dx=-b\cos(x)+a\sin(x)$$

input
```
Integrate[a*Cos[x] + b*Sin[x],x]
```

output
```
-(b*Cos[x]) + a*Sin[x]
```

3.4.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(x)+b\sin(x))\,dx$$

$$\Big\downarrow \text{2009}$$

$$a\sin(x)-b\cos(x)$$

input
```
Int[a*Cos[x] + b*Sin[x],x]
```

output
```
-(b*Cos[x]) + a*Sin[x]
```

3.4.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.4.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
default	$-b\cos(x)+a\sin(x)$	11
risch	$-b\cos(x)+a\sin(x)$	11
parts	$-b\cos(x)+a\sin(x)$	11
parallelrisch	$a\sin(x)-b\cos(x)-b$	14
meijerg	$a\sin(x)+b\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(x)}{\sqrt{\pi}}\right)$	22
norman	$\frac{2a\tan\left(\frac{x}{2}\right)-2b}{1+\tan\left(\frac{x}{2}\right)^{2}}$	23

input
```
int(a*cos(x)+b*sin(x),x,method=_RETURNVERBOSE)
```

output
```
-b*cos(x)+a*sin(x)
```

3.4.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int (a\cos(x) + b\sin(x))\, dx = -b\cos(x) + a\sin(x)$$

input
```
integrate(a*cos(x)+b*sin(x),x, algorithm="fricas")
```

output
```
-b*cos(x) + a*sin(x)
```

3.4.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int (a\cos(x) + b\sin(x))\, dx = a\sin(x) - b\cos(x)$$

input
```
integrate(a*cos(x)+b*sin(x),x)
```

output
```
a*sin(x) - b*cos(x)
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int (a\cos(x) + b\sin(x))\, dx = -b\cos(x) + a\sin(x)$$

input
```
integrate(a*cos(x)+b*sin(x),x, algorithm="maxima")
```

output
```
-b*cos(x) + a*sin(x)
```

3.4.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int (a\cos(x) + b\sin(x))\,dx = -b\cos(x) + a\sin(x)$$

input
```
integrate(a*cos(x)+b*sin(x),x, algorithm="giac")
```

output
```
-b*cos(x) + a*sin(x)
```

3.4.9 Mupad [B] (verification not implemented)

Time = 20.98 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int (a\cos(x) + b\sin(x))\,dx = a\,\sin(x) - b\,\cos(x)$$

input
```
int(a*cos(x) + b*sin(x),x)
```

output
```
a*sin(x) - b*cos(x)
```

3.5 $\int \csc(x)(a\cos(x)+b\sin(x))\,dx$

3.5.1 Optimal result

Integrand size = 12, antiderivative size = 9

$$\int \csc(x)(a\cos(x)+b\sin(x))\,dx = bx + a\log(\sin(x))$$

output
```
b*x+a*ln(sin(x))
```

3.5.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.56

$$\int \csc(x)(a\cos(x)+b\sin(x))\,dx = bx + a\log(\cos(x)) + a\log(\tan(x))$$

input
```
Integrate[Csc[x]*(a*Cos[x] + b*Sin[x]),x]
```

output
```
b*x + a*Log[Cos[x]] + a*Log[Tan[x]]
```

3.5.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3564, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(x)(a\cos(x)+b\sin(x))\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{a\cos(x)+b\sin(x)}{\sin(x)}dx$$
$$\Big\downarrow \text{ 3564}$$
$$\int (a\cot(x)+b)dx$$
$$\Big\downarrow \text{ 2009}$$
$$a\log(\sin(x))+bx$$

input
```
Int[Csc[x]*(a*Cos[x] + b*Sin[x]),x]
```

output
```
b*x + a*Log[Sin[x]]
```

3.5.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3564
```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(b + a*Cot[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

3.5.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

method	result	size
default	$xb + a\ln(\sin(x))$	10
parts	$-a\ln(\csc(x)) + xb$	11
risch	$xb - iax + a\ln(\mathrm{e}^{2ix} - 1)$	20
parallelrisch	$xb + a\ln(\csc(x) - \cot(x)) - a\ln\left(\frac{2}{\cos(x)+1}\right)$	27
norman	$\frac{xb + xb\tan(\frac{x}{2})^2}{1+\tan(\frac{x}{2})^2} + a\ln\left(\tan\left(\frac{x}{2}\right)\right) - a\ln\left(1 + \tan\left(\frac{x}{2}\right)^2\right)$	45

input
```
int(csc(x)*(a*cos(x)+b*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
x*b+a*ln(sin(x))
```

3.5.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int \csc(x)(a\cos(x) + b\sin(x))\,dx = bx + a\log\left(\frac{1}{2}\sin(x)\right)$$

input
```
integrate(csc(x)*(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

output
```
b*x + a*log(1/2*sin(x))
```

3.5.6 Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int \csc(x)(a\cos(x) + b\sin(x))\,dx = a\log(\sin(x)) + bx$$

input
```
integrate(csc(x)*(a*cos(x)+b*sin(x)),x)
```

output
```
a*log(sin(x)) + b*x
```

3.5.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \csc(x)(a\cos(x) + b\sin(x))\,dx = bx + a\log(\sin(x))$$

input
```
integrate(csc(x)*(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

output
```
b*x + a*log(sin(x))
```

3.5.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. 2(9) = 18.

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.67

$$\int \csc(x)(a\cos(x) + b\sin(x))\,dx = bx - a\log\left(\tan\left(\frac{1}{2}x\right)^2 + 1\right) + a\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)$$

input
```
integrate(csc(x)*(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

output
```
b*x - a*log(tan(1/2*x)^2 + 1) + a*log(abs(tan(1/2*x)))
```

3.5.9 Mupad [B] (verification not implemented)

Time = 21.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 6.00

$$\int \csc(x)(a\cos(x) + b\sin(x))\,dx = a\ln\left(\tan\left(\frac{x}{2}\right)\right) - a\ln\left(\tan\left(\frac{x}{2}\right) - \mathrm{i}\right) - a\ln\left(\tan\left(\frac{x}{2}\right) + 1\mathrm{i}\right) - b\ln\left(\tan\left(\frac{x}{2}\right) - \mathrm{i}\right)1\mathrm{i} + b\ln\left(\tan\left(\frac{x}{2}\right) + 1\mathrm{i}\right)1\mathrm{i}$$

input
```
int((a*cos(x) + b*sin(x))/sin(x),x)
```

output
```
a*log(tan(x/2)) - a*log(tan(x/2) - 1i) - a*log(tan(x/2) + 1i) - b*log(tan(
x/2) - 1i)*1i + b*log(tan(x/2) + 1i)*1i
```

3.6 $\int \csc^2(x)(a\cos(x)+b\sin(x))\,dx$

3.6.1 Optimal result

Integrand size = 14, antiderivative size = 12

$$\int \csc^2(x)(a\cos(x)+b\sin(x))\,dx=-b\text{arctanh}(\cos(x))-a\csc(x)$$

output
```
-b*arctanh(cos(x))-a*csc(x)
```

3.6.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 25 vs. 2(12) = 24.

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.08

$$\int \csc^2(x)(a\cos(x)+b\sin(x))\,dx=-a\csc(x)-b\log\left(\cos\left(\frac{x}{2}\right)\right)+b\log\left(\sin\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[Csc[x]^2*(a*Cos[x] + b*Sin[x]),x]
```

output
```
-(a*Csc[x]) - b*Log[Cos[x/2]] + b*Log[Sin[x/2]]
```

3.6.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 3568, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(x)(a\cos(x)+b\sin(x))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\cos(x)+b\sin(x)}{\sin(x)^2}dx$$

$$\Big\downarrow \text{3568}$$

$$\int (a\cot(x)\csc(x)+b\csc(x))dx$$

$$\Big\downarrow \text{2009}$$

$$-a\csc(x)-b\text{arctanh}(\cos(x))$$

input
```
Int[Csc[x]^2*(a*Cos[x] + b*Sin[x]),x]
```

output
```
-(b*ArcTanh[Cos[x]]) - a*Csc[x]
```

3.6.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3568
```
Int[sin[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[sin[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

3.6.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

method	result	size
parallelrisch	$b\ln\left(\csc\left(x\right)-\cot\left(x\right)\right)-a\csc\left(x\right)$	17
parts	$b\ln\left(\csc\left(x\right)-\cot\left(x\right)\right)-a\csc\left(x\right)$	17
default	$-\frac{a}{\sin\left(x\right)}+b\ln\left(\csc\left(x\right)-\cot\left(x\right)\right)$	19
risch	$-\frac{2ia\,{\mathrm e}^{ix}}{{\mathrm e}^{2ix}-1}-b\ln\left({\mathrm e}^{ix}+1\right)+b\ln\left({\mathrm e}^{ix}-1\right)$	41
norman	$\frac{-\frac{a}{2}-\frac{a\tan\left(\frac{x}{2}\right)^{4}}{2}-\tan\left(\frac{x}{2}\right)^{2}a}{\tan\left(\frac{x}{2}\right)\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)}+b\ln\left(\tan\left(\frac{x}{2}\right)\right)$	48

input
```
int(csc(x)^2*(a*cos(x)+b*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
b*ln(csc(x)-cot(x))-a*csc(x)
```

3.6.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(12) = 24.

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.75

$$\int \csc^2(x)(a\cos(x)+b\sin(x))\,dx$$
$$=-\frac{b\log\left(\frac{1}{2}\,\cos\left(x\right)+\frac{1}{2}\right)\sin\left(x\right)-b\log\left(-\frac{1}{2}\,\cos\left(x\right)+\frac{1}{2}\right)\sin\left(x\right)+2\,a}{2\,\sin\left(x\right)}$$

input
```
integrate(csc(x)^2*(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

output
```
-1/2*(b*log(1/2*cos(x) + 1/2)*sin(x) - b*log(-1/2*cos(x) + 1/2)*sin(x) + 2
*a)/sin(x)
```

3.6.6 Sympy [A] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.00

$$\int \csc^2(x)(a\cos(x)+b\sin(x))\,dx=-\frac{a}{\sin{(x)}}+\frac{b\log{(\cos{(x)}-1)}}{2}-\frac{b\log{(\cos{(x)}+1)}}{2}$$

input
```
integrate(csc(x)**2*(a*cos(x)+b*sin(x)),x)
```

output
```
-a/sin(x) + b*log(cos(x) - 1)/2 - b*log(cos(x) + 1)/2
```

3.6.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.00

$$\int \csc^2(x)(a\cos(x)+b\sin(x))\,dx=-\frac{1}{2}\,b(\log{(\cos{(x)}+1)}-\log{(\cos{(x)}-1)})-\frac{a}{\sin{(x)}}$$

input
```
integrate(csc(x)^2*(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

output
```
-1/2*b*(log(cos(x) + 1) - log(cos(x) - 1)) - a/sin(x)
```

3.6.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(12) = 24.

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.75

$$\int \csc^2(x)(a\cos(x)+b\sin(x))\,dx=b\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)-\frac{1}{2}\,a\tan\left(\frac{1}{2}\,x\right)-\frac{2\,b\tan\left(\frac{1}{2}\,x\right)+a}{2\,\tan\left(\frac{1}{2}\,x\right)}$$

input
```
integrate(csc(x)^2*(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

output
```
b*log(abs(tan(1/2*x))) - 1/2*a*tan(1/2*x) - 1/2*(2*b*tan(1/2*x) + a)/tan(1
/2*x)
```

3.6.9 Mupad [B] (verification not implemented)

Time = 20.68 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.00

$$\int \csc^2(x)(a\cos(x)+b\sin(x))\,dx = b\,\ln\left(\tan\left(\frac{x}{2}\right)\right)-\frac{a}{2\tan\left(\frac{x}{2}\right)}-\frac{a\tan\left(\frac{x}{2}\right)}{2}$$

input
```
int((a*cos(x) + b*sin(x))/sin(x)^2,x)
```

output
```
b*log(tan(x/2)) - a/(2*tan(x/2)) - (a*tan(x/2))/2
```

3.7 $\int \csc^3(x)(a\cos(x)+b\sin(x))\,dx$

3.7.1 Optimal result

Integrand size = 14, antiderivative size = 15

$$\int \csc^3(x)(a\cos(x)+b\sin(x))\,dx=-b\cot(x)-\frac{1}{2}a\csc^2(x)$$

output
```
-b*cot(x)-1/2*a*csc(x)^2
```

3.7.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \csc^3(x)(a\cos(x)+b\sin(x))\,dx=-b\cot(x)-\frac{1}{2}a\csc^2(x)$$

input
```
Integrate[Csc[x]^3*(a*Cos[x] + b*Sin[x]),x]
```

output
```
-(b*Cot[x]) - (a*Csc[x]^2)/2
```

3.7.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 3568, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(x)(a\cos(x)+b\sin(x))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\cos(x)+b\sin(x)}{\sin(x)^3}dx$$

$$\Big\downarrow \text{3568}$$

$$\int \left(a\cot(x)\csc^2(x)+b\csc^2(x)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2}a\csc^2(x)-b\cot(x)$$

input
```
Int[Csc[x]^3*(a*Cos[x] + b*Sin[x]),x]
```

output
```
-(b*Cot[x]) - (a*Csc[x]^2)/2
```

3.7.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3568
```
Int[sin[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[sin[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

3.7.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{a}{2\sin(x)^2}-b\cot(x)$	14
parts	$-b\cot(x)-\frac{a\csc(x)^2}{2}$	14
risch	$-\frac{2i(ia\,e^{2ix}+b\,e^{2ix}-b)}{(e^{2ix}-1)^2}$	33
parallelrisch	$\frac{-a\tan(\frac{x}{2})^4+4b\tan(\frac{x}{2})^3-4b\tan(\frac{x}{2})-a}{8\tan(\frac{x}{2})^2}$	38
norman	$\frac{-\frac{a}{8}-\frac{a\tan(\frac{x}{2})^6}{8}-\frac{b\tan(\frac{x}{2})}{2}+\frac{b\tan(\frac{x}{2})^5}{2}}{\tan(\frac{x}{2})^2\left(1+\tan(\frac{x}{2})^2\right)}$	47

input
```
int(csc(x)^3*(a*cos(x)+b*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/sin(x)^2-b*cot(x)
```

3.7.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \csc^3(x)(a\cos(x)+b\sin(x))\,dx=\frac{2\,b\cos(x)\sin(x)+a}{2\left(\cos(x)^2-1\right)}$$

input
```
integrate(csc(x)^3*(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

output
```
1/2*(2*b*cos(x)*sin(x) + a)/(cos(x)^2 - 1)
```

3.7.6 Sympy [A] (verification not implemented)

Time = 1.84 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \csc^3(x)(a\cos(x)+b\sin(x))\,dx = -\frac{a}{2\sin^2{(x)}} - \frac{b\cos{(x)}}{\sin{(x)}}$$

input
```
integrate(csc(x)**3*(a*cos(x)+b*sin(x)),x)
```

output
```
-a/(2*sin(x)**2) - b*cos(x)/sin(x)
```

3.7.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \csc^3(x)(a\cos(x)+b\sin(x))\,dx = -\frac{b}{\tan{(x)}} - \frac{a}{2\,\sin{(x)}^2}$$

input
```
integrate(csc(x)^3*(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

output
```
-b/tan(x) - 1/2*a/sin(x)^2
```

3.7.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \csc^3(x)(a\cos(x)+b\sin(x))\,dx = -\frac{2\,b\tan{(x)}+a}{2\,\tan{(x)}^2}$$

input
```
integrate(csc(x)^3*(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

output
```
-1/2*(2*b*tan(x) + a)/tan(x)^2
```

3.7.9 Mupad [B] (verification not implemented)

Time = 20.82 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \csc^3(x)(a\cos(x)+b\sin(x))\,dx=-\frac{a+b\,\sin\left(2\,x\right)}{2\sin\left(x\right)^2}$$

input
```
int((a*cos(x) + b*sin(x))/sin(x)^3,x)
```

output
```
-(a + b*sin(2*x))/(2*sin(x)^2)
```

3.8 $\int \frac{\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx$

3.8.1 Optimal result

Integrand size = 16, antiderivative size = 91

$$\int \frac{\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{a^2bx}{(a^2+b^2)^2}+\frac{bx}{2(a^2+b^2)}-\frac{a^3\log(a\cos(x)+b\sin(x))}{(a^2+b^2)^2} - \frac{b\cos(x)\sin(x)}{2(a^2+b^2)}-\frac{a\sin^2(x)}{2(a^2+b^2)}$$

output
```
a^2*b*x/(a^2+b^2)^2+1/2*b*x/(a^2+b^2)-a^3*ln(a*cos(x)+b*sin(x))/(a^2+b^2)^
2-1/2*b*cos(x)*sin(x)/(a^2+b^2)-1/2*a*sin(x)^2/(a^2+b^2)
```

3.8.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.03

$$\int \frac{\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{-4ia^3x+6a^2bx+2b^3x+4ia^3\arctan(\tan(x))+a(a^2+b^2)\cos(2x)-2a^3\log\left((a\cos(x)+b\sin(x))^2\right)-a^2}{4(a^2+b^2)^2}$$

input
```
Integrate[Sin[x]^3/(a*Cos[x] + b*Sin[x]),x]
```

output
```
((-4*I)*a^3*x + 6*a^2*b*x + 2*b^3*x + (4*I)*a^3*ArcTan[Tan[x]] + a*(a^2 +
b^2)*Cos[2*x] - 2*a^3*Log[(a*Cos[x] + b*Sin[x])^2] - a^2*b*Sin[2*x] - b^3*
Sin[2*x])/(4*(a^2 + b^2)^2)
```

3.8.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3578, 3042, 3115, 24, 3576, 3042, 3612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)^3}{a\cos(x)+b\sin(x)}dx$$

$$\Big\downarrow \text{3578}$$

$$\frac{b\int \sin^2(x)dx}{a^2+b^2}+\frac{a^2\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}-\frac{a\sin^2(x)}{2\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\int \sin(x)^2dx}{a^2+b^2}+\frac{a^2\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}-\frac{a\sin^2(x)}{2\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{3115}$$

$$\frac{a^2\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\left(\frac{\int 1dx}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}-\frac{a\sin^2(x)}{2\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{24}$$

$$\frac{a^2\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}-\frac{a\sin^2(x)}{2\left(a^2+b^2\right)}+\frac{b\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}$$

$$\Big\downarrow \text{3576}$$

$$\frac{a^2\left(\frac{bx}{a^2+b^2}-\frac{a\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}-\frac{a\sin^2(x)}{2\left(a^2+b^2\right)}+\frac{b\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{a^2\left(\frac{bx}{a^2+b^2}-\frac{a\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}-\frac{a\sin^2(x)}{2(a^2+b^2)}+\frac{b\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}$$

$$\Big\downarrow \; 3612$$

$$-\frac{a\sin^2(x)}{2(a^2+b^2)}+\frac{b\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}+\frac{a^2\left(\frac{bx}{a^2+b^2}-\frac{a\log(a\cos(x)+b\sin(x))}{a^2+b^2}\right)}{a^2+b^2}$$

input
```
Int[Sin[x]^3/(a*Cos[x] + b*Sin[x]),x]
```

output
```
(a^2*((b*x)/(a^2 + b^2) - (a*Log[a*Cos[x] + b*Sin[x]])/(a^2 + b^2)))/(a^2
+ b^2) - (a*Sin[x]^2)/(2*(a^2 + b^2)) + (b*(x/2 - (Cos[x]*Sin[x])/2))/(a^2
 + b^2)
```

3.8.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3576
```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(x/(a^2 + b^2)), x] - Simp[a/(a^2 + b
^2)   Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x
]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3578
```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-a)*(Sin[c + d*x]^(m - 1)/(d*(a^2
 + b^2)*(m - 1))), x] + (Simp[a^2/(a^2 + b^2)   Int[Sin[c + d*x]^(m - 2)/(a
*Cos[c + d*x] + b*Sin[c + d*x]), x], x] + Simp[b/(a^2 + b^2)   Int[Sin[c +
d*x]^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ
[m, 1]
```

rule 3612
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

3.8.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.07

method	result
default	$-\frac{a^{3}\ln\left(a+b\tan\left(x\right)\right)}{\left(a^{2}+b^{2}\right)^{2}}+\frac{\frac{\left(-\frac{1}{2}a^{2}b-\frac{1}{2}b^{3}\right)\tan\left(x\right)+\frac{a^{3}}{2}+\frac{a\,b^{2}}{2}}{1+\tan\left(x\right)^{2}}+\frac{a^{3}\ln\left(1+\tan\left(x\right)^{2}\right)}{2}+\frac{\left(3a^{2}b+b^{3}\right)\arctan\left(\tan\left(x\right)\right)}{2}}{\left(a^{2}+b^{2}\right)^{2}}$
parallelrisch	$\frac{-a^{2}b\sin\left(2x\right)-b^{3}\sin\left(2x\right)+a^{3}\cos\left(2x\right)+a\,b^{2}\cos\left(2x\right)-4a^{3}\ln\left(\frac{-a\cos\left(x\right)-b\sin\left(x\right)}{\cos\left(x\right)+1}\right)+4a^{3}\ln\left(\frac{1}{\cos\left(x\right)+1}\right)+6x\,a^{2}b+2x\,b^{3}-a^{3}-a\,b^{2}}{4\left(a^{2}+b^{2}\right)^{2}}$
risch	$\frac{xb}{4iba-2a^{2}+2b^{2}}+\frac{ixa}{2iba-a^{2}+b^{2}}+\frac{{\mathrm e}^{2ix}}{-8ib+8a}+\frac{{\mathrm e}^{-2ix}}{8ib+8a}+\frac{2ia^{3}x}{a^{4}+2a^{2}b^{2}+b^{4}}-\frac{a^{3}\ln\left({\mathrm e}^{2ix}-\frac{ib+a}{ib-a}\right)}{a^{4}+2a^{2}b^{2}+b^{4}}$
norman	$\frac{\frac{b\tan\left(\frac{x}{2}\right)^{5}}{a^{2}+b^{2}}-\frac{2a\tan\left(\frac{x}{2}\right)^{2}}{a^{2}+b^{2}}-\frac{2a\tan\left(\frac{x}{2}\right)^{4}}{a^{2}+b^{2}}-\frac{b\tan\left(\frac{x}{2}\right)}{a^{2}+b^{2}}+\frac{b\left(3a^{2}+b^{2}\right)x}{2a^{4}+4a^{2}b^{2}+2b^{4}}+\frac{3b\left(3a^{2}+b^{2}\right)x\tan\left(\frac{x}{2}\right)^{2}}{2\left(a^{4}+2a^{2}b^{2}+b^{4}\right)}+\frac{3b\left(3a^{2}+b^{2}\right)x\tan\left(\frac{x}{2}\right)^{4}}{2\left(a^{4}+2a^{2}b^{2}+b^{4}\right)}+\frac{b\left(3a^{2}+b^{2}\right)x}{2a^{4}+4a^{2}b^{2}}}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)^{3}}$

input
```
int(sin(x)^3/(a*cos(x)+b*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
-a^3/(a^2+b^2)^2*ln(a+b*tan(x))+1/(a^2+b^2)^2*(((-1/2*a^2*b-1/2*b^3)*tan(x
)+1/2*a^3+1/2*a*b^2)/(1+tan(x)^2)+1/2*a^3*ln(1+tan(x)^2)+1/2*(3*a^2*b+b^3)
*arctan(tan(x)))
```

3.8.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.02

$$\int \frac{\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx = -\frac{a^3\log\left(2\,ab\cos\left(x\right)\sin\left(x\right)+\left(a^2-b^2\right)\cos\left(x\right)^2+b^2\right)-\left(a^3+ab^2\right)\cos\left(x\right)^2+\left(a^2b+b^3\right)\cos\left(x\right)\sin\left(x\right)-\left(\right.}{2\left(a^4+2\,a^2b^2+b^4\right)}$$

input
```
integrate(sin(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

output
```
-1/2*(a^3*log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) - (a^3 + a
*b^2)*cos(x)^2 + (a^2*b + b^3)*cos(x)*sin(x) - (3*a^2*b + b^3)*x)/(a^4 + 2
*a^2*b^2 + b^4)
```

3.8.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx = \text{Timed out}$$

input
```
integrate(sin(x)**3/(a*cos(x)+b*sin(x)),x)
```

output
```
Timed out
```

3.8.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. 2(85) = 170.

Time = 0.29 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.30

$$\begin{aligned}\int \frac{\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx =& -\frac{a^3\log\left(-a-\frac{2\,b\sin(x)}{\cos(x)+1}+\frac{a\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^4+2\,a^2b^2+b^4} \\ &+\frac{a^3\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{\left(3\,a^2b+b^3\right)\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^4+2\,a^2b^2+b^4} \\ &-\frac{\frac{b\sin(x)}{\cos(x)+1}+\frac{2\,a\sin(x)^2}{(\cos(x)+1)^2}-\frac{b\sin(x)^3}{(\cos(x)+1)^3}}{a^2+b^2+\frac{2\,(a^2+b^2)\sin(x)^2}{(\cos(x)+1)^2}+\frac{(a^2+b^2)\sin(x)^4}{(\cos(x)+1)^4}}\end{aligned}$$

input
```
integrate(sin(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

output
```
-a^3*log(-a - 2*b*sin(x)/(cos(x) + 1) + a*sin(x)^2/(cos(x) + 1)^2)/(a^4 +
2*a^2*b^2 + b^4) + a^3*log(sin(x)^2/(cos(x) + 1)^2 + 1)/(a^4 + 2*a^2*b^2 +
 b^4) + (3*a^2*b + b^3)*arctan(sin(x)/(cos(x) + 1))/(a^4 + 2*a^2*b^2 + b^4
) - (b*sin(x)/(cos(x) + 1) + 2*a*sin(x)^2/(cos(x) + 1)^2 - b*sin(x)^3/(cos
(x) + 1)^3)/(a^2 + b^2 + 2*(a^2 + b^2)*sin(x)^2/(cos(x) + 1)^2 + (a^2 + b^
2)*sin(x)^4/(cos(x) + 1)^4)
```

3.8.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.63

$$\int \frac{\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx = -\frac{a^3b\log\left(|b\tan\left(x\right)+a|\right)}{a^4b+2\,a^2b^3+b^5} + \frac{a^3\log\left(\tan\left(x\right)^2+1\right)}{2\,(a^4+2\,a^2b^2+b^4)} + \frac{(3\,a^2b+b^3)x}{2\,(a^4+2\,a^2b^2+b^4)} - \frac{a^3\tan\left(x\right)^2+a^2b\tan\left(x\right)+b^3\tan\left(x\right)-ab^2}{2\,(a^4+2\,a^2b^2+b^4)\left(\tan\left(x\right)^2+1\right)}$$

input
```
integrate(sin(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

output
```
-a^3*b*log(abs(b*tan(x) + a))/(a^4*b + 2*a^2*b^3 + b^5) + 1/2*a^3*log(tan(
x)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + 1/2*(3*a^2*b + b^3)*x/(a^4 + 2*a^2*b^2
 + b^4) - 1/2*(a^3*tan(x)^2 + a^2*b*tan(x) + b^3*tan(x) - a*b^2)/((a^4 + 2
*a^2*b^2 + b^4)*(tan(x)^2 + 1))
```

3.8.9 Mupad [B] (verification not implemented)

Time = 29.18 (sec) , antiderivative size = 3512, normalized size of antiderivative = 38.59

$$\int \frac{\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx = \text{Too large to display}$$

input
```
int(sin(x)^3/(a*cos(x) + b*sin(x)),x)
```

output
```
(4*a^3*log(1/(cos(x) + 1)))/(4*a^4 + 4*b^4 + 8*a^2*b^2) - ((b*tan(x/2))/(a
^2 + b^2) + (2*a*tan(x/2)^2)/(a^2 + b^2) - (b*tan(x/2)^3)/(a^2 + b^2))/(2*
tan(x/2)^2 + tan(x/2)^4 + 1) - (a^3*log(a + 2*b*tan(x/2) - a*tan(x/2)^2))/
(a^4 + b^4 + 2*a^2*b^2) - (b*atan((tan(x/2)*((((4*a^3*((b*((8*(4*a^2*b^8 -
 8*a^10 + 16*a^4*b^6 + 12*a^6*b^4 - 8*a^8*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3
*a^4*b^2) + (32*a^3*(12*a*b^10 + 48*a^3*b^8 + 72*a^5*b^6 + 48*a^7*b^4 + 12
*a^9*b^2))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2
)))*(3*a^2 + b^2))/(2*(a^4 + b^4 + 2*a^2*b^2)) + (16*a^3*b*(3*a^2 + b^2)*(
12*a*b^10 + 48*a^3*b^8 + 72*a^5*b^6 + 48*a^7*b^4 + 12*a^9*b^2))/((4*a^4 +
4*b^4 + 8*a^2*b^2)*(a^4 + b^4 + 2*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*
b^2))))/(4*a^4 + 4*b^4 + 8*a^2*b^2) + (b*(3*a^2 + b^2)*((8*(2*a*b^8 + 13*a
^3*b^6 + 32*a^5*b^4 + 21*a^7*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (
4*a^3*((8*(4*a^2*b^8 - 8*a^10 + 16*a^4*b^6 + 12*a^6*b^4 - 8*a^8*b^2))/(a^6
 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (32*a^3*(12*a*b^10 + 48*a^3*b^8 + 72*a^5
*b^6 + 48*a^7*b^4 + 12*a^9*b^2))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^6 + b^6 +
 3*a^2*b^4 + 3*a^4*b^2))))/(4*a^4 + 4*b^4 + 8*a^2*b^2)))/(2*(a^4 + b^4 + 2
*a^2*b^2)) - (b^3*(3*a^2 + b^2)^3*(12*a*b^10 + 48*a^3*b^8 + 72*a^5*b^6 + 4
8*a^7*b^4 + 12*a^9*b^2))/((a^4 + b^4 + 2*a^2*b^2)^3*(a^6 + b^6 + 3*a^2*b^4
 + 3*a^4*b^2)))*(16*a^8 + b^8 + 5*a^2*b^6 - 13*a^4*b^4 - 73*a^6*b^2))/(16*
a^8 + b^8 + 7*a^2*b^6 + 15*a^4*b^4 + 25*a^6*b^2)^2 + (2*a*b*(b^6 - 28*a...
```

3.8. $\int \frac{\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx$

3.9 $\int \frac{\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx$

3.9.1 Optimal result

Integrand size = 16, antiderivative size = 68

$$\int \frac{\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx=-\frac{a^2\operatorname{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}-\frac{b\cos(x)}{a^2+b^2}-\frac{a\sin(x)}{a^2+b^2}$$

output
```
-a^2*arctanh((b*cos(x)-a*sin(x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)-b*cos(x)
/(a^2+b^2)-a*sin(x)/(a^2+b^2)
```

3.9.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

$$\int \frac{\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx=\frac{2a^2\operatorname{arctanh}\left(\frac{-b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}-\frac{b\cos(x)+a\sin(x)}{a^2+b^2}$$

input
```
Integrate[Sin[x]^2/(a*Cos[x] + b*Sin[x]),x]
```

output
```
(2*a^2*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2) - (b*
Cos[x] + a*Sin[x])/(a^2 + b^2)
```

3.9.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3042, 3578, 3042, 3118, 3553, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(x)}{a\cos(x)+b\sin(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)^2}{a\cos(x)+b\sin(x)}dx$$

$$\Big\downarrow \text{3578}$$

$$\frac{b\int \sin(x)dx}{a^2+b^2}+\frac{a^2\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}-\frac{a\sin(x)}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\int \sin(x)dx}{a^2+b^2}+\frac{a^2\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}-\frac{a\sin(x)}{a^2+b^2}$$

$$\Big\downarrow \text{3118}$$

$$\frac{a^2\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}-\frac{a\sin(x)}{a^2+b^2}-\frac{b\cos(x)}{a^2+b^2}$$

$$\Big\downarrow \text{3553}$$

$$-\frac{a^2\int \frac{1}{a^2+b^2-(b\cos(x)-a\sin(x))^2}d(b\cos(x)-a\sin(x))}{a^2+b^2}-\frac{a\sin(x)}{a^2+b^2}-\frac{b\cos(x)}{a^2+b^2}$$

$$\Big\downarrow \text{219}$$

$$-\frac{a^2\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}-\frac{a\sin(x)}{a^2+b^2}-\frac{b\cos(x)}{a^2+b^2}$$

input
```
Int[Sin[x]^2/(a*Cos[x] + b*Sin[x]),x]
```

output
```
-((a^2*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2))
- (b*Cos[x])/(a^2 + b^2) - (a*Sin[x])/(a^2 + b^2)
```

3.9.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3578
```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-a)*(Sin[c + d*x]^(m - 1)/(d*(a^2
 + b^2)*(m - 1))), x] + (Simp[a^2/(a^2 + b^2)   Int[Sin[c + d*x]^(m - 2)/(a
*Cos[c + d*x] + b*Sin[c + d*x]), x], x] + Simp[b/(a^2 + b^2)   Int[Sin[c +
d*x]^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ
[m, 1]
```

3.9.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.24

method	result	size
default	$\frac{8a^2 \operatorname{arctanh}\left(\frac{2a\tan\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{(4a^2+4b^2)\sqrt{a^2+b^2}}+\frac{-2a\tan\left(\frac{x}{2}\right)-2b}{(a^2+b^2)\left(1+\tan\left(\frac{x}{2}\right)^2\right)}$	84
risch	$\frac{i e^{ix}}{-2ib+2a}-\frac{i e^{-ix}}{2(ib+a)}-\frac{a^2\ln\left(e^{ix}-\frac{ia^3+ia\,b^2-a^2b-b^3}{(a^2+b^2)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}}+\frac{a^2\ln\left(e^{ix}+\frac{ia^3+ia\,b^2-a^2b-b^3}{(a^2+b^2)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}}$	146

input
```
int(sin(x)^2/(a*cos(x)+b*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
8*a^2/(4*a^2+4*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*x)-2*b)/(a^2+
b^2)^(1/2))+2/(a^2+b^2)*(-a*tan(1/2*x)-b)/(1+tan(1/2*x)^2)
```

3.9.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(64) = 128$.

Time = 0.27 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.12

$$\int \frac{\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{\sqrt{a^2+b^2}a^2\log\left(-\frac{2\,ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}(b\cos(x)-a\sin(x))}{2\,ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2+b^2}\right)-2\left(a^2b+b^3\right)\cos(x)-2\left(a^3\right.}{2\left(a^4+2\,a^2b^2+b^4\right)}$$

input
```
integrate(sin(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

output
```
1/2*(sqrt(a^2 + b^2)*a^2*log(-(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2
- 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos(x)*sin
(x) + (a^2 - b^2)*cos(x)^2 + b^2)) - 2*(a^2*b + b^3)*cos(x) - 2*(a^3 + a*b
^2)*sin(x))/(a^4 + 2*a^2*b^2 + b^4)
```

3.9.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 76.32 (sec) , antiderivative size = 706, normalized size of antiderivative = 10.38

$$\int \frac{\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx = \begin{cases} \tilde{\infty}\cos(x) \\ -\frac{\cos(x)}{b} \\ -\frac{\sin^2(x)}{3ib\sin(x)+3b\cos(x)}-\frac{2i\sin(x)\cos(x)}{3ib\sin(x)+3b\cos(x)}-\frac{2\cos^2(x)}{3ib\sin(x)+3b\cos(x)} \\ -\frac{\sin^2(x)}{-3ib\sin(x)+3b\cos(x)}+\frac{2i\sin(x)\cos(x)}{-3ib\sin(x)+3b\cos(x)}-\frac{2\cos^2(x)}{-3ib\sin(x)+3b\cos(x)} \\ -\frac{a^2\log\left(\tan\left(\frac{x}{2}\right)-\frac{b}{a}-\frac{\sqrt{a^2+b^2}}{a}\right)\tan^2\left(\frac{x}{2}\right)}{a^2\sqrt{a^2+b^2}\tan^2\left(\frac{x}{2}\right)+a^2\sqrt{a^2+b^2}+b^2\sqrt{a^2+b^2}\tan^2\left(\frac{x}{2}\right)+b^2\sqrt{a^2+b^2}}-\frac{a^2\log\left(\tan\left(\frac{x}{2}\right)-\frac{b}{a}-\frac{\sqrt{a^2+b^2}}{a}\right)}{a^2\sqrt{a^2+b^2}\tan^2\left(\frac{x}{2}\right)+a^2\sqrt{a^2+b^2}+b^2\sqrt{a^2+b^2}\tan^2\left(\frac{x}{2}\right)+b^2\sqrt{a^2+b^2}} \end{cases}$$

input
```
integrate(sin(x)**2/(a*cos(x)+b*sin(x)),x)
```

output
```
Piecewise((zoo*cos(x), Eq(a, 0) & Eq(b, 0)), (-cos(x)/b, Eq(a, 0)), (-sin(
x)**2/(3*I*b*sin(x) + 3*b*cos(x)) - 2*I*sin(x)*cos(x)/(3*I*b*sin(x) + 3*b*
cos(x)) - 2*cos(x)**2/(3*I*b*sin(x) + 3*b*cos(x)), Eq(a, -I*b)), (-sin(x)*
*2/(-3*I*b*sin(x) + 3*b*cos(x)) + 2*I*sin(x)*cos(x)/(-3*I*b*sin(x) + 3*b*c
os(x)) - 2*cos(x)**2/(-3*I*b*sin(x) + 3*b*cos(x)), Eq(a, I*b)), (-a**2*log
(tan(x/2) - b/a - sqrt(a**2 + b**2)/a)*tan(x/2)**2/(a**2*sqrt(a**2 + b**2)
*tan(x/2)**2 + a**2*sqrt(a**2 + b**2) + b**2*sqrt(a**2 + b**2)*tan(x/2)**2
 + b**2*sqrt(a**2 + b**2)) - a**2*log(tan(x/2) - b/a - sqrt(a**2 + b**2)/a
)/(a**2*sqrt(a**2 + b**2)*tan(x/2)**2 + a**2*sqrt(a**2 + b**2) + b**2*sqrt
(a**2 + b**2)*tan(x/2)**2 + b**2*sqrt(a**2 + b**2)) + a**2*log(tan(x/2) -
b/a + sqrt(a**2 + b**2)/a)*tan(x/2)**2/(a**2*sqrt(a**2 + b**2)*tan(x/2)**2
 + a**2*sqrt(a**2 + b**2) + b**2*sqrt(a**2 + b**2)*tan(x/2)**2 + b**2*sqrt
(a**2 + b**2)) + a**2*log(tan(x/2) - b/a + sqrt(a**2 + b**2)/a)/(a**2*sqrt
(a**2 + b**2)*tan(x/2)**2 + a**2*sqrt(a**2 + b**2) + b**2*sqrt(a**2 + b**2
)*tan(x/2)**2 + b**2*sqrt(a**2 + b**2)) - 2*a*sqrt(a**2 + b**2)*tan(x/2)/(
a**2*sqrt(a**2 + b**2)*tan(x/2)**2 + a**2*sqrt(a**2 + b**2) + b**2*sqrt(a*
*2 + b**2)*tan(x/2)**2 + b**2*sqrt(a**2 + b**2)) - 2*b*sqrt(a**2 + b**2)/(
a**2*sqrt(a**2 + b**2)*tan(x/2)**2 + a**2*sqrt(a**2 + b**2) + b**2*sqrt(a*
*2 + b**2)*tan(x/2)**2 + b**2*sqrt(a**2 + b**2)), True))
```

3.9.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.56

$$\int \frac{\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx = -\frac{a^2\log\left(\frac{b-\frac{a\sin(x)}{\cos(x)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(x)}{\cos(x)+1}-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}} - \frac{2\left(b+\frac{a\sin(x)}{\cos(x)+1}\right)}{a^2+b^2+\frac{(a^2+b^2)\sin(x)^2}{(\cos(x)+1)^2}}$$

input
```
integrate(sin(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

output
```
-a^2*log((b - a*sin(x)/(cos(x) + 1) + sqrt(a^2 + b^2))/(b - a*sin(x)/(cos(
x) + 1) - sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(b + a*sin(x)/(cos(x) +
1))/(a^2 + b^2 + (a^2 + b^2)*sin(x)^2/(cos(x) + 1)^2)
```

3.9.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.38

$$\int \frac{\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx = -\frac{a^2\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^2+b^2)^{\frac{3}{2}}} - \frac{2\left(a\tan\left(\frac{1}{2}\,x\right)+b\right)}{(a^2+b^2)\left(\tan\left(\frac{1}{2}\,x\right)^2+1\right)}$$

input
```
integrate(sin(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

output
```
-a^2*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*x)
- 2*b + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(a*tan(1/2*x) + b)/((a^2
 + b^2)*(tan(1/2*x)^2 + 1))
```

3.9.9 Mupad [B] (verification not implemented)

Time = 20.94 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.38

$$\int \frac{\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx = -\frac{\frac{2\,b}{a^2+b^2}+\frac{2\,a\tan\left(\frac{x}{2}\right)}{a^2+b^2}}{\tan\left(\frac{x}{2}\right)^2+1} - \frac{2\,a^2\,\mathrm{atanh}\left(\frac{2\,a^2\,b+2\,b^3-2\,a\tan\left(\frac{x}{2}\right)\,\left(a^2+b^2\right)}{2\,\left(a^2+b^2\right)^{3/2}}\right)}{\left(a^2+b^2\right)^{3/2}}$$

input
```
int(sin(x)^2/(a*cos(x) + b*sin(x)),x)
```

output
```
- ((2*b)/(a^2 + b^2) + (2*a*tan(x/2))/(a^2 + b^2))/(tan(x/2)^2 + 1) - (2*a
^2*atanh((2*a^2*b + 2*b^3 - 2*a*tan(x/2)*(a^2 + b^2))/(2*(a^2 + b^2)^(3/2)
)))/(a^2 + b^2)^(3/2)
```

3.10 $\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}\,dx$

3.10.1 Optimal result

Integrand size = 14, antiderivative size = 35

$$\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{bx}{a^2+b^2} - \frac{a\log(a\cos(x)+b\sin(x))}{a^2+b^2}$$

output
```
b*x/(a^2+b^2)-a*ln(a*cos(x)+b*sin(x))/(a^2+b^2)
```

3.10.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.34

$$\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{2(-ia+b)x+2ia\arctan(\tan(x))-a\log\left((a\cos(x)+b\sin(x))^2\right)}{2(a^2+b^2)}$$

input
```
Integrate[Sin[x]/(a*Cos[x] + b*Sin[x]),x]
```

output
```
(2*((-I)*a + b)*x + (2*I)*a*ArcTan[Tan[x]] - a*Log[(a*Cos[x] + b*Sin[x])^2
])/(2*(a^2 + b^2))
```

3.10.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3576, 3042, 3612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}dx$$

$$\Big\downarrow \text{ 3576}$$

$$\frac{bx}{a^2+b^2}-\frac{a\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{bx}{a^2+b^2}-\frac{a\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{ 3612}$$

$$\frac{bx}{a^2+b^2}-\frac{a\log(a\cos(x)+b\sin(x))}{a^2+b^2}$$

input
```
Int[Sin[x]/(a*Cos[x] + b*Sin[x]),x]
```

output
```
(b*x)/(a^2 + b^2) - (a*Log[a*Cos[x] + b*Sin[x]])/(a^2 + b^2)
```

3.10.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3576
```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(x/(a^2 + b^2)), x] - Simp[a/(a^2 + b
^2)   Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x
]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3612
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

3.10.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.31

method	result	size
parallelrisch	$\frac{a \ln\left(\frac{1}{\cos(x)+1}\right)-a \ln\left(\frac{-a\cos(x)-b\sin(x)}{\cos(x)+1}\right)+xb}{a^2+b^2}$	46
default	$-\frac{a\ln(a+b\tan(x))}{a^2+b^2}+\frac{\frac{a\ln\left(1+\tan(x)^2\right)}{2}+b\arctan(\tan(x))}{a^2+b^2}$	47
risch	$\frac{ix}{ib-a}+\frac{2iax}{a^2+b^2}-\frac{a\ln\left(e^{2ix}-\frac{ib+a}{ib-a}\right)}{a^2+b^2}$	67
norman	$\frac{\frac{bx}{a^2+b^2}+\frac{bx\tan\left(\frac{x}{2}\right)^2}{a^2+b^2}}{1+\tan\left(\frac{x}{2}\right)^2}+\frac{a\ln\left(1+\tan\left(\frac{x}{2}\right)^2\right)}{a^2+b^2}-\frac{a\ln\left(\tan\left(\frac{x}{2}\right)^2a-2b\tan\left(\frac{x}{2}\right)-a\right)}{a^2+b^2}$	96

input
```
int(sin(x)/(a*cos(x)+b*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
(a*ln(1/(cos(x)+1))-a*ln((-a*cos(x)-b*sin(x))/(cos(x)+1))+x*b)/(a^2+b^2)
```

3.10.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.31

$$\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{2\,bx - a\log\left(2\,ab\cos\left(x\right)\sin\left(x\right) + \left(a^2-b^2\right)\cos\left(x\right)^2 + b^2\right)}{2\left(a^2+b^2\right)}$$

input
```
integrate(sin(x)/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

output
```
1/2*(2*b*x - a*log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2))/(a^2
 + b^2)
```

3.10.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 165, normalized size of antiderivative = 4.71

$$\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}\,dx = \begin{cases} \tilde{\infty}x & \text{for } a=0 \wedge b=0 \\ -\frac{\log\left(\cos\left(x\right)\right)}{a} & \text{for } b=0 \\ \frac{ix\sin\left(x\right)}{2ib\sin\left(x\right)+2b\cos\left(x\right)} + \frac{x\cos\left(x\right)}{2ib\sin\left(x\right)+2b\cos\left(x\right)} - \frac{\sin\left(x\right)}{2ib\sin\left(x\right)+2b\cos\left(x\right)} & \text{for } a=-ib \\ -\frac{ix\sin\left(x\right)}{-2ib\sin\left(x\right)+2b\cos\left(x\right)} + \frac{x\cos\left(x\right)}{-2ib\sin\left(x\right)+2b\cos\left(x\right)} - \frac{\sin\left(x\right)}{-2ib\sin\left(x\right)+2b\cos\left(x\right)} & \text{for } a=ib \\ -\frac{a\log\left(\frac{a\cos\left(x\right)}{b}+\sin\left(x\right)\right)}{a^2+b^2} + \frac{bx}{a^2+b^2} & \text{otherwise} \end{cases}$$

input
```
integrate(sin(x)/(a*cos(x)+b*sin(x)),x)
```

output
```
Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (-log(cos(x))/a, Eq(b, 0)), (I*x*s
in(x)/(2*I*b*sin(x) + 2*b*cos(x)) + x*cos(x)/(2*I*b*sin(x) + 2*b*cos(x)) -
 sin(x)/(2*I*b*sin(x) + 2*b*cos(x)), Eq(a, -I*b)), (-I*x*sin(x)/(-2*I*b*si
n(x) + 2*b*cos(x)) + x*cos(x)/(-2*I*b*sin(x) + 2*b*cos(x)) - sin(x)/(-2*I*
b*sin(x) + 2*b*cos(x)), Eq(a, I*b)), (-a*log(a*cos(x)/b + sin(x))/(a**2 +
b**2) + b*x/(a**2 + b**2), True))
```

3.10.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(35) = 70.

Time = 0.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.51

$$\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{2\,b\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^2+b^2} - \frac{a\log\left(-a-\frac{2\,b\sin(x)}{\cos(x)+1}+\frac{a\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^2+b^2} + \frac{a\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{a^2+b^2}$$

input
```
integrate(sin(x)/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

output
```
2*b*arctan(sin(x)/(cos(x) + 1))/(a^2 + b^2) - a*log(-a - 2*b*sin(x)/(cos(x
) + 1) + a*sin(x)^2/(cos(x) + 1)^2)/(a^2 + b^2) + a*log(sin(x)^2/(cos(x) +
 1)^2 + 1)/(a^2 + b^2)
```

3.10.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.57

$$\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}\,dx = -\frac{ab\log\left(|b\tan\left(x\right)+a|\right)}{a^2b+b^3} + \frac{bx}{a^2+b^2} + \frac{a\log\left(\tan\left(x\right)^2+1\right)}{2\left(a^2+b^2\right)}$$

input
```
integrate(sin(x)/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

output
```
-a*b*log(abs(b*tan(x) + a))/(a^2*b + b^3) + b*x/(a^2 + b^2) + 1/2*a*log(ta
n(x)^2 + 1)/(a^2 + b^2)
```

3.10.9 Mupad [B] (verification not implemented)

Time = 23.05 (sec) , antiderivative size = 970, normalized size of antiderivative = 27.71

$$\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}\,dx = -\frac{a\,\ln\left(a\,\cos\left(x\right)+b\,\sin\left(x\right)\right)}{a^2+b^2}$$

$$-\frac{2\,b\,\mathrm{atan}\left(\frac{\left(a^4+2\,a^2\,b^2+b^4\right)\left(\tan\left(\frac{x}{2}\right)\left(\frac{\left(4\,a^4-13\,a^2\,b^2+b^4\right)\left(\frac{b\left(64\,a\,b^2+\frac{a\left(32\,a^2\,b^2-64\,a^4+\frac{a\left(96\,a^3\,b^2+96\,a\,b^4\right)}{a^2+b^2}\right)}{a^2+b^2}\right)}{a^2+b^2}-\frac{b^3\left(96\,a^3\,b^2+96\,a\,b^4\right)}{\left(a^2+b^2\right)^3}+\frac{a\cdots}{\cdots}\cdots\right)}{\left(4\,a^4+5\,a^2\,b^2+b^4\right)^2}\cdots\right)\cdots\right)}{\cdots}\cdots\right)}{\cdots}$$

input
```
int(sin(x)/(a*cos(x) + b*sin(x)),x)
```

output
```
- (a*log(a*cos(x) + b*sin(x)))/(a^2 + b^2) - (2*b*atan(((a^4 + b^4 + 2*a^2
*b^2)*(tan(x/2)*(((4*a^4 + b^4 - 13*a^2*b^2)*((b*(64*a*b^2 + (a*(32*a^2*b^
2 - 64*a^4 + (a*(96*a*b^4 + 96*a^3*b^2))/(a^2 + b^2)))/(a^2 + b^2)))/(a^2
+ b^2) - (b^3*(96*a*b^4 + 96*a^3*b^2))/(a^2 + b^2)^3 + (a*((b*(32*a^2*b^2
- 64*a^4 + (a*(96*a*b^4 + 96*a^3*b^2))/(a^2 + b^2)))/(a^2 + b^2) + (a*b*(9
6*a*b^4 + 96*a^3*b^2))/(a^2 + b^2)^2))/(a^2 + b^2)))/(4*a^4 + b^4 + 5*a^2*
b^2)^2 - (6*a*b*(2*a^2 - b^2)*(64*a^2 + (a*(64*a*b^2 + (a*(32*a^2*b^2 - 64
*a^4 + (a*(96*a*b^4 + 96*a^3*b^2))/(a^2 + b^2)))/(a^2 + b^2)))/(a^2 + b^2)
 - (b*((b*(32*a^2*b^2 - 64*a^4 + (a*(96*a*b^4 + 96*a^3*b^2))/(a^2 + b^2)))
/(a^2 + b^2) + (a*b*(96*a*b^4 + 96*a^3*b^2))/(a^2 + b^2)^2))/(a^2 + b^2) -
 (a*b^2*(96*a*b^4 + 96*a^3*b^2))/(a^2 + b^2)^3))/(4*a^4 + b^4 + 5*a^2*b^2)
^2) - ((4*a^4 + b^4 - 13*a^2*b^2)*((b*(32*a^2*b - (a*(64*a^3*b - 32*a*b^3
+ (a*(96*a^4*b + 96*a^2*b^3))/(a^2 + b^2)))/(a^2 + b^2)))/(a^2 + b^2) + (b
^3*(96*a^4*b + 96*a^2*b^3))/(a^2 + b^2)^3 - (a*((b*(64*a^3*b - 32*a*b^3 +
(a*(96*a^4*b + 96*a^2*b^3))/(a^2 + b^2)))/(a^2 + b^2) + (a*b*(96*a^4*b + 9
6*a^2*b^3))/(a^2 + b^2)^2))/(a^2 + b^2)))/(4*a^4 + b^4 + 5*a^2*b^2)^2 + (6
*a*b*(2*a^2 - b^2)*((a*(32*a^2*b - (a*(64*a^3*b - 32*a*b^3 + (a*(96*a^4*b
+ 96*a^2*b^3))/(a^2 + b^2)))/(a^2 + b^2)))/(a^2 + b^2) + (b*((b*(64*a^3*b
- 32*a*b^3 + (a*(96*a^4*b + 96*a^2*b^3))/(a^2 + b^2)))/(a^2 + b^2) + (a*b*
(96*a^4*b + 96*a^2*b^3))/(a^2 + b^2)^2))/(a^2 + b^2) + (a*b^2*(96*a^4*b...
```

3.10. $\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}\,dx$

3.11 $\int \frac{1}{a\cos(x)+b\sin(x)}\,dx$

3.11.1 Optimal result

Integrand size = 11, antiderivative size = 36

$$\int \frac{1}{a\cos(x)+b\sin(x)}\,dx = -\frac{\operatorname{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

output
```
-arctanh((b*cos(x)-a*sin(x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2)
```

3.11.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{1}{a\cos(x)+b\sin(x)}\,dx = \frac{2\operatorname{arctanh}\left(\frac{-b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

input
```
Integrate[(a*Cos[x] + b*Sin[x])^(-1),x]
```

output
```
(2*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2]
```

3.11.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 3553, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a\cos(x)+b\sin(x)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{a\cos(x)+b\sin(x)}dx$$
$$\Big\downarrow \text{3553}$$
$$-\int \frac{1}{a^2+b^2-(b\cos(x)-a\sin(x))^2}d(b\cos(x)-a\sin(x))$$
$$\Big\downarrow \text{219}$$
$$-\frac{\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

input
```
Int[(a*Cos[x] + b*Sin[x])^(-1),x]
```

output
```
-(ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]]/Sqrt[a^2 + b^2])
```

3.11.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

3.11.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{2\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$	35
risch	$\frac{\ln\left(\mathrm{e}^{ix}+\frac{ia-b}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}-\frac{\ln\left(\mathrm{e}^{ix}-\frac{ia-b}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$	74

input
```
int(1/(a*cos(x)+b*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*x)-2*b)/(a^2+b^2)^(1/2))
```

3.11.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(32) = 64$.

Time = 0.26 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.67

$$\int \frac{1}{a\cos(x)+b\sin(x)}\,dx = \frac{\log\left(-\frac{2\,ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}(b\cos(x)-a\sin(x))}{2\,ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2+b^2}\right)}{2\,\sqrt{a^2+b^2}}$$

input
```
integrate(1/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

output
```
1/2*log(-(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 - 2*a^2 - b^2 + 2*sqr
t(a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos
(x)^2 + b^2))/sqrt(a^2 + b^2)
```

3.11.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.57 (sec) , antiderivative size = 112, normalized size of antiderivative = 3.11

$$\int \frac{1}{a\cos(x)+b\sin(x)}\,dx = \begin{cases} \tilde{\infty}\log\left(\tan\left(\frac{x}{2}\right)\right) & \text{for } a=0\wedge b=0 \\ \frac{\log\left(\tan\left(\frac{x}{2}\right)\right)}{b} & \text{for } a=0 \\ -\frac{1}{ib\sin(x)+b\cos(x)} & \text{for } a=-ib \\ -\frac{1}{-ib\sin(x)+b\cos(x)} & \text{for } a=ib \\ -\frac{\log\left(\tan\left(\frac{x}{2}\right)-\frac{b}{a}-\frac{\sqrt{a^2+b^2}}{a}\right)}{\sqrt{a^2+b^2}}+\frac{\log\left(\tan\left(\frac{x}{2}\right)-\frac{b}{a}+\frac{\sqrt{a^2+b^2}}{a}\right)}{\sqrt{a^2+b^2}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a*cos(x)+b*sin(x)),x)
```

output
```
Piecewise((zoo*log(tan(x/2)), Eq(a, 0) & Eq(b, 0)), (log(tan(x/2))/b, Eq(a
, 0)), (-1/(I*b*sin(x) + b*cos(x)), Eq(a, -I*b)), (-1/(-I*b*sin(x) + b*cos
(x)), Eq(a, I*b)), (-log(tan(x/2) - b/a - sqrt(a**2 + b**2)/a)/sqrt(a**2 +
 b**2) + log(tan(x/2) - b/a + sqrt(a**2 + b**2)/a)/sqrt(a**2 + b**2), True
))
```

3.11.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.69

$$\int \frac{1}{a\cos(x)+b\sin(x)}\,dx = -\frac{\log\left(\frac{b-\frac{a\sin(x)}{\cos(x)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(x)}{\cos(x)+1}-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

input
```
integrate(1/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

output
```
-log((b - a*sin(x)/(cos(x) + 1) + sqrt(a^2 + b^2))/(b - a*sin(x)/(cos(x) +
 1) - sqrt(a^2 + b^2)))/sqrt(a^2 + b^2)
```

3.11.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.69

$$\int \frac{1}{a\cos(x)+b\sin(x)}\,dx = -\frac{\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}}$$

input
```
integrate(1/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

output
```
-log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*x) - 2*
b + 2*sqrt(a^2 + b^2)))/sqrt(a^2 + b^2)
```

3.11.9 Mupad [B] (verification not implemented)

Time = 21.37 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \frac{1}{a\cos(x)+b\sin(x)}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{b-a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

input
```
int(1/(a*cos(x) + b*sin(x)),x)
```

output
```
-(2*atanh((b - a*tan(x/2))/(a^2 + b^2)^(1/2)))/(a^2 + b^2)^(1/2)
```

3.12 $\int \frac{\csc(x)}{a\cos(x)+b\sin(x)}\,dx$

3.12.1 Optimal result

Integrand size = 14, antiderivative size = 23

$$\int \frac{\csc(x)}{a\cos(x)+b\sin(x)}\,dx=\frac{\log(\sin(x))}{a}-\frac{\log(a\cos(x)+b\sin(x))}{a}$$

output
```
ln(sin(x))/a-ln(a*cos(x)+b*sin(x))/a
```

3.12.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{\csc(x)}{a\cos(x)+b\sin(x)}\,dx=\frac{\log(\sin(x))-\log(a\cos(x)+b\sin(x))}{a}$$

input
```
Integrate[Csc[x]/(a*Cos[x] + b*Sin[x]),x]
```

output
```
(Log[Sin[x]] - Log[a*Cos[x] + b*Sin[x]])/a
```

3.12.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 3580, 3042, 25, 3612, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(x)}{a\cos(x)+b\sin(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(x)(a\cos(x)+b\sin(x))}dx$$

$$\Big\downarrow \text{3580}$$

$$\frac{\int \cot(x)dx}{a}-\frac{\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int -\tan\left(x+\frac{\pi}{2}\right)dx}{a}-\frac{\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a}-\frac{\int \tan\left(x+\frac{\pi}{2}\right)dx}{a}$$

$$\Big\downarrow \text{3612}$$

$$-\frac{\int \tan\left(x+\frac{\pi}{2}\right)dx}{a}-\frac{\log(a\cos(x)+b\sin(x))}{a}$$

$$\Big\downarrow \text{3956}$$

$$\frac{\log(\sin(x))}{a}-\frac{\log(a\cos(x)+b\sin(x))}{a}$$

input
```
Int[Csc[x]/(a*Cos[x] + b*Sin[x]),x]
```

output
```
Log[Sin[x]]/a - Log[a*Cos[x] + b*Sin[x]]/a
```

3.12.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3580
```
Int[1/(sin[(c_.) + (d_.)*(x_)]*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(
c_.) + (d_.)*(x_)])), x_Symbol] :> Simp[1/a   Int[Cot[c + d*x], x], x] - Si
mp[1/a   Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c +
d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3612
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

3.12.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{\ln(\tan(x))}{a} - \frac{\ln(a+b\tan(x))}{a}$	21
parallelrisch	$\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)-\ln\left(\tan\left(\frac{x}{2}\right)^{2}a-2b\tan\left(\frac{x}{2}\right)-a\right)}{a}$	33
norman	$\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a} - \frac{\ln\left(\tan\left(\frac{x}{2}\right)^{2}a-2b\tan\left(\frac{x}{2}\right)-a\right)}{a}$	36
risch	$-\frac{\ln\left(e^{2ix}-\frac{ib+a}{ib-a}\right)}{a} + \frac{\ln\left(e^{2ix}-1\right)}{a}$	44

input
```
int(csc(x)/(a*cos(x)+b*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
1/a*ln(tan(x))-1/a*ln(a+b*tan(x))
```

3.12.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.91

$$\int \frac{\csc(x)}{a\cos(x)+b\sin(x)}\,dx = -\frac{\log\left(2\,ab\cos\left(x\right)\sin\left(x\right)+\left(a^2-b^2\right)\cos\left(x\right)^2+b^2\right)-\log\left(-\frac{1}{4}\,\cos\left(x\right)^2+\frac{1}{4}\right)}{2\,a}$$

input
```
integrate(csc(x)/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

output
```
-1/2*(log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) - log(-1/4*cos
(x)^2 + 1/4))/a
```

3.12.6 Sympy [F]

$$\int \frac{\csc(x)}{a\cos(x)+b\sin(x)}\,dx = \int \frac{\csc\left(x\right)}{a\cos\left(x\right)+b\sin\left(x\right)}\,dx$$

input
```
integrate(csc(x)/(a*cos(x)+b*sin(x)),x)
```

output
```
Integral(csc(x)/(a*cos(x) + b*sin(x)), x)
```

3.12.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(23) = 46$.

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.09

$$\int \frac{\csc(x)}{a\cos(x)+b\sin(x)}\,dx = -\frac{\log\left(-a-\frac{2\,b\sin(x)}{\cos(x)+1}+\frac{a\sin(x)^2}{(\cos(x)+1)^2}\right)}{a}+\frac{\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a}$$

input
```
integrate(csc(x)/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

output
```
-log(-a - 2*b*sin(x)/(cos(x) + 1) + a*sin(x)^2/(cos(x) + 1)^2)/a + log(sin
(x)/(cos(x) + 1))/a
```

3.12.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\csc(x)}{a\cos(x)+b\sin(x)}\,dx=-\frac{\log\left(|b\tan\left(x\right)+a|\right)}{a}+\frac{\log\left(|\tan\left(x\right)|\right)}{a}$$

input
```
integrate(csc(x)/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

output
```
-log(abs(b*tan(x) + a))/a + log(abs(tan(x)))/a
```

3.12.9 Mupad [B] (verification not implemented)

Time = 21.42 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.39

$$\int \frac{\csc(x)}{a\cos(x)+b\sin(x)}\,dx=-\frac{\ln\left(-a\tan\left(\frac{x}{2}\right)^2+2\,b\tan\left(\frac{x}{2}\right)+a\right)-\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a}$$

input
```
int(1/(sin(x)*(a*cos(x) + b*sin(x))),x)
```

output
```
-(log(a + 2*b*tan(x/2) - a*tan(x/2)^2) - log(tan(x/2)))/a
```

3.13 $\int \frac{\csc^2(x)}{a\cos(x)+b\sin(x)}\,dx$

3.13.1 Optimal result

Integrand size = 16, antiderivative size = 55

$$\int \frac{\csc^2(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{b\text{arctanh}(\cos(x))}{a^2} - \frac{\sqrt{a^2+b^2}\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^2} - \frac{\csc(x)}{a}$$

output
```
b*arctanh(cos(x))/a^2-csc(x)/a-arctanh((b*cos(x)-a*sin(x))/(a^2+b^2)^(1/2)
)*(a^2+b^2)^(1/2)/a^2
```

3.13.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.22

$$\int \frac{\csc^2(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{2\sqrt{a^2+b^2}\text{arctanh}\left(\frac{-b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right) - a\csc(x) + b\left(\log\left(\cos\left(\frac{x}{2}\right)\right) - \log\left(\sin\left(\frac{x}{2}\right)\right)\right)}{a^2}$$

input
```
Integrate[Csc[x]^2/(a*Cos[x] + b*Sin[x]),x]
```

output
```
(2*Sqrt[a^2 + b^2]*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]] - a*Csc[x] +
 b*(Log[Cos[x/2]] - Log[Sin[x/2]]))/a^2
```

3.13.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3042, 3582, 3042, 3553, 219, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(x)}{a\cos(x)+b\sin(x)}dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\sin(x)^2(a\cos(x)+b\sin(x))}dx$$
$$\Big\downarrow \; 3582$$
$$\frac{(a^2+b^2)\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2}-\frac{b\int \csc(x)dx}{a^2}-\frac{\csc(x)}{a}$$
$$\Big\downarrow \; 3042$$
$$\frac{(a^2+b^2)\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2}-\frac{b\int \csc(x)dx}{a^2}-\frac{\csc(x)}{a}$$
$$\Big\downarrow \; 3553$$
$$-\frac{(a^2+b^2)\int \frac{1}{a^2+b^2-(b\cos(x)-a\sin(x))^2}d(b\cos(x)-a\sin(x))}{a^2}-\frac{b\int \csc(x)dx}{a^2}-\frac{\csc(x)}{a}$$
$$\Big\downarrow \; 219$$
$$-\frac{b\int \csc(x)dx}{a^2}-\frac{\sqrt{a^2+b^2}\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^2}-\frac{\csc(x)}{a}$$
$$\Big\downarrow \; 4257$$
$$-\frac{\sqrt{a^2+b^2}\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^2}+\frac{b\text{arctanh}(\cos(x))}{a^2}-\frac{\csc(x)}{a}$$

input
```
Int[Csc[x]^2/(a*Cos[x] + b*Sin[x]),x]
```

output
```
(b*ArcTanh[Cos[x]])/a^2 - (Sqrt[a^2 + b^2]*ArcTanh[(b*Cos[x] - a*Sin[x])/S
qrt[a^2 + b^2]])/a^2 - Csc[x]/a
```

3.13.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3582
```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[Sin[c + d*x]^(m + 1)/(a*d*(m + 1))
, x] + (-Simp[b/a^2   Int[Sin[c + d*x]^(m + 1), x], x] + Simp[(a^2 + b^2)/a
^2   Int[Sin[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /;
 FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.13.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.47

method	result	size
default	$-\frac{\tan\left(\frac{x}{2}\right)}{2a}-\frac{\left(-4a^{2}-4b^{2}\right)\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{x}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{2a^{2}\sqrt{a^{2}+b^{2}}}-\frac{1}{2a\tan\left(\frac{x}{2}\right)}-\frac{b\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a^{2}}$	81
risch	$-\frac{2i\,\mathrm{e}^{ix}}{a\left(\mathrm{e}^{2ix}-1\right)}-\frac{b\ln\left(\mathrm{e}^{ix}-1\right)}{a^{2}}+\frac{b\ln\left(\mathrm{e}^{ix}+1\right)}{a^{2}}-\frac{\sqrt{a^{2}+b^{2}}\,\ln\left(\mathrm{e}^{ix}-\frac{ia-b}{\sqrt{a^{2}+b^{2}}}\right)}{a^{2}}+\frac{\sqrt{a^{2}+b^{2}}\,\ln\left(\mathrm{e}^{ix}+\frac{ia-b}{\sqrt{a^{2}+b^{2}}}\right)}{a^{2}}$	127

input
```
int(csc(x)^2/(a*cos(x)+b*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a*tan(1/2*x)-1/2/a^2*(-4*a^2-4*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*
tan(1/2*x)-2*b)/(a^2+b^2)^(1/2))-1/2/a/tan(1/2*x)-b/a^2*ln(tan(1/2*x))
```

3.13.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. 2(51) = 102.

Time = 0.28 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.42

$$\int \frac{\csc^2(x)}{a\cos(x)+b\sin(x)}\,dx$$
$$= \frac{b\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)\sin(x)-b\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)\sin(x)+\sqrt{a^2+b^2}\log\left(-\frac{2\,ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2-}{2\,ab\cos(x)\sin(x)+}\right.}{2\,a^2\sin(x)}$$

input
```
integrate(csc(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

output
```
1/2*(b*log(1/2*cos(x) + 1/2)*sin(x) - b*log(-1/2*cos(x) + 1/2)*sin(x) + sq
rt(a^2 + b^2)*log(-(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 - 2*a^2 - b
^2 + 2*sqrt(a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos(x)*sin(x) + (a^2
- b^2)*cos(x)^2 + b^2))*sin(x) - 2*a)/(a^2*sin(x))
```

3.13.6 Sympy [F]

$$\int \frac{\csc^2(x)}{a\cos(x)+b\sin(x)}\,dx = \int \frac{\csc^2(x)}{a\cos(x)+b\sin(x)}\,dx$$

input
```
integrate(csc(x)**2/(a*cos(x)+b*sin(x)),x)
```

output
```
Integral(csc(x)**2/(a*cos(x) + b*sin(x)), x)
```

3.13.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(51) = 102$.

Time = 0.30 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.95

$$\int \frac{\csc^2(x)}{a\cos(x)+b\sin(x)}\,dx = -\frac{b\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^2} - \frac{\sqrt{a^2+b^2}\log\left(\frac{b-\frac{a\sin(x)}{\cos(x)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(x)}{\cos(x)+1}-\sqrt{a^2+b^2}}\right)}{a^2} - \frac{\cos(x)+1}{2\,a\sin(x)} - \frac{\sin(x)}{2\,a(\cos(x)+1)}$$

input
```
integrate(csc(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

output
```
-b*log(sin(x)/(cos(x) + 1))/a^2 - sqrt(a^2 + b^2)*log((b - a*sin(x)/(cos(x
) + 1) + sqrt(a^2 + b^2))/(b - a*sin(x)/(cos(x) + 1) - sqrt(a^2 + b^2)))/a
^2 - 1/2*(cos(x) + 1)/(a*sin(x)) - 1/2*sin(x)/(a*(cos(x) + 1))
```

3.13.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(51) = 102$.

Time = 0.32 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.96

$$\int \frac{\csc^2(x)}{a\cos(x)+b\sin(x)}\,dx = -\frac{b\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)}{a^2} - \frac{\tan\left(\frac{1}{2}\,x\right)}{2\,a} - \frac{\sqrt{a^2+b^2}\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{a^2} + \frac{2\,b\tan\left(\frac{1}{2}\,x\right)-a}{2\,a^2\tan\left(\frac{1}{2}\,x\right)}$$

input
```
integrate(csc(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

output
```
-b*log(abs(tan(1/2*x)))/a^2 - 1/2*tan(1/2*x)/a - sqrt(a^2 + b^2)*log(abs(2
*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*x) - 2*b + 2*sqrt
(a^2 + b^2)))/a^2 + 1/2*(2*b*tan(1/2*x) - a)/(a^2*tan(1/2*x))
```

3.13.9 Mupad [B] (verification not implemented)

Time = 22.96 (sec) , antiderivative size = 170, normalized size of antiderivative = 3.09

$$\int \frac{\csc^2(x)}{a\cos(x)+b\sin(x)}\,dx$$
$$= \frac{2\,\mathrm{atanh}\left(\frac{a^3\,\cos\left(\frac{x}{2}\right)\,\sqrt{a^2+b^2}+4\,b^3\,\sin\left(\frac{x}{2}\right)\,\sqrt{a^2+b^2}+3\,a^2\,b\,\sin\left(\frac{x}{2}\right)\,\sqrt{a^2+b^2}+2\,a\,b^2\,\cos\left(\frac{x}{2}\right)\,\sqrt{a^2+b^2}}{\sin\left(\frac{x}{2}\right)\,a^4+2\,\cos\left(\frac{x}{2}\right)\,a^3\,b+5\,\sin\left(\frac{x}{2}\right)\,a^2\,b^2+2\,\cos\left(\frac{x}{2}\right)\,a\,b^3+4\,\sin\left(\frac{x}{2}\right)\,b^4}\right)\,\sqrt{a^2+b^2}}{a^2}$$
$$-\frac{1}{a\,\sin\left(x\right)}-\frac{b\,\ln\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{a^2}$$

input
```
int(1/(sin(x)^2*(a*cos(x) + b*sin(x))),x)
```

output
```
(2*atanh((a^3*cos(x/2)*(a^2 + b^2)^(1/2) + 4*b^3*sin(x/2)*(a^2 + b^2)^(1/2
) + 3*a^2*b*sin(x/2)*(a^2 + b^2)^(1/2) + 2*a*b^2*cos(x/2)*(a^2 + b^2)^(1/2
))/(a^4*sin(x/2) + 4*b^4*sin(x/2) + 5*a^2*b^2*sin(x/2) + 2*a*b^3*cos(x/2)
+ 2*a^3*b*cos(x/2)))*(a^2 + b^2)^(1/2))/a^2 - 1/(a*sin(x)) - (b*log(sin(x/
2)/cos(x/2)))/a^2
```

3.14 $\int \frac{\csc^3(x)}{a\cos(x)+b\sin(x)}\,dx$

3.14.1 Optimal result

Integrand size = 16, antiderivative size = 55

$$\int \frac{\csc^3(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{b\cot(x)}{a^2}-\frac{\csc^2(x)}{2a}+\frac{(a^2+b^2)\log(\sin(x))}{a^3} - \frac{(a^2+b^2)\log(a\cos(x)+b\sin(x))}{a^3}$$

output
```
b*cot(x)/a^2-1/2*csc(x)^2/a+(a^2+b^2)*ln(sin(x))/a^3-(a^2+b^2)*ln(a*cos(x)
+b*sin(x))/a^3
```

3.14.2 Mathematica [A] (verified)

Time = 1.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87

$$\int \frac{\csc^3(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{2ab\cot(x)-a^2\csc^2(x)+2(a^2+b^2)\left(\log(\sin(x))-\log(a\cos(x)+b\sin(x))\right)}{2a^3}$$

input
```
Integrate[Csc[x]^3/(a*Cos[x] + b*Sin[x]),x]
```

output
```
(2*a*b*Cot[x] - a^2*Csc[x]^2 + 2*(a^2 + b^2)*(Log[Sin[x]] - Log[a*Cos[x] +
 b*Sin[x]]))/(2*a^3)
```

3.14.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {3042, 3582, 3042, 3580, 3042, 25, 3612, 3956, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(x)}{a\cos(x)+b\sin(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(x)^3(a\cos(x)+b\sin(x))}dx$$

$$\Big\downarrow \text{3582}$$

$$\frac{(a^2+b^2)\int \frac{\csc(x)}{a\cos(x)+b\sin(x)}dx}{a^2}-\frac{b\int \csc^2(x)dx}{a^2}-\frac{\csc^2(x)}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a^2+b^2)\int \frac{1}{\sin(x)(a\cos(x)+b\sin(x))}dx}{a^2}-\frac{b\int \csc(x)^2dx}{a^2}-\frac{\csc^2(x)}{2a}$$

$$\Big\downarrow \text{3580}$$

$$\frac{(a^2+b^2)\left(\frac{\int \cot(x)dx}{a}-\frac{\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a}\right)}{a^2}-\frac{b\int \csc(x)^2dx}{a^2}-\frac{\csc^2(x)}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a^2+b^2)\left(\frac{\int -\tan\left(x+\frac{\pi}{2}\right)dx}{a}-\frac{\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a}\right)}{a^2}-\frac{b\int \csc(x)^2dx}{a^2}-\frac{\csc^2(x)}{2a}$$

$$\Big\downarrow \text{25}$$

$$\frac{(a^2+b^2)\left(-\frac{\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a}-\frac{\int \tan\left(x+\frac{\pi}{2}\right)dx}{a}\right)}{a^2}-\frac{b\int \csc(x)^2dx}{a^2}-\frac{\csc^2(x)}{2a}$$

$$\Big\downarrow \text{3612}$$

$$\frac{(a^2+b^2)\left(-\frac{\int \tan\left(x+\frac{\pi}{2}\right)dx}{a}-\frac{\log(a\cos(x)+b\sin(x))}{a}\right)}{a^2}-\frac{b\int \csc(x)^2dx}{a^2}-\frac{\csc^2(x)}{2a}$$

$$\Big\downarrow \text{3956}$$

$$-\frac{b\int \csc(x)^2dx}{a^2}+\frac{(a^2+b^2)\left(\frac{\log(\sin(x))}{a}-\frac{\log(a\cos(x)+b\sin(x))}{a}\right)}{a^2}-\frac{\csc^2(x)}{2a}$$

$$\Big\downarrow \; 4254$$

$$\frac{b\int 1d\cot(x)}{a^2}+\frac{(a^2+b^2)\left(\frac{\log(\sin(x))}{a}-\frac{\log(a\cos(x)+b\sin(x))}{a}\right)}{a^2}-\frac{\csc^2(x)}{2a}$$

$$\Big\downarrow \; 24$$

$$\frac{(a^2+b^2)\left(\frac{\log(\sin(x))}{a}-\frac{\log(a\cos(x)+b\sin(x))}{a}\right)}{a^2}+\frac{b\cot(x)}{a^2}-\frac{\csc^2(x)}{2a}$$

input
```
Int[Csc[x]^3/(a*Cos[x] + b*Sin[x]),x]
```

output
```
(b*Cot[x])/a^2 - Csc[x]^2/(2*a) + ((a^2 + b^2)*(Log[Sin[x]]/a - Log[a*Cos[
x] + b*Sin[x]]/a))/a^2
```

3.14.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3580
```
Int[1/(sin[(c_.) + (d_.)*(x_)]*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(
c_.) + (d_.)*(x_)])), x_Symbol] :> Simp[1/a   Int[Cot[c + d*x], x], x] - Si
mp[1/a   Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c +
d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3582
```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[Sin[c + d*x]^(m + 1)/(a*d*(m + 1))
, x] + (-Simp[b/a^2   Int[Sin[c + d*x]^(m + 1), x], x] + Simp[(a^2 + b^2)/a
^2   Int[Sin[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /;
 FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

rule 3612
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.14.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

method	result
default	$-\frac{1}{2a\tan(x)^2}+\frac{(a^2+b^2)\ln(\tan(x))}{a^3}+\frac{b}{a^2\tan(x)}-\frac{(a^2+b^2)\ln(a+b\tan(x))}{a^3}$
norman	$\frac{-\frac{1}{8a}-\frac{\tan(\frac{x}{2})^4}{8a}+\frac{b\tan(\frac{x}{2})}{2a^2}-\frac{b\tan(\frac{x}{2})^3}{2a^2}}{\tan(\frac{x}{2})^2}+\frac{(a^2+b^2)\ln(\tan(\frac{x}{2}))}{a^3}-\frac{(a^2+b^2)\ln\left(\tan(\frac{x}{2})^2a-2b\tan(\frac{x}{2})-a\right)}{a^3}$
parallelrisch	$\frac{-\csc(x)^2a^2-\cot(x)^2a^2+4\cot(x)ab-4a^2\ln\left(\frac{-2a\cos(x)-2b\sin(x)}{\cos(x)+1}\right)-4b^2\ln\left(\frac{-2a\cos(x)-2b\sin(x)}{\cos(x)+1}\right)+4\ln(\csc(x)-\cot(x))a^2+4\ln(}{4a^3}$
risch	$\frac{2i(-ia\,e^{2ix}+b\,e^{2ix}-b)}{(e^{2ix}-1)^2a^2}+\frac{\ln(e^{2ix}-1)}{a}+\frac{\ln(e^{2ix}-1)b^2}{a^3}-\frac{\ln\left(e^{2ix}-\frac{ib+a}{ib-a}\right)}{a}-\frac{\ln\left(e^{2ix}-\frac{ib+a}{ib-a}\right)b^2}{a^3}$

input
```
int(csc(x)^3/(a*cos(x)+b*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a/tan(x)^2+(a^2+b^2)/a^3*ln(tan(x))+b/a^2/tan(x)-(a^2+b^2)/a^3*ln(a+b
*tan(x))
```

3.14.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(53) = 106$.

Time = 0.27 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.13

$$\int \frac{\csc^3(x)}{a\cos(x)+b\sin(x)}\,dx = -\frac{2\,ab\cos\left(x\right)\sin\left(x\right) - a^2 + \left(\left(a^2+b^2\right)\cos\left(x\right)^2 - a^2 - b^2\right)\log\left(2\,ab\cos\left(x\right)\sin\left(x\right) + \left(a^2-b^2\right)\cos\left(x\right)^2 + b^2\right.}{2\left(a^3\cos\left(x\right)^2 - a^3\right)}$$

input
```
integrate(csc(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

output
```
-1/2*(2*a*b*cos(x)*sin(x) - a^2 + ((a^2 + b^2)*cos(x)^2 - a^2 - b^2)*log(2
*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) - ((a^2 + b^2)*cos(x)^2 -
 a^2 - b^2)*log(-1/4*cos(x)^2 + 1/4))/(a^3*cos(x)^2 - a^3)
```

3.14.6 Sympy [F]

$$\int \frac{\csc^3(x)}{a\cos(x)+b\sin(x)}\,dx = \int \frac{\csc^3\left(x\right)}{a\cos\left(x\right)+b\sin\left(x\right)}\,dx$$

input
```
integrate(csc(x)**3/(a*cos(x)+b*sin(x)),x)
```

output
```
Integral(csc(x)**3/(a*cos(x) + b*sin(x)), x)
```

3.14.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(53) = 106$.

Time = 0.20 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.16

$$\int \frac{\csc^3(x)}{a\cos(x)+b\sin(x)}\,dx = -\frac{\frac{4\,b\sin(x)}{\cos(x)+1} + \frac{a\sin(x)^2}{(\cos(x)+1)^2}}{8\,a^2} - \frac{\left(a^2+b^2\right)\log\left(-a - \frac{2\,b\sin(x)}{\cos(x)+1} + \frac{a\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^3} + \frac{\left(a^2+b^2\right)\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^3} - \frac{\left(a - \frac{4\,b\sin(x)}{\cos(x)+1}\right)\left(\cos\left(x\right)+1\right)^2}{8\,a^2\sin\left(x\right)^2}$$

input
```
integrate(csc(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

output
```
-1/8*(4*b*sin(x)/(cos(x) + 1) + a*sin(x)^2/(cos(x) + 1)^2)/a^2 - (a^2 + b^
2)*log(-a - 2*b*sin(x)/(cos(x) + 1) + a*sin(x)^2/(cos(x) + 1)^2)/a^3 + (a^
2 + b^2)*log(sin(x)/(cos(x) + 1))/a^3 - 1/8*(a - 4*b*sin(x)/(cos(x) + 1))*
(cos(x) + 1)^2/(a^2*sin(x)^2)
```

3.14.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.42

$$\int \frac{\csc^3(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{(a^2+b^2)\log(|\tan(x)|)}{a^3} - \frac{(a^2b+b^3)\log(|b\tan(x)+a|)}{a^3b} - \frac{3a^2\tan(x)^2+3b^2\tan(x)^2-2ab\tan(x)+a^2}{2a^3\tan(x)^2}$$

input
```
integrate(csc(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

output
```
(a^2 + b^2)*log(abs(tan(x)))/a^3 - (a^2*b + b^3)*log(abs(b*tan(x) + a))/(a
^3*b) - 1/2*(3*a^2*tan(x)^2 + 3*b^2*tan(x)^2 - 2*a*b*tan(x) + a^2)/(a^3*ta
n(x)^2)
```

3.14.9 Mupad [B] (verification not implemented)

Time = 21.32 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.65

$$\int \frac{\csc^3(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)(a^2+b^2)}{a^3} - \frac{\ln\left(-a\tan\left(\frac{x}{2}\right)^2+2b\tan\left(\frac{x}{2}\right)+a\right)(a^2+b^2)}{a^3} - \frac{\tan\left(\frac{x}{2}\right)^2}{8a} - \frac{b\tan\left(\frac{x}{2}\right)}{2a^2} - \frac{\frac{a}{2}-2b\tan\left(\frac{x}{2}\right)}{4a^2\tan\left(\frac{x}{2}\right)^2}$$

input
```
int(1/(sin(x)^3*(a*cos(x) + b*sin(x))),x)
```

output
```
(log(tan(x/2))*(a^2 + b^2))/a^3 - (log(a + 2*b*tan(x/2) - a*tan(x/2)^2)*(a
^2 + b^2))/a^3 - tan(x/2)^2/(8*a) - (b*tan(x/2))/(2*a^2) - (a/2 - 2*b*tan(
x/2))/(4*a^2*tan(x/2)^2)
```

3.14. $\int \frac{\csc^3(x)}{a\cos(x)+b\sin(x)}\,dx$

3.15 $\int \frac{\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

3.15.1 Optimal result

Integrand size = 16, antiderivative size = 107

$$\int \frac{\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{6a^2b\operatorname{arctanh}\left(\frac{-b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} + \frac{3a(a^2-b^2)+a(a^2+b^2)\cos(2x)-b(a^2+b^2)\sin(2x)}{2\,(a^2+b^2)^2\,(a\cos(x)+b\sin(x))}$$

output
```
6*a^2*b*arctanh((-b+a*tan(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2)+1/2*(3*
a*(a^2-b^2)+a*(a^2+b^2)*cos(2*x)-b*(a^2+b^2)*sin(2*x))/(a^2+b^2)^2/(a*cos(
x)+b*sin(x))
```

3.15.2 Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00

$$\int \frac{\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{6a^2b\operatorname{arctanh}\left(\frac{-b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} + \frac{3a(a^2-b^2)+a(a^2+b^2)\cos(2x)-b(a^2+b^2)\sin(2x)}{2\,(a^2+b^2)^2\,(a\cos(x)+b\sin(x))}$$

input
```
Integrate[Sin[x]^3/(a*Cos[x] + b*Sin[x])^2,x]
```

output
```
(6*a^2*b*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) + (
3*a*(a^2 - b^2) + a*(a^2 + b^2)*Cos[2*x] - b*(a^2 + b^2)*Sin[2*x])/(2*(a^2
 + b^2)^2*(a*Cos[x] + b*Sin[x]))
```

3.15.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 283 vs. 2(107) = 214.

Time = 1.23 (sec) , antiderivative size = 283, normalized size of antiderivative = 2.64, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3042, 4901, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)^3}{(a\cos(x)+b\sin(x))^2}dx$$

$$\Big\downarrow \text{4901}$$

$$\int \left(-\frac{a^3\cos^3(x)}{b^3(a\cos(x)+b\sin(x))^2}+\frac{3a^2\cos^2(x)}{b^3(a\cos(x)+b\sin(x))}-\frac{2a\cos(x)}{b^3}+\frac{\sin(x)}{b^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^2(3a^2+b^2)\,\text{arctanh}\left(\frac{b-a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b\,(a^2+b^2)^{5/2}}-\frac{2a^2b\text{arctanh}\left(\frac{b-a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}-\frac{3a^2\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{b\,(a^2+b^2)^{3/2}}+\frac{3a^2\cos(x)}{b^2\,(a^2+b^2)}+\frac{2a^2\left(a+b\tan\left(\frac{x}{2}\right)\right)}{(a^2+b^2)^2\left(-a\tan^2\left(\frac{x}{2}\right)+a+2b\tan\left(\frac{x}{2}\right)\right)}+\frac{3a^3\sin(x)}{b^3\,(a^2+b^2)}-\frac{2a^3\cos^2\left(\frac{x}{2}\right)\left((a^2-b^2)\tan\left(\frac{x}{2}\right)+2ab\right)}{b^3\,(a^2+b^2)^2}-\frac{2a\sin(x)}{b^3}-\frac{\cos(x)}{b^2}$$

input
```
Int[Sin[x]^3/(a*Cos[x] + b*Sin[x])^2,x]
```

output
```
(-3*a^2*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(b*(a^2 + b^2)^(3/
2)) - (2*a^2*b*ArcTanh[(b - a*Tan[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2
) + (2*a^2*(3*a^2 + b^2)*ArcTanh[(b - a*Tan[x/2])/Sqrt[a^2 + b^2]])/(b*(a^
2 + b^2)^(5/2)) - Cos[x]/b^2 + (3*a^2*Cos[x])/(b^2*(a^2 + b^2)) - (2*a*Sin
[x])/b^3 + (3*a^3*Sin[x])/(b^3*(a^2 + b^2)) - (2*a^3*Cos[x/2]^2*(2*a*b + (
a^2 - b^2)*Tan[x/2]))/(b^3*(a^2 + b^2)^2) + (2*a^2*(a + b*Tan[x/2]))/((a^2
 + b^2)^2*(a + 2*b*Tan[x/2] - a*Tan[x/2]^2))
```

3.15.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4901
```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

3.15.4 Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.31

method	result
default	$-\frac{4\left(\tan\left(\frac{x}{2}\right)ab-\frac{a^2}{2}+\frac{b^2}{2}\right)}{\left(a^4+2a^2b^2+b^4\right)\left(1+\tan\left(\frac{x}{2}\right)^2\right)}+\frac{4a^2\left(\frac{-\frac{b\tan\left(\frac{x}{2}\right)}{2}-\frac{a}{2}}{\tan\left(\frac{x}{2}\right)^2 a-2b\tan\left(\frac{x}{2}\right)-a}+\frac{3b\,\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{2\sqrt{a^2+b^2}}\right)}{a^4+2a^2b^2+b^4}$
risch	$\frac{e^{ix}}{-4iba+2a^2-2b^2}+\frac{e^{-ix}}{4iba+2a^2-2b^2}+\frac{2a^3 e^{ix}}{\left(-ib\,e^{2ix}+a\,e^{2ix}+ib+a\right)\left(ib+a\right)^2\left(-ib+a\right)^2}-\frac{3ib\,a^2\ln\left(e^{ix}+\frac{ib+a}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}\,\left(a^2+b^2\right)^2}+\frac{3ib\,a^2\ln\left(e^{ix}\right.}{\sqrt{-a^2-b^2}}$

input
```
int(sin(x)^3/(a*cos(x)+b*sin(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-4/(a^4+2*a^2*b^2+b^4)*(tan(1/2*x)*a*b-1/2*a^2+1/2*b^2)/(1+tan(1/2*x)^2)+4
*a^2/(a^4+2*a^2*b^2+b^4)*((-1/2*b*tan(1/2*x)-1/2*a)/(tan(1/2*x)^2*a-2*b*ta
n(1/2*x)-a)+3/2*b/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*x)-2*b)/(a^2+b^
2)^(1/2)))
```

3.15.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(99) = 198$.

Time = 0.27 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.24

$$\int \frac{\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{2\,a^5 - 2\,a^3b^2 - 4\,ab^4 + 2\,(a^5 + 2\,a^3b^2 + ab^4)\cos(x)^2 - 2\,(a^4b + 2\,a^2b^3 + b^5)\cos(x)\sin(x) + 3\,(a^3b\cos(x)}{2\,((a^7 + 3\,a^5b^2 + 3\,a^3b^4 + ab^6)\cos(x) + (a^6b +}$$

input
```
integrate(sin(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

output
```
1/2*(2*a^5 - 2*a^3*b^2 - 4*a*b^4 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*cos(x)^2 -
2*(a^4*b + 2*a^2*b^3 + b^5)*cos(x)*sin(x) + 3*(a^3*b*cos(x) + a^2*b^2*sin(
x))*sqrt(a^2 + b^2)*log(-(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 - 2*a
^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos(x)*sin(x) +
 (a^2 - b^2)*cos(x)^2 + b^2))/((a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cos(
x) + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*sin(x))
```

3.15.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \text{Timed out}$$

input
```
integrate(sin(x)**3/(a*cos(x)+b*sin(x))**2,x)
```

output
```
Timed out
```

3.15.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(99) = 198$.

Time = 0.31 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.36

$$\int \frac{\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx = -\frac{3\,a^2b\log\left(\frac{b-\frac{a\sin(x)}{\cos(x)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(x)}{\cos(x)+1}-\sqrt{a^2+b^2}}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}} + \frac{2\left(2\,a^3-ab^2-\frac{3\,ab^2\sin(x)^2}{(\cos(x)+1)^2}+\frac{3\,a^2b\sin(x)^3}{(\cos(x)+1)^3}+\frac{(a^2b-2\,b^3)\sin(x)}{\cos(x)+1}\right)}{a^5+2\,a^3b^2+ab^4+\frac{2\,(a^4b+2\,a^2b^3+b^5)\sin(x)}{\cos(x)+1}+\frac{2\,(a^4b+2\,a^2b^3+b^5)\sin(x)^3}{(\cos(x)+1)^3}-\frac{(a^5+2\,a^3b^2+ab^4)\sin(x)^4}{(\cos(x)+1)^4}}$$

input
```
integrate(sin(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

output
```
-3*a^2*b*log((b - a*sin(x)/(cos(x) + 1) + sqrt(a^2 + b^2))/(b - a*sin(x)/(
cos(x) + 1) - sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2))
+ 2*(2*a^3 - a*b^2 - 3*a*b^2*sin(x)^2/(cos(x) + 1)^2 + 3*a^2*b*sin(x)^3/(c
os(x) + 1)^3 + (a^2*b - 2*b^3)*sin(x)/(cos(x) + 1))/(a^5 + 2*a^3*b^2 + a*b
^4 + 2*(a^4*b + 2*a^2*b^3 + b^5)*sin(x)/(cos(x) + 1) + 2*(a^4*b + 2*a^2*b^
3 + b^5)*sin(x)^3/(cos(x) + 1)^3 - (a^5 + 2*a^3*b^2 + a*b^4)*sin(x)^4/(cos
(x) + 1)^4)
```

3.15.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.74

$$\int \frac{\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx = -\frac{3\,a^2b\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}} - \frac{2\left(3\,a^2b\tan\left(\frac{1}{2}\,x\right)^3-3\,ab^2\tan\left(\frac{1}{2}\,x\right)^2+a^2b\tan\left(\frac{1}{2}\,x\right)-2\,b^3\tan\left(\frac{1}{2}\,x\right)+2\,a^3-ab^2\right)}{\left(a\tan\left(\frac{1}{2}\,x\right)^4-2\,b\tan\left(\frac{1}{2}\,x\right)^3-2\,b\tan\left(\frac{1}{2}\,x\right)-a\right)(a^4+2\,a^2b^2+b^4)}$$

input
```
integrate(sin(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

output
```
-3*a^2*b*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2
*x) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2))
- 2*(3*a^2*b*tan(1/2*x)^3 - 3*a*b^2*tan(1/2*x)^2 + a^2*b*tan(1/2*x) - 2*b^
3*tan(1/2*x) + 2*a^3 - a*b^2)/((a*tan(1/2*x)^4 - 2*b*tan(1/2*x)^3 - 2*b*ta
n(1/2*x) - a)*(a^4 + 2*a^2*b^2 + b^4))
```

3.15.9 Mupad [B] (verification not implemented)

Time = 21.87 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.09

$$\int \frac{\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx = -\frac{\frac{2\,(a\,b^2-2\,a^3)}{a^4+2\,a^2\,b^2+b^4}-\frac{2\tan\left(\frac{x}{2}\right)\,(a^2\,b-2\,b^3)}{a^4+2\,a^2\,b^2+b^4}+\frac{6\,a\,b^2\tan\left(\frac{x}{2}\right)^2}{a^4+2\,a^2\,b^2+b^4}-\frac{6\,a^2\,b\tan\left(\frac{x}{2}\right)^3}{a^4+2\,a^2\,b^2+b^4}}{-a\tan\left(\frac{x}{2}\right)^4+2\,b\tan\left(\frac{x}{2}\right)^3+2\,b\tan\left(\frac{x}{2}\right)+a} - \frac{6\,a^2\,b\,\mathrm{atanh}\left(\frac{2\,a^4\,b+2\,b^5+4\,a^2\,b^3-2\,a\tan\left(\frac{x}{2}\right)\,(a^4+2\,a^2\,b^2+b^4)}{2\,(a^2+b^2)^{5/2}}\right)}{(a^2+b^2)^{5/2}}$$

input
```
int(sin(x)^3/(a*cos(x) + b*sin(x))^2,x)
```

output
```
- ((2*(a*b^2 - 2*a^3))/(a^4 + b^4 + 2*a^2*b^2) - (2*tan(x/2)*(a^2*b - 2*b^
3))/(a^4 + b^4 + 2*a^2*b^2) + (6*a*b^2*tan(x/2)^2)/(a^4 + b^4 + 2*a^2*b^2)
 - (6*a^2*b*tan(x/2)^3)/(a^4 + b^4 + 2*a^2*b^2))/(a + 2*b*tan(x/2) - a*tan
(x/2)^4 + 2*b*tan(x/2)^3) - (6*a^2*b*atanh((2*a^4*b + 2*b^5 + 4*a^2*b^3 -
2*a*tan(x/2)*(a^4 + b^4 + 2*a^2*b^2))/(2*(a^2 + b^2)^(5/2))))/(a^2 + b^2)^
(5/2)
```

3.16 $\int \frac{\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

3.16.1 Optimal result

Integrand size = 16, antiderivative size = 64

$$\int \frac{\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx = -\frac{(a^2-b^2)\,x}{(a^2+b^2)^2}+\frac{a}{(a^2+b^2)\,(b+a\cot(x))} - \frac{2ab\log(a\cos(x)+b\sin(x))}{(a^2+b^2)^2}$$

output
```
-(a^2-b^2)*x/(a^2+b^2)^2+a/(a^2+b^2)/(b+a*cot(x))-2*a*b*ln(a*cos(x)+b*sin(
x))/(a^2+b^2)^2
```

3.16.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.89

$$\int \frac{\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{-a\cos(x)\left((a+ib)^2x+ab\log\left((a\cos(x)+b\sin(x))^2\right)\right)+\left(a^3+ab^2(1-2ix)-a^2bx+b^3x-ab^2\log\left((a\cos\right.\right.}{(a^2+b^2)^2\,(a\cos(x)+b\sin(x))}$$

input
```
Integrate[Sin[x]^2/(a*Cos[x] + b*Sin[x])^2,x]
```

output
```
(-(a*Cos[x]*((a + I*b)^2*x + a*b*Log[(a*Cos[x] + b*Sin[x])^2])) + (a^3 + a
*b^2*(1 - (2*I)*x) - a^2*b*x + b^3*x - a*b^2*Log[(a*Cos[x] + b*Sin[x])^2])
*Sin[x] + (2*I)*a*b*ArcTan[Tan[x]]*(a*Cos[x] + b*Sin[x]))/((a^2 + b^2)^2*(
a*Cos[x] + b*Sin[x]))
```

3.16.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.17, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {3042, 3564, 3042, 3964, 3042, 4014, 25, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)^2}{(a\cos(x)+b\sin(x))^2}dx$$

$$\Big\downarrow \text{3564}$$

$$\int \frac{1}{(a\cot(x)+b)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(b-a\tan\left(x+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3964}$$

$$\frac{\int \frac{b-a\cot(x)}{b+a\cot(x)}dx}{a^2+b^2}+\frac{a}{(a^2+b^2)(a\cot(x)+b)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{b+a\tan\left(x+\frac{\pi}{2}\right)}{b-a\tan\left(x+\frac{\pi}{2}\right)}dx}{a^2+b^2}+\frac{a}{(a^2+b^2)(a\cot(x)+b)}$$

$$\Big\downarrow \text{4014}$$

$$\frac{-\frac{2ab\int -\frac{a-b\cot(x)}{b+a\cot(x)}dx}{a^2+b^2}-\frac{x(a^2-b^2)}{a^2+b^2}}{a^2+b^2}+\frac{a}{(a^2+b^2)(a\cot(x)+b)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{2ab\int \frac{a-b\cot(x)}{b+a\cot(x)}dx}{a^2+b^2}-\frac{x(a^2-b^2)}{a^2+b^2}}{a^2+b^2}+\frac{a}{(a^2+b^2)\,(a\cot(x)+b)}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{2ab\int \frac{a+b\tan\left(x+\frac{\pi}{2}\right)}{b-a\tan\left(x+\frac{\pi}{2}\right)}dx}{a^2+b^2}-\frac{x(a^2-b^2)}{a^2+b^2}}{a^2+b^2}+\frac{a}{(a^2+b^2)\,(a\cot(x)+b)}$$

$$\Big\downarrow \; 4013$$

$$\frac{a}{(a^2+b^2)\,(a\cot(x)+b)}+\frac{-\frac{x(a^2-b^2)}{a^2+b^2}-\frac{2ab\log(a\cos(x)+b\sin(x))}{a^2+b^2}}{a^2+b^2}$$

input
```
Int[Sin[x]^2/(a*Cos[x] + b*Sin[x])^2,x]
```

output
```
a/((a^2 + b^2)*(b + a*Cot[x])) + (-(((a^2 - b^2)*x)/(a^2 + b^2)) - (2*a*b*
Log[a*Cos[x] + b*Sin[x]])/(a^2 + b^2))/(a^2 + b^2)
```

3.16.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3564
```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(b + a*Cot[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

rule 3964
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)
 Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

3.16.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.27

method	result
default	$-\frac{a^{2}}{\left(a^{2}+b^{2}\right) b \left(a+b \tan \left(x \right)\right)}-\frac{2 a b \ln \left(a+b \tan \left(x \right)\right)}{\left(a^{2}+b^{2}\right)^{2}}+\frac{a b \ln \left(1+\tan \left(x \right)^{2}\right)+\left(-a^{2}+b^{2}\right) \arctan \left(\tan \left(x \right)\right)}{\left(a^{2}+b^{2}\right)^{2}}$
parallelrisch	$\frac{\left(-2 a^{2} b \cos \left(x \right)-2 \sin \left(x \right) a \,b^{2}\right) \ln \left(\frac{-a \cos \left(x \right)-b \sin \left(x \right)}{\cos \left(x \right)+1}\right)+\left(2 a^{2} b \cos \left(x \right)+2 \sin \left(x \right) a \,b^{2}\right) \ln \left(\frac{1}{\cos \left(x \right)+1}\right)+\left(-x \,a^{2} b +x \,b^{3}+a^{3}+a \,b^{2}\right) \sin \left(x \right.}{\left(a \cos \left(x \right)+b \sin \left(x \right)\right) \left(a^{2}+b^{2}\right)^{2}}$
risch	$\frac{x}{2 i b a -a^{2}+b^{2}}+\frac{4 i a b x}{a^{4}+2 a^{2} b^{2}+b^{4}}+\frac{2 i a^{2}}{\left(-i b \,{\mathrm e}^{2 i x}+a \,{\mathrm e}^{2 i x}+i b +a \right) \left(i b +a \right) \left(-i b +a \right)^{2}}-\frac{2 a b \ln \left({\mathrm e}^{2 i x}-\frac{i b +a}{i b -a}\right)}{a^{4}+2 a^{2} b^{2}+b^{4}}$
norman	$\frac{\frac{a \left(a^{2}-b^{2}\right) x}{a^{4}+2 a^{2} b^{2}+b^{4}}+\frac{a \left(a^{2}-b^{2}\right) x \tan \left(\frac{x}{2}\right)^{2}}{a^{4}+2 a^{2} b^{2}+b^{4}}-\frac{2 a \tan \left(\frac{x}{2}\right)}{a^{2}+b^{2}}-\frac{4 a \tan \left(\frac{x}{2}\right)^{3}}{a^{2}+b^{2}}-\frac{2 a \tan \left(\frac{x}{2}\right)^{5}}{a^{2}+b^{2}}-\frac{a \left(a^{2}-b^{2}\right) x \tan \left(\frac{x}{2}\right)^{4}}{a^{4}+2 a^{2} b^{2}+b^{4}}-\frac{a \left(a^{2}-b^{2}\right) x \tan \left(\frac{x}{2}\right)^{6}}{a^{4}+2 a^{2} b^{2}+b^{4}}+\frac{2 b \left(a^{2}-\right.}{a^{4}+}}{\left(1+\tan \left(\frac{x}{2}\right)^{2}\right)^{2} \left(\tan \left(\frac{x}{2}\right)^{2} a -2 b \tan \left(\frac{x}{2}\right)-a \right)}$

input
```
int(sin(x)^2/(a*cos(x)+b*sin(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-a^2/(a^2+b^2)/b/(a+b*tan(x))-2*a*b/(a^2+b^2)^2*ln(a+b*tan(x))+1/(a^2+b^2)
^2*(a*b*ln(1+tan(x)^2)+(-a^2+b^2)*arctan(tan(x)))
```

3.16.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(64) = 128$.

Time = 0.26 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.06

$$\int \frac{\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx = -\frac{(a^2b+(a^3-ab^2)x)\cos(x)+(a^2b\cos(x)+ab^2\sin(x))\log\left(2\,ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2+b^2\right)}{(a^5+2\,a^3b^2+ab^4)\cos(x)+(a^4b+2\,a^2b^3+b^5)\sin(x)}$$

input
```
integrate(sin(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

output
```
-((a^2*b + (a^3 - a*b^2)*x)*cos(x) + (a^2*b*cos(x) + a*b^2*sin(x))*log(2*a
*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) - (a^3 - (a^2*b - b^3)*x)*s
in(x))/((a^5 + 2*a^3*b^2 + a*b^4)*cos(x) + (a^4*b + 2*a^2*b^3 + b^5)*sin(x
))
```

3.16.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(sin(x)**2/(a*cos(x)+b*sin(x))**2,x)
```

output
```
Exception raised: AttributeError >> 'NoneType' object has no attribute 'pr
imitive'
```

3.16.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.83

$$\int \frac{\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx = -\frac{2\,ab\log\left(b\tan(x)+a\right)}{a^4+2\,a^2b^2+b^4}+\frac{ab\log\left(\tan(x)^2+1\right)}{a^4+2\,a^2b^2+b^4} - \frac{a^2}{a^3b+ab^3+(a^2b^2+b^4)\tan(x)} - \frac{(a^2-b^2)x}{a^4+2\,a^2b^2+b^4}$$

input
```
integrate(sin(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

output
```
-2*a*b*log(b*tan(x) + a)/(a^4 + 2*a^2*b^2 + b^4) + a*b*log(tan(x)^2 + 1)/(
a^4 + 2*a^2*b^2 + b^4) - a^2/(a^3*b + a*b^3 + (a^2*b^2 + b^4)*tan(x)) - (a
^2 - b^2)*x/(a^4 + 2*a^2*b^2 + b^4)
```

3.16.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(64) = 128.

Time = 0.28 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.17

$$\int \frac{\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx = -\frac{2\,ab^2\log(|b\tan(x)+a|)}{a^4b+2\,a^2b^3+b^5}+\frac{ab\log\left(\tan(x)^2+1\right)}{a^4+2\,a^2b^2+b^4} - \frac{(a^2-b^2)x}{a^4+2\,a^2b^2+b^4}+\frac{2\,ab^3\tan(x)-a^4+a^2b^2}{(a^4b+2\,a^2b^3+b^5)(b\tan(x)+a)}$$

input
```
integrate(sin(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

output
```
-2*a*b^2*log(abs(b*tan(x) + a))/(a^4*b + 2*a^2*b^3 + b^5) + a*b*log(tan(x)
^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) - (a^2 - b^2)*x/(a^4 + 2*a^2*b^2 + b^4) +
(2*a*b^3*tan(x) - a^4 + a^2*b^2)/((a^4*b + 2*a^2*b^3 + b^5)*(b*tan(x) + a)
)
```

3.16.9 Mupad [B] (verification not implemented)

Time = 27.72 (sec) , antiderivative size = 626, normalized size of antiderivative = 9.78

$$\int \frac{\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx$$

$$= \frac{a^3\sin(x)+a\,b^2\sin(x)-2\,a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)\cos(x)+2\,b^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)\sin(x)+2\,a\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)\cos(}{}$$

input
```
int(sin(x)^2/(a*cos(x) + b*sin(x))^2,x)
```

output
```
(a^3*sin(x) + a*b^2*sin(x) - 2*a^3*atan(sin(x/2)/cos(x/2))*cos(x) + 2*b^3*
atan(sin(x/2)/cos(x/2))*sin(x) + 2*a*b^2*atan(sin(x/2)/cos(x/2))*cos(x) -
2*a^2*b*atan(sin(x/2)/cos(x/2))*sin(x) + 2*a^2*b*cos(x)*log((1024*a^14 + 1
024*a^2*b^12 + 26624*a^4*b^10 + 146432*a^6*b^8 - 348160*a^8*b^6 + 146432*a
^10*b^4 + 26624*a^12*b^2)/(a^16/2 + b^16/2 + 4*a^2*b^14 + 14*a^4*b^12 + 28
*a^6*b^10 + 35*a^8*b^8 + 28*a^10*b^6 + 14*a^12*b^4 + 4*a^14*b^2 + (a^16*co
s(x))/2 + (b^16*cos(x))/2 + 4*a^2*b^14*cos(x) + 14*a^4*b^12*cos(x) + 28*a^
6*b^10*cos(x) + 35*a^8*b^8*cos(x) + 28*a^10*b^6*cos(x) + 14*a^12*b^4*cos(x
) + 4*a^14*b^2*cos(x))) + 2*a*b^2*log((1024*a^14 + 1024*a^2*b^12 + 26624*a
^4*b^10 + 146432*a^6*b^8 - 348160*a^8*b^6 + 146432*a^10*b^4 + 26624*a^12*b
^2)/(a^16/2 + b^16/2 + 4*a^2*b^14 + 14*a^4*b^12 + 28*a^6*b^10 + 35*a^8*b^8
 + 28*a^10*b^6 + 14*a^12*b^4 + 4*a^14*b^2 + (a^16*cos(x))/2 + (b^16*cos(x)
)/2 + 4*a^2*b^14*cos(x) + 14*a^4*b^12*cos(x) + 28*a^6*b^10*cos(x) + 35*a^8
*b^8*cos(x) + 28*a^10*b^6*cos(x) + 14*a^12*b^4*cos(x) + 4*a^14*b^2*cos(x))
)*sin(x) - 2*a^2*b*log((a*cos(x) + b*sin(x))/cos(x/2)^2)*cos(x) - 2*a*b^2*
log((a*cos(x) + b*sin(x))/cos(x/2)^2)*sin(x))/(b^5*sin(x) + a^5*cos(x) + a
*b^4*cos(x) + a^4*b*sin(x) + 2*a^3*b^2*cos(x) + 2*a^2*b^3*sin(x))
```

3.16. $\int \frac{\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

3.17 $\int \frac{\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

3.17.1 Optimal result

Integrand size = 14, antiderivative size = 60

$$\int \frac{\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx = -\frac{b\,\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{a}{(a^2+b^2)\,(a\cos(x)+b\sin(x))}$$

output
```
-b*arctanh((b*cos(x)-a*sin(x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)+a/(a^2+b^2
)/(a*cos(x)+b*sin(x))
```

3.17.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.03

$$\int \frac{\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{2b\,\text{arctanh}\left(\frac{-b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{a}{(a^2+b^2)\,(a\cos(x)+b\sin(x))}$$

input
```
Integrate[Sin[x]/(a*Cos[x] + b*Sin[x])^2,x]
```

output
```
(2*b*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2) + a/((a
^2 + b^2)*(a*Cos[x] + b*Sin[x]))
```

3.17.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3042, 3633, 3042, 3553, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)}{(a\cos(x)+b\sin(x))^2}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\sin(x)}{(a\cos(x)+b\sin(x))^2}dx$$
$$\Big\downarrow \text{ 3633}$$
$$\frac{b\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a}{(a^2+b^2)(a\cos(x)+b\sin(x))}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{b\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a}{(a^2+b^2)(a\cos(x)+b\sin(x))}$$
$$\Big\downarrow \text{ 3553}$$
$$\frac{a}{(a^2+b^2)(a\cos(x)+b\sin(x))}-\frac{b\int \frac{1}{a^2+b^2-(b\cos(x)-a\sin(x))^2}d(b\cos(x)-a\sin(x))}{a^2+b^2}$$
$$\Big\downarrow \text{ 219}$$
$$\frac{a}{(a^2+b^2)(a\cos(x)+b\sin(x))}-\frac{b\,\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}$$

input
```
Int[Sin[x]/(a*Cos[x] + b*Sin[x])^2,x]
```

output
```
-((b*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2)) +
a/((a^2 + b^2)*(a*Cos[x] + b*Sin[x]))
```

3.17.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3633
```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[-(b*C + (a*C
- c*A)*Cos[d + e*x] + b*A*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
 e*x] + c*Sin[d + e*x])), x] + Simp[(a*A - c*C)/(a^2 - b^2 - c^2)   Int[1/(
a + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, C}
, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - c*C, 0]
```

3.17.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.62

method	result	size
default	$\frac{8b\tan\left(\frac{x}{2}\right)+8a}{\left(-4a^{2}-4b^{2}\right)\left(\tan\left(\frac{x}{2}\right)^{2}a-2b\tan\left(\frac{x}{2}\right)-a\right)}-\frac{8b\,\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{x}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{\left(-4a^{2}-4b^{2}\right)\sqrt{a^{2}+b^{2}}}$	97
risch	$\frac{2a\,\mathrm{e}^{ix}}{(ib+a)(-ib+a)\left(-ib\,\mathrm{e}^{2ix}+a\,\mathrm{e}^{2ix}+ib+a\right)}+\frac{ib\ln\left(\mathrm{e}^{ix}-\frac{ib+a}{\sqrt{-a^{2}-b^{2}}}\right)}{\sqrt{-a^{2}-b^{2}}\,(a^{2}+b^{2})}-\frac{ib\ln\left(\mathrm{e}^{ix}+\frac{ib+a}{\sqrt{-a^{2}-b^{2}}}\right)}{\sqrt{-a^{2}-b^{2}}\,(a^{2}+b^{2})}$	157

input
```
int(sin(x)/(a*cos(x)+b*sin(x))^2,x,method=_RETURNVERBOSE)
```

output
```
4*(2*b*tan(1/2*x)+2*a)/(-4*a^2-4*b^2)/(tan(1/2*x)^2*a-2*b*tan(1/2*x)-a)-8*
b/(-4*a^2-4*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*x)-2*b)/(a^2+b^2
)^(1/2))
```

3.17.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(56) = 112$.

Time = 0.26 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.73

$$\int \frac{\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx$$
$$= \frac{2\,a^3 + 2\,ab^2 + \left(ab\cos\left(x\right) + b^2\sin\left(x\right)\right)\sqrt{a^2+b^2}\log\left(-\frac{2\,ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}(b\cos(x)-a\,\mathrm{si}}{2\,ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2+b^2}\right.}{2\left(\left(a^5+2\,a^3b^2+ab^4\right)\cos\left(x\right)+\left(a^4b+2\,a^2b^3+b^5\right)\sin\left(x\right)\right)}$$

input
```
integrate(sin(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

output
```
1/2*(2*a^3 + 2*a*b^2 + (a*b*cos(x) + b^2*sin(x))*sqrt(a^2 + b^2)*log(-(2*a
*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*
(b*cos(x) - a*sin(x)))/(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2))
)/((a^5 + 2*a^3*b^2 + a*b^4)*cos(x) + (a^4*b + 2*a^2*b^3 + b^5)*sin(x))
```

3.17.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(sin(x)/(a*cos(x)+b*sin(x))**2,x)
```

output
```
Exception raised: AttributeError >> 'NoneType' object has no attribute 'pr
imitive'
```

3.17.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(56) = 112$.

Time = 0.31 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.13

$$\int \frac{\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx = -\frac{b\log\left(\frac{b-\frac{a\sin(x)}{\cos(x)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(x)}{\cos(x)+1}-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}} + \frac{2\left(a+\frac{b\sin(x)}{\cos(x)+1}\right)}{a^3+ab^2+\frac{2\,(a^2b+b^3)\sin(x)}{\cos(x)+1}-\frac{(a^3+ab^2)\sin(x)^2}{(\cos(x)+1)^2}}$$

input
```
integrate(sin(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

output
```
-b*log((b - a*sin(x)/(cos(x) + 1) + sqrt(a^2 + b^2))/(b - a*sin(x)/(cos(x)
 + 1) - sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) + 2*(a + b*sin(x)/(cos(x) + 1)
)/(a^3 + a*b^2 + 2*(a^2*b + b^3)*sin(x)/(cos(x) + 1) - (a^3 + a*b^2)*sin(x
)^2/(cos(x) + 1)^2)
```

3.17.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.72

$$\int \frac{\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx = -\frac{b\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}} - \frac{2\,\left(b\tan\left(\frac{1}{2}\,x\right)+a\right)}{\left(a\tan\left(\frac{1}{2}\,x\right)^2-2\,b\tan\left(\frac{1}{2}\,x\right)-a\right)\left(a^2+b^2\right)}$$

input
```
integrate(sin(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

output
```
-b*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*x) -
2*b + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(b*tan(1/2*x) + a)/((a*tan
(1/2*x)^2 - 2*b*tan(1/2*x) - a)*(a^2 + b^2))
```

3.17.9 Mupad [B] (verification not implemented)

Time = 21.69 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.43

$$\int \frac{\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{\frac{2\,a}{a^2+b^2}+\frac{2\,b\tan\left(\frac{x}{2}\right)}{a^2+b^2}}{-a\tan\left(\frac{x}{2}\right)^2+2\,b\tan\left(\frac{x}{2}\right)+a} - \frac{2\,b\,\mathrm{atanh}\left(\frac{2\,b-2\,a\tan\left(\frac{x}{2}\right)}{2\,\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}$$

input
```
int(sin(x)/(a*cos(x) + b*sin(x))^2,x)
```

output
```
((2*a)/(a^2 + b^2) + (2*b*tan(x/2))/(a^2 + b^2))/(a + 2*b*tan(x/2) - a*tan
(x/2)^2) - (2*b*atanh((2*b - 2*a*tan(x/2))/(2*(a^2 + b^2)^(1/2))))/(a^2 +
b^2)^(3/2)
```

3.18 $\int \frac{1}{(a\cos(x)+b\sin(x))^2}\,dx$

3.18.1 Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{1}{(a\cos(x)+b\sin(x))^2}\,dx=\frac{\sin(x)}{a(a\cos(x)+b\sin(x))}$$

output
```
sin(x)/a/(a*cos(x)+b*sin(x))
```

3.18.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a\cos(x)+b\sin(x))^2}\,dx=\frac{\sin(x)}{a(a\cos(x)+b\sin(x))}$$

input
```
Integrate[(a*Cos[x] + b*Sin[x])^(-2),x]
```

output
```
Sin[x]/(a*(a*Cos[x] + b*Sin[x]))
```

3.18.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3042, 3554}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cos(x)+b\sin(x))^2}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{(a\cos(x)+b\sin(x))^2}dx$$

$$\Big\downarrow \ 3554$$

$$\frac{\sin(x)}{a(a\cos(x)+b\sin(x))}$$

input
```
Int[(a*Cos[x] + b*Sin[x])^(-2),x]
```

output
```
Sin[x]/(a*(a*Cos[x] + b*Sin[x]))
```

3.18.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3554
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

3.18.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{1}{b(a+b\tan(x))}$	14
parallelrisch	$\frac{\sin(x)}{a(a\cos(x)+b\sin(x))}$	18
norman	$-\frac{2\tan\left(\frac{x}{2}\right)}{a\left(\tan\left(\frac{x}{2}\right)^2 a-2b\tan\left(\frac{x}{2}\right)-a\right)}$	31
risch	$\frac{2i}{\left(-ib\,e^{2ix}+a\,e^{2ix}+ib+a\right)(-ib+a)}$	36

input
```
int(1/(a*cos(x)+b*sin(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/b/(a+b*tan(x))
```

3.18.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(17) = 34.

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.29

$$\int \frac{1}{(a\cos(x)+b\sin(x))^2}\,dx = -\frac{b\cos(x)-a\sin(x)}{(a^3+ab^2)\cos(x)+(a^2b+b^3)\sin(x)}$$

input
```
integrate(1/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

output
```
-(b*cos(x) - a*sin(x))/((a^3 + a*b^2)*cos(x) + (a^2*b + b^3)*sin(x))
```

3.18.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 605 vs. 2(14) = 28.

Time = 157.81 (sec) , antiderivative size = 605, normalized size of antiderivative = 35.59

$$\int \frac{1}{(a\cos(x) + b\sin(x))^2}\,dx$$
$$= \begin{cases} \tilde{\infty}\left(\frac{\tan\left(\frac{x}{2}\right)}{2} - \frac{1}{2\tan\left(\frac{x}{2}\right)}\right) \\ \frac{\frac{\tan\left(\frac{x}{2}\right)}{2} - \frac{1}{2\tan\left(\frac{x}{2}\right)}}{b^2} \\ \frac{x\tan^4\left(\frac{x}{2}\right)}{2b^2\sin^2(x)\tan^4\left(\frac{x}{2}\right) - 4b^2\sin^2(x)\tan^2\left(\frac{x}{2}\right) + 2b^2\sin^2(x) + 8b^2\sin(x)\cos(x)\tan^3\left(\frac{x}{2}\right) - 8b^2\sin(x)\cos(x)\tan\left(\frac{x}{2}\right) + 8b^2\cos^2(x)\tan^2\left(\frac{x}{2}\right)} + \frac{}{2b} \\ -\frac{2\tan\left(\frac{x}{2}\right)}{a^2\tan^2\left(\frac{x}{2}\right) - a^2 - 2ab\tan\left(\frac{x}{2}\right)} \end{cases}$$

input
```
integrate(1/(a*cos(x)+b*sin(x))**2,x)
```

output
```
Piecewise((zoo*(tan(x/2)/2 - 1/(2*tan(x/2))), Eq(a, 0) & Eq(b, 0)), ((tan(
x/2)/2 - 1/(2*tan(x/2)))/b**2, Eq(a, 0)), (x*tan(x/2)**4/(2*b**2*sin(x)**2
*tan(x/2)**4 - 4*b**2*sin(x)**2*tan(x/2)**2 + 2*b**2*sin(x)**2 + 8*b**2*si
n(x)*cos(x)*tan(x/2)**3 - 8*b**2*sin(x)*cos(x)*tan(x/2) + 8*b**2*cos(x)**2
*tan(x/2)**2) + 2*x*tan(x/2)**2/(2*b**2*sin(x)**2*tan(x/2)**4 - 4*b**2*sin
(x)**2*tan(x/2)**2 + 2*b**2*sin(x)**2 + 8*b**2*sin(x)*cos(x)*tan(x/2)**3 -
 8*b**2*sin(x)*cos(x)*tan(x/2) + 8*b**2*cos(x)**2*tan(x/2)**2) + x/(2*b**2
*sin(x)**2*tan(x/2)**4 - 4*b**2*sin(x)**2*tan(x/2)**2 + 2*b**2*sin(x)**2 +
 8*b**2*sin(x)*cos(x)*tan(x/2)**3 - 8*b**2*sin(x)*cos(x)*tan(x/2) + 8*b**2
*cos(x)**2*tan(x/2)**2) - 2*tan(x/2)**3/(2*b**2*sin(x)**2*tan(x/2)**4 - 4*
b**2*sin(x)**2*tan(x/2)**2 + 2*b**2*sin(x)**2 + 8*b**2*sin(x)*cos(x)*tan(x
/2)**3 - 8*b**2*sin(x)*cos(x)*tan(x/2) + 8*b**2*cos(x)**2*tan(x/2)**2) + 2
*tan(x/2)/(2*b**2*sin(x)**2*tan(x/2)**4 - 4*b**2*sin(x)**2*tan(x/2)**2 + 2
*b**2*sin(x)**2 + 8*b**2*sin(x)*cos(x)*tan(x/2)**3 - 8*b**2*sin(x)*cos(x)*
tan(x/2) + 8*b**2*cos(x)**2*tan(x/2)**2), Eq(a, 2*b*tan(x/2)/((tan(x/2) -
1)*(tan(x/2) + 1)))), (-2*tan(x/2)/(a**2*tan(x/2)**2 - a**2 - 2*a*b*tan(x/
2)), True))
```

3.18. $\int \frac{1}{(a\cos(x)+b\sin(x))^2}\,dx$

3.18.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{1}{(a\cos(x)+b\sin(x))^2}\,dx = -\frac{1}{b^2\tan(x)+ab}$$

input
```
integrate(1/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

output
```
-1/(b^2*tan(x) + a*b)
```

3.18.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{1}{(a\cos(x)+b\sin(x))^2}\,dx = -\frac{1}{(b\tan(x)+a)b}$$

input
```
integrate(1/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

output
```
-1/((b*tan(x) + a)*b)
```

3.18.9 Mupad [B] (verification not implemented)

Time = 21.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.71

$$\int \frac{1}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{2\tan\left(\frac{x}{2}\right)}{a\left(-a\tan\left(\frac{x}{2}\right)^2+2\,b\tan\left(\frac{x}{2}\right)+a\right)}$$

input
```
int(1/(a*cos(x) + b*sin(x))^2,x)
```

output
```
(2*tan(x/2))/(a*(a + 2*b*tan(x/2) - a*tan(x/2)^2))
```

3.19 $\int \frac{\csc(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

3.19.1 Optimal result

Integrand size = 14, antiderivative size = 63

$$\int \frac{\csc(x)}{(a\cos(x)+b\sin(x))^2}\,dx = -\frac{\operatorname{arctanh}(\cos(x))}{a^2}+\frac{b\operatorname{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}} + \frac{1}{a(a\cos(x)+b\sin(x))}$$

output
```
-arctanh(cos(x))/a^2+1/a/(a*cos(x)+b*sin(x))+b*arctanh((b*cos(x)-a*sin(x))
/(a^2+b^2)^(1/2))/a^2/(a^2+b^2)^(1/2)
```

3.19.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.14

$$\int \frac{\csc(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{-\frac{2b\operatorname{arctanh}\left(\frac{-b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}+\frac{a\csc(x)}{b+a\cot(x)}-\log\left(\cos\left(\frac{x}{2}\right)\right)+\log\left(\sin\left(\frac{x}{2}\right)\right)}{a^2}$$

input
```
Integrate[Csc[x]/(a*Cos[x] + b*Sin[x])^2,x]
```

output
```
((-2*b*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2] + (a*Cs
c[x])/(b + a*Cot[x]) - Log[Cos[x/2]] + Log[Sin[x/2]])/a^2
```

3.19.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 3572, 3042, 3553, 219, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(x)}{(a\cos(x)+b\sin(x))^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sin(x)(a\cos(x)+b\sin(x))^2}dx$$
$$\Big\downarrow \text{3572}$$
$$-\frac{b\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2}+\frac{\int \csc(x)dx}{a^2}+\frac{1}{a(a\cos(x)+b\sin(x))}$$
$$\Big\downarrow \text{3042}$$
$$-\frac{b\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2}+\frac{\int \csc(x)dx}{a^2}+\frac{1}{a(a\cos(x)+b\sin(x))}$$
$$\Big\downarrow \text{3553}$$
$$\frac{b\int \frac{1}{a^2+b^2-(b\cos(x)-a\sin(x))^2}d(b\cos(x)-a\sin(x))}{a^2}+\frac{\int \csc(x)dx}{a^2}+\frac{1}{a(a\cos(x)+b\sin(x))}$$
$$\Big\downarrow \text{219}$$
$$\frac{\int \csc(x)dx}{a^2}+\frac{b\,\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}}+\frac{1}{a(a\cos(x)+b\sin(x))}$$
$$\Big\downarrow \text{4257}$$
$$\frac{b\,\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}}-\frac{\text{arctanh}(\cos(x))}{a^2}+\frac{1}{a(a\cos(x)+b\sin(x))}$$

input
```
Int[Csc[x]/(a*Cos[x] + b*Sin[x])^2,x]
```

output
```
-(ArcTanh[Cos[x]]/a^2) + (b*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]]
)/(a^2*Sqrt[a^2 + b^2]) + 1/(a*(a*Cos[x] + b*Sin[x]))
```

3.19.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3572
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_)/si
n[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-(a*Cos[c + d*x] + b*Sin[c + d*x])
^(n + 1)/(a*d*(n + 1)), x] + (Simp[1/a^2   Int[(a*Cos[c + d*x] + b*Sin[c +
d*x])^(n + 2)/Sin[c + d*x], x], x] - Simp[b/a^2   Int[(a*Cos[c + d*x] + b*S
in[c + d*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.19.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.35

method	result	size
default	$\frac{\frac{4\left(-\frac{b\tan\left(\frac{x}{2}\right)}{2}-\frac{a}{2}\right)}{\tan\left(\frac{x}{2}\right)^{2}a-2b\tan\left(\frac{x}{2}\right)-a}-\frac{2b\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{x}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{\sqrt{a^{2}+b^{2}}}}{a^{2}}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a^{2}}$	85
risch	$\frac{2\,\mathrm{e}^{ix}}{a\left(-ib\,\mathrm{e}^{2ix}+a\,\mathrm{e}^{2ix}+ib+a\right)}-\frac{ib\ln\left(\mathrm{e}^{ix}-\frac{ib+a}{\sqrt{-a^{2}-b^{2}}}\right)}{\sqrt{-a^{2}-b^{2}}\,a^{2}}+\frac{ib\ln\left(\mathrm{e}^{ix}+\frac{ib+a}{\sqrt{-a^{2}-b^{2}}}\right)}{\sqrt{-a^{2}-b^{2}}\,a^{2}}-\frac{\ln\left(\mathrm{e}^{ix}+1\right)}{a^{2}}+\frac{\ln\left(\mathrm{e}^{ix}-1\right)}{a^{2}}$	156

input
```
int(csc(x)/(a*cos(x)+b*sin(x))^2,x,method=_RETURNVERBOSE)
```

output
```
4/a^2*((-1/2*b*tan(1/2*x)-1/2*a)/(tan(1/2*x)^2*a-2*b*tan(1/2*x)-a)-1/2*b/(
a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*x)-2*b)/(a^2+b^2)^(1/2)))+1/a^2*ln
(tan(1/2*x))
```

3.19.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(59) = 118$.

Time = 0.29 (sec) , antiderivative size = 220, normalized size of antiderivative = 3.49

$$\int \frac{\csc(x)}{(a\cos(x)+b\sin(x))^2}\,dx$$
$$= \frac{2a^3+2ab^2+(ab\cos(x)+b^2\sin(x))\sqrt{a^2+b^2}\log\left(\frac{2ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2-2a^2-b^2-2\sqrt{a^2+b^2}(b\cos(x)-a\sin(x}{2ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2+b^2}\right.}{2\left((a^5+a^3\right.}$$

input
```
integrate(csc(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

output
```
1/2*(2*a^3 + 2*a*b^2 + (a*b*cos(x) + b^2*sin(x))*sqrt(a^2 + b^2)*log((2*a*
b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(
b*cos(x) - a*sin(x)))/(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2))
- ((a^3 + a*b^2)*cos(x) + (a^2*b + b^3)*sin(x))*log(1/2*cos(x) + 1/2) + ((
a^3 + a*b^2)*cos(x) + (a^2*b + b^3)*sin(x))*log(-1/2*cos(x) + 1/2))/((a^5
+ a^3*b^2)*cos(x) + (a^4*b + a^2*b^3)*sin(x))
```

3.19.6 Sympy [F]

$$\int \frac{\csc(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \int \frac{\csc(x)}{(a\cos(x)+b\sin(x))^2}\,dx$$

input
```
integrate(csc(x)/(a*cos(x)+b*sin(x))**2,x)
```

output
```
Integral(csc(x)/(a*cos(x) + b*sin(x))**2, x)
```

3.19.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(59) = 118.

Time = 0.30 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.03

$$\int \frac{\csc(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{2\left(a+\frac{b\sin(x)}{\cos(x)+1}\right)}{a^3+\frac{2a^2b\sin(x)}{\cos(x)+1}-\frac{a^3\sin(x)^2}{(\cos(x)+1)^2}} + \frac{b\log\left(\frac{b-\frac{a\sin(x)}{\cos(x)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(x)}{\cos(x)+1}-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}a^2} + \frac{\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^2}$$

input
```
integrate(csc(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

output
```
2*(a + b*sin(x)/(cos(x) + 1))/(a^3 + 2*a^2*b*sin(x)/(cos(x) + 1) - a^3*sin
(x)^2/(cos(x) + 1)^2) + b*log((b - a*sin(x)/(cos(x) + 1) + sqrt(a^2 + b^2)
)/(b - a*sin(x)/(cos(x) + 1) - sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^2) + l
og(sin(x)/(cos(x) + 1))/a^2
```

3.19.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.73

$$\int \frac{\csc(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{b\log\left(\frac{\left|2a\tan\left(\frac{1}{2}x\right)-2b-2\sqrt{a^2+b^2}\right|}{\left|2a\tan\left(\frac{1}{2}x\right)-2b+2\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}a^2} + \frac{\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{a^2} - \frac{2\left(b\tan\left(\frac{1}{2}x\right)+a\right)}{\left(a\tan\left(\frac{1}{2}x\right)^2-2b\tan\left(\frac{1}{2}x\right)-a\right)a^2}$$

input
```
integrate(csc(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

output
```
b*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*x) - 2
*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^2) + log(abs(tan(1/2*x)))/a^2
- 2*(b*tan(1/2*x) + a)/((a*tan(1/2*x)^2 - 2*b*tan(1/2*x) - a)*a^2)
```

3.19.9 Mupad [B] (verification not implemented)

Time = 21.95 (sec) , antiderivative size = 492, normalized size of antiderivative = 7.81

$$\int \frac{\csc(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{\frac{2}{a}+\frac{2b\tan\left(\frac{x}{2}\right)}{a^2}}{-a\tan\left(\frac{x}{2}\right)^2+2b\tan\left(\frac{x}{2}\right)+a}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a^2}$$

$$+\frac{b\,\mathrm{atan}\left(\frac{\frac{b\sqrt{a^2+b^2}\left(4b+\frac{2\tan\left(\frac{x}{2}\right)\left(a^2+4b^2\right)}{a}+\frac{b\left(2a^2b+\frac{2\tan\left(\frac{x}{2}\right)\left(3a^4+4a^2b^2\right)}{a}\right)\sqrt{a^2+b^2}}{a^4+a^2b^2}\right)1\mathrm{i}}{a^4+a^2b^2}+\frac{b\sqrt{a^2+b^2}\left(4b+\frac{2\tan\left(\frac{x}{2}\right)\left(a^2+4b^2\right)}{a}-\frac{b\left(2a^2b+\cdots\right.}{\cdots}\right.}{a^4+a^2b^2}}{\frac{4b}{a^2}+\frac{b\sqrt{a^2+b^2}\left(4b+\frac{2\tan\left(\frac{x}{2}\right)\left(a^2+4b^2\right)}{a}+\frac{b\left(2a^2b+\frac{2\tan\left(\frac{x}{2}\right)\left(3a^4+4a^2b^2\right)}{a}\right)\sqrt{a^2+b^2}}{a^4+a^2b^2}\right)}{a^4+a^2b^2}-\frac{b\sqrt{a^2+b^2}\left(4b+\frac{2\tan\left(\frac{x}{2}\right)\left(a^2+4b^2\right)}{a}-\frac{b\left(2a^2\cdots\right.}{\cdots}\right.}{a^4+a^2\cdots}}\right)\cdots}{a^4+a^2b^2}$$

input
```
int(1/(sin(x)*(a*cos(x) + b*sin(x))^2),x)
```

output
```
(2/a + (2*b*tan(x/2))/a^2)/(a + 2*b*tan(x/2) - a*tan(x/2)^2) + log(tan(x/2
))/a^2 + (b*atan(((b*(a^2 + b^2)^(1/2)*(4*b + (2*tan(x/2)*(a^2 + 4*b^2))/a
 + (b*(2*a^2*b + (2*tan(x/2)*(3*a^4 + 4*a^2*b^2))/a)*(a^2 + b^2)^(1/2))/(a
^4 + a^2*b^2))*1i)/(a^4 + a^2*b^2) + (b*(a^2 + b^2)^(1/2)*(4*b + (2*tan(x/
2)*(a^2 + 4*b^2))/a - (b*(2*a^2*b + (2*tan(x/2)*(3*a^4 + 4*a^2*b^2))/a)*(a
^2 + b^2)^(1/2))/(a^4 + a^2*b^2))*1i)/(a^4 + a^2*b^2))/((4*b)/a^2 + (b*(a^
2 + b^2)^(1/2)*(4*b + (2*tan(x/2)*(a^2 + 4*b^2))/a + (b*(2*a^2*b + (2*tan(
x/2)*(3*a^4 + 4*a^2*b^2))/a)*(a^2 + b^2)^(1/2))/(a^4 + a^2*b^2)))/(a^4 + a
^2*b^2) - (b*(a^2 + b^2)^(1/2)*(4*b + (2*tan(x/2)*(a^2 + 4*b^2))/a - (b*(2
*a^2*b + (2*tan(x/2)*(3*a^4 + 4*a^2*b^2))/a)*(a^2 + b^2)^(1/2))/(a^4 + a^2
*b^2)))/(a^4 + a^2*b^2)))*(a^2 + b^2)^(1/2)*2i)/(a^4 + a^2*b^2)
```

3.20 $\int \frac{\csc^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

3.20.1 Optimal result

Integrand size = 16, antiderivative size = 49

$$\int \frac{\csc^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx=-\frac{\cot(x)}{a^2}-\frac{2b\log(\tan(x))}{a^3}+\frac{2b\log(a+b\tan(x))}{a^3}-\frac{\frac{1}{b}+\frac{b}{a^2}}{a+b\tan(x)}$$

output
```
-cot(x)/a^2-2*b*ln(tan(x))/a^3+2*b*ln(a+b*tan(x))/a^3+(-1/b-b/a^2)/(a+b*ta
n(x))
```

3.20.2 Mathematica [A] (verified)

Time = 2.43 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.55

$$\int \frac{\csc^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx$$
$$=\frac{a^2+b^2-a^2\cot^2(x)-2b^2\log(\sin(x))-ab\cot(x)(1+2\log(\sin(x))-2\log(a\cos(x)+b\sin(x)))+2b^2\log}{a^3(b+a\cot(x))}$$

input
```
Integrate[Csc[x]^2/(a*Cos[x] + b*Sin[x])^2,x]
```

output
```
(a^2 + b^2 - a^2*Cot[x]^2 - 2*b^2*Log[Sin[x]] - a*b*Cot[x]*(1 + 2*Log[Sin[
x]] - 2*Log[a*Cos[x] + b*Sin[x]]) + 2*b^2*Log[a*Cos[x] + b*Sin[x]])/(a^3*(
b + a*Cot[x]))
```

3.20.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3566, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\sin(x)^2(a\cos(x)+b\sin(x))^2}dx$$

$$\Big\downarrow \ 3566$$

$$\int \frac{\left(\tan^2(x)+1\right)\cot^2(x)}{(a+b\tan(x))^2}d\tan(x)$$

$$\Big\downarrow \ 522$$

$$\int \left(\frac{2b^2}{a^3(a+b\tan(x))}-\frac{2b\cot(x)}{a^3}+\frac{a^2+b^2}{a^2(a+b\tan(x))^2}+\frac{\cot^2(x)}{a^2}\right)d\tan(x)$$

$$\Big\downarrow \ 2009$$

$$-\frac{2b\log(\tan(x))}{a^3}+\frac{2b\log(a+b\tan(x))}{a^3}-\frac{\frac{b}{a^2}+\frac{1}{b}}{a+b\tan(x)}-\frac{\cot(x)}{a^2}$$

input
```
Int[Csc[x]^2/(a*Cos[x] + b*Sin[x])^2,x]
```

output
```
-(Cot[x]/a^2) - (2*b*Log[Tan[x]])/a^3 + (2*b*Log[a + b*Tan[x]])/a^3 - (b^(
-1) + b/a^2)/(a + b*Tan[x])
```

3.20.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3566
```
Int[sin[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[1/d   Subst[Int[x^m*((a + b*
x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Tan[c + d*x]], x] /; FreeQ[{a, b, c
, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n, 0
] && GtQ[m, 1])
```

3.20.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.14

method	result
default	$-\frac{a^{2}+b^{2}}{a^{2} b \left(a+b \tan \left(x\right)\right)}+\frac{2 b \ln \left(a+b \tan \left(x\right)\right)}{a^{3}}-\frac{1}{a^{2} \tan \left(x\right)}-\frac{2 b \ln \left(\tan \left(x\right)\right)}{a^{3}}$
risch	$-\frac{4 \left(b\,{\mathrm e}^{2 i x}-b+i a\right)}{\left({\mathrm e}^{2 i x}-1\right) \left(-i b\,{\mathrm e}^{2 i x}+a\,{\mathrm e}^{2 i x}+i b+a\right) a^{2}}+\frac{2 b \ln \left({\mathrm e}^{2 i x}-\frac{i b+a}{i b-a}\right)}{a^{3}}-\frac{2 b \ln \left({\mathrm e}^{2 i x}-1\right)}{a^{3}}$
norman	$\frac{\frac{1}{2 a}+\frac{\tan \left(\frac{x}{2}\right)^{4}}{2 a}-\frac{\left(3 a^{2}+4 b^{2}\right) \tan \left(\frac{x}{2}\right)^{2}}{a^{3}}}{\tan \left(\frac{x}{2}\right) \left(\tan \left(\frac{x}{2}\right)^{2} a-2 b \tan \left(\frac{x}{2}\right)-a\right)}-\frac{2 b \ln \left(\tan \left(\frac{x}{2}\right)\right)}{a^{3}}+\frac{2 b \ln \left(\tan \left(\frac{x}{2}\right)^{2} a-2 b \tan \left(\frac{x}{2}\right)-a\right)}{a^{3}}$
parallelrisch	$\frac{-2 \cot \left(x\right) \cos \left(x\right) a^{2}+2 a b \cos \left(x\right) \ln \left(\frac{-2 a \cos \left(x\right)-2 b \sin \left(x\right)}{\cos \left(x\right)+1}\right)-2 \ln \left(\csc \left(x\right)-\cot \left(x\right)\right) a b \cos \left(x\right)+2 \sin \left(x\right) b^{2} \ln \left(\frac{-2 a \cos \left(x\right)-2 b \sin \left(x\right)}{\cos \left(x\right)+1}\right)-2 \cdots}{\left(a \cos \left(x\right)+b \sin \left(x\right)\right) a^{3}}$

input
```
int(csc(x)^2/(a*cos(x)+b*sin(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-(a^2+b^2)/a^2/b/(a+b*tan(x))+2*b*ln(a+b*tan(x))/a^3-1/a^2/tan(x)-2*b*ln(t
an(x))/a^3
```

3.20.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(49) = 98.

Time = 0.27 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.73

$$\int \frac{\csc^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{2\,a^2\cos(x)^2 + 2\,ab\cos(x)\sin(x) - a^2 + \left(b^2\cos(x)^2 - ab\cos(x)\sin(x) - b^2\right)\log\left(2\,ab\cos(x)\sin(x) + (a\right.}{a^3b\cos(x)^2 - a^4\cos(x)\sin(x}$$

input
```
integrate(csc(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

output
```
(2*a^2*cos(x)^2 + 2*a*b*cos(x)*sin(x) - a^2 + (b^2*cos(x)^2 - a*b*cos(x)*s
in(x) - b^2)*log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) - (b^2*
cos(x)^2 - a*b*cos(x)*sin(x) - b^2)*log(-1/4*cos(x)^2 + 1/4))/(a^3*b*cos(x
)^2 - a^4*cos(x)*sin(x) - a^3*b)
```

3.20.6 Sympy [F]

$$\int \frac{\csc^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \int \frac{\csc^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx$$

input
```
integrate(csc(x)**2/(a*cos(x)+b*sin(x))**2,x)
```

output
```
Integral(csc(x)**2/(a*cos(x) + b*sin(x))**2, x)
```

3.20.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.27

$$\int \frac{\csc^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx = -\frac{ab + (a^2 + 2\,b^2)\tan(x)}{a^2b^2\tan(x)^2 + a^3b\tan(x)} + \frac{2\,b\log(b\tan(x) + a)}{a^3} - \frac{2\,b\log(\tan(x))}{a^3}$$

input
```
integrate(csc(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

output
```
-(a*b + (a^2 + 2*b^2)*tan(x))/(a^2*b^2*tan(x)^2 + a^3*b*tan(x)) + 2*b*log(
b*tan(x) + a)/a^3 - 2*b*log(tan(x))/a^3
```

3.20.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.29

$$\int \frac{\csc^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{2\,b\log\left(|b\tan\left(x\right)+a|\right)}{a^3}-\frac{2\,b\log\left(|\tan\left(x\right)|\right)}{a^3}-\frac{a^2\tan\left(x\right)+2\,b^2\tan\left(x\right)+ab}{\left(b\tan\left(x\right)^2+a\tan\left(x\right)\right)a^2b}$$

input
```
integrate(csc(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

output
```
2*b*log(abs(b*tan(x) + a))/a^3 - 2*b*log(abs(tan(x)))/a^3 - (a^2*tan(x) +
2*b^2*tan(x) + a*b)/((b*tan(x)^2 + a*tan(x))*a^2*b)
```

3.20.9 Mupad [B] (verification not implemented)

Time = 21.71 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.33

$$\int \frac{\csc^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{\tan\left(\frac{x}{2}\right)}{2\,a^2}-\frac{a+2\,b\tan\left(\frac{x}{2}\right)-\frac{\tan\left(\frac{x}{2}\right)^2\,\left(5\,a^2+4\,b^2\right)}{a}}{-2\,a^3\tan\left(\frac{x}{2}\right)^3+2\,a^3\tan\left(\frac{x}{2}\right)+4\,b\,a^2\tan\left(\frac{x}{2}\right)^2}+\frac{2\,b\,\ln\left(-a\tan\left(\frac{x}{2}\right)^2+2\,b\tan\left(\frac{x}{2}\right)+a\right)}{a^3}-\frac{2\,b\,\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a^3}$$

input
```
int(1/(sin(x)^2*(a*cos(x) + b*sin(x))^2),x)
```

output
```
tan(x/2)/(2*a^2) - (a + 2*b*tan(x/2) - (tan(x/2)^2*(5*a^2 + 4*b^2))/a)/(2*
a^3*tan(x/2) - 2*a^3*tan(x/2)^3 + 4*a^2*b*tan(x/2)^2) + (2*b*log(a + 2*b*t
an(x/2) - a*tan(x/2)^2))/a^3 - (2*b*log(tan(x/2)))/a^3
```

3.21 $\int \frac{\csc^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

3.21.1 Optimal result

Integrand size = 16, antiderivative size = 118

$$\int \frac{\csc^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx = -\frac{\operatorname{arctanh}(\cos(x))}{2a^2}-\frac{2b^2\operatorname{arctanh}(\cos(x))}{a^4}-\frac{(a^2+b^2)\operatorname{arctanh}(\cos(x))}{a^4}+\frac{3b\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^4}+\frac{2b\csc(x)}{a^3}-\frac{\cot(x)\csc(x)}{2a^2}+\frac{a^2+b^2}{a^3(a\cos(x)+b\sin(x))}$$

output
```
-1/2*arctanh(cos(x))/a^2-2*b^2*arctanh(cos(x))/a^4-(a^2+b^2)*arctanh(cos(x
))/a^4+2*b*csc(x)/a^3-1/2*cot(x)*csc(x)/a^2+(a^2+b^2)/a^3/(a*cos(x)+b*sin(
x))+3*b*arctanh((b*cos(x)-a*sin(x))/(a^2+b^2)^(1/2))*(a^2+b^2)^(1/2)/a^4
```

3.21.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 270 vs. 2(118) = 236.

Time = 2.42 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.29

$$\int \frac{\csc^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{-48b\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{-b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)(b+a\cot(x))+8a^3\csc(x)+8ab^2\csc(x)-12a^2b\log\left(\cos\left(\frac{x}{2}\right)\right)-24b^3}{}$$

input
```
Integrate[Csc[x]^3/(a*Cos[x] + b*Sin[x])^2,x]
```

output
```
(-48*b*Sqrt[a^2 + b^2]*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]]*(b + a*C
ot[x]) + 8*a^3*Csc[x] + 8*a*b^2*Csc[x] - 12*a^2*b*Log[Cos[x/2]] - 24*b^3*L
og[Cos[x/2]] - 12*a^3*Cot[x]*Log[Cos[x/2]] - 24*a*b^2*Cot[x]*Log[Cos[x/2]]
 + 12*a^2*b*Log[Sin[x/2]] + 24*b^3*Log[Sin[x/2]] + 12*a^3*Cot[x]*Log[Sin[x
/2]] + 24*a*b^2*Cot[x]*Log[Sin[x/2]] + a^2*b*Sec[x/2]^2 + a^3*Cot[x]*Sec[x
/2]^2 - a*Csc[x/2]^2*(-4*a*b*Cos[x] + a^2*Cot[x] + b*(a - 4*b*Sin[x])) + 8
*a*b^2*Tan[x/2] + 8*a^2*b*Cot[x]*Tan[x/2])/(8*a^4*(b + a*Cot[x]))
```

3.21.3 Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.32, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {3042, 3584, 3042, 3572, 3042, 3553, 219, 3582, 3042, 3553, 219, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(x)}{(a\cos(x)+b\sin(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(x)^3(a\cos(x)+b\sin(x))^2}dx$$

$$\Big\downarrow \text{3584}$$

$$\frac{(a^2+b^2)\int \frac{\csc(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2}-\frac{2b\int \frac{\csc^2(x)}{a\cos(x)+b\sin(x)}dx}{a^2}+\frac{\int \csc^3(x)dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a^2+b^2)\int \frac{1}{\sin(x)(a\cos(x)+b\sin(x))^2}dx}{a^2}-\frac{2b\int \frac{1}{\sin(x)^2(a\cos(x)+b\sin(x))}dx}{a^2}+\frac{\int \csc(x)^3dx}{a^2}$$

$$\Big\downarrow \text{3572}$$

$$\frac{(a^2+b^2)\left(-\frac{b\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2}+\frac{\int \csc(x)dx}{a^2}+\frac{1}{a(a\cos(x)+b\sin(x))}\right)}{a^2}-\frac{2b\int \frac{1}{\sin(x)^2(a\cos(x)+b\sin(x))}dx}{a^2}+\frac{\int \csc(x)^3dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(a^2+b^2\right)\left(-\frac{b\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2}+\frac{\int \csc(x)dx}{a^2}+\frac{1}{a(a\cos(x)+b\sin(x))}\right)}{a^2}-\frac{2b\int \frac{1}{\sin(x)^2(a\cos(x)+b\sin(x))}dx}{a^2}+\frac{\int \csc(x)^3dx}{a^2}$$

$$\Big\downarrow \text{3553}$$

$$\frac{\left(a^2+b^2\right)\left(\frac{b\int \frac{1}{a^2+b^2-(b\cos(x)-a\sin(x))^2}d(b\cos(x)-a\sin(x))}{a^2}+\frac{\int \csc(x)dx}{a^2}+\frac{1}{a(a\cos(x)+b\sin(x))}\right)}{a^2}-\frac{2b\int \frac{1}{\sin(x)^2(a\cos(x)+b\sin(x))}dx}{a^2}+\frac{\int \csc(x)^3dx}{a^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{\left(a^2+b^2\right)\left(\frac{\int \csc(x)dx}{a^2}+\frac{b\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}}+\frac{1}{a(a\cos(x)+b\sin(x))}\right)}{a^2}-\frac{2b\int \frac{1}{\sin(x)^2(a\cos(x)+b\sin(x))}dx}{a^2}+\frac{\int \csc(x)^3dx}{a^2}$$

$$\Big\downarrow \text{3582}$$

$$\frac{\left(a^2+b^2\right)\left(\frac{\int \csc(x)dx}{a^2}+\frac{b\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}}+\frac{1}{a(a\cos(x)+b\sin(x))}\right)}{a^2}-\frac{2b\left(\frac{\left(a^2+b^2\right)\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2}-\frac{b\int \csc(x)dx}{a^2}-\frac{\csc(x)}{a}\right)}{a^2}+\frac{\int \csc(x)^3dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(a^2+b^2\right)\left(\frac{\int \csc(x)dx}{a^2}+\frac{b\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}}+\frac{1}{a(a\cos(x)+b\sin(x))}\right)}{a^2}-\frac{2b\left(\frac{\left(a^2+b^2\right)\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2}-\frac{b\int \csc(x)dx}{a^2}-\frac{\csc(x)}{a}\right)}{a^2}+\frac{\int \csc(x)^3dx}{a^2}$$

$$\Big\downarrow \text{3553}$$

$$\frac{\left(a^2+b^2\right)\left(\frac{\int \csc(x)dx}{a^2}+\frac{b\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}}+\frac{1}{a(a\cos(x)+b\sin(x))}\right)}{a^2}-\frac{2b\left(-\frac{\left(a^2+b^2\right)\int \frac{1}{a^2+b^2-(b\cos(x)-a\sin(x))^2}d(b\cos(x)-a\sin(x))}{a^2}-\frac{b\int \csc(x)dx}{a^2}-\frac{\csc(x)}{a}\right)}{a^2}+\frac{\int \csc(x)^3dx}{a^2}$$

3.21. $\int \frac{\csc^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

$$\downarrow \text{219}$$

$$\frac{(a^2+b^2)\left(\frac{\int \csc(x)dx}{a^2}+\frac{b\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}}+\frac{1}{a(a\cos(x)+b\sin(x))}\right)}{a^2}-\frac{2b\left(-\frac{b\int \csc(x)dx}{a^2}-\frac{\sqrt{a^2+b^2}\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^2}-\frac{\csc(x)}{a}\right)}{a^2}+\frac{\int \csc(x)^3dx}{a^2}$$

$$\downarrow \text{4255}$$

$$\frac{(a^2+b^2)\left(\frac{\int \csc(x)dx}{a^2}+\frac{b\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}}+\frac{1}{a(a\cos(x)+b\sin(x))}\right)}{a^2}-\frac{2b\left(-\frac{b\int \csc(x)dx}{a^2}-\frac{\sqrt{a^2+b^2}\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^2}-\frac{\csc(x)}{a}\right)}{a^2}+\frac{\frac{\int \csc(x)dx}{2}-\frac{1}{2}\cot(x)\csc(x)}{a^2}$$

$$\downarrow \text{3042}$$

$$\frac{(a^2+b^2)\left(\frac{\int \csc(x)dx}{a^2}+\frac{b\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}}+\frac{1}{a(a\cos(x)+b\sin(x))}\right)}{a^2}-\frac{2b\left(-\frac{b\int \csc(x)dx}{a^2}-\frac{\sqrt{a^2+b^2}\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^2}-\frac{\csc(x)}{a}\right)}{a^2}+\frac{\frac{\int \csc(x)dx}{2}-\frac{1}{2}\cot(x)\csc(x)}{a^2}$$

$$\downarrow \text{4257}$$

$$\frac{(a^2+b^2)\left(\frac{b\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}}-\frac{\text{arctanh}(\cos(x))}{a^2}+\frac{1}{a(a\cos(x)+b\sin(x))}\right)}{a^2}-\frac{2b\left(-\frac{\sqrt{a^2+b^2}\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^2}+\frac{b\text{arctanh}(\cos(x))}{a^2}-\frac{\csc(x)}{a}\right)}{a^2}+\frac{-\frac{1}{2}\text{arctanh}(\cos(x))-\frac{1}{2}\cot(x)\csc(x)}{a^2}$$

input
```
Int[Csc[x]^3/(a*Cos[x] + b*Sin[x])^2,x]
```

3.21. $\int \frac{\csc^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

output
```
(-2*b*((b*ArcTanh[Cos[x]])/a^2 - (Sqrt[a^2 + b^2]*ArcTanh[(b*Cos[x] - a*Si
n[x])/Sqrt[a^2 + b^2]])/a^2 - Csc[x]/a))/a^2 + (-1/2*ArcTanh[Cos[x]] - (Co
t[x]*Csc[x])/2)/a^2 + ((a^2 + b^2)*(-(ArcTanh[Cos[x]]/a^2) + (b*ArcTanh[(b
*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2*Sqrt[a^2 + b^2]) + 1/(a*(a*Cos[
x] + b*Sin[x]))))/a^2
```

3.21.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3572
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_)/si
n[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-(a*Cos[c + d*x] + b*Sin[c + d*x])
^(n + 1)/(a*d*(n + 1)), x] + (Simp[1/a^2   Int[(a*Cos[c + d*x] + b*Sin[c +
d*x])^(n + 2)/Sin[c + d*x], x], x] - Simp[b/a^2   Int[(a*Cos[c + d*x] + b*S
in[c + d*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

rule 3582
```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[Sin[c + d*x]^(m + 1)/(a*d*(m + 1))
, x] + (-Simp[b/a^2   Int[Sin[c + d*x]^(m + 1), x], x] + Simp[(a^2 + b^2)/a
^2   Int[Sin[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /;
 FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

3.21. $\int \frac{\csc^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

rule 3584
```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a^2 + b^2)/a^2   Int[Sin[c +
 d*x]^(m + 2)*(a*Cos[c + d*x] + b*Sin[c + d*x])^n, x], x] + (Simp[1/a^2   I
nt[Sin[c + d*x]^m*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] - Simp[
2*(b/a^2)   Int[Sin[c + d*x]^(m + 1)*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n +
 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] &
& LtQ[m, -1]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.21.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.34

method	result
default	$-\frac{1}{8a^2\tan\left(\frac{x}{2}\right)^2}+\frac{(6a^2+12b^2)\ln\left(\tan\left(\frac{x}{2}\right)\right)}{4a^4}+\frac{b}{a^3\tan\left(\frac{x}{2}\right)}+\frac{\frac{\tan\left(\frac{x}{2}\right)^2 a}{2}+4b\tan\left(\frac{x}{2}\right)}{4a^3}+\frac{\frac{4\left(\left(-\frac{1}{2}a^2b-\frac{1}{2}b^3\right)\tan\left(\frac{x}{2}\right)-\frac{(a^2+b^2)a}{2}\right)}{\tan\left(\frac{x}{2}\right)^2a-2b\tan\left(\frac{x}{2}\right)-a}-6b}{a^4}$
risch	$\frac{e^{ix}\left(3iab\,e^{4ix}+3a^2e^{4ix}+6b^2e^{4ix}-2a^2e^{2ix}-12b^2e^{2ix}-3iba+3a^2+6b^2\right)}{\left(e^{2ix}-1\right)^2\left(-ib\,e^{2ix}+a\,e^{2ix}+ib+a\right)a^3}-\frac{3i\sqrt{-a^2-b^2}\,b\ln\left(e^{ix}-\frac{\sqrt{-a^2-b^2}\,(ib+a)}{a^2+b^2}\right)}{a^4}+\frac{3i\sqrt{-a^2-b^2}}{}$

input
```
int(csc(x)^3/(a*cos(x)+b*sin(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8/a^2/tan(1/2*x)^2+1/4/a^4*(6*a^2+12*b^2)*ln(tan(1/2*x))+b/a^3/tan(1/2*
x)+1/4/a^3*(1/2*tan(1/2*x)^2*a+4*b*tan(1/2*x))+4/a^4*(((-1/2*a^2*b-1/2*b^3
)*tan(1/2*x)-1/2*(a^2+b^2)*a)/(tan(1/2*x)^2*a-2*b*tan(1/2*x)-a)-3/2*b*(a^2
+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*x)-2*b)/(a^2+b^2)^(1/2)))
```

3.21.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. 2(110) = 220.

Time = 0.32 (sec) , antiderivative size = 345, normalized size of antiderivative = 2.92

$$\int \frac{\csc^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx =$$
$$-\frac{6\,a^2b\cos\left(x\right)\sin\left(x\right)+4\,a^3+12\,ab^2-6\,(a^3+2\,ab^2)\cos\left(x\right)^2-6\,\left(ab\cos\left(x\right)^3-ab\cos\left(x\right)+\left(b^2\cos\left(x\right)^2-\right.\right.}{}$$

input
```
integrate(csc(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

output
```
-1/4*(6*a^2*b*cos(x)*sin(x) + 4*a^3 + 12*a*b^2 - 6*(a^3 + 2*a*b^2)*cos(x)^
2 - 6*(a*b*cos(x)^3 - a*b*cos(x) + (b^2*cos(x)^2 - b^2)*sin(x))*sqrt(a^2 +
 b^2)*log((2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 - 2*a^2 - b^2 - 2*sq
rt(a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*co
s(x)^2 + b^2)) + 3*((a^3 + 2*a*b^2)*cos(x)^3 - (a^3 + 2*a*b^2)*cos(x) - (a
^2*b + 2*b^3 - (a^2*b + 2*b^3)*cos(x)^2)*sin(x))*log(1/2*cos(x) + 1/2) - 3
*((a^3 + 2*a*b^2)*cos(x)^3 - (a^3 + 2*a*b^2)*cos(x) - (a^2*b + 2*b^3 - (a^
2*b + 2*b^3)*cos(x)^2)*sin(x))*log(-1/2*cos(x) + 1/2))/(a^5*cos(x)^3 - a^5
*cos(x) + (a^4*b*cos(x)^2 - a^4*b)*sin(x))
```

3.21.6 Sympy [F]

$$\int \frac{\csc^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \int \frac{\csc^3\left(x\right)}{\left(a\cos\left(x\right)+b\sin\left(x\right)\right)^2}\,dx$$

input
```
integrate(csc(x)**3/(a*cos(x)+b*sin(x))**2,x)
```

output
```
Integral(csc(x)**3/(a*cos(x) + b*sin(x))**2, x)
```

3.21.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. 2(110) = 220.

Time = 0.31 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.05

$$\int \frac{\csc^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx = -\frac{a^3 - \frac{6\,a^2 b\sin(x)}{\cos(x)+1} - \frac{(17\,a^3+32\,ab^2)\sin(x)^2}{(\cos(x)+1)^2} - \frac{8\,(a^2 b+2\,b^3)\sin(x)^3}{(\cos(x)+1)^3}}{8\left(\frac{a^5\sin(x)^2}{(\cos(x)+1)^2} + \frac{2\,a^4 b\sin(x)^3}{(\cos(x)+1)^3} - \frac{a^5\sin(x)^4}{(\cos(x)+1)^4}\right)} + \frac{\frac{8\,b\sin(x)}{\cos(x)+1} + \frac{a\sin(x)^2}{(\cos(x)+1)^2}}{8\,a^3} + \frac{3\,(a^2+2\,b^2)\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{2\,a^4} + \frac{3\,(a^2 b+b^3)\log\left(\frac{b-\frac{a\sin(x)}{\cos(x)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(x)}{\cos(x)+1}-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,a^4}$$

input
```
integrate(csc(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

output
```
-1/8*(a^3 - 6*a^2*b*sin(x)/(cos(x) + 1) - (17*a^3 + 32*a*b^2)*sin(x)^2/(co
s(x) + 1)^2 - 8*(a^2*b + 2*b^3)*sin(x)^3/(cos(x) + 1)^3)/(a^5*sin(x)^2/(co
s(x) + 1)^2 + 2*a^4*b*sin(x)^3/(cos(x) + 1)^3 - a^5*sin(x)^4/(cos(x) + 1)^
4) + 1/8*(8*b*sin(x)/(cos(x) + 1) + a*sin(x)^2/(cos(x) + 1)^2)/a^3 + 3/2*(
a^2 + 2*b^2)*log(sin(x)/(cos(x) + 1))/a^4 + 3*(a^2*b + b^3)*log((b - a*sin
(x)/(cos(x) + 1) + sqrt(a^2 + b^2))/(b - a*sin(x)/(cos(x) + 1) - sqrt(a^2
+ b^2)))/(sqrt(a^2 + b^2)*a^4)
```

3.21.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.82

$$\int \frac{\csc^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{3\,(a^2+2\,b^2)\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)}{2\,a^4} + \frac{3\,(a^2 b+b^3)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,a^4} + \frac{a^2\tan\left(\frac{1}{2}\,x\right)^2 + 8\,ab\tan\left(\frac{1}{2}\,x\right)}{8\,a^4} - \frac{2\,\left(a^2 b\tan\left(\frac{1}{2}\,x\right) + b^3\tan\left(\frac{1}{2}\,x\right) + a^3 + ab^2\right)}{\left(a\tan\left(\frac{1}{2}\,x\right)^2 - 2\,b\tan\left(\frac{1}{2}\,x\right) - a\right)a^4} - \frac{18\,a^2\tan\left(\frac{1}{2}\,x\right)^2 + 36\,b^2\tan\left(\frac{1}{2}\,x\right)^2 - 8\,ab\tan\left(\frac{1}{2}\,x\right) + a^2}{8\,a^4\tan\left(\frac{1}{2}\,x\right)^2}$$

input
```
integrate(csc(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

output
```
3/2*(a^2 + 2*b^2)*log(abs(tan(1/2*x)))/a^4 + 3*(a^2*b + b^3)*log(abs(2*a*t
an(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*x) - 2*b + 2*sqrt(a^2
 + b^2)))/(sqrt(a^2 + b^2)*a^4) + 1/8*(a^2*tan(1/2*x)^2 + 8*a*b*tan(1/2*x)
)/a^4 - 2*(a^2*b*tan(1/2*x) + b^3*tan(1/2*x) + a^3 + a*b^2)/((a*tan(1/2*x)
^2 - 2*b*tan(1/2*x) - a)*a^4) - 1/8*(18*a^2*tan(1/2*x)^2 + 36*b^2*tan(1/2*
x)^2 - 8*a*b*tan(1/2*x) + a^2)/(a^4*tan(1/2*x)^2)
```

3.21.9 Mupad [B] (verification not implemented)

Time = 22.09 (sec) , antiderivative size = 511, normalized size of antiderivative = 4.33

$$\int \frac{\csc^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{\tan\left(\frac{x}{2}\right)^2\left(\frac{17\,a^2}{2}+16\,b^2\right)-\frac{a^2}{2}+3\,a\,b\tan\left(\frac{x}{2}\right)+\frac{4\tan\left(\frac{x}{2}\right)^3\,(a^2\,b+2\,b^3)}{a}}{-4\,a^4\tan\left(\frac{x}{2}\right)^4+4\,a^4\tan\left(\frac{x}{2}\right)^2+8\,b\,a^3\tan\left(\frac{x}{2}\right)^3} + \frac{\tan\left(\frac{x}{2}\right)^2}{8\,a^2}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)\,(3\,a^2+6\,b^2)}{2\,a^4}+\frac{b\tan\left(\frac{x}{2}\right)}{a^3} - \frac{6\,b\,\mathrm{atanh}\left(\frac{54\,b^2\,\sqrt{a^2+b^2}}{18\,a^2\,b+90\,b^3+\frac{72\,b^5}{a^2}+\frac{216\,b^4\tan\left(\frac{x}{2}\right)}{a}+\frac{144\,b^6\tan\left(\frac{x}{2}\right)}{a^3}+72\,a\,b^2\tan\left(\frac{x}{2}\right)}+\frac{72\,b^4\,\sqrt{a^2+b^2}}{18\,a^4\,b+72\,b^5+90\,a^2\,b^3+72\,a^3\,b^2\tan\left(\frac{x}{2}\right)+\frac{144\,b^6\tan\left(\frac{x}{2}\right)}{a}\cdots}\right.}{}$$

input
```
int(1/(sin(x)^3*(a*cos(x) + b*sin(x))^2),x)
```

output
```
(tan(x/2)^2*((17*a^2)/2 + 16*b^2) - a^2/2 + 3*a*b*tan(x/2) + (4*tan(x/2)^3
*(a^2*b + 2*b^3))/a)/(4*a^4*tan(x/2)^2 - 4*a^4*tan(x/2)^4 + 8*a^3*b*tan(x/
2)^3) + tan(x/2)^2/(8*a^2) + (log(tan(x/2))*(3*a^2 + 6*b^2))/(2*a^4) + (b*
tan(x/2))/a^3 - (6*b*atanh((54*b^2*(a^2 + b^2)^(1/2))/(18*a^2*b + 90*b^3 +
 (72*b^5)/a^2 + (216*b^4*tan(x/2))/a + (144*b^6*tan(x/2))/a^3 + 72*a*b^2*t
an(x/2)) + (72*b^4*(a^2 + b^2)^(1/2))/(18*a^4*b + 72*b^5 + 90*a^2*b^3 + 72
*a^3*b^2*tan(x/2) + (144*b^6*tan(x/2))/a + 216*a*b^4*tan(x/2)) + (144*b^3*
tan(x/2)*(a^2 + b^2)^(1/2))/(216*b^4*tan(x/2) + 90*a*b^3 + 18*a^3*b + (72*
b^5)/a + 72*a^2*b^2*tan(x/2) + (144*b^6*tan(x/2))/a^2) + (144*b^5*tan(x/2)
*(a^2 + b^2)^(1/2))/(144*b^6*tan(x/2) + 72*a*b^5 + 18*a^5*b + 90*a^3*b^3 +
 216*a^2*b^4*tan(x/2) + 72*a^4*b^2*tan(x/2)) + (18*b*tan(x/2)*(a^2 + b^2)^
(1/2))/(18*a*b + 72*b^2*tan(x/2) + (90*b^3)/a + (72*b^5)/a^3 + (216*b^4*ta
n(x/2))/a^2 + (144*b^6*tan(x/2))/a^4))*(a^2 + b^2)^(1/2))/a^4
```

3.22 $\int \frac{\sin^3(x)}{(a\cos(x)+b\sin(x))^3}\,dx$

3.22.1 Optimal result

Integrand size = 16, antiderivative size = 98

$$\int \frac{\sin^3(x)}{(a\cos(x)+b\sin(x))^3}\,dx = -\frac{b(3a^2-b^2)\,x}{(a^2+b^2)^3}+\frac{a}{2\,(a^2+b^2)\,(b+a\cot(x))^2}+\frac{2ab}{(a^2+b^2)^2\,(b+a\cot(x))}+\frac{a(a^2-3b^2)\log(a\cos(x)+b\sin(x))}{(a^2+b^2)^3}$$

output
```
-b*(3*a^2-b^2)*x/(a^2+b^2)^3+1/2*a/(a^2+b^2)/(b+a*cot(x))^2+2*a*b/(a^2+b^2
)^2/(b+a*cot(x))+a*(a^2-3*b^2)*ln(a*cos(x)+b*sin(x))/(a^2+b^2)^3
```

3.22.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.05 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.16

$$\int \frac{\sin^3(x)}{(a\cos(x)+b\sin(x))^3}\,dx = \frac{b(-3a^2+b^2)\,x}{(a^2+b^2)^3}+\frac{a(a^2-3b^2)\log(a\cos(x)+b\sin(x))}{(a^2+b^2)^3}+\frac{a^3}{2(a-ib)^2(a+ib)^2(a\cos(x)+b\sin(x))^2}+\frac{3ab\sin(x)}{(a^2+b^2)^2\,(a\cos(x)+b\sin(x))}$$

input
```
Integrate[Sin[x]^3/(a*Cos[x] + b*Sin[x])^3,x]
```

output
```
(b*(-3*a^2 + b^2)*x)/(a^2 + b^2)^3 + (a*(a^2 - 3*b^2)*Log[a*Cos[x] + b*Sin
[x]])/(a^2 + b^2)^3 + a^3/(2*(a - I*b)^2*(a + I*b)^2*(a*Cos[x] + b*Sin[x])
^2) + (3*a*b*Sin[x])/((a^2 + b^2)^2*(a*Cos[x] + b*Sin[x]))
```

3.22.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.23, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3042, 3564, 3042, 3964, 3042, 4012, 25, 3042, 4014, 25, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(x)}{(a\cos(x)+b\sin(x))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)^3}{(a\cos(x)+b\sin(x))^3}dx$$

$$\Big\downarrow \text{3564}$$

$$\int \frac{1}{(a\cot(x)+b)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(b-a\tan\left(x+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3964}$$

$$\frac{\int \frac{b-a\cot(x)}{(b+a\cot(x))^2}dx}{a^2+b^2}+\frac{a}{2\left(a^2+b^2\right)(a\cot(x)+b)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{b+a\tan\left(x+\frac{\pi}{2}\right)}{\left(b-a\tan\left(x+\frac{\pi}{2}\right)\right)^2}dx}{a^2+b^2}+\frac{a}{2\left(a^2+b^2\right)(a\cot(x)+b)^2}$$

$$\Big\downarrow \text{4012}$$

$$\frac{\frac{\int -\frac{a^2+2b\cot(x)a-b^2}{b+a\cot(x)}dx}{a^2+b^2}+\frac{2ab}{(a^2+b^2)(a\cot(x)+b)}}{a^2+b^2}+\frac{a}{2\left(a^2+b^2\right)(a\cot(x)+b)^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{2ab}{(a^2+b^2)(a\cot(x)+b)}-\frac{\int \frac{a^2+2b\cot(x)a-b^2}{b+a\cot(x)}dx}{a^2+b^2}}{a^2+b^2}+\frac{a}{2\left(a^2+b^2\right)(a\cot(x)+b)^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{2ab}{(a^2+b^2)(a\cot(x)+b)}-\frac{\int \frac{a^2-2b\tan\left(x+\frac{\pi}{2}\right)a-b^2}{b-a\tan\left(x+\frac{\pi}{2}\right)}dx}{a^2+b^2}}{a^2+b^2}+\frac{a}{2\left(a^2+b^2\right)(a\cot(x)+b)^2}$$

$$\Big\downarrow \text{ 4014}$$

$$\frac{\frac{2ab}{(a^2+b^2)(a\cot(x)+b)}-\frac{\frac{bx\left(3a^2-b^2\right)}{a^2+b^2}-\frac{a\left(a^2-3b^2\right)\int -\frac{a-b\cot(x)}{b+a\cot(x)}dx}{a^2+b^2}}{a^2+b^2}}{a^2+b^2}+\frac{a}{2\left(a^2+b^2\right)(a\cot(x)+b)^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{2ab}{(a^2+b^2)(a\cot(x)+b)}-\frac{\frac{a\left(a^2-3b^2\right)\int \frac{a-b\cot(x)}{b+a\cot(x)}dx}{a^2+b^2}+\frac{bx\left(3a^2-b^2\right)}{a^2+b^2}}{a^2+b^2}}{a^2+b^2}+\frac{a}{2\left(a^2+b^2\right)(a\cot(x)+b)^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{2ab}{(a^2+b^2)(a\cot(x)+b)}-\frac{\frac{a\left(a^2-3b^2\right)\int \frac{a+b\tan\left(x+\frac{\pi}{2}\right)}{b-a\tan\left(x+\frac{\pi}{2}\right)}dx}{a^2+b^2}+\frac{bx\left(3a^2-b^2\right)}{a^2+b^2}}{a^2+b^2}}{a^2+b^2}+\frac{a}{2\left(a^2+b^2\right)(a\cot(x)+b)^2}$$

$$\Big\downarrow \text{ 4013}$$

$$\frac{a}{2\left(a^2+b^2\right)(a\cot(x)+b)^2}+\frac{\frac{2ab}{(a^2+b^2)(a\cot(x)+b)}-\frac{\frac{bx\left(3a^2-b^2\right)}{a^2+b^2}-\frac{a\left(a^2-3b^2\right)\log(a\cos(x)+b\sin(x))}{a^2+b^2}}{a^2+b^2}}{a^2+b^2}$$

input
```
Int[Sin[x]^3/(a*Cos[x] + b*Sin[x])^3,x]
```

output
```
a/(2*(a^2 + b^2)*(b + a*Cot[x])^2) + ((2*a*b)/((a^2 + b^2)*(b + a*Cot[x]))
 - ((b*(3*a^2 - b^2)*x)/(a^2 + b^2) - (a*(a^2 - 3*b^2)*Log[a*Cos[x] + b*Si
n[x]])/(a^2 + b^2))/(a^2 + b^2))/(a^2 + b^2)
```

3.22. $\int \frac{\sin^3(x)}{(a\cos(x)+b\sin(x))^3}\,dx$

3.22.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3564
```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(b + a*Cot[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

rule 3964
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)
 Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

3.22. $\int \frac{\sin^3(x)}{(a\cos(x)+b\sin(x))^3}\,dx$

3.22.4 Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.37

method	result
default	$\frac{a(a^2-3b^2)\ln(a+b\tan(x))}{(a^2+b^2)^3}-\frac{a^2(a^2+3b^2)}{(a^2+b^2)^2 b^2(a+b\tan(x))}+\frac{a^3}{2b^2(a^2+b^2)(a+b\tan(x))^2}+\frac{\frac{(-a^3+3ab^2)\ln(1+\tan(x)^2)}{2}+(-3a^2}{(a^2+b^2)^3}$
parallelrisch	$\frac{2a(a^2-3b^2)((a^2-b^2)\cos(2x)+2ba\sin(2x)+a^2+b^2)\ln\left(\frac{-a\cos(x)-b\sin(x)}{\cos(x)+1}\right)-2a(a^2-3b^2)((a^2-b^2)\cos(2x)+2ba\sin(2x)+a^2+b}{2(a^2+b^2)^3((a}$
risch	$-\frac{ix}{3ib\,a^2-ib^3-a^3+3a\,b^2}-\frac{2ia^3x}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{6iax\,b^2}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{2a^2(2iab\,e^{2ix}+a^2e^{2ix}+3b^2e^{2ix}+3iba-3b^2)}{(-ib\,e^{2ix}+a\,e^{2ix}+ib+a)^2(ib+a)^2(-ib+a)^3}$
norman	$\frac{\frac{(2a^5+10a^3b^2)\tan(\frac{x}{2})^2}{a^2(a^4+2a^2b^2+b^4)}+\frac{(2a^5+10a^3b^2)\tan(\frac{x}{2})^8}{a^2(a^4+2a^2b^2+b^4)}-\frac{2(-3a^5-15a^3b^2)\tan(\frac{x}{2})^4}{a^2(a^4+2a^2b^2+b^4)}-\frac{2(-3a^5-15a^3b^2)\tan(\frac{x}{2})^6}{a^2(a^4+2a^2b^2+b^4)}-\frac{(3a^2-b^2)a^2bx}{a^6+3a^4b^2+3a^2b^4+b^6}+}{}$

input
```
int(sin(x)^3/(a*cos(x)+b*sin(x))^3,x,method=_RETURNVERBOSE)
```

output
```
a*(a^2-3*b^2)/(a^2+b^2)^3*ln(a+b*tan(x))-a^2*(a^2+3*b^2)/(a^2+b^2)^2/b^2/(
a+b*tan(x))+1/2*a^3/b^2/(a^2+b^2)/(a+b*tan(x))^2+1/(a^2+b^2)^3*(1/2*(-a^3+
3*a*b^2)*ln(1+tan(x)^2)+(-3*a^2*b+b^3)*arctan(tan(x)))
```

3.22.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. 2(96) = 192.

Time = 0.27 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.88

$$\int \frac{\sin^3(x)}{(a\cos(x)+b\sin(x))^3}\,dx$$
$$=\frac{a^5+7\,a^3b^2-2\,(6\,a^3b^2+(3\,a^4b-4\,a^2b^3+b^5)x)\cos(x)^2+2\,(3\,a^4b-3\,a^2b^3-2\,(3\,a^3b^2-ab^4)x)\cos(x)\sin}{2\,(a^6b^2+3\,a^4b^4+3\,a^2b^6+b^8+(a}$$

input
```
integrate(sin(x)^3/(a*cos(x)+b*sin(x))^3,x, algorithm="fricas")
```

output
```
1/2*(a^5 + 7*a^3*b^2 - 2*(6*a^3*b^2 + (3*a^4*b - 4*a^2*b^3 + b^5)*x)*cos(x
)^2 + 2*(3*a^4*b - 3*a^2*b^3 - 2*(3*a^3*b^2 - a*b^4)*x)*cos(x)*sin(x) - 2*
(3*a^2*b^3 - b^5)*x + (a^3*b^2 - 3*a*b^4 + (a^5 - 4*a^3*b^2 + 3*a*b^4)*cos
(x)^2 + 2*(a^4*b - 3*a^2*b^3)*cos(x)*sin(x))*log(2*a*b*cos(x)*sin(x) + (a^
2 - b^2)*cos(x)^2 + b^2))/(a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8 + (a^8 +
2*a^6*b^2 - 2*a^2*b^6 - b^8)*cos(x)^2 + 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 +
 a*b^7)*cos(x)*sin(x))
```

3.22.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{\sin^3(x)}{(a\cos(x)+b\sin(x))^3}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(sin(x)**3/(a*cos(x)+b*sin(x))**3,x)
```

output
```
Exception raised: AttributeError >> 'NoneType' object has no attribute 'pr
imitive'
```

3.22.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 359 vs. 2(96) = 192.

Time = 0.29 (sec) , antiderivative size = 359, normalized size of antiderivative = 3.66

$$\int \frac{\sin^3(x)}{(a\cos(x)+b\sin(x))^3}\,dx = -\frac{2\,(3\,a^2b-b^3)\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{(a^3-3\,ab^2)\log\left(-a-\frac{2\,b\sin(x)}{\cos(x)+1}+\frac{a\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} - \frac{(a^3-3\,ab^2)\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{2\left(\frac{2\,a^2b\sin(x)}{\cos(x)+1}-\frac{2\,a^2b\sin(x)^3}{(\cos(x)+1)^3}+\frac{(a^3+5\,ab^2)\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^6+2\,a^4b^2+a^2b^4+\frac{4\,(a^5b+2\,a^3b^3+ab^5)\sin(x)}{\cos(x)+1}-\frac{2\,(a^6-3\,a^2b^4-2\,b^6)\sin(x)^2}{(\cos(x)+1)^2}-\frac{4\,(a^5b+2\,a^3b^3+ab^5)\sin(x)^3}{(\cos(x)+1)^3}+\frac{(a^6+2\,a^4b^2+a^2b^4}{(\cos(x)+1}}$$

input
```
integrate(sin(x)^3/(a*cos(x)+b*sin(x))^3,x, algorithm="maxima")
```

output
```
-2*(3*a^2*b - b^3)*arctan(sin(x)/(cos(x) + 1))/(a^6 + 3*a^4*b^2 + 3*a^2*b^
4 + b^6) + (a^3 - 3*a*b^2)*log(-a - 2*b*sin(x)/(cos(x) + 1) + a*sin(x)^2/(
cos(x) + 1)^2)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (a^3 - 3*a*b^2)*log(s
in(x)^2/(cos(x) + 1)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 2*(2*a^2
*b*sin(x)/(cos(x) + 1) - 2*a^2*b*sin(x)^3/(cos(x) + 1)^3 + (a^3 + 5*a*b^2)
*sin(x)^2/(cos(x) + 1)^2)/(a^6 + 2*a^4*b^2 + a^2*b^4 + 4*(a^5*b + 2*a^3*b^
3 + a*b^5)*sin(x)/(cos(x) + 1) - 2*(a^6 - 3*a^2*b^4 - 2*b^6)*sin(x)^2/(cos
(x) + 1)^2 - 4*(a^5*b + 2*a^3*b^3 + a*b^5)*sin(x)^3/(cos(x) + 1)^3 + (a^6
+ 2*a^4*b^2 + a^2*b^4)*sin(x)^4/(cos(x) + 1)^4)
```

3.22.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. 2(96) = 192.

Time = 0.28 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.47

$$\int \frac{\sin^3(x)}{(a\cos(x)+b\sin(x))^3}\,dx = -\frac{(3\,a^2b-b^3)x}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} - \frac{(a^3-3\,ab^2)\log\left(\tan\left(x\right)^2+1\right)}{2\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)} + \frac{(a^3b-3\,ab^3)\log\left(\left|b\tan\left(x\right)+a\right|\right)}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7} - \frac{3\,a^3b^4\tan\left(x\right)^2-9\,ab^6\tan\left(x\right)^2+2\,a^6b\tan\left(x\right)+14\,a^4b^3\tan\left(x\right)-12\,a^2b^5\tan\left(x\right)+a^7+9\,a^5b^2-4\,a^3b^4}{2\left(a^6b^2+3\,a^4b^4+3\,a^2b^6+b^8\right)(b\tan\left(x\right)+a)^2}$$

input
```
integrate(sin(x)^3/(a*cos(x)+b*sin(x))^3,x, algorithm="giac")
```

output
```
-(3*a^2*b - b^3)*x/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 1/2*(a^3 - 3*a*b^
2)*log(tan(x)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (a^3*b - 3*a*b^
3)*log(abs(b*tan(x) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7) - 1/2*(3*a
^3*b^4*tan(x)^2 - 9*a*b^6*tan(x)^2 + 2*a^6*b*tan(x) + 14*a^4*b^3*tan(x) -
12*a^2*b^5*tan(x) + a^7 + 9*a^5*b^2 - 4*a^3*b^4)/((a^6*b^2 + 3*a^4*b^4 + 3
*a^2*b^6 + b^8)*(b*tan(x) + a)^2)
```

3.22.9 Mupad [B] (verification not implemented)

Time = 30.10 (sec) , antiderivative size = 5324, normalized size of antiderivative = 54.33

$$\int \frac{\sin^3(x)}{(a\cos(x)+b\sin(x))^3}\,dx = \text{Too large to display}$$

input
```
int(sin(x)^3/(a*cos(x) + b*sin(x))^3,x)
```

output
```
((2*tan(x/2)^2*(5*a*b^2 + a^3))/(a^4 + b^4 + 2*a^2*b^2) - (4*a^2*b*tan(x/2
)^3)/(a^4 + b^4 + 2*a^2*b^2) + (4*a^2*b*tan(x/2))/(a^4 + b^4 + 2*a^2*b^2))
/(a^2 - tan(x/2)^2*(2*a^2 - 4*b^2) + a^2*tan(x/2)^4 + 4*a*b*tan(x/2) - 4*a
*b*tan(x/2)^3) - (log(a + 2*b*tan(x/2) - a*tan(x/2)^2)*(3*a*b^2 - a^3))/(a
^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (log(1/(cos(x) + 1))*(6*a*b^2 - 2*a^3)
)/(2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (2*b*atan((tan(x/2)*(((((6*a*b
^2 - 2*a^3)*((b*((32*(a^2*b^12 - 2*a^14 + 15*a^4*b^10 + 48*a^6*b^8 + 62*a^
8*b^6 + 33*a^10*b^4 + 3*a^12*b^2))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8
+ 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) - (16*(6*a*b^2 - 2*a^3)*(3*a*b^16
+ 21*a^3*b^14 + 63*a^5*b^12 + 105*a^7*b^10 + 105*a^9*b^8 + 63*a^11*b^6 + 2
1*a^13*b^4 + 3*a^15*b^2))/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)*(a^12 + b^1
2 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(3*a
^2 - b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (16*b*(6*a*b^2 - 2*a^3)*(
3*a^2 - b^2)*(3*a*b^16 + 21*a^3*b^14 + 63*a^5*b^12 + 105*a^7*b^10 + 105*a^
9*b^8 + 63*a^11*b^6 + 21*a^13*b^4 + 3*a^15*b^2))/((a^6 + b^6 + 3*a^2*b^4 +
 3*a^4*b^2)^2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8
*b^4 + 6*a^10*b^2))))/(2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (b*((32*(2
*a*b^10 - 24*a^3*b^8 - 36*a^5*b^6 + 8*a^7*b^4 + 18*a^9*b^2))/(a^12 + b^12
+ 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) - ((6*a*
b^2 - 2*a^3)*((32*(a^2*b^12 - 2*a^14 + 15*a^4*b^10 + 48*a^6*b^8 + 62*a^...
```

3.23 $\int \frac{\sin^2(x)}{(a\cos(x)+b\sin(x))^3}\,dx$

3.23.1 Optimal result

Integrand size = 16, antiderivative size = 92

$$\int \frac{\sin^2(x)}{(a\cos(x)+b\sin(x))^3}\,dx = -\frac{(a^2-2b^2)\,\text{arctanh}\left(\frac{-b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} + \frac{a(3ab\cos(x)+(a^2+4b^2)\sin(x))}{2\,(a^2+b^2)^2\,(a\cos(x)+b\sin(x))^2}$$

output
```
-(a^2-2*b^2)*arctanh((-b+a*tan(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2)+1/
2*a*(3*a*b*cos(x)+(a^2+4*b^2)*sin(x))/(a^2+b^2)^2/(a*cos(x)+b*sin(x))^2
```

3.23.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00

$$\int \frac{\sin^2(x)}{(a\cos(x)+b\sin(x))^3}\,dx = -\frac{(a^2-2b^2)\,\text{arctanh}\left(\frac{-b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} + \frac{a(3ab\cos(x)+(a^2+4b^2)\sin(x))}{2\,(a^2+b^2)^2\,(a\cos(x)+b\sin(x))^2}$$

input
```
Integrate[Sin[x]^2/(a*Cos[x] + b*Sin[x])^3,x]
```

output
```
-(((a^2 - 2*b^2)*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(
5/2)) + (a*(3*a*b*Cos[x] + (a^2 + 4*b^2)*Sin[x]))/(2*(a^2 + b^2)^2*(a*Cos[
x] + b*Sin[x])^2)
```

3.23.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 300 vs. $2(92) = 184$.

Time = 0.92 (sec) , antiderivative size = 300, normalized size of antiderivative = 3.26, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3042, 4901, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(x)}{(a\cos(x)+b\sin(x))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)^2}{(a\cos(x)+b\sin(x))^3}dx$$

$$\Big\downarrow \text{4901}$$

$$\int \left(\frac{a^2\cos^2(x)}{b^2(a\cos(x)+b\sin(x))^3}-\frac{2a\cos(x)}{b^2(a\cos(x)+b\sin(x))^2}+\frac{1}{b^2(a\cos(x)+b\sin(x))}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2\left(2a^2-b^2\right)\text{arctanh}\left(\frac{b-a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\left(a^2+b^2\right)^{5/2}}+\frac{2a^2\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{b^2\left(a^2+b^2\right)^{3/2}}-\frac{\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}+\frac{2\left(\left(a^2+2b^2\right)\tan\left(\frac{x}{2}\right)+ab\right)}{a\left(a^2+b^2\right)\left(-a\tan^2\left(\frac{x}{2}\right)+a+2b\tan\left(\frac{x}{2}\right)\right)^2}+\frac{2a}{b\left(a^2+b^2\right)\left(a\cos(x)+b\sin(x)\right)}-\frac{4a^4+ab\left(5a^2+2b^2\right)\tan\left(\frac{x}{2}\right)+3a^2b^2+2b^4}{ab\left(a^2+b^2\right)^2\left(-a\tan^2\left(\frac{x}{2}\right)+a+2b\tan\left(\frac{x}{2}\right)\right)}$$

input
```
Int[Sin[x]^2/(a*Cos[x] + b*Sin[x])^3,x]
```

output
```
(2*a^2*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(b^2*(a^2 + b^2)^(3
/2)) - ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]]/(b^2*Sqrt[a^2 + b^2]
) - (a^2*(2*a^2 - b^2)*ArcTanh[(b - a*Tan[x/2])/Sqrt[a^2 + b^2]])/(b^2*(a^
2 + b^2)^(5/2)) + (2*a)/(b*(a^2 + b^2)*(a*Cos[x] + b*Sin[x])) + (2*(a*b +
(a^2 + 2*b^2)*Tan[x/2]))/(a*(a^2 + b^2)*(a + 2*b*Tan[x/2] - a*Tan[x/2]^2)^
2) - (4*a^4 + 3*a^2*b^2 + 2*b^4 + a*b*(5*a^2 + 2*b^2)*Tan[x/2])/(a*b*(a^2
+ b^2)^2*(a + 2*b*Tan[x/2] - a*Tan[x/2]^2))
```

3.23.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4901
```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

3.23.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(84) = 168$.

Time = 0.60 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.30

method	result
default	$-\frac{8\left(-\frac{a\left(a^{2}-2b^{2}\right)\tan\left(\frac{x}{2}\right)^{3}}{8\left(a^{4}+2a^{2}b^{2}+b^{4}\right)}+\frac{3b\left(a^{2}-2b^{2}\right)\tan\left(\frac{x}{2}\right)^{2}}{8\left(a^{4}+2a^{2}b^{2}+b^{4}\right)}-\frac{\left(a^{2}+10b^{2}\right)a\tan\left(\frac{x}{2}\right)}{8\left(a^{4}+2a^{2}b^{2}+b^{4}\right)}-\frac{3a^{2}b}{8\left(a^{4}+2a^{2}b^{2}+b^{4}\right)}\right)}{\left(\tan\left(\frac{x}{2}\right)^{2}a-2b\tan\left(\frac{x}{2}\right)-a\right)^{2}}-\frac{\left(a^{2}-2b^{2}\right)\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{x}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{\left(a^{4}+2a^{2}b^{2}+b^{4}\right)\sqrt{a^{2}+b^{2}}}$
risch	$-\frac{ia\,\mathrm{e}^{ix}\left(3iab\,\mathrm{e}^{2ix}+a^{2}\mathrm{e}^{2ix}+4b^{2}\mathrm{e}^{2ix}+3iba-a^{2}-4b^{2}\right)}{\left(-ib\,\mathrm{e}^{2ix}+a\,\mathrm{e}^{2ix}+ib+a\right)^{2}\left(ib+a\right)^{2}\left(-ib+a\right)^{2}}-\frac{\ln\left(\mathrm{e}^{ix}+\frac{ia^{5}+2ia^{3}b^{2}+ia\,b^{4}-a^{4}b-2a^{2}b^{3}-b^{5}}{\left(a^{2}+b^{2}\right)^{\frac{5}{2}}}\right)a^{2}}{2\left(a^{2}+b^{2}\right)^{\frac{5}{2}}}+\frac{\ln\left(\mathrm{e}^{ix}+\frac{ia^{5}+2ia^{3}b^{2}+ia\,b}{\left(a^{2}+\right.}\right.}{\left(a^{2}+b^{\right.}}$

input
```
int(sin(x)^2/(a*cos(x)+b*sin(x))^3,x,method=_RETURNVERBOSE)
```

output
```
-8*(-1/8*a*(a^2-2*b^2)/(a^4+2*a^2*b^2+b^4)*tan(1/2*x)^3+3/8*b*(a^2-2*b^2)/
(a^4+2*a^2*b^2+b^4)*tan(1/2*x)^2-1/8*(a^2+10*b^2)*a/(a^4+2*a^2*b^2+b^4)*ta
n(1/2*x)-3/8*a^2*b/(a^4+2*a^2*b^2+b^4))/(tan(1/2*x)^2*a-2*b*tan(1/2*x)-a)^
2-(a^2-2*b^2)/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2
*x)-2*b)/(a^2+b^2)^(1/2))
```

3.23.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(84) = 168$.

Time = 0.25 (sec) , antiderivative size = 282, normalized size of antiderivative = 3.07

$$\int \frac{\sin^2(x)}{(a\cos(x)+b\sin(x))^3}\,dx = -\frac{\left(a^2b^2-2\,b^4+(a^4-3\,a^2b^2+2\,b^4)\cos\left(x\right)^2+2\,(a^3b-2\,ab^3)\cos\left(x\right)\sin\left(x\right)\right)\sqrt{a^2+b^2}\log\left(-\frac{2\,ab\cos(x)\sin(x}{}\right.}{4\left(a^6b^2+3\,a^4b^4+3\,a^2b^6+b^8+(a^8+2\,a^6b^2-2\,a^2b^6-b^8)\right.}$$

input
```
integrate(sin(x)^2/(a*cos(x)+b*sin(x))^3,x, algorithm="fricas")
```

output
```
-1/4*((a^2*b^2 - 2*b^4 + (a^4 - 3*a^2*b^2 + 2*b^4)*cos(x)^2 + 2*(a^3*b - 2
*a*b^3)*cos(x)*sin(x))*sqrt(a^2 + b^2)*log(-(2*a*b*cos(x)*sin(x) + (a^2 -
b^2)*cos(x)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*
a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2)) - 6*(a^4*b + a^2*b^3)*cos
(x) - 2*(a^5 + 5*a^3*b^2 + 4*a*b^4)*sin(x))/(a^6*b^2 + 3*a^4*b^4 + 3*a^2*b
^6 + b^8 + (a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*cos(x)^2 + 2*(a^7*b + 3*a^5
*b^3 + 3*a^3*b^5 + a*b^7)*cos(x)*sin(x))
```

3.23.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^2(x)}{(a\cos(x)+b\sin(x))^3}\,dx = \text{Timed out}$$

input
```
integrate(sin(x)**2/(a*cos(x)+b*sin(x))**3,x)
```

output
```
Timed out
```

3.23.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. 2(84) = 168.

Time = 0.31 (sec) , antiderivative size = 299, normalized size of antiderivative = 3.25

$$\int \frac{\sin^2(x)}{(a\cos(x)+b\sin(x))^3}\,dx = \frac{(a^2-2\,b^2)\log\left(\frac{b-\frac{a\sin(x)}{\cos(x)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(x)}{\cos(x)+1}-\sqrt{a^2+b^2}}\right)}{2\,(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}} + \frac{3\,a^2b+\frac{(a^3+10\,ab^2)\sin(x)}{\cos(x)+1}-\frac{3\,(a^2b-2\,b^3)\sin(x)^2}{(\cos(x)+1)^2}+\frac{(a^3-2\,ab^2)\sin(x)^3}{(\cos(x)+1)^3}}{a^6+2\,a^4b^2+a^2b^4+\frac{4\,(a^5b+2\,a^3b^3+ab^5)\sin(x)}{\cos(x)+1}-\frac{2\,(a^6-3\,a^2b^4-2\,b^6)\sin(x)^2}{(\cos(x)+1)^2}-\frac{4\,(a^5b+2\,a^3b^3+ab^5)\sin(x)^3}{(\cos(x)+1)^3}+\frac{(a^6+2\,a^4b^2+a^2b^4}{(\cos(x)+1}$$

input
```
integrate(sin(x)^2/(a*cos(x)+b*sin(x))^3,x, algorithm="maxima")
```

output
```
1/2*(a^2 - 2*b^2)*log((b - a*sin(x)/(cos(x) + 1) + sqrt(a^2 + b^2))/(b - a
*sin(x)/(cos(x) + 1) - sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2
 + b^2)) + (3*a^2*b + (a^3 + 10*a*b^2)*sin(x)/(cos(x) + 1) - 3*(a^2*b - 2*
b^3)*sin(x)^2/(cos(x) + 1)^2 + (a^3 - 2*a*b^2)*sin(x)^3/(cos(x) + 1)^3)/(a
^6 + 2*a^4*b^2 + a^2*b^4 + 4*(a^5*b + 2*a^3*b^3 + a*b^5)*sin(x)/(cos(x) +
1) - 2*(a^6 - 3*a^2*b^4 - 2*b^6)*sin(x)^2/(cos(x) + 1)^2 - 4*(a^5*b + 2*a^
3*b^3 + a*b^5)*sin(x)^3/(cos(x) + 1)^3 + (a^6 + 2*a^4*b^2 + a^2*b^4)*sin(x
)^4/(cos(x) + 1)^4)
```

3.23.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(84) = 168.

Time = 0.31 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.14

$$\int \frac{\sin^2(x)}{(a\cos(x)+b\sin(x))^3}\,dx = \frac{(a^2-2\,b^2)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{2\,(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}} + \frac{a^3\tan\left(\frac{1}{2}\,x\right)^3-2\,ab^2\tan\left(\frac{1}{2}\,x\right)^3-3\,a^2b\tan\left(\frac{1}{2}\,x\right)^2+6\,b^3\tan\left(\frac{1}{2}\,x\right)^2+a^3\tan\left(\frac{1}{2}\,x\right)+10\,ab^2\tan\left(\frac{1}{2}\,x\right)+3}{(a^4+2\,a^2b^2+b^4)\left(a\tan\left(\frac{1}{2}\,x\right)^2-2\,b\tan\left(\frac{1}{2}\,x\right)-a\right)^2}$$

input
```
integrate(sin(x)^2/(a*cos(x)+b*sin(x))^3,x, algorithm="giac")
```

output
```
1/2*(a^2 - 2*b^2)*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*
a*tan(1/2*x) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2
 + b^2)) + (a^3*tan(1/2*x)^3 - 2*a*b^2*tan(1/2*x)^3 - 3*a^2*b*tan(1/2*x)^2
 + 6*b^3*tan(1/2*x)^2 + a^3*tan(1/2*x) + 10*a*b^2*tan(1/2*x) + 3*a^2*b)/((
a^4 + 2*a^2*b^2 + b^4)*(a*tan(1/2*x)^2 - 2*b*tan(1/2*x) - a)^2)
```

3.23.9 Mupad [B] (verification not implemented)

Time = 21.14 (sec) , antiderivative size = 263, normalized size of antiderivative = 2.86

$$\int \frac{\sin^2(x)}{(a\cos(x)+b\sin(x))^3}\,dx = \frac{\frac{3a^2b}{a^4+2a^2b^2+b^4}+\frac{a\tan\left(\frac{x}{2}\right)(a^2+10b^2)}{a^4+2a^2b^2+b^4}+\frac{a\tan\left(\frac{x}{2}\right)^3(a^2-2b^2)}{a^4+2a^2b^2+b^4}-\frac{3b\tan\left(\frac{x}{2}\right)^2(a^2-2b^2)}{a^4+2a^2b^2+b^4}}{a^2-\tan\left(\frac{x}{2}\right)^2(2a^2-4b^2)+a^2\tan\left(\frac{x}{2}\right)^4+4ab\tan\left(\frac{x}{2}\right)-4ab\tan\left(\frac{x}{2}\right)^3} + \frac{\operatorname{atanh}\left(\frac{2a^4b+4a^2b^3+2b^5}{2(a^2+b^2)^{5/2}}-\frac{a\tan\left(\frac{x}{2}\right)(a^4+2a^2b^2+b^4)}{(a^2+b^2)^{5/2}}\right)(a^2-2b^2)}{(a^2+b^2)^{5/2}}$$

input
```
int(sin(x)^2/(a*cos(x) + b*sin(x))^3,x)
```

output
```
((3*a^2*b)/(a^4 + b^4 + 2*a^2*b^2) + (a*tan(x/2)*(a^2 + 10*b^2))/(a^4 + b^
4 + 2*a^2*b^2) + (a*tan(x/2)^3*(a^2 - 2*b^2))/(a^4 + b^4 + 2*a^2*b^2) - (3
*b*tan(x/2)^2*(a^2 - 2*b^2))/(a^4 + b^4 + 2*a^2*b^2))/(a^2 - tan(x/2)^2*(2
*a^2 - 4*b^2) + a^2*tan(x/2)^4 + 4*a*b*tan(x/2) - 4*a*b*tan(x/2)^3) + (ata
nh((2*a^4*b + 2*b^5 + 4*a^2*b^3)/(2*(a^2 + b^2)^(5/2)) - (a*tan(x/2)*(a^4
+ b^4 + 2*a^2*b^2))/(a^2 + b^2)^(5/2))*(a^2 - 2*b^2))/(a^2 + b^2)^(5/2)
```

3.24 $\int \frac{\sin(x)}{(a\cos(x)+b\sin(x))^3}\,dx$

3.24.1 Optimal result

Integrand size = 14, antiderivative size = 15

$$\int \frac{\sin(x)}{(a\cos(x)+b\sin(x))^3}\,dx = \frac{1}{2a(b+a\cot(x))^2}$$

output
```
1/2/a/(b+a*cot(x))^2
```

3.24.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 47 vs. 2(15) = 30.

Time = 0.17 (sec) , antiderivative size = 47, normalized size of antiderivative = 3.13

$$\int \frac{\sin(x)}{(a\cos(x)+b\sin(x))^3}\,dx = \frac{2b^2\sin^2(x)+a(a+b\sin(2x))}{2a\,(a^2+b^2)\,(a\cos(x)+b\sin(x))^2}$$

input
```
Integrate[Sin[x]/(a*Cos[x] + b*Sin[x])^3,x]
```

output
```
(2*b^2*Sin[x]^2 + a*(a + b*Sin[2*x]))/(2*a*(a^2 + b^2)*(a*Cos[x] + b*Sin[x
])^2)
```

3.24.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 3566, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)}{(a\cos(x)+b\sin(x))^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(x)}{(a\cos(x)+b\sin(x))^3}dx$$

$$\Big\downarrow \text{ 3566}$$

$$\int \frac{\tan(x)}{(a+b\tan(x))^3}d\tan(x)$$

$$\Big\downarrow \text{ 48}$$

$$\frac{\tan^2(x)}{2a(a+b\tan(x))^2}$$

input
```
Int[Sin[x]/(a*Cos[x] + b*Sin[x])^3,x]
```

output
```
Tan[x]^2/(2*a*(a + b*Tan[x])^2)
```

3.24.3.1 Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3566
```
Int[sin[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[1/d   Subst[Int[x^m*((a + b*
x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Tan[c + d*x]], x] /; FreeQ[{a, b, c
, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n, 0
] && GtQ[m, 1])
```

3.24.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(13) = 26$.

Time = 0.44 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.93

method	result	size
default	$-\frac{1}{b^{2}(a+b\tan(x))}+\frac{a}{2b^{2}(a+b\tan(x))^{2}}$	29
parallelrisch	$\frac{1-\cos(2x)}{2a((a^{2}-b^{2})\cos(2x)+2ba\sin(2x)+a^{2}+b^{2})}$	45
norman	$\frac{\frac{2\tan\left(\frac{x}{2}\right)^{2}}{a}+\frac{2\tan\left(\frac{x}{2}\right)^{4}}{a}}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)\left(\tan\left(\frac{x}{2}\right)^{2}a-2b\tan\left(\frac{x}{2}\right)-a\right)^{2}}$	56
risch	$-\frac{2i(ia\,e^{2ix}+b\,e^{2ix}-b)}{(b\,e^{2ix}+ia\,e^{2ix}-b+ia)^{2}(ia+b)^{2}}$	58

input
```
int(sin(x)/(a*cos(x)+b*sin(x))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/b^2/(a+b*tan(x))+1/2*a/b^2/(a+b*tan(x))^2
```

3.24.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(13) = 26$.

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 7.73

$$\int\frac{\sin(x)}{(a\cos(x)+b\sin(x))^{3}}\,dx = -\frac{4\,ab^{2}\cos\left(x\right)^{2}-a^{3}-3\,ab^{2}-2\,(a^{2}b-b^{3})\cos\left(x\right)\sin\left(x\right)}{2\,\left(a^{4}b^{2}+2\,a^{2}b^{4}+b^{6}+(a^{6}+a^{4}b^{2}-a^{2}b^{4}-b^{6})\cos\left(x\right)^{2}+2\,(a^{5}b+2\,a^{3}b^{3}+ab^{5})\cos\left(x\right)\sin\left(x\right)\right)}$$

input
```
integrate(sin(x)/(a*cos(x)+b*sin(x))^3,x, algorithm="fricas")
```

output
```
-1/2*(4*a*b^2*cos(x)^2 - a^3 - 3*a*b^2 - 2*(a^2*b - b^3)*cos(x)*sin(x))/(a
^4*b^2 + 2*a^2*b^4 + b^6 + (a^6 + a^4*b^2 - a^2*b^4 - b^6)*cos(x)^2 + 2*(a
^5*b + 2*a^3*b^3 + a*b^5)*cos(x)*sin(x))
```

3.24.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin(x)}{(a\cos(x)+b\sin(x))^3}\,dx = \text{Timed out}$$

input
```
integrate(sin(x)/(a*cos(x)+b*sin(x))**3,x)
```

output
```
Timed out
```

3.24.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(13) = 26.

Time = 0.24 (sec) , antiderivative size = 84, normalized size of antiderivative = 5.60

$$\int \frac{\sin(x)}{(a\cos(x)+b\sin(x))^3}\,dx = \frac{2\sin(x)^2}{\left(a^3+\frac{4a^2b\sin(x)}{\cos(x)+1}-\frac{4a^2b\sin(x)^3}{(\cos(x)+1)^3}+\frac{a^3\sin(x)^4}{(\cos(x)+1)^4}-\frac{2(a^3-2ab^2)\sin(x)^2}{(\cos(x)+1)^2}\right)(\cos(x)+1)^2}$$

input
```
integrate(sin(x)/(a*cos(x)+b*sin(x))^3,x, algorithm="maxima")
```

output
```
2*sin(x)^2/((a^3 + 4*a^2*b*sin(x)/(cos(x) + 1) - 4*a^2*b*sin(x)^3/(cos(x)
+ 1)^3 + a^3*sin(x)^4/(cos(x) + 1)^4 - 2*(a^3 - 2*a*b^2)*sin(x)^2/(cos(x)
+ 1)^2)*(cos(x) + 1)^2)
```

3.24.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \frac{\sin(x)}{(a\cos(x)+b\sin(x))^3}\,dx = -\frac{2\,b\tan(x)+a}{2\,(b\tan(x)+a)^2b^2}$$

input
```
integrate(sin(x)/(a*cos(x)+b*sin(x))^3,x, algorithm="giac")
```

output
```
-1/2*(2*b*tan(x) + a)/((b*tan(x) + a)^2*b^2)
```

3.24.9 Mupad [B] (verification not implemented)

Time = 21.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 3.20

$$\int \frac{\sin(x)}{(a\cos(x)+b\sin(x))^3}\,dx = \frac{\tan\left(\frac{x}{2}\right)^2\left(a-\frac{2\,a^2-4\,b^2}{2\,a}\right)}{b^2\left(-a\tan\left(\frac{x}{2}\right)^2+2\,b\tan\left(\frac{x}{2}\right)+a\right)^2}$$

input
```
int(sin(x)/(a*cos(x) + b*sin(x))^3,x)
```

output
```
(tan(x/2)^2*(a - (2*a^2 - 4*b^2)/(2*a)))/(b^2*(a + 2*b*tan(x/2) - a*tan(x/
2)^2)^2)
```

3.25 $\int \frac{1}{(a\cos(x)+b\sin(x))^3}\,dx$

3.25.1 Optimal result

Integrand size = 11, antiderivative size = 73

$$\int \frac{1}{(a\cos(x)+b\sin(x))^3}\,dx=-\frac{\operatorname{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{3/2}}-\frac{b\cos(x)-a\sin(x)}{2\left(a^2+b^2\right)(a\cos(x)+b\sin(x))^2}$$

output
```
-1/2*arctanh((b*cos(x)-a*sin(x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)+1/2*(-b*
cos(x)+a*sin(x))/(a^2+b^2)/(a*cos(x)+b*sin(x))^2
```

3.25.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.38

$$\int \frac{1}{(a\cos(x)+b\sin(x))^3}\,dx=\frac{\left(a^2+b^2\right)(-b\cos(x)+a\sin(x))+2\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{-b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)(a\cos(x)+b\sin(x))^2}{2(a-ib)^2(a+ib)^2(a\cos(x)+b\sin(x))^2}$$

input
```
Integrate[(a*Cos[x] + b*Sin[x])^(-3),x]
```

output
```
((a^2 + b^2)*(-(b*Cos[x]) + a*Sin[x]) + 2*Sqrt[a^2 + b^2]*ArcTanh[(-b + a*
Tan[x/2])/Sqrt[a^2 + b^2]]*(a*Cos[x] + b*Sin[x])^2)/(2*(a - I*b)^2*(a + I*
b)^2*(a*Cos[x] + b*Sin[x])^2)
```

3.25.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {3042, 3555, 3042, 3553, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cos(x)+b\sin(x))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a\cos(x)+b\sin(x))^3}dx$$

$$\Big\downarrow \text{3555}$$

$$\frac{\int \frac{1}{a\cos(x)+b\sin(x)}dx}{2(a^2+b^2)}-\frac{b\cos(x)-a\sin(x)}{2(a^2+b^2)(a\cos(x)+b\sin(x))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{a\cos(x)+b\sin(x)}dx}{2(a^2+b^2)}-\frac{b\cos(x)-a\sin(x)}{2(a^2+b^2)(a\cos(x)+b\sin(x))^2}$$

$$\Big\downarrow \text{3553}$$

$$-\frac{\int \frac{1}{a^2+b^2-(b\cos(x)-a\sin(x))^2}d(b\cos(x)-a\sin(x))}{2(a^2+b^2)}-\frac{b\cos(x)-a\sin(x)}{2(a^2+b^2)(a\cos(x)+b\sin(x))^2}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{3/2}}-\frac{b\cos(x)-a\sin(x)}{2(a^2+b^2)(a\cos(x)+b\sin(x))^2}$$

input
```
Int[(a*Cos[x] + b*Sin[x])^(-3),x]
```

output
```
-1/2*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]]/(a^2 + b^2)^(3/2) - (b
*Cos[x] - a*Sin[x])/(2*(a^2 + b^2)*(a*Cos[x] + b*Sin[x])^2)
```

3.25.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3555
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(b*Cos[c + d*x] - a*Sin[c + d*x])*((a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Simp[(n + 2)/((n + 1)*(a^
2 + b^2))   Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

3.25.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(65) = 130$.

Time = 0.53 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.15

method	result	size
default	$-\frac{2\left(-\frac{\left(a^2+2b^2\right)\tan\left(\frac{x}{2}\right)^3}{2\left(a^2+b^2\right)a}-\frac{b\left(a^2-2b^2\right)\tan\left(\frac{x}{2}\right)^2}{2\left(a^2+b^2\right)a^2}-\frac{\left(a^2-2b^2\right)\tan\left(\frac{x}{2}\right)}{2\left(a^2+b^2\right)a}+\frac{b}{2a^2+2b^2}\right)}{\left(\tan\left(\frac{x}{2}\right)^2a-2b\tan\left(\frac{x}{2}\right)-a\right)^2}+\frac{\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}$	157
risch	$\frac{e^{ix}\left(ia\,e^{2ix}+b\,e^{2ix}-ia+b\right)}{\left(b\,e^{2ix}+ia\,e^{2ix}-b+ia\right)^2(-ia+b)(ia+b)}+\frac{\ln\left(e^{ix}+\frac{ia^3+ia\,b^2-a^2b-b^3}{\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{2\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{\ln\left(e^{ix}-\frac{ia^3+ia\,b^2-a^2b-b^3}{\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{2\left(a^2+b^2\right)^{\frac{3}{2}}}$	187

input
```
int(1/(a*cos(x)+b*sin(x))^3,x,method=_RETURNVERBOSE)
```

output
```
-2*(-1/2*(a^2+2*b^2)/(a^2+b^2)/a*tan(1/2*x)^3-1/2*b*(a^2-2*b^2)/(a^2+b^2)/
a^2*tan(1/2*x)^2-1/2*(a^2-2*b^2)/(a^2+b^2)/a*tan(1/2*x)+1/2*b/(a^2+b^2))/(
tan(1/2*x)^2*a-2*b*tan(1/2*x)-a)^2+1/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tan(
1/2*x)-2*b)/(a^2+b^2)^(1/2))
```

3.25.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(65) = 130$.

Time = 0.26 (sec) , antiderivative size = 225, normalized size of antiderivative = 3.08

$$\int \frac{1}{(a\cos(x)+b\sin(x))^3}\,dx = \frac{\left(2\,ab\cos\left(x\right)\sin\left(x\right)+\left(a^2-b^2\right)\cos\left(x\right)^2+b^2\right)\sqrt{a^2+b^2}\log\left(-\frac{2\,ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}(b\,c}{2\,ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2+b^2}\right.}{4\left(a^4b^2+2\,a^2b^4+b^6+\left(a^6+a^4b^2-a^2b^4-b^6\right)\cos\left(x\right)^2+2\left(a^5b+2\,a^3b^3\right.\right.}$$

input
```
integrate(1/(a*cos(x)+b*sin(x))^3,x, algorithm="fricas")
```

output
```
1/4*((2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2)*sqrt(a^2 + b^2)*lo
g(-(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 - 2*a^2 - b^2 + 2*sqrt(a^2
+ b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2
+ b^2)) - 2*(a^2*b + b^3)*cos(x) + 2*(a^3 + a*b^2)*sin(x))/(a^4*b^2 + 2*a^
2*b^4 + b^6 + (a^6 + a^4*b^2 - a^2*b^4 - b^6)*cos(x)^2 + 2*(a^5*b + 2*a^3*
b^3 + a*b^5)*cos(x)*sin(x))
```

3.25.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{(a\cos(x)+b\sin(x))^3}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(1/(a*cos(x)+b*sin(x))**3,x)
```

output
```
Exception raised: AttributeError >> 'NoneType' object has no attribute 'pr
imitive'
```

3.25.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(65) = 130$.

Time = 0.33 (sec) , antiderivative size = 250, normalized size of antiderivative = 3.42

$$\int \frac{1}{(a\cos(x)+b\sin(x))^3}\,dx = -\frac{a^2b - \frac{(a^3-2\,ab^2)\sin(x)}{\cos(x)+1} - \frac{(a^2b-2\,b^3)\sin(x)^2}{(\cos(x)+1)^2} - \frac{(a^3+2\,ab^2)\sin(x)^3}{(\cos(x)+1)^3}}{a^6+a^4b^2+\frac{4\,(a^5b+a^3b^3)\sin(x)}{\cos(x)+1} - \frac{2\,(a^6-a^4b^2-2\,a^2b^4)\sin(x)^2}{(\cos(x)+1)^2} - \frac{4\,(a^5b+a^3b^3)\sin(x)^3}{(\cos(x)+1)^3} + \frac{(a^6+a^4b^2)\sin(x)^4}{(\cos(x)+1)^4}} - \frac{\log\left(\frac{b-\frac{a\sin(x)}{\cos(x)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(x)}{\cos(x)+1}-\sqrt{a^2+b^2}}\right)}{2\,(a^2+b^2)^{\frac{3}{2}}}$$

input
```
integrate(1/(a*cos(x)+b*sin(x))^3,x, algorithm="maxima")
```

output
```
-(a^2*b - (a^3 - 2*a*b^2)*sin(x)/(cos(x) + 1) - (a^2*b - 2*b^3)*sin(x)^2/(
cos(x) + 1)^2 - (a^3 + 2*a*b^2)*sin(x)^3/(cos(x) + 1)^3)/(a^6 + a^4*b^2 +
4*(a^5*b + a^3*b^3)*sin(x)/(cos(x) + 1) - 2*(a^6 - a^4*b^2 - 2*a^2*b^4)*si
n(x)^2/(cos(x) + 1)^2 - 4*(a^5*b + a^3*b^3)*sin(x)^3/(cos(x) + 1)^3 + (a^6
 + a^4*b^2)*sin(x)^4/(cos(x) + 1)^4) - 1/2*log((b - a*sin(x)/(cos(x) + 1)
+ sqrt(a^2 + b^2))/(b - a*sin(x)/(cos(x) + 1) - sqrt(a^2 + b^2)))/(a^2 + b
^2)^(3/2)
```

3.25.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(65) = 130$.

Time = 0.30 (sec) , antiderivative size = 166, normalized size of antiderivative = 2.27

$$\int \frac{1}{(a\cos(x)+b\sin(x))^3}\,dx = -\frac{\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{2\,(a^2+b^2)^{\frac{3}{2}}} + \frac{a^3\tan\left(\frac{1}{2}\,x\right)^3+2\,ab^2\tan\left(\frac{1}{2}\,x\right)^3+a^2b\tan\left(\frac{1}{2}\,x\right)^2-2\,b^3\tan\left(\frac{1}{2}\,x\right)^2+a^3\tan\left(\frac{1}{2}\,x\right)-2\,ab^2\tan\left(\frac{1}{2}\,x\right)-a^2b}{(a^4+a^2b^2)\left(a\tan\left(\frac{1}{2}\,x\right)^2-2\,b\tan\left(\frac{1}{2}\,x\right)-a\right)^2}$$

input
```
integrate(1/(a*cos(x)+b*sin(x))^3,x, algorithm="giac")
```

output
```
-1/2*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*x)
- 2*b + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) + (a^3*tan(1/2*x)^3 + 2*a*b^
2*tan(1/2*x)^3 + a^2*b*tan(1/2*x)^2 - 2*b^3*tan(1/2*x)^2 + a^3*tan(1/2*x)
- 2*a*b^2*tan(1/2*x) - a^2*b)/((a^4 + a^2*b^2)*(a*tan(1/2*x)^2 - 2*b*tan(1
/2*x) - a)^2)
```

3.25.9 Mupad [B] (verification not implemented)

Time = 21.69 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.96

$$\int \frac{1}{(a\cos(x)+b\sin(x))^3}\,dx = \frac{\frac{\tan\left(\frac{x}{2}\right)^3\,(a^2+2\,b^2)}{a\,(a^2+b^2)}-\frac{b}{a^2+b^2}+\frac{\tan\left(\frac{x}{2}\right)\,(a^2-2\,b^2)}{a\,(a^2+b^2)}+\frac{b\tan\left(\frac{x}{2}\right)^2\,(a^2-2\,b^2)}{a^2\,(a^2+b^2)}}{a^2-\tan\left(\frac{x}{2}\right)^2\,(2\,a^2-4\,b^2)+a^2\tan\left(\frac{x}{2}\right)^4+4\,a\,b\tan\left(\frac{x}{2}\right)-4\,a\,b\tan\left(\frac{x}{2}\right)^3} - \frac{\operatorname{atanh}\left(-\frac{\left(2\,a\tan\left(\frac{x}{2}\right)-\frac{2\,a^2\,b+2\,b^3}{a^2+b^2}\right)\left(\frac{a^2}{2}+\frac{b^2}{2}\right)}{(a^2+b^2)^{3/2}}\right)}{(a^2+b^2)^{3/2}}$$

input
```
int(1/(a*cos(x) + b*sin(x))^3,x)
```

output
```
((tan(x/2)^3*(a^2 + 2*b^2))/(a*(a^2 + b^2)) - b/(a^2 + b^2) + (tan(x/2)*(a
^2 - 2*b^2))/(a*(a^2 + b^2)) + (b*tan(x/2)^2*(a^2 - 2*b^2))/(a^2*(a^2 + b^
2)))/(a^2 - tan(x/2)^2*(2*a^2 - 4*b^2) + a^2*tan(x/2)^4 + 4*a*b*tan(x/2) -
 4*a*b*tan(x/2)^3) - atanh(-((2*a*tan(x/2) - (2*a^2*b + 2*b^3)/(a^2 + b^2)
)*(a^2/2 + b^2/2))/(a^2 + b^2)^(3/2))/(a^2 + b^2)^(3/2)
```

3.26 $\int \frac{\csc(x)}{(a\cos(x)+b\sin(x))^3}\,dx$

3.26.1 Optimal result

Integrand size = 14, antiderivative size = 59

$$\int \frac{\csc(x)}{(a\cos(x)+b\sin(x))^3}\,dx = \frac{\log(\tan(x))}{a^3} - \frac{\log(a+b\tan(x))}{a^3} + \frac{\frac{1}{a}+\frac{a}{b^2}}{2(a+b\tan(x))^2} + \frac{\frac{1}{a^2}-\frac{1}{b^2}}{a+b\tan(x)}$$

output
```
ln(tan(x))/a^3-ln(a+b*tan(x))/a^3+1/2*(1/a+a/b^2)/(a+b*tan(x))^2+(1/a^2-1/
b^2)/(a+b*tan(x))
```

3.26.2 Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.63

$$\int \frac{\csc(x)}{(a\cos(x)+b\sin(x))^3}\,dx = \frac{a^2\csc^2(x)+2ab\cot(x)(-1+2\log(\sin(x))-2\log(a\cos(x)+b\sin(x)))+2b^2(-1+\log(\sin(x))-\log(a\,\mathrm{co}}{2a^3(b+a\cot(x))^2}$$

input
```
Integrate[Csc[x]/(a*Cos[x] + b*Sin[x])^3,x]
```

output
```
(a^2*Csc[x]^2 + 2*a*b*Cot[x]*(-1 + 2*Log[Sin[x]] - 2*Log[a*Cos[x] + b*Sin[
x]]) + 2*b^2*(-1 + Log[Sin[x]] - Log[a*Cos[x] + b*Sin[x]]) + 2*a^2*Cot[x]^
2*(Log[Sin[x]] - Log[a*Cos[x] + b*Sin[x]]))/(2*a^3*(b + a*Cot[x])^2)
```

3.26.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3566, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(x)}{(a\cos(x)+b\sin(x))^3}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin(x)(a\cos(x)+b\sin(x))^3}dx$$

$$\Big\downarrow \text{ 3566}$$

$$\int \frac{\left(\tan^2(x)+1\right)\cot(x)}{(a+b\tan(x))^3}d\tan(x)$$

$$\Big\downarrow \text{ 522}$$

$$\int \left(-\frac{b}{a^3(a+b\tan(x))}+\frac{\cot(x)}{a^3}+\frac{a^2-b^2}{a^2b(a+b\tan(x))^2}+\frac{-a^2-b^2}{ab(a+b\tan(x))^3}\right)d\tan(x)$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\log(a+b\tan(x))}{a^3}+\frac{\log(\tan(x))}{a^3}+\frac{\frac{1}{a^2}-\frac{1}{b^2}}{a+b\tan(x)}+\frac{\frac{a}{b^2}+\frac{1}{a}}{2(a+b\tan(x))^2}$$

input
```
Int[Csc[x]/(a*Cos[x] + b*Sin[x])^3,x]
```

output
```
Log[Tan[x]]/a^3 - Log[a + b*Tan[x]]/a^3 + (a^(-1) + a/b^2)/(2*(a + b*Tan[x
])^2) + (a^(-2) - b^(-2))/(a + b*Tan[x])
```

3.26.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3566
```
Int[sin[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[1/d   Subst[Int[x^m*((a + b*
x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Tan[c + d*x]], x] /; FreeQ[{a, b, c
, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n, 0
] && GtQ[m, 1])
```

3.26.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.24

method	result
default	$\frac{\ln(\tan(x))}{a^3}-\frac{-a^2-b^2}{2a\,b^2(a+b\tan(x))^2}-\frac{a^2-b^2}{a^2b^2(a+b\tan(x))}-\frac{\ln(a+b\tan(x))}{a^3}$
norman	$\frac{-\frac{2\left(-a^2+3b^2\right)\tan\left(\frac{x}{2}\right)^2}{a^3}-\frac{4b\tan\left(\frac{x}{2}\right)}{a^2}+\frac{4b\tan\left(\frac{x}{2}\right)^3}{a^2}}{\left(\tan\left(\frac{x}{2}\right)^2a-2b\tan\left(\frac{x}{2}\right)-a\right)^2}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a^3}-\frac{\ln\left(\tan\left(\frac{x}{2}\right)^2a-2b\tan\left(\frac{x}{2}\right)-a\right)}{a^3}$
risch	$\frac{2a^2e^{2ix}-2b^2e^{2ix}-4iab\,e^{2ix}+2b^2-2iba}{(-ib\,e^{2ix}+a\,e^{2ix}+ib+a)^2a^2(-ib+a)}+\frac{\ln(e^{2ix}-1)}{a^3}-\frac{\ln\left(e^{2ix}-\frac{ib+a}{ib-a}\right)}{a^3}$
parallelrisch	$\frac{\left(\left(-2a^2+2b^2\right)\cos(2x)-4ba\sin(2x)-2a^2-2b^2\right)\ln\left(\frac{-2a\cos(x)-2b\sin(x)}{\cos(x)+1}\right)+\left(\left(2a^2-2b^2\right)\cos(2x)+4ba\sin(2x)+2a^2+2b^2\right)\ln(\csc(x}{2a^3\left(\left(a^2-b^2\right)\cos(2x)+2ba\sin(2x)+a^2+b^2\right)}$

input
```
int(csc(x)/(a*cos(x)+b*sin(x))^3,x,method=_RETURNVERBOSE)
```

output
```
ln(tan(x))/a^3-1/2*(-a^2-b^2)/a/b^2/(a+b*tan(x))^2-(a^2-b^2)/a^2/b^2/(a+b*
tan(x))-ln(a+b*tan(x))/a^3
```

3.26.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 220 vs. 2(57) = 114.

Time = 0.27 (sec) , antiderivative size = 220, normalized size of antiderivative = 3.73

$$\int \frac{\csc(x)}{(a\cos(x)+b\sin(x))^3}\,dx = \frac{4\,a^2b^2\cos\left(x\right)^2+a^4-a^2b^2-2\left(a^3b-ab^3\right)\cos\left(x\right)\sin\left(x\right)-\left(a^2b^2+b^4+\left(a^4-b^4\right)\cos\left(x\right)^2+2\left(a^3b+ab^3\right)}{2\left(a^5b^2+a^3}$$

input
```
integrate(csc(x)/(a*cos(x)+b*sin(x))^3,x, algorithm="fricas")
```

output
```
1/2*(4*a^2*b^2*cos(x)^2 + a^4 - a^2*b^2 - 2*(a^3*b - a*b^3)*cos(x)*sin(x)
- (a^2*b^2 + b^4 + (a^4 - b^4)*cos(x)^2 + 2*(a^3*b + a*b^3)*cos(x)*sin(x))
*log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) + (a^2*b^2 + b^4 +
(a^4 - b^4)*cos(x)^2 + 2*(a^3*b + a*b^3)*cos(x)*sin(x))*log(-1/4*cos(x)^2
+ 1/4))/(a^5*b^2 + a^3*b^4 + (a^7 - a^3*b^4)*cos(x)^2 + 2*(a^6*b + a^4*b^3
)*cos(x)*sin(x))
```

3.26.6 Sympy [F]

$$\int \frac{\csc(x)}{(a\cos(x)+b\sin(x))^3}\,dx = \int \frac{\csc{(x)}}{(a\cos{(x)}+b\sin{(x)})^3}\,dx$$

input
```
integrate(csc(x)/(a*cos(x)+b*sin(x))**3,x)
```

output
```
Integral(csc(x)/(a*cos(x) + b*sin(x))**3, x)
```

3.26.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(57) = 114$.

Time = 0.23 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.92

$$\int \frac{\csc(x)}{(a\cos(x)+b\sin(x))^3}\,dx = -\frac{2\left(\frac{2ab\sin(x)}{\cos(x)+1}-\frac{2ab\sin(x)^3}{(\cos(x)+1)^3}-\frac{(a^2-3b^2)\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^5+\frac{4a^4b\sin(x)}{\cos(x)+1}-\frac{4a^4b\sin(x)^3}{(\cos(x)+1)^3}+\frac{a^5\sin(x)^4}{(\cos(x)+1)^4}-\frac{2(a^5-2a^3b^2)\sin(x)^2}{(\cos(x)+1)^2}} - \frac{\log\left(-a-\frac{2b\sin(x)}{\cos(x)+1}+\frac{a\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^3}+\frac{\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^3}$$

input
```
integrate(csc(x)/(a*cos(x)+b*sin(x))^3,x, algorithm="maxima")
```

output
```
-2*(2*a*b*sin(x)/(cos(x) + 1) - 2*a*b*sin(x)^3/(cos(x) + 1)^3 - (a^2 - 3*b
^2)*sin(x)^2/(cos(x) + 1)^2)/(a^5 + 4*a^4*b*sin(x)/(cos(x) + 1) - 4*a^4*b*
sin(x)^3/(cos(x) + 1)^3 + a^5*sin(x)^4/(cos(x) + 1)^4 - 2*(a^5 - 2*a^3*b^2
)*sin(x)^2/(cos(x) + 1)^2) - log(-a - 2*b*sin(x)/(cos(x) + 1) + a*sin(x)^2
/(cos(x) + 1)^2)/a^3 + log(sin(x)/(cos(x) + 1))/a^3
```

3.26.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.31

$$\int \frac{\csc(x)}{(a\cos(x)+b\sin(x))^3}\,dx = -\frac{\log(|b\tan(x)+a|)}{a^3}+\frac{\log(|\tan(x)|)}{a^3} + \frac{3\,b^4\tan(x)^2-2\,a^3b\tan(x)+8\,ab^3\tan(x)-a^4+6\,a^2b^2}{2\,(b\tan(x)+a)^2a^3b^2}$$

input
```
integrate(csc(x)/(a*cos(x)+b*sin(x))^3,x, algorithm="giac")
```

output
```
-log(abs(b*tan(x) + a))/a^3 + log(abs(tan(x)))/a^3 + 1/2*(3*b^4*tan(x)^2 -
 2*a^3*b*tan(x) + 8*a*b^3*tan(x) - a^4 + 6*a^2*b^2)/((b*tan(x) + a)^2*a^3*
b^2)
```

3.26.9 Mupad [B] (verification not implemented)

Time = 21.56 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.22

$$\int \frac{\csc(x)}{(a\cos(x)+b\sin(x))^3}\,dx = \frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a^3}-\frac{\ln\left(-a\tan\left(\frac{x}{2}\right)^2+2\,b\tan\left(\frac{x}{2}\right)+a\right)}{a^3} + \frac{\frac{2\tan\left(\frac{x}{2}\right)^2\,(a^2-3\,b^2)}{a^3}+\frac{4\,b\tan\left(\frac{x}{2}\right)^3}{a^2}-\frac{4\,b\tan\left(\frac{x}{2}\right)}{a^2}}{a^2-\tan\left(\frac{x}{2}\right)^2\,(2\,a^2-4\,b^2)+a^2\tan\left(\frac{x}{2}\right)^4+4\,a\,b\tan\left(\frac{x}{2}\right)-4\,a\,b\tan\left(\frac{x}{2}\right)^3}$$

input
```
int(1/(sin(x)*(a*cos(x) + b*sin(x))^3),x)
```

output
```
log(tan(x/2))/a^3 - log(a + 2*b*tan(x/2) - a*tan(x/2)^2)/a^3 + ((2*tan(x/2
)^2*(a^2 - 3*b^2))/a^3 + (4*b*tan(x/2)^3)/a^2 - (4*b*tan(x/2))/a^2)/(a^2 -
 tan(x/2)^2*(2*a^2 - 4*b^2) + a^2*tan(x/2)^4 + 4*a*b*tan(x/2) - 4*a*b*tan(
x/2)^3)
```

3.27 $\int \frac{\csc^2(x)}{(a\cos(x)+b\sin(x))^3}\,dx$

3.27.1 Optimal result

Integrand size = 16, antiderivative size = 184

$$\int \frac{\csc^2(x)}{(a\cos(x)+b\sin(x))^3}\,dx = \frac{3b\text{arctanh}(\cos(x))}{a^4} - \frac{\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{2a^2\sqrt{a^2+b^2}} - \frac{2b^2\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^4\sqrt{a^2+b^2}} - \frac{\sqrt{a^2+b^2}\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^4} - \frac{\csc(x)}{a^3} - \frac{b\cos(x)-a\sin(x)}{2a^2(a\cos(x)+b\sin(x))^2} - \frac{2b}{a^3(a\cos(x)+b\sin(x))}$$

output
```
3*b*arctanh(cos(x))/a^4-csc(x)/a^3+1/2*(-b*cos(x)+a*sin(x))/a^2/(a*cos(x)+
b*sin(x))^2-2*b/a^3/(a*cos(x)+b*sin(x))-1/2*arctanh((b*cos(x)-a*sin(x))/(a
^2+b^2)^(1/2))/a^2/(a^2+b^2)^(1/2)-2*b^2*arctanh((b*cos(x)-a*sin(x))/(a^2+
b^2)^(1/2))/a^4/(a^2+b^2)^(1/2)-arctanh((b*cos(x)-a*sin(x))/(a^2+b^2)^(1/2
))*(a^2+b^2)^(1/2)/a^4
```

3.27.2 Mathematica [A] (verified)

Time = 1.29 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.05

$$\int \frac{\csc^2(x)}{(a\cos(x)+b\sin(x))^3}\,dx = \frac{\csc^3(x)(a\cos(x)+b\sin(x))\left(a(a^2+b^2)\sin(x)-5ab(a\cos(x)+b\sin(x))+\frac{6(a^2+2b^2)\operatorname{arctanh}\left(\frac{-b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)(\cdots}{\sqrt{a^2+b^2}}\right.}{\cdots}$$

input
```
Integrate[Csc[x]^2/(a*Cos[x] + b*Sin[x])^3,x]
```

output
```
(Csc[x]^3*(a*Cos[x] + b*Sin[x])*(a*(a^2 + b^2)*Sin[x] - 5*a*b*(a*Cos[x] +
b*Sin[x]) + (6*(a^2 + 2*b^2)*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]]*(a
*Cos[x] + b*Sin[x])^2)/Sqrt[a^2 + b^2] - a*Cot[x/2]*(a*Cos[x] + b*Sin[x])^
2 + 6*b*Log[Cos[x/2]]*(a*Cos[x] + b*Sin[x])^2 - 6*b*Log[Sin[x/2]]*(a*Cos[x
] + b*Sin[x])^2 - a*(a*Cos[x] + b*Sin[x])^2*Tan[x/2]))/(2*a^4*(b + a*Cot[x
])^3)
```

3.27.3 Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.16, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 3584, 3042, 3555, 3042, 3553, 219, 3572, 3042, 3553, 219, 3582, 3042, 3553, 219, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(x)}{(a\cos(x)+b\sin(x))^3}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\sin(x)^2(a\cos(x)+b\sin(x))^3}dx$$

$$\Big\downarrow \ 3584$$

$$\frac{(a^2+b^2)\int \frac{1}{(a\cos(x)+b\sin(x))^3}dx}{a^2}+\frac{\int \frac{\csc^2(x)}{a\cos(x)+b\sin(x)}dx}{a^2}-\frac{2b\int \frac{\csc(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a^2+b^2)\int \frac{1}{(a\cos(x)+b\sin(x))^3}dx}{a^2}-\frac{2b\int \frac{1}{\sin(x)(a\cos(x)+b\sin(x))^2}dx}{a^2}+\frac{\int \frac{1}{\sin(x)^2(a\cos(x)+b\sin(x))}dx}{a^2}$$

$$\Big\downarrow \text{3555}$$

$$\frac{(a^2+b^2)\left(\frac{\int \frac{1}{a\cos(x)+b\sin(x)}dx}{2(a^2+b^2)}-\frac{b\cos(x)-a\sin(x)}{2(a^2+b^2)(a\cos(x)+b\sin(x))^2}\right)}{a^2}-\frac{2b\int \frac{1}{\sin(x)(a\cos(x)+b\sin(x))^2}dx}{a^2}+\frac{\int \frac{1}{\sin(x)^2(a\cos(x)+b\sin(x))}dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a^2+b^2)\left(\frac{\int \frac{1}{a\cos(x)+b\sin(x)}dx}{2(a^2+b^2)}-\frac{b\cos(x)-a\sin(x)}{2(a^2+b^2)(a\cos(x)+b\sin(x))^2}\right)}{a^2}-\frac{2b\int \frac{1}{\sin(x)(a\cos(x)+b\sin(x))^2}dx}{a^2}+\frac{\int \frac{1}{\sin(x)^2(a\cos(x)+b\sin(x))}dx}{a^2}$$

$$\Big\downarrow \text{3553}$$

$$\frac{(a^2+b^2)\left(-\frac{\int \frac{1}{a^2+b^2-(b\cos(x)-a\sin(x))^2}d(b\cos(x)-a\sin(x))}{2(a^2+b^2)}-\frac{b\cos(x)-a\sin(x)}{2(a^2+b^2)(a\cos(x)+b\sin(x))^2}\right)}{a^2}-\frac{2b\int \frac{1}{\sin(x)(a\cos(x)+b\sin(x))^2}dx}{a^2}+\frac{\int \frac{1}{\sin(x)^2(a\cos(x)+b\sin(x))}dx}{a^2}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2b\int \frac{1}{\sin(x)(a\cos(x)+b\sin(x))^2}dx}{a^2}+\frac{\int \frac{1}{\sin(x)^2(a\cos(x)+b\sin(x))}dx}{a^2}+\frac{(a^2+b^2)\left(-\frac{\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{3/2}}-\frac{b\cos(x)-a\sin(x)}{2(a^2+b^2)(a\cos(x)+b\sin(x))^2}\right)}{a^2}$$

$$\Big\downarrow \text{3572}$$

$$\frac{\int \frac{1}{\sin(x)^2(a\cos(x)+b\sin(x))}dx}{a^2}-\frac{2b\left(-\frac{b\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2}+\frac{\int \csc(x)dx}{a^2}+\frac{1}{a(a\cos(x)+b\sin(x))}\right)}{a^2}+\frac{(a^2+b^2)\left(-\frac{\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{3/2}}-\frac{b\cos(x)-a\sin(x)}{2(a^2+b^2)(a\cos(x)+b\sin(x))^2}\right)}{a^2}$$

$$\Big\downarrow \text{3042}$$

3.27. $\int \frac{\csc^2(x)}{(a\cos(x)+b\sin(x))^3}\,dx$

$$\frac{\int \frac{1}{\sin(x)^2(a\cos(x)+b\sin(x))}dx}{a^2}-\frac{2b\left(-\frac{b\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2}+\frac{\int \csc(x)dx}{a^2}+\frac{1}{a(a\cos(x)+b\sin(x))}\right)}{a^2}+\frac{(a^2+b^2)\left(-\frac{\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{3/2}}-\frac{b\cos(x)-a\sin(x)}{2(a^2+b^2)(a\cos(x)+b\sin(x))^2}\right)}{a^2}$$

$\downarrow$ 3553

$$-\frac{2b\left(\frac{b\int \frac{1}{a^2+b^2-(b\cos(x)-a\sin(x))^2}d(b\cos(x)-a\sin(x))}{a^2}+\frac{\int \csc(x)dx}{a^2}+\frac{1}{a(a\cos(x)+b\sin(x))}\right)}{a^2}+\frac{\int \frac{1}{\sin(x)^2(a\cos(x)+b\sin(x))}dx}{a^2}+\frac{(a^2+b^2)\left(-\frac{\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{3/2}}-\frac{b\cos(x)-a\sin(x)}{2(a^2+b^2)(a\cos(x)+b\sin(x))^2}\right)}{a^2}$$

$\downarrow$ 219

$$-\frac{2b\left(\frac{\int \csc(x)dx}{a^2}+\frac{b\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}}+\frac{1}{a(a\cos(x)+b\sin(x))}\right)}{a^2}+\frac{\int \frac{1}{\sin(x)^2(a\cos(x)+b\sin(x))}dx}{a^2}+\frac{(a^2+b^2)\left(-\frac{\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{3/2}}-\frac{b\cos(x)-a\sin(x)}{2(a^2+b^2)(a\cos(x)+b\sin(x))^2}\right)}{a^2}$$

$\downarrow$ 3582

$$-\frac{2b\left(\frac{\int \csc(x)dx}{a^2}+\frac{b\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}}+\frac{1}{a(a\cos(x)+b\sin(x))}\right)}{a^2}+\frac{\frac{(a^2+b^2)\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2}-\frac{b\int \csc(x)dx}{a^2}-\frac{\csc(x)}{a}}{a^2}+\frac{(a^2+b^2)\left(-\frac{\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{3/2}}-\frac{b\cos(x)-a\sin(x)}{2(a^2+b^2)(a\cos(x)+b\sin(x))^2}\right)}{a^2}$$

$\downarrow$ 3042

3.27. $\int \frac{\csc^2(x)}{(a\cos(x)+b\sin(x))^3}\,dx$

$$-\frac{2b\left(\frac{\int \csc(x)dx}{a^2}+\frac{b\operatorname{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}}+\frac{1}{a(a\cos(x)+b\sin(x))}\right)}{a^2}+\frac{\frac{(a^2+b^2)\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2}-\frac{b\int \csc(x)dx}{a^2}-\frac{\csc(x)}{a}}{a^2}+\frac{(a^2+b^2)\left(-\frac{\operatorname{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{3/2}}-\frac{b\cos(x)-a\sin(x)}{2(a^2+b^2)(a\cos(x)+b\sin(x))^2}\right)}{a^2}$$

$$\Big\downarrow \; 3553$$

$$-\frac{2b\left(\frac{\int \csc(x)dx}{a^2}+\frac{b\operatorname{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}}+\frac{1}{a(a\cos(x)+b\sin(x))}\right)}{a^2}+\frac{-\frac{(a^2+b^2)\int \frac{1}{a^2+b^2-(b\cos(x)-a\sin(x))^2}d(b\cos(x)-a\sin(x))}{a^2}-\frac{b\int \csc(x)dx}{a^2}-\frac{\csc(x)}{a}}{a^2}+\frac{(a^2+b^2)\left(-\frac{\operatorname{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{3/2}}-\frac{b\cos(x)-a\sin(x)}{2(a^2+b^2)(a\cos(x)+b\sin(x))^2}\right)}{a^2}$$

$$\Big\downarrow \; 219$$

$$-\frac{2b\left(\frac{\int \csc(x)dx}{a^2}+\frac{b\operatorname{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}}+\frac{1}{a(a\cos(x)+b\sin(x))}\right)}{a^2}+\frac{-\frac{b\int \csc(x)dx}{a^2}-\frac{\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^2}-\frac{\csc(x)}{a}}{a^2}+\frac{(a^2+b^2)\left(-\frac{\operatorname{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{3/2}}-\frac{b\cos(x)-a\sin(x)}{2(a^2+b^2)(a\cos(x)+b\sin(x))^2}\right)}{a^2}$$

$$\Big\downarrow \; 4257$$

3.27. $\int \frac{\csc^2(x)}{(a\cos(x)+b\sin(x))^3}\,dx$

$$\frac{(a^2+b^2)\left(-\frac{\operatorname{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{3/2}}-\frac{b\cos(x)-a\sin(x)}{2(a^2+b^2)(a\cos(x)+b\sin(x))^2}\right)}{a^2}-\frac{2b\left(\frac{b\operatorname{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}}-\frac{\operatorname{arctanh}(\cos(x))}{a^2}+\frac{1}{a(a\cos(x)+b\sin(x))}\right)}{a^2}+\frac{-\frac{\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^2}+\frac{b\operatorname{arctanh}(\cos(x))}{a^2}-\frac{\csc(x)}{a}}{a^2}$$

input
```
Int[Csc[x]^2/(a*Cos[x] + b*Sin[x])^3,x]
```

output
```
((b*ArcTanh[Cos[x]])/a^2 - (Sqrt[a^2 + b^2]*ArcTanh[(b*Cos[x] - a*Sin[x])/
Sqrt[a^2 + b^2]])/a^2 - Csc[x]/a)/a^2 + ((a^2 + b^2)*(-1/2*ArcTanh[(b*Cos[
x] - a*Sin[x])/Sqrt[a^2 + b^2]]/(a^2 + b^2)^(3/2) - (b*Cos[x] - a*Sin[x])/
(2*(a^2 + b^2)*(a*Cos[x] + b*Sin[x])^2)))/a^2 - (2*b*(-(ArcTanh[Cos[x]]/a^
2) + (b*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2*Sqrt[a^2 + b^
2]) + 1/(a*(a*Cos[x] + b*Sin[x]))))/a^2
```

3.27.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3555
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(b*Cos[c + d*x] - a*Sin[c + d*x])*((a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Simp[(n + 2)/((n + 1)*(a^
2 + b^2))   Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

rule 3572
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_)/si
n[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-(a*Cos[c + d*x] + b*Sin[c + d*x])
^(n + 1)/(a*d*(n + 1)), x] + (Simp[1/a^2   Int[(a*Cos[c + d*x] + b*Sin[c +
d*x])^(n + 2)/Sin[c + d*x], x], x] - Simp[b/a^2   Int[(a*Cos[c + d*x] + b*S
in[c + d*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0
] && LtQ[n, -1]
```

rule 3582
```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[Sin[c + d*x]^(m + 1)/(a*d*(m + 1))
, x] + (-Simp[b/a^2   Int[Sin[c + d*x]^(m + 1), x], x] + Simp[(a^2 + b^2)/a
^2   Int[Sin[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /;
 FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

rule 3584
```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a^2 + b^2)/a^2   Int[Sin[c +
 d*x]^(m + 2)*(a*Cos[c + d*x] + b*Sin[c + d*x])^n, x], x] + (Simp[1/a^2   I
nt[Sin[c + d*x]^m*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] - Simp[
2*(b/a^2)   Int[Sin[c + d*x]^(m + 1)*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n +
 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] &
& LtQ[m, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.27. $\int \frac{\csc^2(x)}{(a\cos(x)+b\sin(x))^3}\,dx$

3.27.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.89

method	result
default	$-\frac{\tan\left(\frac{x}{2}\right)}{2a^3}-\frac{4\left(\frac{-\frac{a\left(a^2+6b^2\right)\tan\left(\frac{x}{2}\right)^3}{4}-\frac{5b\left(a^2-2b^2\right)\tan\left(\frac{x}{2}\right)^2}{4}-\frac{a\left(a^2-14b^2\right)\tan\left(\frac{x}{2}\right)}{4}+\frac{5a^2b}{4}}{\left(\tan\left(\frac{x}{2}\right)^2a-2b\tan\left(\frac{x}{2}\right)-a\right)^2}-\frac{3\left(a^2+2b^2\right)\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{4\sqrt{a^2+b^2}}\right)}{a^4}-$
risch	$-\frac{ie^{ix}\left(9iab\,e^{4ix}-3a^2e^{4ix}+6b^2e^{4ix}-2a^2e^{2ix}-12b^2e^{2ix}-9iba-3a^2+6b^2\right)}{\left(e^{2ix}-1\right)\left(b\,e^{2ix}+ia\,e^{2ix}-b+ia\right)^2a^3}-\frac{3\ln\left(e^{ix}-\frac{ia-b}{\sqrt{a^2+b^2}}\right)}{2\sqrt{a^2+b^2}\,a^2}-\frac{3\ln\left(e^{ix}-\frac{ia-b}{\sqrt{a^2+b^2}}\right)b^2}{\sqrt{a^2+b^2}\,a^4}+$ 3 l

input
```
int(csc(x)^2/(a*cos(x)+b*sin(x))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*tan(1/2*x)/a^3-4/a^4*((-1/4*a*(a^2+6*b^2)*tan(1/2*x)^3-5/4*b*(a^2-2*b
^2)*tan(1/2*x)^2-1/4*a*(a^2-14*b^2)*tan(1/2*x)+5/4*a^2*b)/(tan(1/2*x)^2*a-
2*b*tan(1/2*x)-a)^2-3/4*(a^2+2*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1
/2*x)-2*b)/(a^2+b^2)^(1/2)))-1/2/a^3/tan(1/2*x)-3/a^4*b*ln(tan(1/2*x))
```

3.27.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 463 vs. 2(168) = 336.

Time = 0.34 (sec) , antiderivative size = 463, normalized size of antiderivative = 2.52

$$\int \frac{\csc^2(x)}{(a\cos(x)+b\sin(x))^3}\,dx =$$
$$-\frac{2\,a^5-10\,a^3b^2-12\,ab^4-6\left(a^5-a^3b^2-2\,ab^4\right)\cos(x)^2-18\left(a^4b+a^2b^3\right)\cos(x)\sin(x)-3\left(2\left(a^3b+2\,a\right.\right.}{}$$

input
```
integrate(csc(x)^2/(a*cos(x)+b*sin(x))^3,x, algorithm="fricas")
```

output
```
-1/4*(2*a^5 - 10*a^3*b^2 - 12*a*b^4 - 6*(a^5 - a^3*b^2 - 2*a*b^4)*cos(x)^2
 - 18*(a^4*b + a^2*b^3)*cos(x)*sin(x) - 3*(2*(a^3*b + 2*a*b^3)*cos(x)^3 -
2*(a^3*b + 2*a*b^3)*cos(x) - (a^2*b^2 + 2*b^4 + (a^4 + a^2*b^2 - 2*b^4)*co
s(x)^2)*sin(x))*sqrt(a^2 + b^2)*log(-(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*co
s(x)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos
(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2)) - 6*(2*(a^3*b^2 + a*b^4)*cos(x)^
3 - 2*(a^3*b^2 + a*b^4)*cos(x) - (a^2*b^3 + b^5 + (a^4*b - b^5)*cos(x)^2)*
sin(x))*log(1/2*cos(x) + 1/2) + 6*(2*(a^3*b^2 + a*b^4)*cos(x)^3 - 2*(a^3*b
^2 + a*b^4)*cos(x) - (a^2*b^3 + b^5 + (a^4*b - b^5)*cos(x)^2)*sin(x))*log(
-1/2*cos(x) + 1/2))/(2*(a^7*b + a^5*b^3)*cos(x)^3 - 2*(a^7*b + a^5*b^3)*co
s(x) - (a^6*b^2 + a^4*b^4 + (a^8 - a^4*b^4)*cos(x)^2)*sin(x))
```

3.27.6 Sympy [F]

$$\int \frac{\csc^2(x)}{(a\cos(x)+b\sin(x))^3}\,dx = \int \frac{\csc^2{(x)}}{(a\cos{(x)}+b\sin{(x)})^3}\,dx$$

input
```
integrate(csc(x)**2/(a*cos(x)+b*sin(x))**3,x)
```

output
```
Integral(csc(x)**2/(a*cos(x) + b*sin(x))**3, x)
```

3.27.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.50

$$\int \frac{\csc^2(x)}{(a\cos(x)+b\sin(x))^3}\,dx = -\frac{a^3+\frac{14\,a^2b\sin(x)}{\cos(x)+1}-\frac{4\,(a^3-8\,ab^2)\sin(x)^2}{(\cos(x)+1)^2}-\frac{2\,(7\,a^2b-10\,b^3)\sin(x)^3}{(\cos(x)+1)^3}-\frac{(a^3+12\,ab^2)\sin(x)^4}{(\cos(x)+1)^4}}{2\left(\frac{a^6\sin(x)}{\cos(x)+1}+\frac{4\,a^5b\sin(x)^2}{(\cos(x)+1)^2}-\frac{4\,a^5b\sin(x)^4}{(\cos(x)+1)^4}+\frac{a^6\sin(x)^5}{(\cos(x)+1)^5}-\frac{2\,(a^6-2\,a^4b^2)\sin(x)^3}{(\cos(x)+1)^3}\right)} - \frac{3\,b\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^4} - \frac{3\,(a^2+2\,b^2)\log\left(\frac{b-\frac{a\sin(x)}{\cos(x)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(x)}{\cos(x)+1}-\sqrt{a^2+b^2}}\right)}{2\,\sqrt{a^2+b^2}a^4} - \frac{\sin{(x)}}{2\,a^3(\cos{(x)}+1)}$$

input
```
integrate(csc(x)^2/(a*cos(x)+b*sin(x))^3,x, algorithm="maxima")
```

output
```
-1/2*(a^3 + 14*a^2*b*sin(x)/(cos(x) + 1) - 4*(a^3 - 8*a*b^2)*sin(x)^2/(cos
(x) + 1)^2 - 2*(7*a^2*b - 10*b^3)*sin(x)^3/(cos(x) + 1)^3 - (a^3 + 12*a*b^
2)*sin(x)^4/(cos(x) + 1)^4)/(a^6*sin(x)/(cos(x) + 1) + 4*a^5*b*sin(x)^2/(c
os(x) + 1)^2 - 4*a^5*b*sin(x)^4/(cos(x) + 1)^4 + a^6*sin(x)^5/(cos(x) + 1)
^5 - 2*(a^6 - 2*a^4*b^2)*sin(x)^3/(cos(x) + 1)^3) - 3*b*log(sin(x)/(cos(x)
 + 1))/a^4 - 3/2*(a^2 + 2*b^2)*log((b - a*sin(x)/(cos(x) + 1) + sqrt(a^2 +
 b^2))/(b - a*sin(x)/(cos(x) + 1) - sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^4
) - 1/2*sin(x)/(a^3*(cos(x) + 1))
```

3.27.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.15

$$\int \frac{\csc^2(x)}{(a\cos(x)+b\sin(x))^3}\,dx = -\frac{3\,b\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)}{a^4} - \frac{\tan\left(\frac{1}{2}\,x\right)}{2\,a^3} - \frac{3\,(a^2+2\,b^2)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{2\,\sqrt{a^2+b^2}a^4} + \frac{6\,b\tan\left(\frac{1}{2}\,x\right)-a}{2\,a^4\tan\left(\frac{1}{2}\,x\right)} + \frac{a^3\tan\left(\frac{1}{2}\,x\right)^3+6\,ab^2\tan\left(\frac{1}{2}\,x\right)^3+5\,a^2b\tan\left(\frac{1}{2}\,x\right)^2-10\,b^3\tan\left(\frac{1}{2}\,x\right)^2+a^3\tan\left(\frac{1}{2}\,x\right)-14\,ab^2\tan\left(\frac{1}{2}\,x\right)-}{\left(a\tan\left(\frac{1}{2}\,x\right)^2-2\,b\tan\left(\frac{1}{2}\,x\right)-a\right)^2a^4}$$

input
```
integrate(csc(x)^2/(a*cos(x)+b*sin(x))^3,x, algorithm="giac")
```

output
```
-3*b*log(abs(tan(1/2*x)))/a^4 - 1/2*tan(1/2*x)/a^3 - 3/2*(a^2 + 2*b^2)*log
(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*x) - 2*b +
2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^4) + 1/2*(6*b*tan(1/2*x) - a)/(a^4*
tan(1/2*x)) + (a^3*tan(1/2*x)^3 + 6*a*b^2*tan(1/2*x)^3 + 5*a^2*b*tan(1/2*x
)^2 - 10*b^3*tan(1/2*x)^2 + a^3*tan(1/2*x) - 14*a*b^2*tan(1/2*x) - 5*a^2*b
)/((a*tan(1/2*x)^2 - 2*b*tan(1/2*x) - a)^2*a^4)
```

3.27.9 Mupad [B] (verification not implemented)

Time = 21.74 (sec) , antiderivative size = 813, normalized size of antiderivative = 4.42

$$\int \frac{\csc^2(x)}{(a\cos(x)+b\sin(x))^3}\,dx$$

$$= \frac{\tan\left(\frac{x}{2}\right)^4(a^2+12b^2)+\tan\left(\frac{x}{2}\right)^2(4a^2-32b^2)-a^2-14ab\tan\left(\frac{x}{2}\right)+\frac{2\tan\left(\frac{x}{2}\right)^3(7a^2b-10b^3)}{a}}{2a^5\tan\left(\frac{x}{2}\right)-\tan\left(\frac{x}{2}\right)^3(4a^5-8a^3b^2)+2a^5\tan\left(\frac{x}{2}\right)^5+8a^4b\tan\left(\frac{x}{2}\right)^2-8a^4b\tan\left(\frac{x}{2}\right)^4}$$

$$-\frac{\tan\left(\frac{x}{2}\right)}{2a^3}-\frac{3b\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a^4}$$

$$-\frac{\operatorname{atan}\Bigg(\frac{\frac{(a^2+2b^2)\sqrt{a^2+b^2}\Big(\frac{3a^6+12a^4b^2}{a^6}+\frac{\tan\left(\frac{x}{2}\right)(12a^4b+24a^2b^3)}{a^5}-\frac{3(a^2+2b^2)\left(2a^2b+\frac{\tan\left(\frac{x}{2}\right)(6a^8+8a^6b^2)}{a^5}\right)\sqrt{a^2+b^2}}{2(a^6+a^4b^2)}\cdots}{2(a^6+a^4b^2)}}{\frac{2(9a^2b+18b^3)}{a^6}-\frac{2\tan\left(\frac{x}{2}\right)(9a^2+18b^2)}{a^5}-\frac{3(a^2+2b^2)\sqrt{a^2+b^2}\Big(\frac{3a^6+12a^4b^2}{a^6}+\frac{\tan\left(\frac{x}{2}\right)(12a^4b+24a^2b^3)}{a^5}-\frac{3(a^2+2b^2)\left(2a^2b+\frac{\tan\left(\frac{x}{2}\right)(6\cdots\right.}{2(a^6+a^4b\cdots}}{2(a^6+a^4b^2)}\cdots}\cdots}{\cdots}$$

input
```
int(1/(sin(x)^2*(a*cos(x) + b*sin(x))^3),x)
```

output
```
(tan(x/2)^4*(a^2 + 12*b^2) + tan(x/2)^2*(4*a^2 - 32*b^2) - a^2 - 14*a*b*ta
n(x/2) + (2*tan(x/2)^3*(7*a^2*b - 10*b^3))/a)/(2*a^5*tan(x/2) - tan(x/2)^3
*(4*a^5 - 8*a^3*b^2) + 2*a^5*tan(x/2)^5 + 8*a^4*b*tan(x/2)^2 - 8*a^4*b*tan
(x/2)^4) - tan(x/2)/(2*a^3) - (3*b*log(tan(x/2)))/a^4 - (atan((((a^2 + 2*b
^2)*(a^2 + b^2)^(1/2)*((3*a^6 + 12*a^4*b^2)/a^6 + (tan(x/2)*(12*a^4*b + 24
*a^2*b^3))/a^5 - (3*(a^2 + 2*b^2)*(2*a^2*b + (tan(x/2)*(6*a^8 + 8*a^6*b^2)
)/a^5)*(a^2 + b^2)^(1/2))/(2*(a^6 + a^4*b^2)))*3i)/(2*(a^6 + a^4*b^2)) + (
(a^2 + 2*b^2)*(a^2 + b^2)^(1/2)*((3*a^6 + 12*a^4*b^2)/a^6 + (tan(x/2)*(12*
a^4*b + 24*a^2*b^3))/a^5 + (3*(a^2 + 2*b^2)*(2*a^2*b + (tan(x/2)*(6*a^8 +
8*a^6*b^2))/a^5)*(a^2 + b^2)^(1/2))/(2*(a^6 + a^4*b^2)))*3i)/(2*(a^6 + a^4
*b^2)))/((2*(9*a^2*b + 18*b^3))/a^6 - (2*tan(x/2)*(9*a^2 + 18*b^2))/a^5 -
(3*(a^2 + 2*b^2)*(a^2 + b^2)^(1/2)*((3*a^6 + 12*a^4*b^2)/a^6 + (tan(x/2)*(
12*a^4*b + 24*a^2*b^3))/a^5 - (3*(a^2 + 2*b^2)*(2*a^2*b + (tan(x/2)*(6*a^8
 + 8*a^6*b^2))/a^5)*(a^2 + b^2)^(1/2))/(2*(a^6 + a^4*b^2))))/(2*(a^6 + a^4
*b^2)) + (3*(a^2 + 2*b^2)*(a^2 + b^2)^(1/2)*((3*a^6 + 12*a^4*b^2)/a^6 + (t
an(x/2)*(12*a^4*b + 24*a^2*b^3))/a^5 + (3*(a^2 + 2*b^2)*(2*a^2*b + (tan(x/
2)*(6*a^8 + 8*a^6*b^2))/a^5)*(a^2 + b^2)^(1/2))/(2*(a^6 + a^4*b^2))))/(2*(
a^6 + a^4*b^2))))*(a^2 + 2*b^2)*(a^2 + b^2)^(1/2)*3i)/(a^6 + a^4*b^2)
```

3.28 $\int \frac{\csc^3(x)}{(a\cos(x)+b\sin(x))^3}\,dx$

3.28.1 Optimal result

Integrand size = 16, antiderivative size = 117

$$\int \frac{\csc^3(x)}{(a\cos(x)+b\sin(x))^3}\,dx = \frac{3b\cot(x)}{a^4}-\frac{\cot^2(x)}{2a^3}+\frac{2(a^2+3b^2)\log(\tan(x))}{a^5} - \frac{2(a^2+3b^2)\log(a+b\tan(x))}{a^5} + \frac{(a^2+b^2)^2}{2a^3b^2(a+b\tan(x))^2}-\frac{(a^2-3b^2)(a^2+b^2)}{a^4b^2(a+b\tan(x))}$$

output
```
3*b*cot(x)/a^4-1/2*cot(x)^2/a^3+2*(a^2+3*b^2)*ln(tan(x))/a^5-2*(a^2+3*b^2)
*ln(a+b*tan(x))/a^5+1/2*(a^2+b^2)^2/a^3/b^2/(a+b*tan(x))^2-(a^2-3*b^2)*(a^
2+b^2)/a^4/b^2/(a+b*tan(x))
```

3.28.2 Mathematica [A] (verified)

Time = 2.02 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.78

$$\int \frac{\csc^3(x)}{(a\cos(x)+b\sin(x))^3}\,dx = \frac{6a^3b\cot^3(x)+a^4\csc^2(x)-2ab\cot(x)\left(3a^2+a^2\csc^2(x)-4(a^2+3b^2)\log(\sin(x))+4a^2\log(a\cos(x)+b\sin\cdots\right.}{}$$

input
```
Integrate[Csc[x]^3/(a*Cos[x] + b*Sin[x])^3,x]
```

output
```
(6*a^3*b*Cot[x]^3 + a^4*Csc[x]^2 - 2*a*b*Cot[x]*(3*a^2 + a^2*Csc[x]^2 - 4*
(a^2 + 3*b^2)*Log[Sin[x]] + 4*a^2*Log[a*Cos[x] + b*Sin[x]] + 12*b^2*Log[a*
Cos[x] + b*Sin[x]]) + 2*b^2*(-3*(a^2 + b^2) + 2*(a^2 + 3*b^2)*Log[Sin[x]]
- 2*(a^2 + 3*b^2)*Log[a*Cos[x] + b*Sin[x]]) + Cot[x]^2*(-(a^4*Csc[x]^2) +
4*a^2*(3*b^2 + (a^2 + 3*b^2)*Log[Sin[x]] - (a^2 + 3*b^2)*Log[a*Cos[x] + b*
Sin[x]])))/(2*a^5*(b + a*Cot[x])^2)
```

3.28.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3566, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(x)}{(a\cos(x)+b\sin(x))^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin(x)^3(a\cos(x)+b\sin(x))^3}dx$$

$$\Big\downarrow \text{ 3566}$$

$$\int \frac{\left(\tan^2(x)+1\right)^2\cot^3(x)}{(a+b\tan(x))^3}d\tan(x)$$

$$\Big\downarrow \text{ 522}$$

$$\int \left(-\frac{3b\cot^2(x)}{a^4}+\frac{\cot^3(x)}{a^3}-\frac{2b\left(a^2+3b^2\right)}{a^5(a+b\tan(x))}+\frac{2\left(a^2+3b^2\right)\cot(x)}{a^5}+\frac{a^4-2a^2b^2-3b^4}{a^4b(a+b\tan(x))^2}-\frac{\left(a^2+b^2\right)^2}{a^3b(a+b\tan(x))^3}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{3b\cot(x)}{a^4}-\frac{\cot^2(x)}{2a^3}+\frac{2\left(a^2+3b^2\right)\log(\tan(x))}{a^5}-\frac{2\left(a^2+3b^2\right)\log(a+b\tan(x))}{a^5}-\frac{\left(a^2-3b^2\right)\left(a^2+b^2\right)}{a^4b^2(a+b\tan(x))}+\frac{\left(a^2+b^2\right)^2}{2a^3b^2(a+b\tan(x))^2}$$

input
```
Int[Csc[x]^3/(a*Cos[x] + b*Sin[x])^3,x]
```

output
```
(3*b*Cot[x])/a^4 - Cot[x]^2/(2*a^3) + (2*(a^2 + 3*b^2)*Log[Tan[x]])/a^5 -
(2*(a^2 + 3*b^2)*Log[a + b*Tan[x]])/a^5 + (a^2 + b^2)^2/(2*a^3*b^2*(a + b*
Tan[x])^2) - ((a^2 - 3*b^2)*(a^2 + b^2))/(a^4*b^2*(a + b*Tan[x]))
```

3.28.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3566
```
Int[sin[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[1/d   Subst[Int[x^m*((a + b*
x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Tan[c + d*x]], x] /; FreeQ[{a, b, c
, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n, 0
] && GtQ[m, 1])
```

3.28.4 Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.09

method	result
default	$-\frac{-a^4-2a^2b^2-b^4}{2a^3b^2(a+b\tan(x))^2}-\frac{a^4-2a^2b^2-3b^4}{a^4b^2(a+b\tan(x))}-\frac{2(a^2+3b^2)\ln(a+b\tan(x))}{a^5}-\frac{1}{2a^3\tan(x)^2}+\frac{(2a^2+6b^2)\ln(\tan(x))}{a^5}+\frac{}{a^4}$
norman	$\frac{\frac{b\tan(\frac{x}{2})}{a^2}+\frac{b(-13a^2-24b^2)\tan(\frac{x}{2})^3}{a^4}-\frac{1}{8a}-\frac{\tan(\frac{x}{2})^8}{8a}-\frac{(-9a^4+56a^2b^2+144b^4)\tan(\frac{x}{2})^4}{4a^5}-\frac{b\tan(\frac{x}{2})^7}{a^2}-\frac{b(-13a^2-24b^2)\tan(\frac{x}{2})^5}{a^4}}{\tan(\frac{x}{2})^2\left(\tan(\frac{x}{2})^2a-2b\tan(\frac{x}{2})-a\right)^2}+$
parallelrisch	$\frac{-4(a^2+3b^2)\left((a^2-b^2)\cos(2x)+2ba\sin(2x)+a^2+b^2\right)\ln\left(\frac{-2a\cos(x)-2b\sin(x)}{\cos(x)+1}\right)+4(a^2+3b^2)\left((a^2-b^2)\cos(2x)+2ba\sin(2x)+a^2+\right.}{2((a^2-b^2)\mathrm{co}}$
risch	$\frac{4i(6ia\,b^2+3a^2b-3b^3+3ia\,b^2e^{6ix}+3b^3e^{6ix}-9b^3e^{4ix}+a^2b\,e^{6ix}-3a^2b\,e^{4ix}+ia^3e^{6ix}+ia^3e^{2ix}-a^2b\,e^{2ix}-9ia\,b^2e^{2ix}+9b^3e^{2ix})}{(e^{2ix}-1)^2(b\,e^{2ix}+ia\,e^{2ix}-b+ia)^2a^4}-$

input
```
int(csc(x)^3/(a*cos(x)+b*sin(x))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-a^4-2*a^2*b^2-b^4)/a^3/b^2/(a+b*tan(x))^2-(a^4-2*a^2*b^2-3*b^4)/a^4
/b^2/(a+b*tan(x))-2*(a^2+3*b^2)*ln(a+b*tan(x))/a^5-1/2/a^3/tan(x)^2+(2*a^2
+6*b^2)/a^5*ln(tan(x))+3/a^4*b/tan(x)
```

3.28.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(113) = 226$.

Time = 0.28 (sec) , antiderivative size = 385, normalized size of antiderivative = 3.29

$$\int \frac{\csc^3(x)}{(a\cos(x)+b\sin(x))^3}\,dx = -\frac{24\,a^2b^2\cos{(x)}^4 - a^4 + 6\,a^2b^2 + 2\,(a^4 - 15\,a^2b^2)\cos{(x)}^2 - 2\,((a^4 + 2\,a^2b^2 - 3\,b^4)\cos{(x)}^4 - a^2b^2 - 3\,b^4}{}$$

input
```
integrate(csc(x)^3/(a*cos(x)+b*sin(x))^3,x, algorithm="fricas")
```

output
```
-1/2*(24*a^2*b^2*cos(x)^4 - a^4 + 6*a^2*b^2 + 2*(a^4 - 15*a^2*b^2)*cos(x)^
2 - 2*((a^4 + 2*a^2*b^2 - 3*b^4)*cos(x)^4 - a^2*b^2 - 3*b^4 - (a^4 + a^2*b
^2 - 6*b^4)*cos(x)^2 + 2*((a^3*b + 3*a*b^3)*cos(x)^3 - (a^3*b + 3*a*b^3)*c
os(x))*sin(x))*log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) + 2*(
(a^4 + 2*a^2*b^2 - 3*b^4)*cos(x)^4 - a^2*b^2 - 3*b^4 - (a^4 + a^2*b^2 - 6*
b^4)*cos(x)^2 + 2*((a^3*b + 3*a*b^3)*cos(x)^3 - (a^3*b + 3*a*b^3)*cos(x))*
sin(x))*log(-1/4*cos(x)^2 + 1/4) - 4*(3*(a^3*b - a*b^3)*cos(x)^3 - (2*a^3*
b - 3*a*b^3)*cos(x))*sin(x))/(a^5*b^2 - (a^7 - a^5*b^2)*cos(x)^4 + (a^7 -
2*a^5*b^2)*cos(x)^2 - 2*(a^6*b*cos(x)^3 - a^6*b*cos(x))*sin(x))
```

3.28.6 Sympy [F]

$$\int \frac{\csc^3(x)}{(a\cos(x)+b\sin(x))^3}\,dx = \int \frac{\csc^3{(x)}}{(a\cos{(x)}+b\sin{(x)})^3}\,dx$$

input
```
integrate(csc(x)**3/(a*cos(x)+b*sin(x))**3,x)
```

output
```
Integral(csc(x)**3/(a*cos(x) + b*sin(x))**3, x)
```

3.28.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(113) = 226$.

Time = 0.27 (sec) , antiderivative size = 308, normalized size of antiderivative = 2.63

$$\int \frac{\csc^3(x)}{(a\cos(x)+b\sin(x))^3}\,dx = -\frac{a^4 - \frac{8a^3b\sin(x)}{\cos(x)+1} - \frac{2(a^4+22a^2b^2)\sin(x)^2}{(\cos(x)+1)^2} + \frac{4(21a^3b+4ab^3)\sin(x)^3}{(\cos(x)+1)^3} - \frac{(15a^4-144a^2b^2-112b^4)\sin(x)^4}{(\cos(x)+1)^4} - \frac{4(19a^3b+16ab^3)\sin(x}{(\cos(x)+1)^5}}{8\left(\frac{a^7\sin(x)^2}{(\cos(x)+1)^2} + \frac{4a^6b\sin(x)^3}{(\cos(x)+1)^3} - \frac{4a^6b\sin(x)^5}{(\cos(x)+1)^5} + \frac{a^7\sin(x)^6}{(\cos(x)+1)^6} - \frac{2(a^7-2a^5b^2)\sin(x)^4}{(\cos(x)+1)^4}\right)} - \frac{\frac{12b\sin(x)}{\cos(x)+1} + \frac{a\sin(x)^2}{(\cos(x)+1)^2}}{8a^4} - \frac{2(a^2+3b^2)\log\left(-a - \frac{2b\sin(x)}{\cos(x)+1} + \frac{a\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^5} + \frac{2(a^2+3b^2)\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^5}$$

input
```
integrate(csc(x)^3/(a*cos(x)+b*sin(x))^3,x, algorithm="maxima")
```

output
```
-1/8*(a^4 - 8*a^3*b*sin(x)/(cos(x) + 1) - 2*(a^4 + 22*a^2*b^2)*sin(x)^2/(c
os(x) + 1)^2 + 4*(21*a^3*b + 4*a*b^3)*sin(x)^3/(cos(x) + 1)^3 - (15*a^4 -
144*a^2*b^2 - 112*b^4)*sin(x)^4/(cos(x) + 1)^4 - 4*(19*a^3*b + 16*a*b^3)*s
in(x)^5/(cos(x) + 1)^5)/(a^7*sin(x)^2/(cos(x) + 1)^2 + 4*a^6*b*sin(x)^3/(c
os(x) + 1)^3 - 4*a^6*b*sin(x)^5/(cos(x) + 1)^5 + a^7*sin(x)^6/(cos(x) + 1)
^6 - 2*(a^7 - 2*a^5*b^2)*sin(x)^4/(cos(x) + 1)^4) - 1/8*(12*b*sin(x)/(cos(
x) + 1) + a*sin(x)^2/(cos(x) + 1)^2)/a^4 - 2*(a^2 + 3*b^2)*log(-a - 2*b*si
n(x)/(cos(x) + 1) + a*sin(x)^2/(cos(x) + 1)^2)/a^5 + 2*(a^2 + 3*b^2)*log(s
in(x)/(cos(x) + 1))/a^5
```

3.28.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.25

$$\int \frac{\csc^3(x)}{(a\cos(x)+b\sin(x))^3}\,dx = \frac{2(a^2+3b^2)\log(|\tan(x)|)}{a^5} - \frac{2(a^2b+3b^3)\log(|b\tan(x)+a|)}{a^5b} - \frac{2a^4b\tan(x)^3 - 4a^2b^3\tan(x)^3 - 12b^5\tan(x)^3 + a^5\tan(x)^2 - 6a^3b^2\tan(x)^2 - 18ab^4\tan(x)^2 - 4a^2b^3}{2\left(b\tan(x)^2 + a\tan(x)\right)^2 a^4b^2}$$

input
```
integrate(csc(x)^3/(a*cos(x)+b*sin(x))^3,x, algorithm="giac")
```

output
```
2*(a^2 + 3*b^2)*log(abs(tan(x)))/a^5 - 2*(a^2*b + 3*b^3)*log(abs(b*tan(x)
+ a))/(a^5*b) - 1/2*(2*a^4*b*tan(x)^3 - 4*a^2*b^3*tan(x)^3 - 12*b^5*tan(x)
^3 + a^5*tan(x)^2 - 6*a^3*b^2*tan(x)^2 - 18*a*b^4*tan(x)^2 - 4*a^2*b^3*tan
(x) + a^3*b^2)/((b*tan(x)^2 + a*tan(x))^2*a^4*b^2)
```

3.28.9 Mupad [B] (verification not implemented)

Time = 21.08 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.16

$$\int \frac{\csc^3(x)}{(a\cos(x)+b\sin(x))^3}\,dx = \frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)\,\left(2\,a^2+6\,b^2\right)}{a^5}$$
$$-\frac{\tan\left(\frac{x}{2}\right)^3\left(42\,a^2\,b+8\,b^3\right)-\tan\left(\frac{x}{2}\right)^5\left(38\,a^2\,b+32\,b^3\right)-\tan\left(\frac{x}{2}\right)^2\left(a^3+22\,a\,b^2\right)+\frac{a^3}{2}+\frac{\tan\left(\frac{x}{2}\right)^4\left(-15\,a^4+144\,a^2\,b}{2\,a}}{4\,a^6\tan\left(\frac{x}{2}\right)^2-\tan\left(\frac{x}{2}\right)^4\left(8\,a^6-16\,a^4\,b^2\right)+4\,a^6\tan\left(\frac{x}{2}\right)^6+16\,a^5\,b\tan\left(\frac{x}{2}\right)^3-16\,a^5\,b\tan}$$
$$-\frac{\ln\left(-a\tan\left(\frac{x}{2}\right)^2+2\,b\tan\left(\frac{x}{2}\right)+a\right)\,\left(2\,a^2+6\,b^2\right)}{a^5}-\frac{\tan\left(\frac{x}{2}\right)^2}{8\,a^3}-\frac{3\,b\tan\left(\frac{x}{2}\right)}{2\,a^4}$$

input
```
int(1/(sin(x)^3*(a*cos(x) + b*sin(x))^3),x)
```

output
```
(log(tan(x/2))*(2*a^2 + 6*b^2))/a^5 - (tan(x/2)^3*(42*a^2*b + 8*b^3) - tan
(x/2)^5*(38*a^2*b + 32*b^3) - tan(x/2)^2*(22*a*b^2 + a^3) + a^3/2 + (tan(x
/2)^4*(112*b^4 - 15*a^4 + 144*a^2*b^2))/(2*a) - 4*a^2*b*tan(x/2))/(4*a^6*t
an(x/2)^2 - tan(x/2)^4*(8*a^6 - 16*a^4*b^2) + 4*a^6*tan(x/2)^6 + 16*a^5*b*
tan(x/2)^3 - 16*a^5*b*tan(x/2)^5) - (log(a + 2*b*tan(x/2) - a*tan(x/2)^2)*
(2*a^2 + 6*b^2))/a^5 - tan(x/2)^2/(8*a^3) - (3*b*tan(x/2))/(2*a^4)
```

3.29 $\int \sin^{-n}(c+dx)(a\cos(c+dx)+ia\sin(c+dx))^n\,dx$

3.29.1 Optimal result

Integrand size = 33, antiderivative size = 66

$$\int \sin^{-n}(c+dx)(a\cos(c+dx)+ia\sin(c+dx))^n\,dx = -\frac{i\,\text{Hypergeometric2F1}\left(1,n,1+n,-\frac{1}{2}i(i+\cot(c+dx))\right)\sin^{-n}(c+dx)(a\cos(c+dx)+ia\sin(c+dx))^{n}}{2dn}$$

output
```
-1/2*I*hypergeom([1, n],[1+n],-1/2*I*(I+cot(d*x+c)))*(a*cos(d*x+c)+I*a*sin
(d*x+c))^n/d/n/(sin(d*x+c)^n)
```

3.29.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 5.00 (sec) , antiderivative size = 367, normalized size of antiderivative = 5.56

$$\int \sin^{-n}(c+dx)(a\cos(c+dx)+ia\sin(c+dx))^n\,dx = -\frac{4\cos\left(\frac{1}{2}(c+dx)\right)\left(\text{AppellF1}\left(1-n,-2n,1,2-n,-i\tan\left(\frac{1}{2}(c+dx)\right),\right.\right.}{d(-1+n)\left(2\,\text{AppellF1}\left(1-n,-2n,1,2-n,-i\tan\left(\frac{1}{2}(c+dx)\right),i\tan\left(\frac{1}{2}(c+dx)\right)\right)+\frac{\left(-2n\,\text{AppellF1}(2-\right.}{}\right.}$$

input
```
Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^n/Sin[c + d*x]^n,x]
```

output
```
(-4*Cos[(c + d*x)/2]*(AppellF1[1 - n, -2*n, 1, 2 - n, (-I)*Tan[(c + d*x)/2
], I*Tan[(c + d*x)/2]] + Hypergeometric2F1[1 - 2*n, 1 - n, 2 - n, (-I)*Tan
[(c + d*x)/2]])*Sin[(c + d*x)/2]*(a*(Cos[c + d*x] + I*Sin[c + d*x]))^n)/(d
*(-1 + n)*Sin[c + d*x]^n*(2*AppellF1[1 - n, -2*n, 1, 2 - n, (-I)*Tan[(c +
d*x)/2], I*Tan[(c + d*x)/2]] + ((-2*n*AppellF1[2 - n, 1 - 2*n, 1, 3 - n, (
-I)*Tan[(c + d*x)/2], I*Tan[(c + d*x)/2]]*(-1 + Cos[c + d*x] + I*Sin[c + d
*x]) - AppellF1[2 - n, -2*n, 2, 3 - n, (-I)*Tan[(c + d*x)/2], I*Tan[(c + d
*x)/2]]*(-1 + Cos[c + d*x] + I*Sin[c + d*x]) + (-2 + n)*(1 + Cos[c + d*x])
*(1 + I*Tan[(c + d*x)/2])^(2*n))*(1 - I*Tan[(c + d*x)/2]))/(-2 + n)))
```

3.29.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {3042, 3562}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{-n}(c+dx)(a\cos(c+dx)+ia\sin(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(c+dx)^{-n}(a\cos(c+dx)+ia\sin(c+dx))^n dx$$

$$\Big\downarrow \text{3562}$$

$$-\frac{i\sin^{-n}(c+dx)\,\text{Hypergeometric2F1}\left(1,n,n+1,-\frac{1}{2}i(\cot(c+dx)+i)\right)(a\cos(c+dx)+ia\sin(c+dx))^n}{2dn}$$

input
```
Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^n/Sin[c + d*x]^n,x]
```

output
```
((-1/2*I)*Hypergeometric2F1[1, n, 1 + n, (-1/2*I)*(I + Cot[c + d*x])]*(a*C
os[c + d*x] + I*a*Sin[c + d*x])^n)/(d*n*Sin[c + d*x]^n)
```

3.29.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3562
```
Int[sin[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c
 + d*x])^n/(2*b*d*n*Sin[c + d*x]^n))*Hypergeometric2F1[1, n, n + 1, (b + a*
Cot[c + d*x])/(2*b)], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[m + n, 0] && E
qQ[a^2 + b^2, 0] &&  !IntegerQ[n]
```

3.29.4 Maple [F]

$$\int (\cos (dx + c) a + ia \sin (dx + c))^n \sin (dx + c)^{-n} \, dx$$

input
```
int((cos(d*x+c)*a+I*a*sin(d*x+c))^n/(sin(d*x+c)^n),x)
```

output
```
int((cos(d*x+c)*a+I*a*sin(d*x+c))^n/(sin(d*x+c)^n),x)
```

3.29.5 Fricas [F]

$$\int \sin^{-n}(c+dx)(a\cos(c+dx)+ia\sin(c+dx))^n \, dx = \int \frac{(a \cos (dx + c) + i\, a \sin (dx + c))^n}{\sin (dx + c)^n} \, dx$$

input
```
integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^n/(sin(d*x+c)^n),x, algorithm="fri
cas")
```

output
```
integral(e^(I*d*n*x + I*c*n + n*log(a))/(1/2*(-I*e^(2*I*d*x + 2*I*c) + I)*
e^(-I*d*x - I*c))^n, x)
```

3.29.6 Sympy [F]

$$\int \sin^{-n}(c+dx)(a\cos(c+dx)+ia\sin(c+dx))^n\,dx = \int (a(i\sin(c+dx)+\cos(c+dx)))^n \sin^{-n}(c+dx)\,dx$$

input
```
integrate((a*cos(d*x+c)+I*a*sin(d*x+c))**n/(sin(d*x+c)**n),x)
```

output
```
Integral((a*(I*sin(c + d*x) + cos(c + d*x)))**n/sin(c + d*x)**n, x)
```

3.29.7 Maxima [F]

$$\int \sin^{-n}(c+dx)(a\cos(c+dx)+ia\sin(c+dx))^n\,dx = \int \frac{(a\cos(dx+c)+i\,a\sin(dx+c))^n}{\sin(dx+c)^n}\,dx$$

input
```
integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^n/(sin(d*x+c)^n),x, algorithm="max
ima")
```

output
```
integrate((a*cos(d*x + c) + I*a*sin(d*x + c))^n*sin(d*x + c)^(-n), x)
```

3.29.8 Giac [F]

$$\int \sin^{-n}(c+dx)(a\cos(c+dx)+ia\sin(c+dx))^n\,dx = \int \frac{(a\cos(dx+c)+i\,a\sin(dx+c))^n}{\sin(dx+c)^n}\,dx$$

input
```
integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^n/(sin(d*x+c)^n),x, algorithm="gia
c")
```

output
```
integrate((a*cos(d*x + c) + I*a*sin(d*x + c))^n/sin(d*x + c)^n, x)
```

3.29.9 Mupad [F(-1)]

Timed out.

$$\int \sin^{-n}(c+dx)(a\cos(c+dx)+ia\sin(c+dx))^n\,dx$$
$$=\int \frac{\left(a\,\cos\left(c+d\,x\right)+a\,\sin\left(c+d\,x\right)\,1\mathrm{i}\right)^n}{\sin\left(c+d\,x\right)^n}\,dx$$

input
```
int((a*cos(c + d*x) + a*sin(c + d*x)*1i)^n/sin(c + d*x)^n,x)
```

output
```
int((a*cos(c + d*x) + a*sin(c + d*x)*1i)^n/sin(c + d*x)^n, x)
```

3.30 $\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$

3.30.1 Optimal result

Integrand size = 26, antiderivative size = 87

$$\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = \frac{5ax}{16}-\frac{b\cos^6(c+dx)}{6d}+\frac{5a\cos(c+dx)\sin(c+dx)}{16d}+\frac{5a\cos^3(c+dx)\sin(c+dx)}{24d}+\frac{a\cos^5(c+dx)\sin(c+dx)}{6d}$$

output
```
5/16*a*x-1/6*b*cos(d*x+c)^6/d+5/16*a*cos(d*x+c)*sin(d*x+c)/d+5/24*a*cos(d*
x+c)^3*sin(d*x+c)/d+1/6*a*cos(d*x+c)^5*sin(d*x+c)/d
```

3.30.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.66

$$\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = \frac{-32b\cos^6(c+dx)+a(60c+60dx+45\sin(2(c+dx))+9\sin(4(c+dx))+\sin(6(c+dx)))}{192d}$$

input
```
Integrate[Cos[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

output
```
(-32*b*Cos[c + d*x]^6 + a*(60*c + 60*d*x + 45*Sin[2*(c + d*x)] + 9*Sin[4*(
c + d*x)] + Sin[6*(c + d*x)]))/(192*d)
```

3.30.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(c+dx)^5(a\cos(c+dx)+b\sin(c+dx))dx$$

$$\Big\downarrow \text{3569}$$

$$\int \left(a\cos^6(c+dx)+b\sin(c+dx)\cos^5(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{5a\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{5a\sin(c+dx)\cos(c+dx)}{16d}+\frac{5ax}{16}-\frac{b\cos^6(c+dx)}{6d}$$

input
```
Int[Cos[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

output
```
(5*a*x)/16 - (b*Cos[c + d*x]^6)/(6*d) + (5*a*Cos[c + d*x]*Sin[c + d*x])/(1
6*d) + (5*a*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (a*Cos[c + d*x]^5*Sin[c
+ d*x])/(6*d)
```

3.30.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

3.30.4 Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.71

method	result
derivativedivides	$\frac{a\left(\frac{\left(\cos(dx+c)^5+\frac{5\cos(dx+c)^3}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)-\frac{b\cos(dx+c)^6}{6}}{d}$
default	$\frac{a\left(\frac{\left(\cos(dx+c)^5+\frac{5\cos(dx+c)^3}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)-\frac{b\cos(dx+c)^6}{6}}{d}$
parts	$\frac{a\left(\frac{\left(\cos(dx+c)^5+\frac{5\cos(dx+c)^3}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)}{d}-\frac{b\cos(dx+c)^6}{6d}$
parallelrisch	$\frac{60axd-b\cos(6dx+6c)+a\sin(6dx+6c)+9a\sin(4dx+4c)+45a\sin(2dx+2c)-6b\cos(4dx+4c)-15b\cos(2dx+2c)+22b}{192d}$
risch	$\frac{5ax}{16}-\frac{b\cos(6dx+6c)}{192d}+\frac{a\sin(6dx+6c)}{192d}-\frac{b\cos(4dx+4c)}{32d}+\frac{3a\sin(4dx+4c)}{64d}-\frac{5b\cos(2dx+2c)}{64d}+\frac{15a\sin(2dx+2}{64d}$
norman	$\frac{\frac{5ax}{16}+\frac{11a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}-\frac{5a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{24d}+\frac{15a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{4d}-\frac{15a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{4d}+\frac{5a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{24d}-\frac{11a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{8d}+15ax}{}$

input
```
int(cos(d*x+c)^5*(cos(d*x+c)*a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/1
6*d*x+5/16*c)-1/6*b*cos(d*x+c)^6)
```

3.30.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.71

$$\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = -\frac{8\,b\cos\left(dx+c\right)^6-15\,adx-\left(8\,a\cos\left(dx+c\right)^5+10\,a\cos\left(dx+c\right)^3+15\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{48\,d}$$

input
```
integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/48*(8*b*cos(d*x + c)^6 - 15*a*d*x - (8*a*cos(d*x + c)^5 + 10*a*cos(d*x
+ c)^3 + 15*a*cos(d*x + c))*sin(d*x + c))/d
```

3.30.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(82) = 164.

Time = 0.36 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.01

$$\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = \begin{cases} \frac{5ax\sin^6(c+dx)}{16}+\frac{15ax\sin^4(c+dx)\cos^2(c+dx)}{16}+\frac{15ax\sin^2(c+dx)\cos^4(c+dx)}{16}+\frac{5ax\cos^6(c+dx)}{16}+\frac{5a\sin^5(c+dx)\cos(c+dx)}{16d}+5a \\ x(a\cos(c)+b\sin(c))\cos^5(c) \end{cases}$$

input
```
integrate(cos(d*x+c)**5*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

output
```
Piecewise((5*a*x*sin(c + d*x)**6/16 + 15*a*x*sin(c + d*x)**4*cos(c + d*x)*
*2/16 + 15*a*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*a*x*cos(c + d*x)**6/
16 + 5*a*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 5*a*sin(c + d*x)**3*cos(c +
 d*x)**3/(6*d) + 11*a*sin(c + d*x)*cos(c + d*x)**5/(16*d) - b*cos(c + d*x)
**6/(6*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))*cos(c)**5, True))
```

3.30.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.71

$$\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = -\frac{32\,b\cos\left(dx+c\right)^6+\left(4\,\sin\left(2\,dx+2\,c\right)^3-60\,dx-60\,c-9\,\sin\left(4\,dx+4\,c\right)-48\,\sin\left(2\,dx+2\,c\right)\right)a}{192\,d}$$

input
```
integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/192*(32*b*cos(d*x + c)^6 + (4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*si
n(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a)/d
```

3.30.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.09

$$\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = \frac{5}{16}\,ax-\frac{b\cos\left(6\,dx+6\,c\right)}{192\,d}-\frac{b\cos\left(4\,dx+4\,c\right)}{32\,d}-\frac{5\,b\cos\left(2\,dx+2\,c\right)}{64\,d}+\frac{a\sin\left(6\,dx+6\,c\right)}{192\,d}+\frac{3\,a\sin\left(4\,dx+4\,c\right)}{64\,d}+\frac{15\,a\sin\left(2\,dx+2\,c\right)}{64\,d}$$

input
```
integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
5/16*a*x - 1/192*b*cos(6*d*x + 6*c)/d - 1/32*b*cos(4*d*x + 4*c)/d - 5/64*b
*cos(2*d*x + 2*c)/d + 1/192*a*sin(6*d*x + 6*c)/d + 3/64*a*sin(4*d*x + 4*c)
/d + 15/64*a*sin(2*d*x + 2*c)/d
```

3.30.9 Mupad [B] (verification not implemented)

Time = 24.58 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.71

$$\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = \frac{5\,a\,x}{16} + \frac{-\frac{11\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8} + 2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10} + \frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24} - \frac{15\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{4} + \frac{20\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}}{3} + \frac{15\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{4}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^6}$$

input
```
int(cos(c + d*x)^5*(a*cos(c + d*x) + b*sin(c + d*x)),x)
```

output
```
(5*a*x)/16 + ((11*a*tan(c/2 + (d*x)/2))/8 - (5*a*tan(c/2 + (d*x)/2)^3)/24
+ (15*a*tan(c/2 + (d*x)/2)^5)/4 - (15*a*tan(c/2 + (d*x)/2)^7)/4 + (5*a*tan
(c/2 + (d*x)/2)^9)/24 - (11*a*tan(c/2 + (d*x)/2)^11)/8 + 2*b*tan(c/2 + (d*
x)/2)^2 + (20*b*tan(c/2 + (d*x)/2)^6)/3 + 2*b*tan(c/2 + (d*x)/2)^10)/(d*(t
an(c/2 + (d*x)/2)^2 + 1)^6)
```

3.31 $\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$

3.31.1 Optimal result

Integrand size = 26, antiderivative size = 60

$$\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = -\frac{b\cos^5(c+dx)}{5d}+\frac{a\sin(c+dx)}{d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin^5(c+dx)}{5d}$$

output
```
-1/5*b*cos(d*x+c)^5/d+a*sin(d*x+c)/d-2/3*a*sin(d*x+c)^3/d+1/5*a*sin(d*x+c)
^5/d
```

3.31.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = -\frac{b\cos^5(c+dx)}{5d}+\frac{a\sin(c+dx)}{d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin^5(c+dx)}{5d}$$

input
```
Integrate[Cos[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

output
```
-1/5*(b*Cos[c + d*x]^5)/d + (a*Sin[c + d*x])/d - (2*a*Sin[c + d*x]^3)/(3*d
) + (a*Sin[c + d*x]^5)/(5*d)
```

3.31.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(c+dx)^4(a\cos(c+dx)+b\sin(c+dx))dx$$

$$\Big\downarrow \text{3569}$$

$$\int \left(a\cos^5(c+dx)+b\sin(c+dx)\cos^4(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\sin^5(c+dx)}{5d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{b\cos^5(c+dx)}{5d}$$

input
```
Int[Cos[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

output
```
-1/5*(b*Cos[c + d*x]^5)/d + (a*Sin[c + d*x])/d - (2*a*Sin[c + d*x]^3)/(3*d
) + (a*Sin[c + d*x]^5)/(5*d)
```

3.31.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

3.31.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.77

method	result
derivativedivides	$\frac{\frac{a\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}-\frac{\cos(dx+c)^5 b}{5}}{d}$
default	$\frac{\frac{a\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}-\frac{\cos(dx+c)^5 b}{5}}{d}$
parts	$\frac{a\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5d}-\frac{b\cos(dx+c)^5}{5d}$
risch	$-\frac{b\cos(dx+c)}{8d}+\frac{5a\sin(dx+c)}{8d}-\frac{b\cos(5dx+5c)}{80d}+\frac{a\sin(5dx+5c)}{80d}-\frac{b\cos(3dx+3c)}{16d}+\frac{5a\sin(3dx+3c)}{48d}$
parallelrisch	$\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8 b+\frac{8a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{3}+\frac{116a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{15}-4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4 b+\frac{8a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}+2a\tan\left(\frac{dx}{2}\cdots\right.}{d\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^5}$
norman	$\frac{-\frac{2b}{5d}+\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{8a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}+\frac{116a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{15d}+\frac{8a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{3d}+\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{d}-\frac{4b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{d}-\frac{2b\tan\left(\cdots\right.}{\cdots}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^5}$

input
```
int(cos(d*x+c)^4*(cos(d*x+c)*a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/5*a*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)-1/5*cos(d*x+c)^5
*b)
```

3.31.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

$$\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$$
$$=-\frac{3\,b\cos\left(dx+c\right)^5-\left(3\,a\cos\left(dx+c\right)^4+4\,a\cos\left(dx+c\right)^2+8\,a\right)\sin\left(dx+c\right)}{15\,d}$$

input
```
integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/15*(3*b*cos(d*x + c)^5 - (3*a*cos(d*x + c)^4 + 4*a*cos(d*x + c)^2 + 8*a
)*sin(d*x + c))/d
```

3.31.6 Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.45

$$\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = \begin{cases} \frac{8a\sin^5(c+dx)}{15d} + \frac{4a\sin^3(c+dx)\cos^2(c+dx)}{3d} + \frac{a\sin(c+dx)\cos^4(c+dx)}{d} - \frac{b\cos^5(c+dx)}{5d} & \text{for } d \neq 0 \\ x(a\cos(c)+b\sin(c))\cos^4(c) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**4*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

output
```
Piecewise((8*a*sin(c + d*x)**5/(15*d) + 4*a*sin(c + d*x)**3*cos(c + d*x)**
2/(3*d) + a*sin(c + d*x)*cos(c + d*x)**4/d - b*cos(c + d*x)**5/(5*d), Ne(d
, 0)), (x*(a*cos(c) + b*sin(c))*cos(c)**4, True))
```

3.31.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.82

$$\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = -\frac{3\,b\cos(dx+c)^5 - \left(3\,\sin(dx+c)^5 - 10\,\sin(dx+c)^3 + 15\,\sin(dx+c)\right)a}{15\,d}$$

input
```
integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/15*(3*b*cos(d*x + c)^5 - (3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin
(d*x + c))*a)/d
```

3.31.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.42

$$\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = -\frac{b\cos(5\,dx+5\,c)}{80\,d} - \frac{b\cos(3\,dx+3\,c)}{16\,d} - \frac{b\cos(dx+c)}{8\,d} + \frac{a\sin(5\,dx+5\,c)}{80\,d} + \frac{5\,a\sin(3\,dx+3\,c)}{48\,d} + \frac{5\,a\sin(dx+c)}{8\,d}$$

input
```
integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
-1/80*b*cos(5*d*x + 5*c)/d - 1/16*b*cos(3*d*x + 3*c)/d - 1/8*b*cos(d*x + c
)/d + 1/80*a*sin(5*d*x + 5*c)/d + 5/48*a*sin(3*d*x + 3*c)/d + 5/8*a*sin(d*
x + c)/d
```

3.31.9 Mupad [B] (verification not implemented)

Time = 22.66 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.12

$$\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = \frac{8\,a\,\sin\left(c+d\,x\right)}{15\,d}-\frac{b\cos\left(c+d\,x\right)^5}{5\,d}+\frac{4\,a\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{15\,d}+\frac{a\cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)}{5\,d}$$

input
```
int(cos(c + d*x)^4*(a*cos(c + d*x) + b*sin(c + d*x)),x)
```

output
```
(8*a*sin(c + d*x))/(15*d) - (b*cos(c + d*x)^5)/(5*d) + (4*a*cos(c + d*x)^2
*sin(c + d*x))/(15*d) + (a*cos(c + d*x)^4*sin(c + d*x))/(5*d)
```

3.32 $\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$

3.32.1 Optimal result

Integrand size = 26, antiderivative size = 65

$$\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = \frac{3ax}{8}-\frac{b\cos^4(c+dx)}{4d}+\frac{3a\cos(c+dx)\sin(c+dx)}{8d}+\frac{a\cos^3(c+dx)\sin(c+dx)}{4d}$$

output
```
3/8*a*x-1/4*b*cos(d*x+c)^4/d+3/8*a*cos(d*x+c)*sin(d*x+c)/d+1/4*a*cos(d*x+c
)^3*sin(d*x+c)/d
```

3.32.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.95

$$\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = \frac{3a(c+dx)}{8d}-\frac{b\cos^4(c+dx)}{4d}+\frac{a\sin(2(c+dx))}{4d}+\frac{a\sin(4(c+dx))}{32d}$$

input
```
Integrate[Cos[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

output
```
(3*a*(c + d*x))/(8*d) - (b*Cos[c + d*x]^4)/(4*d) + (a*Sin[2*(c + d*x)])/(4
*d) + (a*Sin[4*(c + d*x)])/(32*d)
```

3.32.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \cos(c+dx)^3(a\cos(c+dx)+b\sin(c+dx))dx$$

$$\Big\downarrow \ 3569$$

$$\int \left(a\cos^4(c+dx)+b\sin(c+dx)\cos^3(c+dx)\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a\sin(c+dx)\cos(c+dx)}{8d}+\frac{3ax}{8}-\frac{b\cos^4(c+dx)}{4d}$$

input
```
Int[Cos[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

output
```
(3*a*x)/8 - (b*Cos[c + d*x]^4)/(4*d) + (3*a*Cos[c + d*x]*Sin[c + d*x])/(8*
d) + (a*Cos[c + d*x]^3*Sin[c + d*x])/(4*d)
```

3.32.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

3.32.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.80

method	result
derivativedivides	$\frac{a\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)-\frac{\cos(dx+c)^4 b}{4}}{d}$
default	$\frac{a\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)-\frac{\cos(dx+c)^4 b}{4}}{d}$
parts	$\frac{a\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}-\frac{b\cos(dx+c)^4}{4d}$
parallelrisch	$\frac{12axd-b\cos(4dx+4c)+a\sin(4dx+4c)+8a\sin(2dx+2c)-4b\cos(2dx+2c)+5b}{32d}$
risch	$\frac{3ax}{8}-\frac{b\cos(4dx+4c)}{32d}+\frac{a\sin(4dx+4c)}{32d}-\frac{b\cos(2dx+2c)}{8d}+\frac{a\sin(2dx+2c)}{4d}$
norman	$\frac{\frac{3ax}{8}+\frac{5a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}-\frac{3a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{4d}+\frac{3a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{4d}-\frac{5a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{4d}+\frac{3ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2}+\frac{9ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{4}+3ax\tan}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4}$

input
```
int(cos(d*x+c)^3*(cos(d*x+c)*a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)-1/4*co
s(d*x+c)^4*b)
```

3.32.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.78

$$\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$$
$$=-\frac{2\,b\cos\left(dx+c\right)^4-3\,adx-\left(2\,a\cos\left(dx+c\right)^3+3\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{8\,d}$$

input
```
integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/8*(2*b*cos(d*x + c)^4 - 3*a*d*x - (2*a*cos(d*x + c)^3 + 3*a*cos(d*x + c
))*sin(d*x + c))/d
```

3.32.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(60) = 120$.

Time = 0.18 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.97

$$\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$$
$$= \begin{cases} \frac{3ax\sin^4(c+dx)}{8} + \frac{3ax\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{3ax\cos^4(c+dx)}{8} + \frac{3a\sin^3(c+dx)\cos(c+dx)}{8d} + \frac{5a\sin(c+dx)\cos^3(c+dx)}{8d} - \frac{b\cos^4(\ldots}{4d} \\ x(a\cos(c)+b\sin(c))\cos^3(c) \end{cases}$$

input
```
integrate(cos(d*x+c)**3*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

output
```
Piecewise((3*a*x*sin(c + d*x)**4/8 + 3*a*x*sin(c + d*x)**2*cos(c + d*x)**2
/4 + 3*a*x*cos(c + d*x)**4/8 + 3*a*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 5*
a*sin(c + d*x)*cos(c + d*x)**3/(8*d) - b*cos(c + d*x)**4/(4*d), Ne(d, 0)),
 (x*(a*cos(c) + b*sin(c))*cos(c)**3, True))
```

3.32.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

$$\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$$
$$= -\frac{8\,b\cos(dx+c)^4 - (12\,dx + 12\,c + \sin(4\,dx+4\,c) + 8\,\sin(2\,dx+2\,c))a}{32\,d}$$

input
```
integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/32*(8*b*cos(d*x + c)^4 - (12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*
x + 2*c))*a)/d
```

3.32.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$$
$$=\frac{3}{8}\,ax-\frac{b\cos\left(4\,dx+4\,c\right)}{32\,d}-\frac{b\cos\left(2\,dx+2\,c\right)}{8\,d}+\frac{a\sin\left(4\,dx+4\,c\right)}{32\,d}+\frac{a\sin\left(2\,dx+2\,c\right)}{4\,d}$$

input
```
integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
3/8*a*x - 1/32*b*cos(4*d*x + 4*c)/d - 1/8*b*cos(2*d*x + 2*c)/d + 1/32*a*si
n(4*d*x + 4*c)/d + 1/4*a*sin(2*d*x + 2*c)/d
```

3.32.9 Mupad [B] (verification not implemented)

Time = 24.89 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.65

$$\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx=\frac{3\,a\,x}{8}$$
$$+\frac{-\frac{5\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\frac{3\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}-\frac{3\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4}+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{5\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^4}$$

input
```
int(cos(c + d*x)^3*(a*cos(c + d*x) + b*sin(c + d*x)),x)
```

output
```
(3*a*x)/8 + ((5*a*tan(c/2 + (d*x)/2))/4 - (3*a*tan(c/2 + (d*x)/2)^3)/4 + (
3*a*tan(c/2 + (d*x)/2)^5)/4 - (5*a*tan(c/2 + (d*x)/2)^7)/4 + 2*b*tan(c/2 +
 (d*x)/2)^2 + 2*b*tan(c/2 + (d*x)/2)^6)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^4)
```

3.33 $\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$

3.33.1 Optimal result

Integrand size = 26, antiderivative size = 44

$$\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = -\frac{b\cos^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{a\sin^3(c+dx)}{3d}$$

output
```
-1/3*b*cos(d*x+c)^3/d+a*sin(d*x+c)/d-1/3*a*sin(d*x+c)^3/d
```

3.33.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = -\frac{b\cos^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{a\sin^3(c+dx)}{3d}$$

input
```
Integrate[Cos[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

output
```
-1/3*(b*Cos[c + d*x]^3)/d + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d)
```

3.33.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))dx$$

$$\Big\downarrow \text{3569}$$

$$\int \left(a\cos^3(c+dx)+b\sin(c+dx)\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{b\cos^3(c+dx)}{3d}$$

input
```
Int[Cos[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

output
```
-1/3*(b*Cos[c + d*x]^3)/d + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d)
```

3.33.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

3.33.4 Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{\frac{a\left(2+\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)}{3}-\frac{\cos\left(dx+c\right)^{3}b}{3}}{d}$	36
default	$\frac{\frac{a\left(2+\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)}{3}-\frac{\cos\left(dx+c\right)^{3}b}{3}}{d}$	36
parts	$\frac{a\left(2+\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)}{3d}-\frac{b\cos\left(dx+c\right)^{3}}{3d}$	38
risch	$-\frac{b\cos\left(dx+c\right)}{4d}+\frac{3a\sin\left(dx+c\right)}{4d}-\frac{b\cos\left(3dx+3c\right)}{12d}+\frac{a\sin\left(3dx+3c\right)}{12d}$	56
parallelrisch	$\frac{6a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}-6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}b+4a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+6a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{3d\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{3}}$	79
norman	$\frac{-\frac{2b}{3d}+\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{4a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3d}+\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{d}-\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{3}}$	90

input
```
int(cos(d*x+c)^2*(cos(d*x+c)*a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/3*a*(2+cos(d*x+c)^2)*sin(d*x+c)-1/3*cos(d*x+c)^3*b)
```

3.33.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = -\frac{b\cos\left(dx+c\right)^{3}-\left(a\cos\left(dx+c\right)^{2}+2\,a\right)\sin\left(dx+c\right)}{3\,d}$$

input
```
integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/3*(b*cos(d*x + c)^3 - (a*cos(d*x + c)^2 + 2*a)*sin(d*x + c))/d
```

3.33.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.43

$$\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = \begin{cases} \frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)\cos^2(c+dx)}{d}-\frac{b\cos^3(c+dx)}{3d} & \text{for } d\neq 0 \\ x(a\cos(c)+b\sin(c))\cos^2(c) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**2*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

output
```
Piecewise((2*a*sin(c + d*x)**3/(3*d) + a*sin(c + d*x)*cos(c + d*x)**2/d -
b*cos(c + d*x)**3/(3*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))*cos(c)**2, Tr
ue))
```

3.33.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

$$\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = -\frac{b\cos(dx+c)^3+\left(\sin(dx+c)^3-3\sin(dx+c)\right)a}{3d}$$

input
```
integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/3*(b*cos(d*x + c)^3 + (sin(d*x + c)^3 - 3*sin(d*x + c))*a)/d
```

3.33.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.25

$$\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = -\frac{b\cos(3dx+3c)}{12d}-\frac{b\cos(dx+c)}{4d}+\frac{a\sin(3dx+3c)}{12d}+\frac{3a\sin(dx+c)}{4d}$$

input
```
integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
-1/12*b*cos(3*d*x + 3*c)/d - 1/4*b*cos(d*x + c)/d + 1/12*a*sin(3*d*x + 3*c
)/d + 3/4*a*sin(d*x + c)/d
```

3.33.9 Mupad [B] (verification not implemented)

Time = 21.31 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.07

$$\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$$
$$=\frac{2\,a\,\sin\left(c+d\,x\right)}{3\,d}-\frac{b\cos\left(c+d\,x\right)^3}{3\,d}+\frac{a\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{3\,d}$$

input
```
int(cos(c + d*x)^2*(a*cos(c + d*x) + b*sin(c + d*x)),x)
```

output
```
(2*a*sin(c + d*x))/(3*d) - (b*cos(c + d*x)^3)/(3*d) + (a*cos(c + d*x)^2*si
n(c + d*x))/(3*d)
```

3.34 $\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$

3.34.1 Optimal result

Integrand size = 24, antiderivative size = 43

$$\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = \frac{ax}{2}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}+\frac{b\sin^2(c+dx)}{2d}$$

output
```
1/2*a*x+1/2*a*cos(d*x+c)*sin(d*x+c)/d+1/2*b*sin(d*x+c)^2/d
```

3.34.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07

$$\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = \frac{a(c+dx)}{2d}-\frac{b\cos^2(c+dx)}{2d}+\frac{a\sin(2(c+dx))}{4d}$$

input
```
Integrate[Cos[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

output
```
(a*(c + d*x))/(2*d) - (b*Cos[c + d*x]^2)/(2*d) + (a*Sin[2*(c + d*x)])/(4*d
)
```

3.34.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))dx$$

$$\Big\downarrow \ 3569$$

$$\int \left(a\cos^2(c+dx)+b\sin(c+dx)\cos(c+dx)\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}+\frac{b\sin^2(c+dx)}{2d}$$

input
```
Int[Cos[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

output
```
(a*x)/2 + (a*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (b*Sin[c + d*x]^2)/(2*d)
```

3.34.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

3.34.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
risch	$\frac{ax}{2}-\frac{b\cos(2dx+2c)}{4d}+\frac{a\sin(2dx+2c)}{4d}$	36
parallelrisch	$\frac{2axd-b\cos(2dx+2c)+a\sin(2dx+2c)+b}{4d}$	36
derivativedivides	$\frac{a\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-\frac{\cos(dx+c)^2 b}{2}}{d}$	41
default	$\frac{a\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-\frac{\cos(dx+c)^2 b}{2}}{d}$	41
parts	$\frac{a\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{b\sin(dx+c)^2}{2d}$	43
norman	$\frac{\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\frac{ax}{2}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}+\frac{ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{2}+\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}$	99

input
```
int(cos(d*x+c)*(cos(d*x+c)*a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x-1/4*b/d*cos(2*d*x+2*c)+1/4*a/d*sin(2*d*x+2*c)
```

3.34.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = \frac{adx-b\cos(dx+c)^2+a\cos(dx+c)\sin(dx+c)}{2\,d}$$

input
```
integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(a*d*x - b*cos(d*x + c)^2 + a*cos(d*x + c)*sin(d*x + c))/d
```

3.34.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(36) = 72$.

Time = 0.10 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.70

$$\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = \begin{cases} \frac{ax\sin^2{(c+dx)}}{2} + \frac{ax\cos^2{(c+dx)}}{2} + \frac{a\sin{(c+dx)}\cos{(c+dx)}}{2d} - \frac{b\cos^2{(c+dx)}}{2d} & \text{for } d \neq 0 \\ x(a\cos{(c)} + b\sin{(c)})\cos{(c)} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

output
```
Piecewise((a*x*sin(c + d*x)**2/2 + a*x*cos(c + d*x)**2/2 + a*sin(c + d*x)*
cos(c + d*x)/(2*d) - b*cos(c + d*x)**2/(2*d), Ne(d, 0)), (x*(a*cos(c) + b*
sin(c))*cos(c), True))
```

3.34.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = -\frac{2\,b\cos{(dx+c)}^2 - (2\,dx + 2\,c + \sin{(2\,dx+2\,c)})a}{4\,d}$$

input
```
integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/4*(2*b*cos(d*x + c)^2 - (2*d*x + 2*c + sin(2*d*x + 2*c))*a)/d
```

3.34.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = \frac{1}{2}\,ax - \frac{b\cos{(2\,dx+2\,c)}}{4\,d} + \frac{a\sin{(2\,dx+2\,c)}}{4\,d}$$

input
```
integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
1/2*a*x - 1/4*b*cos(2*d*x + 2*c)/d + 1/4*a*sin(2*d*x + 2*c)/d
```

3.34.9 Mupad [B] (verification not implemented)

Time = 21.44 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = \frac{a\,x}{2}-\frac{b\,\cos\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{a\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}$$

input
```
int(cos(c + d*x)*(a*cos(c + d*x) + b*sin(c + d*x)),x)
```

output
```
(a*x)/2 - (b*cos(2*c + 2*d*x))/(4*d) + (a*sin(2*c + 2*d*x))/(4*d)
```

3.35 $\int (a\cos(c+dx)+b\sin(c+dx))\,dx$

3.35.1 Optimal result

Integrand size = 17, antiderivative size = 24

$$\int (a\cos(c+dx)+b\sin(c+dx))\,dx=-\frac{b\cos(c+dx)}{d}+\frac{a\sin(c+dx)}{d}$$

output
```
-b*cos(d*x+c)/d+a*sin(d*x+c)/d
```

3.35.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.92

$$\int (a\cos(c+dx)+b\sin(c+dx))\,dx=-\frac{b\cos(c)\cos(dx)}{d}+\frac{a\cos(dx)\sin(c)}{d}+\frac{a\cos(c)\sin(dx)}{d}+\frac{b\sin(c)\sin(dx)}{d}$$

input
```
Integrate[a*Cos[c + d*x] + b*Sin[c + d*x],x]
```

output
```
-((b*Cos[c]*Cos[d*x])/d) + (a*Cos[d*x]*Sin[c])/d + (a*Cos[c]*Sin[d*x])/d +
 (b*Sin[c]*Sin[d*x])/d
```

3.35.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(c+dx)+b\sin(c+dx))\,dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a\sin(c+dx)}{d}-\frac{b\cos(c+dx)}{d}$$

input
```
Int[a*Cos[c + d*x] + b*Sin[c + d*x],x]
```

output
```
-((b*Cos[c + d*x])/d) + (a*Sin[c + d*x])/d
```

3.35.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.35.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{-\cos(dx+c)b+\sin(dx+c)a}{d}$	23
parallelrisch	$\frac{b-\cos(dx+c)b+\sin(dx+c)a}{d}$	24
default	$-\frac{b\cos(dx+c)}{d}+\frac{a\sin(dx+c)}{d}$	25
risch	$-\frac{b\cos(dx+c)}{d}+\frac{a\sin(dx+c)}{d}$	25
parts	$-\frac{b\cos(dx+c)}{d}+\frac{a\sin(dx+c)}{d}$	25
norman	$\frac{\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{d}+\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}$	50
meijerg	$\frac{(\sqrt{\pi}\,\cos(c)a+\sqrt{\pi}\,\sin(c)b)\sin(dx)}{\sqrt{\pi}\,d}+\frac{(\sqrt{\pi}\,\cos(c)b-\sqrt{\pi}\,\sin(c)a)\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(dx)}{\sqrt{\pi}}\right)}{d}$	61

input
```
int(cos(d*x+c)*a+b*sin(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-cos(d*x+c)*b+sin(d*x+c)*a)
```

3.35.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int (a\cos(c+dx)+b\sin(c+dx))\,dx = -\frac{b\cos(dx+c)-a\sin(dx+c)}{d}$$

input
```
integrate(a*cos(d*x+c)+b*sin(d*x+c),x, algorithm="fricas")
```

output
```
-(b*cos(d*x + c) - a*sin(d*x + c))/d
```

3.35.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.29

$$\int (a\cos(c+dx)+b\sin(c+dx))\,dx = a\left(\begin{cases}\frac{\sin(c+dx)}{d} & \text{for } d\neq 0\\ x\cos(c) & \text{otherwise}\end{cases}\right) + b\left(\begin{cases}-\frac{\cos(c+dx)}{d} & \text{for } d\neq 0\\ x\sin(c) & \text{otherwise}\end{cases}\right)$$

input
```
integrate(a*cos(d*x+c)+b*sin(d*x+c),x)
```

output
```
a*Piecewise((sin(c + d*x)/d, Ne(d, 0)), (x*cos(c), True)) + b*Piecewise((-
cos(c + d*x)/d, Ne(d, 0)), (x*sin(c), True))
```

3.35.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int (a\cos(c+dx)+b\sin(c+dx))\,dx=-\frac{b\cos{(dx+c)}}{d}+\frac{a\sin{(dx+c)}}{d}$$

input
```
integrate(a*cos(d*x+c)+b*sin(d*x+c),x, algorithm="maxima")
```

output
```
-b*cos(d*x + c)/d + a*sin(d*x + c)/d
```

3.35.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int (a\cos(c+dx)+b\sin(c+dx))\,dx=-\frac{b\cos{(dx+c)}}{d}+\frac{a\sin{(dx+c)}}{d}$$

input
```
integrate(a*cos(d*x+c)+b*sin(d*x+c),x, algorithm="giac")
```

output
```
-b*cos(d*x + c)/d + a*sin(d*x + c)/d
```

3.35.9 Mupad [B] (verification not implemented)

Time = 20.79 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.58

$$\int (a\cos(c+dx)+b\sin(c+dx))\,dx=-\frac{2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(b\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-a\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}$$

input
```
int(a*cos(c + d*x) + b*sin(c + d*x),x)
```

output
```
-(2*cos(c/2 + (d*x)/2)*(b*cos(c/2 + (d*x)/2) - a*sin(c/2 + (d*x)/2)))/d
```

3.36 $\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$

3.36.1 Optimal result

Integrand size = 24, antiderivative size = 17

$$\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = ax - \frac{b\log(\cos(c+dx))}{d}$$

output
```
a*x-b*ln(cos(d*x+c))/d
```

3.36.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = ax - \frac{b\log(\cos(c+dx))}{d}$$

input
```
Integrate[Sec[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

output
```
a*x - (b*Log[Cos[c + d*x]])/d
```

3.36.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3042, 3565, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\cos(c+dx)+b\sin(c+dx)}{\cos(c+dx)}dx$$

$$\Big\downarrow \text{3565}$$

$$\int (a+b\tan(c+dx))dx$$

$$\Big\downarrow \text{2009}$$

$$ax-\frac{b\log(\cos(c+dx))}{d}$$

input
```
Int[Sec[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

output
```
a*x - (b*Log[Cos[c + d*x]])/d
```

3.36.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3565
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(a + b*Tan[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

3.36.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.35

method	result	size
derivativedivides	$\frac{-b\ln(\cos(dx+c))+a(dx+c)}{d}$	23
default	$\frac{-b\ln(\cos(dx+c))+a(dx+c)}{d}$	23
parts	$\frac{a(dx+c)}{d}+\frac{b\ln(\sec(dx+c))}{d}$	24
risch	$ibx+ax+\frac{2ibc}{d}-\frac{b\ln(e^{2i(dx+c)}+1)}{d}$	36
parallelrisch	$\frac{axd-b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+b\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}{d}$	54
norman	$\frac{ax+ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}+\frac{b\ln\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}{d}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}$	91

input
```
int(sec(d*x+c)*(cos(d*x+c)*a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-b*ln(cos(d*x+c))+a*(d*x+c))
```

3.36.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.24

$$\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx=\frac{adx-b\log\left(-\cos\left(dx+c\right)\right)}{d}$$

input
```
integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
(a*d*x - b*log(-cos(d*x + c)))/d
```

3.36.6 Sympy [F]

$$\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$$
$$=\int (a\cos(c+dx)+b\sin(c+dx))\sec(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

output
```
Integral((a*cos(c + d*x) + b*sin(c + d*x))*sec(c + d*x), x)
```

3.36.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.76

$$\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = \frac{2\,(dx+c)a-b\log\left(-\sin\left(dx+c\right)^2+1\right)}{2\,d}$$

input
```
integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*(2*(d*x + c)*a - b*log(-sin(d*x + c)^2 + 1))/d
```

3.36.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = \frac{2\,(dx+c)a+b\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d}$$

input
```
integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(2*(d*x + c)*a + b*log(tan(d*x + c)^2 + 1))/d
```

3.36.9 Mupad [B] (verification not implemented)

Time = 20.92 (sec) , antiderivative size = 70, normalized size of antiderivative = 4.12

$$\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$$
$$=\frac{b\ln\left(\frac{1}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}\right)}{d}+\frac{2\,a\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{b\ln\left(\frac{\cos(c+d\,x)}{\cos(c+d\,x)+1}\right)}{d}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))/cos(c + d*x),x)
```

output
```
(b*log(1/cos(c/2 + (d*x)/2)^2))/d + (2*a*atan(sin(c/2 + (d*x)/2)/cos(c/2 +
 (d*x)/2)))/d - (b*log(cos(c + d*x)/(cos(c + d*x) + 1)))/d
```

3.37 $\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$

3.37.1 Optimal result

Integrand size = 26, antiderivative size = 24

$$\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = \frac{a\text{arctanh}(\sin(c+dx))}{d}+\frac{b\sec(c+dx)}{d}$$

output
```
a*arctanh(sin(d*x+c))/d+b*sec(d*x+c)/d
```

3.37.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = \frac{a\text{arctanh}(\sin(c+dx))}{d}+\frac{b\sec(c+dx)}{d}$$

input
```
Integrate[Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

output
```
(a*ArcTanh[Sin[c + d*x]])/d + (b*Sec[c + d*x])/d
```

3.37.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\cos(c+dx)+b\sin(c+dx)}{\cos(c+dx)^2}dx$$

$$\Big\downarrow \text{3569}$$

$$\int (a\sec(c+dx)+b\tan(c+dx)\sec(c+dx))dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\text{arctanh}(\sin(c+dx))}{d}+\frac{b\sec(c+dx)}{d}$$

input
```
Int[Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

output
```
(a*ArcTanh[Sin[c + d*x]])/d + (b*Sec[c + d*x])/d
```

3.37.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

3.37.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

method	result	size
derivativedivides	$\frac{a\ln(\sec(dx+c)+\tan(dx+c))+\frac{b}{\cos(dx+c)}}{d}$	32
default	$\frac{a\ln(\sec(dx+c)+\tan(dx+c))+\frac{b}{\cos(dx+c)}}{d}$	32
parts	$\frac{a\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{b\sec(dx+c)}{d}$	32
parallelrisch	$\frac{-\cos(dx+c)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a+\cos(dx+c)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)a+\cos(dx+c)b+b}{d\cos(dx+c)}$	64
risch	$\frac{2b\,\mathrm{e}^{i(dx+c)}}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)}+\frac{a\ln\left(i+\mathrm{e}^{i(dx+c)}\right)}{d}-\frac{a\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{d}$	67
norman	$\frac{-\frac{2b}{d}-\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}$	92

input
```
int(sec(d*x+c)^2*(cos(d*x+c)*a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*ln(sec(d*x+c)+tan(d*x+c))+1/cos(d*x+c)*b)
```

3.37.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(24) = 48.

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.25

$$\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = \frac{a\cos(dx+c)\log(\sin(dx+c)+1)-a\cos(dx+c)\log(-\sin(dx+c)+1)+2\,b}{2\,d\cos(dx+c)}$$

input
```
integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(a*cos(d*x + c)*log(sin(d*x + c) + 1) - a*cos(d*x + c)*log(-sin(d*x +
c) + 1) + 2*b)/(d*cos(d*x + c))
```

3.37.6 Sympy [F]

$$\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = \int (a\cos(c+dx)+b\sin(c+dx))\sec^2(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**2*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

output
```
Integral((a*cos(c + d*x) + b*sin(c + d*x))*sec(c + d*x)**2, x)
```

3.37.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.67

$$\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = \frac{a(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+\frac{2b}{\cos(dx+c)}}{2d}$$

input
```
integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*(a*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 2*b/cos(d*x + c))
/d
```

3.37.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(24) = 48.

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.25

$$\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = \frac{a\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-a\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)-\frac{2b}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1}}{d}$$

input
```
integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
(a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a*log(abs(tan(1/2*d*x + 1/2*c) - 1
)) - 2*b/(tan(1/2*d*x + 1/2*c)^2 - 1))/d
```

3.37.9 Mupad [B] (verification not implemented)

Time = 21.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.58

$$\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = \frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{2\,b}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))/cos(c + d*x)^2,x)
```

output
```
(2*a*atanh(tan(c/2 + (d*x)/2)))/d - (2*b)/(d*(tan(c/2 + (d*x)/2)^2 - 1))
```

3.38 $\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$

3.38.1 Optimal result

Integrand size = 26, antiderivative size = 28

$$\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx=\frac{b\sec^2(c+dx)}{2d}+\frac{a\tan(c+dx)}{d}$$

output
```
1/2*b*sec(d*x+c)^2/d+a*tan(d*x+c)/d
```

3.38.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx=\frac{b\sec^2(c+dx)}{2d}+\frac{a\tan(c+dx)}{d}$$

input
```
Integrate[Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

output
```
(b*Sec[c + d*x]^2)/(2*d) + (a*Tan[c + d*x])/d
```

3.38.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{a\cos(c+dx)+b\sin(c+dx)}{\cos(c+dx)^3}dx$$

$$\Big\downarrow \text{ 3569}$$

$$\int \left(a\sec^2(c+dx)+b\tan(c+dx)\sec^2(c+dx)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a\tan(c+dx)}{d}+\frac{b\sec^2(c+dx)}{2d}$$

input
```
Int[Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

output
```
(b*Sec[c + d*x]^2)/(2*d) + (a*Tan[c + d*x])/d
```

3.38.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

3.38.4 Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{a\tan(dx+c)+\frac{b}{2\cos(dx+c)^2}}{d}$	25
default	$\frac{a\tan(dx+c)+\frac{b}{2\cos(dx+c)^2}}{d}$	25
parts	$\frac{b\sec(dx+c)^2}{2d}+\frac{a\tan(dx+c)}{d}$	27
risch	$\frac{2ia\,e^{2i(dx+c)}+2b\,e^{2i(dx+c)}+2ia}{d\left(e^{2i(dx+c)}+1\right)^2}$	48
parallelrisch	$-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}$	70
norman	$\frac{\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d}+\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{d}+\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)^2\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}$	99

input
```
int(sec(d*x+c)^3*(cos(d*x+c)*a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*tan(d*x+c)+1/2*b/cos(d*x+c)^2)
```

3.38.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx=\frac{2\,a\cos\left(dx+c\right)\sin\left(dx+c\right)+b}{2\,d\cos\left(dx+c\right)^2}$$

input
```
integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(2*a*cos(d*x + c)*sin(d*x + c) + b)/(d*cos(d*x + c)^2)
```

3.38.6 Sympy [F]

$$\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$$
$$=\int (a\cos(c+dx)+b\sin(c+dx))\sec^3(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**3*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

output
```
Integral((a*cos(c + d*x) + b*sin(c + d*x))*sec(c + d*x)**3, x)
```

3.38.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx=\frac{2\,a\tan(dx+c)-\frac{b}{\sin(dx+c)^2-1}}{2\,d}$$

input
```
integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*(2*a*tan(d*x + c) - b/(sin(d*x + c)^2 - 1))/d
```

3.38.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx=\frac{b\tan(dx+c)^2+2\,a\tan(dx+c)}{2\,d}$$

input
```
integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(b*tan(d*x + c)^2 + 2*a*tan(d*x + c))/d
```

3.38.9 Mupad [B] (verification not implemented)

Time = 20.95 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

$$\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = \frac{\tan(c+d\,x)\ (2\,a+b\tan(c+d\,x))}{2\,d}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))/cos(c + d*x)^3,x)
```

output
```
(tan(c + d*x)*(2*a + b*tan(c + d*x)))/(2*d)
```

3.39 $\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$

3.39.1 Optimal result

Integrand size = 26, antiderivative size = 52

$$\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = \frac{a\text{arctanh}(\sin(c+dx))}{2d}+\frac{b\sec^3(c+dx)}{3d}+\frac{a\sec(c+dx)\tan(c+dx)}{2d}$$

output
```
1/2*a*arctanh(sin(d*x+c))/d+1/3*b*sec(d*x+c)^3/d+1/2*a*sec(d*x+c)*tan(d*x+
c)/d
```

3.39.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = \frac{a\text{arctanh}(\sin(c+dx))}{2d}+\frac{b\sec^3(c+dx)}{3d}+\frac{a\sec(c+dx)\tan(c+dx)}{2d}$$

input
```
Integrate[Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

output
```
(a*ArcTanh[Sin[c + d*x]])/(2*d) + (b*Sec[c + d*x]^3)/(3*d) + (a*Sec[c + d*
x]*Tan[c + d*x])/(2*d)
```

3.39.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\cos(c+dx)+b\sin(c+dx)}{\cos(c+dx)^4}dx$$

$$\Big\downarrow \text{3569}$$

$$\int \left(a\sec^3(c+dx)+b\tan(c+dx)\sec^3(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\text{arctanh}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)\sec(c+dx)}{2d}+\frac{b\sec^3(c+dx)}{3d}$$

input
```
Int[Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

output
```
(a*ArcTanh[Sin[c + d*x]])/(2*d) + (b*Sec[c + d*x]^3)/(3*d) + (a*Sec[c + d*
x]*Tan[c + d*x])/(2*d)
```

3.39.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

3.39.4 Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{b}{3\cos(dx+c)^3}}{d}$
default	$\frac{a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{b}{3\cos(dx+c)^3}}{d}$
parts	$\frac{a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}+\frac{b\sec(dx+c)^3}{3d}$
risch	$\frac{-3ia\,e^{5i(dx+c)}+8b\,e^{3i(dx+c)}+3ia\,e^{i(dx+c)}}{3d\left(e^{2i(dx+c)}+1\right)^3}+\frac{a\ln\left(i+e^{i(dx+c)}\right)}{2d}-\frac{a\ln\left(e^{i(dx+c)}-i\right)}{2d}$
parallelrisch	$\frac{-9a\left(\frac{\cos(3dx+3c)}{3}+\cos(dx+c)\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+9a\left(\frac{\cos(3dx+3c)}{3}+\cos(dx+c)\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+6a\sin(2dx+2}{6d(\cos(3dx+3c)+3\cos(dx+c))}$
norman	$\frac{\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d}+\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{d}-\frac{2b}{3d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}-\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{d}-\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{3d}-\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{d}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)^3\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}$

input
```
int(sec(d*x+c)^4*(cos(d*x+c)*a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+1/3*b/cos
(d*x+c)^3)
```

3.39.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.42

$$\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$$
$$=\frac{3\,a\cos(dx+c)^3\log(\sin(dx+c)+1)-3\,a\cos(dx+c)^3\log(-\sin(dx+c)+1)+6\,a\cos(dx+c)\sin(dx}{12\,d\cos(dx+c)^3}$$

input
```
integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/12*(3*a*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*a*cos(d*x + c)^3*log(-s
in(d*x + c) + 1) + 6*a*cos(d*x + c)*sin(d*x + c) + 4*b)/(d*cos(d*x + c)^3)
```

3.39.6 Sympy [F]

$$\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$$
$$=\int (a\cos(c+dx)+b\sin(c+dx))\sec^4(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**4*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

output
```
Integral((a*cos(c + d*x) + b*sin(c + d*x))*sec(c + d*x)**4, x)
```

3.39.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.17

$$\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$$
$$=-\frac{3\,a\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)-\frac{4\,b}{\cos(dx+c)^3}}{12\,d}$$

input
```
integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/12*(3*a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) +
log(sin(d*x + c) - 1)) - 4*b/cos(d*x + c)^3)/d
```

3.39.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(46) = 92.

Time = 0.30 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.90

$$\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$$
$$=\frac{3\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-6\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-3\,a\tan}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3}}{6\,d}$$

input
```
integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
1/6*(3*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a*log(abs(tan(1/2*d*x + 1/
2*c) - 1)) + 2*(3*a*tan(1/2*d*x + 1/2*c)^5 - 6*b*tan(1/2*d*x + 1/2*c)^4 -
3*a*tan(1/2*d*x + 1/2*c) - 2*b)/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.39.9 Mupad [B] (verification not implemented)

Time = 22.91 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.02

$$\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$$
$$=\frac{a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{2\,b}{3}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))/cos(c + d*x)^4,x)
```

output
```
(a*atanh(tan(c/2 + (d*x)/2)))/d - ((2*b)/3 + a*tan(c/2 + (d*x)/2) - a*tan(
c/2 + (d*x)/2)^5 + 2*b*tan(c/2 + (d*x)/2)^4)/(d*(3*tan(c/2 + (d*x)/2)^2 -
3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

3.40 $\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$

3.40.1 Optimal result

Integrand size = 26, antiderivative size = 44

$$\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = \frac{b\sec^4(c+dx)}{4d}+\frac{a\tan(c+dx)}{d}+\frac{a\tan^3(c+dx)}{3d}$$

output
```
1/4*b*sec(d*x+c)^4/d+a*tan(d*x+c)/d+1/3*a*tan(d*x+c)^3/d
```

3.40.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = \frac{b\sec^4(c+dx)}{4d}+\frac{a\left(\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)\right)}{d}$$

input
```
Integrate[Sec[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

output
```
(b*Sec[c + d*x]^4)/(4*d) + (a*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

3.40.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\cos(c+dx)+b\sin(c+dx)}{\cos(c+dx)^5}dx$$

$$\Big\downarrow \text{3569}$$

$$\int \left(a\sec^4(c+dx)+b\tan(c+dx)\sec^4(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{b\sec^4(c+dx)}{4d}$$

input
```
Int[Sec[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

output
```
(b*Sec[c + d*x]^4)/(4*d) + (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

3.40.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

3.40.4 Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{-a\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+\frac{b}{4\cos(dx+c)^4}}{d}$
default	$\frac{-a\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+\frac{b}{4\cos(dx+c)^4}}{d}$
parts	$-\frac{a\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}+\frac{b\sec(dx+c)^4}{4d}$
risch	$\frac{4ia\,e^{4i(dx+c)}+4b\,e^{4i(dx+c)}+\frac{16ia\,e^{2i(dx+c)}}{3}+\frac{4ia}{3}}{d\left(e^{2i(dx+c)}+1\right)^4}$
parallelrisch	$-\frac{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6 a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5 b-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a}{3}+\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a}{3}-b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}$
norman	$\frac{\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d}+\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{d}+\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{4a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}+\frac{4a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{3d}-\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{d}+\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{d}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)^4\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}$

input
```
int(sec(d*x+c)^5*(cos(d*x+c)*a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+1/4*b/cos(d*x+c)^4)
```

3.40.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.02

$$\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = \frac{4\left(2\,a\cos\left(dx+c\right)^3+a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)+3\,b}{12\,d\cos\left(dx+c\right)^4}$$

input
```
integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/12*(4*(2*a*cos(d*x + c)^3 + a*cos(d*x + c))*sin(d*x + c) + 3*b)/(d*cos(d
*x + c)^4)
```

3.40.6 Sympy [F]

$$\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = \int (a\cos(c+dx)+b\sin(c+dx))\sec^5(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**5*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

output
```
Integral((a*cos(c + d*x) + b*sin(c + d*x))*sec(c + d*x)**5, x)
```

3.40.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = \frac{4\left(\tan(dx+c)^3+3\tan(dx+c)\right)a+\frac{3b}{\left(\sin(dx+c)^2-1\right)^2}}{12d}$$

input
```
integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*a + 3*b/(sin(d*x + c)^2 - 1)^2)/
d
```

3.40.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.09

$$\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = \frac{3b\tan(dx+c)^4+4a\tan(dx+c)^3+6b\tan(dx+c)^2+12a\tan(dx+c)}{12d}$$

input
```
integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
1/12*(3*b*tan(d*x + c)^4 + 4*a*tan(d*x + c)^3 + 6*b*tan(d*x + c)^2 + 12*a*
tan(d*x + c))/d
```

3.40.9 Mupad [B] (verification not implemented)

Time = 21.96 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91

$$\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = \frac{\frac{b}{4}+\frac{a\sin(2\,c+2\,d\,x)}{3}+\frac{a\sin(4\,c+4\,d\,x)}{12}}{d\cos\left(c+d\,x\right)^4}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))/cos(c + d*x)^5,x)
```

output
```
(b/4 + (a*sin(2*c + 2*d*x))/3 + (a*sin(4*c + 4*d*x))/12)/(d*cos(c + d*x)^4
)
```

3.41 $\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$

3.41.1 Optimal result

Integrand size = 26, antiderivative size = 74

$$\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = \frac{3a\text{arctanh}(\sin(c+dx))}{8d}+\frac{b\sec^5(c+dx)}{5d}+\frac{3a\sec(c+dx)\tan(c+dx)}{8d}+\frac{a\sec^3(c+dx)\tan(c+dx)}{4d}$$

output
```
3/8*a*arctanh(sin(d*x+c))/d+1/5*b*sec(d*x+c)^5/d+3/8*a*sec(d*x+c)*tan(d*x+
c)/d+1/4*a*sec(d*x+c)^3*tan(d*x+c)/d
```

3.41.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = \frac{3a\text{arctanh}(\sin(c+dx))}{8d}+\frac{b\sec^5(c+dx)}{5d}+\frac{3a\sec(c+dx)\tan(c+dx)}{8d}+\frac{a\sec^3(c+dx)\tan(c+dx)}{4d}$$

input
```
Integrate[Sec[c + d*x]^6*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

output
```
(3*a*ArcTanh[Sin[c + d*x]])/(8*d) + (b*Sec[c + d*x]^5)/(5*d) + (3*a*Sec[c
+ d*x]*Tan[c + d*x])/(8*d) + (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

3.41.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\cos(c+dx)+b\sin(c+dx)}{\cos(c+dx)^6}dx$$

$$\Big\downarrow \text{3569}$$

$$\int \left(a\sec^5(c+dx)+b\tan(c+dx)\sec^5(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3a\text{arctanh}(\sin(c+dx))}{8d}+\frac{a\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3a\tan(c+dx)\sec(c+dx)}{8d}+\frac{b\sec^5(c+dx)}{5d}$$

input
```
Int[Sec[c + d*x]^6*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

output

```
(3*a*ArcTanh[Sin[c + d*x]])/(8*d) + (b*Sec[c + d*x]^5)/(5*d) + (3*a*Sec[c
+ d*x]*Tan[c + d*x])/(8*d) + (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

3.41.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

3.41.4 Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{a\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{b}{5\cos(dx+c)^5}}{d}$
default	$\frac{a\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{b}{5\cos(dx+c)^5}}{d}$
parts	$\frac{a\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}+\frac{b\sec(dx+c)^5}{5d}$
risch	$\frac{-15ia\,e^{9i(dx+c)}-70ia\,e^{7i(dx+c)}+128b\,e^{5i(dx+c)}+70ia\,e^{3i(dx+c)}+15ia\,e^{i(dx+c)}}{20d\left(e^{2i(dx+c)}+1\right)^5}-\frac{3a\ln\left(e^{i(dx+c)}-i\right)}{8d}+\frac{3a\ln\left(i+e^{i(dx}\right.}{8d}$
parallelrisch	$\frac{-75a\left(\frac{\cos(5dx+5c)}{10}+\frac{\cos(3dx+3c)}{2}+\cos(dx+c)\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+75a\left(\frac{\cos(5dx+5c)}{10}+\frac{\cos(3dx+3c)}{2}+\cos(dx+c)\right)\ln\left(\tan\right.}{20d(\cos(5dx+5c)+5\cos(3dx+}$
norman	$\frac{-\frac{2b}{5d}-\frac{5a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}-\frac{3a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{4d}+\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{2d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{2d}+\frac{3a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{4d}+\frac{5a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{4d}-\frac{4b\tan\left(\frac{dx}{2}\right.}{d}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)^5\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}$

input
```
int(sec(d*x+c)^6*(cos(d*x+c)*a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+t
an(d*x+c)))+1/5*b/cos(d*x+c)^5)
```

3.41.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.19

$$\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$$
$$=\frac{15\,a\cos\left(dx+c\right)^5\log\left(\sin\left(dx+c\right)+1\right)-15\,a\cos\left(dx+c\right)^5\log\left(-\sin\left(dx+c\right)+1\right)+10\left(3\,a\cos\left(dx+c\right)^3\right.}{80\,d\cos\left(dx+c\right)^5}$$

input
```
integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/80*(15*a*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*a*cos(d*x + c)^5*log(
-sin(d*x + c) + 1) + 10*(3*a*cos(d*x + c)^3 + 2*a*cos(d*x + c))*sin(d*x +
c) + 16*b)/(d*cos(d*x + c)^5)
```

3.41.6 Sympy [F(-1)]

Timed out.

$$\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx=\text{Timed out}$$

input
```
integrate(sec(d*x+c)**6*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

output
```
Timed out
```

3.41.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.16

$$\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx=$$
$$-\frac{5\,a\left(\frac{2\left(3\,\sin\left(dx+c\right)^3-5\,\sin\left(dx+c\right)\right)}{\sin\left(dx+c\right)^4-2\,\sin\left(dx+c\right)^2+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)-\frac{16\,b}{\cos\left(dx+c\right)^5}}{80\,d}$$

input
```
integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/80*(5*a*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(
d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 16*
b/cos(d*x + c)^5)/d
```

3.41.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(66) = 132$.

Time = 0.32 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.91

$$\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$$
$$= \frac{15\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(25\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-40\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8-1\right.}{}}{40\,d}$$

input
```
integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
1/40*(15*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*a*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) + 2*(25*a*tan(1/2*d*x + 1/2*c)^9 - 40*b*tan(1/2*d*x + 1/2*c)
^8 - 10*a*tan(1/2*d*x + 1/2*c)^7 - 80*b*tan(1/2*d*x + 1/2*c)^4 + 10*a*tan(
1/2*d*x + 1/2*c)^3 - 25*a*tan(1/2*d*x + 1/2*c) - 8*b)/(tan(1/2*d*x + 1/2*c
)^2 - 1)^5)/d
```

3.41.9 Mupad [B] (verification not implemented)

Time = 26.17 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.36

$$\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = \frac{3\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}$$
$$-\frac{-\frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4}+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{2}+4\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}+\frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}+\frac{2}{5}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))/cos(c + d*x)^6,x)
```

output
```
(3*a*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((2*b)/5 + (5*a*tan(c/2 + (d*x)/2)
)/4 - (a*tan(c/2 + (d*x)/2)^3)/2 + (a*tan(c/2 + (d*x)/2)^7)/2 - (5*a*tan(c
/2 + (d*x)/2)^9)/4 + 4*b*tan(c/2 + (d*x)/2)^4 + 2*b*tan(c/2 + (d*x)/2)^8)/
(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/
2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

3.41. $\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$

3.42 $\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$

3.42.1 Optimal result

Integrand size = 26, antiderivative size = 60

$$\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = \frac{b\sec^6(c+dx)}{6d}+\frac{a\tan(c+dx)}{d}+\frac{2a\tan^3(c+dx)}{3d}+\frac{a\tan^5(c+dx)}{5d}$$

output
```
1/6*b*sec(d*x+c)^6/d+a*tan(d*x+c)/d+2/3*a*tan(d*x+c)^3/d+1/5*a*tan(d*x+c)^
5/d
```

3.42.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.88

$$\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = \frac{b\sec^6(c+dx)}{6d}+\frac{a\left(\tan(c+dx)+\frac{2}{3}\tan^3(c+dx)+\frac{1}{5}\tan^5(c+dx)\right)}{d}$$

input
```
Integrate[Sec[c + d*x]^7*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

output
```
(b*Sec[c + d*x]^6)/(6*d) + (a*(Tan[c + d*x] + (2*Tan[c + d*x]^3)/3 + Tan[c
 + d*x]^5/5))/d
```

3.42.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{a\cos(c+dx)+b\sin(c+dx)}{\cos(c+dx)^7}dx$$

$$\Big\downarrow \; 3569$$

$$\int \left(a\sec^6(c+dx)+b\tan(c+dx)\sec^6(c+dx)\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{a\tan^5(c+dx)}{5d}+\frac{2a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{b\sec^6(c+dx)}{6d}$$

input
```
Int[Sec[c + d*x]^7*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

output
```
(b*Sec[c + d*x]^6)/(6*d) + (a*Tan[c + d*x])/d + (2*a*Tan[c + d*x]^3)/(3*d)
 + (a*Tan[c + d*x]^5)/(5*d)
```

3.42.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

3.42.4 Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.80

method	result
derivativedivides	$\frac{-a\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)+\frac{b}{6\cos(dx+c)^6}}{d}$
default	$\frac{-a\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)+\frac{b}{6\cos(dx+c)^6}}{d}$
parts	$-\frac{a\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)}{d}+\frac{b\sec(dx+c)^6}{6d}$
risch	$\frac{\frac{32ia\,e^{6i(dx+c)}}{3}+\frac{32b\,e^{6i(dx+c)}}{3}+16ia\,e^{4i(dx+c)}+\frac{32ia\,e^{2i(dx+c)}}{5}+\frac{16ia}{15}}{d\left(e^{2i(dx+c)}+1\right)^6}$
parallelrisch	$-\frac{2\left(a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9 b-\frac{7a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{3}+\frac{26\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6 a}{5}-\frac{10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5 b}{3}-\frac{26\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a}{5}+\frac{7\tan(\cdots}{\cdots}\cdots\right.}{d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)^6\cdots}$
norman	$\frac{\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d}+\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{d}+\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{d}+\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{d}+\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{8a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}+\frac{86a\tan\left(\frac{dx}{2}+\frac{c}{2}\cdots\right.}{15d}\cdots}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)^6\left(1+\tan\left(\frac{dx}{2}\cdots\right.\right.}$

input
```
int(sec(d*x+c)^7*(cos(d*x+c)*a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+1/6*b/cos(d*
x+c)^6)
```

3.42.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

$$\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = \frac{2\left(8\,a\cos\left(dx+c\right)^5+4\,a\cos\left(dx+c\right)^3+3\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)+5\,b}{30\,d\cos\left(dx+c\right)^6}$$

input
```
integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/30*(2*(8*a*cos(d*x + c)^5 + 4*a*cos(d*x + c)^3 + 3*a*cos(d*x + c))*sin(d
*x + c) + 5*b)/(d*cos(d*x + c)^6)
```

3.42.6 Sympy [F(-1)]

Timed out.

$$\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**7*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

output
```
Timed out
```

3.42.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.88

$$\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = \frac{2\left(3\tan\left(dx+c\right)^5+10\tan\left(dx+c\right)^3+15\tan\left(dx+c\right)\right)a-\frac{5\,b}{\left(\sin(dx+c)^2-1\right)^3}}{30\,d}$$

input
```
integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
1/30*(2*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a - 5*b/(
sin(d*x + c)^2 - 1)^3)/d
```

3.42.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.17

$$\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = \frac{5\,b\tan\left(dx+c\right)^6+6\,a\tan\left(dx+c\right)^5+15\,b\tan\left(dx+c\right)^4+20\,a\tan\left(dx+c\right)^3+15\,b\tan\left(dx+c\right)^2+30\,a\,\mathrm{ta}}{30\,d}$$

input
```
integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
1/30*(5*b*tan(d*x + c)^6 + 6*a*tan(d*x + c)^5 + 15*b*tan(d*x + c)^4 + 20*a
*tan(d*x + c)^3 + 15*b*tan(d*x + c)^2 + 30*a*tan(d*x + c))/d
```

3.42.9 Mupad [B] (verification not implemented)

Time = 21.17 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.08

$$\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$$
$$=\frac{\frac{8\,a\,\sin(c+d\,x)\cos(c+d\,x)^5}{15}+\frac{4\,a\,\sin(c+d\,x)\cos(c+d\,x)^3}{15}+\frac{a\,\sin(c+d\,x)\,\cos(c+d\,x)}{5}+\frac{b}{6}}{d\cos\left(c+d\,x\right)^6}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))/cos(c + d*x)^7,x)
```

output
```
(b/6 + (a*cos(c + d*x)*sin(c + d*x))/5 + (4*a*cos(c + d*x)^3*sin(c + d*x))
/15 + (8*a*cos(c + d*x)^5*sin(c + d*x))/15)/(d*cos(c + d*x)^6)
```

3.43 $\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$

3.43.1 Optimal result

Integrand size = 28, antiderivative size = 137

$$\begin{aligned}\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx &= -\frac{2ab\cos^7(c+dx)}{7d}+\frac{a^2\sin(c+dx)}{d}-\frac{a^2\sin^3(c+dx)}{d}+\frac{b^2\sin^3(c+dx)}{3d}\\&\quad+\frac{3a^2\sin^5(c+dx)}{5d}-\frac{2b^2\sin^5(c+dx)}{5d}-\frac{a^2\sin^7(c+dx)}{7d}+\frac{b^2\sin^7(c+dx)}{7d}\end{aligned}$$

output
```
-2/7*a*b*cos(d*x+c)^7/d+a^2*sin(d*x+c)/d-a^2*sin(d*x+c)^3/d+1/3*b^2*sin(d*
x+c)^3/d+3/5*a^2*sin(d*x+c)^5/d-2/5*b^2*sin(d*x+c)^5/d-1/7*a^2*sin(d*x+c)^
7/d+1/7*b^2*sin(d*x+c)^7/d
```

3.43.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.67

$$\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = \frac{-30ab\cos^7(c+dx)+105a^2\sin(c+dx)-35(3a^2-b^2)\sin^3(c+dx)+21(3a^2-2b^2)\sin^5(c+dx)-15(a^2}{105d}$$

input
```
Integrate[Cos[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

output
```
(-30*a*b*Cos[c + d*x]^7 + 105*a^2*Sin[c + d*x] - 35*(3*a^2 - b^2)*Sin[c +
d*x]^3 + 21*(3*a^2 - 2*b^2)*Sin[c + d*x]^5 - 15*(a^2 - b^2)*Sin[c + d*x]^7
)/(105*d)
```

3.43.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(c+dx)^5(a\cos(c+dx)+b\sin(c+dx))^2dx$$

$$\Big\downarrow \text{3569}$$

$$\int \left(a^2\cos^7(c+dx)+2ab\sin(c+dx)\cos^6(c+dx)+b^2\sin^2(c+dx)\cos^5(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2\sin^7(c+dx)}{7d}+\frac{3a^2\sin^5(c+dx)}{5d}-\frac{a^2\sin^3(c+dx)}{d}+\frac{a^2\sin(c+dx)}{d}-\frac{2ab\cos^7(c+dx)}{7d}+\frac{b^2\sin^7(c+dx)}{7d}-\frac{2b^2\sin^5(c+dx)}{5d}+\frac{b^2\sin^3(c+dx)}{3d}$$

input
```
Int[Cos[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

output
```
(-2*a*b*Cos[c + d*x]^7)/(7*d) + (a^2*Sin[c + d*x])/d - (a^2*Sin[c + d*x]^3
)/d + (b^2*Sin[c + d*x]^3)/(3*d) + (3*a^2*Sin[c + d*x]^5)/(5*d) - (2*b^2*S
in[c + d*x]^5)/(5*d) - (a^2*Sin[c + d*x]^7)/(7*d) + (b^2*Sin[c + d*x]^7)/(
7*d)
```

3.43.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

3.43.4 Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.72

method	result
parts	$\frac{a^2\left(\frac{16}{5}+\cos(dx+c)^6+\frac{6\cos(dx+c)^4}{5}+\frac{8\cos(dx+c)^2}{5}\right)\sin(dx+c)}{7d}+\frac{b^2\left(\frac{\sin(dx+c)^7}{7}-\frac{2\sin(dx+c)^5}{5}+\frac{\sin(dx+c)^3}{3}\right)}{d}-\frac{2ab\cos}{}$
derivativedivides	$\frac{\frac{a^2\left(\frac{16}{5}+\cos(dx+c)^6+\frac{6\cos(dx+c)^4}{5}+\frac{8\cos(dx+c)^2}{5}\right)\sin(dx+c)}{7}-\frac{2ab\cos(dx+c)^7}{7}+b^2\left(-\frac{\cos(dx+c)^6\sin(dx+c)}{7}+\frac{\left(\frac{8}{3}+\cos(dx+c)^4\right.}{}\right.}{d}$
default	$\frac{\frac{a^2\left(\frac{16}{5}+\cos(dx+c)^6+\frac{6\cos(dx+c)^4}{5}+\frac{8\cos(dx+c)^2}{5}\right)\sin(dx+c)}{7}-\frac{2ab\cos(dx+c)^7}{7}+b^2\left(-\frac{\cos(dx+c)^6\sin(dx+c)}{7}+\frac{\left(\frac{8}{3}+\cos(dx+c)^4\right.}{}\right.}{d}$
risch	$-\frac{5ab\cos(dx+c)}{32d}+\frac{35a^2\sin(dx+c)}{64d}+\frac{5b^2\sin(dx+c)}{64d}-\frac{ab\cos(7dx+7c)}{224d}+\frac{\sin(7dx+7c)a^2}{448d}-\frac{\sin(7dx+7c)b^2}{448d}-$
norman	$\frac{-\frac{4ab}{7d}+\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{13}}{d}+\frac{4(3a^2+2b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}+\frac{4(3a^2+2b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{3d}+\frac{8(53a^2+38b^2)\tan\left(\frac{d}{2}\right.}{35d}}{\left(1+\tan\right.}$
parallelrisch	$\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{13}-4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}ab+4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}a^2+\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}b^2}{3}+\frac{86\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9a^2}{5}-\frac{32\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9b^2}{15}}{}$

input
```
int(cos(d*x+c)^5*(cos(d*x+c)*a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/7*a^2/d*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c)
+b^2/d*(1/7*sin(d*x+c)^7-2/5*sin(d*x+c)^5+1/3*sin(d*x+c)^3)-2/7*a*b*cos(d*
x+c)^7/d
```

3.43.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.69

$$\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx =$$
$$-\frac{30\,ab\cos\left(dx+c\right)^7-\left(15\left(a^2-b^2\right)\cos\left(dx+c\right)^6+3\left(6\,a^2+b^2\right)\cos\left(dx+c\right)^4+4\left(6\,a^2+b^2\right)\cos\left(dx+c\right)\right.}{105\,d}$$

input
```
integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas"
)
```

output
```
-1/105*(30*a*b*cos(d*x + c)^7 - (15*(a^2 - b^2)*cos(d*x + c)^6 + 3*(6*a^2
+ b^2)*cos(d*x + c)^4 + 4*(6*a^2 + b^2)*cos(d*x + c)^2 + 48*a^2 + 8*b^2)*s
in(d*x + c))/d
```

3.43.6 Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.36

$$\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$$
$$=\begin{cases}\frac{16a^2\sin^7(c+dx)}{35d}+\frac{8a^2\sin^5(c+dx)\cos^2(c+dx)}{5d}+\frac{2a^2\sin^3(c+dx)\cos^4(c+dx)}{d}+\frac{a^2\sin(c+dx)\cos^6(c+dx)}{d}-\frac{2ab\cos^7(c+dx)}{7d}+\frac{8b^2 s}{} \\ x(a\cos(c)+b\sin(c))^2\cos^5(c)\end{cases}$$

input
```
integrate(cos(d*x+c)**5*(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

output
```
Piecewise((16*a**2*sin(c + d*x)**7/(35*d) + 8*a**2*sin(c + d*x)**5*cos(c +
 d*x)**2/(5*d) + 2*a**2*sin(c + d*x)**3*cos(c + d*x)**4/d + a**2*sin(c + d
*x)*cos(c + d*x)**6/d - 2*a*b*cos(c + d*x)**7/(7*d) + 8*b**2*sin(c + d*x)*
*7/(105*d) + 4*b**2*sin(c + d*x)**5*cos(c + d*x)**2/(15*d) + b**2*sin(c +
d*x)**3*cos(c + d*x)**4/(3*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**2*cos(
c)**5, True))
```

3.43.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.72

$$\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = -\frac{30\,ab\cos\left(dx+c\right)^7+3\left(5\,\sin\left(dx+c\right)^7-21\,\sin\left(dx+c\right)^5+35\,\sin\left(dx+c\right)^3-35\,\sin\left(dx+c\right)\right)a^2-(15}{105\,d}$$

input
```
integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima"
)
```

output
```
-1/105*(30*a*b*cos(d*x + c)^7 + 3*(5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 +
35*sin(d*x + c)^3 - 35*sin(d*x + c))*a^2 - (15*sin(d*x + c)^7 - 42*sin(d*x
 + c)^5 + 35*sin(d*x + c)^3)*b^2)/d
```

3.43.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.13

$$\begin{aligned}\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx &= -\frac{ab\cos\left(7\,dx+7\,c\right)}{224\,d}-\frac{ab\cos\left(5\,dx+5\,c\right)}{32\,d}-\frac{3\,ab\cos\left(3\,dx+3\,c\right)}{32\,d}\\ &-\frac{5\,ab\cos\left(dx+c\right)}{32\,d}+\frac{\left(a^2-b^2\right)\sin\left(7\,dx+7\,c\right)}{448\,d}+\frac{\left(7\,a^2-3\,b^2\right)\sin\left(5\,dx+5\,c\right)}{320\,d}\\ &+\frac{\left(21\,a^2-b^2\right)\sin\left(3\,dx+3\,c\right)}{192\,d}+\frac{5\left(7\,a^2+b^2\right)\sin\left(dx+c\right)}{64\,d}\end{aligned}$$

input
```
integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/224*a*b*cos(7*d*x + 7*c)/d - 1/32*a*b*cos(5*d*x + 5*c)/d - 3/32*a*b*cos
(3*d*x + 3*c)/d - 5/32*a*b*cos(d*x + c)/d + 1/448*(a^2 - b^2)*sin(7*d*x +
7*c)/d + 1/320*(7*a^2 - 3*b^2)*sin(5*d*x + 5*c)/d + 1/192*(21*a^2 - b^2)*s
in(3*d*x + 3*c)/d + 5/64*(7*a^2 + b^2)*sin(d*x + c)/d
```

3.43.9 Mupad [B] (verification not implemented)

Time = 21.81 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.28

$$\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$$
$$=\frac{16\,a^2\,\sin\left(c+d\,x\right)}{35\,d}+\frac{8\,b^2\,\sin\left(c+d\,x\right)}{105\,d}+\frac{8\,a^2\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{35\,d}+\frac{6\,a^2\cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)}{35\,d}+\frac{a^2\cos\left(c+d\,x\right)^6\,\sin\left(c+d\,x\right)}{7\,d}+\frac{4\,b^2\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{105\,d}+\frac{b^2\cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)}{35\,d}-\frac{b^2\cos\left(c+d\,x\right)^6\,\sin\left(c+d\,x\right)}{7\,d}-\frac{2\,a\,b\cos\left(c+d\,x\right)^7}{7\,d}$$

input
```
int(cos(c + d*x)^5*(a*cos(c + d*x) + b*sin(c + d*x))^2,x)
```

output
```
(16*a^2*sin(c + d*x))/(35*d) + (8*b^2*sin(c + d*x))/(105*d) + (8*a^2*cos(c
 + d*x)^2*sin(c + d*x))/(35*d) + (6*a^2*cos(c + d*x)^4*sin(c + d*x))/(35*d
) + (a^2*cos(c + d*x)^6*sin(c + d*x))/(7*d) + (4*b^2*cos(c + d*x)^2*sin(c
+ d*x))/(105*d) + (b^2*cos(c + d*x)^4*sin(c + d*x))/(35*d) - (b^2*cos(c +
d*x)^6*sin(c + d*x))/(7*d) - (2*a*b*cos(c + d*x)^7)/(7*d)
```

3.44 $\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$

3.44.1 Optimal result

Integrand size = 28, antiderivative size = 174

$$\begin{aligned}\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx &= \frac{5a^2x}{16}+\frac{b^2x}{16}-\frac{ab\cos^6(c+dx)}{3d}+\frac{5a^2\cos(c+dx)\sin(c+dx)}{16d}\\ &\quad+\frac{b^2\cos(c+dx)\sin(c+dx)}{16d}+\frac{5a^2\cos^3(c+dx)\sin(c+dx)}{24d}\\ &\quad+\frac{b^2\cos^3(c+dx)\sin(c+dx)}{24d}+\frac{a^2\cos^5(c+dx)\sin(c+dx)}{6d}-\frac{b^2\cos^5(c+dx)\sin(c+dx)}{6d}\end{aligned}$$

output
```
5/16*a^2*x+1/16*b^2*x-1/3*a*b*cos(d*x+c)^6/d+5/16*a^2*cos(d*x+c)*sin(d*x+c
)/d+1/16*b^2*cos(d*x+c)*sin(d*x+c)/d+5/24*a^2*cos(d*x+c)^3*sin(d*x+c)/d+1/
24*b^2*cos(d*x+c)^3*sin(d*x+c)/d+1/6*a^2*cos(d*x+c)^5*sin(d*x+c)/d-1/6*b^2
*cos(d*x+c)^5*sin(d*x+c)/d
```

3.44.2 Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.84

$$\begin{aligned}\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx &= \frac{(5a^2+b^2)(c+dx)}{16d}-\frac{5ab\cos(2(c+dx))}{32d}-\frac{ab\cos(4(c+dx))}{16d}-\frac{ab\cos(6(c+dx))}{96d}\\ &\quad+\frac{(15a^2+b^2)\sin(2(c+dx))}{64d}+\frac{(3a^2-b^2)\sin(4(c+dx))}{64d}+\frac{(a^2-b^2)\sin(6(c+dx))}{192d}\end{aligned}$$

input
```
Integrate[Cos[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

output
```
((5*a^2 + b^2)*(c + d*x))/(16*d) - (5*a*b*Cos[2*(c + d*x)])/(32*d) - (a*b*
Cos[4*(c + d*x)])/(16*d) - (a*b*Cos[6*(c + d*x)])/(96*d) + ((15*a^2 + b^2)
*Sin[2*(c + d*x)])/(64*d) + ((3*a^2 - b^2)*Sin[4*(c + d*x)])/(64*d) + ((a^
2 - b^2)*Sin[6*(c + d*x)])/(192*d)
```

3.44.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \cos(c+dx)^4(a\cos(c+dx)+b\sin(c+dx))^2dx$$

$$\Big\downarrow \text{ 3569}$$

$$\int \left(a^2\cos^6(c+dx)+2ab\sin(c+dx)\cos^5(c+dx)+b^2\sin^2(c+dx)\cos^4(c+dx)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^2\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{5a^2\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{5a^2\sin(c+dx)\cos(c+dx)}{16d}+\frac{5a^2x}{16}-\frac{ab\cos^6(c+dx)}{3d}-\frac{b^2\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{b^2\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{b^2\sin(c+dx)\cos(c+dx)}{16d}+\frac{b^2x}{16}$$

input
```
Int[Cos[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

output
```
(5*a^2*x)/16 + (b^2*x)/16 - (a*b*Cos[c + d*x]^6)/(3*d) + (5*a^2*Cos[c + d*
x]*Sin[c + d*x])/(16*d) + (b^2*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (5*a^2*
Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (b^2*Cos[c + d*x]^3*Sin[c + d*x])/(2
4*d) + (a^2*Cos[c + d*x]^5*Sin[c + d*x])/(6*d) - (b^2*Cos[c + d*x]^5*Sin[c
 + d*x])/(6*d)
```

3.44.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

3.44.4 Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.68

method	result
derivativedivides	$\frac{a^2\left(\frac{\left(\cos(dx+c)^5+\frac{5\cos(dx+c)^3}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)-\frac{ab\cos(dx+c)^6}{3}+b^2\left(-\frac{\cos(dx+c)^5\sin(dx+c)}{6}+\frac{(\cos(d}{}\right.}{d}$
default	$\frac{a^2\left(\frac{\left(\cos(dx+c)^5+\frac{5\cos(dx+c)^3}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)-\frac{ab\cos(dx+c)^6}{3}+b^2\left(-\frac{\cos(dx+c)^5\sin(dx+c)}{6}+\frac{(\cos(d}{}\right.}{d}$
parts	$\frac{a^2\left(\frac{\left(\cos(dx+c)^5+\frac{5\cos(dx+c)^3}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)}{d}+\frac{b^2\left(-\frac{\cos(dx+c)^5\sin(dx+c)}{6}+\frac{\left(\cos(dx+c)^3+\frac{3\cos(}{}\right.}{24}\right.}{d}$
parallelrisch	$\frac{(45a^2+3b^2)\sin(2dx+2c)+(9a^2-3b^2)\sin(4dx+4c)+(a^2-b^2)\sin(6dx+6c)+60a^2xd+12b^2dx-30ab\cos(2dx+2c)-12ab\,c}{192d}$
risch	$\frac{5a^2x}{16}+\frac{x\,b^2}{16}-\frac{ab\cos(6dx+6c)}{96d}+\frac{\sin(6dx+6c)a^2}{192d}-\frac{\sin(6dx+6c)b^2}{192d}-\frac{ab\cos(4dx+4c)}{16d}+\frac{3\sin(4dx+4c)a^2}{64d}-\frac{s}{}$
norman	$\left(\frac{5a^2}{16}+\frac{b^2}{16}\right)x+\left(\frac{5a^2}{16}+\frac{b^2}{16}\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}+\left(\frac{15a^2}{8}+\frac{3b^2}{8}\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\left(\frac{15a^2}{8}+\frac{3b^2}{8}\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+\left(\frac{25a^2}{4}+\frac{5b^2}{4}\right.$

input
```
int(cos(d*x+c)^4*(cos(d*x+c)*a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5
/16*d*x+5/16*c)-1/3*a*b*cos(d*x+c)^6+b^2*(-1/6*cos(d*x+c)^5*sin(d*x+c)+1/2
4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1/16*d*x+1/16*c))
```

3.44.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.55

$$\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = -\frac{16\,ab\cos(dx+c)^6-3\,(5\,a^2+b^2)dx-\left(8\,(a^2-b^2)\cos(dx+c)^5+2\,(5\,a^2+b^2)\cos(dx+c)^3+3\,(5\,a^2+\right.}{48\,d}$$

input
```
integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas"
)
```

output
```
-1/48*(16*a*b*cos(d*x + c)^6 - 3*(5*a^2 + b^2)*d*x - (8*(a^2 - b^2)*cos(d*
x + c)^5 + 2*(5*a^2 + b^2)*cos(d*x + c)^3 + 3*(5*a^2 + b^2)*cos(d*x + c))*
sin(d*x + c))/d
```

3.44.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(162) = 324$.

Time = 0.37 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.95

$$\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$$
$$= \begin{cases} \frac{5a^2x\sin^6(c+dx)}{16} + \frac{15a^2x\sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{15a^2x\sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{5a^2x\cos^6(c+dx)}{16} + \frac{5a^2\sin^5(c+dx)\cos(c+dx)}{16d} \\ x(a\cos(c)+b\sin(c))^2\cos^4(c) \end{cases}$$

input
```
integrate(cos(d*x+c)**4*(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

output
```
Piecewise((5*a**2*x*sin(c + d*x)**6/16 + 15*a**2*x*sin(c + d*x)**4*cos(c +
 d*x)**2/16 + 15*a**2*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*a**2*x*cos(
c + d*x)**6/16 + 5*a**2*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 5*a**2*sin(c
 + d*x)**3*cos(c + d*x)**3/(6*d) + 11*a**2*sin(c + d*x)*cos(c + d*x)**5/(1
6*d) - a*b*cos(c + d*x)**6/(3*d) + b**2*x*sin(c + d*x)**6/16 + 3*b**2*x*si
n(c + d*x)**4*cos(c + d*x)**2/16 + 3*b**2*x*sin(c + d*x)**2*cos(c + d*x)**
4/16 + b**2*x*cos(c + d*x)**6/16 + b**2*sin(c + d*x)**5*cos(c + d*x)/(16*d
) + b**2*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) - b**2*sin(c + d*x)*cos(c +
 d*x)**5/(16*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**2*cos(c)**4, True))
```

3.44.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.59

$$\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx =$$
$$-\frac{64\,ab\cos(dx+c)^6 + \left(4\sin(2\,dx+2\,c)^3 - 60\,dx - 60\,c - 9\sin(4\,dx+4\,c) - 48\sin(2\,dx+2\,c)\right)a^2 - (}{192\,d}$$

input
```
integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima"
)
```

output
```
-1/192*(64*a*b*cos(d*x + c)^6 + (4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*
sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a^2 - (4*sin(2*d*x + 2*c)^3 + 12*d
*x + 12*c - 3*sin(4*d*x + 4*c))*b^2)/d
```

3.44.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.76

$$\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$$
$$=\frac{1}{16}\left(5a^2+b^2\right)x-\frac{ab\cos(6dx+6c)}{96d}-\frac{ab\cos(4dx+4c)}{16d}-\frac{5ab\cos(2dx+2c)}{32d}+\frac{(a^2-b^2)\sin(6dx+6c)}{192d}+\frac{(3a^2-b^2)\sin(4dx+4c)}{64d}+\frac{(15a^2+b^2)\sin(2dx+2c)}{64d}$$

input
```
integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
1/16*(5*a^2 + b^2)*x - 1/96*a*b*cos(6*d*x + 6*c)/d - 1/16*a*b*cos(4*d*x +
4*c)/d - 5/32*a*b*cos(2*d*x + 2*c)/d + 1/192*(a^2 - b^2)*sin(6*d*x + 6*c)/
d + 1/64*(3*a^2 - b^2)*sin(4*d*x + 4*c)/d + 1/64*(15*a^2 + b^2)*sin(2*d*x
+ 2*c)/d
```

3.44.9 Mupad [B] (verification not implemented)

Time = 21.95 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.90

$$\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$$
$$=\frac{5a^2x}{16}+\frac{b^2x}{16}+\frac{5a^2\cos(c+dx)^3\sin(c+dx)}{24d}+\frac{a^2\cos(c+dx)^5\sin(c+dx)}{6d}+\frac{b^2\cos(c+dx)^3\sin(c+dx)}{24d}-\frac{b^2\cos(c+dx)^5\sin(c+dx)}{6d}-\frac{ab\cos(c+dx)^6}{3d}+\frac{5a^2\cos(c+dx)\sin(c+dx)}{16d}+\frac{b^2\cos(c+dx)\sin(c+dx)}{16d}$$

input
```
int(cos(c + d*x)^4*(a*cos(c + d*x) + b*sin(c + d*x))^2,x)
```

output
```
(5*a^2*x)/16 + (b^2*x)/16 + (5*a^2*cos(c + d*x)^3*sin(c + d*x))/(24*d) + (
a^2*cos(c + d*x)^5*sin(c + d*x))/(6*d) + (b^2*cos(c + d*x)^3*sin(c + d*x))
/(24*d) - (b^2*cos(c + d*x)^5*sin(c + d*x))/(6*d) - (a*b*cos(c + d*x)^6)/(
3*d) + (5*a^2*cos(c + d*x)*sin(c + d*x))/(16*d) + (b^2*cos(c + d*x)*sin(c
+ d*x))/(16*d)
```

3.45 $\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$

3.45.1 Optimal result

Integrand size = 28, antiderivative size = 103

$$\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = -\frac{2ab\cos^5(c+dx)}{5d}+\frac{a^2\sin(c+dx)}{d}-\frac{2a^2\sin^3(c+dx)}{3d}+\frac{b^2\sin^3(c+dx)}{3d}+\frac{a^2\sin^5(c+dx)}{5d}-\frac{b^2\sin^5(c+dx)}{5d}$$

output
```
-2/5*a*b*cos(d*x+c)^5/d+a^2*sin(d*x+c)/d-2/3*a^2*sin(d*x+c)^3/d+1/3*b^2*si
n(d*x+c)^3/d+1/5*a^2*sin(d*x+c)^5/d-1/5*b^2*sin(d*x+c)^5/d
```

3.45.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.67

$$\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = \frac{-6ab\cos^5(c+dx)+15a^2\sin(c+dx)+5(-2a^2+b^2)\sin^3(c+dx)+3(a^2-b^2)\sin^5(c+dx)}{15d}$$

input
```
Integrate[Cos[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

output
```
(-6*a*b*Cos[c + d*x]^5 + 15*a^2*Sin[c + d*x] + 5*(-2*a^2 + b^2)*Sin[c + d*
x]^3 + 3*(a^2 - b^2)*Sin[c + d*x]^5)/(15*d)
```

3.45.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(c+dx)^3(a\cos(c+dx)+b\sin(c+dx))^2dx$$

$$\Big\downarrow \text{3569}$$

$$\int \left(a^2\cos^5(c+dx)+2ab\sin(c+dx)\cos^4(c+dx)+b^2\sin^2(c+dx)\cos^3(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2\sin^5(c+dx)}{5d}-\frac{2a^2\sin^3(c+dx)}{3d}+\frac{a^2\sin(c+dx)}{d}-\frac{2ab\cos^5(c+dx)}{5d}-\frac{b^2\sin^5(c+dx)}{5d}+\frac{b^2\sin^3(c+dx)}{3d}$$

input
```
Int[Cos[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

output
```
(-2*a*b*Cos[c + d*x]^5)/(5*d) + (a^2*Sin[c + d*x])/d - (2*a^2*Sin[c + d*x]
^3)/(3*d) + (b^2*Sin[c + d*x]^3)/(3*d) + (a^2*Sin[c + d*x]^5)/(5*d) - (b^2
*Sin[c + d*x]^5)/(5*d)
```

3.45.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

3.45.4 Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.77

method	result
parts	$\frac{a^{2}\left(\frac{8}{3}+\cos(dx+c)^{4}+\frac{4\cos(dx+c)^{2}}{3}\right)\sin(dx+c)}{5d}+\frac{b^{2}\left(-\frac{\sin(dx+c)^{5}}{5}+\frac{\sin(dx+c)^{3}}{3}\right)}{d}-\frac{2ab\cos(dx+c)^{5}}{5d}$
derivativedivides	$\frac{\frac{a^{2}\left(\frac{8}{3}+\cos(dx+c)^{4}+\frac{4\cos(dx+c)^{2}}{3}\right)\sin(dx+c)}{5}-\frac{2ab\cos(dx+c)^{5}}{5}+b^{2}\left(-\frac{\cos(dx+c)^{4}\sin(dx+c)}{5}+\frac{\left(2+\cos(dx+c)^{2}\right)\sin(dx+c)}{15}\right)}{d}$
default	$\frac{\frac{a^{2}\left(\frac{8}{3}+\cos(dx+c)^{4}+\frac{4\cos(dx+c)^{2}}{3}\right)\sin(dx+c)}{5}-\frac{2ab\cos(dx+c)^{5}}{5}+b^{2}\left(-\frac{\cos(dx+c)^{4}\sin(dx+c)}{5}+\frac{\left(2+\cos(dx+c)^{2}\right)\sin(dx+c)}{15}\right)}{d}$
risch	$-\frac{ab\cos(dx+c)}{4d}+\frac{5a^{2}\sin(dx+c)}{8d}+\frac{b^{2}\sin(dx+c)}{8d}-\frac{ab\cos(5dx+5c)}{40d}+\frac{\sin(5dx+5c)a^{2}}{80d}-\frac{\sin(5dx+5c)b^{2}}{80d}-ab\,c$
parallelrisch	$\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}a^{2}-4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}ab+\frac{8\left(a^{2}+b^{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{3}+\frac{4\left(29a^{2}-4b^{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{15}-8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}ab+8\left(a^{2}+b^{2}\right.}{d\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{5}}$
norman	$\frac{-\frac{4ab}{5d}+\frac{2a^{2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2a^{2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{d}+\frac{8\left(a^{2}+b^{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3d}+\frac{8\left(a^{2}+b^{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{3d}+\frac{4\left(29a^{2}-4b^{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{15d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{5}}$

input
```
int(cos(d*x+c)^3*(cos(d*x+c)*a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/5*a^2/d*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+b^2/d*(-1/5*sin(d
*x+c)^5+1/3*sin(d*x+c)^3)-2/5*a*b*cos(d*x+c)^5/d
```

3.45.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.72

$$\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = -\frac{6\,ab\cos\left(dx+c\right)^5-\left(3\left(a^2-b^2\right)\cos\left(dx+c\right)^4+\left(4\,a^2+b^2\right)\cos\left(dx+c\right)^2+8\,a^2+2\,b^2\right)\sin\left(dx+c\right)}{15\,d}$$

input
```
integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas"
)
```

output
```
-1/15*(6*a*b*cos(d*x + c)^5 - (3*(a^2 - b^2)*cos(d*x + c)^4 + (4*a^2 + b^2
)*cos(d*x + c)^2 + 8*a^2 + 2*b^2)*sin(d*x + c))/d
```

3.45.6 Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.34

$$\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = \begin{cases} \frac{8a^2\sin^5(c+dx)}{15d}+\frac{4a^2\sin^3(c+dx)\cos^2(c+dx)}{3d}+\frac{a^2\sin(c+dx)\cos^4(c+dx)}{d}-\frac{2ab\cos^5(c+dx)}{5d}+\frac{2b^2\sin^5(c+dx)}{15d}+\frac{b^2\sin^3(c+dx)\cos^{}}{3d} \\ x(a\cos(c)+b\sin(c))^2\cos^3(c) \end{cases}$$

input
```
integrate(cos(d*x+c)**3*(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

output
```
Piecewise((8*a**2*sin(c + d*x)**5/(15*d) + 4*a**2*sin(c + d*x)**3*cos(c +
d*x)**2/(3*d) + a**2*sin(c + d*x)*cos(c + d*x)**4/d - 2*a*b*cos(c + d*x)**
5/(5*d) + 2*b**2*sin(c + d*x)**5/(15*d) + b**2*sin(c + d*x)**3*cos(c + d*x
)**2/(3*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**2*cos(c)**3, True))
```

3.45.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.75

$$\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx =$$
$$-\frac{6\,ab\cos\left(dx+c\right)^5-\left(3\,\sin\left(dx+c\right)^5-10\,\sin\left(dx+c\right)^3+15\,\sin\left(dx+c\right)\right)a^2+\left(3\,\sin\left(dx+c\right)^5-5\,\sin\left(\right.\right.}{15\,d}$$

input
```
integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima"
)
```

output
```
-1/15*(6*a*b*cos(d*x + c)^5 - (3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*s
in(d*x + c))*a^2 + (3*sin(d*x + c)^5 - 5*sin(d*x + c)^3)*b^2)/d
```

3.45.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.11

$$\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$$
$$= -\frac{ab\cos\left(5\,dx+5\,c\right)}{40\,d}-\frac{ab\cos\left(3\,dx+3\,c\right)}{8\,d}-\frac{ab\cos\left(dx+c\right)}{4\,d}$$
$$+\frac{\left(a^2-b^2\right)\sin\left(5\,dx+5\,c\right)}{80\,d}+\frac{\left(5\,a^2-b^2\right)\sin\left(3\,dx+3\,c\right)}{48\,d}+\frac{\left(5\,a^2+b^2\right)\sin\left(dx+c\right)}{8\,d}$$

input
```
integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/40*a*b*cos(5*d*x + 5*c)/d - 1/8*a*b*cos(3*d*x + 3*c)/d - 1/4*a*b*cos(d*
x + c)/d + 1/80*(a^2 - b^2)*sin(5*d*x + 5*c)/d + 1/48*(5*a^2 - b^2)*sin(3*
d*x + 3*c)/d + 1/8*(5*a^2 + b^2)*sin(d*x + c)/d
```

3.45.9 Mupad [B] (verification not implemented)

Time = 21.69 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.12

$$\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$$
$$= \frac{2\left(\frac{3\sin(c+d\,x)\,a^2\cos(c+d\,x)^4}{2}+2\sin\left(c+d\,x\right)\,a^2\cos\left(c+d\,x\right)^2+4\sin\left(c+d\,x\right)\,a^2-3\,a\,b\cos\left(c+d\,x\right)^5-\frac{3\sin(\right.}{15\,d}$$

input
```
int(cos(c + d*x)^3*(a*cos(c + d*x) + b*sin(c + d*x))^2,x)
```

output
```
(2*(4*a^2*sin(c + d*x) + b^2*sin(c + d*x) + 2*a^2*cos(c + d*x)^2*sin(c + d
*x) + (3*a^2*cos(c + d*x)^4*sin(c + d*x))/2 + (b^2*cos(c + d*x)^2*sin(c +
d*x))/2 - (3*b^2*cos(c + d*x)^4*sin(c + d*x))/2 - 3*a*b*cos(c + d*x)^5))/(
15*d)
```

3.46 $\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$

3.46.1 Optimal result

Integrand size = 28, antiderivative size = 126

$$\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = \frac{3a^2x}{8}+\frac{b^2x}{8}-\frac{ab\cos^4(c+dx)}{2d}+\frac{3a^2\cos(c+dx)\sin(c+dx)}{8d}+\frac{b^2\cos(c+dx)\sin(c+dx)}{8d}+\frac{a^2\cos^3(c+dx)\sin(c+dx)}{4d}-\frac{b^2\cos^3(c+dx)\sin(c+dx)}{4d}$$

output
```
3/8*a^2*x+1/8*b^2*x-1/2*a*b*cos(d*x+c)^4/d+3/8*a^2*cos(d*x+c)*sin(d*x+c)/d
+1/8*b^2*cos(d*x+c)*sin(d*x+c)/d+1/4*a^2*cos(d*x+c)^3*sin(d*x+c)/d-1/4*b^2
*cos(d*x+c)^3*sin(d*x+c)/d
```

3.46.2 Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.78

$$\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = \frac{(3a^2+b^2)(c+dx)}{8d}-\frac{ab\cos(2(c+dx))}{4d}-\frac{ab\cos(4(c+dx))}{16d}+\frac{a^2\sin(2(c+dx))}{4d}+\frac{(a^2-b^2)\sin(4(c+dx))}{32d}$$

input
```
Integrate[Cos[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

output
```
((3*a^2 + b^2)*(c + d*x))/(8*d) - (a*b*Cos[2*(c + d*x)])/(4*d) - (a*b*Cos[
4*(c + d*x)])/(16*d) + (a^2*Sin[2*(c + d*x)])/(4*d) + ((a^2 - b^2)*Sin[4*(
c + d*x)])/(32*d)
```

3.46.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))^2dx$$

$$\Big\downarrow \text{3569}$$

$$\int \left(a^2\cos^4(c+dx)+2ab\sin(c+dx)\cos^3(c+dx)+b^2\sin^2(c+dx)\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a^2\sin(c+dx)\cos(c+dx)}{8d}+\frac{3a^2x}{8}-\frac{ab\cos^4(c+dx)}{2d}-\frac{b^2\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{b^2\sin(c+dx)\cos(c+dx)}{8d}+\frac{b^2x}{8}$$

input
```
Int[Cos[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

output
```
(3*a^2*x)/8 + (b^2*x)/8 - (a*b*Cos[c + d*x]^4)/(2*d) + (3*a^2*Cos[c + d*x]
*Sin[c + d*x])/(8*d) + (b^2*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^2*Cos[c
+ d*x]^3*Sin[c + d*x])/(4*d) - (b^2*Cos[c + d*x]^3*Sin[c + d*x])/(4*d)
```

3.46.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

3.46.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.67

method	result
parallelrisch	$\frac{(a^2-b^2)\sin(4dx+4c)+12a^2xd+4b^2dx+8\sin(2dx+2c)a^2-8ab\cos(2dx+2c)-2ab\cos(4dx+4c)+10ab}{32d}$
derivativedivides	$\frac{a^2\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)-\frac{ab\cos(dx+c)^4}{2}+b^2\left(-\frac{\sin(dx+c)\cos(dx+c)^3}{4}+\frac{\sin(dx+c)\cos(dx+c)}{8}+\frac{d}{8}\right.}{d}$
default	$\frac{a^2\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)-\frac{ab\cos(dx+c)^4}{2}+b^2\left(-\frac{\sin(dx+c)\cos(dx+c)^3}{4}+\frac{\sin(dx+c)\cos(dx+c)}{8}+\frac{d}{8}\right.}{d}$
risch	$\frac{3a^2x}{8}+\frac{x\,b^2}{8}-\frac{ab\cos(4dx+4c)}{16d}+\frac{\sin(4dx+4c)a^2}{32d}-\frac{\sin(4dx+4c)b^2}{32d}-\frac{ab\cos(2dx+2c)}{4d}+\frac{\sin(2dx+2c)a^2}{4d}$
parts	$\frac{a^2\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}+\frac{b^2\left(-\frac{\sin(dx+c)\cos(dx+c)^3}{4}+\frac{\sin(dx+c)\cos(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)}{d}-\frac{a}{}$
norman	$\frac{\left(\frac{3a^2}{8}+\frac{b^2}{8}\right)x+\left(\frac{3a^2}{2}+\frac{b^2}{2}\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\left(\frac{3a^2}{2}+\frac{b^2}{2}\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+\left(\frac{3a^2}{8}+\frac{b^2}{8}\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8+\left(\frac{9a^2}{4}+\frac{3b^2}{4}\right)x\tan\left(\right.}{}$

input
```
int(cos(d*x+c)^2*(cos(d*x+c)*a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/32*((a^2-b^2)*sin(4*d*x+4*c)+12*a^2*x*d+4*b^2*d*x+8*sin(2*d*x+2*c)*a^2-8
*a*b*cos(2*d*x+2*c)-2*a*b*cos(4*d*x+4*c)+10*a*b)/d
```

3.46.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.60

$$\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx =$$
$$-\frac{4\,ab\cos\left(dx+c\right)^4-(3\,a^2+b^2)dx-\left(2\left(a^2-b^2\right)\cos\left(dx+c\right)^3+\left(3\,a^2+b^2\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{8\,d}$$

input
```
integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas"
)
```

output
```
-1/8*(4*a*b*cos(d*x + c)^4 - (3*a^2 + b^2)*d*x - (2*(a^2 - b^2)*cos(d*x +
c)^3 + (3*a^2 + b^2)*cos(d*x + c))*sin(d*x + c))/d
```

3.46.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(116) = 232.

Time = 0.20 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.89

$$\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$$
$$=\begin{cases}\frac{3a^2x\sin^4(c+dx)}{8}+\frac{3a^2x\sin^2(c+dx)\cos^2(c+dx)}{4}+\frac{3a^2x\cos^4(c+dx)}{8}+\frac{3a^2\sin^3(c+dx)\cos(c+dx)}{8d}+\frac{5a^2\sin(c+dx)\cos^3(c+dx)}{8d}-\frac{a}{} \\ x(a\cos(c)+b\sin(c))^2\cos^2(c)\end{cases}$$

input
```
integrate(cos(d*x+c)**2*(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

output
```
Piecewise((3*a**2*x*sin(c + d*x)**4/8 + 3*a**2*x*sin(c + d*x)**2*cos(c + d
*x)**2/4 + 3*a**2*x*cos(c + d*x)**4/8 + 3*a**2*sin(c + d*x)**3*cos(c + d*x
)/(8*d) + 5*a**2*sin(c + d*x)*cos(c + d*x)**3/(8*d) - a*b*cos(c + d*x)**4/
(2*d) + b**2*x*sin(c + d*x)**4/8 + b**2*x*sin(c + d*x)**2*cos(c + d*x)**2/
4 + b**2*x*cos(c + d*x)**4/8 + b**2*sin(c + d*x)**3*cos(c + d*x)/(8*d) - b
**2*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c)
)**2*cos(c)**2, True))
```

3.46.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.60

$$\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx =$$
$$-\frac{16\,ab\cos\left(dx+c\right)^4-\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)a^2-\left(4\,dx+4\,c-\sin\left(4\,dx+\right.\right.}{32\,d}$$

input
```
integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima"
)
```

output
```
-1/32*(16*a*b*cos(d*x + c)^4 - (12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2
*d*x + 2*c))*a^2 - (4*d*x + 4*c - sin(4*d*x + 4*c))*b^2)/d
```

3.46.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.67

$$\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$$
$$=\frac{1}{8}\left(3\,a^2+b^2\right)x-\frac{ab\cos\left(4\,dx+4\,c\right)}{16\,d}-\frac{ab\cos\left(2\,dx+2\,c\right)}{4\,d}$$
$$+\frac{a^2\sin\left(2\,dx+2\,c\right)}{4\,d}+\frac{\left(a^2-b^2\right)\sin\left(4\,dx+4\,c\right)}{32\,d}$$

input
```
integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
1/8*(3*a^2 + b^2)*x - 1/16*a*b*cos(4*d*x + 4*c)/d - 1/4*a*b*cos(2*d*x + 2*
c)/d + 1/4*a^2*sin(2*d*x + 2*c)/d + 1/32*(a^2 - b^2)*sin(4*d*x + 4*c)/d
```

3.46.9 Mupad [B] (verification not implemented)

Time = 20.90 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.71

$$\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$$
$$=\frac{4\,a^2\,\sin\left(2\,c+2\,d\,x\right)+\frac{a^2\,\sin(4\,c+4\,d\,x)}{2}-\frac{b^2\,\sin(4\,c+4\,d\,x)}{2}+2\,a\,b\sin\left(2\,c+2\,d\,x\right)^2+8\,a\,b\sin\left(c+d\,x\right)^2+6\,a^2\,d\,x}{16\,d}$$

input
```
int(cos(c + d*x)^2*(a*cos(c + d*x) + b*sin(c + d*x))^2,x)
```

output
```
(4*a^2*sin(2*c + 2*d*x) + (a^2*sin(4*c + 4*d*x))/2 - (b^2*sin(4*c + 4*d*x)
)/2 + 2*a*b*sin(2*c + 2*d*x)^2 + 8*a*b*sin(c + d*x)^2 + 6*a^2*d*x + 2*b^2*
d*x)/(16*d)
```

3.47 $\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$

3.47.1 Optimal result

Integrand size = 26, antiderivative size = 67

$$\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = -\frac{2ab\cos^3(c+dx)}{3d}+\frac{a^2\sin(c+dx)}{d}-\frac{a^2\sin^3(c+dx)}{3d}+\frac{b^2\sin^3(c+dx)}{3d}$$

output
```
-2/3*a*b*cos(d*x+c)^3/d+a^2*sin(d*x+c)/d-1/3*a^2*sin(d*x+c)^3/d+1/3*b^2*si
n(d*x+c)^3/d
```

3.47.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = -\frac{2ab\cos^3(c+dx)}{3d}+\frac{a^2\sin(c+dx)}{d}-\frac{a^2\sin^3(c+dx)}{3d}+\frac{b^2\sin^3(c+dx)}{3d}$$

input
```
Integrate[Cos[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

output
```
(-2*a*b*Cos[c + d*x]^3)/(3*d) + (a^2*Sin[c + d*x])/d - (a^2*Sin[c + d*x]^3
)/(3*d) + (b^2*Sin[c + d*x]^3)/(3*d)
```

3.47.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2dx$$

$$\Big\downarrow \text{ 3569}$$

$$\int \left(a^2\cos^3(c+dx)+2ab\sin(c+dx)\cos^2(c+dx)+b^2\sin^2(c+dx)\cos(c+dx)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^2\sin^3(c+dx)}{3d}+\frac{a^2\sin(c+dx)}{d}-\frac{2ab\cos^3(c+dx)}{3d}+\frac{b^2\sin^3(c+dx)}{3d}$$

input
```
Int[Cos[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

output
```
(-2*a*b*Cos[c + d*x]^3)/(3*d) + (a^2*Sin[c + d*x])/d - (a^2*Sin[c + d*x]^3
)/(3*d) + (b^2*Sin[c + d*x]^3)/(3*d)
```

3.47.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

3.47.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{\frac{a^2\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}-\frac{2ab\cos(dx+c)^3}{3}+\frac{b^2\sin(dx+c)^3}{3}}{d}$	52
default	$\frac{\frac{a^2\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}-\frac{2ab\cos(dx+c)^3}{3}+\frac{b^2\sin(dx+c)^3}{3}}{d}$	52
parts	$\frac{a^2\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3d}+\frac{b^2\sin(dx+c)^3}{3d}-\frac{2ab\cos(dx+c)^3}{3d}$	57
risch	$-\frac{ab\cos(dx+c)}{2d}+\frac{3a^2\sin(dx+c)}{4d}+\frac{b^2\sin(dx+c)}{4d}-\frac{ab\cos(3dx+3c)}{6d}+\frac{\sin(3dx+3c)a^2}{12d}-\frac{\sin(3dx+3c)b^2}{12d}$	93
parallelrisch	$\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5a^2-4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4ab+\frac{4\left(a^2+2b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}+2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{4ab}{3}}{d\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}$	93
norman	$\frac{-\frac{4ab}{3d}+\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d}+\frac{4\left(a^2+2b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}-\frac{4ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}$	104

input
```
int(cos(d*x+c)*(cos(d*x+c)*a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/3*a^2*(2+cos(d*x+c)^2)*sin(d*x+c)-2/3*a*b*cos(d*x+c)^3+1/3*b^2*sin(
d*x+c)^3)
```

3.47.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.79

$$\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$$
$$= -\frac{2\,ab\cos\left(dx+c\right)^3-\left(\left(a^2-b^2\right)\cos\left(dx+c\right)^2+2\,a^2+b^2\right)\sin\left(dx+c\right)}{3\,d}$$

input
```
integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/3*(2*a*b*cos(d*x + c)^3 - ((a^2 - b^2)*cos(d*x + c)^2 + 2*a^2 + b^2)*si
n(d*x + c))/d
```

3.47.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.27

$$\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$$
$$= \begin{cases} \frac{2a^2\sin^3(c+dx)}{3d} + \frac{a^2\sin(c+dx)\cos^2(c+dx)}{d} - \frac{2ab\cos^3(c+dx)}{3d} + \frac{b^2\sin^3(c+dx)}{3d} & \text{for } d \neq 0 \\ x(a\cos(c)+b\sin(c))^2\cos(c) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

output
```
Piecewise((2*a**2*sin(c + d*x)**3/(3*d) + a**2*sin(c + d*x)*cos(c + d*x)**
2/d - 2*a*b*cos(c + d*x)**3/(3*d) + b**2*sin(c + d*x)**3/(3*d), Ne(d, 0)),
 (x*(a*cos(c) + b*sin(c))**2*cos(c), True))
```

3.47.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

$$\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$$
$$= -\frac{2\,ab\cos(dx+c)^3 - b^2\sin(dx+c)^3 + \left(\sin(dx+c)^3 - 3\,\sin(dx+c)\right)a^2}{3\,d}$$

input
```
integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/3*(2*a*b*cos(d*x + c)^3 - b^2*sin(d*x + c)^3 + (sin(d*x + c)^3 - 3*sin(
d*x + c))*a^2)/d
```

3.47.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.09

$$\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$$
$$= -\frac{ab\cos(3\,dx+3\,c)}{6\,d} - \frac{ab\cos(dx+c)}{2\,d} + \frac{(a^2-b^2)\sin(3\,dx+3\,c)}{12\,d} + \frac{(3\,a^2+b^2)\sin(dx+c)}{4\,d}$$

input
```
integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/6*a*b*cos(3*d*x + 3*c)/d - 1/2*a*b*cos(d*x + c)/d + 1/12*(a^2 - b^2)*si
n(3*d*x + 3*c)/d + 1/4*(3*a^2 + b^2)*sin(d*x + c)/d
```

3.47.9 Mupad [B] (verification not implemented)

Time = 21.75 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.15

$$\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$$
$$=\frac{2\left(\frac{\sin(c+d\,x)\,a^2\cos(c+d\,x)^2}{2}+\sin\left(c+d\,x\right)\,a^2-a\,b\cos\left(c+d\,x\right)^3-\frac{\sin(c+d\,x)\,b^2\cos(c+d\,x)^2}{2}+\frac{\sin(c+d\,x)\,b^2}{2}\right)}{3\,d}$$

input
```
int(cos(c + d*x)*(a*cos(c + d*x) + b*sin(c + d*x))^2,x)
```

output
```
(2*(a^2*sin(c + d*x) + (b^2*sin(c + d*x))/2 + (a^2*cos(c + d*x)^2*sin(c +
d*x))/2 - (b^2*cos(c + d*x)^2*sin(c + d*x))/2 - a*b*cos(c + d*x)^3))/(3*d)
```

3.48 $\int (a\cos(c+dx)+b\sin(c+dx))^2\,dx$

3.48.1 Optimal result

Integrand size = 19, antiderivative size = 55

$$\int (a\cos(c+dx)+b\sin(c+dx))^2\,dx = \frac{1}{2}\left(a^2+b^2\right)x-\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))}{2d}$$

output
```
1/2*(a^2+b^2)*x-1/2*(b*cos(d*x+c)-a*sin(d*x+c))*(a*cos(d*x+c)+b*sin(d*x+c)
)/d
```

3.48.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

$$\int (a\cos(c+dx)+b\sin(c+dx))^2\,dx = \frac{2\left(a^2+b^2\right)(c+dx)-2ab\cos(2(c+dx))+\left(a^2-b^2\right)\sin(2(c+dx))}{4d}$$

input
```
Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

output
```
(2*(a^2 + b^2)*(c + d*x) - 2*a*b*Cos[2*(c + d*x)] + (a^2 - b^2)*Sin[2*(c +
 d*x)])/(4*d)
```

3.48.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3042, 3552, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(c+dx)+b\sin(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\cos(c+dx)+b\sin(c+dx))^2dx$$

$$\Big\downarrow \text{3552}$$

$$\frac{1}{2}(a^2+b^2)\int 1dx-\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))}{2d}$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{2}x(a^2+b^2)-\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))}{2d}$$

input
```
Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

output
```
((a^2 + b^2)*x)/2 - ((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b
*Sin[c + d*x]))/(2*d)
```

3.48.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3552
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(-(b*Cos[c + d*x] - a*Sin[c + d*x]))*((a*Cos[c + d*x] + b*
Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[(n - 1)*((a^2 + b^2)/n)   Int[(a*Co
s[c + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

3.48.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.04

method	result
parallelrisch	$\frac{\sin(2dx+2c)(a^2-b^2)+2a^2xd+2b^2dx-2ab\cos(2dx+2c)+2ab}{4d}$
risch	$\frac{a^2x}{2}+\frac{x\,b^2}{2}-\frac{ab\cos(2dx+2c)}{2d}+\frac{\sin(2dx+2c)a^2}{4d}-\frac{\sin(2dx+2c)b^2}{4d}$
derivativedivides	$\frac{a^2\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-ab\cos(dx+c)^2+b^2\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
default	$\frac{a^2\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-ab\cos(dx+c)^2+b^2\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
parts	$\frac{a^2\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{b^2\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{ab\sin(dx+c)^2}{d}$
norman	$\frac{\left(\frac{a^2}{2}+\frac{b^2}{2}\right)x+\left(\frac{a^2}{2}+\frac{b^2}{2}\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\frac{\left(a^2-b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+(a^2+b^2)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-\frac{\left(a^2-b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}+4ab\tan\ldots}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}$

input
```
int((cos(d*x+c)*a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(sin(2*d*x+2*c)*(a^2-b^2)+2*a^2*x*d+2*b^2*d*x-2*a*b*cos(2*d*x+2*c)+2*a
*b)/d
```

3.48.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

$$\int (a\cos(c+dx)+b\sin(c+dx))^2\,dx = -\frac{2\,ab\cos\left(dx+c\right)^2-(a^2+b^2)dx-(a^2-b^2)\cos\left(dx+c\right)\sin\left(dx+c\right)}{2\,d}$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/2*(2*a*b*cos(d*x + c)^2 - (a^2 + b^2)*d*x - (a^2 - b^2)*cos(d*x + c)*si
n(d*x + c))/d
```

3.48.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(49) = 98$.

Time = 0.10 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.33

$$\int (a\cos(c+dx)+b\sin(c+dx))^2\,dx = \begin{cases} \frac{a^2x\sin^2(c+dx)}{2} + \frac{a^2x\cos^2(c+dx)}{2} + \frac{a^2\sin(c+dx)\cos(c+dx)}{2d} - \frac{ab\cos^2(c+dx)}{d} + \frac{b^2x\sin^2(c+dx)}{2} + \frac{b^2x\cos^2(c+dx)}{2} - \frac{b^2\sin(c+dx \\ x(a\cos(c)+b\sin(c))^2 \end{cases}$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

output
```
Piecewise((a**2*x*sin(c + d*x)**2/2 + a**2*x*cos(c + d*x)**2/2 + a**2*sin(
c + d*x)*cos(c + d*x)/(2*d) - a*b*cos(c + d*x)**2/d + b**2*x*sin(c + d*x)*
*2/2 + b**2*x*cos(c + d*x)**2/2 - b**2*sin(c + d*x)*cos(c + d*x)/(2*d), Ne
(d, 0)), (x*(a*cos(c) + b*sin(c))**2, True))
```

3.48.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.24

$$\int (a\cos(c+dx)+b\sin(c+dx))^2\,dx = -\frac{ab\cos(dx+c)^2}{d} + \frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))a^2}{4\,d} + \frac{(2\,dx+2\,c-\sin(2\,dx+2\,c))b^2}{4\,d}$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
-a*b*cos(d*x + c)^2/d + 1/4*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^2/d + 1/4*(
2*d*x + 2*c - sin(2*d*x + 2*c))*b^2/d
```

3.48.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int (a\cos(c+dx)+b\sin(c+dx))^2\,dx = \frac{1}{2}\left(a^2+b^2\right)x - \frac{ab\cos\left(2\,dx+2\,c\right)}{2\,d} + \frac{\left(a^2-b^2\right)\sin\left(2\,dx+2\,c\right)}{4\,d}$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
1/2*(a^2 + b^2)*x - 1/2*a*b*cos(2*d*x + 2*c)/d + 1/4*(a^2 - b^2)*sin(2*d*x
 + 2*c)/d
```

3.48.9 Mupad [B] (verification not implemented)

Time = 21.16 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.15

$$\int (a\cos(c+dx)+b\sin(c+dx))^2\,dx = \frac{a^2\,x}{2} + \frac{b^2\,x}{2} + \frac{a^2\,\sin\left(2\,c+2\,d\,x\right)}{4\,d} - \frac{b^2\,\sin\left(2\,c+2\,d\,x\right)}{4\,d} - \frac{a\,b\,\cos\left(2\,c+2\,d\,x\right)}{2\,d}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))^2,x)
```

output
```
(a^2*x)/2 + (b^2*x)/2 + (a^2*sin(2*c + 2*d*x))/(4*d) - (b^2*sin(2*c + 2*d*
x))/(4*d) - (a*b*cos(2*c + 2*d*x))/(2*d)
```

3.49 $\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$

3.49.1 Optimal result

Integrand size = 26, antiderivative size = 55

$$\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = \frac{b^2\operatorname{arctanh}(\sin(c+dx))}{d} - \frac{2ab\cos(c+dx)}{d} + \frac{a^2\sin(c+dx)}{d} - \frac{b^2\sin(c+dx)}{d}$$

output
```
b^2*arctanh(sin(d*x+c))/d-2*a*b*cos(d*x+c)/d+a^2*sin(d*x+c)/d-b^2*sin(d*x+
c)/d
```

3.49.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.53

$$\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = \frac{-2ab\cos(c+dx)+b^2\left(-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right.\right.}{d}$$

input
```
Integrate[Sec[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

output
```
(-2*a*b*Cos[c + d*x] + b^2*(-Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + Lo
g[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + (a^2 - b^2)*Sin[c + d*x])/d
```

3.49.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\cos(c+dx)+b\sin(c+dx))^2}{\cos(c+dx)}dx$$

$$\Big\downarrow \text{3569}$$

$$\int \left(a^2\cos(c+dx)+2ab\sin(c+dx)+b^2\sin(c+dx)\tan(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2\sin(c+dx)}{d}-\frac{2ab\cos(c+dx)}{d}+\frac{b^2\text{arctanh}(\sin(c+dx))}{d}-\frac{b^2\sin(c+dx)}{d}$$

input
```
Int[Sec[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

output
```
(b^2*ArcTanh[Sin[c + d*x]])/d - (2*a*b*Cos[c + d*x])/d + (a^2*Sin[c + d*x]
)/d - (b^2*Sin[c + d*x])/d
```

3.49.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

3.49.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{\sin(dx+c)a^2-2ab\cos(dx+c)+b^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))}{d}$
default	$\frac{\sin(dx+c)a^2-2ab\cos(dx+c)+b^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))}{d}$
parts	$\frac{a^2\sin(dx+c)}{d}+\frac{b^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))}{d}-\frac{2ab\cos(dx+c)}{d}$
parallelrisch	$\frac{-b^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+b^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\sin(dx+c)(a^2-b^2)-2ab(\cos(dx+c)+1)}{d}$
norman	$\frac{\frac{4ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{d}+\frac{2(a^2-b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2(a^2-b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}+\frac{4ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}+\frac{b^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{b^2\mathrm{l}}{}$
risch	$-\frac{e^{i(dx+c)}ba}{d}-\frac{ie^{i(dx+c)}a^2}{2d}+\frac{ie^{i(dx+c)}b^2}{2d}-\frac{e^{-i(dx+c)}ba}{d}+\frac{ie^{-i(dx+c)}a^2}{2d}-\frac{ie^{-i(dx+c)}b^2}{2d}+\frac{b^2\ln(i+e^{i(dx+c)})}{d}$

input
```
int(sec(d*x+c)*(cos(d*x+c)*a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(sin(d*x+c)*a^2-2*a*b*cos(d*x+c)+b^2*(-sin(d*x+c)+ln(sec(d*x+c)+tan(d*
x+c))))
```

3.49.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.13

$$\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = -\frac{4\,ab\cos(dx+c)-b^2\log(\sin(dx+c)+1)+b^2\log(-\sin(dx+c)+1)-2\,(a^2-b^2)\sin(dx+c)}{2\,d}$$

input
```
integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/2*(4*a*b*cos(d*x + c) - b^2*log(sin(d*x + c) + 1) + b^2*log(-sin(d*x +
c) + 1) - 2*(a^2 - b^2)*sin(d*x + c))/d
```

3.49.6 Sympy [F]

$$\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$$
$$=\int (a\cos{(c+dx)}+b\sin{(c+dx)})^2\sec{(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

output
```
Integral((a*cos(c + d*x) + b*sin(c + d*x))**2*sec(c + d*x), x)
```

3.49.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09

$$\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$$
$$=\frac{b^2(\log{(\sin{(dx+c)}+1)}-\log{(\sin{(dx+c)}-1)}-2\sin{(dx+c)})-4\,ab\cos{(dx+c)}+2\,a^2\sin{(dx+c)}}{2\,d}$$

input
```
integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/2*(b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*x + c))
- 4*a*b*cos(d*x + c) + 2*a^2*sin(d*x + c))/d
```

3.49.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.62

$$\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$$
$$=\frac{b^2\log{\left(\left|\tan{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+1\right|\right)}-b^2\log{\left(\left|\tan{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-1\right|\right)}+\frac{2\,\left(a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,ab\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}}{d}$$

input
```
integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
(b^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - b^2*log(abs(tan(1/2*d*x + 1/2*c)
 - 1)) + 2*(a^2*tan(1/2*d*x + 1/2*c) - b^2*tan(1/2*d*x + 1/2*c) - 2*a*b)/(
tan(1/2*d*x + 1/2*c)^2 + 1))/d
```

3.49.9 Mupad [B] (verification not implemented)

Time = 21.41 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.20

$$\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = \frac{2\,b^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} - \frac{4\,a\,b-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^2-2\,b^2\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))^2/cos(c + d*x),x)
```

output
```
(2*b^2*atanh(tan(c/2 + (d*x)/2)))/d - (4*a*b - tan(c/2 + (d*x)/2)*(2*a^2 -
 2*b^2))/(d*(tan(c/2 + (d*x)/2)^2 + 1))
```

3.50 $\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$

3.50.1 Optimal result

Integrand size = 28, antiderivative size = 39

$$\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = \left(a^2-b^2\right)x-\frac{2ab\log(\cos(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}$$

output
```
(a^2-b^2)*x-2*a*b*ln(cos(d*x+c))/d+b^2*tan(d*x+c)/d
```

3.50.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.77

$$\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = \frac{-i\left((a+ib)^2\log(i-\tan(c+dx))-(a-ib)^2\log(i+\tan(c+dx))\right)+2b^2\tan(c+dx)}{2d}$$

input
```
Integrate[Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

output
```
((-I)*((a + I*b)^2*Log[I - Tan[c + d*x]] - (a - I*b)^2*Log[I + Tan[c + d*x
]]) + 2*b^2*Tan[c + d*x])/(2*d)
```

3.50.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 3565, 3042, 3958, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\cos(c+dx)+b\sin(c+dx))^2}{\cos(c+dx)^2}dx$$

$$\Big\downarrow \text{3565}$$

$$\int (a+b\tan(c+dx))^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(c+dx))^2dx$$

$$\Big\downarrow \text{3958}$$

$$2ab\int \tan(c+dx)dx+x\left(a^2-b^2\right)+\frac{b^2\tan(c+dx)}{d}$$

$$\Big\downarrow \text{3042}$$

$$2ab\int \tan(c+dx)dx+x\left(a^2-b^2\right)+\frac{b^2\tan(c+dx)}{d}$$

$$\Big\downarrow \text{3956}$$

$$x\left(a^2-b^2\right)-\frac{2ab\log(\cos(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}$$

input
```
Int[Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

output
```
(a^2 - b^2)*x - (2*a*b*Log[Cos[c + d*x]])/d + (b^2*Tan[c + d*x])/d
```

3.50.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3565
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(a + b*Tan[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 3958
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Simp[b^2*(Tan[c + d*x]/d), x] + Simp[2*a*b   Int[Tan[c + d*x], x]
, x]) /; FreeQ[{a, b, c, d}, x]
```

3.50.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.13

method	result
derivativedivides	$\frac{a^{2}(dx+c)-2ab\ln(\cos(dx+c))+b^{2}(\tan(dx+c)-dx-c)}{d}$
default	$\frac{a^{2}(dx+c)-2ab\ln(\cos(dx+c))+b^{2}(\tan(dx+c)-dx-c)}{d}$
parts	$\frac{a^{2}(dx+c)}{d}+\frac{b^{2}(\tan(dx+c)-dx-c)}{d}+\frac{2ab\ln(\sec(dx+c))}{d}$
risch	$2iabx+a^{2}x-x\,b^{2}+\frac{4iabc}{d}+\frac{2ib^{2}}{d\left(e^{2i(dx+c)}+1\right)}-\frac{2ab\ln\left(e^{2i(dx+c)}+1\right)}{d}$
parallelrisch	$\frac{2ab\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)\cos(dx+c)-2ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)-2ab\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+dx(a-b)(c}{d\cos(dx+c)}$
norman	$\frac{(-a^{2}+b^{2})x+(-a^{2}+b^{2})x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+(a^{2}-b^{2})x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+(a^{2}-b^{2})x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}-\frac{2b^{2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{4b^{2}\tan(}{d}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{2}}$

input
```
int(sec(d*x+c)^2*(cos(d*x+c)*a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(d*x+c)-2*a*b*ln(cos(d*x+c))+b^2*(tan(d*x+c)-d*x-c))
```

3.50.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.54

$$\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = \frac{(a^2-b^2)dx\cos(dx+c)-2\,ab\cos(dx+c)\log(-\cos(dx+c))+b^2\sin(dx+c)}{d\cos(dx+c)}$$

input
```
integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas"
)
```

output
```
((a^2 - b^2)*d*x*cos(d*x + c) - 2*a*b*cos(d*x + c)*log(-cos(d*x + c)) + b^
2*sin(d*x + c))/(d*cos(d*x + c))
```

3.50.6 Sympy [F]

$$\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = \int (a\cos(c+dx)+b\sin(c+dx))^2\sec^2(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**2*(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

output
```
Integral((a*cos(c + d*x) + b*sin(c + d*x))**2*sec(c + d*x)**2, x)
```

3.50.7 Maxima [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26

$$\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = \frac{(dx+c)a^2-(dx+c-\tan(dx+c))b^2-ab\log\left(-\sin(dx+c)^2+1\right)}{d}$$

input
```
integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima"
)
```

output
```
((d*x + c)*a^2 - (d*x + c - tan(d*x + c))*b^2 - a*b*log(-sin(d*x + c)^2 +
1))/d
```

3.50.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.13

$$\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = \frac{ab\log\left(\tan(dx+c)^2+1\right)+b^2\tan(dx+c)+(a^2-b^2)(dx+c)}{d}$$

input
```
integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
(a*b*log(tan(d*x + c)^2 + 1) + b^2*tan(d*x + c) + (a^2 - b^2)*(d*x + c))/d
```

3.50.9 Mupad [B] (verification not implemented)

Time = 23.22 (sec) , antiderivative size = 118, normalized size of antiderivative = 3.03

$$\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = \frac{b^2\tan(c+d\,x)}{d}+\frac{2\,a^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{2\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} - \frac{2\,a\,b\,\ln\left(\frac{\cos(c+d\,x)}{\cos(c+d\,x)+1}\right)}{d}+\frac{2\,a\,b\,\ln\left(\frac{1}{{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}\right)}{d}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))^2/cos(c + d*x)^2,x)
```

output
```
(b^2*tan(c + d*x))/d + (2*a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))
/d - (2*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (2*a*b*log(co
s(c + d*x)/(cos(c + d*x) + 1)))/d + (2*a*b*log(1/cos(c/2 + (d*x)/2)^2))/d
```

3.50. $\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$

3.51 $\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$

3.51.1 Optimal result

Integrand size = 28, antiderivative size = 67

$$\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = \frac{a^2\text{arctanh}(\sin(c+dx))}{d}-\frac{b^2\text{arctanh}(\sin(c+dx))}{2d}+\frac{2ab\sec(c+dx)}{d}+\frac{b^2\sec(c+dx)\tan(c+dx)}{2d}$$

output
```
a^2*arctanh(sin(d*x+c))/d-1/2*b^2*arctanh(sin(d*x+c))/d+2*a*b*sec(d*x+c)/d
+1/2*b^2*sec(d*x+c)*tan(d*x+c)/d
```

3.51.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = \frac{a^2\text{arctanh}(\sin(c+dx))}{d}-\frac{b^2\text{arctanh}(\sin(c+dx))}{2d}+\frac{2ab\sec(c+dx)}{d}+\frac{b^2\sec(c+dx)\tan(c+dx)}{2d}$$

input
```
Integrate[Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

output
```
(a^2*ArcTanh[Sin[c + d*x]])/d - (b^2*ArcTanh[Sin[c + d*x]])/(2*d) + (2*a*b
*Sec[c + d*x])/d + (b^2*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

3.51.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\cos(c+dx)+b\sin(c+dx))^2}{\cos(c+dx)^3}dx$$

$$\Big\downarrow \text{3569}$$

$$\int \left(a^2\sec(c+dx)+2ab\tan(c+dx)\sec(c+dx)+b^2\tan^2(c+dx)\sec(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2\text{arctanh}(\sin(c+dx))}{d}+\frac{2ab\sec(c+dx)}{d}-\frac{b^2\text{arctanh}(\sin(c+dx))}{2d}+\frac{b^2\tan(c+dx)\sec(c+dx)}{2d}$$

input
```
Int[Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

output

```
(a^2*ArcTanh[Sin[c + d*x]])/d - (b^2*ArcTanh[Sin[c + d*x]])/(2*d) + (2*a*b
*Sec[c + d*x])/d + (b^2*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

3.51.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

3.51.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.24

method	result
derivativedivides	$\frac{a^2\ln(\sec(dx+c)+\tan(dx+c))+\frac{2ab}{\cos(dx+c)}+b^2\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
default	$\frac{a^2\ln(\sec(dx+c)+\tan(dx+c))+\frac{2ab}{\cos(dx+c)}+b^2\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
parts	$\frac{a^2\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{b^2\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}+\frac{2ab\sec(dx+c)}{d}$
parallelrisch	$\frac{-(1+\cos(2dx+2c))\left(a^2-\frac{b^2}{2}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+(1+\cos(2dx+2c))\left(a^2-\frac{b^2}{2}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+4\left(\cos(dx+c)a-\right.}{d(1+\cos(2dx+2c))}$
risch	$\frac{b\,e^{i(dx+c)}\left(-ib\,e^{2i(dx+c)}+4\,e^{2i(dx+c)}a+ib+4a\right)}{d\left(e^{2i(dx+c)}+1\right)^2}-\frac{\ln\left(e^{i(dx+c)}-i\right)a^2}{d}+\frac{b^2\ln\left(e^{i(dx+c)}-i\right)}{2d}+\frac{\ln\left(i+e^{i(dx+c)}\right)a^2}{d}-$
norman	$\frac{\frac{4ab}{d}+\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{d}+\frac{3b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}+\frac{3b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d}-\frac{4ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{d}+\frac{4ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d}-}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)^2\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}$

input
```
int(sec(d*x+c)^3*(cos(d*x+c)*a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*ln(sec(d*x+c)+tan(d*x+c))+2*a*b/cos(d*x+c)+b^2*(1/2*sin(d*x+c)^3/
cos(d*x+c)^2+1/2*sin(d*x+c)-1/2*ln(sec(d*x+c)+tan(d*x+c))))
```

3.51.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.43

$$\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$$
$$=\frac{(2\,a^2-b^2)\cos{(dx+c)}^2\log{(\sin{(dx+c)}+1)}-(2\,a^2-b^2)\cos{(dx+c)}^2\log{(-\sin{(dx+c)}+1)}+8\,ab\cos}{4\,d\cos{(dx+c)}^2}$$

input
```
integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas"
)
```

output
```
1/4*((2*a^2 - b^2)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (2*a^2 - b^2)*co
s(d*x + c)^2*log(-sin(d*x + c) + 1) + 8*a*b*cos(d*x + c) + 2*b^2*sin(d*x +
 c))/(d*cos(d*x + c)^2)
```

3.51.6 Sympy [F]

$$\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$$
$$=\int (a\cos{(c+dx)}+b\sin{(c+dx)})^2\sec^3{(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**3*(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

output
```
Integral((a*cos(c + d*x) + b*sin(c + d*x))**2*sec(c + d*x)**3, x)
```

3.51.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.33

$$\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx =$$
$$-\frac{b^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}+\log{(\sin{(dx+c)}+1)}-\log{(\sin{(dx+c)}-1)}\right)-2\,a^2(\log{(\sin{(dx+c)}+1)}-\log{(\sin}}{4\,d}$$

input
```
integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima"
)
```

output
```
-1/4*(b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - l
og(sin(d*x + c) - 1)) - 2*a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) -
1)) - 8*a*b/cos(d*x + c))/d
```

3.51.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.82

$$\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$$
$$=\frac{(2\,a^2-b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-(2\,a^2-b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,ab\tan}{\left(\tan\left(\frac{1}{2}\right.\right.}}{2\,d}$$

input
```
integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
1/2*((2*a^2 - b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (2*a^2 - b^2)*log(
abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(b^2*tan(1/2*d*x + 1/2*c)^3 - 4*a*b*tan
(1/2*d*x + 1/2*c)^2 + b^2*tan(1/2*d*x + 1/2*c) + 4*a*b)/(tan(1/2*d*x + 1/2
*c)^2 - 1)^2)/d
```

3.51.9 Mupad [B] (verification not implemented)

Time = 22.22 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.58

$$\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$$
$$=\frac{b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+4\,a\,b}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$
$$+\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(2\,a^2-b^2\right)}{d}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))^2/cos(c + d*x)^3,x)
```

output
```
(4*a*b + b^2*tan(c/2 + (d*x)/2)^3 + b^2*tan(c/2 + (d*x)/2) - 4*a*b*tan(c/2
 + (d*x)/2)^2)/(d*(tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^2 + 1)) + (
atanh(tan(c/2 + (d*x)/2))*(2*a^2 - b^2))/d
```

3.51. $\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$

3.52 $\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$

3.52.1 Optimal result

Integrand size = 28, antiderivative size = 30

$$\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx=\frac{(b+a\cot(c+dx))^3\tan^3(c+dx)}{3bd}$$

output
```
1/3*(b+a*cot(d*x+c))^3*tan(d*x+c)^3/b/d
```

3.52.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.53

$$\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$$
$$=\frac{a^2\tan(c+dx)}{d}+\frac{ab\tan^2(c+dx)}{d}+\frac{b^2\tan^3(c+dx)}{3d}$$

input
```
Integrate[Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

output
```
(a^2*Tan[c + d*x])/d + (a*b*Tan[c + d*x]^2)/d + (b^2*Tan[c + d*x]^3)/(3*d)
```

3.52.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3042, 3567, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(a\cos(c+dx)+b\sin(c+dx))^2}{\cos(c+dx)^4}dx$$

$$\Big\downarrow \ 3567$$

$$-\frac{\int (b+a\cot(c+dx))^2\tan^4(c+dx)d\cot(c+dx)}{d}$$

$$\Big\downarrow \ 48$$

$$\frac{\tan^3(c+dx)(a\cot(c+dx)+b)^3}{3bd}$$

input
```
Int[Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

output
```
((b + a*Cot[c + d*x])^3*Tan[c + d*x]^3)/(3*b*d)
```

3.52.3.1 Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3567
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[x^m*((b
+ a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a,
b, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[
n, 0] && GtQ[m, 1])
```

3.52.4 Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.60

method	result
derivativedivides	$\frac{a^2\tan(dx+c)+\frac{ab}{\cos(dx+c)^2}+\frac{b^2\sin(dx+c)^3}{3\cos(dx+c)^3}}{d}$
default	$\frac{a^2\tan(dx+c)+\frac{ab}{\cos(dx+c)^2}+\frac{b^2\sin(dx+c)^3}{3\cos(dx+c)^3}}{d}$
parts	$\frac{a^2\tan(dx+c)}{d}+\frac{b^2\sin(dx+c)^3}{3d\cos(dx+c)^3}+\frac{ab\sec(dx+c)^2}{d}$
risch	$-\frac{2i\left(6iab\,e^{4i(dx+c)}-3a^2e^{4i(dx+c)}+3b^2e^{4i(dx+c)}+6iab\,e^{2i(dx+c)}-6a^2e^{2i(dx+c)}-3a^2+b^2\right)}{3d\left(e^{2i(dx+c)}+1\right)^3}$
parallelrisch	$-\frac{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4a^2-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3ab+\left(-2a^2+\frac{4b^2}{3}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)ab+a^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}$
norman	$\frac{\frac{4ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{d}-\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{d}-\frac{8b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}-\frac{8b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{3d}+\frac{4(3a^2-4b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{3d}-}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)^3\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}$

input
```
int(sec(d*x+c)^4*(cos(d*x+c)*a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*tan(d*x+c)+a*b/cos(d*x+c)^2+1/3*b^2*sin(d*x+c)^3/cos(d*x+c)^3)
```

3.52.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.83

$$\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = \frac{3\,ab\cos(dx+c)+\left((3\,a^2-b^2)\cos(dx+c)^2+b^2\right)\sin(dx+c)}{3\,d\cos(dx+c)^3}$$

input
```
integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas"
)
```

output
```
1/3*(3*a*b*cos(d*x + c) + ((3*a^2 - b^2)*cos(d*x + c)^2 + b^2)*sin(d*x + c
))/(d*cos(d*x + c)^3)
```

3.52.6 Sympy [F]

$$\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$$
$$=\int (a\cos{(c+dx)}+b\sin{(c+dx)})^2\sec^4{(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**4*(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

output
```
Integral((a*cos(c + d*x) + b*sin(c + d*x))**2*sec(c + d*x)**4, x)
```

3.52.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.50

$$\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$$
$$=\frac{b^2\tan{(dx+c)}^3+3\,a^2\tan{(dx+c)}-\frac{3\,ab}{\sin(dx+c)^2-1}}{3\,d}$$

input
```
integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima"
)
```

output
```
1/3*(b^2*tan(d*x + c)^3 + 3*a^2*tan(d*x + c) - 3*a*b/(sin(d*x + c)^2 - 1))
/d
```

3.52.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.37

$$\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = \frac{b^2\tan(dx+c)^3+3\,ab\tan(dx+c)^2+3\,a^2\tan(dx+c)}{3\,d}$$

input
```
integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
1/3*(b^2*tan(d*x + c)^3 + 3*a*b*tan(d*x + c)^2 + 3*a^2*tan(d*x + c))/d
```

3.52.9 Mupad [B] (verification not implemented)

Time = 21.59 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.27

$$\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = \frac{\frac{b^2\sin(c+d\,x)}{3}+\frac{\cos(c+d\,x)^2\sin(c+d\,x)\,(3\,a^2-b^2)}{3}+a\,b\cos(c+d\,x)\sin(c+d\,x)^2}{d\cos(c+d\,x)^3}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))^2/cos(c + d*x)^4,x)
```

output
```
((b^2*sin(c + d*x))/3 + (cos(c + d*x)^2*sin(c + d*x)*(3*a^2 - b^2))/3 + a*
b*cos(c + d*x)*sin(c + d*x)^2)/(d*cos(c + d*x)^3)
```

3.53 $\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$

3.53.1 Optimal result

Integrand size = 28, antiderivative size = 120

$$\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = \frac{a^2\operatorname{arctanh}(\sin(c+dx))}{2d}-\frac{b^2\operatorname{arctanh}(\sin(c+dx))}{8d}+\frac{2ab\sec^3(c+dx)}{3d}+\frac{a^2\sec(c+dx)\tan(c+dx)}{2d}-\frac{b^2\sec(c+dx)\tan(c+dx)}{8d}+\frac{b^2\sec^3(c+dx)\tan(c+dx)}{4d}$$

output
```
1/2*a^2*arctanh(sin(d*x+c))/d-1/8*b^2*arctanh(sin(d*x+c))/d+2/3*a*b*sec(d*
x+c)^3/d+1/2*a^2*sec(d*x+c)*tan(d*x+c)/d-1/8*b^2*sec(d*x+c)*tan(d*x+c)/d+1
/4*b^2*sec(d*x+c)^3*tan(d*x+c)/d
```

3.53.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00

$$\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = \frac{a^2\operatorname{arctanh}(\sin(c+dx))}{2d}-\frac{b^2\operatorname{arctanh}(\sin(c+dx))}{8d}+\frac{2ab\sec^3(c+dx)}{3d}+\frac{a^2\sec(c+dx)\tan(c+dx)}{2d}-\frac{b^2\sec(c+dx)\tan(c+dx)}{8d}+\frac{b^2\sec^3(c+dx)\tan(c+dx)}{4d}$$

input
```
Integrate[Sec[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

output
```
(a^2*ArcTanh[Sin[c + d*x]])/(2*d) - (b^2*ArcTanh[Sin[c + d*x]])/(8*d) + (2
*a*b*Sec[c + d*x]^3)/(3*d) + (a^2*Sec[c + d*x]*Tan[c + d*x])/(2*d) - (b^2*
Sec[c + d*x]*Tan[c + d*x])/(8*d) + (b^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

3.53.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\cos(c+dx)+b\sin(c+dx))^2}{\cos(c+dx)^5}dx$$

$$\Big\downarrow \text{3569}$$

$$\int \left(a^2\sec^3(c+dx)+2ab\tan(c+dx)\sec^3(c+dx)+b^2\tan^2(c+dx)\sec^3(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2\text{arctanh}(\sin(c+dx))}{2d}+\frac{a^2\tan(c+dx)\sec(c+dx)}{2d}+\frac{2ab\sec^3(c+dx)}{3d}-\frac{b^2\text{arctanh}(\sin(c+dx))}{8d}+\frac{b^2\tan(c+dx)\sec^3(c+dx)}{4d}-\frac{b^2\tan(c+dx)\sec(c+dx)}{8d}$$

input
```
Int[Sec[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

output
```
(a^2*ArcTanh[Sin[c + d*x]])/(2*d) - (b^2*ArcTanh[Sin[c + d*x]])/(8*d) + (2
*a*b*Sec[c + d*x]^3)/(3*d) + (a^2*Sec[c + d*x]*Tan[c + d*x])/(2*d) - (b^2*
Sec[c + d*x]*Tan[c + d*x])/(8*d) + (b^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

3.53.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

3.53.4 Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{2ab}{3\cos(dx+c)^3}+b^2\left(\frac{\sin(dx+c)^3}{4\cos(dx+c)^4}+\frac{\sin(dx+c)^3}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx+}{}\right.}{d}$
default	$\frac{a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{2ab}{3\cos(dx+c)^3}+b^2\left(\frac{\sin(dx+c)^3}{4\cos(dx+c)^4}+\frac{\sin(dx+c)^3}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx+}{}\right.}{d}$
parts	$\frac{a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}+\frac{b^2\left(\frac{\sin(dx+c)^3}{4\cos(dx+c)^4}+\frac{\sin(dx+c)^3}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx+c)+\tan(dx+}{8}\right.}{d}$
parallelrisch	$\frac{-48\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\cos(2dx+2c)\right)\left(a-\frac{b}{2}\right)\left(a+\frac{b}{2}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+48\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\cos(2dx+2c)\right)\left(a-\frac{b}{2}\right)\left(\right.}{24d(3+}$
risch	$-\frac{ie^{i(dx+c)}\left(12a^2e^{6i(dx+c)}-3b^2e^{6i(dx+c)}+12a^2e^{4i(dx+c)}+21b^2e^{4i(dx+c)}+64iab\,e^{4i(dx+c)}-12a^2e^{2i(dx+c)}-21b^2e^{2i(dx+}\right.}{12d\left(e^{2i(dx+c)}+1\right)^4}$
norman	$\frac{\frac{4ab}{3d}-\frac{(4a^2-11b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{2d}-\frac{(4a^2-11b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{2d}+\frac{(4a^2+b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{(4a^2+b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{4d}+\frac{(4a^2+9b^2)}{}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right.}$

input
```
int(sec(d*x+c)^5*(cos(d*x+c)*a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+2/3*a*b
/cos(d*x+c)^3+b^2*(1/4*sin(d*x+c)^3/cos(d*x+c)^4+1/8*sin(d*x+c)^3/cos(d*x+
c)^2+1/8*sin(d*x+c)-1/8*ln(sec(d*x+c)+tan(d*x+c))))
```

3.53.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00

$$\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$$
$$= \frac{3\,(4\,a^2-b^2)\cos{(dx+c)}^4\log{(\sin{(dx+c)}+1)}-3\,(4\,a^2-b^2)\cos{(dx+c)}^4\log{(-\sin{(dx+c)}+1)}+32\,a}{48\,d\cos{(dx+c)}^4}$$

input
```
integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas"
)
```

output
```
1/48*(3*(4*a^2 - b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(4*a^2 - b^
2)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 32*a*b*cos(d*x + c) + 6*((4*a^2
 - b^2)*cos(d*x + c)^2 + 2*b^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

3.53.6 Sympy [F(-1)]

Timed out.

$$\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**5*(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

output
```
Timed out
```

3.53.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.08

$$\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$$
$$= \frac{3\,b^2\left(\frac{2\left(\sin(dx+c)^3+\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-\log{(\sin{(dx+c)}+1)}+\log{(\sin{(dx+c)}-1)}\right)-12\,a^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log{(s}\right.}{48\,d}$$

input
```
integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima"
)
```

output
```
1/48*(3*b^2*(2*(sin(d*x + c)^3 + sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x
 + c)^2 + 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 12*a^2*(2*
sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x +
c) - 1)) + 32*a*b/cos(d*x + c)^3)/d
```

3.53.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(108) = 216$.

Time = 0.33 (sec) , antiderivative size = 249, normalized size of antiderivative = 2.08

$$\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$$

$$= \frac{3\,(4\,a^2-b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(4\,a^2-b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(12\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+\right.}{}}{}$$

input
```
integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
1/24*(3*(4*a^2 - b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(4*a^2 - b^2)
*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(12*a^2*tan(1/2*d*x + 1/2*c)^7 + 3
*b^2*tan(1/2*d*x + 1/2*c)^7 - 48*a*b*tan(1/2*d*x + 1/2*c)^6 - 12*a^2*tan(1
/2*d*x + 1/2*c)^5 + 21*b^2*tan(1/2*d*x + 1/2*c)^5 + 48*a*b*tan(1/2*d*x + 1
/2*c)^4 - 12*a^2*tan(1/2*d*x + 1/2*c)^3 + 21*b^2*tan(1/2*d*x + 1/2*c)^3 -
16*a*b*tan(1/2*d*x + 1/2*c)^2 + 12*a^2*tan(1/2*d*x + 1/2*c) + 3*b^2*tan(1/
2*d*x + 1/2*c) + 16*a*b)/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.53.9 Mupad [B] (verification not implemented)

Time = 24.79 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.80

$$\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$$

$$= \frac{\left(a^2+\frac{b^2}{4}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\left(\frac{7\,b^2}{4}-a^2\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\left(\frac{7\,b^2}{4}-\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\right.}$$

$$+\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(a^2-\frac{b^2}{4}\right)}{d}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))^2/cos(c + d*x)^5,x)
```

output
```
((4*a*b)/3 + tan(c/2 + (d*x)/2)*(a^2 + b^2/4) + tan(c/2 + (d*x)/2)^7*(a^2
+ b^2/4) - tan(c/2 + (d*x)/2)^3*(a^2 - (7*b^2)/4) - tan(c/2 + (d*x)/2)^5*(
a^2 - (7*b^2)/4) - (4*a*b*tan(c/2 + (d*x)/2)^2)/3 + 4*a*b*tan(c/2 + (d*x)/
2)^4 - 4*a*b*tan(c/2 + (d*x)/2)^6)/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2
+ (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) + (atan
h(tan(c/2 + (d*x)/2))*(a^2 - b^2/4))/d
```

3.54 $\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$

3.54.1 Optimal result

Integrand size = 28, antiderivative size = 85

$$\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = \frac{a^2\tan(c+dx)}{d}+\frac{ab\tan^2(c+dx)}{d}+\frac{(a^2+b^2)\tan^3(c+dx)}{3d}+\frac{ab\tan^4(c+dx)}{2d}+\frac{b^2\tan^5(c+dx)}{5d}$$

output
```
a^2*tan(d*x+c)/d+a*b*tan(d*x+c)^2/d+1/3*(a^2+b^2)*tan(d*x+c)^3/d+1/2*a*b*t
an(d*x+c)^4/d+1/5*b^2*tan(d*x+c)^5/d
```

3.54.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.64

$$\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = \frac{(a+b\tan(c+dx))^3\left(a^2+10b^2-3ab\tan(c+dx)+6b^2\tan^2(c+dx)\right)}{30b^3d}$$

input
```
Integrate[Sec[c + d*x]^6*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

output
```
((a + b*Tan[c + d*x])^3*(a^2 + 10*b^2 - 3*a*b*Tan[c + d*x] + 6*b^2*Tan[c +
 d*x]^2))/(30*b^3*d)
```

3.54.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3567, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\cos(c+dx)+b\sin(c+dx))^2}{\cos(c+dx)^6}dx$$

$$\Big\downarrow \text{3567}$$

$$-\frac{\int (b+a\cot(c+dx))^2\left(\cot^2(c+dx)+1\right)\tan^6(c+dx)d\cot(c+dx)}{d}$$

$$\Big\downarrow \text{522}$$

$$-\frac{\int \left(b^2\tan^6(c+dx)+2ab\tan^5(c+dx)+\left(a^2+b^2\right)\tan^4(c+dx)+2ab\tan^3(c+dx)+a^2\tan^2(c+dx)\right)d\cot(c+}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{1}{3}\left(a^2+b^2\right)\tan^3(c+dx)-a^2\tan(c+dx)-\frac{1}{2}ab\tan^4(c+dx)-ab\tan^2(c+dx)-\frac{1}{5}b^2\tan^5(c+dx)}{d}$$

input
```
Int[Sec[c + d*x]^6*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

output
```
-((-(a^2*Tan[c + d*x]) - a*b*Tan[c + d*x]^2 - ((a^2 + b^2)*Tan[c + d*x]^3)
/3 - (a*b*Tan[c + d*x]^4)/2 - (b^2*Tan[c + d*x]^5)/5)/d)
```

3.54.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3567
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[x^m*((b
+ a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a,
b, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[
n, 0] && GtQ[m, 1])
```

3.54.4 Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{-a^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+\frac{ab}{2\cos(dx+c)^4}+b^2\left(\frac{\sin(dx+c)^3}{5\cos(dx+c)^5}+\frac{2\sin(dx+c)^3}{15\cos(dx+c)^3}\right)}{d}$
default	$\frac{-a^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+\frac{ab}{2\cos(dx+c)^4}+b^2\left(\frac{\sin(dx+c)^3}{5\cos(dx+c)^5}+\frac{2\sin(dx+c)^3}{15\cos(dx+c)^3}\right)}{d}$
parts	$-\frac{a^2\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}+\frac{b^2\left(\frac{\sin(dx+c)^3}{5\cos(dx+c)^5}+\frac{2\sin(dx+c)^3}{15\cos(dx+c)^3}\right)}{d}+\frac{ab\sec(dx+c)^4}{2d}$
risch	$\frac{4i\left(-30iab\,e^{6i(dx+c)}+15a^2e^{6i(dx+c)}-15b^2e^{6i(dx+c)}-30iab\,e^{4i(dx+c)}+35a^2e^{4i(dx+c)}+5b^2e^{4i(dx+c)}+25a^2e^{2i(dx+c)}-5b^2 \ldots\right.}{15d\left(e^{2i(dx+c)}+1\right)^5}$
parallelrisch	$-\frac{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8a^2-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7ab+\frac{4\left(-2a^2+b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{3}+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5ab+\frac{2\left(5a^2+\frac{4b^2}{5}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{3}-2 \ldots\right.}{d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1 \ldots\right.}$
norman	$\frac{\frac{4ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{d}+\frac{4ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{d}-\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{13}}{d}+\frac{4\left(a^2-2b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}+\frac{4\left(a^2-2b^2\right)\tan\left(\frac{dx}{2} \ldots\right.}{3d}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2- \ldots\right.}$

input
```
int(sec(d*x+c)^6*(cos(d*x+c)*a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a^2*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+1/2*a*b/cos(d*x+c)^4+b^2*(1/5
*sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c)^3/cos(d*x+c)^3))
```

3.54.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.93

$$\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = \frac{15\,ab\cos(dx+c)+2\left(2\left(5\,a^2-b^2\right)\cos(dx+c)^4+\left(5\,a^2-b^2\right)\cos(dx+c)^2+3\,b^2\right)\sin(dx+c)}{30\,d\cos(dx+c)^5}$$

input
```
integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas"
)
```

output
```
1/30*(15*a*b*cos(d*x + c) + 2*(2*(5*a^2 - b^2)*cos(d*x + c)^4 + (5*a^2 - b
^2)*cos(d*x + c)^2 + 3*b^2)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

3.54.6 Sympy [F(-1)]

Timed out.

$$\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**6*(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

output
```
Timed out
```

3.54.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.82

$$\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = \frac{10\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)a^2+2\left(3\,\tan\left(dx+c\right)^5+5\,\tan\left(dx+c\right)^3\right)b^2+\frac{15\,ab}{\left(\sin(dx+c)^2-1\right)^2}}{30\,d}$$

input
```
integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima"
)
```

output
```
1/30*(10*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2 + 2*(3*tan(d*x + c)^5 + 5*t
an(d*x + c)^3)*b^2 + 15*a*b/(sin(d*x + c)^2 - 1)^2)/d
```

3.54.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.94

$$\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = \frac{6\,b^2\tan\left(dx+c\right)^5+15\,ab\tan\left(dx+c\right)^4+10\,a^2\tan\left(dx+c\right)^3+10\,b^2\tan\left(dx+c\right)^3+30\,ab\tan\left(dx+c\right)^2+}{30\,d}$$

input
```
integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
1/30*(6*b^2*tan(d*x + c)^5 + 15*a*b*tan(d*x + c)^4 + 10*a^2*tan(d*x + c)^3
 + 10*b^2*tan(d*x + c)^3 + 30*a*b*tan(d*x + c)^2 + 30*a^2*tan(d*x + c))/d
```

3.54.9 Mupad [B] (verification not implemented)

Time = 22.48 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.15

$$\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = \frac{\frac{b^2\,\sin(c+d\,x)}{5}+\cos\left(c+d\,x\right)^2\left(\frac{a^2\,\sin(c+d\,x)}{3}-\frac{b^2\,\sin(c+d\,x)}{15}\right)+\cos\left(c+d\,x\right)^4\left(\frac{2\,a^2\,\sin(c+d\,x)}{3}-\frac{2\,b^2\,\sin(c+d\,x)}{15}\right)+\frac{a\,b\,c}{}}{d\cos\left(c+d\,x\right)^5}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))^2/cos(c + d*x)^6,x)
```

output
```
((b^2*sin(c + d*x))/5 + cos(c + d*x)^2*((a^2*sin(c + d*x))/3 - (b^2*sin(c
+ d*x))/15) + cos(c + d*x)^4*((2*a^2*sin(c + d*x))/3 - (2*b^2*sin(c + d*x)
)/15) + (a*b*cos(c + d*x))/2)/(d*cos(c + d*x)^5)
```

3.54. $\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$

3.55 $\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$

3.55.1 Optimal result

Integrand size = 28, antiderivative size = 168

$$\begin{aligned}&\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx\\&=\frac{3a^2\text{arctanh}(\sin(c+dx))}{8d}-\frac{b^2\text{arctanh}(\sin(c+dx))}{16d}+\frac{2ab\sec^5(c+dx)}{5d}\\&\quad+\frac{3a^2\sec(c+dx)\tan(c+dx)}{8d}-\frac{b^2\sec(c+dx)\tan(c+dx)}{16d}+\frac{a^2\sec^3(c+dx)\tan(c+dx)}{4d}\\&\quad-\frac{b^2\sec^3(c+dx)\tan(c+dx)}{24d}+\frac{b^2\sec^5(c+dx)\tan(c+dx)}{6d}\end{aligned}$$

output
```
3/8*a^2*arctanh(sin(d*x+c))/d-1/16*b^2*arctanh(sin(d*x+c))/d+2/5*a*b*sec(d
*x+c)^5/d+3/8*a^2*sec(d*x+c)*tan(d*x+c)/d-1/16*b^2*sec(d*x+c)*tan(d*x+c)/d
+1/4*a^2*sec(d*x+c)^3*tan(d*x+c)/d-1/24*b^2*sec(d*x+c)^3*tan(d*x+c)/d+1/6*
b^2*sec(d*x+c)^5*tan(d*x+c)/d
```

3.55.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00

$$\begin{aligned}&\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx\\&=\frac{3a^2\text{arctanh}(\sin(c+dx))}{8d}-\frac{b^2\text{arctanh}(\sin(c+dx))}{16d}+\frac{2ab\sec^5(c+dx)}{5d}\\&\quad+\frac{3a^2\sec(c+dx)\tan(c+dx)}{8d}-\frac{b^2\sec(c+dx)\tan(c+dx)}{16d}+\frac{a^2\sec^3(c+dx)\tan(c+dx)}{4d}\\&\quad-\frac{b^2\sec^3(c+dx)\tan(c+dx)}{24d}+\frac{b^2\sec^5(c+dx)\tan(c+dx)}{6d}\end{aligned}$$

input
```
Integrate[Sec[c + d*x]^7*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

output
```
(3*a^2*ArcTanh[Sin[c + d*x]])/(8*d) - (b^2*ArcTanh[Sin[c + d*x]])/(16*d) +
 (2*a*b*Sec[c + d*x]^5)/(5*d) + (3*a^2*Sec[c + d*x]*Tan[c + d*x])/(8*d) -
(b^2*Sec[c + d*x]*Tan[c + d*x])/(16*d) + (a^2*Sec[c + d*x]^3*Tan[c + d*x])
/(4*d) - (b^2*Sec[c + d*x]^3*Tan[c + d*x])/(24*d) + (b^2*Sec[c + d*x]^5*Ta
n[c + d*x])/(6*d)
```

3.55.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\cos(c+dx)+b\sin(c+dx))^2}{\cos(c+dx)^7}dx$$

$$\Big\downarrow \text{3569}$$

$$\int \left(a^2\sec^5(c+dx)+2ab\tan(c+dx)\sec^5(c+dx)+b^2\tan^2(c+dx)\sec^5(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3a^2\text{arctanh}(\sin(c+dx))}{8d}+\frac{a^2\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3a^2\tan(c+dx)\sec(c+dx)}{8d}+\frac{2ab\sec^5(c+dx)}{5d}-\frac{b^2\text{arctanh}(\sin(c+dx))}{16d}+\frac{b^2\tan(c+dx)\sec^5(c+dx)}{6d}-\frac{b^2\tan(c+dx)\sec^3(c+dx)}{24d}-\frac{b^2\tan(c+dx)\sec(c+dx)}{16d}$$

input
```
Int[Sec[c + d*x]^7*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

output
```
(3*a^2*ArcTanh[Sin[c + d*x]])/(8*d) - (b^2*ArcTanh[Sin[c + d*x]])/(16*d) +
 (2*a*b*Sec[c + d*x]^5)/(5*d) + (3*a^2*Sec[c + d*x]*Tan[c + d*x])/(8*d) -
(b^2*Sec[c + d*x]*Tan[c + d*x])/(16*d) + (a^2*Sec[c + d*x]^3*Tan[c + d*x])
/(4*d) - (b^2*Sec[c + d*x]^3*Tan[c + d*x])/(24*d) + (b^2*Sec[c + d*x]^5*Ta
n[c + d*x])/(6*d)
```

3.55.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

3.55.4 Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{a^2\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{2ab}{5\cos(dx+c)^5}+b^2\left(\frac{\sin(dx+c)^3}{6\cos(dx+c)^6}+\frac{\sin(dx+c)^3}{8\cos(dx+c)}\right.}{d}$
default	$\frac{a^2\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{2ab}{5\cos(dx+c)^5}+b^2\left(\frac{\sin(dx+c)^3}{6\cos(dx+c)^6}+\frac{\sin(dx+c)^3}{8\cos(dx+c)}\right.}{d}$
parts	$\frac{a^2\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}+\frac{b^2\left(\frac{\sin(dx+c)^3}{6\cos(dx+c)^6}+\frac{\sin(dx+c)^3}{8\cos(dx+c)^4}+\frac{\sin(dx}{16\cos(d}\right.}{}$
parallelrisch	$\frac{-1350\left(\frac{\cos(6dx+6c)}{15}+\frac{2\cos(4dx+4c)}{5}+\cos(2dx+2c)+\frac{2}{3}\right)\left(a^2-\frac{b^2}{6}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+1350\left(\frac{\cos(6dx+6c)}{15}+\frac{2\cos(4dx+4c)}{5}\right.}{}$
risch	$-\frac{ie^{i(dx+c)}\left(90a^2e^{10i(dx+c)}-15b^2e^{10i(dx+c)}+510a^2e^{8i(dx+c)}-85b^2e^{8i(dx+c)}+420a^2e^{6i(dx+c)}+570b^2e^{6i(dx+c)}+1536ia\right.}{120d\left(e^{2i(dx+}\right.}$
norman	$\frac{4ab}{5d}-\frac{\left(6a^2-281b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{24d}-\frac{\left(6a^2-281b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{24d}-\frac{7\left(6a^2-25b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{24d}-\frac{7\left(6a^2-25b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{24d}+($

input
```
int(sec(d*x+c)^7*(cos(d*x+c)*a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)
+tan(d*x+c)))+2/5*a*b/cos(d*x+c)^5+b^2*(1/6*sin(d*x+c)^3/cos(d*x+c)^6+1/8*
sin(d*x+c)^3/cos(d*x+c)^4+1/16*sin(d*x+c)^3/cos(d*x+c)^2+1/16*sin(d*x+c)-1
/16*ln(sec(d*x+c)+tan(d*x+c))))
```

3.55.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.85

$$\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = \frac{15\,(6\,a^2-b^2)\cos{(dx+c)}^6\log{(\sin{(dx+c)}+1)}-15\,(6\,a^2-b^2)\cos{(dx+c)}^6\log{(-\sin{(dx+c)}+1)}+1}{480\,d\cos{(dx}}$$

input
```
integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas"
)
```

output
```
1/480*(15*(6*a^2 - b^2)*cos(d*x + c)^6*log(sin(d*x + c) + 1) - 15*(6*a^2 -
 b^2)*cos(d*x + c)^6*log(-sin(d*x + c) + 1) + 192*a*b*cos(d*x + c) + 10*(3
*(6*a^2 - b^2)*cos(d*x + c)^4 + 2*(6*a^2 - b^2)*cos(d*x + c)^2 + 8*b^2)*si
n(d*x + c))/(d*cos(d*x + c)^6)
```

3.55.6 Sympy [F(-1)]

Timed out.

$$\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**7*(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

output
```
Timed out
```

3.55.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.07

$$\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$$
$$= \frac{5\,b^2\left(\frac{2\left(3\,\sin(dx+c)^5-8\,\sin(dx+c)^3-3\,\sin(dx+c)\right)}{\sin(dx+c)^6-3\,\sin(dx+c)^4+3\,\sin(dx+c)^2-1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)-30\,a^2\left(\frac{2\left(\right.}{\mathrm{si}}\right.}{480\,d}$$

input
```
integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima"
)
```

output
```
1/480*(5*b^2*(2*(3*sin(d*x + c)^5 - 8*sin(d*x + c)^3 - 3*sin(d*x + c))/(si
n(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 3*log(sin(d*x +
c) + 1) + 3*log(sin(d*x + c) - 1)) - 30*a^2*(2*(3*sin(d*x + c)^3 - 5*sin(d
*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1)
 + 3*log(sin(d*x + c) - 1)) + 192*a*b/cos(d*x + c)^5)/d
```

3.55.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. 2(152) = 304.

Time = 0.34 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.04

$$\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$$
$$= \frac{15\,(6\,a^2-b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,(6\,a^2-b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(150\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.}{}}{}$$

input
```
integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
1/240*(15*(6*a^2 - b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(6*a^2 - b
^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(150*a^2*tan(1/2*d*x + 1/2*c)^1
1 + 15*b^2*tan(1/2*d*x + 1/2*c)^11 - 480*a*b*tan(1/2*d*x + 1/2*c)^10 - 210
*a^2*tan(1/2*d*x + 1/2*c)^9 + 235*b^2*tan(1/2*d*x + 1/2*c)^9 + 480*a*b*tan
(1/2*d*x + 1/2*c)^8 + 60*a^2*tan(1/2*d*x + 1/2*c)^7 + 390*b^2*tan(1/2*d*x
+ 1/2*c)^7 - 960*a*b*tan(1/2*d*x + 1/2*c)^6 + 60*a^2*tan(1/2*d*x + 1/2*c)^
5 + 390*b^2*tan(1/2*d*x + 1/2*c)^5 + 960*a*b*tan(1/2*d*x + 1/2*c)^4 - 210*
a^2*tan(1/2*d*x + 1/2*c)^3 + 235*b^2*tan(1/2*d*x + 1/2*c)^3 - 96*a*b*tan(1
/2*d*x + 1/2*c)^2 + 150*a^2*tan(1/2*d*x + 1/2*c) + 15*b^2*tan(1/2*d*x + 1/
2*c) + 96*a*b)/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

3.55.9 Mupad [B] (verification not implemented)

Time = 25.04 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.95

$$\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$$

$$= \frac{\left(\frac{5a^2}{4}+\frac{b^2}{8}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}-4ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+\left(\frac{47b^2}{24}-\frac{7a^2}{4}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}+4ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}+\left(\frac{a}{2}\right.}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-6\,\mathrm{ta}\right.}$$

$$+\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(\frac{3a^2}{4}-\frac{b^2}{8}\right)}{d}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))^2/cos(c + d*x)^7,x)
```

output
```
((4*a*b)/5 + tan(c/2 + (d*x)/2)^5*(a^2/2 + (13*b^2)/4) + tan(c/2 + (d*x)/2
)^7*(a^2/2 + (13*b^2)/4) + tan(c/2 + (d*x)/2)^11*((5*a^2)/4 + b^2/8) - tan
(c/2 + (d*x)/2)^3*((7*a^2)/4 - (47*b^2)/24) - tan(c/2 + (d*x)/2)^9*((7*a^2
)/4 - (47*b^2)/24) + tan(c/2 + (d*x)/2)*((5*a^2)/4 + b^2/8) - (4*a*b*tan(c
/2 + (d*x)/2)^2)/5 + 8*a*b*tan(c/2 + (d*x)/2)^4 - 8*a*b*tan(c/2 + (d*x)/2)
^6 + 4*a*b*tan(c/2 + (d*x)/2)^8 - 4*a*b*tan(c/2 + (d*x)/2)^10)/(d*(15*tan(
c/2 + (d*x)/2)^4 - 6*tan(c/2 + (d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*t
an(c/2 + (d*x)/2)^8 - 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1)
) + (atanh(tan(c/2 + (d*x)/2))*((3*a^2)/4 - b^2/8))/d
```

3.56 $\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$

3.56.1 Optimal result

Integrand size = 28, antiderivative size = 125

$$\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = \frac{a^2\tan(c+dx)}{d}+\frac{ab\tan^2(c+dx)}{d}+\frac{(2a^2+b^2)\tan^3(c+dx)}{3d}+\frac{ab\tan^4(c+dx)}{d}+\frac{(a^2+2b^2)\tan^5(c+dx)}{5d}+\frac{ab\tan^6(c+dx)}{3d}+\frac{b^2\tan^7(c+dx)}{7d}$$

output
```
a^2*tan(d*x+c)/d+a*b*tan(d*x+c)^2/d+1/3*(2*a^2+b^2)*tan(d*x+c)^3/d+a*b*tan
(d*x+c)^4/d+1/5*(a^2+2*b^2)*tan(d*x+c)^5/d+1/3*a*b*tan(d*x+c)^6/d+1/7*b^2*
tan(d*x+c)^7/d
```

3.56.2 Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.83

$$\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = \frac{\tan(c+dx)\left(105a^2+105ab\tan(c+dx)+35(2a^2+b^2)\tan^2(c+dx)+105ab\tan^3(c+dx)+21(a^2+2b^2)\right.}{105d}$$

input
```
Integrate[Sec[c + d*x]^8*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

output
```
(Tan[c + d*x]*(105*a^2 + 105*a*b*Tan[c + d*x] + 35*(2*a^2 + b^2)*Tan[c + d
*x]^2 + 105*a*b*Tan[c + d*x]^3 + 21*(a^2 + 2*b^2)*Tan[c + d*x]^4 + 35*a*b*
Tan[c + d*x]^5 + 15*b^2*Tan[c + d*x]^6))/(105*d)
```

3.56.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3567, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\cos(c+dx)+b\sin(c+dx))^2}{\cos(c+dx)^8}dx$$

$$\Big\downarrow \text{3567}$$

$$-\frac{\int (b+a\cot(c+dx))^2\left(\cot^2(c+dx)+1\right)^2\tan^8(c+dx)d\cot(c+dx)}{d}$$

$$\Big\downarrow \text{522}$$

$$-\frac{\int \left(b^2\tan^8(c+dx)+2ab\tan^7(c+dx)+\left(a^2+2b^2\right)\tan^6(c+dx)+4ab\tan^5(c+dx)+\left(2a^2+b^2\right)\tan^4(c+dx)+\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{1}{5}\left(a^2+2b^2\right)\tan^5(c+dx)-\frac{1}{3}\left(2a^2+b^2\right)\tan^3(c+dx)-a^2\tan(c+dx)-\frac{1}{3}ab\tan^6(c+dx)-ab\tan^4(c+dx)-}{d}$$

input
```
Int[Sec[c + d*x]^8*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

output
```
-((-(a^2*Tan[c + d*x]) - a*b*Tan[c + d*x]^2 - ((2*a^2 + b^2)*Tan[c + d*x]^
3)/3 - a*b*Tan[c + d*x]^4 - ((a^2 + 2*b^2)*Tan[c + d*x]^5)/5 - (a*b*Tan[c
+ d*x]^6)/3 - (b^2*Tan[c + d*x]^7)/7)/d)
```

3.56.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3567
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[x^m*((b
+ a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a,
b, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[
n, 0] && GtQ[m, 1])
```

3.56.4 Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{-a^{2}\left(-\frac{8}{15}-\frac{\sec(dx+c)^{4}}{5}-\frac{4\sec(dx+c)^{2}}{15}\right)\tan(dx+c)+\frac{ab}{3\cos(dx+c)^{6}}+b^{2}\left(\frac{\sin(dx+c)^{3}}{7\cos(dx+c)^{7}}+\frac{4\sin(dx+c)^{3}}{35\cos(dx+c)^{5}}+\frac{8\sin(dx+c)^{3}}{105\cos(dx+c)^{3}}\right)}{d}$
default	$\frac{-a^{2}\left(-\frac{8}{15}-\frac{\sec(dx+c)^{4}}{5}-\frac{4\sec(dx+c)^{2}}{15}\right)\tan(dx+c)+\frac{ab}{3\cos(dx+c)^{6}}+b^{2}\left(\frac{\sin(dx+c)^{3}}{7\cos(dx+c)^{7}}+\frac{4\sin(dx+c)^{3}}{35\cos(dx+c)^{5}}+\frac{8\sin(dx+c)^{3}}{105\cos(dx+c)^{3}}\right)}{d}$
parts	$-\frac{a^{2}\left(-\frac{8}{15}-\frac{\sec(dx+c)^{4}}{5}-\frac{4\sec(dx+c)^{2}}{15}\right)\tan(dx+c)}{d}+\frac{b^{2}\left(\frac{\sin(dx+c)^{3}}{7\cos(dx+c)^{7}}+\frac{4\sin(dx+c)^{3}}{35\cos(dx+c)^{5}}+\frac{8\sin(dx+c)^{3}}{105\cos(dx+c)^{3}}\right)}{d}+\frac{ab\sec(dx-}{3d}$
risch	$\frac{16i\left(-140iab\,e^{8i(dx+c)}+70a^{2}e^{8i(dx+c)}-70b^{2}e^{8i(dx+c)}-140iab\,e^{6i(dx+c)}+175a^{2}e^{6i(dx+c)}+35b^{2}e^{6i(dx+c)}+147a^{2}e^{4i(dx+}\right.}{105d\left(e^{2i(dx+c)}+1\right)^{7}}$
parallelrisch	$-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(105a^{2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}-210\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}ab-350\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}a^{2}+140\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}b^{2}+210\tan\left(\right.\right.}{}$

input
```
int(sec(d*x+c)^8*(cos(d*x+c)*a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a^2*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+1/3*a*b/co
s(d*x+c)^6+b^2*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*sin(d*x+c)^3/cos(d*x+c)
^5+8/105*sin(d*x+c)^3/cos(d*x+c)^3))
```

3.56.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.80

$$\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = \frac{35\,ab\cos(dx+c)+\left(8\left(7\,a^2-b^2\right)\cos(dx+c)^6+4\left(7\,a^2-b^2\right)\cos(dx+c)^4+3\left(7\,a^2-b^2\right)\cos(dx+c)^2+\right.}{105\,d\cos(dx+c)^7}$$

input
```
integrate(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas"
)
```

output
```
1/105*(35*a*b*cos(d*x + c) + (8*(7*a^2 - b^2)*cos(d*x + c)^6 + 4*(7*a^2 -
b^2)*cos(d*x + c)^4 + 3*(7*a^2 - b^2)*cos(d*x + c)^2 + 15*b^2)*sin(d*x + c
))/(d*cos(d*x + c)^7)
```

3.56.6 Sympy [F(-1)]

Timed out.

$$\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**8*(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

output
```
Timed out
```

3.56.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.73

$$\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$$
$$=\frac{7\left(3\tan\left(dx+c\right)^5+10\tan\left(dx+c\right)^3+15\tan\left(dx+c\right)\right)a^2+\left(15\tan\left(dx+c\right)^7+42\tan\left(dx+c\right)^5+35\text{ t}\right.}{105\,d}$$

input
```
integrate(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima"
)
```

output
```
1/105*(7*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^2 + (1
5*tan(d*x + c)^7 + 42*tan(d*x + c)^5 + 35*tan(d*x + c)^3)*b^2 - 35*a*b/(si
n(d*x + c)^2 - 1)^3)/d
```

3.56.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.94

$$\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$$
$$=\frac{15\,b^2\tan\left(dx+c\right)^7+35\,ab\tan\left(dx+c\right)^6+21\,a^2\tan\left(dx+c\right)^5+42\,b^2\tan\left(dx+c\right)^5+105\,ab\tan\left(dx+c\right)^4}{105\,d}$$

input
```
integrate(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
1/105*(15*b^2*tan(d*x + c)^7 + 35*a*b*tan(d*x + c)^6 + 21*a^2*tan(d*x + c)
^5 + 42*b^2*tan(d*x + c)^5 + 105*a*b*tan(d*x + c)^4 + 70*a^2*tan(d*x + c)^
3 + 35*b^2*tan(d*x + c)^3 + 105*a*b*tan(d*x + c)^2 + 105*a^2*tan(d*x + c))
/d
```

3.56.9 Mupad [B] (verification not implemented)

Time = 22.70 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.04

$$\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$$
$$= \frac{\frac{b^2\sin(c+d\,x)}{7}+\cos\left(c+d\,x\right)^2\left(\frac{a^2\sin(c+d\,x)}{5}-\frac{b^2\sin(c+d\,x)}{35}\right)+\cos\left(c+d\,x\right)^4\left(\frac{4\,a^2\sin(c+d\,x)}{15}-\frac{4\,b^2\sin(c+d\,x)}{105}\right)+\cos}{d\cos\left(c+d\,x\right)^7}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))^2/cos(c + d*x)^8,x)
```

output
```
((b^2*sin(c + d*x))/7 + cos(c + d*x)^2*((a^2*sin(c + d*x))/5 - (b^2*sin(c
+ d*x))/35) + cos(c + d*x)^4*((4*a^2*sin(c + d*x))/15 - (4*b^2*sin(c + d*x
))/105) + cos(c + d*x)^6*((8*a^2*sin(c + d*x))/15 - (8*b^2*sin(c + d*x))/1
05) + (a*b*cos(c + d*x))/3)/(d*cos(c + d*x)^7)
```

3.57 $\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

3.57.1 Optimal result

Integrand size = 28, antiderivative size = 265

$$\begin{aligned}
&\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx \\
&= \frac{35a^3x}{128}+\frac{15}{128}ab^2x-\frac{b^3\cos^6(c+dx)}{6d}-\frac{3a^2b\cos^8(c+dx)}{8d}+\frac{b^3\cos^8(c+dx)}{8d} \\
&\quad+\frac{35a^3\cos(c+dx)\sin(c+dx)}{128d}+\frac{15ab^2\cos(c+dx)\sin(c+dx)}{128d} \\
&\quad+\frac{35a^3\cos^3(c+dx)\sin(c+dx)}{192d}+\frac{5ab^2\cos^3(c+dx)\sin(c+dx)}{64d} \\
&\quad+\frac{7a^3\cos^5(c+dx)\sin(c+dx)}{48d}+\frac{ab^2\cos^5(c+dx)\sin(c+dx)}{16d} \\
&\quad+\frac{a^3\cos^7(c+dx)\sin(c+dx)}{8d}-\frac{3ab^2\cos^7(c+dx)\sin(c+dx)}{8d}
\end{aligned}$$

output
```
35/128*a^3*x+15/128*a*b^2*x-1/6*b^3*cos(d*x+c)^6/d-3/8*a^2*b*cos(d*x+c)^8/
d+1/8*b^3*cos(d*x+c)^8/d+35/128*a^3*cos(d*x+c)*sin(d*x+c)/d+15/128*a*b^2*c
os(d*x+c)*sin(d*x+c)/d+35/192*a^3*cos(d*x+c)^3*sin(d*x+c)/d+5/64*a*b^2*cos
(d*x+c)^3*sin(d*x+c)/d+7/48*a^3*cos(d*x+c)^5*sin(d*x+c)/d+1/16*a*b^2*cos(d
*x+c)^5*sin(d*x+c)/d+1/8*a^3*cos(d*x+c)^7*sin(d*x+c)/d-3/8*a*b^2*cos(d*x+c
)^7*sin(d*x+c)/d
```

3.57.2 Mathematica [A] (verified)

Time = 1.35 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.89

$$\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$
$$= \frac{5a(7a^2+3b^2)(c+dx)}{128d} - \frac{3b(7a^2+b^2)\cos(2(c+dx))}{128d} - \frac{b(21a^2+b^2)\cos(4(c+dx))}{256d} - \frac{b(9a^2-b^2)\cos(6(c+dx))}{384d} - \frac{b(3a^2-b^2)\cos(8(c+dx))}{1024d} + \frac{a(14a^2+3b^2)\sin(2(c+dx))}{64d} + \frac{a(7a^2-3b^2)\sin(4(c+dx))}{128d} + \frac{a(2a^2-3b^2)\sin(6(c+dx))}{192d} + \frac{a(a^2-3b^2)\sin(8(c+dx))}{1024d}$$

input
```
Integrate[Cos[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

output
```
(5*a*(7*a^2 + 3*b^2)*(c + d*x))/(128*d) - (3*b*(7*a^2 + b^2)*Cos[2*(c + d*
x)])/(128*d) - (b*(21*a^2 + b^2)*Cos[4*(c + d*x)])/(256*d) - (b*(9*a^2 - b
^2)*Cos[6*(c + d*x)])/(384*d) - (b*(3*a^2 - b^2)*Cos[8*(c + d*x)])/(1024*d
) + (a*(14*a^2 + 3*b^2)*Sin[2*(c + d*x)])/(64*d) + (a*(7*a^2 - 3*b^2)*Sin[
4*(c + d*x)])/(128*d) + (a*(2*a^2 - 3*b^2)*Sin[6*(c + d*x)])/(192*d) + (a*
(a^2 - 3*b^2)*Sin[8*(c + d*x)])/(1024*d)
```

3.57.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \cos(c+dx)^5(a\cos(c+dx)+b\sin(c+dx))^3dx$$
$$\Big\downarrow \ 3569$$
$$\int \left(a^3\cos^8(c+dx)+3a^2b\sin(c+dx)\cos^7(c+dx)+3ab^2\sin^2(c+dx)\cos^6(c+dx)+b^3\sin^3(c+dx)\cos^5(c+dx)\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3\sin(c+dx)\cos^7(c+dx)}{8d}+\frac{7a^3\sin(c+dx)\cos^5(c+dx)}{48d}+\frac{35a^3\sin(c+dx)\cos^3(c+dx)}{192d}+\frac{35a^3\sin(c+dx)\cos(c+dx)}{128d}+\frac{35a^3x}{128}-\frac{3a^2b\cos^8(c+dx)}{8d}-\frac{3ab^2\sin(c+dx)\cos^7(c+dx)}{8d}+\frac{ab^2\sin(c+dx)\cos^5(c+dx)}{16d}+\frac{5ab^2\sin(c+dx)\cos^3(c+dx)}{64d}+\frac{15ab^2\sin(c+dx)\cos(c+dx)}{128d}+\frac{15}{128}ab^2x+\frac{b^3\cos^8(c+dx)}{8d}-\frac{b^3\cos^6(c+dx)}{6d}$$

input
```
Int[Cos[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

output
```
(35*a^3*x)/128 + (15*a*b^2*x)/128 - (b^3*Cos[c + d*x]^6)/(6*d) - (3*a^2*b*
Cos[c + d*x]^8)/(8*d) + (b^3*Cos[c + d*x]^8)/(8*d) + (35*a^3*Cos[c + d*x]*
Sin[c + d*x])/(128*d) + (15*a*b^2*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (35
*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(192*d) + (5*a*b^2*Cos[c + d*x]^3*Sin[c
+ d*x])/(64*d) + (7*a^3*Cos[c + d*x]^5*Sin[c + d*x])/(48*d) + (a*b^2*Cos[c
 + d*x]^5*Sin[c + d*x])/(16*d) + (a^3*Cos[c + d*x]^7*Sin[c + d*x])/(8*d) -
 (3*a*b^2*Cos[c + d*x]^7*Sin[c + d*x])/(8*d)
```

3.57.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

3.57.4 Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.66

method	result
derivativedivides	$\frac{a^3\left(\frac{\left(\cos(dx+c)^7+\frac{7\cos(dx+c)^5}{6}+\frac{35\cos(dx+c)^3}{24}+\frac{35\cos(dx+c)}{16}\right)\sin(dx+c)}{8}+\frac{35dx}{128}+\frac{35c}{128}\right)-\frac{3a^2b\cos(dx+c)^8}{8}+3ab^2\left(-\frac{\cos(dx-}{}\right.}{}$
default	$\frac{a^3\left(\frac{\left(\cos(dx+c)^7+\frac{7\cos(dx+c)^5}{6}+\frac{35\cos(dx+c)^3}{24}+\frac{35\cos(dx+c)}{16}\right)\sin(dx+c)}{8}+\frac{35dx}{128}+\frac{35c}{128}\right)-\frac{3a^2b\cos(dx+c)^8}{8}+3ab^2\left(-\frac{\cos(dx-}{}\right.}{}$
parts	$\frac{a^3\left(\frac{\left(\cos(dx+c)^7+\frac{7\cos(dx+c)^5}{6}+\frac{35\cos(dx+c)^3}{24}+\frac{35\cos(dx+c)}{16}\right)\sin(dx+c)}{8}+\frac{35dx}{128}+\frac{35c}{128}\right)}{d}+\frac{b^3\left(\frac{\cos(dx+c)^8}{8}-\frac{\cos(dx+c)^6}{6}\right)}{d}$
parallelrisch	$\frac{(-504a^2b-72b^3)\cos(2dx+2c)+(-252a^2b-12b^3)\cos(4dx+4c)+(-72a^2b+8b^3)\cos(6dx+6c)+(-9a^2b+3b^3)\cos(8dx+8}{}$
risch	$\frac{35a^3x}{128}+\frac{15ab^2x}{128}-\frac{3b\cos(8dx+8c)a^2}{1024d}+\frac{b^3\cos(8dx+8c)}{1024d}+\frac{a^3\sin(8dx+8c)}{1024d}-\frac{3a\sin(8dx+8c)b^2}{1024d}-\frac{3b\cos(6dx+}{128d}$
norman	$\frac{(\frac{35}{128}a^3+\frac{15}{128}ab^2)x+(\frac{35}{16}a^3+\frac{15}{16}ab^2)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+(\frac{35}{16}a^3+\frac{15}{16}ab^2)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{14}+(\frac{35}{128}a^3+\frac{15}{128}ab^2)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right.}{}$

input
```
int(cos(d*x+c)^5*(cos(d*x+c)*a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(1/8*(cos(d*x+c)^7+7/6*cos(d*x+c)^5+35/24*cos(d*x+c)^3+35/16*cos(
d*x+c))*sin(d*x+c)+35/128*d*x+35/128*c)-3/8*a^2*b*cos(d*x+c)^8+3*a*b^2*(-1
/8*cos(d*x+c)^7*sin(d*x+c)+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*
x+c))*sin(d*x+c)+5/128*d*x+5/128*c)+b^3*(-1/8*sin(d*x+c)^2*cos(d*x+c)^6-1/
24*cos(d*x+c)^6))
```

3.57.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.57

$$\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx =$$
$$-\frac{64\,b^3\cos(dx+c)^6+48\,(3\,a^2b-b^3)\cos(dx+c)^8-15\,(7\,a^3+3\,ab^2)dx-\left(48\,(a^3-3\,ab^2)\cos(dx+c)^7\right.}{3}$$

input
```
integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas"
)
```

output
```
-1/384*(64*b^3*cos(d*x + c)^6 + 48*(3*a^2*b - b^3)*cos(d*x + c)^8 - 15*(7*
a^3 + 3*a*b^2)*d*x - (48*(a^3 - 3*a*b^2)*cos(d*x + c)^7 + 8*(7*a^3 + 3*a*b
^2)*cos(d*x + c)^5 + 10*(7*a^3 + 3*a*b^2)*cos(d*x + c)^3 + 15*(7*a^3 + 3*a
*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

3.57.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 532 vs. $2(257) = 514$.

Time = 0.78 (sec) , antiderivative size = 532, normalized size of antiderivative = 2.01

$$\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$
$$= \begin{cases} \frac{35a^3x\sin^8(c+dx)}{128} + \frac{35a^3x\sin^6(c+dx)\cos^2(c+dx)}{32} + \frac{105a^3x\sin^4(c+dx)\cos^4(c+dx)}{64} + \frac{35a^3x\sin^2(c+dx)\cos^6(c+dx)}{32} + \frac{35a^3x\cos^8(c\ldots}{128} \\ x(a\cos(c)+b\sin(c))^3\cos^5(c) \end{cases}$$

input
```
integrate(cos(d*x+c)**5*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

output
```
Piecewise((35*a**3*x*sin(c + d*x)**8/128 + 35*a**3*x*sin(c + d*x)**6*cos(c
 + d*x)**2/32 + 105*a**3*x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 35*a**3*x*
sin(c + d*x)**2*cos(c + d*x)**6/32 + 35*a**3*x*cos(c + d*x)**8/128 + 35*a*
*3*sin(c + d*x)**7*cos(c + d*x)/(128*d) + 385*a**3*sin(c + d*x)**5*cos(c +
 d*x)**3/(384*d) + 511*a**3*sin(c + d*x)**3*cos(c + d*x)**5/(384*d) + 93*a
**3*sin(c + d*x)*cos(c + d*x)**7/(128*d) - 3*a**2*b*cos(c + d*x)**8/(8*d)
+ 15*a*b**2*x*sin(c + d*x)**8/128 + 15*a*b**2*x*sin(c + d*x)**6*cos(c + d*
x)**2/32 + 45*a*b**2*x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 15*a*b**2*x*si
n(c + d*x)**2*cos(c + d*x)**6/32 + 15*a*b**2*x*cos(c + d*x)**8/128 + 15*a*
b**2*sin(c + d*x)**7*cos(c + d*x)/(128*d) + 55*a*b**2*sin(c + d*x)**5*cos(
c + d*x)**3/(128*d) + 73*a*b**2*sin(c + d*x)**3*cos(c + d*x)**5/(128*d) -
15*a*b**2*sin(c + d*x)*cos(c + d*x)**7/(128*d) + b**3*sin(c + d*x)**8/(24*
d) + b**3*sin(c + d*x)**6*cos(c + d*x)**2/(6*d) + b**3*sin(c + d*x)**4*cos
(c + d*x)**4/(4*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**3*cos(c)**5, True
))
```

3.57.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.62

$$\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx =$$

$$-\frac{1152\,a^2b\cos\left(dx+c\right)^8+\left(128\,\sin\left(2\,dx+2\,c\right)^3-840\,dx-840\,c-3\,\sin\left(8\,dx+8\,c\right)-168\,\sin\left(4\,dx+4\right.}{}$$

input
```
integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima"
)
```

output
```
-1/3072*(1152*a^2*b*cos(d*x + c)^8 + (128*sin(2*d*x + 2*c)^3 - 840*d*x - 8
40*c - 3*sin(8*d*x + 8*c) - 168*sin(4*d*x + 4*c) - 768*sin(2*d*x + 2*c))*a
^3 - 3*(64*sin(2*d*x + 2*c)^3 + 120*d*x + 120*c - 3*sin(8*d*x + 8*c) - 24*
sin(4*d*x + 4*c))*a*b^2 - 128*(3*sin(d*x + c)^8 - 8*sin(d*x + c)^6 + 6*sin
(d*x + c)^4)*b^3)/d
```

3.57.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.82

$$\begin{aligned}&\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx\\&=\frac{5}{128}\left(7\,a^3+3\,ab^2\right)x-\frac{\left(3\,a^2b-b^3\right)\cos\left(8\,dx+8\,c\right)}{1024\,d}-\frac{\left(9\,a^2b-b^3\right)\cos\left(6\,dx+6\,c\right)}{384\,d}\\&\quad-\frac{\left(21\,a^2b+b^3\right)\cos\left(4\,dx+4\,c\right)}{256\,d}-\frac{3\left(7\,a^2b+b^3\right)\cos\left(2\,dx+2\,c\right)}{128\,d}\\&\quad+\frac{\left(a^3-3\,ab^2\right)\sin\left(8\,dx+8\,c\right)}{1024\,d}+\frac{\left(2\,a^3-3\,ab^2\right)\sin\left(6\,dx+6\,c\right)}{192\,d}\\&\quad+\frac{\left(7\,a^3-3\,ab^2\right)\sin\left(4\,dx+4\,c\right)}{128\,d}+\frac{\left(14\,a^3+3\,ab^2\right)\sin\left(2\,dx+2\,c\right)}{64\,d}\end{aligned}$$

input
```
integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
5/128*(7*a^3 + 3*a*b^2)*x - 1/1024*(3*a^2*b - b^3)*cos(8*d*x + 8*c)/d - 1/
384*(9*a^2*b - b^3)*cos(6*d*x + 6*c)/d - 1/256*(21*a^2*b + b^3)*cos(4*d*x
+ 4*c)/d - 3/128*(7*a^2*b + b^3)*cos(2*d*x + 2*c)/d + 1/1024*(a^3 - 3*a*b^
2)*sin(8*d*x + 8*c)/d + 1/192*(2*a^3 - 3*a*b^2)*sin(6*d*x + 6*c)/d + 1/128
*(7*a^3 - 3*a*b^2)*sin(4*d*x + 4*c)/d + 1/64*(14*a^3 + 3*a*b^2)*sin(2*d*x
+ 2*c)/d
```

3.57.9 Mupad [B] (verification not implemented)

Time = 24.22 (sec) , antiderivative size = 523, normalized size of antiderivative = 1.97

$$\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$

$$= \frac{4\,b^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{15\,a\,b^2}{64}-\frac{93\,a^3}{64}\right)+\frac{40\,b^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{3}+4\,b^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{}$$

$$+\frac{5\,a\,\mathrm{atan}\left(\frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(7\,a^2+3\,b^2)}{64\,\left(\frac{35\,a^3}{64}+\frac{15\,a\,b^2}{64}\right)}\right)\,(7\,a^2+3\,b^2)}{64\,d}$$

$$-\frac{5\,a\,(7\,a^2+3\,b^2)\,\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)}{64\,d}$$

input
```
int(cos(c + d*x)^5*(a*cos(c + d*x) + b*sin(c + d*x))^3,x)
```

output
```
(4*b^3*tan(c/2 + (d*x)/2)^4 - tan(c/2 + (d*x)/2)*((15*a*b^2)/64 - (93*a^3)
/64) + (40*b^3*tan(c/2 + (d*x)/2)^8)/3 + 4*b^3*tan(c/2 + (d*x)/2)^12 + tan
(c/2 + (d*x)/2)^15*((15*a*b^2)/64 - (93*a^3)/64) + tan(c/2 + (d*x)/2)^3*((
397*a*b^2)/64 + (91*a^3)/192) - tan(c/2 + (d*x)/2)^13*((397*a*b^2)/64 + (9
1*a^3)/192) - tan(c/2 + (d*x)/2)^5*((895*a*b^2)/64 - (1799*a^3)/192) + tan
(c/2 + (d*x)/2)^11*((895*a*b^2)/64 - (1799*a^3)/192) + tan(c/2 + (d*x)/2)^
7*((1765*a*b^2)/64 - (1085*a^3)/192) - tan(c/2 + (d*x)/2)^9*((1765*a*b^2)/
64 - (1085*a^3)/192) + tan(c/2 + (d*x)/2)^6*(42*a^2*b - (16*b^3)/3) + tan(
c/2 + (d*x)/2)^10*(42*a^2*b - (16*b^3)/3) + 6*a^2*b*tan(c/2 + (d*x)/2)^2 +
 6*a^2*b*tan(c/2 + (d*x)/2)^14)/(d*(8*tan(c/2 + (d*x)/2)^2 + 28*tan(c/2 +
(d*x)/2)^4 + 56*tan(c/2 + (d*x)/2)^6 + 70*tan(c/2 + (d*x)/2)^8 + 56*tan(c/
2 + (d*x)/2)^10 + 28*tan(c/2 + (d*x)/2)^12 + 8*tan(c/2 + (d*x)/2)^14 + tan
(c/2 + (d*x)/2)^16 + 1)) + (5*a*atan((5*a*tan(c/2 + (d*x)/2)*(7*a^2 + 3*b^
2))/(64*((15*a*b^2)/64 + (35*a^3)/64))*(7*a^2 + 3*b^2))/(64*d) - (5*a*(7*
a^2 + 3*b^2)*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2))/(64*d)
```

3.58 $\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

3.58.1 Optimal result

Integrand size = 28, antiderivative size = 175

$$\begin{aligned}\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx &= -\frac{b^3\cos^5(c+dx)}{5d}-\frac{3a^2b\cos^7(c+dx)}{7d}+\frac{b^3\cos^7(c+dx)}{7d}+\frac{a^3\sin(c+dx)}{d}\\&\quad-\frac{a^3\sin^3(c+dx)}{d}+\frac{ab^2\sin^3(c+dx)}{d}+\frac{3a^3\sin^5(c+dx)}{5d}\\&\quad-\frac{6ab^2\sin^5(c+dx)}{5d}-\frac{a^3\sin^7(c+dx)}{7d}+\frac{3ab^2\sin^7(c+dx)}{7d}\end{aligned}$$

output
```
-1/5*b^3*cos(d*x+c)^5/d-3/7*a^2*b*cos(d*x+c)^7/d+1/7*b^3*cos(d*x+c)^7/d+a^
3*sin(d*x+c)/d-a^3*sin(d*x+c)^3/d+a*b^2*sin(d*x+c)^3/d+3/5*a^3*sin(d*x+c)^
5/d-6/5*a*b^2*sin(d*x+c)^5/d-1/7*a^3*sin(d*x+c)^7/d+3/7*a*b^2*sin(d*x+c)^7
/d
```

3.58.2 Mathematica [A] (verified)

Time = 3.16 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.81

$$\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = \frac{-30a^2b\cos^7(c+dx)+b^3\cos^5(c+dx)(-9+5\cos(2(c+dx)))+4b^3\sqrt{\cos^2(c+dx)}\sec(c+dx)+2a\sin(c}{70d}$$

input
```
Integrate[Cos[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

output
```
(-30*a^2*b*Cos[c + d*x]^7 + b^3*Cos[c + d*x]^5*(-9 + 5*Cos[2*(c + d*x)]) +
 4*b^3*Sqrt[Cos[c + d*x]^2]*Sec[c + d*x] + 2*a*Sin[c + d*x]*(35*a^2 - 35*(
a^2 - b^2)*Sin[c + d*x]^2 + 21*(a^2 - 2*b^2)*Sin[c + d*x]^4 - 5*(a^2 - 3*b
^2)*Sin[c + d*x]^6))/(70*d)
```

3.58.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(c+dx)^4(a\cos(c+dx)+b\sin(c+dx))^3dx$$

$$\Big\downarrow \text{3569}$$

$$\int \left(a^3\cos^7(c+dx)+3a^2b\sin(c+dx)\cos^6(c+dx)+3ab^2\sin^2(c+dx)\cos^5(c+dx)+b^3\sin^3(c+dx)\cos^4(c+dx)\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3\sin^7(c+dx)}{7d}+\frac{3a^3\sin^5(c+dx)}{5d}-\frac{a^3\sin^3(c+dx)}{d}+\frac{a^3\sin(c+dx)}{d}-\frac{3a^2b\cos^7(c+dx)}{7d}+\frac{3ab^2\sin^7(c+dx)}{7d}-\frac{6ab^2\sin^5(c+dx)}{5d}+\frac{ab^2\sin^3(c+dx)}{d}+\frac{b^3\cos^7(c+dx)}{7d}-\frac{b^3\cos^5(c+dx)}{5d}$$

input
```
Int[Cos[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

output
```
-1/5*(b^3*Cos[c + d*x]^5)/d - (3*a^2*b*Cos[c + d*x]^7)/(7*d) + (b^3*Cos[c
+ d*x]^7)/(7*d) + (a^3*Sin[c + d*x])/d - (a^3*Sin[c + d*x]^3)/d + (a*b^2*S
in[c + d*x]^3)/d + (3*a^3*Sin[c + d*x]^5)/(5*d) - (6*a*b^2*Sin[c + d*x]^5)
/(5*d) - (a^3*Sin[c + d*x]^7)/(7*d) + (3*a*b^2*Sin[c + d*x]^7)/(7*d)
```

3.58.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

3.58.4 Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.75

method	result
parts	$\frac{a^3\left(\frac{16}{5}+\cos(dx+c)^6+\frac{6\cos(dx+c)^4}{5}+\frac{8\cos(dx+c)^2}{5}\right)\sin(dx+c)}{7d}+\frac{b^3\left(\frac{\cos(dx+c)^7}{7}-\frac{\cos(dx+c)^5}{5}\right)}{d}+\frac{3ab^2\left(\frac{\sin(dx+c)^7}{7}-\cdots\right.}{\cdots}$
derivativedivides	$\frac{\frac{a^3\left(\frac{16}{5}+\cos(dx+c)^6+\frac{6\cos(dx+c)^4}{5}+\frac{8\cos(dx+c)^2}{5}\right)\sin(dx+c)}{7}-\frac{3a^2b\cos(dx+c)^7}{7}+3ab^2\left(-\frac{\cos(dx+c)^6\sin(dx+c)}{7}+\frac{\left(\frac{8}{3}+\cos(dx+\cdots\right.}{\cdots}\right.}{d}$
default	$\frac{\frac{a^3\left(\frac{16}{5}+\cos(dx+c)^6+\frac{6\cos(dx+c)^4}{5}+\frac{8\cos(dx+c)^2}{5}\right)\sin(dx+c)}{7}-\frac{3a^2b\cos(dx+c)^7}{7}+3ab^2\left(-\frac{\cos(dx+c)^6\sin(dx+c)}{7}+\frac{\left(\frac{8}{3}+\cos(dx+\cdots\right.}{\cdots}\right.}{d}$
risch	$-\frac{15a^2b\cos(dx+c)}{64d}-\frac{3b^3\cos(dx+c)}{64d}+\frac{35a^3\sin(dx+c)}{64d}+\frac{15ab^2\sin(dx+c)}{64d}-\frac{3b\cos(7dx+7c)a^2}{448d}+\frac{b^3\cos(7dx+\cdots}{448d}$
norman	$\frac{-\frac{30a^2b+4b^3}{35d}+\frac{2a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{13}}{d}-\frac{4b^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{5d}-\frac{4b^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{d}-\frac{8b^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{d}-\frac{6a^2b\tan\left(\frac{dx}{2}+\cdots\right.}{d}}{\cdots}$
parallelrisch	$\frac{-\frac{6a^2b}{7}-\frac{4b^3}{35}-\frac{32\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9ab^2}{5}+\frac{912\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7ab^2}{35}-\frac{32\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5ab^2}{5}+8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3ab^2+\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4b^3}{5}-6\cdots}{\cdots}$

input
```
int(cos(d*x+c)^4*(cos(d*x+c)*a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/7*a^3/d*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c)
+b^3/d*(1/7*cos(d*x+c)^7-1/5*cos(d*x+c)^5)+3*a*b^2/d*(1/7*sin(d*x+c)^7-2/5
*sin(d*x+c)^5+1/3*sin(d*x+c)^3)-3/7*a^2*b*cos(d*x+c)^7/d
```

3.58.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.70

$$\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx =$$
$$-\frac{7\,b^3\cos{(dx+c)}^5+5\,(3\,a^2b-b^3)\cos{(dx+c)}^7-\left(5\,(a^3-3\,ab^2)\cos{(dx+c)}^6+3\,(2\,a^3+ab^2)\cos{(dx+c)}\right.}{35\,d}$$

input
```
integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas"
)
```

output
```
-1/35*(7*b^3*cos(d*x + c)^5 + 5*(3*a^2*b - b^3)*cos(d*x + c)^7 - (5*(a^3 -
 3*a*b^2)*cos(d*x + c)^6 + 3*(2*a^3 + a*b^2)*cos(d*x + c)^4 + 16*a^3 + 8*a
*b^2 + 4*(2*a^3 + a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/d
```

3.58.6 Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.33

$$\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$
$$=\begin{cases}\frac{16a^3\sin^7{(c+dx)}}{35d}+\frac{8a^3\sin^5{(c+dx)}\cos^2{(c+dx)}}{5d}+\frac{2a^3\sin^3{(c+dx)}\cos^4{(c+dx)}}{d}+\frac{a^3\sin{(c+dx)}\cos^6{(c+dx)}}{d}-\frac{3a^2b\cos^7{(c+dx)}}{7d}+\frac{8ab}{} \\ x(a\cos{(c)}+b\sin{(c)})^3\cos^4{(c)}\end{cases}$$

input
```
integrate(cos(d*x+c)**4*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

output
```
Piecewise((16*a**3*sin(c + d*x)**7/(35*d) + 8*a**3*sin(c + d*x)**5*cos(c +
 d*x)**2/(5*d) + 2*a**3*sin(c + d*x)**3*cos(c + d*x)**4/d + a**3*sin(c + d
*x)*cos(c + d*x)**6/d - 3*a**2*b*cos(c + d*x)**7/(7*d) + 8*a*b**2*sin(c +
d*x)**7/(35*d) + 4*a*b**2*sin(c + d*x)**5*cos(c + d*x)**2/(5*d) + a*b**2*s
in(c + d*x)**3*cos(c + d*x)**4/d - b**3*sin(c + d*x)**2*cos(c + d*x)**5/(5
*d) - 2*b**3*cos(c + d*x)**7/(35*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**
3*cos(c)**4, True))
```

3.58.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.72

$$\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx =$$
$$-\frac{15\,a^2b\cos\left(dx+c\right)^7+\left(5\,\sin\left(dx+c\right)^7-21\,\sin\left(dx+c\right)^5+35\,\sin\left(dx+c\right)^3-35\,\sin\left(dx+c\right)\right)a^3-(15}{35\,d}$$

input
```
integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima"
)
```

output
```
-1/35*(15*a^2*b*cos(d*x + c)^7 + (5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 3
5*sin(d*x + c)^3 - 35*sin(d*x + c))*a^3 - (15*sin(d*x + c)^7 - 42*sin(d*x
+ c)^5 + 35*sin(d*x + c)^3)*a*b^2 - (5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*
b^3)/d
```

3.58.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.13

$$\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$
$$= -\frac{(3\,a^2b-b^3)\cos\left(7\,dx+7\,c\right)}{448\,d}-\frac{(15\,a^2b-b^3)\cos\left(5\,dx+5\,c\right)}{320\,d}-\frac{(9\,a^2b+b^3)\cos\left(3\,dx+3\,c\right)}{64\,d}-\frac{3\,(5\,a^2b+b^3)\cos\left(dx+c\right)}{64\,d}+\frac{(a^3-3\,ab^2)\sin\left(7\,dx+7\,c\right)}{448\,d}+\frac{(7\,a^3-9\,ab^2)\sin\left(5\,dx+5\,c\right)}{320\,d}+\frac{(7\,a^3-ab^2)\sin\left(3\,dx+3\,c\right)}{64\,d}+\frac{5\,(7\,a^3+3\,ab^2)\sin\left(dx+c\right)}{64\,d}$$

input
```
integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/448*(3*a^2*b - b^3)*cos(7*d*x + 7*c)/d - 1/320*(15*a^2*b - b^3)*cos(5*d
*x + 5*c)/d - 1/64*(9*a^2*b + b^3)*cos(3*d*x + 3*c)/d - 3/64*(5*a^2*b + b^
3)*cos(d*x + c)/d + 1/448*(a^3 - 3*a*b^2)*sin(7*d*x + 7*c)/d + 1/320*(7*a^
3 - 9*a*b^2)*sin(5*d*x + 5*c)/d + 1/64*(7*a^3 - a*b^2)*sin(3*d*x + 3*c)/d
+ 5/64*(7*a^3 + 3*a*b^2)*sin(d*x + c)/d
```

3.58.9 Mupad [B] (verification not implemented)

Time = 24.04 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.22

$$\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$
$$= \frac{16\,a^3\,\sin(c+d\,x)}{35\,d} - \frac{b^3\cos(c+d\,x)^5}{5\,d} + \frac{b^3\cos(c+d\,x)^7}{7\,d} - \frac{3\,a^2\,b\cos(c+d\,x)^7}{7\,d} + \frac{8\,a^3\cos(c+d\,x)^2\,\sin(c+d\,x)}{35\,d} + \frac{6\,a^3\cos(c+d\,x)^4\,\sin(c+d\,x)}{35\,d} + \frac{a^3\cos(c+d\,x)^6\,\sin(c+d\,x)}{7\,d} + \frac{8\,a\,b^2\,\sin(c+d\,x)}{35\,d} + \frac{4\,a\,b^2\cos(c+d\,x)^2\,\sin(c+d\,x)}{35\,d} + \frac{3\,a\,b^2\cos(c+d\,x)^4\,\sin(c+d\,x)}{35\,d} - \frac{3\,a\,b^2\cos(c+d\,x)^6\,\sin(c+d\,x)}{7\,d}$$

input
```
int(cos(c + d*x)^4*(a*cos(c + d*x) + b*sin(c + d*x))^3,x)
```

output
```
(16*a^3*sin(c + d*x))/(35*d) - (b^3*cos(c + d*x)^5)/(5*d) + (b^3*cos(c + d
*x)^7)/(7*d) - (3*a^2*b*cos(c + d*x)^7)/(7*d) + (8*a^3*cos(c + d*x)^2*sin(
c + d*x))/(35*d) + (6*a^3*cos(c + d*x)^4*sin(c + d*x))/(35*d) + (a^3*cos(c
 + d*x)^6*sin(c + d*x))/(7*d) + (8*a*b^2*sin(c + d*x))/(35*d) + (4*a*b^2*c
os(c + d*x)^2*sin(c + d*x))/(35*d) + (3*a*b^2*cos(c + d*x)^4*sin(c + d*x))
/(35*d) - (3*a*b^2*cos(c + d*x)^6*sin(c + d*x))/(7*d)
```

3.59 $\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

3.59.1 Optimal result

Integrand size = 28, antiderivative size = 216

$$\begin{aligned}\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx &= \frac{5a^3x}{16}+\frac{3}{16}ab^2x-\frac{a^2b\cos^6(c+dx)}{2d}+\frac{5a^3\cos(c+dx)\sin(c+dx)}{16d}\\ &+\frac{3ab^2\cos(c+dx)\sin(c+dx)}{16d}+\frac{5a^3\cos^3(c+dx)\sin(c+dx)}{24d}\\ &+\frac{ab^2\cos^3(c+dx)\sin(c+dx)}{8d}+\frac{a^3\cos^5(c+dx)\sin(c+dx)}{6d}\\ &-\frac{ab^2\cos^5(c+dx)\sin(c+dx)}{2d}+\frac{b^3\sin^4(c+dx)}{4d}-\frac{b^3\sin^6(c+dx)}{6d}\end{aligned}$$

output
```
5/16*a^3*x+3/16*a*b^2*x-1/2*a^2*b*cos(d*x+c)^6/d+5/16*a^3*cos(d*x+c)*sin(d
*x+c)/d+3/16*a*b^2*cos(d*x+c)*sin(d*x+c)/d+5/24*a^3*cos(d*x+c)^3*sin(d*x+c
)/d+1/8*a*b^2*cos(d*x+c)^3*sin(d*x+c)/d+1/6*a^3*cos(d*x+c)^5*sin(d*x+c)/d-
1/2*a*b^2*cos(d*x+c)^5*sin(d*x+c)/d+1/4*b^3*sin(d*x+c)^4/d-1/6*b^3*sin(d*x
+c)^6/d
```

3.59.2 Mathematica [A] (verified)

Time = 1.90 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.79

$$\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = \frac{a(5a^2+3b^2)\,(c+dx)}{16d} - \frac{3b(5a^2+b^2)\cos(2(c+dx))}{64d} - \frac{3a^2b\cos(4(c+dx))}{32d} - \frac{b(3a^2-b^2)\cos(6(c+dx))}{192d} + \frac{3a(5a^2+b^2)\sin(2(c+dx))}{64d} + \frac{3a(a^2-b^2)\sin(4(c+dx))}{64d} + \frac{a(a^2-3b^2)\sin(6(c+dx))}{192d}$$

input
```
Integrate[Cos[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

output
```
(a*(5*a^2 + 3*b^2)*(c + d*x))/(16*d) - (3*b*(5*a^2 + b^2)*Cos[2*(c + d*x)]
)/(64*d) - (3*a^2*b*Cos[4*(c + d*x)])/(32*d) - (b*(3*a^2 - b^2)*Cos[6*(c +
 d*x)])/(192*d) + (3*a*(5*a^2 + b^2)*Sin[2*(c + d*x)])/(64*d) + (3*a*(a^2
- b^2)*Sin[4*(c + d*x)])/(64*d) + (a*(a^2 - 3*b^2)*Sin[6*(c + d*x)])/(192*
d)
```

3.59.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \cos(c+dx)^3(a\cos(c+dx)+b\sin(c+dx))^3dx$$

$$\Big\downarrow \; 3569$$

$$\int \left(a^3\cos^6(c+dx)+3a^2b\sin(c+dx)\cos^5(c+dx)+3ab^2\sin^2(c+dx)\cos^4(c+dx)+b^3\sin^3(c+dx)\cos^3(c+dx\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{a^3\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{5a^3\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{5a^3\sin(c+dx)\cos(c+dx)}{16d}+\frac{5a^3x}{16}-\frac{a^2b\cos^6(c+dx)}{2d}-\frac{ab^2\sin(c+dx)\cos^5(c+dx)}{2d}+\frac{ab^2\sin(c+dx)\cos^3(c+dx)}{8d}+\frac{3ab^2\sin(c+dx)\cos(c+dx)}{16d}+\frac{3}{16}ab^2x-\frac{b^3\sin^6(c+dx)}{6d}+\frac{b^3\sin^4(c+dx)}{4d}$$

input
```
Int[Cos[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

output
```
(5*a^3*x)/16 + (3*a*b^2*x)/16 - (a^2*b*Cos[c + d*x]^6)/(2*d) + (5*a^3*Cos[
c + d*x]*Sin[c + d*x])/(16*d) + (3*a*b^2*Cos[c + d*x]*Sin[c + d*x])/(16*d)
 + (5*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (a*b^2*Cos[c + d*x]^3*Sin[
c + d*x])/(8*d) + (a^3*Cos[c + d*x]^5*Sin[c + d*x])/(6*d) - (a*b^2*Cos[c +
 d*x]^5*Sin[c + d*x])/(2*d) + (b^3*Sin[c + d*x]^4)/(4*d) - (b^3*Sin[c + d*
x]^6)/(6*d)
```

3.59.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

3.59.4 Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.71

method	result
parallelrisch	$\frac{\left(-45a^2b-9b^3\right)\cos\left(2dx+2c\right)+\left(-3a^2b+b^3\right)\cos\left(6dx+6c\right)+\left(45a^3+9a\,b^2\right)\sin\left(2dx+2c\right)+\left(9a^3-9a\,b^2\right)\sin\left(4dx+4c\right)+\left(a^3-\right.}{192d}$
derivativedivides	$\frac{a^3\left(\frac{\left(\cos\left(dx+c\right)^5+\frac{5\cos\left(dx+c\right)^3}{4}+\frac{15\cos\left(dx+c\right)}{8}\right)\sin\left(dx+c\right)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)-\frac{a^2b\cos\left(dx+c\right)^6}{2}+3a\,b^2\left(-\frac{\cos\left(dx+c\right)^5\sin\left(dx+c\right)}{6}+\right.}{d}$
default	$\frac{a^3\left(\frac{\left(\cos\left(dx+c\right)^5+\frac{5\cos\left(dx+c\right)^3}{4}+\frac{15\cos\left(dx+c\right)}{8}\right)\sin\left(dx+c\right)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)-\frac{a^2b\cos\left(dx+c\right)^6}{2}+3a\,b^2\left(-\frac{\cos\left(dx+c\right)^5\sin\left(dx+c\right)}{6}+\right.}{d}$
parts	$\frac{a^3\left(\frac{\left(\cos\left(dx+c\right)^5+\frac{5\cos\left(dx+c\right)^3}{4}+\frac{15\cos\left(dx+c\right)}{8}\right)\sin\left(dx+c\right)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)}{d}+\frac{b^3\left(-\frac{\sin\left(dx+c\right)^6}{6}+\frac{\sin\left(dx+c\right)^4}{4}\right)}{d}+\frac{3a\,b^2\left(-\right.}{}$
risch	$\frac{5a^3x}{16}+\frac{3a\,b^2x}{16}-\frac{b\cos\left(6dx+6c\right)a^2}{64d}+\frac{b^3\cos\left(6dx+6c\right)}{192d}+\frac{a^3\sin\left(6dx+6c\right)}{192d}-\frac{a\sin\left(6dx+6c\right)b^2}{64d}-\frac{3b\cos\left(4dx+4c\right)a}{32d}$
norman	$\frac{\left(\frac{5}{16}a^3+\frac{3}{16}a\,b^2\right)x+\left(\frac{5}{16}a^3+\frac{3}{16}a\,b^2\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}+\left(\frac{15}{8}a^3+\frac{9}{8}a\,b^2\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\left(\frac{15}{8}a^3+\frac{9}{8}a\,b^2\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+\left(\right.}{}$

input
```
int(cos(d*x+c)^3*(cos(d*x+c)*a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/192*((-45*a^2*b-9*b^3)*cos(2*d*x+2*c)+(-3*a^2*b+b^3)*cos(6*d*x+6*c)+(45*
a^3+9*a*b^2)*sin(2*d*x+2*c)+(9*a^3-9*a*b^2)*sin(4*d*x+4*c)+(a^3-3*a*b^2)*s
in(6*d*x+6*c)+60*a^3*x*d+36*a*b^2*d*x-18*cos(4*d*x+4*c)*a^2*b+66*a^2*b+8*b
^3)/d
```

3.59.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.59

$$\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx =$$
$$-\frac{12\,b^3\cos\left(dx+c\right)^4+8\left(3\,a^2b-b^3\right)\cos\left(dx+c\right)^6-3\left(5\,a^3+3\,ab^2\right)dx-\left(8\left(a^3-3\,ab^2\right)\cos\left(dx+c\right)^5+2\right.}{48\,d}$$

input
```
integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas"
)
```

output
```
-1/48*(12*b^3*cos(d*x + c)^4 + 8*(3*a^2*b - b^3)*cos(d*x + c)^6 - 3*(5*a^3
 + 3*a*b^2)*d*x - (8*(a^3 - 3*a*b^2)*cos(d*x + c)^5 + 2*(5*a^3 + 3*a*b^2)*
cos(d*x + c)^3 + 3*(5*a^3 + 3*a*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

3.59.6 Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.85

$$\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$
$$=\begin{cases} \frac{5a^3x\sin^6(c+dx)}{16}+\frac{15a^3x\sin^4(c+dx)\cos^2(c+dx)}{16}+\frac{15a^3x\sin^2(c+dx)\cos^4(c+dx)}{16}+\frac{5a^3x\cos^6(c+dx)}{16}+\frac{5a^3\sin^5(c+dx)\cos(c+dx)}{16d} \\ x(a\cos(c)+b\sin(c))^3\cos^3(c) \end{cases}$$

input
```
integrate(cos(d*x+c)**3*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

output
```
Piecewise((5*a**3*x*sin(c + d*x)**6/16 + 15*a**3*x*sin(c + d*x)**4*cos(c +
 d*x)**2/16 + 15*a**3*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*a**3*x*cos(
c + d*x)**6/16 + 5*a**3*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 5*a**3*sin(c
 + d*x)**3*cos(c + d*x)**3/(6*d) + 11*a**3*sin(c + d*x)*cos(c + d*x)**5/(1
6*d) - a**2*b*cos(c + d*x)**6/(2*d) + 3*a*b**2*x*sin(c + d*x)**6/16 + 9*a*
b**2*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 9*a*b**2*x*sin(c + d*x)**2*cos
(c + d*x)**4/16 + 3*a*b**2*x*cos(c + d*x)**6/16 + 3*a*b**2*sin(c + d*x)**5
*cos(c + d*x)/(16*d) + a*b**2*sin(c + d*x)**3*cos(c + d*x)**3/(2*d) - 3*a*
b**2*sin(c + d*x)*cos(c + d*x)**5/(16*d) + b**3*sin(c + d*x)**6/(12*d) + b
**3*sin(c + d*x)**4*cos(c + d*x)**2/(4*d), Ne(d, 0)), (x*(a*cos(c) + b*sin
(c))**3*cos(c)**3, True))
```

3.59.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.61

$$\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx =$$
$$-\frac{96\,a^2b\cos(dx+c)^6+\left(4\,\sin(2\,dx+2\,c)^3-60\,dx-60\,c-9\,\sin(4\,dx+4\,c)-48\,\sin(2\,dx+2\,c)\right)a^3-}{192}$$

input
```
integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima"
)
```

output
```
-1/192*(96*a^2*b*cos(d*x + c)^6 + (4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c -
9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a^3 - 3*(4*sin(2*d*x + 2*c)^3 +
12*d*x + 12*c - 3*sin(4*d*x + 4*c))*a*b^2 + 16*(2*sin(d*x + c)^6 - 3*sin(d
*x + c)^4)*b^3)/d
```

3.59.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.73

$$\begin{aligned}\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx &= -\frac{3\,a^2b\cos\left(4\,dx+4\,c\right)}{32\,d}+\frac{1}{16}\left(5\,a^3+3\,ab^2\right)x-\frac{\left(3\,a^2b-b^3\right)\cos\left(6\,dx+6\,c\right)}{192\,d}\\ &\quad-\frac{3\left(5\,a^2b+b^3\right)\cos\left(2\,dx+2\,c\right)}{64\,d}+\frac{\left(a^3-3\,ab^2\right)\sin\left(6\,dx+6\,c\right)}{192\,d}\\ &\quad+\frac{3\left(a^3-ab^2\right)\sin\left(4\,dx+4\,c\right)}{64\,d}+\frac{3\left(5\,a^3+ab^2\right)\sin\left(2\,dx+2\,c\right)}{64\,d}\end{aligned}$$

input
```
integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
-3/32*a^2*b*cos(4*d*x + 4*c)/d + 1/16*(5*a^3 + 3*a*b^2)*x - 1/192*(3*a^2*b
 - b^3)*cos(6*d*x + 6*c)/d - 3/64*(5*a^2*b + b^3)*cos(2*d*x + 2*c)/d + 1/1
92*(a^3 - 3*a*b^2)*sin(6*d*x + 6*c)/d + 3/64*(a^3 - a*b^2)*sin(4*d*x + 4*c
)/d + 3/64*(5*a^3 + a*b^2)*sin(2*d*x + 2*c)/d
```

3.59.9 Mupad [B] (verification not implemented)

Time = 24.12 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.88

$$\begin{aligned}\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx &= \frac{4\,b^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{3\,a\,b^2}{8}-\frac{11\,a^3}{8}\right)+4\,b^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}\,\left(\frac{3\,a\,b^2}{8}-\frac{11\,a^3}{8}\right)-\mathrm{ta}}{d\,\left(\tan\left(\frac{c}{2}+\right.\right.}\\ &\quad+\frac{a\,\mathrm{atan}\left(\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(5\,a^2+3\,b^2\right)}{8\,\left(\frac{5\,a^3}{8}+\frac{3\,a\,b^2}{8}\right)}\right)\,\left(5\,a^2+3\,b^2\right)}{8\,d}-\frac{a\,\left(5\,a^2+3\,b^2\right)\,\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)}{8\,d}\end{aligned}$$

input
```
int(cos(c + d*x)^3*(a*cos(c + d*x) + b*sin(c + d*x))^3,x)
```

output
```
(4*b^3*tan(c/2 + (d*x)/2)^4 - tan(c/2 + (d*x)/2)*((3*a*b^2)/8 - (11*a^3)/8
) + 4*b^3*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^11*((3*a*b^2)/8 - (11*
a^3)/8) - tan(c/2 + (d*x)/2)^5*((39*a*b^2)/4 - (15*a^3)/4) + tan(c/2 + (d*
x)/2)^7*((39*a*b^2)/4 - (15*a^3)/4) + tan(c/2 + (d*x)/2)^3*((47*a*b^2)/8 -
 (5*a^3)/24) - tan(c/2 + (d*x)/2)^9*((47*a*b^2)/8 - (5*a^3)/24) + tan(c/2
+ (d*x)/2)^6*(20*a^2*b - (8*b^3)/3) + 6*a^2*b*tan(c/2 + (d*x)/2)^2 + 6*a^2
*b*tan(c/2 + (d*x)/2)^10)/(d*(6*tan(c/2 + (d*x)/2)^2 + 15*tan(c/2 + (d*x)/
2)^4 + 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 + 6*tan(c/2 + (d*
x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1)) + (a*atan((a*tan(c/2 + (d*x)/2)*(5*
a^2 + 3*b^2))/(8*((3*a*b^2)/8 + (5*a^3)/8)))*(5*a^2 + 3*b^2))/(8*d) - (a*(
5*a^2 + 3*b^2)*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2))/(8*d)
```

3.59. $\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

3.60 $\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

3.60.1 Optimal result

Integrand size = 28, antiderivative size = 140

$$\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = -\frac{b^3\cos^3(c+dx)}{3d}-\frac{3a^2b\cos^5(c+dx)}{5d}+\frac{b^3\cos^5(c+dx)}{5d}+\frac{a^3\sin(c+dx)}{d} -\frac{2a^3\sin^3(c+dx)}{3d}+\frac{ab^2\sin^3(c+dx)}{d}+\frac{a^3\sin^5(c+dx)}{5d}-\frac{3ab^2\sin^5(c+dx)}{5d}$$

output
```
-1/3*b^3*cos(d*x+c)^3/d-3/5*a^2*b*cos(d*x+c)^5/d+1/5*b^3*cos(d*x+c)^5/d+a^
3*sin(d*x+c)/d-2/3*a^3*sin(d*x+c)^3/d+a*b^2*sin(d*x+c)^3/d+1/5*a^3*sin(d*x
+c)^5/d-3/5*a*b^2*sin(d*x+c)^5/d
```

3.60.2 Mathematica [A] (verified)

Time = 1.22 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.86

$$\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = \frac{-9a^2b\cos^5(c+dx)+15a^3\sin(c+dx)-5a(2a^2-3b^2)\sin^3(c+dx)+3a(a^2-3b^2)\sin^5(c+dx)+b^3\cos(}{15d}$$

input
```
Integrate[Cos[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

output
```
(-9*a^2*b*Cos[c + d*x]^5 + 15*a^3*Sin[c + d*x] - 5*a*(2*a^2 - 3*b^2)*Sin[c
 + d*x]^3 + 3*a*(a^2 - 3*b^2)*Sin[c + d*x]^5 + b^3*Cos[c + d*x]*(-2 + 2/Sq
rt[Cos[c + d*x]^2] - Sin[c + d*x]^2 + 3*Sin[c + d*x]^4))/(15*d)
```

3.60.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))^3dx$$

$$\Big\downarrow \; 3569$$

$$\int \left(a^3\cos^5(c+dx)+3a^2b\sin(c+dx)\cos^4(c+dx)+3ab^2\sin^2(c+dx)\cos^3(c+dx)+b^3\sin^3(c+dx)\cos^2(c+dx)\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{a^3\sin^5(c+dx)}{5d}-\frac{2a^3\sin^3(c+dx)}{3d}+\frac{a^3\sin(c+dx)}{d}-\frac{3a^2b\cos^5(c+dx)}{5d}-\frac{3ab^2\sin^5(c+dx)}{5d}+\frac{ab^2\sin^3(c+dx)}{d}+\frac{b^3\cos^5(c+dx)}{5d}-\frac{b^3\cos^3(c+dx)}{3d}$$

input
```
Int[Cos[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

output
```
-1/3*(b^3*Cos[c + d*x]^3)/d - (3*a^2*b*Cos[c + d*x]^5)/(5*d) + (b^3*Cos[c
+ d*x]^5)/(5*d) + (a^3*Sin[c + d*x])/d - (2*a^3*Sin[c + d*x]^3)/(3*d) + (a
*b^2*Sin[c + d*x]^3)/d + (a^3*Sin[c + d*x]^5)/(5*d) - (3*a*b^2*Sin[c + d*x
]^5)/(5*d)
```

3.60.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

3.60.4 Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.79

method	result
parts	$\frac{a^3\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5d}+\frac{b^3\left(\frac{\cos(dx+c)^5}{5}-\frac{\cos(dx+c)^3}{3}\right)}{d}+\frac{3a\,b^2\left(-\frac{\sin(dx+c)^5}{5}+\frac{\sin(dx+c)^3}{3}\right)}{d}$
derivativedivides	$\frac{\frac{a^3\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}-\frac{3a^2b\cos(dx+c)^5}{5}+3a\,b^2\left(-\frac{\sin(dx+c)\cos(dx+c)^4}{5}+\frac{\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{15}\right)}{d}$
default	$\frac{\frac{a^3\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}-\frac{3a^2b\cos(dx+c)^5}{5}+3a\,b^2\left(-\frac{\sin(dx+c)\cos(dx+c)^4}{5}+\frac{\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{15}\right)}{d}$
risch	$-\frac{3a^2b\cos(dx+c)}{8d}-\frac{b^3\cos(dx+c)}{8d}+\frac{5a^3\sin(dx+c)}{8d}+\frac{3a\,b^2\sin(dx+c)}{8d}-\frac{3b\cos(5dx+5c)a^2}{80d}+\frac{b^3\cos(5dx+5c)}{80d}$
parallelrisch	$\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9a^3-6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8a^2b+\frac{8\left(a^3+3a\,b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{3}-4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6b^3+\frac{4\left(29a^3-12a\,b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{15}+\frac{4(}{}}{d\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}$
norman	$\frac{-\frac{18a^2b+4b^3}{15d}+\frac{2a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{d}-\frac{4b^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{d}-\frac{4b^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{3d}-\frac{6a^2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{d}-\frac{2\left(18a^2b-2b^3\right.}{}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^5}$

input
```
int(cos(d*x+c)^2*(cos(d*x+c)*a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/5*a^3/d*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+b^3/d*(1/5*cos(d*
x+c)^5-1/3*cos(d*x+c)^3)+3*a*b^2/d*(-1/5*sin(d*x+c)^5+1/3*sin(d*x+c)^3)-3/
5*a^2*b*cos(d*x+c)^5/d
```

3.60.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.73

$$\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx =$$
$$-\frac{5\,b^3\cos\left(dx+c\right)^3+3\left(3\,a^2b-b^3\right)\cos\left(dx+c\right)^5-\left(3\left(a^3-3\,ab^2\right)\cos\left(dx+c\right)^4+8\,a^3+6\,ab^2+\left(4\,a^3+3\right.\right.}{15\,d}$$

input
```
integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas"
)
```

output
```
-1/15*(5*b^3*cos(d*x + c)^3 + 3*(3*a^2*b - b^3)*cos(d*x + c)^5 - (3*(a^3 -
 3*a*b^2)*cos(d*x + c)^4 + 8*a^3 + 6*a*b^2 + (4*a^3 + 3*a*b^2)*cos(d*x + c
)^2)*sin(d*x + c))/d
```

3.60.6 Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.30

$$\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$
$$=\begin{cases}\frac{8a^3\sin^5(c+dx)}{15d}+\frac{4a^3\sin^3(c+dx)\cos^2(c+dx)}{3d}+\frac{a^3\sin(c+dx)\cos^4(c+dx)}{d}-\frac{3a^2b\cos^5(c+dx)}{5d}+\frac{2ab^2\sin^5(c+dx)}{5d}+\frac{ab^2\sin^3(c+dx)}{d}\\ x(a\cos(c)+b\sin(c))^3\cos^2(c)\end{cases}$$

input
```
integrate(cos(d*x+c)**2*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

output
```
Piecewise((8*a**3*sin(c + d*x)**5/(15*d) + 4*a**3*sin(c + d*x)**3*cos(c +
d*x)**2/(3*d) + a**3*sin(c + d*x)*cos(c + d*x)**4/d - 3*a**2*b*cos(c + d*x
)**5/(5*d) + 2*a*b**2*sin(c + d*x)**5/(5*d) + a*b**2*sin(c + d*x)**3*cos(c
 + d*x)**2/d - b**3*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - 2*b**3*cos(c +
 d*x)**5/(15*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**3*cos(c)**2, True))
```

3.60.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.76

$$\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx =$$
$$-\frac{9\,a^2b\cos{(dx+c)}^5 - \left(3\,\sin{(dx+c)}^5 - 10\,\sin{(dx+c)}^3 + 15\,\sin{(dx+c)}\right)a^3 + 3\left(3\,\sin{(dx+c)}^5 - 5\,\mathrm{s}\right.}{15\,d}$$

input
```
integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima"
)
```

output
```
-1/15*(9*a^2*b*cos(d*x + c)^5 - (3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15
*sin(d*x + c))*a^3 + 3*(3*sin(d*x + c)^5 - 5*sin(d*x + c)^3)*a*b^2 - (3*co
s(d*x + c)^5 - 5*cos(d*x + c)^3)*b^3)/d
```

3.60.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.04

$$\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$
$$= -\frac{(3\,a^2b-b^3)\cos{(5\,dx+5\,c)}}{80\,d} - \frac{(9\,a^2b+b^3)\cos{(3\,dx+3\,c)}}{48\,d} - \frac{(3\,a^2b+b^3)\cos{(dx+c)}}{8\,d} + \frac{(a^3-3\,ab^2)\sin{(5\,dx+5\,c)}}{80\,d} + \frac{(5\,a^3-3\,ab^2)\sin{(3\,dx+3\,c)}}{48\,d} + \frac{(5\,a^3+3\,ab^2)\sin{(dx+c)}}{8\,d}$$

input
```
integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/80*(3*a^2*b - b^3)*cos(5*d*x + 5*c)/d - 1/48*(9*a^2*b + b^3)*cos(3*d*x
+ 3*c)/d - 1/8*(3*a^2*b + b^3)*cos(d*x + c)/d + 1/80*(a^3 - 3*a*b^2)*sin(5
*d*x + 5*c)/d + 1/48*(5*a^3 - 3*a*b^2)*sin(3*d*x + 3*c)/d + 1/8*(5*a^3 + 3
*a*b^2)*sin(d*x + c)/d
```

3.60.9 Mupad [B] (verification not implemented)

Time = 22.84 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.05

$$\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$
$$=\frac{2\left(\frac{3\sin(c+d\,x)\,a^3\cos(c+d\,x)^4}{2}+2\sin\left(c+d\,x\right)\,a^3\cos\left(c+d\,x\right)^2+4\sin\left(c+d\,x\right)\,a^3-\frac{9\,a^2\,b\cos(c+d\,x)^5}{2}-\frac{9\sin(c+d\,x)}{}\right.}{15\,d}$$

input
```
int(cos(c + d*x)^2*(a*cos(c + d*x) + b*sin(c + d*x))^3,x)
```

output
```
(2*(4*a^3*sin(c + d*x) - (5*b^3*cos(c + d*x)^3)/2 + (3*b^3*cos(c + d*x)^5)
/2 - (9*a^2*b*cos(c + d*x)^5)/2 + 2*a^3*cos(c + d*x)^2*sin(c + d*x) + (3*a
^3*cos(c + d*x)^4*sin(c + d*x))/2 + 3*a*b^2*sin(c + d*x) + (3*a*b^2*cos(c
+ d*x)^2*sin(c + d*x))/2 - (9*a*b^2*cos(c + d*x)^4*sin(c + d*x))/2))/(15*d
)
```

3.61 $\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

3.61.1 Optimal result

Integrand size = 26, antiderivative size = 78

$$\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = \frac{3}{8}a\left(a^2+b^2\right)x+\frac{3a(b+a\cot(c+dx))(a-b\cot(c+dx))\sin^2(c+dx)}{8d} + \frac{(b+a\cot(c+dx))^3\sin^4(c+dx)}{4d}$$

output
```
3/8*a*(a^2+b^2)*x+3/8*a*(b+a*cot(d*x+c))*(a-b*cot(d*x+c))*sin(d*x+c)^2/d+1
/4*(b+a*cot(d*x+c))^3*sin(d*x+c)^4/d
```

3.61.2 Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.21

$$\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = \frac{12a\left(a^2+b^2\right)(c+dx)-4\left(3a^2b+b^3\right)\cos(2(c+dx))+\left(-3a^2b+b^3\right)\cos(4(c+dx))+8a^3\sin(2(c+dx))+}{32d}$$

input
```
Integrate[Cos[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

output
```
(12*a*(a^2 + b^2)*(c + d*x) - 4*(3*a^2*b + b^3)*Cos[2*(c + d*x)] + (-3*a^2
*b + b^3)*Cos[4*(c + d*x)] + 8*a^3*Sin[2*(c + d*x)] + a*(a^2 - 3*b^2)*Sin[
4*(c + d*x)])/(32*d)
```

3.61.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.22, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 3567, 531, 27, 487, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3dx$$

$$\Big\downarrow \text{3567}$$

$$-\frac{\int \frac{\cot(c+dx)(b+a\cot(c+dx))^3}{(\cot^2(c+dx)+1)^3}d\cot(c+dx)}{d}$$

$$\Big\downarrow \text{531}$$

$$-\frac{-\frac{1}{4}\int -\frac{3a(b+a\cot(c+dx))^2}{(\cot^2(c+dx)+1)^2}d\cot(c+dx)-\frac{(a\cot(c+dx)+b)^3}{4(\cot^2(c+dx)+1)^2}}{d}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{3}{4}a\int \frac{(b+a\cot(c+dx))^2}{(\cot^2(c+dx)+1)^2}d\cot(c+dx)-\frac{(a\cot(c+dx)+b)^3}{4(\cot^2(c+dx)+1)^2}}{d}$$

$$\Big\downarrow \text{487}$$

$$-\frac{\frac{3}{4}a\left(\frac{1}{2}(a^2+b^2)\int \frac{1}{\cot^2(c+dx)+1}d\cot(c+dx)-\frac{(a\cot(c+dx)+b)(a-b\cot(c+dx))}{2(\cot^2(c+dx)+1)}\right)-\frac{(a\cot(c+dx)+b)^3}{4(\cot^2(c+dx)+1)^2}}{d}$$

$$\Big\downarrow \text{216}$$

$$-\frac{\frac{3}{4}a\left(\frac{1}{2}(a^2+b^2)\arctan(\cot(c+dx))-\frac{(a\cot(c+dx)+b)(a-b\cot(c+dx))}{2(\cot^2(c+dx)+1)}\right)-\frac{(a\cot(c+dx)+b)^3}{4(\cot^2(c+dx)+1)^2}}{d}$$

input
```
Int[Cos[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

output
```
-((-1/4*(b + a*Cot[c + d*x])^3/(1 + Cot[c + d*x]^2)^2 + (3*a*(((a^2 + b^2)
*ArcTan[Cot[c + d*x]])/2 - ((b + a*Cot[c + d*x])*(a - b*Cot[c + d*x]))/(2*
(1 + Cot[c + d*x]^2))))/4)/d)
```

3.61.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 487
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n - 1)*(a*d - b*c*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] +
 Simp[(2*p + 3)*((b*c^2 + a*d^2)/(2*a*b*(p + 1)))   Int[(c + d*x)^(n - 2)*(
a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[n + 2*p + 2, 0]
 && LtQ[p, -1]
```

rule 531
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symb
ol] :> With[{Qx = PolynomialQuotient[x^m, a + b*x^2, x], e = Coeff[Polynomi
alRemainder[x^m, a + b*x^2, x], x, 0], f = Coeff[PolynomialRemainder[x^m, a
 + b*x^2, x], x, 1]}, Simp[(c + d*x)^n*(a*f - b*e*x)*((a + b*x^2)^(p + 1)/(
2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b
*x^2)^(p + 1)*ExpandToSum[2*a*b*(p + 1)*(c + d*x)*Qx - a*d*f*n + b*c*e*(2*p
 + 3) + b*d*e*(n + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGt
Q[n, 0] && IGtQ[m, 0] && LtQ[p, -1] && GtQ[n, 1] && IntegerQ[2*p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3567
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[x^m*((b
+ a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a,
b, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[
n, 0] && GtQ[m, 1])
```

3.61.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.40

method	result
parallelrisch	$\frac{(-12a^2b-4b^3)\cos(2dx+2c)+(-3a^2b+b^3)\cos(4dx+4c)+(a^3-3a\,b^2)\sin(4dx+4c)+12a^3xd+12a\,b^2dx+8\sin(2dx+2c)a^3}{32d}$
derivativedivides	$\frac{a^3\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)-\frac{3a^2b\cos(dx+c)^4}{4}+3a\,b^2\left(-\frac{\sin(dx+c)\cos(dx+c)^3}{4}+\frac{\sin(dx+c)\cos(dx+c}{8}\right.}{d}$
default	$\frac{a^3\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)-\frac{3a^2b\cos(dx+c)^4}{4}+3a\,b^2\left(-\frac{\sin(dx+c)\cos(dx+c)^3}{4}+\frac{\sin(dx+c)\cos(dx+c}{8}\right.}{d}$
parts	$\frac{a^3\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}+\frac{b^3\sin(dx+c)^4}{4d}+\frac{3a\,b^2\left(-\frac{\sin(dx+c)\cos(dx+c)^3}{4}+\frac{\sin(dx+c)\cos(}{8}\right.}{d}$
risch	$\frac{3a^3x}{8}+\frac{3a\,b^2x}{8}-\frac{3b\cos(4dx+4c)a^2}{32d}+\frac{b^3\cos(4dx+4c)}{32d}+\frac{a^3\sin(4dx+4c)}{32d}-\frac{3a\sin(4dx+4c)b^2}{32d}-\frac{3b\cos(2dx+2c}{8d}$
norman	$(\frac{3}{8}a^3+\frac{3}{8}a\,b^2)x+(\frac{3}{2}a^3+\frac{3}{2}a\,b^2)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+(\frac{3}{2}a^3+\frac{3}{2}a\,b^2)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+(\frac{3}{8}a^3+\frac{3}{8}a\,b^2)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8+(\frac{9}{4}a^3+\frac{9}{4}a$

input
```
int(cos(d*x+c)*(cos(d*x+c)*a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/32*((-12*a^2*b-4*b^3)*cos(2*d*x+2*c)+(-3*a^2*b+b^3)*cos(4*d*x+4*c)+(a^3-
3*a*b^2)*sin(4*d*x+4*c)+12*a^3*x*d+12*a*b^2*d*x+8*sin(2*d*x+2*c)*a^3+15*a^
2*b+3*b^3)/d
```

3.61.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.28

$$\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = -\frac{4\,b^3\cos\left(dx+c\right)^2+2\,(3\,a^2b-b^3)\cos\left(dx+c\right)^4-3\,(a^3+ab^2)dx-\left(2\,(a^3-3\,ab^2)\cos\left(dx+c\right)^3+3\,(a^3\right.}{8\,d}$$

input
```
integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/8*(4*b^3*cos(d*x + c)^2 + 2*(3*a^2*b - b^3)*cos(d*x + c)^4 - 3*(a^3 + a
*b^2)*d*x - (2*(a^3 - 3*a*b^2)*cos(d*x + c)^3 + 3*(a^3 + a*b^2)*cos(d*x +
c))*sin(d*x + c))/d
```

3.61.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. 2(71) = 142.

Time = 0.21 (sec) , antiderivative size = 272, normalized size of antiderivative = 3.49

$$\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$
$$= \begin{cases} \frac{3a^3x\sin^4(c+dx)}{8} + \frac{3a^3x\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{3a^3x\cos^4(c+dx)}{8} + \frac{3a^3\sin^3(c+dx)\cos(c+dx)}{8d} + \frac{5a^3\sin(c+dx)\cos^3(c+dx)}{8d} - 3 \\ x(a\cos(c)+b\sin(c))^3\cos(c) \end{cases}$$

input
```
integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

output
```
Piecewise((3*a**3*x*sin(c + d*x)**4/8 + 3*a**3*x*sin(c + d*x)**2*cos(c + d
*x)**2/4 + 3*a**3*x*cos(c + d*x)**4/8 + 3*a**3*sin(c + d*x)**3*cos(c + d*x
)/(8*d) + 5*a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) - 3*a**2*b*cos(c + d*x
)**4/(4*d) + 3*a*b**2*x*sin(c + d*x)**4/8 + 3*a*b**2*x*sin(c + d*x)**2*cos
(c + d*x)**2/4 + 3*a*b**2*x*cos(c + d*x)**4/8 + 3*a*b**2*sin(c + d*x)**3*c
os(c + d*x)/(8*d) - 3*a*b**2*sin(c + d*x)*cos(c + d*x)**3/(8*d) + b**3*sin
(c + d*x)**4/(4*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**3*cos(c), True))
```

3.61.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.17

$$\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx =$$
$$-\frac{24\,a^2b\cos(dx+c)^4 - 8\,b^3\sin(dx+c)^4 - (12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))a^3 - 3\,(4\,c}{32\,d}$$

input
```
integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/32*(24*a^2*b*cos(d*x + c)^4 - 8*b^3*sin(d*x + c)^4 - (12*d*x + 12*c + s
in(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^3 - 3*(4*d*x + 4*c - sin(4*d*x + 4
*c))*a*b^2)/d
```

3.61.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.33

$$\begin{aligned}&\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx\\&=\frac{a^3\sin(2\,dx+2\,c)}{4\,d}+\frac{3}{8}\left(a^3+ab^2\right)x-\frac{\left(3\,a^2b-b^3\right)\cos(4\,dx+4\,c)}{32\,d}\\&\quad-\frac{\left(3\,a^2b+b^3\right)\cos(2\,dx+2\,c)}{8\,d}+\frac{\left(a^3-3\,ab^2\right)\sin(4\,dx+4\,c)}{32\,d}\end{aligned}$$

input
```
integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
1/4*a^3*sin(2*d*x + 2*c)/d + 3/8*(a^3 + a*b^2)*x - 1/32*(3*a^2*b - b^3)*co
s(4*d*x + 4*c)/d - 1/8*(3*a^2*b + b^3)*cos(2*d*x + 2*c)/d + 1/32*(a^3 - 3*
a*b^2)*sin(4*d*x + 4*c)/d
```

3.61.9 Mupad [B] (verification not implemented)

Time = 23.85 (sec) , antiderivative size = 281, normalized size of antiderivative = 3.60

$$\begin{aligned}&\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx\\&=\frac{4\,b^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{3\,a\,b^2}{4}-\frac{5\,a^3}{4}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\left(\frac{3\,a\,b^2}{4}-\frac{5\,a^3}{4}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(\frac{21\,a\,b^2}{4}-3\right.}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.\right.}\\&\quad-\frac{3\,a\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)\left(a^2+b^2\right)}{4\,d}+\frac{3\,a\,\mathrm{atan}\left(\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(a^2+b^2\right)}{4\left(\frac{3\,a^3}{4}+\frac{3\,a\,b^2}{4}\right)}\right)\left(a^2+b^2\right)}{4\,d}\end{aligned}$$

input
```
int(cos(c + d*x)*(a*cos(c + d*x) + b*sin(c + d*x))^3,x)
```

output
```
(4*b^3*tan(c/2 + (d*x)/2)^4 - tan(c/2 + (d*x)/2)*((3*a*b^2)/4 - (5*a^3)/4)
 + tan(c/2 + (d*x)/2)^7*((3*a*b^2)/4 - (5*a^3)/4) + tan(c/2 + (d*x)/2)^3*(
(21*a*b^2)/4 - (3*a^3)/4) - tan(c/2 + (d*x)/2)^5*((21*a*b^2)/4 - (3*a^3)/4
) + 6*a^2*b*tan(c/2 + (d*x)/2)^2 + 6*a^2*b*tan(c/2 + (d*x)/2)^6)/(d*(4*tan
(c/2 + (d*x)/2)^2 + 6*tan(c/2 + (d*x)/2)^4 + 4*tan(c/2 + (d*x)/2)^6 + tan(
c/2 + (d*x)/2)^8 + 1)) - (3*a*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2)*(a^2 +
b^2))/(4*d) + (3*a*atan((3*a*tan(c/2 + (d*x)/2)*(a^2 + b^2))/(4*((3*a*b^2)
/4 + (3*a^3)/4)))*(a^2 + b^2))/(4*d)
```

3.62 $\int (a\cos(c+dx)+b\sin(c+dx))^3\,dx$

3.62.1 Optimal result

Integrand size = 19, antiderivative size = 58

$$\int (a\cos(c+dx)+b\sin(c+dx))^3\,dx=-\frac{(a^2+b^2)\,(b\cos(c+dx)-a\sin(c+dx))}{d}+\frac{(b\cos(c+dx)-a\sin(c+dx))^3}{3d}$$

output
```
-(a^2+b^2)*(b*cos(d*x+c)-a*sin(d*x+c))/d+1/3*(b*cos(d*x+c)-a*sin(d*x+c))^3
/d
```

3.62.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.40

$$\int (a\cos(c+dx)+b\sin(c+dx))^3\,dx$$
$$=\frac{-9b(a^2+b^2)\cos(c+dx)+(-3a^2b+b^3)\cos(3(c+dx))+2a(5a^2+3b^2+(a^2-3b^2)\cos(2(c+dx)))\sin(c}{12d}$$

input
```
Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

output
```
(-9*b*(a^2 + b^2)*Cos[c + d*x] + (-3*a^2*b + b^3)*Cos[3*(c + d*x)] + 2*a*(
5*a^2 + 3*b^2 + (a^2 - 3*b^2)*Cos[2*(c + d*x)])*Sin[c + d*x])/(12*d)
```

3.62.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3042, 3551, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(c+dx)+b\sin(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\cos(c+dx)+b\sin(c+dx))^3dx$$

$$\Big\downarrow \text{3551}$$

$$-\frac{\int \left(a^2+b^2-(b\cos(c+dx)-a\sin(c+dx))^2\right)d(b\cos(c+dx)-a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(a^2+b^2)\,(b\cos(c+dx)-a\sin(c+dx))-\frac{1}{3}(b\cos(c+dx)-a\sin(c+dx))^3}{d}$$

input
```
Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

output
```
-(((a^2 + b^2)*(b*Cos[c + d*x] - a*Sin[c + d*x]) - (b*Cos[c + d*x] - a*Sin
[c + d*x])^3/3)/d)
```

3.62.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3551
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[(a^2 + b^2 - x^2)^((n - 1)/2), x], x,
b*Cos[c + d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
 b^2, 0] && IGtQ[(n - 1)/2, 0]
```

3.62.4 Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.29

method	result
derivativedivides	$\frac{\frac{a^3(2+\cos(dx+c)^2)\sin(dx+c)}{3}-a^2b\cos(dx+c)^3+a\,b^2\sin(dx+c)^3-\frac{b^3(2+\sin(dx+c)^2)\cos(dx+c)}{3}}{d}$
default	$\frac{\frac{a^3(2+\cos(dx+c)^2)\sin(dx+c)}{3}-a^2b\cos(dx+c)^3+a\,b^2\sin(dx+c)^3-\frac{b^3(2+\sin(dx+c)^2)\cos(dx+c)}{3}}{d}$
parts	$\frac{a^3(2+\cos(dx+c)^2)\sin(dx+c)}{3d}-\frac{b^3(2+\sin(dx+c)^2)\cos(dx+c)}{3d}+\frac{a\,b^2\sin(dx+c)^3}{d}-\frac{a^2b\cos(dx+c)^3}{d}$
parallelrisch	$\frac{2a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5-6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4a^2b+\frac{4(a^3+6a\,b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}-4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b^3+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a^3-2a^2b-\frac{4b^3}{3}}{d\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}$
risch	$-\frac{3a^2b\cos(dx+c)}{4d}-\frac{3b^3\cos(dx+c)}{4d}+\frac{3a^3\sin(dx+c)}{4d}+\frac{3a\,b^2\sin(dx+c)}{4d}-\frac{b\cos(3dx+3c)a^2}{4d}+\frac{b^3\cos(3dx+3c)}{12d}-$
norman	$\frac{-\frac{6a^2b+4b^3}{3d}+\frac{2a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d}-\frac{4b^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d}-\frac{6a^2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{d}+\frac{4a(a^2+6b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}$

input
```
int((cos(d*x+c)*a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/3*a^3*(2+cos(d*x+c)^2)*sin(d*x+c)-a^2*b*cos(d*x+c)^3+a*b^2*sin(d*x+
c)^3-1/3*b^3*(2+sin(d*x+c)^2)*cos(d*x+c))
```

3.62.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.33

$$\int (a\cos(c+dx)+b\sin(c+dx))^3\,dx = -\frac{3\,b^3\cos(dx+c)+(3\,a^2b-b^3)\cos(dx+c)^3-\left(2\,a^3+3\,ab^2+(a^3-3\,ab^2)\cos(dx+c)^2\right)\sin(dx+c)}{3\,d}$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/3*(3*b^3*cos(d*x + c) + (3*a^2*b - b^3)*cos(d*x + c)^3 - (2*a^3 + 3*a*b
^2 + (a^3 - 3*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/d
```

3.62.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(48) = 96.

Time = 0.13 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.02

$$\int (a\cos(c+dx)+b\sin(c+dx))^3\,dx$$
$$=\begin{cases}\frac{2a^3\sin^3(c+dx)}{3d}+\frac{a^3\sin(c+dx)\cos^2(c+dx)}{d}-\frac{a^2b\cos^3(c+dx)}{d}+\frac{ab^2\sin^3(c+dx)}{d}-\frac{b^3\sin^2(c+dx)\cos(c+dx)}{d}-\frac{2b^3\cos^3(c+dx)}{3d} & \text{fo} \\ x(a\cos(c)+b\sin(c))^3 & \text{c}\end{cases}$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

output
```
Piecewise((2*a**3*sin(c + d*x)**3/(3*d) + a**3*sin(c + d*x)*cos(c + d*x)**
2/d - a**2*b*cos(c + d*x)**3/d + a*b**2*sin(c + d*x)**3/d - b**3*sin(c + d
*x)**2*cos(c + d*x)/d - 2*b**3*cos(c + d*x)**3/(3*d), Ne(d, 0)), (x*(a*cos
(c) + b*sin(c))**3, True))
```

3.62.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.45

$$\int (a\cos(c+dx)+b\sin(c+dx))^3\,dx=-\frac{a^2b\cos(dx+c)^3}{d}+\frac{ab^2\sin(dx+c)^3}{d}-\frac{\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)a^3}{3\,d}+\frac{\left(\cos(dx+c)^3-3\,\cos(dx+c)\right)b^3}{3\,d}$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
-a^2*b*cos(d*x + c)^3/d + a*b^2*sin(d*x + c)^3/d - 1/3*(sin(d*x + c)^3 - 3
*sin(d*x + c))*a^3/d + 1/3*(cos(d*x + c)^3 - 3*cos(d*x + c))*b^3/d
```

3.62.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.57

$$\int (a\cos(c+dx)+b\sin(c+dx))^3\,dx = -\frac{(3\,a^2b-b^3)\cos(3\,dx+3\,c)}{12\,d} - \frac{3\,(a^2b+b^3)\cos(dx+c)}{4\,d} + \frac{(a^3-3\,ab^2)\sin(3\,dx+3\,c)}{12\,d} + \frac{3\,(a^3+ab^2)\sin(dx+c)}{4\,d}$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/12*(3*a^2*b - b^3)*cos(3*d*x + 3*c)/d - 3/4*(a^2*b + b^3)*cos(d*x + c)/
d + 1/12*(a^3 - 3*a*b^2)*sin(3*d*x + 3*c)/d + 3/4*(a^3 + a*b^2)*sin(d*x +
c)/d
```

3.62.9 Mupad [B] (verification not implemented)

Time = 22.40 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.79

$$\int (a\cos(c+dx)+b\sin(c+dx))^3\,dx$$
$$= \frac{\frac{\sin(c+d\,x)\,a^3\cos(c+d\,x)^2}{3}+\frac{2\,\sin(c+d\,x)\,a^3}{3}-a^2\,b\cos\left(c+d\,x\right)^3-\sin\left(c+d\,x\right)\,a\,b^2\cos\left(c+d\,x\right)^2+\sin\left(c+d\,x\right)\,a\,b}{d}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))^3,x)
```

output
```
((2*a^3*sin(c + d*x))/3 - b^3*cos(c + d*x) + (b^3*cos(c + d*x)^3)/3 - a^2*
b*cos(c + d*x)^3 + (a^3*cos(c + d*x)^2*sin(c + d*x))/3 + a*b^2*sin(c + d*x
) - a*b^2*cos(c + d*x)^2*sin(c + d*x))/d
```

3.63 $\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

3.63.1 Optimal result

Integrand size = 26, antiderivative size = 91

$$\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = \frac{1}{2}a\left(a^2+3b^2\right)x-\frac{b^3\log(\sin(c+dx))}{d}+\frac{b^3\log(\tan(c+dx))}{d} + \frac{(b(3a^2-b^2)+a(a^2-3b^2)\cot(c+dx))\sin^2(c+dx)}{2d}$$

output
```
1/2*a*(a^2+3*b^2)*x-b^3*ln(sin(d*x+c))/d+b^3*ln(tan(d*x+c))/d+1/2*(b*(3*a^
2-b^2)+a*(a^2-3*b^2)*cot(d*x+c))*sin(d*x+c)^2/d
```

3.63.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 401 vs. 2(91) = 182.

Time = 0.63 (sec) , antiderivative size = 401, normalized size of antiderivative = 4.41

$$\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = \frac{5a^4b^2+2a^2b^4-b^6+(-3a^4b^2-2a^2b^4+b^6)\cos(2(c+dx))+2a^2b^4\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)+2b^6\log}{}$$

input
```
Integrate[Sec[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

output
```
(5*a^4*b^2 + 2*a^2*b^4 - b^6 + (-3*a^4*b^2 - 2*a^2*b^4 + b^6)*Cos[2*(c + d
*x)] + 2*a^2*b^4*Log[Sqrt[-b^2] - b*Tan[c + d*x]] + 2*b^6*Log[Sqrt[-b^2] -
 b*Tan[c + d*x]] - a^5*Sqrt[-b^2]*Log[Sqrt[-b^2] - b*Tan[c + d*x]] + 4*a^3
*(-b^2)^(3/2)*Log[Sqrt[-b^2] - b*Tan[c + d*x]] - 3*a*(-b^2)^(5/2)*Log[Sqrt
[-b^2] - b*Tan[c + d*x]] + 2*a^2*b^4*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + 2*
b^6*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + a^5*Sqrt[-b^2]*Log[Sqrt[-b^2] + b*T
an[c + d*x]] + 3*a*b^4*Sqrt[-b^2]*Log[Sqrt[-b^2] + b*Tan[c + d*x]] - 4*a^3
*(-b^2)^(3/2)*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + a*b*(a^4 - 2*a^2*b^2 - 3*
b^4)*Sin[2*(c + d*x)])/(4*b*(a^2 + b^2)*d)
```

3.63.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 3567, 532, 25, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a\cos(c+dx)+b\sin(c+dx))^3}{\cos(c+dx)}dx$$

$$\Big\downarrow \; 3567$$

$$-\frac{\int \frac{(b+a\cot(c+dx))^3\tan(c+dx)}{\left(\cot^2(c+dx)+1\right)^2}d\cot(c+dx)}{d}$$

$$\Big\downarrow \; 532$$

$$-\frac{-\frac{1}{2}\int -\frac{\left(2b^3+a\left(a^2+3b^2\right)\cot(c+dx)\right)\tan(c+dx)}{\cot^2(c+dx)+1}d\cot(c+dx)-\frac{a\left(a^2-3b^2\right)\cot(c+dx)+b\left(3a^2-b^2\right)}{2\left(\cot^2(c+dx)+1\right)}}{d}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{1}{2}\int \frac{\left(2b^3+a\left(a^2+3b^2\right)\cot(c+dx)\right)\tan(c+dx)}{\cot^2(c+dx)+1}d\cot(c+dx)-\frac{a\left(a^2-3b^2\right)\cot(c+dx)+b\left(3a^2-b^2\right)}{2\left(\cot^2(c+dx)+1\right)}}{d}$$

$$\Big\downarrow \; 523$$

$$-\frac{\frac{1}{2}\int \left(2\tan(c+dx)b^3+\frac{a^3+3b^2a-2b^3\cot(c+dx)}{\cot^2(c+dx)+1}\right)d\cot(c+dx)-\frac{a\left(a^2-3b^2\right)\cot(c+dx)+b\left(3a^2-b^2\right)}{2\left(\cot^2(c+dx)+1\right)}}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{1}{2}\left(a\left(a^2+3b^2\right)\arctan(\cot(c+dx))-b^3\log\left(\cot^2(c+dx)+1\right)+2b^3\log(\cot(c+dx))\right)-\frac{a\left(a^2-3b^2\right)\cot(c+dx)+b\left(3a^2\right.}{2\left(\cot^2(c+dx)+1\right)}}{d}$$

input
```
Int[Sec[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

output
```
-((-1/2*(b*(3*a^2 - b^2) + a*(a^2 - 3*b^2)*Cot[c + d*x])/(1 + Cot[c + d*x]
^2) + (a*(a^2 + 3*b^2)*ArcTan[Cot[c + d*x]] + 2*b^3*Log[Cot[c + d*x]] - b^
3*Log[1 + Cot[c + d*x]^2])/2)/d)
```

3.63.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3567
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[x^m*((b
+ a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a,
b, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[
n, 0] && GtQ[m, 1])
```

3.63.4 Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{a^3\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-\frac{3\cos(dx+c)^2a^2b}{2}+3a\,b^2\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+b^3\left(-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+\right.}{d}$
default	$\frac{a^3\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-\frac{3\cos(dx+c)^2a^2b}{2}+3a\,b^2\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+b^3\left(-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+\right.}{d}$
parts	$\frac{a^3\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{b^3\left(-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)}{d}-\frac{3a^2b}{2d\sec(dx+c)^2}+\frac{3a\,b^2\left(-\frac{\sin(dx+c)\cos(dx-\right.}{d}$
parallelrisch	$\frac{4b^3\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)-4b^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-4b^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\left(-3a^2b+b^3\right)\cos(2dx+2c)+\left(a^3-3a\,b^2\right)\sin(}{4d}$
risch	$ix\,b^3+\frac{a^3x}{2}+\frac{3a\,b^2x}{2}-\frac{3\,e^{2i(dx+c)}a^2b}{8d}+\frac{e^{2i(dx+c)}b^3}{8d}-\frac{ie^{2i(dx+c)}a^3}{8d}+\frac{3ie^{2i(dx+c)}a\,b^2}{8d}-\frac{3\,e^{-2i(dx+c)}a^2b}{8d}$
norman	$\frac{\left(\frac{1}{2}a^3+\frac{3}{2}a\,b^2\right)x+\left(\frac{1}{2}a^3+\frac{3}{2}a\,b^2\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+\left(\frac{3}{2}a^3+\frac{9}{2}a\,b^2\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\left(\frac{3}{2}a^3+\frac{9}{2}a\,b^2\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\frac{(6a^2b-2b^3}{}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}$

input
```
int(sec(d*x+c)*(cos(d*x+c)*a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)-3/2*cos(d*x+c)^2*a^2*b+
3*a*b^2*(-1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)+b^3*(-1/2*sin(d*x+c)^2-
ln(cos(d*x+c))))
```

3.63.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.87

$$\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = -\frac{2\,b^3\log\left(-\cos\left(dx+c\right)\right) - (a^3+3\,ab^2)dx + (3\,a^2b-b^3)\cos\left(dx+c\right)^2 - (a^3-3\,ab^2)\cos\left(dx+c\right)\sin\left(dx\right.}{2\,d}$$

input
```
integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/2*(2*b^3*log(-cos(d*x + c)) - (a^3 + 3*a*b^2)*d*x + (3*a^2*b - b^3)*cos
(d*x + c)^2 - (a^3 - 3*a*b^2)*cos(d*x + c)*sin(d*x + c))/d
```

3.63.6 Sympy [F]

$$\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = \int \left(a\cos\left(c+dx\right)+b\sin\left(c+dx\right)\right)^3\sec\left(c+dx\right)dx$$

input
```
integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

output
```
Integral((a*cos(c + d*x) + b*sin(c + d*x))**3*sec(c + d*x), x)
```

3.63.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00

$$\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = \frac{6\,a^2b\sin\left(dx+c\right)^2 + (2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))a^3 + 3\left(2\,dx+2\,c-\sin\left(2\,dx+2\,c\right)\right)ab^2 - 2\left(\sin\left(dx+c\right.\right.}{4\,d}$$

input
```
integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/4*(6*a^2*b*sin(d*x + c)^2 + (2*d*x + 2*c + sin(2*d*x + 2*c))*a^3 + 3*(2*
d*x + 2*c - sin(2*d*x + 2*c))*a*b^2 - 2*(sin(d*x + c)^2 + log(sin(d*x + c)
^2 - 1))*b^3)/d
```

3.63.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.02

$$\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$
$$=\frac{b^3\log\left(\tan\left(dx+c\right)^2+1\right)+(a^3+3\,ab^2)(dx+c)-\frac{b^3\tan(dx+c)^2-a^3\tan(dx+c)+3\,ab^2\tan(dx+c)+3\,a^2b}{\tan(dx+c)^2+1}}{2\,d}$$

input
```
integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
1/2*(b^3*log(tan(d*x + c)^2 + 1) + (a^3 + 3*a*b^2)*(d*x + c) - (b^3*tan(d*
x + c)^2 - a^3*tan(d*x + c) + 3*a*b^2*tan(d*x + c) + 3*a^2*b)/(tan(d*x + c
)^2 + 1))/d
```

3.63.9 Mupad [B] (verification not implemented)

Time = 23.26 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.71

$$\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$
$$=\frac{b^3\,\ln\left(\frac{1}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}\right)-b^3\,\ln\left(\frac{\cos(c+d\,x)}{\cos(c+d\,x)+1}\right)+a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)+\frac{b^3\,\cos(2\,c+2\,d\,x)}{4}+\frac{a^3\,\sin(2\,c+2\,d\,x)}{4}+3\,a\,b^2\,\mathrm{a}}{d}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))^3/cos(c + d*x),x)
```

output
```
(b^3*log(1/cos(c/2 + (d*x)/2)^2) - b^3*log(cos(c + d*x)/(cos(c + d*x) + 1)
) + a^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + (b^3*cos(2*c + 2*d*x
))/4 + (a^3*sin(2*c + 2*d*x))/4 + 3*a*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/2
+ (d*x)/2)) - (3*a^2*b*cos(2*c + 2*d*x))/4 - (3*a*b^2*sin(2*c + 2*d*x))/4)
/d
```

3.64 $\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

3.64.1 Optimal result

Integrand size = 28, antiderivative size = 86

$$\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = \frac{3ab^2\text{arctanh}(\sin(c+dx))}{d} - \frac{3a^2b\cos(c+dx)}{d} + \frac{b^3\cos(c+dx)}{d} + \frac{b^3\sec(c+dx)}{d} + \frac{a^3\sin(c+dx)}{d} - \frac{3ab^2\sin(c+dx)}{d}$$

output
```
3*a*b^2*arctanh(sin(d*x+c))/d-3*a^2*b*cos(d*x+c)/d+b^3*cos(d*x+c)/d+b^3*se
c(d*x+c)/d+a^3*sin(d*x+c)/d-3*a*b^2*sin(d*x+c)/d
```

3.64.2 Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.52

$$\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = \frac{\sec(c+dx)\left(-3a^2b+3b^3+(-3a^2b+b^3)\cos(2(c+dx))-6ab^2\cos(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}\right.\right.\right.\right.}{2d}$$

input
```
Integrate[Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

output
```
(Sec[c + d*x]*(-3*a^2*b + 3*b^3 + (-3*a^2*b + b^3)*Cos[2*(c + d*x)] - 6*a*
b^2*Cos[c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c +
d*x)/2] + Sin[(c + d*x)/2]]) + a^3*Sin[2*(c + d*x)] - 3*a*b^2*Sin[2*(c + d
*x)]))/(2*d)
```

3.64.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\cos(c+dx)+b\sin(c+dx))^3}{\cos(c+dx)^2}dx$$

$$\Big\downarrow \text{3569}$$

$$\int \left(a^3\cos(c+dx)+3a^2b\sin(c+dx)+3ab^2\sin(c+dx)\tan(c+dx)+b^3\sin(c+dx)\tan^2(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3\sin(c+dx)}{d}-\frac{3a^2b\cos(c+dx)}{d}+\frac{3ab^2\text{arctanh}(\sin(c+dx))}{d}-\frac{3ab^2\sin(c+dx)}{d}+\frac{b^3\cos(c+dx)}{d}+\frac{b^3\sec(c+dx)}{d}$$

input
```
Int[Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

output
```
(3*a*b^2*ArcTanh[Sin[c + d*x]])/d - (3*a^2*b*Cos[c + d*x])/d + (b^3*Cos[c
+ d*x])/d + (b^3*Sec[c + d*x])/d + (a^3*Sin[c + d*x])/d - (3*a*b^2*Sin[c +
 d*x])/d
```

3.64.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

3.64.4 Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.12

method	result
derivativedivides	$\frac{a^3\sin(dx+c)-3\cos(dx+c)a^2b+3a\,b^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+b^3\left(\frac{\sin(dx+c)^4}{\cos(dx+c)}+\left(2+\sin(dx+c)^2\right)\cos(\right.}{d}$
default	$\frac{a^3\sin(dx+c)-3\cos(dx+c)a^2b+3a\,b^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+b^3\left(\frac{\sin(dx+c)^4}{\cos(dx+c)}+\left(2+\sin(dx+c)^2\right)\cos(\right.}{d}$
parts	$\frac{a^3\sin(dx+c)}{d}+\frac{b^3\left(\frac{\sin(dx+c)^4}{\cos(dx+c)}+\left(2+\sin(dx+c)^2\right)\cos(dx+c)\right)}{d}+\frac{3a\,b^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))}{d}-$
parallelrisch	$\frac{3\left(-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+1\right)b^2a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+3b^2a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4-1\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+2(a^3-3a\,b^2)\tan\left(\frac{dx}{2}+\right.}{d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4-1\right)}$
risch	$-\frac{3\,e^{i(dx+c)}a^2b}{2d}+\frac{e^{i(dx+c)}b^3}{2d}-\frac{ie^{i(dx+c)}a^3}{2d}+\frac{3ie^{i(dx+c)}a\,b^2}{2d}-\frac{3\,e^{-i(dx+c)}a^2b}{2d}+\frac{e^{-i(dx+c)}b^3}{2d}+\frac{ie^{-i(dx+c)}a^3}{2d}$
norman	$\frac{\frac{6a^2b-4b^3}{d}-\frac{6a^2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{d}+\frac{2\left(3a^2b-4b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d}-\frac{2a\left(a^2-3b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2a\left(a^2-3b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}+\frac{2a\left(a^2-3\right.}{}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}$

input
```
int(sec(d*x+c)^2*(cos(d*x+c)*a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*sin(d*x+c)-3*cos(d*x+c)*a^2*b+3*a*b^2*(-sin(d*x+c)+ln(sec(d*x+c)+
tan(d*x+c)))+b^3*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c)))
```

3.64.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.27

$$\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$
$$= \frac{3\,ab^2\cos(dx+c)\log(\sin(dx+c)+1) - 3\,ab^2\cos(dx+c)\log(-\sin(dx+c)+1) + 2\,b^3 - 2\,(3\,a^2b-b^3)}{2\,d\cos(dx+c)}$$

input
```
integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas"
)
```

output
```
1/2*(3*a*b^2*cos(d*x + c)*log(sin(d*x + c) + 1) - 3*a*b^2*cos(d*x + c)*log
(-sin(d*x + c) + 1) + 2*b^3 - 2*(3*a^2*b - b^3)*cos(d*x + c)^2 + 2*(a^3 -
3*a*b^2)*cos(d*x + c)*sin(d*x + c))/(d*cos(d*x + c))
```

3.64.6 Sympy [F]

$$\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$
$$= \int (a\cos(c+dx)+b\sin(c+dx))^3\sec^2(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**2*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

output
```
Integral((a*cos(c + d*x) + b*sin(c + d*x))**3*sec(c + d*x)**2, x)
```

3.64.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.98

$$\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$
$$= \frac{2\,b^3\left(\frac{1}{\cos(dx+c)}+\cos(dx+c)\right) + 3\,ab^2(\log(\sin(dx+c)+1) - \log(\sin(dx+c)-1) - 2\,\sin(dx+c)) - 6}{2\,d}$$

input
```
integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima"
)
```

output
```
1/2*(2*b^3*(1/cos(d*x + c) + cos(d*x + c)) + 3*a*b^2*(log(sin(d*x + c) + 1
) - log(sin(d*x + c) - 1) - 2*sin(d*x + c)) - 6*a^2*b*cos(d*x + c) + 2*a^3
*sin(d*x + c))/d
```

3.64.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.74

$$\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$
$$=\frac{3\,ab^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,ab^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-\cdots\right.}{\cdots}}{d}$$

input
```
integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
(3*a*b^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a*b^2*log(abs(tan(1/2*d*x
+ 1/2*c) - 1)) + 2*(a^3*tan(1/2*d*x + 1/2*c)^3 - 3*a*b^2*tan(1/2*d*x + 1/2
*c)^3 - 3*a^2*b*tan(1/2*d*x + 1/2*c)^2 - a^3*tan(1/2*d*x + 1/2*c) + 3*a*b^
2*tan(1/2*d*x + 1/2*c) + 3*a^2*b - 2*b^3)/(tan(1/2*d*x + 1/2*c)^4 - 1))/d
```

3.64.9 Mupad [B] (verification not implemented)

Time = 22.86 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.35

$$\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx=\frac{6\,a\,b^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$
$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(6\,a\,b^2-2\,a^3\right)-6\,a^2\,b-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(6\,a\,b^2-2\,a^3\right)+4\,b^3+6\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-1\right)}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))^3/cos(c + d*x)^2,x)
```

output
```
(6*a*b^2*atanh(tan(c/2 + (d*x)/2)))/d - (tan(c/2 + (d*x)/2)^3*(6*a*b^2 - 2
*a^3) - 6*a^2*b - tan(c/2 + (d*x)/2)*(6*a*b^2 - 2*a^3) + 4*b^3 + 6*a^2*b*t
an(c/2 + (d*x)/2)^2)/(d*(tan(c/2 + (d*x)/2)^4 - 1))
```

3.65 $\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

3.65.1 Optimal result

Integrand size = 28, antiderivative size = 72

$$\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = a(a^2-3b^2)\,x-\frac{b(3a^2-b^2)\log(\cos(c+dx))}{d}+\frac{2ab^2\tan(c+dx)}{d}+\frac{b(a+b\tan(c+dx))^2}{2d}$$

output
```
a*(a^2-3*b^2)*x-b*(3*a^2-b^2)*ln(cos(d*x+c))/d+2*a*b^2*tan(d*x+c)/d+1/2*b*
(a+b*tan(d*x+c))^2/d
```

3.65.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.10

$$\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = \frac{(ia-b)^3\log(i-\tan(c+dx))-(ia+b)^3\log(i+\tan(c+dx))+6ab^2\tan(c+dx)+b^3\tan^2(c+dx)}{2d}$$

input
```
Integrate[Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

output
```
((I*a - b)^3*Log[I - Tan[c + d*x]] - (I*a + b)^3*Log[I + Tan[c + d*x]] + 6
*a*b^2*Tan[c + d*x] + b^3*Tan[c + d*x]^2)/(2*d)
```

3.65.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3565, 3042, 3963, 3042, 4008, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\cos(c+dx)+b\sin(c+dx))^3}{\cos(c+dx)^3}dx$$

$$\Big\downarrow \text{3565}$$

$$\int (a+b\tan(c+dx))^3dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(c+dx))^3dx$$

$$\Big\downarrow \text{3963}$$

$$\int (a+b\tan(c+dx))\left(a^2+2b\tan(c+dx)a-b^2\right)dx+\frac{b(a+b\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(c+dx))\left(a^2+2b\tan(c+dx)a-b^2\right)dx+\frac{b(a+b\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{4008}$$

$$b\left(3a^2-b^2\right)\int \tan(c+dx)dx+ax\left(a^2-3b^2\right)+\frac{2ab^2\tan(c+dx)}{d}+\frac{b(a+b\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{3042}$$

$$b\left(3a^2-b^2\right)\int \tan(c+dx)dx+ax\left(a^2-3b^2\right)+\frac{2ab^2\tan(c+dx)}{d}+\frac{b(a+b\tan(c+dx))^2}{2d}$$

$$\Big\downarrow \text{3956}$$

$$-\frac{b\left(3a^2-b^2\right)\log(\cos(c+dx))}{d}+ax\left(a^2-3b^2\right)+\frac{2ab^2\tan(c+dx)}{d}+\frac{b(a+b\tan(c+dx))^2}{2d}$$

input
```
Int[Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

output
```
a*(a^2 - 3*b^2)*x - (b*(3*a^2 - b^2)*Log[Cos[c + d*x]])/d + (2*a*b^2*Tan[c
 + d*x])/d + (b*(a + b*Tan[c + d*x])^2)/(2*d)
```

3.65.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3565
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(a + b*Tan[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 3963
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d
*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2
+ b^2, 0] && GtQ[n, 1]
```

rule 4008
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Simp[b*d*(Tan[e + f*x]/f),
x] + Simp[(b*c + a*d)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

3.65.4 Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{a^3(dx+c)-3a^2b\ln(\cos(dx+c))+3a\,b^2(\tan(dx+c)-dx-c)+b^3\left(\frac{\tan(dx+c)^2}{2}+\ln(\cos(dx+c))\right)}{d}$
default	$\frac{a^3(dx+c)-3a^2b\ln(\cos(dx+c))+3a\,b^2(\tan(dx+c)-dx-c)+b^3\left(\frac{\tan(dx+c)^2}{2}+\ln(\cos(dx+c))\right)}{d}$
parts	$\frac{a^3(dx+c)}{d}+\frac{b^3\left(\frac{\tan(dx+c)^2}{2}+\ln(\cos(dx+c))\right)}{d}+\frac{3a\,b^2(\tan(dx+c)-dx-c)}{d}+\frac{3a^2b\ln(\sec(dx+c))}{d}$
risch	$3ia^2bx-ix\,b^3+a^3x-3a\,b^2x+\frac{6ib\,a^2c}{d}-\frac{2ib^3c}{d}+\frac{2b^2(3ia\,e^{2i(dx+c)}+b\,e^{2i(dx+c)}+3ia)}{d(e^{2i(dx+c)}+1)^2}-\frac{3b\ln(e^{2i(dx+}{d}$
parallelrisch	$\frac{6b(1+\cos(2dx+2c))\left(a^2-\frac{b^2}{3}\right)\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)-6b(1+\cos(2dx+2c))\left(a^2-\frac{b^2}{3}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-6b(1+\cos(2dx+2}{2d(1+\cos(2dx}$
norman	$\frac{(a^3-3a\,b^2)x+(-2a^3+6a\,b^2)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+(-2a^3+6a\,b^2)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+(a^3-3a\,b^2)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+(a^3-3a\,b^2)x}{}$

input
```
int(sec(d*x+c)^3*(cos(d*x+c)*a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(d*x+c)-3*a^2*b*ln(cos(d*x+c))+3*a*b^2*(tan(d*x+c)-d*x-c)+b^3*(1/
2*tan(d*x+c)^2+ln(cos(d*x+c))))
```

3.65.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.22

$$\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$
$$=\frac{2\,(a^3-3\,ab^2)dx\cos\left(dx+c\right)^2+6\,ab^2\cos\left(dx+c\right)\sin\left(dx+c\right)-2\,(3\,a^2b-b^3)\cos\left(dx+c\right)^2\log\left(-\cos\left(dx\right.\right.}{2\,d\cos\left(dx+c\right)^2}$$

input
```
integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas"
)
```

output
```
1/2*(2*(a^3 - 3*a*b^2)*d*x*cos(d*x + c)^2 + 6*a*b^2*cos(d*x + c)*sin(d*x +
 c) - 2*(3*a^2*b - b^3)*cos(d*x + c)^2*log(-cos(d*x + c)) + b^3)/(d*cos(d*
x + c)^2)
```

3.65.6 Sympy [F]

$$\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$
$$=\int \left(a\cos\left(c+dx\right)+b\sin\left(c+dx\right)\right)^3\sec^3\left(c+dx\right)dx$$

input
```
integrate(sec(d*x+c)**3*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

output
```
Integral((a*cos(c + d*x) + b*sin(c + d*x))**3*sec(c + d*x)**3, x)
```

3.65.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.18

$$\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$
$$=\frac{2\left(dx+c\right)a^3-6\left(dx+c-\tan\left(dx+c\right)\right)ab^2-b^3\left(\frac{1}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)^2-1\right)\right)-3\,a^2b\log\left(-\sin\right.}{2\,d}$$

input
```
integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima"
)
```

output
```
1/2*(2*(d*x + c)*a^3 - 6*(d*x + c - tan(d*x + c))*a*b^2 - b^3*(1/(sin(d*x
+ c)^2 - 1) - log(sin(d*x + c)^2 - 1)) - 3*a^2*b*log(-sin(d*x + c)^2 + 1))
/d
```

3.65.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.99

$$\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$
$$=\frac{b^3\tan\left(dx+c\right)^2+6\,ab^2\tan\left(dx+c\right)+2\left(a^3-3\,ab^2\right)\left(dx+c\right)+\left(3\,a^2b-b^3\right)\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d}$$

input
```
integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
1/2*(b^3*tan(d*x + c)^2 + 6*a*b^2*tan(d*x + c) + 2*(a^3 - 3*a*b^2)*(d*x +
c) + (3*a^2*b - b^3)*log(tan(d*x + c)^2 + 1))/d
```

3.65.9 Mupad [B] (verification not implemented)

Time = 23.58 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.54

$$\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = \frac{2\left(\frac{b^3\ln\left(\frac{\cos(c+dx)}{\cos(c+dx)+1}\right)}{2}-\frac{b^3\ln\left(\frac{1}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2}\right)}{2}+a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)+\frac{3a^2b\ln\left(\frac{1}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2}\right)}{2}-3ab^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\cdots\right)}{d} + \frac{\frac{b^3}{2}+\frac{3a\sin(2c+2dx)b^2}{2}}{d\left(\frac{\cos(2c+2dx)}{2}+\frac{1}{2}\right)}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))^3/cos(c + d*x)^3,x)
```

output
```
(2*((b^3*log(cos(c + d*x)/(cos(c + d*x) + 1)))/2 - (b^3*log(1/cos(c/2 + (d
*x)/2)^2))/2 + a^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + (3*a^2*b*
log(1/cos(c/2 + (d*x)/2)^2))/2 - 3*a*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 +
 (d*x)/2)) - (3*a^2*b*log(cos(c + d*x)/(cos(c + d*x) + 1)))/2))/d + (b^3/2
 + (3*a*b^2*sin(2*c + 2*d*x))/2)/(d*(cos(2*c + 2*d*x)/2 + 1/2))
```

3.66 $\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

3.66.1 Optimal result

Integrand size = 28, antiderivative size = 103

$$\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = \frac{a^3\text{arctanh}(\sin(c+dx))}{d}-\frac{3ab^2\text{arctanh}(\sin(c+dx))}{2d}+\frac{3a^2b\sec(c+dx)}{d}-\frac{b^3\sec(c+dx)}{d}+\frac{b^3\sec^3(c+dx)}{3d}+\frac{3ab^2\sec(c+dx)\tan(c+dx)}{2d}$$

output
```
a^3*arctanh(sin(d*x+c))/d-3/2*a*b^2*arctanh(sin(d*x+c))/d+3*a^2*b*sec(d*x+
c)/d-b^3*sec(d*x+c)/d+1/3*b^3*sec(d*x+c)^3/d+3/2*a*b^2*sec(d*x+c)*tan(d*x+
c)/d
```

3.66.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 293 vs. 2(103) = 206.

Time = 1.69 (sec) , antiderivative size = 293, normalized size of antiderivative = 2.84

$$\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = \frac{36a^2b-10b^3-6a(2a^2-3b^2)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+12a^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

output
```
(36*a^2*b - 10*b^3 - 6*a*(2*a^2 - 3*b^2)*Log[Cos[(c + d*x)/2] - Sin[(c + d
*x)/2]] + 12*a^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 18*a*b^2*Log[C
os[(c + d*x)/2] + Sin[(c + d*x)/2]] + (9*a*b^2)/(Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2])^2 + b^3/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + 2*b*(18*a^2
- b^2 + 2*b^2*Cos[c + d*x] + (18*a^2 - 5*b^2)*Cos[2*(c + d*x)])*Sec[c + d*
x]^3*Sin[(c + d*x)/2]^2 - (9*a*b^2)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^
2 + b^3/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(12*d)
```

3.66.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a\cos(c+dx)+b\sin(c+dx))^3}{\cos(c+dx)^4}dx$$

$$\Big\downarrow \text{ 3569}$$

$$\int \left(a^3\sec(c+dx)+3a^2b\tan(c+dx)\sec(c+dx)+3ab^2\tan^2(c+dx)\sec(c+dx)+b^3\tan^3(c+dx)\sec(c+dx)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^3\text{arctanh}(\sin(c+dx))}{d}+\frac{3a^2b\sec(c+dx)}{d}-\frac{3ab^2\text{arctanh}(\sin(c+dx))}{2d}+\frac{3ab^2\tan(c+dx)\sec(c+dx)}{2d}+\frac{b^3\sec^3(c+dx)}{3d}-\frac{b^3\sec(c+dx)}{d}$$

input
```
Int[Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

output
```
(a^3*ArcTanh[Sin[c + d*x]])/d - (3*a*b^2*ArcTanh[Sin[c + d*x]])/(2*d) + (3
*a^2*b*Sec[c + d*x])/d - (b^3*Sec[c + d*x])/d + (b^3*Sec[c + d*x]^3)/(3*d)
 + (3*a*b^2*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

3.66.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

3.66.4 Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.13

method	result
parts	$\frac{a^3 \ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{b^3\left(\frac{\sec(dx+c)^3}{3}-\sec(dx+c)\right)}{d}+\frac{3a\,b^2\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx\cdots}{2}\right.}{d}$
derivativedivides	$\frac{a^3 \ln(\sec(dx+c)+\tan(dx+c))+\frac{3a^2b}{\cos(dx+c)}+3a\,b^2\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+b^3\left(\frac{\sin(dx+c)^4}{3\cos(dx+c)^3}\cdots\right.}{d}$
default	$\frac{a^3 \ln(\sec(dx+c)+\tan(dx+c))+\frac{3a^2b}{\cos(dx+c)}+3a\,b^2\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+b^3\left(\frac{\sin(dx+c)^4}{3\cos(dx+c)^3}\cdots\right.}{d}$
parallelrisch	$\frac{-18a\left(a^2-\frac{3b^2}{2}\right)\left(\frac{\cos(3dx+3c)}{3}+\cos(dx+c)\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+18a\left(a^2-\frac{3b^2}{2}\right)\left(\frac{\cos(3dx+3c)}{3}+\cos(dx+c)\right)\ln\left(\tan\left(\frac{dx}{2}\cdots\right.\right.}{6d(\cos(3dx\cdots}$
risch	$-\frac{b\,e^{i(dx+c)}\left(9iab\,e^{4i(dx+c)}-18a^2e^{4i(dx+c)}+6b^2e^{4i(dx+c)}-36a^2e^{2i(dx+c)}+4b^2e^{2i(dx+c)}-9iab-18a^2+6b^2\right)}{3d\left(e^{2i(dx+c)}+1\right)^3}+\frac{a^3\ln(i+\cdots}{}$
norman	$\frac{\frac{\left(12a^2b-8b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{d}-\frac{18a^2b-4b^3}{3d}-\frac{\left(6a^2b+4b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{d}-\frac{3a\,b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{9a\,b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}-\frac{6a\,b^2\tan\left(\frac{dx}{2}+\cdots\right.}{d}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\cdots\right.}$

input
```
int(sec(d*x+c)^4*(cos(d*x+c)*a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
a^3/d*ln(sec(d*x+c)+tan(d*x+c))+b^3/d*(1/3*sec(d*x+c)^3-sec(d*x+c))+3*a*b^
2/d*(1/2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*sin(d*x+c)-1/2*ln(sec(d*x+c)+tan(d*
x+c)))+3*a^2*b*sec(d*x+c)/d
```

3.66.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.19

$$\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$
$$=\frac{3\,(2\,a^3-3\,ab^2)\cos{(dx+c)}^3\log{(\sin{(dx+c)}+1)}-3\,(2\,a^3-3\,ab^2)\cos{(dx+c)}^3\log{(-\sin{(dx+c)}+1)}}{12\,d\cos{(dx+c)}^3}$$

input
```
integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas"
)
```

output
```
1/12*(3*(2*a^3 - 3*a*b^2)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(2*a^3
- 3*a*b^2)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 18*a*b^2*cos(d*x + c)*s
in(d*x + c) + 4*b^3 + 12*(3*a^2*b - b^3)*cos(d*x + c)^2)/(d*cos(d*x + c)^3
)
```

3.66.6 Sympy [F(-1)]

Timed out.

$$\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx=\text{Timed out}$$

input
```
integrate(sec(d*x+c)**4*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

output
```
Timed out
```

3.66.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.15

$$\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx =$$

$$-\frac{9\,ab^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}+\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)\right)-6\,a^3(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\right.}{12\,d}$$

input
```
integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima"
)
```

output
```
-1/12*(9*a*b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1
) - log(sin(d*x + c) - 1)) - 6*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x +
c) - 1)) - 36*a^2*b/cos(d*x + c) + 4*(3*cos(d*x + c)^2 - 1)*b^3/cos(d*x +
c)^3)/d
```

3.66.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.66

$$\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$

$$=\frac{3\left(2\,a^3-3\,ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(2\,a^3-3\,ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(9\,ab^2\tan\left(\frac{1}{2}\,dx+\right.\right.}{}}{6\,d}$$

input
```
integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
1/6*(3*(2*a^3 - 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(2*a^3 - 3
*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(9*a*b^2*tan(1/2*d*x + 1/2*
c)^5 - 18*a^2*b*tan(1/2*d*x + 1/2*c)^4 + 36*a^2*b*tan(1/2*d*x + 1/2*c)^2 -
 12*b^3*tan(1/2*d*x + 1/2*c)^2 - 9*a*b^2*tan(1/2*d*x + 1/2*c) - 18*a^2*b +
 4*b^3)/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.66.9 Mupad [B] (verification not implemented)

Time = 25.26 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.55

$$\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = -\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\,(3\,a\,b^2-2\,a^3)}{d}$$
$$-\frac{6\,a^2\,b-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(12\,a^2\,b-4\,b^3)-\frac{4\,b^3}{3}+3\,a\,b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+6\,a^2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-3\,a\,b^2\tan\left(\frac{c}{2}+\right.}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))^3/cos(c + d*x)^4,x)
```

output
```
- (atanh(tan(c/2 + (d*x)/2))*(3*a*b^2 - 2*a^3))/d - (6*a^2*b - tan(c/2 + (
d*x)/2)^2*(12*a^2*b - 4*b^3) - (4*b^3)/3 + 3*a*b^2*tan(c/2 + (d*x)/2) + 6*
a^2*b*tan(c/2 + (d*x)/2)^4 - 3*a*b^2*tan(c/2 + (d*x)/2)^5)/(d*(3*tan(c/2 +
 (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

3.67 $\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

3.67.1 Optimal result

Integrand size = 28, antiderivative size = 30

$$\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx=\frac{(b+a\cot(c+dx))^4\tan^4(c+dx)}{4bd}$$

output
```
1/4*(b+a*cot(d*x+c))^4*tan(d*x+c)^4/b/d
```

3.67.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.90

$$\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$
$$=\frac{\tan(c+dx)\left(4a^3+6a^2b\tan(c+dx)+4ab^2\tan^2(c+dx)+b^3\tan^3(c+dx)\right)}{4d}$$

input
```
Integrate[Sec[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

output
```
(Tan[c + d*x]*(4*a^3 + 6*a^2*b*Tan[c + d*x] + 4*a*b^2*Tan[c + d*x]^2 + b^3
*Tan[c + d*x]^3))/(4*d)
```

3.67.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3042, 3567, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\cos(c+dx)+b\sin(c+dx))^3}{\cos(c+dx)^5}dx$$

$$\Big\downarrow \text{3567}$$

$$-\frac{\int (b+a\cot(c+dx))^3\tan^5(c+dx)d\cot(c+dx)}{d}$$

$$\Big\downarrow \text{48}$$

$$\frac{\tan^4(c+dx)(a\cot(c+dx)+b)^4}{4bd}$$

input
```
Int[Sec[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

output
```
((b + a*Cot[c + d*x])^4*Tan[c + d*x]^4)/(4*b*d)
```

3.67.3.1 Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3567
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[x^m*((b
+ a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a,
b, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[
n, 0] && GtQ[m, 1])
```

3.67.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(28) = 56$.

Time = 1.21 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.40

method	result
derivativedivides	$\frac{a^3\tan(dx+c)+\frac{3a^2b}{2\cos(dx+c)^2}+\frac{a\,b^2\sin(dx+c)^3}{\cos(dx+c)^3}+\frac{b^3\sin(dx+c)^4}{4\cos(dx+c)^4}}{d}$
default	$\frac{a^3\tan(dx+c)+\frac{3a^2b}{2\cos(dx+c)^2}+\frac{a\,b^2\sin(dx+c)^3}{\cos(dx+c)^3}+\frac{b^3\sin(dx+c)^4}{4\cos(dx+c)^4}}{d}$
parts	$\frac{a^3\tan(dx+c)}{d}+\frac{b^3\left(\frac{\sec(dx+c)^4}{4}-\frac{\sec(dx+c)^2}{2}\right)}{d}+\frac{a\,b^2\sin(dx+c)^3}{d\cos(dx+c)^3}+\frac{3a^2b\sec(dx+c)^2}{2d}$
parallelrisch	$-\frac{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4a^2-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3ab+\left(-2a^2+2b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+2\tan\left(\frac{dx}{2}\right.}{d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}$
risch	$-\frac{2\left(-ia^3e^{6i(dx+c)}+3ia\,b^2e^{6i(dx+c)}-3a^2b\,e^{6i(dx+c)}+b^3e^{6i(dx+c)}-3ia^3e^{4i(dx+c)}+3ia\,b^2e^{4i(dx+c)}-6a^2b\,e^{4i(dx+c)}-3ia\right.}{d\left(e^{2i(dx+c)}+1\right)^4}$
norman	$\frac{6a^2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d}+\frac{6a^2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{d}+\frac{\left(6a^2b+4b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{d}+\frac{\left(6a^2b+4b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{d}+\frac{2a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2a^3\tan}{}$

input
```
int(sec(d*x+c)^5*(cos(d*x+c)*a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*tan(d*x+c)+3/2*a^2*b/cos(d*x+c)^2+a*b^2*sin(d*x+c)^3/cos(d*x+c)^3
+1/4*b^3*sin(d*x+c)^4/cos(d*x+c)^4)
```

3.67.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(28) = 56$.

Time = 0.24 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.60

$$\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = \frac{b^3+2\,(3\,a^2b-b^3)\cos{(dx+c)}^2+4\,\left(ab^2\cos{(dx+c)}+(a^3-ab^2)\cos{(dx+c)}^3\right)\sin{(dx+c)}}{4\,d\cos{(dx+c)}^4}$$

input
```
integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas"
)
```

output
```
1/4*(b^3 + 2*(3*a^2*b - b^3)*cos(d*x + c)^2 + 4*(a*b^2*cos(d*x + c) + (a^3
 - a*b^2)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

3.67.6 Sympy [F(-1)]

Timed out.

$$\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**5*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

output
```
Timed out
```

3.67.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(28) = 56$.

Time = 0.23 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.90

$$\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = \frac{4\,ab^2\tan{(dx+c)}^3+4\,a^3\tan{(dx+c)}+\frac{\left(2\,\sin(dx+c)^2-1\right)b^3}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-\frac{6\,a^2b}{\sin(dx+c)^2-1}}{4\,d}$$

input
```
integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima"
)
```

output
```
1/4*(4*a*b^2*tan(d*x + c)^3 + 4*a^3*tan(d*x + c) + (2*sin(d*x + c)^2 - 1)*
b^3/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 6*a^2*b/(sin(d*x + c)^2 - 1)
)/d
```

3.67.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(28) = 56$.

Time = 0.37 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.90

$$\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = \frac{b^3\tan{(dx+c)}^4+4\,ab^2\tan{(dx+c)}^3+6\,a^2b\tan{(dx+c)}^2+4\,a^3\tan{(dx+c)}}{4\,d}$$

input
```
integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
1/4*(b^3*tan(d*x + c)^4 + 4*a*b^2*tan(d*x + c)^3 + 6*a^2*b*tan(d*x + c)^2
+ 4*a^3*tan(d*x + c))/d
```

3.67.9 Mupad [B] (verification not implemented)

Time = 22.63 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.93

$$\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = \frac{\cos{(c+d\,x)}^3\,\left(a^3\,\sin{(c+d\,x)}-a\,b^2\,\sin{(c+d\,x)}\right)+\cos{(c+d\,x)}^2\,\left(\frac{3\,a^2\,b}{2}-\frac{b^3}{2}\right)+\frac{b^3}{4}+a\,b^2\,\cos{(c+d\,x)}\,\mathrm{s}}{d\cos{(c+d\,x)}^4}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))^3/cos(c + d*x)^5,x)
```

output
```
(cos(c + d*x)^3*(a^3*sin(c + d*x) - a*b^2*sin(c + d*x)) + cos(c + d*x)^2*(
(3*a^2*b)/2 - b^3/2) + b^3/4 + a*b^2*cos(c + d*x)*sin(c + d*x))/(d*cos(c +
 d*x)^4)
```

3.68 $\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

3.68.1 Optimal result

Integrand size = 28, antiderivative size = 158

$$\begin{aligned}\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx &= \frac{a^3\operatorname{arctanh}(\sin(c+dx))}{2d}-\frac{3ab^2\operatorname{arctanh}(\sin(c+dx))}{8d}+\frac{a^2b\sec^3(c+dx)}{d}\\ &\quad-\frac{b^3\sec^3(c+dx)}{3d}+\frac{b^3\sec^5(c+dx)}{5d}+\frac{a^3\sec(c+dx)\tan(c+dx)}{2d}\\ &\quad-\frac{3ab^2\sec(c+dx)\tan(c+dx)}{8d}+\frac{3ab^2\sec^3(c+dx)\tan(c+dx)}{4d}\end{aligned}$$

output
```
1/2*a^3*arctanh(sin(d*x+c))/d-3/8*a*b^2*arctanh(sin(d*x+c))/d+a^2*b*sec(d*
x+c)^3/d-1/3*b^3*sec(d*x+c)^3/d+1/5*b^3*sec(d*x+c)^5/d+1/2*a^3*sec(d*x+c)*
tan(d*x+c)/d-3/8*a*b^2*sec(d*x+c)*tan(d*x+c)/d+3/4*a*b^2*sec(d*x+c)^3*tan(
d*x+c)/d
```

3.68.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 464 vs. 2(158) = 316.

Time = 1.32 (sec) , antiderivative size = 464, normalized size of antiderivative = 2.94

$$\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = \frac{\sec^5(c+dx)\left(960a^2b+64b^3+320(3a^2b-b^3)\cos(2(c+dx))-300a^3\cos(3(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]^6*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

output
```
(Sec[c + d*x]^5*(960*a^2*b + 64*b^3 + 320*(3*a^2*b - b^3)*Cos[2*(c + d*x)]
 - 300*a^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 225
*a*b^2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 60*a^3*
Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 45*a*b^2*Cos[5
*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 150*a*(4*a^2 - 3*b^
2)*Cos[c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]]) + 300*a^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2
] + Sin[(c + d*x)/2]] - 225*a*b^2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] +
Sin[(c + d*x)/2]] + 60*a^3*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2]] - 45*a*b^2*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x
)/2]] + 240*a^3*Sin[2*(c + d*x)] + 540*a*b^2*Sin[2*(c + d*x)] + 120*a^3*Si
n[4*(c + d*x)] - 90*a*b^2*Sin[4*(c + d*x)]))/(1920*d)
```

3.68.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\cos(c+dx)+b\sin(c+dx))^3}{\cos(c+dx)^6}dx$$

$$\Big\downarrow \text{3569}$$

$$\int \left(a^3\sec^3(c+dx)+3a^2b\tan(c+dx)\sec^3(c+dx)+3ab^2\tan^2(c+dx)\sec^3(c+dx)+b^3\tan^3(c+dx)\sec^3(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3\text{arctanh}(\sin(c+dx))}{2d}+\frac{a^3\tan(c+dx)\sec(c+dx)}{2d}+\frac{a^2b\sec^3(c+dx)}{d}-\frac{3ab^2\text{arctanh}(\sin(c+dx))}{8d}+\frac{3ab^2\tan(c+dx)\sec^3(c+dx)}{4d}-\frac{3ab^2\tan(c+dx)\sec(c+dx)}{8d}+\frac{b^3\sec^5(c+dx)}{5d}-\frac{b^3\sec^3(c+dx)}{3d}$$

input
```
Int[Sec[c + d*x]^6*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

output
```
(a^3*ArcTanh[Sin[c + d*x]])/(2*d) - (3*a*b^2*ArcTanh[Sin[c + d*x]])/(8*d)
+ (a^2*b*Sec[c + d*x]^3)/d - (b^3*Sec[c + d*x]^3)/(3*d) + (b^3*Sec[c + d*x
]^5)/(5*d) + (a^3*Sec[c + d*x]*Tan[c + d*x])/(2*d) - (3*a*b^2*Sec[c + d*x]
*Tan[c + d*x])/(8*d) + (3*a*b^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

3.68.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

3.68.4 Maple [A] (verified)

Time = 1.62 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.97

method	result
parts	$\frac{a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}+\frac{b^3\left(\frac{\sec(dx+c)^5}{5}-\frac{\sec(dx+c)^3}{3}\right)}{d}+\frac{3a\,b^2\left(\frac{\sin(dx+c)^3}{4\cos(dx+c)^4}+\frac{\sin(dx+c)}{8\cos(dx+c}\right.}{}$
derivativedivides	$\frac{a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{a^2b}{\cos(dx+c)^3}+3a\,b^2\left(\frac{\sin(dx+c)^3}{4\cos(dx+c)^4}+\frac{\sin(dx+c)^3}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\ln(\sec(dx\right.}{d}$
default	$\frac{a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{a^2b}{\cos(dx+c)^3}+3a\,b^2\left(\frac{\sin(dx+c)^3}{4\cos(dx+c)^4}+\frac{\sin(dx+c)^3}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\ln(\sec(dx\right.}{d}$
parallelrisch	$\frac{-600\left(a^2-\frac{3b^2}{4}\right)\left(\frac{\cos(5dx+5c)}{10}+\frac{\cos(3dx+3c)}{2}+\cos(dx+c)\right)a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+600\left(a^2-\frac{3b^2}{4}\right)\left(\frac{\cos(5dx+5c)}{10}+\frac{\cos(3dx+3}{2}\right.}{}$
risch	$-\frac{e^{i(dx+c)}\left(60ia^3e^{8i(dx+c)}-45ia\,b^2e^{8i(dx+c)}+120ia^3e^{6i(dx+c)}+270ia\,b^2e^{6i(dx+c)}-480a^2b\,e^{6i(dx+c)}+160b^3e^{6i(dx+c)}-\right.}{60d(e^{2}}$
norman	$\frac{-\frac{30a^2b-4b^3}{15d}-\frac{6a^2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{14}}{d}-\frac{2\left(3a^2b+2b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{d}+\frac{2\left(15a^2b-20b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{3d}-\frac{2\left(15a^2b+4b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{15d}}{}$

input
```
int(sec(d*x+c)^6*(cos(d*x+c)*a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
a^3/d*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+b^3/d*(1/5
*sec(d*x+c)^5-1/3*sec(d*x+c)^3)+3*a*b^2/d*(1/4*sin(d*x+c)^3/cos(d*x+c)^4+1
/8*sin(d*x+c)^3/cos(d*x+c)^2+1/8*sin(d*x+c)-1/8*ln(sec(d*x+c)+tan(d*x+c)))
+a^2*b*sec(d*x+c)^3/d
```

3.68.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.93

$$\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$
$$=\frac{15\left(4a^3-3ab^2\right)\cos\left(dx+c\right)^5\log\left(\sin\left(dx+c\right)+1\right)-15\left(4a^3-3ab^2\right)\cos\left(dx+c\right)^5\log\left(-\sin\left(dx+c\right)+1\right.}{240\,d\cos}$$

input
```
integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas"
)
```

output
```
1/240*(15*(4*a^3 - 3*a*b^2)*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*(4*a
^3 - 3*a*b^2)*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 48*b^3 + 80*(3*a^2*b
 - b^3)*cos(d*x + c)^2 + 30*(6*a*b^2*cos(d*x + c) + (4*a^3 - 3*a*b^2)*cos(
d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

3.68.6 Sympy [F(-1)]

Timed out.

$$\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**6*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

output
```
Timed out
```

3.68.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.99

$$\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$
$$= \frac{45\,ab^2\left(\frac{2\left(\sin(dx+c)^3+\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1} - \log\left(\sin\left(dx+c\right)+1\right) + \log\left(\sin\left(dx+c\right)-1\right)\right) - 60\,a^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} - \log\right.}{240\,d}$$

input
```
integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima"
)
```

output
```
1/240*(45*a*b^2*(2*(sin(d*x + c)^3 + sin(d*x + c))/(sin(d*x + c)^4 - 2*sin
(d*x + c)^2 + 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 60*a^3
*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*
x + c) - 1)) + 240*a^2*b/cos(d*x + c)^3 - 16*(5*cos(d*x + c)^2 - 3)*b^3/co
s(d*x + c)^5)/d
```

3.68.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. 2(144) = 288.

Time = 0.38 (sec) , antiderivative size = 333, normalized size of antiderivative = 2.11

$$\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$
$$= \frac{15\left(4\,a^3-3\,ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 15\left(4\,a^3-3\,ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{2\left(60\,a^3\tan\left(\frac{1}{2}\right.\right.}{}}{}$$

input
```
integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
1/120*(15*(4*a^3 - 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(4*a^3
 - 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(60*a^3*tan(1/2*d*x + 1
/2*c)^9 + 45*a*b^2*tan(1/2*d*x + 1/2*c)^9 - 360*a^2*b*tan(1/2*d*x + 1/2*c)
^8 - 120*a^3*tan(1/2*d*x + 1/2*c)^7 + 270*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 7
20*a^2*b*tan(1/2*d*x + 1/2*c)^6 - 240*b^3*tan(1/2*d*x + 1/2*c)^6 - 480*a^2
*b*tan(1/2*d*x + 1/2*c)^4 - 80*b^3*tan(1/2*d*x + 1/2*c)^4 + 120*a^3*tan(1/
2*d*x + 1/2*c)^3 - 270*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 240*a^2*b*tan(1/2*d*
x + 1/2*c)^2 - 80*b^3*tan(1/2*d*x + 1/2*c)^2 - 60*a^3*tan(1/2*d*x + 1/2*c)
 - 45*a*b^2*tan(1/2*d*x + 1/2*c) - 120*a^2*b + 16*b^3)/(tan(1/2*d*x + 1/2*
c)^2 - 1)^5)/d
```

3.68.9 Mupad [B] (verification not implemented)

Time = 26.53 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.85

$$\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$
$$= \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9\left(a^3+\frac{3\,a\,b^2}{4}\right)-2\,a^2\,b-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{9\,a\,b^2}{2}-2\,a^3\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7\left(\frac{9\,a\,b^2}{2}-2\,a^3\right)+\tan\left(\frac{c}{2}\right.}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\right.\right.}$$
$$-\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(\frac{3\,a\,b^2}{4}-a^3\right)}{d}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))^3/cos(c + d*x)^6,x)
```

output
```
(tan(c/2 + (d*x)/2)^9*((3*a*b^2)/4 + a^3) - 2*a^2*b - tan(c/2 + (d*x)/2)^3
*((9*a*b^2)/2 - 2*a^3) + tan(c/2 + (d*x)/2)^7*((9*a*b^2)/2 - 2*a^3) + tan(
c/2 + (d*x)/2)^2*(4*a^2*b - (4*b^3)/3) - tan(c/2 + (d*x)/2)^4*(8*a^2*b + (
4*b^3)/3) + tan(c/2 + (d*x)/2)^6*(12*a^2*b - 4*b^3) + (4*b^3)/15 - tan(c/2
 + (d*x)/2)*((3*a*b^2)/4 + a^3) - 6*a^2*b*tan(c/2 + (d*x)/2)^8)/(d*(5*tan(
c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*t
an(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1)) - (atanh(tan(c/2 + (d*x)
/2))*((3*a*b^2)/4 - a^3))/d
```

3.69 $\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

3.69.1 Optimal result

Integrand size = 28, antiderivative size = 120

$$\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = \frac{a^3\tan(c+dx)}{d}+\frac{3a^2b\tan^2(c+dx)}{2d}+\frac{a(a^2+3b^2)\tan^3(c+dx)}{3d}+\frac{b(3a^2+b^2)\tan^4(c+dx)}{4d}+\frac{3ab^2\tan^5(c+dx)}{5d}+\frac{b^3\tan^6(c+dx)}{6d}$$

output
```
a^3*tan(d*x+c)/d+3/2*a^2*b*tan(d*x+c)^2/d+1/3*a*(a^2+3*b^2)*tan(d*x+c)^3/d
+1/4*b*(3*a^2+b^2)*tan(d*x+c)^4/d+3/5*a*b^2*tan(d*x+c)^5/d+1/6*b^3*tan(d*x
+c)^6/d
```

3.69.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.45

$$\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = \frac{(a+b\tan(c+dx))^4\left(a^2+15b^2-4ab\tan(c+dx)+10b^2\tan^2(c+dx)\right)}{60b^3d}$$

input
```
Integrate[Sec[c + d*x]^7*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

output
```
((a + b*Tan[c + d*x])^4*(a^2 + 15*b^2 - 4*a*b*Tan[c + d*x] + 10*b^2*Tan[c
+ d*x]^2))/(60*b^3*d)
```

3.69.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3567, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a\cos(c+dx)+b\sin(c+dx))^3}{\cos(c+dx)^7}dx$$

$$\Big\downarrow \; 3567$$

$$-\frac{\int (b+a\cot(c+dx))^3\left(\cot^2(c+dx)+1\right)\tan^7(c+dx)d\cot(c+dx)}{d}$$

$$\Big\downarrow \; 522$$

$$-\frac{\int \left(b^3\tan^7(c+dx)+3ab^2\tan^6(c+dx)+\left(b^3+3a^2b\right)\tan^5(c+dx)+\left(a^3+3b^2a\right)\tan^4(c+dx)+3a^2b\tan^3(c+d}{d}$$

$$\Big\downarrow \; 2009$$

$$-\frac{-a^3\tan(c+dx)-\frac{1}{4}b\left(3a^2+b^2\right)\tan^4(c+dx)-\frac{1}{3}a\left(a^2+3b^2\right)\tan^3(c+dx)-\frac{3}{2}a^2b\tan^2(c+dx)-\frac{3}{5}ab^2\tan^5(c+}{d}$$

input
```
Int[Sec[c + d*x]^7*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

output
```
-((-(a^3*Tan[c + d*x]) - (3*a^2*b*Tan[c + d*x]^2)/2 - (a*(a^2 + 3*b^2)*Tan
[c + d*x]^3)/3 - (b*(3*a^2 + b^2)*Tan[c + d*x]^4)/4 - (3*a*b^2*Tan[c + d*x
]^5)/5 - (b^3*Tan[c + d*x]^6)/6)/d)
```

3.69.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3567
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[x^m*((b
+ a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a,
b, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[
n, 0] && GtQ[m, 1])
```

3.69.4 Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.99

method	result
parts	$-\frac{a^{3}\left(-\frac{2}{3}-\frac{\sec(dx+c)^{2}}{3}\right)\tan(dx+c)}{d}+\frac{b^{3}\left(\frac{\sec(dx+c)^{6}}{6}-\frac{\sec(dx+c)^{4}}{4}\right)}{d}+\frac{3a\,b^{2}\left(\frac{\sin(dx+c)^{3}}{5\cos(dx+c)^{5}}+\frac{2\sin(dx+c)^{3}}{15\cos(dx+c)^{3}}\right)}{d}+\frac{3a^{2}b}{}$
derivativedivides	$\frac{-a^{3}\left(-\frac{2}{3}-\frac{\sec(dx+c)^{2}}{3}\right)\tan(dx+c)+\frac{3a^{2}b}{4\cos(dx+c)^{4}}+3a\,b^{2}\left(\frac{\sin(dx+c)^{3}}{5\cos(dx+c)^{5}}+\frac{2\sin(dx+c)^{3}}{15\cos(dx+c)^{3}}\right)+b^{3}\left(\frac{\sin(dx+c)^{4}}{6\cos(dx+c)^{6}}+\frac{\sin(dx+c)^{4}}{12\cos(dx+c)^{4}}\right.}{d}$
default	$\frac{-a^{3}\left(-\frac{2}{3}-\frac{\sec(dx+c)^{2}}{3}\right)\tan(dx+c)+\frac{3a^{2}b}{4\cos(dx+c)^{4}}+3a\,b^{2}\left(\frac{\sin(dx+c)^{3}}{5\cos(dx+c)^{5}}+\frac{2\sin(dx+c)^{3}}{15\cos(dx+c)^{3}}\right)+b^{3}\left(\frac{\sin(dx+c)^{4}}{6\cos(dx+c)^{6}}+\frac{\sin(dx+c)^{4}}{12\cos(dx+c)^{4}}\right.}{d}$
risch	$-\frac{4\left(-15ia^{3}e^{8i(dx+c)}+45ia\,b^{2}e^{8i(dx+c)}-45a^{2}b\,e^{8i(dx+c)}+15b^{3}e^{8i(dx+c)}-50ia^{3}e^{6i(dx+c)}+30ia\,b^{2}e^{6i(dx+c)}-90a^{2}b\,e^{6i(\right.}{15d\left(e^{2i}\right.}$
parallelrisch	$-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(15\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}a^{3}-45\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}a^{2}b-55\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}a^{3}+60\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}a\,b^{2}+90\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{}$

input
```
int(sec(d*x+c)^7*(cos(d*x+c)*a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-a^3/d*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+b^3/d*(1/6*sec(d*x+c)^6-1/4*sec(
d*x+c)^4)+3*a*b^2/d*(1/5*sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c)^3/cos(d
*x+c)^3)+3/4*a^2*b/d*sec(d*x+c)^4
```

3.69.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.88

$$\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$
$$=\frac{10\,b^3+15\,(3\,a^2b-b^3)\cos{(dx+c)}^2+4\,\left(2\,(5\,a^3-3\,ab^2)\cos{(dx+c)}^5+9\,ab^2\cos{(dx+c)}+(5\,a^3-3\,ab^2)\right.}{60\,d\cos{(dx+c)}^6}$$

input
```
integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas"
)
```

output
```
1/60*(10*b^3 + 15*(3*a^2*b - b^3)*cos(d*x + c)^2 + 4*(2*(5*a^3 - 3*a*b^2)*
cos(d*x + c)^5 + 9*a*b^2*cos(d*x + c) + (5*a^3 - 3*a*b^2)*cos(d*x + c)^3)*
sin(d*x + c))/(d*cos(d*x + c)^6)
```

3.69.6 Sympy [F(-1)]

Timed out.

$$\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx=\text{Timed out}$$

input
```
integrate(sec(d*x+c)**7*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

output
```
Timed out
```

3.69.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.02

$$\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$
$$= \frac{20\left(\tan(dx+c)^3+3\tan(dx+c)\right)a^3+12\left(3\tan(dx+c)^5+5\tan(dx+c)^3\right)ab^2-\frac{5\left(3\sin(dx+c)^2-\right.}{\sin(dx+c)^6-3\sin(dx+c)^4+}}{60\,d}$$

input
```
integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima"
)
```

output
```
1/60*(20*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^3 + 12*(3*tan(d*x + c)^5 + 5*
tan(d*x + c)^3)*a*b^2 - 5*(3*sin(d*x + c)^2 - 1)*b^3/(sin(d*x + c)^6 - 3*s
in(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) + 45*a^2*b/(sin(d*x + c)^2 - 1)^2)/d
```

3.69.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.93

$$\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$
$$= \frac{10\,b^3\tan(dx+c)^6+36\,ab^2\tan(dx+c)^5+45\,a^2b\tan(dx+c)^4+15\,b^3\tan(dx+c)^4+20\,a^3\tan(dx+c)^3}{60\,d}$$

input
```
integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
1/60*(10*b^3*tan(d*x + c)^6 + 36*a*b^2*tan(d*x + c)^5 + 45*a^2*b*tan(d*x +
 c)^4 + 15*b^3*tan(d*x + c)^4 + 20*a^3*tan(d*x + c)^3 + 60*a*b^2*tan(d*x +
 c)^3 + 90*a^2*b*tan(d*x + c)^2 + 60*a^3*tan(d*x + c))/d
```

3.69.9 Mupad [B] (verification not implemented)

Time = 22.48 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.02

$$\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$
$$=\frac{\cos(c+d\,x)^3\left(\frac{a^3\,\sin(c+d\,x)}{3}-\frac{a\,b^2\,\sin(c+d\,x)}{5}\right)+\cos(c+d\,x)^5\left(\frac{2\,a^3\,\sin(c+d\,x)}{3}-\frac{2\,a\,b^2\,\sin(c+d\,x)}{5}\right)+\cos(c+d\,x)^2\,(}{d\cos(c+d\,x)^6}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))^3/cos(c + d*x)^7,x)
```

output
```
(cos(c + d*x)^3*((a^3*sin(c + d*x))/3 - (a*b^2*sin(c + d*x))/5) + cos(c +
d*x)^5*((2*a^3*sin(c + d*x))/3 - (2*a*b^2*sin(c + d*x))/5) + cos(c + d*x)^
2*((3*a^2*b)/4 - b^3/4) + b^3/6 + (3*a*b^2*cos(c + d*x)*sin(c + d*x))/5)/(
d*cos(c + d*x)^6)
```

3.70 $\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

3.70.1 Optimal result

Integrand size = 28, antiderivative size = 210

$$\begin{aligned}\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx &= \frac{3a^3\operatorname{arctanh}(\sin(c+dx))}{8d}-\frac{3ab^2\operatorname{arctanh}(\sin(c+dx))}{16d}+\frac{3a^2b\sec^5(c+dx)}{5d}\\ &-\frac{b^3\sec^5(c+dx)}{5d}+\frac{b^3\sec^7(c+dx)}{7d}+\frac{3a^3\sec(c+dx)\tan(c+dx)}{8d}\\ &-\frac{3ab^2\sec(c+dx)\tan(c+dx)}{16d}+\frac{a^3\sec^3(c+dx)\tan(c+dx)}{4d}\\ &-\frac{ab^2\sec^3(c+dx)\tan(c+dx)}{8d}+\frac{ab^2\sec^5(c+dx)\tan(c+dx)}{2d}\end{aligned}$$

output
```
3/8*a^3*arctanh(sin(d*x+c))/d-3/16*a*b^2*arctanh(sin(d*x+c))/d+3/5*a^2*b*s
ec(d*x+c)^5/d-1/5*b^3*sec(d*x+c)^5/d+1/7*b^3*sec(d*x+c)^7/d+3/8*a^3*sec(d*
x+c)*tan(d*x+c)/d-3/16*a*b^2*sec(d*x+c)*tan(d*x+c)/d+1/4*a^3*sec(d*x+c)^3*
tan(d*x+c)/d-1/8*a*b^2*sec(d*x+c)^3*tan(d*x+c)/d+1/2*a*b^2*sec(d*x+c)^5*ta
n(d*x+c)/d
```

3.70.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 637 vs. $2(210) = 420$.

Time = 2.01 (sec) , antiderivative size = 637, normalized size of antiderivative = 3.03

$$\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$
$$= \frac{\sec^7(c+dx)\left(10752a^2b+1536b^3+3584(3a^2b-b^3)\cos(2(c+dx))-4410a^3\cos(3(c+dx))\log\left(\cos\left(\frac{1}{2}(c+\right.\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]^8*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

output
```
(Sec[c + d*x]^7*(10752*a^2*b + 1536*b^3 + 3584*(3*a^2*b - b^3)*Cos[2*(c +
d*x)] - 4410*a^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
 + 2205*a*b^2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] -
1470*a^3*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 735*a
*b^2*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 210*a^3*C
os[7*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 105*a*b^2*Cos[7
*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 3675*a*(2*a^2 - b^2
)*Cos[c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*
x)/2] + Sin[(c + d*x)/2]]) + 4410*a^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2
] + Sin[(c + d*x)/2]] - 2205*a*b^2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2]] + 1470*a^3*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[
(c + d*x)/2]] - 735*a*b^2*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2]] + 210*a^3*Cos[7*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/
2]] - 105*a*b^2*Cos[7*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]
+ 4340*a^3*Sin[2*(c + d*x)] + 6790*a*b^2*Sin[2*(c + d*x)] + 2800*a^3*Sin[4
*(c + d*x)] - 1400*a*b^2*Sin[4*(c + d*x)] + 420*a^3*Sin[6*(c + d*x)] - 210
*a*b^2*Sin[6*(c + d*x)]))/(35840*d)
```

3.70.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\cos(c+dx)+b\sin(c+dx))^3}{\cos(c+dx)^8}dx$$

$$\Big\downarrow \text{3569}$$

$$\int \left(a^3\sec^5(c+dx)+3a^2b\tan(c+dx)\sec^5(c+dx)+3ab^2\tan^2(c+dx)\sec^5(c+dx)+b^3\tan^3(c+dx)\sec^5(c+dx\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{3a^3\text{arctanh}(\sin(c+dx))}{8d}+\frac{a^3\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3a^3\tan(c+dx)\sec(c+dx)}{8d}+\frac{3a^2b\sec^5(c+dx)}{5d}-\frac{3ab^2\text{arctanh}(\sin(c+dx))}{16d}+\frac{ab^2\tan(c+dx)\sec^5(c+dx)}{2d}-\frac{ab^2\tan(c+dx)\sec^3(c+dx)}{8d}-\frac{3ab^2\tan(c+dx)\sec(c+dx)}{16d}+\frac{b^3\sec^7(c+dx)}{7d}-\frac{b^3\sec^5(c+dx)}{5d}$$

input
```
Int[Sec[c + d*x]^8*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

output
```
(3*a^3*ArcTanh[Sin[c + d*x]])/(8*d) - (3*a*b^2*ArcTanh[Sin[c + d*x]])/(16*
d) + (3*a^2*b*Sec[c + d*x]^5)/(5*d) - (b^3*Sec[c + d*x]^5)/(5*d) + (b^3*Se
c[c + d*x]^7)/(7*d) + (3*a^3*Sec[c + d*x]*Tan[c + d*x])/(8*d) - (3*a*b^2*S
ec[c + d*x]*Tan[c + d*x])/(16*d) + (a^3*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
 - (a*b^2*Sec[c + d*x]^3*Tan[c + d*x])/(8*d) + (a*b^2*Sec[c + d*x]^5*Tan[c
 + d*x])/(2*d)
```

3.70.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

3.70.4 Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.89

method	result
parts	$\frac{a^{3}\left(-\left(-\frac{\sec(dx+c)^{3}}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}+\frac{b^{3}\left(\frac{\sec(dx+c)^{7}}{7}-\frac{\sec(dx+c)^{5}}{5}\right)}{d}+\frac{3a\,b^{2}\left(\right.}{}$
derivativedivides	$\frac{a^{3}\left(-\left(-\frac{\sec(dx+c)^{3}}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{3a^{2}b}{5\cos(dx+c)^{5}}+3a\,b^{2}\left(\frac{\sin(dx+c)^{3}}{6\cos(dx+c)^{6}}+\frac{\sin(dx+}{8\cos(dx}\right.}{}$
default	$\frac{a^{3}\left(-\left(-\frac{\sec(dx+c)^{3}}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{3a^{2}b}{5\cos(dx+c)^{5}}+3a\,b^{2}\left(\frac{\sin(dx+c)^{3}}{6\cos(dx+c)^{6}}+\frac{\sin(dx+}{8\cos(dx}\right.}{}$
parallelrisch	$\frac{-210\left(a^{2}-\frac{b^{2}}{2}\right)a(\cos(7dx+7c)+7\cos(5dx+5c)+21\cos(3dx+3c)+35\cos(dx+c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+210\left(a^{2}-\frac{b^{2}}{2}\right)a(\cos}{}$
risch	$-\frac{e^{i(dx+c)}\left(-210ia^{3}+3395ia\,b^{2}e^{8i(dx+c)}+210ia^{3}e^{12i(dx+c)}+2170ia^{3}e^{8i(dx+c)}-700ia\,b^{2}e^{10i(dx+c)}-3395ia\,b^{2}e^{4i(dx+c)}\right.}{}$

input
```
int(sec(d*x+c)^8*(cos(d*x+c)*a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
a^3/d*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+ta
n(d*x+c)))+b^3/d*(1/7*sec(d*x+c)^7-1/5*sec(d*x+c)^5)+3*a*b^2/d*(1/6*sin(d*
x+c)^3/cos(d*x+c)^6+1/8*sin(d*x+c)^3/cos(d*x+c)^4+1/16*sin(d*x+c)^3/cos(d*
x+c)^2+1/16*sin(d*x+c)-1/16*ln(sec(d*x+c)+tan(d*x+c)))+3/5*a^2*b*sec(d*x+c
)^5/d
```

3.70.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.81

$$\int \sec^{8}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^{3}\,dx$$
$$=\frac{105\,(2\,a^{3}-ab^{2})\cos{(dx+c)}^{7}\log{(\sin{(dx+c)}+1)}-105\,(2\,a^{3}-ab^{2})\cos{(dx+c)}^{7}\log{(-\sin{(dx+c)}+1}}{}$$

input
```
integrate(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas"
)
```

output
```
1/1120*(105*(2*a^3 - a*b^2)*cos(d*x + c)^7*log(sin(d*x + c) + 1) - 105*(2*
a^3 - a*b^2)*cos(d*x + c)^7*log(-sin(d*x + c) + 1) + 160*b^3 + 224*(3*a^2*
b - b^3)*cos(d*x + c)^2 + 70*(3*(2*a^3 - a*b^2)*cos(d*x + c)^5 + 8*a*b^2*c
os(d*x + c) + 2*(2*a^3 - a*b^2)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x +
 c)^7)
```

3.70.6 Sympy [F(-1)]

Timed out.

$$\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**8*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

output
```
Timed out
```

3.70.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.99

$$\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$
$$= \frac{35\,ab^2\left(\frac{2\left(3\,\sin(dx+c)^5-8\,\sin(dx+c)^3-3\,\sin(dx+c)\right)}{\sin(dx+c)^6-3\,\sin(dx+c)^4+3\,\sin(dx+c)^2-1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)-70\,a^3\left(\right.}{1}$$

input
```
integrate(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima"
)
```

output
```
1/1120*(35*a*b^2*(2*(3*sin(d*x + c)^5 - 8*sin(d*x + c)^3 - 3*sin(d*x + c))
/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 3*log(sin(d*
x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 70*a^3*(2*(3*sin(d*x + c)^3 - 5*s
in(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c)
+ 1) + 3*log(sin(d*x + c) - 1)) + 672*a^2*b/cos(d*x + c)^5 - 32*(7*cos(d*x
 + c)^2 - 5)*b^3/cos(d*x + c)^7)/d
```

3.70.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 465 vs. $2(190) = 380$.

Time = 0.41 (sec) , antiderivative size = 465, normalized size of antiderivative = 2.21

$$\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$
$$= \frac{105\,(2\,a^3-ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-105\,(2\,a^3-ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(350\,a^3\tan\left(\frac{1}{2}\right.\right.}{}}{}$$

input
```
integrate(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
1/560*(105*(2*a^3 - a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 105*(2*a^3
 - a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(350*a^3*tan(1/2*d*x + 1/
2*c)^13 + 105*a*b^2*tan(1/2*d*x + 1/2*c)^13 - 1680*a^2*b*tan(1/2*d*x + 1/2
*c)^12 - 840*a^3*tan(1/2*d*x + 1/2*c)^11 + 1540*a*b^2*tan(1/2*d*x + 1/2*c)
^11 + 3360*a^2*b*tan(1/2*d*x + 1/2*c)^10 - 1120*b^3*tan(1/2*d*x + 1/2*c)^1
0 + 630*a^3*tan(1/2*d*x + 1/2*c)^9 + 1085*a*b^2*tan(1/2*d*x + 1/2*c)^9 - 5
040*a^2*b*tan(1/2*d*x + 1/2*c)^8 - 1120*b^3*tan(1/2*d*x + 1/2*c)^8 + 6720*
a^2*b*tan(1/2*d*x + 1/2*c)^6 - 2240*b^3*tan(1/2*d*x + 1/2*c)^6 - 630*a^3*t
an(1/2*d*x + 1/2*c)^5 - 1085*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 3696*a^2*b*tan
(1/2*d*x + 1/2*c)^4 - 448*b^3*tan(1/2*d*x + 1/2*c)^4 + 840*a^3*tan(1/2*d*x
 + 1/2*c)^3 - 1540*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 672*a^2*b*tan(1/2*d*x +
1/2*c)^2 - 224*b^3*tan(1/2*d*x + 1/2*c)^2 - 350*a^3*tan(1/2*d*x + 1/2*c) -
 105*a*b^2*tan(1/2*d*x + 1/2*c) - 336*a^2*b + 32*b^3)/(tan(1/2*d*x + 1/2*c
)^2 - 1)^7)/d
```

3.70.9 Mupad [B] (verification not implemented)

Time = 26.80 (sec) , antiderivative size = 423, normalized size of antiderivative = 2.01

$$\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = \frac{3\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(2\,a^2-b^2\right)}{8\,d}$$
$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{5\,a^3}{4}+\frac{3\,a\,b^2}{8}\right)+\frac{6\,a^2\,b}{5}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{11\,a\,b^2}{2}-3\,a^3\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}\,\left(\frac{11\,a\,b^2}{2}-3\,a^3\right)-\mathrm{ta}}{}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))^3/cos(c + d*x)^8,x)
```

output
```
(3*a*atanh(tan(c/2 + (d*x)/2))*(2*a^2 - b^2))/(8*d) - (tan(c/2 + (d*x)/2)*
((3*a*b^2)/8 + (5*a^3)/4) + (6*a^2*b)/5 + tan(c/2 + (d*x)/2)^3*((11*a*b^2)
/2 - 3*a^3) - tan(c/2 + (d*x)/2)^11*((11*a*b^2)/2 - 3*a^3) - tan(c/2 + (d*
x)/2)^13*((3*a*b^2)/8 + (5*a^3)/4) + tan(c/2 + (d*x)/2)^5*((31*a*b^2)/8 +
(9*a^3)/4) - tan(c/2 + (d*x)/2)^9*((31*a*b^2)/8 + (9*a^3)/4) - tan(c/2 + (
d*x)/2)^10*(12*a^2*b - 4*b^3) - tan(c/2 + (d*x)/2)^2*((12*a^2*b)/5 - (4*b^
3)/5) + tan(c/2 + (d*x)/2)^8*(18*a^2*b + 4*b^3) - tan(c/2 + (d*x)/2)^6*(24
*a^2*b - 8*b^3) + tan(c/2 + (d*x)/2)^4*((66*a^2*b)/5 + (8*b^3)/5) - (4*b^3
)/35 + 6*a^2*b*tan(c/2 + (d*x)/2)^12)/(d*(7*tan(c/2 + (d*x)/2)^2 - 21*tan(
c/2 + (d*x)/2)^4 + 35*tan(c/2 + (d*x)/2)^6 - 35*tan(c/2 + (d*x)/2)^8 + 21*
tan(c/2 + (d*x)/2)^10 - 7*tan(c/2 + (d*x)/2)^12 + tan(c/2 + (d*x)/2)^14 -
1))
```

3.70. $\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

3.71 $\int \sec^9(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

3.71.1 Optimal result

Integrand size = 28, antiderivative size = 174

$$\begin{aligned}\int \sec^9(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx &= \frac{a^3\tan(c+dx)}{d}+\frac{3a^2b\tan^2(c+dx)}{2d}+\frac{a(2a^2+3b^2)\tan^3(c+dx)}{3d}\\ &\quad+\frac{b(6a^2+b^2)\tan^4(c+dx)}{4d}+\frac{a(a^2+6b^2)\tan^5(c+dx)}{5d}\\ &\quad+\frac{b(3a^2+2b^2)\tan^6(c+dx)}{6d}+\frac{3ab^2\tan^7(c+dx)}{7d}+\frac{b^3\tan^8(c+dx)}{8d}\end{aligned}$$

output
```
a^3*tan(d*x+c)/d+3/2*a^2*b*tan(d*x+c)^2/d+1/3*a*(2*a^2+3*b^2)*tan(d*x+c)^3
/d+1/4*b*(6*a^2+b^2)*tan(d*x+c)^4/d+1/5*a*(a^2+6*b^2)*tan(d*x+c)^5/d+1/6*b
*(3*a^2+2*b^2)*tan(d*x+c)^6/d+3/7*a*b^2*tan(d*x+c)^7/d+1/8*b^3*tan(d*x+c)^
8/d
```

3.71.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.66

$$\begin{aligned}&\int \sec^9(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx\\ &=\frac{\frac{1}{4}(a^2+b^2)^2(a+b\tan(c+dx))^4-\frac{4}{5}a(a^2+b^2)(a+b\tan(c+dx))^5+\frac{1}{3}(3a^2+b^2)(a+b\tan(c+dx))^6-\ldots}{b^5d}\end{aligned}$$

input
```
Integrate[Sec[c + d*x]^9*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

output
```
(((a^2 + b^2)^2*(a + b*Tan[c + d*x])^4)/4 - (4*a*(a^2 + b^2)*(a + b*Tan[c
+ d*x])^5)/5 + ((3*a^2 + b^2)*(a + b*Tan[c + d*x])^6)/3 - (4*a*(a + b*Tan[
c + d*x])^7)/7 + (a + b*Tan[c + d*x])^8/8)/(b^5*d)
```

3.71.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3567, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^9(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\cos(c+dx)+b\sin(c+dx))^3}{\cos(c+dx)^9}dx$$

$$\Big\downarrow \text{3567}$$

$$-\frac{\int (b+a\cot(c+dx))^3\left(\cot^2(c+dx)+1\right)^2\tan^9(c+dx)d\cot(c+dx)}{d}$$

$$\Big\downarrow \text{522}$$

$$-\frac{\int \left(b^3\tan^9(c+dx)+3ab^2\tan^8(c+dx)+\left(2b^3+3a^2b\right)\tan^7(c+dx)+\left(a^3+6b^2a\right)\tan^6(c+dx)+\left(b^3+6a^2b\right)\text{ta}}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-a^3\tan(c+dx)-\frac{1}{6}b\left(3a^2+2b^2\right)\tan^6(c+dx)-\frac{1}{5}a\left(a^2+6b^2\right)\tan^5(c+dx)-\frac{1}{4}b\left(6a^2+b^2\right)\tan^4(c+dx)-\frac{1}{3}a(2}{d}$$

input
```
Int[Sec[c + d*x]^9*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

output
```
-((-(a^3*Tan[c + d*x]) - (3*a^2*b*Tan[c + d*x]^2)/2 - (a*(2*a^2 + 3*b^2)*T
an[c + d*x]^3)/3 - (b*(6*a^2 + b^2)*Tan[c + d*x]^4)/4 - (a*(a^2 + 6*b^2)*T
an[c + d*x]^5)/5 - (b*(3*a^2 + 2*b^2)*Tan[c + d*x]^6)/6 - (3*a*b^2*Tan[c +
 d*x]^7)/7 - (b^3*Tan[c + d*x]^8)/8)/d)
```

3.71.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3567
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[x^m*((b
+ a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a,
b, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[
n, 0] && GtQ[m, 1])
```

3.71.4 Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.84

method	result
parts	$-\frac{a^3\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)}{d}+\frac{b^3\left(\frac{\sec(dx+c)^8}{8}-\frac{\sec(dx+c)^6}{6}\right)}{d}+\frac{3a\,b^2\left(\frac{\sin(dx+c)^3}{7\cos(dx+c)^7}+\frac{4\sin(dx+c)^3}{35\cos(dx+c)^5}\right.}{d}$
derivativedivides	$\frac{-a^3\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)+\frac{a^2b}{2\cos(dx+c)^6}+3a\,b^2\left(\frac{\sin(dx+c)^3}{7\cos(dx+c)^7}+\frac{4\sin(dx+c)^3}{35\cos(dx+c)^5}+\frac{8\sin(dx+c)^3}{105\cos(dx+c)^3}\right)+}{d}$
default	$\frac{-a^3\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)+\frac{a^2b}{2\cos(dx+c)^6}+3a\,b^2\left(\frac{\sin(dx+c)^3}{7\cos(dx+c)^7}+\frac{4\sin(dx+c)^3}{35\cos(dx+c)^5}+\frac{8\sin(dx+c)^3}{105\cos(dx+c)^3}\right)+}{d}$
risch	$-\frac{16\left(-7ia^3-322ia^3\mathrm{e}^{6i(dx+c)}-210a^2b\,\mathrm{e}^{10i(dx+c)}+70b^3\mathrm{e}^{10i(dx+c)}-70ia^3\mathrm{e}^{10i(dx+c)}-42ia\,b^2\mathrm{e}^{6i(dx+c)}-420a^2b\,\mathrm{e}^{8i(dx+c)}\right.}{}$
parallelrisch	$-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(-105a^3+84\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4a\,b^2-280\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9b^3-700\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7b^3-280\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5b^3-210\right.}{}$

input
```
int(sec(d*x+c)^9*(cos(d*x+c)*a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-a^3/d*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+b^3/d*(1/8*se
c(d*x+c)^8-1/6*sec(d*x+c)^6)+3*a*b^2/d*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35
*sin(d*x+c)^3/cos(d*x+c)^5+8/105*sin(d*x+c)^3/cos(d*x+c)^3)+1/2*a^2*b/d*se
c(d*x+c)^6
```

3.71.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.74

$$\int \sec^9(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = \frac{105\,b^3+140\,(3\,a^2b-b^3)\cos{(dx+c)}^2+8\,(8\,(7\,a^3-3\,ab^2)\cos{(dx+c)}^7+4\,(7\,a^3-3\,ab^2)\cos{(dx+c)}^5+}{840\,d\cos{(dx+c)}^8}$$

input
```
integrate(sec(d*x+c)^9*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas"
)
```

output
```
1/840*(105*b^3 + 140*(3*a^2*b - b^3)*cos(d*x + c)^2 + 8*(8*(7*a^3 - 3*a*b^
2)*cos(d*x + c)^7 + 4*(7*a^3 - 3*a*b^2)*cos(d*x + c)^5 + 45*a*b^2*cos(d*x
+ c) + 3*(7*a^3 - 3*a*b^2)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^8
)
```

3.71.6 Sympy [F(-1)]

Timed out.

$$\int \sec^9(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**9*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

output
```
Timed out
```

3.71.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.89

$$\int \sec^9(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$

$$= \frac{56\left(3\,\tan\left(dx+c\right)^5+10\,\tan\left(dx+c\right)^3+15\,\tan\left(dx+c\right)\right)a^3+24\left(15\,\tan\left(dx+c\right)^7+42\,\tan\left(dx+c\right)^5+\right.}{840\,d}$$

input
```
integrate(sec(d*x+c)^9*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima"
)
```

output
```
1/840*(56*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^3 + 2
4*(15*tan(d*x + c)^7 + 42*tan(d*x + c)^5 + 35*tan(d*x + c)^3)*a*b^2 + 35*(
4*sin(d*x + c)^2 - 1)*b^3/(sin(d*x + c)^8 - 4*sin(d*x + c)^6 + 6*sin(d*x +
 c)^4 - 4*sin(d*x + c)^2 + 1) - 420*a^2*b/(sin(d*x + c)^2 - 1)^3)/d
```

3.71.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.95

$$\int \sec^9(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$

$$= \frac{105\,b^3\tan\left(dx+c\right)^8+360\,ab^2\tan\left(dx+c\right)^7+420\,a^2b\tan\left(dx+c\right)^6+280\,b^3\tan\left(dx+c\right)^6+168\,a^3\tan\left(dx\right.}{}$$

input
```
integrate(sec(d*x+c)^9*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
1/840*(105*b^3*tan(d*x + c)^8 + 360*a*b^2*tan(d*x + c)^7 + 420*a^2*b*tan(d
*x + c)^6 + 280*b^3*tan(d*x + c)^6 + 168*a^3*tan(d*x + c)^5 + 1008*a*b^2*t
an(d*x + c)^5 + 1260*a^2*b*tan(d*x + c)^4 + 210*b^3*tan(d*x + c)^4 + 560*a
^3*tan(d*x + c)^3 + 840*a*b^2*tan(d*x + c)^3 + 1260*a^2*b*tan(d*x + c)^2 +
 840*a^3*tan(d*x + c))/d
```

3.71.9 Mupad [B] (verification not implemented)

Time = 23.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.90

$$\int \sec^9(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$
$$=\frac{\cos\left(c+d\,x\right)^3\,\left(\frac{a^3\,\sin(c+d\,x)}{5}-\frac{3\,a\,b^2\,\sin(c+d\,x)}{35}\right)+\cos\left(c+d\,x\right)^5\,\left(\frac{4\,a^3\,\sin(c+d\,x)}{15}-\frac{4\,a\,b^2\,\sin(c+d\,x)}{35}\right)+\cos\left(c+d\,x\right)^7}{d\cos\left(c+d\,x\right)^8}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))^3/cos(c + d*x)^9,x)
```

output
```
(cos(c + d*x)^3*((a^3*sin(c + d*x))/5 - (3*a*b^2*sin(c + d*x))/35) + cos(c
 + d*x)^5*((4*a^3*sin(c + d*x))/15 - (4*a*b^2*sin(c + d*x))/35) + cos(c +
d*x)^7*((8*a^3*sin(c + d*x))/15 - (8*a*b^2*sin(c + d*x))/35) + cos(c + d*x
)^2*((a^2*b)/2 - b^3/6) + b^3/8 + (3*a*b^2*cos(c + d*x)*sin(c + d*x))/7)/(
d*cos(c + d*x)^8)
```

3.72 $\int \sec^{10}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

3.72.1 Optimal result

Integrand size = 28, antiderivative size = 259

$$\begin{aligned}
&\int \sec^{10}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx \\
&= \frac{5a^3\text{arctanh}(\sin(c+dx))}{16d} - \frac{15ab^2\text{arctanh}(\sin(c+dx))}{128d} + \frac{3a^2b\sec^7(c+dx)}{7d} \\
&\quad - \frac{b^3\sec^7(c+dx)}{7d} + \frac{b^3\sec^9(c+dx)}{9d} + \frac{5a^3\sec(c+dx)\tan(c+dx)}{16d} \\
&\quad - \frac{15ab^2\sec(c+dx)\tan(c+dx)}{128d} + \frac{5a^3\sec^3(c+dx)\tan(c+dx)}{24d} \\
&\quad - \frac{5ab^2\sec^3(c+dx)\tan(c+dx)}{64d} + \frac{a^3\sec^5(c+dx)\tan(c+dx)}{6d} \\
&\quad - \frac{ab^2\sec^5(c+dx)\tan(c+dx)}{16d} + \frac{3ab^2\sec^7(c+dx)\tan(c+dx)}{8d}
\end{aligned}$$

output
```
5/16*a^3*arctanh(sin(d*x+c))/d-15/128*a*b^2*arctanh(sin(d*x+c))/d+3/7*a^2*
b*sec(d*x+c)^7/d-1/7*b^3*sec(d*x+c)^7/d+1/9*b^3*sec(d*x+c)^9/d+5/16*a^3*se
c(d*x+c)*tan(d*x+c)/d-15/128*a*b^2*sec(d*x+c)*tan(d*x+c)/d+5/24*a^3*sec(d*
x+c)^3*tan(d*x+c)/d-5/64*a*b^2*sec(d*x+c)^3*tan(d*x+c)/d+1/6*a^3*sec(d*x+c
)^5*tan(d*x+c)/d-1/16*a*b^2*sec(d*x+c)^5*tan(d*x+c)/d+3/8*a*b^2*sec(d*x+c)
^7*tan(d*x+c)/d
```

3.72.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 810 vs. $2(259) = 518$.

Time = 5.46 (sec) , antiderivative size = 810, normalized size of antiderivative = 3.13

$$\int \sec^{10}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$
$$= \frac{\sec^9(c+dx)\,\big(442368a^2b+81920b^3+147456(3a^2b-b^3)\cos(2(c+dx))-211680a^3\cos(3(c+dx))\log\big(\mathrm{co}$$

input
```
Integrate[Sec[c + d*x]^10*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

output
```
(Sec[c + d*x]^9*(442368*a^2*b + 81920*b^3 + 147456*(3*a^2*b - b^3)*Cos[2*(
c + d*x)] - 211680*a^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*
x)/2]] + 79380*a*b^2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)
/2]] - 90720*a^3*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
 + 34020*a*b^2*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] -
 22680*a^3*Cos[7*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 850
5*a*b^2*Cos[7*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 2520*a
^3*Cos[9*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 945*a*b^2*C
os[9*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 39690*a*(8*a^2
- 3*b^2)*Cos[c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[
(c + d*x)/2] + Sin[(c + d*x)/2]]) + 211680*a^3*Cos[3*(c + d*x)]*Log[Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2]] - 79380*a*b^2*Cos[3*(c + d*x)]*Log[Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]] + 90720*a^3*Cos[5*(c + d*x)]*Log[Cos[(c + d*x
)/2] + Sin[(c + d*x)/2]] - 34020*a*b^2*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/
2] + Sin[(c + d*x)/2]] + 22680*a^3*Cos[7*(c + d*x)]*Log[Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2]] - 8505*a*b^2*Cos[7*(c + d*x)]*Log[Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]] + 2520*a^3*Cos[9*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2]] - 945*a*b^2*Cos[9*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*
x)/2]] + 223776*a^3*Sin[2*(c + d*x)] + 303156*a*b^2*Sin[2*(c + d*x)] + 167
328*a^3*Sin[4*(c + d*x)] - 62748*a*b^2*Sin[4*(c + d*x)] + 43680*a^3*Sin...
```

3.72.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{10}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\cos(c+dx)+b\sin(c+dx))^3}{\cos(c+dx)^{10}}dx$$

$$\Big\downarrow \text{3569}$$

$$\int \left(a^3\sec^7(c+dx)+3a^2b\tan(c+dx)\sec^7(c+dx)+3ab^2\tan^2(c+dx)\sec^7(c+dx)+b^3\tan^3(c+dx)\sec^7(c+dx\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{5a^3\text{arctanh}(\sin(c+dx))}{16d}+\frac{a^3\tan(c+dx)\sec^5(c+dx)}{6d}+\frac{5a^3\tan(c+dx)\sec^3(c+dx)}{24d}+\frac{5a^3\tan(c+dx)\sec(c+dx)}{16d}+\frac{3a^2b\sec^7(c+dx)}{7d}-\frac{15ab^2\text{arctanh}(\sin(c+dx))}{128d}+\frac{3ab^2\tan(c+dx)\sec^7(c+dx)}{8d}-\frac{ab^2\tan(c+dx)\sec^5(c+dx)}{16d}-\frac{5ab^2\tan(c+dx)\sec^3(c+dx)}{64d}-\frac{15ab^2\tan(c+dx)\sec(c+dx)}{128d}+\frac{b^3\sec^9(c+dx)}{9d}-\frac{b^3\sec^7(c+dx)}{7d}$$

input
```
Int[Sec[c + d*x]^10*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

output
```
(5*a^3*ArcTanh[Sin[c + d*x]])/(16*d) - (15*a*b^2*ArcTanh[Sin[c + d*x]])/(1
28*d) + (3*a^2*b*Sec[c + d*x]^7)/(7*d) - (b^3*Sec[c + d*x]^7)/(7*d) + (b^3
*Sec[c + d*x]^9)/(9*d) + (5*a^3*Sec[c + d*x]*Tan[c + d*x])/(16*d) - (15*a*
b^2*Sec[c + d*x]*Tan[c + d*x])/(128*d) + (5*a^3*Sec[c + d*x]^3*Tan[c + d*x
])/(24*d) - (5*a*b^2*Sec[c + d*x]^3*Tan[c + d*x])/(64*d) + (a^3*Sec[c + d*
x]^5*Tan[c + d*x])/(6*d) - (a*b^2*Sec[c + d*x]^5*Tan[c + d*x])/(16*d) + (3
*a*b^2*Sec[c + d*x]^7*Tan[c + d*x])/(8*d)
```

3.72.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

3.72.4 Maple [A] (verified)

Time = 2.14 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.83

method	result
parts	$\frac{a^3\left(-\left(-\frac{\sec(dx+c)^5}{6}-\frac{5\sec(dx+c)^3}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)}{d}+\frac{b^3\left(\frac{\sec(dx+c)^9}{9}-\frac{\sec(dx+c)}{7}\right.}{d}$
derivativedivides	$\frac{a^3\left(-\left(-\frac{\sec(dx+c)^5}{6}-\frac{5\sec(dx+c)^3}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)+\frac{3a^2b}{7\cos(dx+c)^7}+3a\,b^2\left(\frac{\sin(dx+}{8\cos(dx-}\right.}{}$
default	$\frac{a^3\left(-\left(-\frac{\sec(dx+c)^5}{6}-\frac{5\sec(dx+c)^3}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)+\frac{3a^2b}{7\cos(dx+c)^7}+3a\,b^2\left(\frac{\sin(dx+}{8\cos(dx-}\right.}{}$
parallelrisch	$\frac{-22680\left(\frac{\cos(9dx+9c)}{9}+\cos(7dx+7c)+4\cos(5dx+5c)+\frac{28\cos(3dx+3c)}{3}+14\cos(dx+c)\right)\left(a^2-\frac{3b^2}{8}\right)a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+}{}$
risch	$-\frac{e^{i(dx+c)}\left(2520ia^3e^{16i(dx+c)}-2520ia^3-111888ia^3e^{6i(dx+c)}+21840ia^3e^{14i(dx+c)}-31374ia\,b^2e^{12i(dx+c)}-151578ia\,b^2e\right.}{}$

input
```
int(sec(d*x+c)^10*(cos(d*x+c)*a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
a^3/d*(-(-1/6*sec(d*x+c)^5-5/24*sec(d*x+c)^3-5/16*sec(d*x+c))*tan(d*x+c)+5
/16*ln(sec(d*x+c)+tan(d*x+c)))+b^3/d*(1/9*sec(d*x+c)^9-1/7*sec(d*x+c)^7)+3
*a*b^2/d*(1/8*sin(d*x+c)^3/cos(d*x+c)^8+5/48*sin(d*x+c)^3/cos(d*x+c)^6+5/6
4*sin(d*x+c)^3/cos(d*x+c)^4+5/128*sin(d*x+c)^3/cos(d*x+c)^2+5/128*sin(d*x+
c)-5/128*ln(sec(d*x+c)+tan(d*x+c)))+3/7*a^2*b*sec(d*x+c)^7/d
```

3.72.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.74

$$\int \sec^{10}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$
$$= \frac{315\,(8\,a^3-3\,ab^2)\cos{(dx+c)}^9\log{(\sin{(dx+c)}+1)}-315\,(8\,a^3-3\,ab^2)\cos{(dx+c)}^9\log{(-\sin{(dx+c)}+\dots}}{\dots}$$

input
```
integrate(sec(d*x+c)^10*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

output
```
1/16128*(315*(8*a^3 - 3*a*b^2)*cos(d*x + c)^9*log(sin(d*x + c) + 1) - 315*
(8*a^3 - 3*a*b^2)*cos(d*x + c)^9*log(-sin(d*x + c) + 1) + 1792*b^3 + 2304*
(3*a^2*b - b^3)*cos(d*x + c)^2 + 42*(15*(8*a^3 - 3*a*b^2)*cos(d*x + c)^7 +
 10*(8*a^3 - 3*a*b^2)*cos(d*x + c)^5 + 144*a*b^2*cos(d*x + c) + 8*(8*a^3 -
 3*a*b^2)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^9)
```

3.72.6 Sympy [F(-1)]

Timed out.

$$\int \sec^{10}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**10*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

output
```
Timed out
```

3.72.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.96

$$\int \sec^{10}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$
$$= \frac{63\,ab^2\left(\frac{2\left(15\,\sin(dx+c)^7-55\,\sin(dx+c)^5+73\,\sin(dx+c)^3+15\,\sin(dx+c)\right)}{\sin(dx+c)^8-4\,\sin(dx+c)^6+6\,\sin(dx+c)^4-4\,\sin(dx+c)^2+1}-15\log{(\sin{(dx+c)}+1)}+15\log{(\sin{(dx+c}\dots}\right.}{\dots}$$

input
```
integrate(sec(d*x+c)^10*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

output
```
1/16128*(63*a*b^2*(2*(15*sin(d*x + c)^7 - 55*sin(d*x + c)^5 + 73*sin(d*x +
 c)^3 + 15*sin(d*x + c))/(sin(d*x + c)^8 - 4*sin(d*x + c)^6 + 6*sin(d*x +
c)^4 - 4*sin(d*x + c)^2 + 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x +
 c) - 1)) - 168*a^3*(2*(15*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*sin(d*x
 + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15*log
(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) + 6912*a^2*b/cos(d*x + c)^7
 - 256*(9*cos(d*x + c)^2 - 7)*b^3/cos(d*x + c)^9)/d
```

3.72.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 597 vs. $2(235) = 470$.

Time = 0.42 (sec) , antiderivative size = 597, normalized size of antiderivative = 2.31

$$\int \sec^{10}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$

$$= \frac{315\,(8\,a^3-3\,ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-315\,(8\,a^3-3\,ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(5544\,a^3\,\mathrm{ta}\right.}{}}{}$$

input
```
integrate(sec(d*x+c)^10*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
1/8064*(315*(8*a^3 - 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 315*(8*
a^3 - 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(5544*a^3*tan(1/2*d*
x + 1/2*c)^17 + 945*a*b^2*tan(1/2*d*x + 1/2*c)^17 - 24192*a^2*b*tan(1/2*d*
x + 1/2*c)^16 - 15792*a^3*tan(1/2*d*x + 1/2*c)^15 + 24066*a*b^2*tan(1/2*d*
x + 1/2*c)^15 + 48384*a^2*b*tan(1/2*d*x + 1/2*c)^14 - 16128*b^3*tan(1/2*d*
x + 1/2*c)^14 + 29232*a^3*tan(1/2*d*x + 1/2*c)^13 + 31374*a*b^2*tan(1/2*d*
x + 1/2*c)^13 - 145152*a^2*b*tan(1/2*d*x + 1/2*c)^12 - 26880*b^3*tan(1/2*d
*x + 1/2*c)^12 - 33264*a^3*tan(1/2*d*x + 1/2*c)^11 + 54810*a*b^2*tan(1/2*d
*x + 1/2*c)^11 + 241920*a^2*b*tan(1/2*d*x + 1/2*c)^10 - 80640*b^3*tan(1/2*
d*x + 1/2*c)^10 - 193536*a^2*b*tan(1/2*d*x + 1/2*c)^8 - 48384*b^3*tan(1/2*
d*x + 1/2*c)^8 + 33264*a^3*tan(1/2*d*x + 1/2*c)^7 - 54810*a*b^2*tan(1/2*d*
x + 1/2*c)^7 + 145152*a^2*b*tan(1/2*d*x + 1/2*c)^6 - 48384*b^3*tan(1/2*d*x
 + 1/2*c)^6 - 29232*a^3*tan(1/2*d*x + 1/2*c)^5 - 31374*a*b^2*tan(1/2*d*x +
 1/2*c)^5 - 76032*a^2*b*tan(1/2*d*x + 1/2*c)^4 - 6912*b^3*tan(1/2*d*x + 1/
2*c)^4 + 15792*a^3*tan(1/2*d*x + 1/2*c)^3 - 24066*a*b^2*tan(1/2*d*x + 1/2*
c)^3 + 6912*a^2*b*tan(1/2*d*x + 1/2*c)^2 - 2304*b^3*tan(1/2*d*x + 1/2*c)^2
 - 5544*a^3*tan(1/2*d*x + 1/2*c) - 945*a*b^2*tan(1/2*d*x + 1/2*c) - 3456*a
^2*b + 256*b^3)/(tan(1/2*d*x + 1/2*c)^2 - 1)^9)/d
```

3.72.9 Mupad [B] (verification not implemented)

Time = 26.78 (sec) , antiderivative size = 547, normalized size of antiderivative = 2.11

$$\int \sec^{10}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = -\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(\frac{15\,a\,b^2}{64}-\frac{5\,a^3}{8}\right)}{d}$$

$$-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{11\,a^3}{8}+\frac{15\,a\,b^2}{64}\right)+\frac{6\,a^2\,b}{7}-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{17}\left(\frac{11\,a^3}{8}+\frac{15\,a\,b^2}{64}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{191\,a\,b^2}{32}-\frac{47\,a^3}{12}\right)+}{}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))^3/cos(c + d*x)^10,x)
```

output
```
- (atanh(tan(c/2 + (d*x)/2))*((15*a*b^2)/64 - (5*a^3)/8))/d - (tan(c/2 + (
d*x)/2)*((15*a*b^2)/64 + (11*a^3)/8) + (6*a^2*b)/7 - tan(c/2 + (d*x)/2)^17
*((15*a*b^2)/64 + (11*a^3)/8) + tan(c/2 + (d*x)/2)^3*((191*a*b^2)/32 - (47
*a^3)/12) - tan(c/2 + (d*x)/2)^15*((191*a*b^2)/32 - (47*a^3)/12) + tan(c/2
 + (d*x)/2)^5*((249*a*b^2)/32 + (29*a^3)/4) - tan(c/2 + (d*x)/2)^13*((249*
a*b^2)/32 + (29*a^3)/4) + tan(c/2 + (d*x)/2)^7*((435*a*b^2)/32 - (33*a^3)/
4) - tan(c/2 + (d*x)/2)^11*((435*a*b^2)/32 - (33*a^3)/4) - tan(c/2 + (d*x)
/2)^14*(12*a^2*b - 4*b^3) - tan(c/2 + (d*x)/2)^2*((12*a^2*b)/7 - (4*b^3)/7
) - tan(c/2 + (d*x)/2)^6*(36*a^2*b - 12*b^3) + tan(c/2 + (d*x)/2)^8*(48*a^
2*b + 12*b^3) + tan(c/2 + (d*x)/2)^12*(36*a^2*b + (20*b^3)/3) - tan(c/2 +
(d*x)/2)^10*(60*a^2*b - 20*b^3) + tan(c/2 + (d*x)/2)^4*((132*a^2*b)/7 + (1
2*b^3)/7) - (4*b^3)/63 + 6*a^2*b*tan(c/2 + (d*x)/2)^16)/(d*(9*tan(c/2 + (d
*x)/2)^2 - 36*tan(c/2 + (d*x)/2)^4 + 84*tan(c/2 + (d*x)/2)^6 - 126*tan(c/2
 + (d*x)/2)^8 + 126*tan(c/2 + (d*x)/2)^10 - 84*tan(c/2 + (d*x)/2)^12 + 36*
tan(c/2 + (d*x)/2)^14 - 9*tan(c/2 + (d*x)/2)^16 + tan(c/2 + (d*x)/2)^18 -
1))
```

3.72. $\int \sec^{10}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

3.73 $\int \sec^{11}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

3.73.1 Optimal result

Integrand size = 28, antiderivative size = 213

$$\int \sec^{11}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = \frac{a^3\tan(c+dx)}{d}+\frac{3a^2b\tan^2(c+dx)}{2d}+\frac{a(a^2+b^2)\tan^3(c+dx)}{d}+\frac{b(9a^2+b^2)\tan^4(c+dx)}{4d}+\frac{3a(a^2+3b^2)\tan^5(c+dx)}{5d}+\frac{b(3a^2+b^2)\tan^6(c+dx)}{2d}+\frac{a(a^2+9b^2)\tan^7(c+dx)}{7d}+\frac{3b(a^2+b^2)\tan^8(c+dx)}{8d}+\frac{ab^2\tan^9(c+dx)}{3d}+\frac{b^3\tan^{10}(c+dx)}{10d}$$

output
```
a^3*tan(d*x+c)/d+3/2*a^2*b*tan(d*x+c)^2/d+a*(a^2+b^2)*tan(d*x+c)^3/d+1/4*b
*(9*a^2+b^2)*tan(d*x+c)^4/d+3/5*a*(a^2+3*b^2)*tan(d*x+c)^5/d+1/2*b*(3*a^2+
b^2)*tan(d*x+c)^6/d+1/7*a*(a^2+9*b^2)*tan(d*x+c)^7/d+3/8*b*(a^2+b^2)*tan(d
*x+c)^8/d+1/3*a*b^2*tan(d*x+c)^9/d+1/10*b^3*tan(d*x+c)^10/d
```

3.73.2 Mathematica [A] (verified)

Time = 2.17 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.83

$$\int \sec^{11}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = \frac{\frac{1}{4}(a^2+b^2)^3(a+b\tan(c+dx))^4-\frac{6}{5}a(a^2+b^2)^2(a+b\tan(c+dx))^5+\frac{1}{2}(a^2+b^2)(5a^2+b^2)(a+b\tan(c+$$

input
```
Integrate[Sec[c + d*x]^11*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

output
```
(((a^2 + b^2)^3*(a + b*Tan[c + d*x])^4)/4 - (6*a*(a^2 + b^2)^2*(a + b*Tan[
c + d*x])^5)/5 + ((a^2 + b^2)*(5*a^2 + b^2)*(a + b*Tan[c + d*x])^6)/2 - (4
*a*(5*a^2 + 3*b^2)*(a + b*Tan[c + d*x])^7)/7 + (3*(5*a^2 + b^2)*(a + b*Tan
[c + d*x])^8)/8 - (2*a*(a + b*Tan[c + d*x])^9)/3 + (a + b*Tan[c + d*x])^10
/10)/(b^7*d)
```

3.73.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3567, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{11}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\cos(c+dx)+b\sin(c+dx))^3}{\cos(c+dx)^{11}}dx$$

$$\Big\downarrow \text{3567}$$

$$-\frac{\int (b+a\cot(c+dx))^3\left(\cot^2(c+dx)+1\right)^3\tan^{11}(c+dx)d\cot(c+dx)}{d}$$

$$\Big\downarrow \text{522}$$

$$-\frac{\int \left(b^3\tan^{11}(c+dx)+3ab^2\tan^{10}(c+dx)+3b\left(a^2+b^2\right)\tan^9(c+dx)+\left(a^3+9b^2a\right)\tan^8(c+dx)+3\left(b^3+3a^2b\right)\right.}{}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-a^3\tan(c+dx)-\frac{3}{8}b\left(a^2+b^2\right)\tan^8(c+dx)-\frac{1}{7}a\left(a^2+9b^2\right)\tan^7(c+dx)-\frac{1}{2}b\left(3a^2+b^2\right)\tan^6(c+dx)-\frac{3}{5}a\left(a^2\right.}{}$$

input
```
Int[Sec[c + d*x]^11*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

output
```
-((-(a^3*Tan[c + d*x]) - (3*a^2*b*Tan[c + d*x]^2)/2 - a*(a^2 + b^2)*Tan[c
+ d*x]^3 - (b*(9*a^2 + b^2)*Tan[c + d*x]^4)/4 - (3*a*(a^2 + 3*b^2)*Tan[c +
 d*x]^5)/5 - (b*(3*a^2 + b^2)*Tan[c + d*x]^6)/2 - (a*(a^2 + 9*b^2)*Tan[c +
 d*x]^7)/7 - (3*b*(a^2 + b^2)*Tan[c + d*x]^8)/8 - (a*b^2*Tan[c + d*x]^9)/3
 - (b^3*Tan[c + d*x]^10)/10)/d)
```

3.73.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3567
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[x^m*((b
+ a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a,
b, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[
n, 0] && GtQ[m, 1])
```

3.73.4 Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.82

method	result
parts	$-\frac{a^3\left(-\frac{16}{35}-\frac{\sec(dx+c)^6}{7}-\frac{6\sec(dx+c)^4}{35}-\frac{8\sec(dx+c)^2}{35}\right)\tan(dx+c)}{d}+\frac{b^3\left(\frac{\sec(dx+c)^{10}}{10}-\frac{\sec(dx+c)^8}{8}\right)}{d}+\frac{3a\,b^2\left(\frac{\sin(dx+}{9\cos(dx+}\right.}{}$
derivativedivides	$\frac{-a^3\left(-\frac{16}{35}-\frac{\sec(dx+c)^6}{7}-\frac{6\sec(dx+c)^4}{35}-\frac{8\sec(dx+c)^2}{35}\right)\tan(dx+c)+\frac{3a^2b}{8\cos(dx+c)^8}+3a\,b^2\left(\frac{\sin(dx+c)^3}{9\cos(dx+c)^9}+\frac{2\sin(dx+c)^3}{21\cos(dx+c)^7}+\frac{8\,\mathrm{s}}{105}\right.}{d}$
default	$\frac{-a^3\left(-\frac{16}{35}-\frac{\sec(dx+c)^6}{7}-\frac{6\sec(dx+c)^4}{35}-\frac{8\sec(dx+c)^2}{35}\right)\tan(dx+c)+\frac{3a^2b}{8\cos(dx+c)^8}+3a\,b^2\left(\frac{\sin(dx+c)^3}{9\cos(dx+c)^9}+\frac{2\sin(dx+c)^3}{21\cos(dx+c)^7}+\frac{8\,\mathrm{s}}{105}\right.}{d}$
risch	$-\frac{32\left(-3ia^3-105ia^3\mathrm{e}^{12i(dx+c)}-315a^2b\,\mathrm{e}^{12i(dx+c)}+105b^3\mathrm{e}^{12i(dx+c)}-360ia^3\mathrm{e}^{6i(dx+c)}+126ia\,b^2\mathrm{e}^{10i(dx+c)}-630a^2b\,\mathrm{e}^{10i}\right.}{}$
parallelrisch	$-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(-315\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{17}a^2b+420\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{16}a\,b^2-105a^3-84\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4a\,b^2-1176\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9b^3\right.}{}$

input
```
int(sec(d*x+c)^11*(cos(d*x+c)*a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-a^3/d*(-16/35-1/7*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*tan(d
*x+c)+b^3/d*(1/10*sec(d*x+c)^10-1/8*sec(d*x+c)^8)+3*a*b^2/d*(1/9*sin(d*x+c
)^3/cos(d*x+c)^9+2/21*sin(d*x+c)^3/cos(d*x+c)^7+8/105*sin(d*x+c)^3/cos(d*x
+c)^5+16/315*sin(d*x+c)^3/cos(d*x+c)^3)+3/8*a^2*b/d*sec(d*x+c)^8
```

3.73.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.70

$$\int \sec^{11}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$
$$=\frac{84\,b^3+105\,(3\,a^2b-b^3)\cos{(dx+c)}^2+8\,\left(16\,(3\,a^3-ab^2)\cos{(dx+c)}^9+8\,(3\,a^3-ab^2)\cos{(dx+c)}^7+6\,(3\right.}{840\,d\cos{(dx+c)}^{10}}$$

input
```
integrate(sec(d*x+c)^11*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas
")
```

output
```
1/840*(84*b^3 + 105*(3*a^2*b - b^3)*cos(d*x + c)^2 + 8*(16*(3*a^3 - a*b^2)
*cos(d*x + c)^9 + 8*(3*a^3 - a*b^2)*cos(d*x + c)^7 + 6*(3*a^3 - a*b^2)*cos
(d*x + c)^5 + 35*a*b^2*cos(d*x + c) + 5*(3*a^3 - a*b^2)*cos(d*x + c)^3)*si
n(d*x + c))/(d*cos(d*x + c)^10)
```

3.73.6 Sympy [F(-1)]

Timed out.

$$\int \sec^{11}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**11*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

output
```
Timed out
```

3.73.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.86

$$\int \sec^{11}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$
$$= \frac{24\left(5\tan\left(dx+c\right)^7+21\tan\left(dx+c\right)^5+35\tan\left(dx+c\right)^3+35\tan\left(dx+c\right)\right)a^3+8\left(35\tan\left(dx+c\right)^9+13\right.}{}$$

input
```
integrate(sec(d*x+c)^11*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima
")
```

output
```
1/840*(24*(5*tan(d*x + c)^7 + 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 35*t
an(d*x + c))*a^3 + 8*(35*tan(d*x + c)^9 + 135*tan(d*x + c)^7 + 189*tan(d*x
 + c)^5 + 105*tan(d*x + c)^3)*a*b^2 - 21*(5*sin(d*x + c)^2 - 1)*b^3/(sin(d
*x + c)^10 - 5*sin(d*x + c)^8 + 10*sin(d*x + c)^6 - 10*sin(d*x + c)^4 + 5*
sin(d*x + c)^2 - 1) + 315*a^2*b/(sin(d*x + c)^2 - 1)^4)/d
```

3.73.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.03

$$\int \sec^{11}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$
$$= \frac{84\,b^3\tan\left(dx+c\right)^{10}+280\,ab^2\tan\left(dx+c\right)^9+315\,a^2b\tan\left(dx+c\right)^8+315\,b^3\tan\left(dx+c\right)^8+120\,a^3\tan\left(dx\right.}{}$$

input
```
integrate(sec(d*x+c)^11*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
1/840*(84*b^3*tan(d*x + c)^10 + 280*a*b^2*tan(d*x + c)^9 + 315*a^2*b*tan(d
*x + c)^8 + 315*b^3*tan(d*x + c)^8 + 120*a^3*tan(d*x + c)^7 + 1080*a*b^2*t
an(d*x + c)^7 + 1260*a^2*b*tan(d*x + c)^6 + 420*b^3*tan(d*x + c)^6 + 504*a
^3*tan(d*x + c)^5 + 1512*a*b^2*tan(d*x + c)^5 + 1890*a^2*b*tan(d*x + c)^4
+ 210*b^3*tan(d*x + c)^4 + 840*a^3*tan(d*x + c)^3 + 840*a*b^2*tan(d*x + c)
^3 + 1260*a^2*b*tan(d*x + c)^2 + 840*a^3*tan(d*x + c))/d
```

3.73.9 Mupad [B] (verification not implemented)

Time = 23.92 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.89

$$\int \sec^{11}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$$
$$= \frac{\cos(c+d\,x)^3\left(\frac{a^3\,\sin(c+d\,x)}{7}-\frac{a\,b^2\,\sin(c+d\,x)}{21}\right)+\cos(c+d\,x)^5\left(\frac{6\,a^3\,\sin(c+d\,x)}{35}-\frac{2\,a\,b^2\,\sin(c+d\,x)}{35}\right)+\cos(c+d\,x)^7\,(}{}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))^3/cos(c + d*x)^11,x)
```

output
```
(cos(c + d*x)^3*((a^3*sin(c + d*x))/7 - (a*b^2*sin(c + d*x))/21) + cos(c +
 d*x)^5*((6*a^3*sin(c + d*x))/35 - (2*a*b^2*sin(c + d*x))/35) + cos(c + d*
x)^7*((8*a^3*sin(c + d*x))/35 - (8*a*b^2*sin(c + d*x))/105) + cos(c + d*x)
^9*((16*a^3*sin(c + d*x))/35 - (16*a*b^2*sin(c + d*x))/105) + cos(c + d*x)
^2*((3*a^2*b)/8 - b^3/8) + b^3/10 + (a*b^2*cos(c + d*x)*sin(c + d*x))/3)/(
d*cos(c + d*x)^10)
```

3.74 $\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

3.74.1 Optimal result

Integrand size = 28, antiderivative size = 279

$$\begin{aligned}
&\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx\\
&= -\frac{4ab^3\cos^7(c+dx)}{7d}-\frac{4a^3b\cos^9(c+dx)}{9d}+\frac{4ab^3\cos^9(c+dx)}{9d}\\
&\quad+\frac{a^4\sin(c+dx)}{d}-\frac{4a^4\sin^3(c+dx)}{3d}+\frac{2a^2b^2\sin^3(c+dx)}{d}+\frac{6a^4\sin^5(c+dx)}{5d}\\
&\quad-\frac{18a^2b^2\sin^5(c+dx)}{5d}+\frac{b^4\sin^5(c+dx)}{5d}-\frac{4a^4\sin^7(c+dx)}{7d}+\frac{18a^2b^2\sin^7(c+dx)}{7d}\\
&\quad-\frac{2b^4\sin^7(c+dx)}{7d}+\frac{a^4\sin^9(c+dx)}{9d}-\frac{2a^2b^2\sin^9(c+dx)}{3d}+\frac{b^4\sin^9(c+dx)}{9d}
\end{aligned}$$

output
```
-4/7*a*b^3*cos(d*x+c)^7/d-4/9*a^3*b*cos(d*x+c)^9/d+4/9*a*b^3*cos(d*x+c)^9/
d+a^4*sin(d*x+c)/d-4/3*a^4*sin(d*x+c)^3/d+2*a^2*b^2*sin(d*x+c)^3/d+6/5*a^4
*sin(d*x+c)^5/d-18/5*a^2*b^2*sin(d*x+c)^5/d+1/5*b^4*sin(d*x+c)^5/d-4/7*a^4
*sin(d*x+c)^7/d+18/7*a^2*b^2*sin(d*x+c)^7/d-2/7*b^4*sin(d*x+c)^7/d+1/9*a^4
*sin(d*x+c)^9/d-2/3*a^2*b^2*sin(d*x+c)^9/d+1/9*b^4*sin(d*x+c)^9/d
```

3.74.2 Mathematica [A] (verified)

Time = 6.16 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.33

$$\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = -\frac{4a^3b\cos^9(c+dx)}{9d}+\frac{a^4\sin(c+dx)}{d} - \frac{4a^4\sin^3(c+dx)}{3d}+\frac{6a^4\sin^5(c+dx)}{5d}-\frac{4a^4\sin^7(c+dx)}{7d}+\frac{a^4\sin^9(c+dx)}{9d} + \frac{2a^2b^2(105\sin^3(c+dx)-189\sin^5(c+dx)+135\sin^7(c+dx)-35\sin^9(c+dx))}{105d} + \frac{b^4(63\sin^5(c+dx)-90\sin^7(c+dx)+35\sin^9(c+dx))}{315d} + \frac{4ab^3\cos(c+dx)\sin^8(c+dx)\left(2\csc^8(c+dx)+7\sqrt{1-\sin^2(c+dx)}-19\csc^2(c+dx)\sqrt{1-\sin^2(c+dx)}\right.}{63d\sqrt{}}$$

input
```
Integrate[Cos[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

output
```
(-4*a^3*b*Cos[c + d*x]^9)/(9*d) + (a^4*Sin[c + d*x])/d - (4*a^4*Sin[c + d*
x]^3)/(3*d) + (6*a^4*Sin[c + d*x]^5)/(5*d) - (4*a^4*Sin[c + d*x]^7)/(7*d)
+ (a^4*Sin[c + d*x]^9)/(9*d) + (2*a^2*b^2*(105*Sin[c + d*x]^3 - 189*Sin[c
+ d*x]^5 + 135*Sin[c + d*x]^7 - 35*Sin[c + d*x]^9))/(105*d) + (b^4*(63*Sin
[c + d*x]^5 - 90*Sin[c + d*x]^7 + 35*Sin[c + d*x]^9))/(315*d) + (4*a*b^3*C
os[c + d*x]*Sin[c + d*x]^8*(2*Csc[c + d*x]^8 + 7*Sqrt[1 - Sin[c + d*x]^2]
- 19*Csc[c + d*x]^2*Sqrt[1 - Sin[c + d*x]^2] + 15*Csc[c + d*x]^4*Sqrt[1 -
Sin[c + d*x]^2] - Csc[c + d*x]^6*Sqrt[1 - Sin[c + d*x]^2] - 2*Csc[c + d*x]
^8*Sqrt[1 - Sin[c + d*x]^2]))/(63*d*Sqrt[Cos[c + d*x]^2])
```

3.74.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(c+dx)^5(a\cos(c+dx)+b\sin(c+dx))^4dx$$

$$\Big\downarrow \text{3569}$$

$$\int \left(a^4\cos^9(c+dx)+4a^3b\sin(c+dx)\cos^8(c+dx)+6a^2b^2\sin^2(c+dx)\cos^7(c+dx)+4ab^3\sin^3(c+dx)\cos^6(c+\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^4\sin^9(c+dx)}{9d}-\frac{4a^4\sin^7(c+dx)}{7d}+\frac{6a^4\sin^5(c+dx)}{5d}-\frac{4a^4\sin^3(c+dx)}{3d}+\frac{a^4\sin(c+dx)}{d}-\frac{4a^3b\cos^9(c+dx)}{9d}-\frac{2a^2b^2\sin^9(c+dx)}{3d}+\frac{18a^2b^2\sin^7(c+dx)}{7d}-\frac{18a^2b^2\sin^5(c+dx)}{5d}+\frac{2a^2b^2\sin^3(c+dx)}{d}+\frac{4ab^3\cos^9(c+dx)}{9d}-\frac{4ab^3\cos^7(c+dx)}{7d}+\frac{b^4\sin^9(c+dx)}{9d}-\frac{2b^4\sin^7(c+dx)}{7d}+\frac{b^4\sin^5(c+dx)}{5d}$$

input
```
Int[Cos[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

output
```
(-4*a*b^3*Cos[c + d*x]^7)/(7*d) - (4*a^3*b*Cos[c + d*x]^9)/(9*d) + (4*a*b^
3*Cos[c + d*x]^9)/(9*d) + (a^4*Sin[c + d*x])/d - (4*a^4*Sin[c + d*x]^3)/(3
*d) + (2*a^2*b^2*Sin[c + d*x]^3)/d + (6*a^4*Sin[c + d*x]^5)/(5*d) - (18*a^
2*b^2*Sin[c + d*x]^5)/(5*d) + (b^4*Sin[c + d*x]^5)/(5*d) - (4*a^4*Sin[c +
d*x]^7)/(7*d) + (18*a^2*b^2*Sin[c + d*x]^7)/(7*d) - (2*b^4*Sin[c + d*x]^7)
/(7*d) + (a^4*Sin[c + d*x]^9)/(9*d) - (2*a^2*b^2*Sin[c + d*x]^9)/(3*d) + (
b^4*Sin[c + d*x]^9)/(9*d)
```

3.74.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

3.74.4 Maple [A] (verified)

Time = 1.73 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.69

method	result
parts	$\frac{a^{4}\left(\frac{128}{35}+\cos(dx+c)^{8}+\frac{8\cos(dx+c)^{6}}{7}+\frac{48\cos(dx+c)^{4}}{35}+\frac{64\cos(dx+c)^{2}}{35}\right)\sin(dx+c)}{9d}+\frac{b^{4}\left(\frac{\sin(dx+c)^{9}}{9}-\frac{2\sin(dx+c)^{7}}{7}+\frac{\sin(dx-}{5}\right.}{d}$
derivativedivides	$\frac{\frac{a^{4}\left(\frac{128}{35}+\cos(dx+c)^{8}+\frac{8\cos(dx+c)^{6}}{7}+\frac{48\cos(dx+c)^{4}}{35}+\frac{64\cos(dx+c)^{2}}{35}\right)\sin(dx+c)}{9}-\frac{4a^{3}b\cos(dx+c)^{9}}{9}+6a^{2}b^{2}\left(-\frac{\sin(dx+c)\cos(dx+}{9}\right.}{}$
default	$\frac{\frac{a^{4}\left(\frac{128}{35}+\cos(dx+c)^{8}+\frac{8\cos(dx+c)^{6}}{7}+\frac{48\cos(dx+c)^{4}}{35}+\frac{64\cos(dx+c)^{2}}{35}\right)\sin(dx+c)}{9}-\frac{4a^{3}b\cos(dx+c)^{9}}{9}+6a^{2}b^{2}\left(-\frac{\sin(dx+c)\cos(dx+}{9}\right.}{}$
risch	$-\frac{7a^{3}b\cos(dx+c)}{32d}-\frac{3a\,b^{3}\cos(dx+c)}{32d}+\frac{63a^{4}\sin(dx+c)}{128d}+\frac{21a^{2}b^{2}\sin(dx+c)}{64d}+\frac{3b^{4}\sin(dx+c)}{128d}-\frac{a^{3}b\cos(9dx+9c}{576d}$
parallelrisch	$630a^{4}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{17}-2520\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{16}a^{3}b+(1680a^{4}+5040a^{2}b^{2})\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{15}-5040\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{14}a\,b^{3}+(9576a^{4}$
norman	$-\frac{56a^{3}b+16a\,b^{3}}{63d}+\frac{2a^{4}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2a^{4}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{17}}{d}-\frac{16a\,b^{3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{14}}{d}-\frac{48a\,b^{3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}}{d}-\frac{16a\,b^{3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{7d}-\frac{8}{}$

input
```
int(cos(d*x+c)^5*(cos(d*x+c)*a+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/9*a^4/d*(128/35+cos(d*x+c)^8+8/7*cos(d*x+c)^6+48/35*cos(d*x+c)^4+64/35*c
os(d*x+c)^2)*sin(d*x+c)+b^4/d*(1/9*sin(d*x+c)^9-2/7*sin(d*x+c)^7+1/5*sin(d
*x+c)^5)+4*a*b^3/d*(1/9*cos(d*x+c)^9-1/7*cos(d*x+c)^7)-4/9*a^3*b*cos(d*x+c
)^9/d-6*a^2*b^2/d*(1/9*sin(d*x+c)^9-3/7*sin(d*x+c)^7+3/5*sin(d*x+c)^5-1/3*
sin(d*x+c)^3)
```

3.74.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.63

$$\int \cos^{5}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^{4}\,dx =$$

$$-\frac{180\,ab^{3}\cos\left(dx+c\right)^{7}+140\left(a^{3}b-ab^{3}\right)\cos\left(dx+c\right)^{9}-\left(35\left(a^{4}-6\,a^{2}b^{2}+b^{4}\right)\cos\left(dx+c\right)^{8}+10\left(4\,a^{4}+\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas"
)
```

output
```
-1/315*(180*a*b^3*cos(d*x + c)^7 + 140*(a^3*b - a*b^3)*cos(d*x + c)^9 - (3
5*(a^4 - 6*a^2*b^2 + b^4)*cos(d*x + c)^8 + 10*(4*a^4 + 3*a^2*b^2 - 5*b^4)*
cos(d*x + c)^6 + 3*(16*a^4 + 12*a^2*b^2 + b^4)*cos(d*x + c)^4 + 128*a^4 +
96*a^2*b^2 + 8*b^4 + 4*(16*a^4 + 12*a^2*b^2 + b^4)*cos(d*x + c)^2)*sin(d*x
 + c))/d
```

3.74.6 Sympy [A] (verification not implemented)

Time = 1.07 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.32

$$\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$
$$= \begin{cases} \frac{128a^4\sin^9(c+dx)}{315d} + \frac{64a^4\sin^7(c+dx)\cos^2(c+dx)}{35d} + \frac{16a^4\sin^5(c+dx)\cos^4(c+dx)}{5d} + \frac{8a^4\sin^3(c+dx)\cos^6(c+dx)}{3d} + \frac{a^4\sin(c+dx)\cos^8(}{d} \\ x(a\cos(c)+b\sin(c))^4\cos^5(c) \end{cases}$$

input
```
integrate(cos(d*x+c)**5*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

output
```
Piecewise((128*a**4*sin(c + d*x)**9/(315*d) + 64*a**4*sin(c + d*x)**7*cos(
c + d*x)**2/(35*d) + 16*a**4*sin(c + d*x)**5*cos(c + d*x)**4/(5*d) + 8*a**
4*sin(c + d*x)**3*cos(c + d*x)**6/(3*d) + a**4*sin(c + d*x)*cos(c + d*x)**
8/d - 4*a**3*b*cos(c + d*x)**9/(9*d) + 32*a**2*b**2*sin(c + d*x)**9/(105*d
) + 48*a**2*b**2*sin(c + d*x)**7*cos(c + d*x)**2/(35*d) + 12*a**2*b**2*sin
(c + d*x)**5*cos(c + d*x)**4/(5*d) + 2*a**2*b**2*sin(c + d*x)**3*cos(c + d
*x)**6/d - 4*a*b**3*sin(c + d*x)**2*cos(c + d*x)**7/(7*d) - 8*a*b**3*cos(c
 + d*x)**9/(63*d) + 8*b**4*sin(c + d*x)**9/(315*d) + 4*b**4*sin(c + d*x)**
7*cos(c + d*x)**2/(35*d) + b**4*sin(c + d*x)**5*cos(c + d*x)**4/(5*d), Ne(
d, 0)), (x*(a*cos(c) + b*sin(c))**4*cos(c)**5, True))
```

3.74.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.67

$$\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx =$$
$$-\frac{140\,a^3b\cos(dx+c)^9 - (35\,\sin(dx+c)^9 - 180\,\sin(dx+c)^7 + 378\,\sin(dx+c)^5 - 420\,\sin(dx+c)^3 +}{}$$

input
```
integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima"
)
```

output
```
-1/315*(140*a^3*b*cos(d*x + c)^9 - (35*sin(d*x + c)^9 - 180*sin(d*x + c)^7
 + 378*sin(d*x + c)^5 - 420*sin(d*x + c)^3 + 315*sin(d*x + c))*a^4 + 6*(35
*sin(d*x + c)^9 - 135*sin(d*x + c)^7 + 189*sin(d*x + c)^5 - 105*sin(d*x +
c)^3)*a^2*b^2 - 20*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^7)*a*b^3 - (35*sin(d
*x + c)^9 - 90*sin(d*x + c)^7 + 63*sin(d*x + c)^5)*b^4)/d
```

3.74.8 Giac [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.96

$$\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = -\frac{a^3b\cos(5\,dx+5\,c)}{16\,d} - \frac{(a^3b-ab^3)\cos(9\,dx+9\,c)}{576\,d} - \frac{(7\,a^3b-3\,ab^3)\cos(7\,dx+7\,c)}{448\,d} - \frac{(7\,a^3b+2\,ab^3)\cos(3\,dx+3\,c)}{48\,d} - \frac{(7\,a^3b+3\,ab^3)\cos(dx+c)}{32\,d} + \frac{(a^4-6\,a^2b^2+b^4)\sin(9\,dx+9\,c)}{2304\,d} + \frac{(9\,a^4-30\,a^2b^2+b^4)\sin(7\,dx+7\,c)}{1792\,d} + \frac{(9\,a^4-12\,a^2b^2-b^4)\sin(5\,dx+5\,c)}{320\,d} + \frac{(21\,a^4-b^4)\sin(3\,dx+3\,c)}{192\,d} + \frac{3\,(21\,a^4+14\,a^2b^2+b^4)\sin(dx+c)}{128\,d}$$

input
```
integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/16*a^3*b*cos(5*d*x + 5*c)/d - 1/576*(a^3*b - a*b^3)*cos(9*d*x + 9*c)/d
- 1/448*(7*a^3*b - 3*a*b^3)*cos(7*d*x + 7*c)/d - 1/48*(7*a^3*b + 2*a*b^3)*
cos(3*d*x + 3*c)/d - 1/32*(7*a^3*b + 3*a*b^3)*cos(d*x + c)/d + 1/2304*(a^4
 - 6*a^2*b^2 + b^4)*sin(9*d*x + 9*c)/d + 1/1792*(9*a^4 - 30*a^2*b^2 + b^4)
*sin(7*d*x + 7*c)/d + 1/320*(9*a^4 - 12*a^2*b^2 - b^4)*sin(5*d*x + 5*c)/d
+ 1/192*(21*a^4 - b^4)*sin(3*d*x + 3*c)/d + 3/128*(21*a^4 + 14*a^2*b^2 + b
^4)*sin(d*x + c)/d
```

3.74.9 Mupad [B] (verification not implemented)

Time = 24.67 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.20

$$\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx =$$

$$-\frac{\frac{b^4\sin(3c+3dx)}{192}-\frac{3b^4\sin(c+dx)}{128}-\frac{7a^4\sin(3c+3dx)}{64}-\frac{9a^4\sin(5c+5dx)}{320}-\frac{9a^4\sin(7c+7dx)}{1792}-\frac{a^4\sin(9c+9dx)}{2304}-\frac{63a^4\sin}{12}}{}$$

input
```
int(cos(c + d*x)^5*(a*cos(c + d*x) + b*sin(c + d*x))^4,x)
```

output
```
-((b^4*sin(3*c + 3*d*x))/192 - (3*b^4*sin(c + d*x))/128 - (7*a^4*sin(3*c +
 3*d*x))/64 - (9*a^4*sin(5*c + 5*d*x))/320 - (9*a^4*sin(7*c + 7*d*x))/1792
 - (a^4*sin(9*c + 9*d*x))/2304 - (63*a^4*sin(c + d*x))/128 + (b^4*sin(5*c
+ 5*d*x))/320 - (b^4*sin(7*c + 7*d*x))/1792 - (b^4*sin(9*c + 9*d*x))/2304
+ (a*b^3*cos(3*c + 3*d*x))/24 + (7*a^3*b*cos(3*c + 3*d*x))/48 + (a^3*b*cos
(5*c + 5*d*x))/16 - (3*a*b^3*cos(7*c + 7*d*x))/448 + (a^3*b*cos(7*c + 7*d*
x))/64 - (a*b^3*cos(9*c + 9*d*x))/576 + (a^3*b*cos(9*c + 9*d*x))/576 - (21
*a^2*b^2*sin(c + d*x))/64 + (3*a^2*b^2*sin(5*c + 5*d*x))/80 + (15*a^2*b^2*
sin(7*c + 7*d*x))/896 + (a^2*b^2*sin(9*c + 9*d*x))/384 + (3*a*b^3*cos(c +
d*x))/32 + (7*a^3*b*cos(c + d*x))/32)/d
```

3.75 $\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

3.75.1 Optimal result

Integrand size = 28, antiderivative size = 381

$$\begin{aligned}
&\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx \\
&= \frac{35a^4x}{128}+\frac{15}{64}a^2b^2x+\frac{3b^4x}{128}-\frac{2ab^3\cos^6(c+dx)}{3d}-\frac{a^3b\cos^8(c+dx)}{2d} \\
&\quad+\frac{ab^3\cos^8(c+dx)}{2d}+\frac{35a^4\cos(c+dx)\sin(c+dx)}{128d}+\frac{15a^2b^2\cos(c+dx)\sin(c+dx)}{64d} \\
&\quad+\frac{3b^4\cos(c+dx)\sin(c+dx)}{128d}+\frac{35a^4\cos^3(c+dx)\sin(c+dx)}{192d} \\
&\quad+\frac{5a^2b^2\cos^3(c+dx)\sin(c+dx)}{32d}+\frac{b^4\cos^3(c+dx)\sin(c+dx)}{64d} \\
&\quad+\frac{7a^4\cos^5(c+dx)\sin(c+dx)}{48d}+\frac{a^2b^2\cos^5(c+dx)\sin(c+dx)}{8d} \\
&\quad-\frac{b^4\cos^5(c+dx)\sin(c+dx)}{16d}+\frac{a^4\cos^7(c+dx)\sin(c+dx)}{8d} \\
&\quad-\frac{3a^2b^2\cos^7(c+dx)\sin(c+dx)}{4d}-\frac{b^4\cos^5(c+dx)\sin^3(c+dx)}{8d}
\end{aligned}$$

output
```
35/128*a^4*x+15/64*a^2*b^2*x+3/128*b^4*x-2/3*a*b^3*cos(d*x+c)^6/d-1/2*a^3*
b*cos(d*x+c)^8/d+1/2*a*b^3*cos(d*x+c)^8/d+35/128*a^4*cos(d*x+c)*sin(d*x+c)
/d+15/64*a^2*b^2*cos(d*x+c)*sin(d*x+c)/d+3/128*b^4*cos(d*x+c)*sin(d*x+c)/d
+35/192*a^4*cos(d*x+c)^3*sin(d*x+c)/d+5/32*a^2*b^2*cos(d*x+c)^3*sin(d*x+c)
/d+1/64*b^4*cos(d*x+c)^3*sin(d*x+c)/d+7/48*a^4*cos(d*x+c)^5*sin(d*x+c)/d+1
/8*a^2*b^2*cos(d*x+c)^5*sin(d*x+c)/d-1/16*b^4*cos(d*x+c)^5*sin(d*x+c)/d+1/
8*a^4*cos(d*x+c)^7*sin(d*x+c)/d-3/4*a^2*b^2*cos(d*x+c)^7*sin(d*x+c)/d-1/8*
b^4*cos(d*x+c)^5*sin(d*x+c)^3/d
```

3.75.2 Mathematica [A] (verified)

Time = 3.12 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.58

$$\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$
$$= \frac{24(35a^4+30a^2b^2+3b^4)(c+dx)-96ab(7a^2+3b^2)\cos(2(c+dx))-48ab(7a^2+b^2)\cos(4(c+dx))-32a}{}$$

input
```
Integrate[Cos[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

output
```
(24*(35*a^4 + 30*a^2*b^2 + 3*b^4)*(c + d*x) - 96*a*b*(7*a^2 + 3*b^2)*Cos[2
*(c + d*x)] - 48*a*b*(7*a^2 + b^2)*Cos[4*(c + d*x)] - 32*a*b*(3*a^2 - b^2)
*Cos[6*(c + d*x)] - 12*a*b*(a^2 - b^2)*Cos[8*(c + d*x)] + 96*a^2*(7*a^2 +
3*b^2)*Sin[2*(c + d*x)] + 24*(7*a^4 - 6*a^2*b^2 - b^4)*Sin[4*(c + d*x)] +
32*a^2*(a^2 - 3*b^2)*Sin[6*(c + d*x)] + 3*(a^4 - 6*a^2*b^2 + b^4)*Sin[8*(c
 + d*x)])/(3072*d)
```

3.75.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \cos(c+dx)^4(a\cos(c+dx)+b\sin(c+dx))^4dx$$
$$\Big\downarrow \ 3569$$
$$\int \left(a^4\cos^8(c+dx)+4a^3b\sin(c+dx)\cos^7(c+dx)+6a^2b^2\sin^2(c+dx)\cos^6(c+dx)+4ab^3\sin^3(c+dx)\cos^5(c+\right.$$
$$\Big\downarrow \ 2009$$

$$\frac{a^4\sin(c+dx)\cos^7(c+dx)}{8d}+\frac{7a^4\sin(c+dx)\cos^5(c+dx)}{48d}+\frac{35a^4\sin(c+dx)\cos^3(c+dx)}{192d}+\frac{35a^4\sin(c+dx)\cos(c+dx)}{128d}+\frac{35a^4x}{128}-\frac{a^3b\cos^8(c+dx)}{2d}-\frac{3a^2b^2\sin(c+dx)\cos^7(c+dx)}{4d}+\frac{a^2b^2\sin(c+dx)\cos^5(c+dx)}{8d}+\frac{5a^2b^2\sin(c+dx)\cos^3(c+dx)}{32d}+\frac{15a^2b^2\sin(c+dx)\cos(c+dx)}{64d}+\frac{15}{64}a^2b^2x+\frac{ab^3\cos^8(c+dx)}{2d}-\frac{2ab^3\cos^6(c+dx)}{3d}-\frac{b^4\sin^3(c+dx)\cos^5(c+dx)}{8d}-\frac{b^4\sin(c+dx)\cos^5(c+dx)}{16d}+\frac{b^4\sin(c+dx)\cos^3(c+dx)}{64d}+\frac{3b^4\sin(c+dx)\cos(c+dx)}{128d}+\frac{3b^4x}{128}$$

input
```
Int[Cos[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

output
```
(35*a^4*x)/128 + (15*a^2*b^2*x)/64 + (3*b^4*x)/128 - (2*a*b^3*Cos[c + d*x]
^6)/(3*d) - (a^3*b*Cos[c + d*x]^8)/(2*d) + (a*b^3*Cos[c + d*x]^8)/(2*d) +
(35*a^4*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (15*a^2*b^2*Cos[c + d*x]*Sin[
c + d*x])/(64*d) + (3*b^4*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (35*a^4*Cos
[c + d*x]^3*Sin[c + d*x])/(192*d) + (5*a^2*b^2*Cos[c + d*x]^3*Sin[c + d*x]
)/(32*d) + (b^4*Cos[c + d*x]^3*Sin[c + d*x])/(64*d) + (7*a^4*Cos[c + d*x]^
5*Sin[c + d*x])/(48*d) + (a^2*b^2*Cos[c + d*x]^5*Sin[c + d*x])/(8*d) - (b^
4*Cos[c + d*x]^5*Sin[c + d*x])/(16*d) + (a^4*Cos[c + d*x]^7*Sin[c + d*x])/
(8*d) - (3*a^2*b^2*Cos[c + d*x]^7*Sin[c + d*x])/(4*d) - (b^4*Cos[c + d*x]^
5*Sin[c + d*x]^3)/(8*d)
```

3.75.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

3.75.4 Maple [A] (verified)

Time = 1.62 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.63

method	result
parallelrisch	$\frac{24(7a^4-6a^2b^2-b^4)\sin(4dx+4c)+3(a^4-6a^2b^2+b^4)\sin(8dx+8c)+96(-7a^3b-3a\,b^3)\cos(2dx+2c)+48(-7a^3b-a\,b^3)\cos}{}$
derivativedivides	$\frac{a^4\left(\frac{\left(\cos(dx+c)^7+\frac{7\cos(dx+c)^5}{6}+\frac{35\cos(dx+c)^3}{24}+\frac{35\cos(dx+c)}{16}\right)\sin(dx+c)}{8}+\frac{35dx}{128}+\frac{35c}{128}\right)-\frac{a^3b\cos(dx+c)^8}{2}+6a^2b^2\left(-\frac{\sin(dx+}{}\right.}{}$
default	$\frac{a^4\left(\frac{\left(\cos(dx+c)^7+\frac{7\cos(dx+c)^5}{6}+\frac{35\cos(dx+c)^3}{24}+\frac{35\cos(dx+c)}{16}\right)\sin(dx+c)}{8}+\frac{35dx}{128}+\frac{35c}{128}\right)-\frac{a^3b\cos(dx+c)^8}{2}+6a^2b^2\left(-\frac{\sin(dx+}{}\right.}{}$
parts	$\frac{a^4\left(\frac{\left(\cos(dx+c)^7+\frac{7\cos(dx+c)^5}{6}+\frac{35\cos(dx+c)^3}{24}+\frac{35\cos(dx+c)}{16}\right)\sin(dx+c)}{8}+\frac{35dx}{128}+\frac{35c}{128}\right)}{d}+\frac{b^4\left(-\frac{\sin(dx+c)^3\cos(dx+c)^5}{8}-\right.}{}$
risch	$\frac{35a^4x}{128}+\frac{15a^2b^2x}{64}+\frac{3b^4x}{128}-\frac{a^3b\cos(8dx+8c)}{256d}+\frac{a\,b^3\cos(8dx+8c)}{256d}+\frac{\sin(8dx+8c)a^4}{1024d}-\frac{3\sin(8dx+8c)a^2b^2}{512d}+$
norman	$\frac{(\frac{35}{128}a^4+\frac{15}{64}a^2b^2+\frac{3}{128}b^4)x+(\frac{35}{16}a^4+\frac{15}{8}a^2b^2+\frac{3}{16}b^4)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+(\frac{35}{16}a^4+\frac{15}{8}a^2b^2+\frac{3}{16}b^4)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{14}+(\frac{35}{128}a^4+}{}$

input
```
int(cos(d*x+c)^4*(cos(d*x+c)*a+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/3072*(24*(7*a^4-6*a^2*b^2-b^4)*sin(4*d*x+4*c)+3*(a^4-6*a^2*b^2+b^4)*sin(
8*d*x+8*c)+96*(-7*a^3*b-3*a*b^3)*cos(2*d*x+2*c)+48*(-7*a^3*b-a*b^3)*cos(4*
d*x+4*c)+32*(-3*a^3*b+a*b^3)*cos(6*d*x+6*c)+12*(-a^3*b+a*b^3)*cos(8*d*x+8*
c)+96*(7*a^4+3*a^2*b^2)*sin(2*d*x+2*c)+32*(a^4-3*a^2*b^2)*sin(6*d*x+6*c)+8
40*a^4*d*x+720*a^2*b^2*d*x+72*b^4*d*x+1116*a^3*b+292*a*b^3)/d
```

3.75.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.48

$$\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx =$$
$$-\frac{256\,ab^3\cos(dx+c)^6+192\,(a^3b-ab^3)\cos(dx+c)^8-3\,(35\,a^4+30\,a^2b^2+3\,b^4)dx-\left(48\,(a^4-6\,a^2b^2+\right.}{}$$

input
```
integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas"
)
```

output
```
-1/384*(256*a*b^3*cos(d*x + c)^6 + 192*(a^3*b - a*b^3)*cos(d*x + c)^8 - 3*
(35*a^4 + 30*a^2*b^2 + 3*b^4)*d*x - (48*(a^4 - 6*a^2*b^2 + b^4)*cos(d*x +
c)^7 + 8*(7*a^4 + 6*a^2*b^2 - 9*b^4)*cos(d*x + c)^5 + 2*(35*a^4 + 30*a^2*b
^2 + 3*b^4)*cos(d*x + c)^3 + 3*(35*a^4 + 30*a^2*b^2 + 3*b^4)*cos(d*x + c))
*sin(d*x + c))/d
```

3.75.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 760 vs. $2(367) = 734$.

Time = 0.81 (sec) , antiderivative size = 760, normalized size of antiderivative = 1.99

$$\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$
$$= \begin{cases} \frac{35a^4x\sin^8(c+dx)}{128} + \frac{35a^4x\sin^6(c+dx)\cos^2(c+dx)}{32} + \frac{105a^4x\sin^4(c+dx)\cos^4(c+dx)}{64} + \frac{35a^4x\sin^2(c+dx)\cos^6(c+dx)}{32} + \frac{35a^4x\cos^8(\ldots}{128} \\ x(a\cos(c)+b\sin(c))^4\cos^4(c) \end{cases}$$

input
```
integrate(cos(d*x+c)**4*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

output
```
Piecewise((35*a**4*x*sin(c + d*x)**8/128 + 35*a**4*x*sin(c + d*x)**6*cos(c
 + d*x)**2/32 + 105*a**4*x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 35*a**4*x*
sin(c + d*x)**2*cos(c + d*x)**6/32 + 35*a**4*x*cos(c + d*x)**8/128 + 35*a*
*4*sin(c + d*x)**7*cos(c + d*x)/(128*d) + 385*a**4*sin(c + d*x)**5*cos(c +
 d*x)**3/(384*d) + 511*a**4*sin(c + d*x)**3*cos(c + d*x)**5/(384*d) + 93*a
**4*sin(c + d*x)*cos(c + d*x)**7/(128*d) - a**3*b*cos(c + d*x)**8/(2*d) +
15*a**2*b**2*x*sin(c + d*x)**8/64 + 15*a**2*b**2*x*sin(c + d*x)**6*cos(c +
 d*x)**2/16 + 45*a**2*b**2*x*sin(c + d*x)**4*cos(c + d*x)**4/32 + 15*a**2*
b**2*x*sin(c + d*x)**2*cos(c + d*x)**6/16 + 15*a**2*b**2*x*cos(c + d*x)**8
/64 + 15*a**2*b**2*sin(c + d*x)**7*cos(c + d*x)/(64*d) + 55*a**2*b**2*sin(
c + d*x)**5*cos(c + d*x)**3/(64*d) + 73*a**2*b**2*sin(c + d*x)**3*cos(c +
d*x)**5/(64*d) - 15*a**2*b**2*sin(c + d*x)*cos(c + d*x)**7/(64*d) + a*b**3
*sin(c + d*x)**8/(6*d) + 2*a*b**3*sin(c + d*x)**6*cos(c + d*x)**2/(3*d) +
a*b**3*sin(c + d*x)**4*cos(c + d*x)**4/d + 3*b**4*x*sin(c + d*x)**8/128 +
3*b**4*x*sin(c + d*x)**6*cos(c + d*x)**2/32 + 9*b**4*x*sin(c + d*x)**4*cos
(c + d*x)**4/64 + 3*b**4*x*sin(c + d*x)**2*cos(c + d*x)**6/32 + 3*b**4*x*c
os(c + d*x)**8/128 + 3*b**4*sin(c + d*x)**7*cos(c + d*x)/(128*d) + 11*b**4
*sin(c + d*x)**5*cos(c + d*x)**3/(128*d) - 11*b**4*sin(c + d*x)**3*cos(c +
 d*x)**5/(128*d) - 3*b**4*sin(c + d*x)*cos(c + d*x)**7/(128*d), Ne(d, 0)),
 (x*(a*cos(c) + b*sin(c))**4*cos(c)**4, True))
```

3.75.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.52

$$\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx =$$

$$-\frac{1536\,a^3b\cos\left(dx+c\right)^8+\left(128\,\sin\left(2\,dx+2\,c\right)^3-840\,dx-840\,c-3\,\sin\left(8\,dx+8\,c\right)-168\,\sin\left(4\,dx+4\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima"
)
```

output
```
-1/3072*(1536*a^3*b*cos(d*x + c)^8 + (128*sin(2*d*x + 2*c)^3 - 840*d*x - 8
40*c - 3*sin(8*d*x + 8*c) - 168*sin(4*d*x + 4*c) - 768*sin(2*d*x + 2*c))*a
^4 - 6*(64*sin(2*d*x + 2*c)^3 + 120*d*x + 120*c - 3*sin(8*d*x + 8*c) - 24*
sin(4*d*x + 4*c))*a^2*b^2 - 512*(3*sin(d*x + c)^8 - 8*sin(d*x + c)^6 + 6*s
in(d*x + c)^4)*a*b^3 - 3*(24*d*x + 24*c + sin(8*d*x + 8*c) - 8*sin(4*d*x +
 4*c))*b^4)/d
```

3.75.8 Giac [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.64

$$\begin{aligned}\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx &= \frac{1}{128}\left(35\,a^4+30\,a^2b^2+3\,b^4\right)x-\frac{\left(a^3b-ab^3\right)\cos\left(8\,dx+8\,c\right)}{256\,d}\\ &\quad-\frac{\left(3\,a^3b-ab^3\right)\cos\left(6\,dx+6\,c\right)}{96\,d}-\frac{\left(7\,a^3b+ab^3\right)\cos\left(4\,dx+4\,c\right)}{64\,d}\\ &\quad-\frac{\left(7\,a^3b+3\,ab^3\right)\cos\left(2\,dx+2\,c\right)}{32\,d}\\ &\quad+\frac{\left(a^4-6\,a^2b^2+b^4\right)\sin\left(8\,dx+8\,c\right)}{1024\,d}+\frac{\left(a^4-3\,a^2b^2\right)\sin\left(6\,dx+6\,c\right)}{96\,d}\\ &\quad+\frac{\left(7\,a^4-6\,a^2b^2-b^4\right)\sin\left(4\,dx+4\,c\right)}{128\,d}+\frac{\left(7\,a^4+3\,a^2b^2\right)\sin\left(2\,dx+2\,c\right)}{32\,d}\end{aligned}$$

input
```
integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

output
```
1/128*(35*a^4 + 30*a^2*b^2 + 3*b^4)*x - 1/256*(a^3*b - a*b^3)*cos(8*d*x +
8*c)/d - 1/96*(3*a^3*b - a*b^3)*cos(6*d*x + 6*c)/d - 1/64*(7*a^3*b + a*b^3
)*cos(4*d*x + 4*c)/d - 1/32*(7*a^3*b + 3*a*b^3)*cos(2*d*x + 2*c)/d + 1/102
4*(a^4 - 6*a^2*b^2 + b^4)*sin(8*d*x + 8*c)/d + 1/96*(a^4 - 3*a^2*b^2)*sin(
6*d*x + 6*c)/d + 1/128*(7*a^4 - 6*a^2*b^2 - b^4)*sin(4*d*x + 4*c)/d + 1/32
*(7*a^4 + 3*a^2*b^2)*sin(2*d*x + 2*c)/d
```

3.75.9 Mupad [B] (verification not implemented)

Time = 23.87 (sec) , antiderivative size = 343, normalized size of antiderivative = 0.90

$$\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \frac{35\,a^4\,x}{128}+\frac{3\,b^4\,x}{128}+\frac{15\,a^2\,b^2\,x}{64}-\frac{2\,a\,b^3\cos\left(c+d\,x\right)^6}{3\,d}+\frac{a\,b^3\cos\left(c+d\,x\right)^8}{2\,d} -\frac{a^3\,b\cos\left(c+d\,x\right)^8}{2\,d}+\frac{35\,a^4\cos\left(c+d\,x\right)^3\,\sin\left(c+d\,x\right)}{192\,d}+\frac{7\,a^4\cos\left(c+d\,x\right)^5\,\sin\left(c+d\,x\right)}{48\,d} +\frac{a^4\cos\left(c+d\,x\right)^7\,\sin\left(c+d\,x\right)}{8\,d}+\frac{b^4\cos\left(c+d\,x\right)^3\,\sin\left(c+d\,x\right)}{64\,d} -\frac{3\,b^4\cos\left(c+d\,x\right)^5\,\sin\left(c+d\,x\right)}{16\,d}+\frac{b^4\cos\left(c+d\,x\right)^7\,\sin\left(c+d\,x\right)}{8\,d} +\frac{35\,a^4\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{128\,d}+\frac{3\,b^4\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{128\,d} +\frac{15\,a^2\,b^2\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{64\,d}+\frac{5\,a^2\,b^2\cos\left(c+d\,x\right)^3\,\sin\left(c+d\,x\right)}{32\,d} +\frac{a^2\,b^2\cos\left(c+d\,x\right)^5\,\sin\left(c+d\,x\right)}{8\,d}-\frac{3\,a^2\,b^2\cos\left(c+d\,x\right)^7\,\sin\left(c+d\,x\right)}{4\,d}$$

input
```
int(cos(c + d*x)^4*(a*cos(c + d*x) + b*sin(c + d*x))^4,x)
```

output
```
(35*a^4*x)/128 + (3*b^4*x)/128 + (15*a^2*b^2*x)/64 - (2*a*b^3*cos(c + d*x)
^6)/(3*d) + (a*b^3*cos(c + d*x)^8)/(2*d) - (a^3*b*cos(c + d*x)^8)/(2*d) +
(35*a^4*cos(c + d*x)^3*sin(c + d*x))/(192*d) + (7*a^4*cos(c + d*x)^5*sin(c
 + d*x))/(48*d) + (a^4*cos(c + d*x)^7*sin(c + d*x))/(8*d) + (b^4*cos(c + d
*x)^3*sin(c + d*x))/(64*d) - (3*b^4*cos(c + d*x)^5*sin(c + d*x))/(16*d) +
(b^4*cos(c + d*x)^7*sin(c + d*x))/(8*d) + (35*a^4*cos(c + d*x)*sin(c + d*x
))/(128*d) + (3*b^4*cos(c + d*x)*sin(c + d*x))/(128*d) + (15*a^2*b^2*cos(c
 + d*x)*sin(c + d*x))/(64*d) + (5*a^2*b^2*cos(c + d*x)^3*sin(c + d*x))/(32
*d) + (a^2*b^2*cos(c + d*x)^5*sin(c + d*x))/(8*d) - (3*a^2*b^2*cos(c + d*x
)^7*sin(c + d*x))/(4*d)
```

3.76 $\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

3.76.1 Optimal result

Integrand size = 28, antiderivative size = 220

$$\begin{aligned}\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx\\ &= -\frac{4ab^3\cos^5(c+dx)}{5d}-\frac{4a^3b\cos^7(c+dx)}{7d}+\frac{4ab^3\cos^7(c+dx)}{7d}+\frac{a^4\sin(c+dx)}{d}\\ &\quad -\frac{a^4\sin^3(c+dx)}{d}+\frac{2a^2b^2\sin^3(c+dx)}{d}+\frac{3a^4\sin^5(c+dx)}{5d}-\frac{12a^2b^2\sin^5(c+dx)}{5d}\\ &\quad +\frac{b^4\sin^5(c+dx)}{5d}-\frac{a^4\sin^7(c+dx)}{7d}+\frac{6a^2b^2\sin^7(c+dx)}{7d}-\frac{b^4\sin^7(c+dx)}{7d}\end{aligned}$$

output
```
-4/5*a*b^3*cos(d*x+c)^5/d-4/7*a^3*b*cos(d*x+c)^7/d+4/7*a*b^3*cos(d*x+c)^7/
d+a^4*sin(d*x+c)/d-a^4*sin(d*x+c)^3/d+2*a^2*b^2*sin(d*x+c)^3/d+3/5*a^4*sin
(d*x+c)^5/d-12/5*a^2*b^2*sin(d*x+c)^5/d+1/5*b^4*sin(d*x+c)^5/d-1/7*a^4*sin
(d*x+c)^7/d+6/7*a^2*b^2*sin(d*x+c)^7/d-1/7*b^4*sin(d*x+c)^7/d
```

3.76.2 Mathematica [A] (verified)

Time = 2.95 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.75

$$\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \frac{-20a^3b\cos^7(c+dx)+35a^4\sin(c+dx)-35a^2(a^2-2b^2)\sin^3(c+dx)+7(3a^4-12a^2b^2+b^4)\sin^5(c+dx}{}$$

input
```
Integrate[Cos[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

output
```
(-20*a^3*b*Cos[c + d*x]^7 + 35*a^4*Sin[c + d*x] - 35*a^2*(a^2 - 2*b^2)*Sin
[c + d*x]^3 + 7*(3*a^4 - 12*a^2*b^2 + b^4)*Sin[c + d*x]^5 - 5*(a^4 - 6*a^2
*b^2 + b^4)*Sin[c + d*x]^7 + 4*a*b^3*Cos[c + d*x]*(-2 + 2/Sqrt[Cos[c + d*x
]^2] - Sin[c + d*x]^2 + 8*Sin[c + d*x]^4 - 5*Sin[c + d*x]^6))/(35*d)
```

3.76.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(c+dx)^3(a\cos(c+dx)+b\sin(c+dx))^4dx$$

$$\Big\downarrow \text{3569}$$

$$\int \left(a^4\cos^7(c+dx)+4a^3b\sin(c+dx)\cos^6(c+dx)+6a^2b^2\sin^2(c+dx)\cos^5(c+dx)+4ab^3\sin^3(c+dx)\cos^4(c+\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^4\sin^7(c+dx)}{7d}+\frac{3a^4\sin^5(c+dx)}{5d}-\frac{a^4\sin^3(c+dx)}{d}+\frac{a^4\sin(c+dx)}{d}-\frac{4a^3b\cos^7(c+dx)}{7d}+\frac{6a^2b^2\sin^7(c+dx)}{7d}-\frac{12a^2b^2\sin^5(c+dx)}{5d}+\frac{2a^2b^2\sin^3(c+dx)}{d}+\frac{4ab^3\cos^7(c+dx)}{7d}-\frac{4ab^3\cos^5(c+dx)}{5d}-\frac{b^4\sin^7(c+dx)}{7d}+\frac{b^4\sin^5(c+dx)}{5d}$$

input
```
Int[Cos[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

output
```
(-4*a*b^3*Cos[c + d*x]^5)/(5*d) - (4*a^3*b*Cos[c + d*x]^7)/(7*d) + (4*a*b^
3*Cos[c + d*x]^7)/(7*d) + (a^4*Sin[c + d*x])/d - (a^4*Sin[c + d*x]^3)/d +
(2*a^2*b^2*Sin[c + d*x]^3)/d + (3*a^4*Sin[c + d*x]^5)/(5*d) - (12*a^2*b^2*
Sin[c + d*x]^5)/(5*d) + (b^4*Sin[c + d*x]^5)/(5*d) - (a^4*Sin[c + d*x]^7)/
(7*d) + (6*a^2*b^2*Sin[c + d*x]^7)/(7*d) - (b^4*Sin[c + d*x]^7)/(7*d)
```

3.76.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

3.76.4 Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.74

method	result
parts	$\frac{a^4\left(\frac{16}{5}+\cos(dx+c)^6+\frac{6\cos(dx+c)^4}{5}+\frac{8\cos(dx+c)^2}{5}\right)\sin(dx+c)}{7d}+\frac{b^4\left(-\frac{\sin(dx+c)^7}{7}+\frac{\sin(dx+c)^5}{5}\right)}{d}+\frac{4a\,b^3\left(\frac{\cos(dx+c)^7}{7}-\right.}{d}$
derivativedivides	$\frac{\frac{a^4\left(\frac{16}{5}+\cos(dx+c)^6+\frac{6\cos(dx+c)^4}{5}+\frac{8\cos(dx+c)^2}{5}\right)\sin(dx+c)}{7}-\frac{4a^3b\cos(dx+c)^7}{7}+6a^2b^2\left(-\frac{\sin(dx+c)\cos(dx+c)^6}{7}+\frac{\left(\frac{8}{3}+\cos(dx\right.}{}\right.}{}$
default	$\frac{\frac{a^4\left(\frac{16}{5}+\cos(dx+c)^6+\frac{6\cos(dx+c)^4}{5}+\frac{8\cos(dx+c)^2}{5}\right)\sin(dx+c)}{7}-\frac{4a^3b\cos(dx+c)^7}{7}+6a^2b^2\left(-\frac{\sin(dx+c)\cos(dx+c)^6}{7}+\frac{\left(\frac{8}{3}+\cos(dx\right.}{}\right.}{}$
parallelrisch	$\frac{70\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{13}a^4-280\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}a^3b+(140a^4+560a^2b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}-560\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}a\,b^3+(602a^4-448a}{}$
risch	$-\frac{5a^3b\cos(dx+c)}{16d}-\frac{3a\,b^3\cos(dx+c)}{16d}+\frac{35a^4\sin(dx+c)}{64d}+\frac{15a^2b^2\sin(dx+c)}{32d}+\frac{3b^4\sin(dx+c)}{64d}-\frac{a^3b\cos(7dx+7c}{112d}$
norman	$-\frac{40a^3b+16a\,b^3}{35d}+\frac{2a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{13}}{d}-\frac{16a\,b^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{5d}-\frac{16a\,b^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{d}-\frac{32a\,b^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{d}-\frac{8}{}$

input
```
int(cos(d*x+c)^3*(cos(d*x+c)*a+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/7*a^4/d*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c)
+b^4/d*(-1/7*sin(d*x+c)^7+1/5*sin(d*x+c)^5)+4*a*b^3/d*(1/7*cos(d*x+c)^7-1/
5*cos(d*x+c)^5)-4/7*a^3*b*cos(d*x+c)^7/d+6*a^2*b^2/d*(1/7*sin(d*x+c)^7-2/5
*sin(d*x+c)^5+1/3*sin(d*x+c)^3)
```

3.76.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.68

$$\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = -\frac{28\,ab^3\cos\left(dx+c\right)^5+20\left(a^3b-ab^3\right)\cos\left(dx+c\right)^7-\left(5\left(a^4-6\,a^2b^2+b^4\right)\cos\left(dx+c\right)^6+2\left(3\,a^4+3\,a^2b\right.\right.}{35\,d}$$

input
```
integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas"
)
```

output
```
-1/35*(28*a*b^3*cos(d*x + c)^5 + 20*(a^3*b - a*b^3)*cos(d*x + c)^7 - (5*(a
^4 - 6*a^2*b^2 + b^4)*cos(d*x + c)^6 + 2*(3*a^4 + 3*a^2*b^2 - 4*b^4)*cos(d
*x + c)^4 + 16*a^4 + 16*a^2*b^2 + 2*b^4 + (8*a^4 + 8*a^2*b^2 + b^4)*cos(d*
x + c)^2)*sin(d*x + c))/d
```

3.76.6 Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.30

$$\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \begin{cases} \frac{16a^4\sin^7(c+dx)}{35d} + \frac{8a^4\sin^5(c+dx)\cos^2(c+dx)}{5d} + \frac{2a^4\sin^3(c+dx)\cos^4(c+dx)}{d} + \frac{a^4\sin(c+dx)\cos^6(c+dx)}{d} - \frac{4a^3b\cos^7(c+dx)}{7d} + \frac{16a \\ x(a\cos(c)+b\sin(c))^4\cos^3(c) \end{cases}$$

input
```
integrate(cos(d*x+c)**3*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

output
```
Piecewise((16*a**4*sin(c + d*x)**7/(35*d) + 8*a**4*sin(c + d*x)**5*cos(c +
 d*x)**2/(5*d) + 2*a**4*sin(c + d*x)**3*cos(c + d*x)**4/d + a**4*sin(c + d
*x)*cos(c + d*x)**6/d - 4*a**3*b*cos(c + d*x)**7/(7*d) + 16*a**2*b**2*sin(
c + d*x)**7/(35*d) + 8*a**2*b**2*sin(c + d*x)**5*cos(c + d*x)**2/(5*d) + 2
*a**2*b**2*sin(c + d*x)**3*cos(c + d*x)**4/d - 4*a*b**3*sin(c + d*x)**2*co
s(c + d*x)**5/(5*d) - 8*a*b**3*cos(c + d*x)**7/(35*d) + 2*b**4*sin(c + d*x
)**7/(35*d) + b**4*sin(c + d*x)**5*cos(c + d*x)**2/(5*d), Ne(d, 0)), (x*(a
*cos(c) + b*sin(c))**4*cos(c)**3, True))
```

3.76.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.70

$$\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = -\frac{20\,a^3b\cos(dx+c)^7 + \left(5\,\sin(dx+c)^7 - 21\,\sin(dx+c)^5 + 35\,\sin(dx+c)^3 - 35\,\sin(dx+c)\right)a^4 - 2\,(1}{}$$

input
```
integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima"
)
```

output
```
-1/35*(20*a^3*b*cos(d*x + c)^7 + (5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 3
5*sin(d*x + c)^3 - 35*sin(d*x + c))*a^4 - 2*(15*sin(d*x + c)^7 - 42*sin(d*
x + c)^5 + 35*sin(d*x + c)^3)*a^2*b^2 - 4*(5*cos(d*x + c)^7 - 7*cos(d*x +
c)^5)*a*b^3 + (5*sin(d*x + c)^7 - 7*sin(d*x + c)^5)*b^4)/d
```

3.76.8 Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.04

$$\begin{aligned}\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx &= -\frac{(a^3b-ab^3)\cos(7\,dx+7\,c)}{112\,d} - \frac{(5\,a^3b-ab^3)\cos(5\,dx+5\,c)}{80\,d}\\ &\quad - \frac{(3\,a^3b+ab^3)\cos(3\,dx+3\,c)}{16\,d} - \frac{(5\,a^3b+3\,ab^3)\cos(dx+c)}{16\,d}\\ &\quad + \frac{(a^4-6\,a^2b^2+b^4)\sin(7\,dx+7\,c)}{448\,d} + \frac{(7\,a^4-18\,a^2b^2-b^4)\sin(5\,dx+5\,c)}{320\,d}\\ &\quad + \frac{(7\,a^4-2\,a^2b^2-b^4)\sin(3\,dx+3\,c)}{64\,d} + \frac{(35\,a^4+30\,a^2b^2+3\,b^4)\sin(dx+c)}{64\,d}\end{aligned}$$

input
```
integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/112*(a^3*b - a*b^3)*cos(7*d*x + 7*c)/d - 1/80*(5*a^3*b - a*b^3)*cos(5*d
*x + 5*c)/d - 1/16*(3*a^3*b + a*b^3)*cos(3*d*x + 3*c)/d - 1/16*(5*a^3*b +
3*a*b^3)*cos(d*x + c)/d + 1/448*(a^4 - 6*a^2*b^2 + b^4)*sin(7*d*x + 7*c)/d
 + 1/320*(7*a^4 - 18*a^2*b^2 - b^4)*sin(5*d*x + 5*c)/d + 1/64*(7*a^4 - 2*a
^2*b^2 - b^4)*sin(3*d*x + 3*c)/d + 1/64*(35*a^4 + 30*a^2*b^2 + 3*b^4)*sin(
d*x + c)/d
```

3.76.9 Mupad [B] (verification not implemented)

Time = 23.25 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.32

$$\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = -\frac{\frac{b^4\sin(3\,c+3\,d\,x)}{64} - \frac{3\,b^4\sin(c+d\,x)}{64} - \frac{7\,a^4\sin(3\,c+3\,d\,x)}{64} - \frac{7\,a^4\sin(5\,c+5\,d\,x)}{320} - \frac{a^4\sin(7\,c+7\,d\,x)}{448} - \frac{35\,a^4\sin(c+d\,x)}{64} + \frac{b^4\sin(5\,c+}{320}}{}$$

input
```
int(cos(c + d*x)^3*(a*cos(c + d*x) + b*sin(c + d*x))^4,x)
```

output
```
-((b^4*sin(3*c + 3*d*x))/64 - (3*b^4*sin(c + d*x))/64 - (7*a^4*sin(3*c + 3
*d*x))/64 - (7*a^4*sin(5*c + 5*d*x))/320 - (a^4*sin(7*c + 7*d*x))/448 - (3
5*a^4*sin(c + d*x))/64 + (b^4*sin(5*c + 5*d*x))/320 - (b^4*sin(7*c + 7*d*x
))/448 + (a*b^3*cos(3*c + 3*d*x))/16 + (3*a^3*b*cos(3*c + 3*d*x))/16 - (a*
b^3*cos(5*c + 5*d*x))/80 + (a^3*b*cos(5*c + 5*d*x))/16 - (a*b^3*cos(7*c +
7*d*x))/112 + (a^3*b*cos(7*c + 7*d*x))/112 - (15*a^2*b^2*sin(c + d*x))/32
+ (a^2*b^2*sin(3*c + 3*d*x))/32 + (9*a^2*b^2*sin(5*c + 5*d*x))/160 + (3*a^
2*b^2*sin(7*c + 7*d*x))/224 + (3*a*b^3*cos(c + d*x))/16 + (5*a^3*b*cos(c +
 d*x))/16)/d
```

3.76. $\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

3.77 $\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

3.77.1 Optimal result

Integrand size = 28, antiderivative size = 301

$$\begin{aligned}
&\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx \\
&= \frac{5a^4x}{16}+\frac{3}{8}a^2b^2x+\frac{b^4x}{16}-\frac{2a^3b\cos^6(c+dx)}{3d} \\
&\quad+\frac{5a^4\cos(c+dx)\sin(c+dx)}{16d}+\frac{3a^2b^2\cos(c+dx)\sin(c+dx)}{8d} \\
&\quad+\frac{b^4\cos(c+dx)\sin(c+dx)}{16d}+\frac{5a^4\cos^3(c+dx)\sin(c+dx)}{24d} \\
&\quad+\frac{a^2b^2\cos^3(c+dx)\sin(c+dx)}{4d}-\frac{b^4\cos^3(c+dx)\sin(c+dx)}{8d} \\
&\quad+\frac{a^4\cos^5(c+dx)\sin(c+dx)}{6d}-\frac{a^2b^2\cos^5(c+dx)\sin(c+dx)}{d} \\
&\quad-\frac{b^4\cos^3(c+dx)\sin^3(c+dx)}{6d}+\frac{ab^3\sin^4(c+dx)}{d}-\frac{2ab^3\sin^6(c+dx)}{3d}
\end{aligned}$$

output
```
5/16*a^4*x+3/8*a^2*b^2*x+1/16*b^4*x-2/3*a^3*b*cos(d*x+c)^6/d+5/16*a^4*cos(
d*x+c)*sin(d*x+c)/d+3/8*a^2*b^2*cos(d*x+c)*sin(d*x+c)/d+1/16*b^4*cos(d*x+c
)*sin(d*x+c)/d+5/24*a^4*cos(d*x+c)^3*sin(d*x+c)/d+1/4*a^2*b^2*cos(d*x+c)^3
*sin(d*x+c)/d-1/8*b^4*cos(d*x+c)^3*sin(d*x+c)/d+1/6*a^4*cos(d*x+c)^5*sin(d
*x+c)/d-a^2*b^2*cos(d*x+c)^5*sin(d*x+c)/d-1/6*b^4*cos(d*x+c)^3*sin(d*x+c)^
3/d+a*b^3*sin(d*x+c)^4/d-2/3*a*b^3*sin(d*x+c)^6/d
```

3.77.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.87 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.59

$$\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \frac{12(a-ib)(a+ib)\left(5a^2+b^2\right)(c+dx)-12ab(5a^2+3b^2)\cos(2(c+dx))-24a^3b\cos(4(c+dx))-4ab(a^2-\cdots}{\cdots}$$

input
```
Integrate[Cos[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

output
```
(12*(a - I*b)*(a + I*b)*(5*a^2 + b^2)*(c + d*x) - 12*a*b*(5*a^2 + 3*b^2)*C
os[2*(c + d*x)] - 24*a^3*b*Cos[4*(c + d*x)] - 4*a*b*(a^2 - b^2)*Cos[6*(c +
 d*x)] + 3*(15*a^4 + 6*a^2*b^2 - b^4)*Sin[2*(c + d*x)] + 3*(3*a^4 - 6*a^2*
b^2 - b^4)*Sin[4*(c + d*x)] + (a^4 - 6*a^2*b^2 + b^4)*Sin[6*(c + d*x)])/(1
92*d)
```

3.77.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))^4dx$$

$$\Big\downarrow \text{3569}$$

$$\int \left(a^4\cos^6(c+dx)+4a^3b\sin(c+dx)\cos^5(c+dx)+6a^2b^2\sin^2(c+dx)\cos^4(c+dx)+4ab^3\sin^3(c+dx)\cos^3(c+\cdots\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^4\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{5a^4\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{5a^4\sin(c+dx)\cos(c+dx)}{16d}+\frac{5a^4x}{16}-\frac{2a^3b\cos^6(c+dx)}{3d}-\frac{a^2b^2\sin(c+dx)\cos^5(c+dx)}{d}+\frac{a^2b^2\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a^2b^2\sin(c+dx)\cos(c+dx)}{8d}+\frac{3}{8}a^2b^2x-\frac{2ab^3\sin^6(c+dx)}{3d}+\frac{ab^3\sin^4(c+dx)}{d}-\frac{b^4\sin^3(c+dx)\cos^3(c+dx)}{6d}-\frac{b^4\sin(c+dx)\cos^3(c+dx)}{8d}+\frac{b^4\sin(c+dx)\cos(c+dx)}{16d}+\frac{b^4x}{16}$$

input
```
Int[Cos[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

output
```
(5*a^4*x)/16 + (3*a^2*b^2*x)/8 + (b^4*x)/16 - (2*a^3*b*Cos[c + d*x]^6)/(3*
d) + (5*a^4*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (3*a^2*b^2*Cos[c + d*x]*Si
n[c + d*x])/(8*d) + (b^4*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (5*a^4*Cos[c
+ d*x]^3*Sin[c + d*x])/(24*d) + (a^2*b^2*Cos[c + d*x]^3*Sin[c + d*x])/(4*d
) - (b^4*Cos[c + d*x]^3*Sin[c + d*x])/(8*d) + (a^4*Cos[c + d*x]^5*Sin[c +
d*x])/(6*d) - (a^2*b^2*Cos[c + d*x]^5*Sin[c + d*x])/d - (b^4*Cos[c + d*x]^
3*Sin[c + d*x]^3)/(6*d) + (a*b^3*Sin[c + d*x]^4)/d - (2*a*b^3*Sin[c + d*x]
^6)/(3*d)
```

3.77.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

3.77.4 Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.62

method	result
parallelrisch	$\frac{(45a^4+18a^2b^2-3b^4)\sin(2dx+2c)+(9a^4-18a^2b^2-3b^4)\sin(4dx+4c)+(a^4-6a^2b^2+b^4)\sin(6dx+6c)+(-60a^3b-36a\,b^3)}{192d}$
derivativedivides	$\frac{a^4\left(\frac{\left(\cos(dx+c)^5+\frac{5\cos(dx+c)^3}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)-\frac{2a^3b\cos(dx+c)^6}{3}+6a^2b^2\left(-\frac{\cos(dx+c)^5\sin(dx+c)}{6}+\right.}{}$
default	$\frac{a^4\left(\frac{\left(\cos(dx+c)^5+\frac{5\cos(dx+c)^3}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)-\frac{2a^3b\cos(dx+c)^6}{3}+6a^2b^2\left(-\frac{\cos(dx+c)^5\sin(dx+c)}{6}+\right.}{}$
parts	$\frac{a^4\left(\frac{\left(\cos(dx+c)^5+\frac{5\cos(dx+c)^3}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)}{d}+\frac{b^4\left(-\frac{\cos(dx+c)^3\sin(dx+c)^3}{6}-\frac{\sin(dx+c)\cos(dx+c)}{8}\right.}{d}$
risch	$\frac{5a^4x}{16}+\frac{3a^2b^2x}{8}+\frac{b^4x}{16}-\frac{a^3b\cos(6dx+6c)}{48d}+\frac{a\,b^3\cos(6dx+6c)}{48d}+\frac{a^4\sin(6dx+6c)}{192d}-\frac{a^2\sin(6dx+6c)b^2}{32d}+\frac{\sin(6}{}$
norman	$\frac{(\frac{5}{16}a^4+\frac{3}{8}a^2b^2+\frac{1}{16}b^4)x+(\frac{5}{16}a^4+\frac{3}{8}a^2b^2+\frac{1}{16}b^4)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}+(\frac{15}{8}a^4+\frac{9}{4}a^2b^2+\frac{3}{8}b^4)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+(\frac{15}{8}a^4+\frac{9}{4}a^2b^2-}{}$

input
```
int(cos(d*x+c)^2*(cos(d*x+c)*a+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/192*((45*a^4+18*a^2*b^2-3*b^4)*sin(2*d*x+2*c)+(9*a^4-18*a^2*b^2-3*b^4)*s
in(4*d*x+4*c)+(a^4-6*a^2*b^2+b^4)*sin(6*d*x+6*c)+(-60*a^3*b-36*a*b^3)*cos(
2*d*x+2*c)+(-4*a^3*b+4*a*b^3)*cos(6*d*x+6*c)+60*a^4*d*x+72*a^2*b^2*d*x+12*
b^4*d*x-24*cos(4*d*x+4*c)*a^3*b+88*a^3*b+32*a*b^3)/d
```

3.77.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.50

$$\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx =$$
$$-\frac{48\,ab^3\cos{(dx+c)}^4+32\,(a^3b-ab^3)\cos{(dx+c)}^6-3\,(5\,a^4+6\,a^2b^2+b^4)dx-\left(8\,(a^4-6\,a^2b^2+b^4)\cos\right.}{48\,d}$$

input
```
integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas"
)
```

output
```
-1/48*(48*a*b^3*cos(d*x + c)^4 + 32*(a^3*b - a*b^3)*cos(d*x + c)^6 - 3*(5*
a^4 + 6*a^2*b^2 + b^4)*d*x - (8*(a^4 - 6*a^2*b^2 + b^4)*cos(d*x + c)^5 + 2
*(5*a^4 + 6*a^2*b^2 - 7*b^4)*cos(d*x + c)^3 + 3*(5*a^4 + 6*a^2*b^2 + b^4)*
cos(d*x + c))*sin(d*x + c))/d
```

3.77.6 Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 563, normalized size of antiderivative = 1.87

$$\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$
$$= \begin{cases} \frac{5a^4x\sin^6(c+dx)}{16} + \frac{15a^4x\sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{15a^4x\sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{5a^4x\cos^6(c+dx)}{16} + \frac{5a^4\sin^5(c+dx)\cos(c+dx)}{16d} \\ x(a\cos(c)+b\sin(c))^4\cos^2(c) \end{cases}$$

input
```
integrate(cos(d*x+c)**2*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

output
```
Piecewise((5*a**4*x*sin(c + d*x)**6/16 + 15*a**4*x*sin(c + d*x)**4*cos(c +
 d*x)**2/16 + 15*a**4*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*a**4*x*cos(
c + d*x)**6/16 + 5*a**4*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 5*a**4*sin(c
 + d*x)**3*cos(c + d*x)**3/(6*d) + 11*a**4*sin(c + d*x)*cos(c + d*x)**5/(1
6*d) - 2*a**3*b*cos(c + d*x)**6/(3*d) + 3*a**2*b**2*x*sin(c + d*x)**6/8 +
9*a**2*b**2*x*sin(c + d*x)**4*cos(c + d*x)**2/8 + 9*a**2*b**2*x*sin(c + d*
x)**2*cos(c + d*x)**4/8 + 3*a**2*b**2*x*cos(c + d*x)**6/8 + 3*a**2*b**2*si
n(c + d*x)**5*cos(c + d*x)/(8*d) + a**2*b**2*sin(c + d*x)**3*cos(c + d*x)*
*3/d - 3*a**2*b**2*sin(c + d*x)*cos(c + d*x)**5/(8*d) + a*b**3*sin(c + d*x
)**6/(3*d) + a*b**3*sin(c + d*x)**4*cos(c + d*x)**2/d + b**4*x*sin(c + d*x
)**6/16 + 3*b**4*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 3*b**4*x*sin(c + d
*x)**2*cos(c + d*x)**4/16 + b**4*x*cos(c + d*x)**6/16 + b**4*sin(c + d*x)*
*5*cos(c + d*x)/(16*d) - b**4*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) - b**4
*sin(c + d*x)*cos(c + d*x)**5/(16*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))*
*4*cos(c)**2, True))
```

3.77.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.56

$$\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = -\frac{128\,a^3b\cos(dx+c)^6+\left(4\,\sin(2\,dx+2\,c)^3-60\,dx-60\,c-9\,\sin(4\,dx+4\,c)-48\,\sin(2\,dx+2\,c)\right)a^4-$$

input
```
integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima"
)
```

output
```
-1/192*(128*a^3*b*cos(d*x + c)^6 + (4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c -
 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a^4 - 6*(4*sin(2*d*x + 2*c)^3 +
 12*d*x + 12*c - 3*sin(4*d*x + 4*c))*a^2*b^2 + 64*(2*sin(d*x + c)^6 - 3*si
n(d*x + c)^4)*a*b^3 + (4*sin(2*d*x + 2*c)^3 - 12*d*x - 12*c + 3*sin(4*d*x
+ 4*c))*b^4)/d
```

3.77.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.62

$$\begin{aligned}\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx &= -\frac{a^3b\cos(4\,dx+4\,c)}{8\,d}+\frac{1}{16}\left(5\,a^4+6\,a^2b^2+b^4\right)x-\frac{(a^3b-ab^3)\cos(6\,dx+6\,c)}{48\,d}\\&\quad-\frac{(5\,a^3b+3\,ab^3)\cos(2\,dx+2\,c)}{16\,d}+\frac{(a^4-6\,a^2b^2+b^4)\sin(6\,dx+6\,c)}{192\,d}\\&\quad+\frac{(3\,a^4-6\,a^2b^2-b^4)\sin(4\,dx+4\,c)}{64\,d}+\frac{(15\,a^4+6\,a^2b^2-b^4)\sin(2\,dx+2\,c)}{64\,d}\end{aligned}$$

input
```
integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/8*a^3*b*cos(4*d*x + 4*c)/d + 1/16*(5*a^4 + 6*a^2*b^2 + b^4)*x - 1/48*(a
^3*b - a*b^3)*cos(6*d*x + 6*c)/d - 1/16*(5*a^3*b + 3*a*b^3)*cos(2*d*x + 2*
c)/d + 1/192*(a^4 - 6*a^2*b^2 + b^4)*sin(6*d*x + 6*c)/d + 1/64*(3*a^4 - 6*
a^2*b^2 - b^4)*sin(4*d*x + 4*c)/d + 1/64*(15*a^4 + 6*a^2*b^2 - b^4)*sin(2*
d*x + 2*c)/d
```

3.77.9 Mupad [B] (verification not implemented)

Time = 24.59 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.56

$$\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$
$$= \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\left(-\frac{11a^4}{8}+\frac{3a^2b^2}{4}+\frac{b^4}{8}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\left(\frac{15a^4}{4}-\frac{39a^2b^2}{2}+\frac{19b^4}{4}\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7\left(\frac{15a^4}{4}-\frac{39a}{2}\right.}{}$$
$$-\frac{\left(\operatorname{atan}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)-\frac{dx}{2}\right)(5a^4+6a^2b^2+b^4)}{8d}$$
$$+\frac{\operatorname{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(5a^2+b^2)(a^2+b^2)}{8\left(\frac{5a^4}{8}+\frac{3a^2b^2}{4}+\frac{b^4}{8}\right)}\right)(5a^2+b^2)(a^2+b^2)}{8d}$$

input
```
int(cos(c + d*x)^2*(a*cos(c + d*x) + b*sin(c + d*x))^4,x)
```

output
```
(tan(c/2 + (d*x)/2)^11*(b^4/8 - (11*a^4)/8 + (3*a^2*b^2)/4) + tan(c/2 + (d
*x)/2)^5*((15*a^4)/4 + (19*b^4)/4 - (39*a^2*b^2)/2) - tan(c/2 + (d*x)/2)^7
*((15*a^4)/4 + (19*b^4)/4 - (39*a^2*b^2)/2) - tan(c/2 + (d*x)/2)^3*((5*a^4
)/24 + (17*b^4)/24 - (47*a^2*b^2)/4) + tan(c/2 + (d*x)/2)^9*((5*a^4)/24 +
(17*b^4)/24 - (47*a^2*b^2)/4) - tan(c/2 + (d*x)/2)*(b^4/8 - (11*a^4)/8 + (
3*a^2*b^2)/4) - tan(c/2 + (d*x)/2)^6*((32*a*b^3)/3 - (80*a^3*b)/3) + 8*a^3
*b*tan(c/2 + (d*x)/2)^2 + 16*a*b^3*tan(c/2 + (d*x)/2)^4 + 16*a*b^3*tan(c/2
 + (d*x)/2)^8 + 8*a^3*b*tan(c/2 + (d*x)/2)^10)/(d*(6*tan(c/2 + (d*x)/2)^2
+ 15*tan(c/2 + (d*x)/2)^4 + 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2
)^8 + 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1)) - ((atan(tan(c
/2 + (d*x)/2)) - (d*x)/2)*(5*a^4 + b^4 + 6*a^2*b^2))/(8*d) + (atan((tan(c/
2 + (d*x)/2)*(5*a^2 + b^2)*(a^2 + b^2))/(8*((5*a^4)/8 + b^4/8 + (3*a^2*b^2
)/4)))*(5*a^2 + b^2)*(a^2 + b^2))/(8*d)
```

3.78 $\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

3.78.1 Optimal result

Integrand size = 26, antiderivative size = 165

$$\begin{aligned}\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx &= -\frac{4ab^3\cos^3(c+dx)}{3d}-\frac{4a^3b\cos^5(c+dx)}{5d}+\frac{4ab^3\cos^5(c+dx)}{5d}\\ &\quad+\frac{a^4\sin(c+dx)}{d}-\frac{2a^4\sin^3(c+dx)}{3d}+\frac{2a^2b^2\sin^3(c+dx)}{d}\\ &\quad+\frac{a^4\sin^5(c+dx)}{5d}-\frac{6a^2b^2\sin^5(c+dx)}{5d}+\frac{b^4\sin^5(c+dx)}{5d}\end{aligned}$$

output
```
-4/3*a*b^3*cos(d*x+c)^3/d-4/5*a^3*b*cos(d*x+c)^5/d+4/5*a*b^3*cos(d*x+c)^5/
d+a^4*sin(d*x+c)/d-2/3*a^4*sin(d*x+c)^3/d+2*a^2*b^2*sin(d*x+c)^3/d+1/5*a^4
*sin(d*x+c)^5/d-6/5*a^2*b^2*sin(d*x+c)^5/d+1/5*b^4*sin(d*x+c)^5/d
```

3.78.2 Mathematica [A] (verified)

Time = 1.46 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.78

$$\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$
$$=\frac{-12a^3b\cos^5(c+dx)+15a^4\sin(c+dx)-10a^2(a^2-3b^2)\sin^3(c+dx)+3(a^4-6a^2b^2+b^4)\sin^5(c+dx)+}{15d}$$

input
```
Integrate[Cos[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

output
```
(-12*a^3*b*Cos[c + d*x]^5 + 15*a^4*Sin[c + d*x] - 10*a^2*(a^2 - 3*b^2)*Sin
[c + d*x]^3 + 3*(a^4 - 6*a^2*b^2 + b^4)*Sin[c + d*x]^5 + 4*a*b^3*Cos[c + d
*x]*(-2 + 2/Sqrt[Cos[c + d*x]^2] - Sin[c + d*x]^2 + 3*Sin[c + d*x]^4))/(15
*d)
```

3.78.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4dx$$

$$\Big\downarrow \ 3569$$

$$\int \left(a^4\cos^5(c+dx)+4a^3b\sin(c+dx)\cos^4(c+dx)+6a^2b^2\sin^2(c+dx)\cos^3(c+dx)+4ab^3\sin^3(c+dx)\cos^2(c+\right.$$

$$\Big\downarrow \ 2009$$

$$\frac{a^4\sin^5(c+dx)}{5d}-\frac{2a^4\sin^3(c+dx)}{3d}+\frac{a^4\sin(c+dx)}{d}-\frac{4a^3b\cos^5(c+dx)}{5d}-\frac{6a^2b^2\sin^5(c+dx)}{5d}+\frac{2a^2b^2\sin^3(c+dx)}{d}+\frac{4ab^3\cos^5(c+dx)}{5d}-\frac{4ab^3\cos^3(c+dx)}{3d}+\frac{b^4\sin^5(c+dx)}{5d}$$

input
```
Int[Cos[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

output
```
(-4*a*b^3*Cos[c + d*x]^3)/(3*d) - (4*a^3*b*Cos[c + d*x]^5)/(5*d) + (4*a*b^
3*Cos[c + d*x]^5)/(5*d) + (a^4*Sin[c + d*x])/d - (2*a^4*Sin[c + d*x]^3)/(3
*d) + (2*a^2*b^2*Sin[c + d*x]^3)/d + (a^4*Sin[c + d*x]^5)/(5*d) - (6*a^2*b
^2*Sin[c + d*x]^5)/(5*d) + (b^4*Sin[c + d*x]^5)/(5*d)
```

3.78.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

3.78.4 Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.79

method	result
parts	$\frac{a^4\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5d}+\frac{b^4\sin(dx+c)^5}{5d}+\frac{4a\,b^3\left(\frac{\cos(dx+c)^5}{5}-\frac{\cos(dx+c)^3}{3}\right)}{d}-\frac{4a^3b\cos(dx+}{5d}$
derivativedivides	$\frac{\frac{a^4\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}-\frac{4a^3b\cos(dx+c)^5}{5}+6a^2b^2\left(-\frac{\sin(dx+c)\cos(dx+c)^4}{5}+\frac{\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{15}\right)}{d}$
default	$\frac{\frac{a^4\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}-\frac{4a^3b\cos(dx+c)^5}{5}+6a^2b^2\left(-\frac{\sin(dx+c)\cos(dx+c)^4}{5}+\frac{\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{15}\right)}{d}$
parallelrisch	$\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9a^4-8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8a^3b+\frac{8\left(a^4+6a^2b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{3}-16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6a\,b^3+\frac{4\left(29a^4-24a^2b^2+24b^4\right)\tan\left(\frac{dx}{2}\right.}{15}}{d\left(1+\tan\left(\right.\right.}$
norman	$\frac{-\frac{24a^3b+16a\,b^3}{15d}+\frac{2a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{d}-\frac{16a\,b^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{d}-\frac{16a\,b^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{3d}-\frac{8a^3b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{d}+\frac{4(29}{}}{\left(1+\tan\left(\frac{dx}{2}\right.\right.}$
risch	$-\frac{a^3b\cos(dx+c)}{2d}-\frac{a\,b^3\cos(dx+c)}{2d}+\frac{5a^4\sin(dx+c)}{8d}+\frac{3a^2b^2\sin(dx+c)}{4d}+\frac{b^4\sin(dx+c)}{8d}-\frac{a^3b\cos(5dx+5c)}{20d}+$

input
```
int(cos(d*x+c)*(cos(d*x+c)*a+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/5*a^4/d*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+1/5*b^4*sin(d*x+c
)^5/d+4*a*b^3/d*(1/5*cos(d*x+c)^5-1/3*cos(d*x+c)^3)-4/5*a^3*b*cos(d*x+c)^5
/d+6*a^2*b^2/d*(-1/5*sin(d*x+c)^5+1/3*sin(d*x+c)^3)
```

3.78.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.75

$$\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx =$$
$$-\frac{20\,ab^3\cos{(dx+c)}^3+12\,(a^3b-ab^3)\cos{(dx+c)}^5-\left(3\,(a^4-6\,a^2b^2+b^4)\cos{(dx+c)}^4+8\,a^4+12\,a^2b^2}{15\,d}$$

input
```
integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

output
```
-1/15*(20*a*b^3*cos(d*x + c)^3 + 12*(a^3*b - a*b^3)*cos(d*x + c)^5 - (3*(a
^4 - 6*a^2*b^2 + b^4)*cos(d*x + c)^4 + 8*a^4 + 12*a^2*b^2 + 3*b^4 + 2*(2*a
^4 + 3*a^2*b^2 - 3*b^4)*cos(d*x + c)^2)*sin(d*x + c))/d
```

3.78.6 Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.25

$$\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$
$$=\begin{cases} \frac{8a^4\sin^5{(c+dx)}}{15d}+\frac{4a^4\sin^3{(c+dx)}\cos^2{(c+dx)}}{3d}+\frac{a^4\sin{(c+dx)}\cos^4{(c+dx)}}{d}-\frac{4a^3b\cos^5{(c+dx)}}{5d}+\frac{4a^2b^2\sin^5{(c+dx)}}{5d}+\frac{2a^2b^2\sin^3{(c+} \\ x(a\cos{(c)}+b\sin{(c)})^4\cos{(c)} \end{cases}$$

input
```
integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

output
```
Piecewise((8*a**4*sin(c + d*x)**5/(15*d) + 4*a**4*sin(c + d*x)**3*cos(c +
d*x)**2/(3*d) + a**4*sin(c + d*x)*cos(c + d*x)**4/d - 4*a**3*b*cos(c + d*x
)**5/(5*d) + 4*a**2*b**2*sin(c + d*x)**5/(5*d) + 2*a**2*b**2*sin(c + d*x)*
*3*cos(c + d*x)**2/d - 4*a*b**3*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - 8*
a*b**3*cos(c + d*x)**5/(15*d) + b**4*sin(c + d*x)**5/(5*d), Ne(d, 0)), (x*
(a*cos(c) + b*sin(c))**4*cos(c), True))
```

3.78.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.75

$$\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx =$$
$$-\frac{12\,a^3b\cos\left(dx+c\right)^5-3\,b^4\sin\left(dx+c\right)^5-\left(3\,\sin\left(dx+c\right)^5-10\,\sin\left(dx+c\right)^3+15\,\sin\left(dx+c\right)\right)a^4+6\,(}{15\,d}$$

input
```
integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/15*(12*a^3*b*cos(d*x + c)^5 - 3*b^4*sin(d*x + c)^5 - (3*sin(d*x + c)^5
- 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^4 + 6*(3*sin(d*x + c)^5 - 5*sin(d
*x + c)^3)*a^2*b^2 - 4*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a*b^3)/d
```

3.78.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00

$$\begin{aligned}\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx &= -\frac{(a^3b-ab^3)\cos\left(5\,dx+5\,c\right)}{20\,d}-\frac{(3\,a^3b+ab^3)\cos\left(3\,dx+3\,c\right)}{12\,d}\\&\quad-\frac{(a^3b+ab^3)\cos\left(dx+c\right)}{2\,d}+\frac{(a^4-6\,a^2b^2+b^4)\sin\left(5\,dx+5\,c\right)}{80\,d}\\&\quad+\frac{(5\,a^4-6\,a^2b^2-3\,b^4)\sin\left(3\,dx+3\,c\right)}{48\,d}+\frac{(5\,a^4+6\,a^2b^2+b^4)\sin\left(dx+c\right)}{8\,d}\end{aligned}$$

input
```
integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/20*(a^3*b - a*b^3)*cos(5*d*x + 5*c)/d - 1/12*(3*a^3*b + a*b^3)*cos(3*d*
x + 3*c)/d - 1/2*(a^3*b + a*b^3)*cos(d*x + c)/d + 1/80*(a^4 - 6*a^2*b^2 +
b^4)*sin(5*d*x + 5*c)/d + 1/48*(5*a^4 - 6*a^2*b^2 - 3*b^4)*sin(3*d*x + 3*c
)/d + 1/8*(5*a^4 + 6*a^2*b^2 + b^4)*sin(d*x + c)/d
```

3.78.9 Mupad [B] (verification not implemented)

Time = 22.84 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.24

$$\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$
$$= \frac{2\left(\frac{3\sin(c+dx)\,a^4\cos(c+dx)^4}{2}+2\sin(c+dx)\ a^4\cos(c+dx)^2+4\sin(c+dx)\ a^4-6\,a^3\,b\cos(c+dx)^5-9\,\mathrm{si}\right.}{}$$

input
```
int(cos(c + d*x)*(a*cos(c + d*x) + b*sin(c + d*x))^4,x)
```

output
```
(2*(4*a^4*sin(c + d*x) + (3*b^4*sin(c + d*x))/2 - 10*a*b^3*cos(c + d*x)^3
+ 6*a*b^3*cos(c + d*x)^5 - 6*a^3*b*cos(c + d*x)^5 + 2*a^4*cos(c + d*x)^2*s
in(c + d*x) + (3*a^4*cos(c + d*x)^4*sin(c + d*x))/2 + 6*a^2*b^2*sin(c + d*
x) - 3*b^4*cos(c + d*x)^2*sin(c + d*x) + (3*b^4*cos(c + d*x)^4*sin(c + d*x
))/2 + 3*a^2*b^2*cos(c + d*x)^2*sin(c + d*x) - 9*a^2*b^2*cos(c + d*x)^4*si
n(c + d*x)))/(15*d)
```

3.79 $\int (a\cos(c+dx)+b\sin(c+dx))^4\,dx$

3.79.1 Optimal result

Integrand size = 19, antiderivative size = 108

$$\begin{aligned}\int (a\cos(c+dx)+b\sin(c+dx))^4\,dx &= \frac{3}{8}\left(a^2+b^2\right)^2x-\frac{3(a^2+b^2)\,(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))}{8d}\\&\quad-\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^3}{4d}\end{aligned}$$

output
```
3/8*(a^2+b^2)^2*x-3/8*(a^2+b^2)*(b*cos(d*x+c)-a*sin(d*x+c))*(a*cos(d*x+c)+
b*sin(d*x+c))/d-1/4*(b*cos(d*x+c)-a*sin(d*x+c))*(a*cos(d*x+c)+b*sin(d*x+c)
)^3/d
```

3.79.2 Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.99

$$\int (a\cos(c+dx)+b\sin(c+dx))^4\,dx = \frac{12\left(a^2+b^2\right)^2(c+dx)-16ab(a^2+b^2)\cos(2(c+dx))-4ab(a^2-b^2)\cos(4(c+dx))+8(a^4-b^4)\sin(2(c+}{32d}$$

input
```
Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

output
```
(12*(a^2 + b^2)^2*(c + d*x) - 16*a*b*(a^2 + b^2)*Cos[2*(c + d*x)] - 4*a*b*
(a^2 - b^2)*Cos[4*(c + d*x)] + 8*(a^4 - b^4)*Sin[2*(c + d*x)] + (a^4 - 6*a
^2*b^2 + b^4)*Sin[4*(c + d*x)])/(32*d)
```

3.79.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3042, 3552, 3042, 3552, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(c+dx)+b\sin(c+dx))^4\,dx$$

$$\Big\downarrow \ 3042$$

$$\int (a\cos(c+dx)+b\sin(c+dx))^4dx$$

$$\Big\downarrow \ 3552$$

$$\frac{3}{4}\left(a^2+b^2\right)\int (a\cos(c+dx)+b\sin(c+dx))^2dx-\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^3}{4d}$$

$$\Big\downarrow \ 3042$$

$$\frac{3}{4}\left(a^2+b^2\right)\int (a\cos(c+dx)+b\sin(c+dx))^2dx-\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^3}{4d}$$

$$\Big\downarrow \ 3552$$

$$\frac{3}{4}\left(a^2+b^2\right)\left(\frac{1}{2}\left(a^2+b^2\right)\int 1dx-\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))}{2d}\right)-\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^3}{4d}$$

$$\Big\downarrow \ 24$$

$$\frac{3}{4}\left(a^2+b^2\right)\left(\frac{1}{2}x\left(a^2+b^2\right)-\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))}{2d}\right)-\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^3}{4d}$$

input
```
Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

output
```
-1/4*((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin[c + d*x])^
3)/d + (3*(a^2 + b^2)*(((a^2 + b^2)*x)/2 - ((b*Cos[c + d*x] - a*Sin[c + d*
x])*(a*Cos[c + d*x] + b*Sin[c + d*x]))/(2*d)))/4
```

3.79.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3552
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(-(b*Cos[c + d*x] - a*Sin[c + d*x]))*((a*Cos[c + d*x] + b*
Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[(n - 1)*((a^2 + b^2)/n)   Int[(a*Co
s[c + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

3.79.4 Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.25

method	result
parallelrisch	$\frac{(a^4-6a^2b^2+b^4)\sin(4dx+4c)+16(-a^3b-a\,b^3)\cos(2dx+2c)+4(-a^3b+a\,b^3)\cos(4dx+4c)+8(a^4-b^4)\sin(2dx+2c)+12a^4}{32d}$
derivativedivides	$\frac{a^4\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)-a^3b\cos(dx+c)^4+6a^2b^2\left(-\frac{\sin(dx+c)\cos(dx+c)^3}{4}+\frac{\sin(dx+c)\cos(dx+}{8}\right.}{d}$
default	$\frac{a^4\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)-a^3b\cos(dx+c)^4+6a^2b^2\left(-\frac{\sin(dx+c)\cos(dx+c)^3}{4}+\frac{\sin(dx+c)\cos(dx+}{8}\right.}{d}$
parts	$\frac{a^4\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}+\frac{b^4\left(-\frac{\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}+\frac{a\,b^3}{}$
risch	$\frac{3a^4x}{8}+\frac{3a^2b^2x}{4}+\frac{3b^4x}{8}-\frac{a^3b\cos(4dx+4c)}{8d}+\frac{a\,b^3\cos(4dx+4c)}{8d}+\frac{\sin(4dx+4c)a^4}{32d}-\frac{3\sin(4dx+4c)a^2b^2}{16d}+\sin$
norman	$\frac{(\frac{3}{8}a^4+\frac{3}{4}a^2b^2+\frac{3}{8}b^4)x+(\frac{3}{2}a^4+3a^2b^2+\frac{3}{2}b^4)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+(\frac{3}{2}a^4+3a^2b^2+\frac{3}{2}b^4)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+(\frac{3}{8}a^4+\frac{3}{4}a^2b^2+\frac{3}{8}b^4)x\,t}{}$

input
```
int((cos(d*x+c)*a+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/32*((a^4-6*a^2*b^2+b^4)*sin(4*d*x+4*c)+16*(-a^3*b-a*b^3)*cos(2*d*x+2*c)+
4*(-a^3*b+a*b^3)*cos(4*d*x+4*c)+8*(a^4-b^4)*sin(2*d*x+2*c)+12*a^4*d*x+24*a
^2*b^2*d*x+12*b^4*d*x+20*a^3*b+12*a*b^3)/d
```

3.79.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.12

$$\int (a\cos(c+dx)+b\sin(c+dx))^4\,dx =$$
$$-\frac{16\,ab^3\cos(dx+c)^2+8\,(a^3b-ab^3)\cos(dx+c)^4-3\,(a^4+2\,a^2b^2+b^4)dx-\left(2\,(a^4-6\,a^2b^2+b^4)\cos(d\right.}{8\,d}$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

output
```
-1/8*(16*a*b^3*cos(d*x + c)^2 + 8*(a^3*b - a*b^3)*cos(d*x + c)^4 - 3*(a^4
+ 2*a^2*b^2 + b^4)*d*x - (2*(a^4 - 6*a^2*b^2 + b^4)*cos(d*x + c)^3 + (3*a^
4 + 6*a^2*b^2 - 5*b^4)*cos(d*x + c))*sin(d*x + c))/d
```

3.79.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(102) = 204$.

Time = 0.21 (sec) , antiderivative size = 381, normalized size of antiderivative = 3.53

$$\int (a\cos(c+dx)+b\sin(c+dx))^4\,dx = \begin{cases} \frac{3a^4x\sin^4(c+dx)}{8} + \frac{3a^4x\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{3a^4x\cos^4(c+dx)}{8} + \frac{3a^4\sin^3(c+dx)\cos(c+dx)}{8d} + \frac{5a^4\sin(c+dx)\cos^3(c+dx)}{8d} - \frac{a}{} \\ x(a\cos(c)+b\sin(c))^4 \end{cases}$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

output
```
Piecewise((3*a**4*x*sin(c + d*x)**4/8 + 3*a**4*x*sin(c + d*x)**2*cos(c + d
*x)**2/4 + 3*a**4*x*cos(c + d*x)**4/8 + 3*a**4*sin(c + d*x)**3*cos(c + d*x
)/(8*d) + 5*a**4*sin(c + d*x)*cos(c + d*x)**3/(8*d) - a**3*b*cos(c + d*x)*
*4/d + 3*a**2*b**2*x*sin(c + d*x)**4/4 + 3*a**2*b**2*x*sin(c + d*x)**2*cos
(c + d*x)**2/2 + 3*a**2*b**2*x*cos(c + d*x)**4/4 + 3*a**2*b**2*sin(c + d*x
)**3*cos(c + d*x)/(4*d) - 3*a**2*b**2*sin(c + d*x)*cos(c + d*x)**3/(4*d) +
 a*b**3*sin(c + d*x)**4/d + 3*b**4*x*sin(c + d*x)**4/8 + 3*b**4*x*sin(c +
d*x)**2*cos(c + d*x)**2/4 + 3*b**4*x*cos(c + d*x)**4/8 - 5*b**4*sin(c + d*
x)**3*cos(c + d*x)/(8*d) - 3*b**4*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d
, 0)), (x*(a*cos(c) + b*sin(c))**4, True))
```

3.79.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.26

$$\begin{aligned}\int (a\cos(c+dx)+b\sin(c+dx))^4\,dx &= -\frac{a^3b\cos(dx+c)^4}{d} + \frac{ab^3\sin(dx+c)^4}{d} \\ &+ \frac{(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))a^4}{32\,d} \\ &+ \frac{3\,(4\,dx+4\,c-\sin(4\,dx+4\,c))a^2b^2}{16\,d} \\ &+ \frac{(12\,dx+12\,c+\sin(4\,dx+4\,c)-8\,\sin(2\,dx+2\,c))b^4}{32\,d}\end{aligned}$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

output
```
-a^3*b*cos(d*x + c)^4/d + a*b^3*sin(d*x + c)^4/d + 1/32*(12*d*x + 12*c + s
in(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^4/d + 3/16*(4*d*x + 4*c - sin(4*d*
x + 4*c))*a^2*b^2/d + 1/32*(12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*d*x
 + 2*c))*b^4/d
```

3.79.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.13

$$\int (a\cos(c+dx)+b\sin(c+dx))^4\,dx = \frac{3}{8}\left(a^4+2a^2b^2+b^4\right)x - \frac{(a^3b-ab^3)\cos(4dx+4c)}{8d} - \frac{(a^3b+ab^3)\cos(2dx+2c)}{2d} + \frac{(a^4-6a^2b^2+b^4)\sin(4dx+4c)}{32d} + \frac{(a^4-b^4)\sin(2dx+2c)}{4d}$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

output
```
3/8*(a^4 + 2*a^2*b^2 + b^4)*x - 1/8*(a^3*b - a*b^3)*cos(4*d*x + 4*c)/d - 1
/2*(a^3*b + a*b^3)*cos(2*d*x + 2*c)/d + 1/32*(a^4 - 6*a^2*b^2 + b^4)*sin(4
*d*x + 4*c)/d + 1/4*(a^4 - b^4)*sin(2*d*x + 2*c)/d
```

3.79.9 Mupad [B] (verification not implemented)

Time = 24.14 (sec) , antiderivative size = 320, normalized size of antiderivative = 2.96

$$\int (a\cos(c+dx)+b\sin(c+dx))^4\,dx = \frac{3\,\mathrm{atan}\left(\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(a^2+b^2\right)^2}{4\left(\frac{3a^4}{4}+\frac{3a^2b^2}{2}+\frac{3b^4}{4}\right)}\right)\left(a^2+b^2\right)^2}{4d} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7\left(-\frac{5a^4}{4}+\frac{3a^2b^2}{2}+\frac{3b^4}{4}\right) - \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{3a^4}{4}-\frac{21a^2b^2}{2}+\frac{11b^4}{4}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\left(\frac{3a^4}{4}-\frac{21a}{2}\right.}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+4\tan\left(\frac{c}{2}+\right.\right.} - \frac{3\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)-\frac{dx}{2}\right)\left(a^2+b^2\right)^2}{4d}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))^4,x)
```

output
```
(3*atan((3*tan(c/2 + (d*x)/2)*(a^2 + b^2)^2)/(4*((3*a^4)/4 + (3*b^4)/4 + (
3*a^2*b^2)/2)))*(a^2 + b^2)^2)/(4*d) + (tan(c/2 + (d*x)/2)^7*((3*b^4)/4 -
(5*a^4)/4 + (3*a^2*b^2)/2) - tan(c/2 + (d*x)/2)^3*((3*a^4)/4 + (11*b^4)/4
- (21*a^2*b^2)/2) + tan(c/2 + (d*x)/2)^5*((3*a^4)/4 + (11*b^4)/4 - (21*a^2
*b^2)/2) - tan(c/2 + (d*x)/2)*((3*b^4)/4 - (5*a^4)/4 + (3*a^2*b^2)/2) + 8*
a^3*b*tan(c/2 + (d*x)/2)^2 + 16*a*b^3*tan(c/2 + (d*x)/2)^4 + 8*a^3*b*tan(c
/2 + (d*x)/2)^6)/(d*(4*tan(c/2 + (d*x)/2)^2 + 6*tan(c/2 + (d*x)/2)^4 + 4*t
an(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) - (3*(atan(tan(c/2 + (d*x
)/2)) - (d*x)/2)*(a^2 + b^2)^2)/(4*d)
```

3.79. $\int (a\cos(c+dx)+b\sin(c+dx))^4\,dx$

3.80 $\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

3.80.1 Optimal result

Integrand size = 26, antiderivative size = 150

$$\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \frac{b^4\operatorname{arctanh}(\sin(c+dx))}{d}-\frac{4ab^3\cos(c+dx)}{d}-\frac{4a^3b\cos^3(c+dx)}{3d}+\frac{4ab^3\cos^3(c+dx)}{3d}+\frac{a^4\sin(c+dx)}{d}-\frac{b^4\sin(c+dx)}{d}-\frac{a^4\sin^3(c+dx)}{3d}+\frac{2a^2b^2\sin^3(c+dx)}{d}-\frac{b^4\sin^3(c+dx)}{3d}$$

output
```
b^4*arctanh(sin(d*x+c))/d-4*a*b^3*cos(d*x+c)/d-4/3*a^3*b*cos(d*x+c)^3/d+4/
3*a*b^3*cos(d*x+c)^3/d+a^4*sin(d*x+c)/d-b^4*sin(d*x+c)/d-1/3*a^4*sin(d*x+c
)^3/d+2*a^2*b^2*sin(d*x+c)^3/d-1/3*b^4*sin(d*x+c)^3/d
```

3.80.2 Mathematica [A] (verified)

Time = 2.42 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.21

$$\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \frac{-12ab(a^2+3b^2)\cos(c+dx)+(-4a^3b+4ab^3)\cos(3(c+dx))-12b^4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

output
```
(-12*a*b*(a^2 + 3*b^2)*Cos[c + d*x] + (-4*a^3*b + 4*a*b^3)*Cos[3*(c + d*x)
] - 12*b^4*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 12*b^4*Log[Cos[(c +
d*x)/2] + Sin[(c + d*x)/2]] + 9*a^4*Sin[c + d*x] + 18*a^2*b^2*Sin[c + d*x]
 - 15*b^4*Sin[c + d*x] + a^4*Sin[3*(c + d*x)] - 6*a^2*b^2*Sin[3*(c + d*x)]
 + b^4*Sin[3*(c + d*x)])/(12*d)
```

3.80.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\cos(c+dx)+b\sin(c+dx))^4}{\cos(c+dx)}dx$$

$$\Big\downarrow \text{3569}$$

$$\int \left(a^4\cos^3(c+dx)+4a^3b\sin(c+dx)\cos^2(c+dx)+6a^2b^2\sin^2(c+dx)\cos(c+dx)+4ab^3\sin^3(c+dx)+b^4\sin^3(\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^4\sin^3(c+dx)}{3d}+\frac{a^4\sin(c+dx)}{d}-\frac{4a^3b\cos^3(c+dx)}{3d}+\frac{2a^2b^2\sin^3(c+dx)}{d}+\frac{4ab^3\cos^3(c+dx)}{3d}-\frac{4ab^3\cos(c+dx)}{d}+\frac{b^4\text{arctanh}(\sin(c+dx))}{d}-\frac{b^4\sin^3(c+dx)}{3d}-\frac{b^4\sin(c+dx)}{d}$$

input
```
Int[Sec[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

output
```
(b^4*ArcTanh[Sin[c + d*x]])/d - (4*a*b^3*Cos[c + d*x])/d - (4*a^3*b*Cos[c
+ d*x]^3)/(3*d) + (4*a*b^3*Cos[c + d*x]^3)/(3*d) + (a^4*Sin[c + d*x])/d -
(b^4*Sin[c + d*x])/d - (a^4*Sin[c + d*x]^3)/(3*d) + (2*a^2*b^2*Sin[c + d*x
]^3)/d - (b^4*Sin[c + d*x]^3)/(3*d)
```

3.80.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

3.80.4 Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.77

method	result
derivativedivides	$\frac{\frac{a^4\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}-\frac{4a^3b\cos(dx+c)^3}{3}+2a^2b^2\sin(dx+c)^3-\frac{4a\,b^3\left(2+\sin(dx+c)^2\right)\cos(dx+c)}{3}+b^4\left(-\frac{\sin(dx+c)^3}{3}-\sin(\right.}{d}$
default	$\frac{\frac{a^4\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3}-\frac{4a^3b\cos(dx+c)^3}{3}+2a^2b^2\sin(dx+c)^3-\frac{4a\,b^3\left(2+\sin(dx+c)^2\right)\cos(dx+c)}{3}+b^4\left(-\frac{\sin(dx+c)^3}{3}-\sin(\right.}{d}$
parts	$\frac{a^4\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{3d}+\frac{b^4\left(-\frac{\sin(dx+c)^3}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)}{d}-\frac{4a^3b}{3\sec(dx+c)^3d}+\frac{4a}{}$
parallelrisch	$\frac{-12b^4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+12b^4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\left(a^4-6a^2b^2+b^4\right)\sin(3dx+3c)+\left(-4a^3b+4a\,b^3\right)\cos(3dx+3c)+(9}{12d}$
norman	$\frac{-\frac{8a^3b+16a\,b^3}{3d}+\frac{2\left(a^4-b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2\left(a^4-b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{d}-\frac{8a^3b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{d}+\frac{2\left(5a^4+24a^2b^2-13b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3d}+}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4}$
risch	$-\frac{e^{i(dx+c)}a^3b}{2d}-\frac{3\,e^{i(dx+c)}a\,b^3}{2d}-\frac{3ie^{i(dx+c)}a^4}{8d}-\frac{5ie^{-i(dx+c)}b^4}{8d}+\frac{3ie^{-i(dx+c)}a^4}{8d}-\frac{e^{-i(dx+c)}a^3b}{2d}-\frac{3\,e^{-i(dx-}}{2d}$

input
```
int(sec(d*x+c)*(cos(d*x+c)*a+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/3*a^4*(2+cos(d*x+c)^2)*sin(d*x+c)-4/3*a^3*b*cos(d*x+c)^3+2*a^2*b^2*
sin(d*x+c)^3-4/3*a*b^3*(2+sin(d*x+c)^2)*cos(d*x+c)+b^4*(-1/3*sin(d*x+c)^3-
sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c))))
```

3.80.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.81

$$\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx =$$
$$-\frac{24\,ab^3\cos\left(dx+c\right)-3\,b^4\log\left(\sin\left(dx+c\right)+1\right)+3\,b^4\log\left(-\sin\left(dx+c\right)+1\right)+8\left(a^3b-ab^3\right)\cos\left(dx+c\right.}{6\,d}$$

input
```
integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

output
```
-1/6*(24*a*b^3*cos(d*x + c) - 3*b^4*log(sin(d*x + c) + 1) + 3*b^4*log(-sin
(d*x + c) + 1) + 8*(a^3*b - a*b^3)*cos(d*x + c)^3 - 2*(2*a^4 + 6*a^2*b^2 -
 4*b^4 + (a^4 - 6*a^2*b^2 + b^4)*cos(d*x + c)^2)*sin(d*x + c))/d
```

3.80.6 Sympy [F]

$$\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$
$$=\int \left(a\cos\left(c+dx\right)+b\sin\left(c+dx\right)\right)^4\sec\left(c+dx\right)dx$$

input
```
integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

output
```
Integral((a*cos(c + d*x) + b*sin(c + d*x))**4*sec(c + d*x), x)
```

3.80.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.84

$$\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx =$$
$$-\frac{8\,a^3b\cos\left(dx+c\right)^3-12\,a^2b^2\sin\left(dx+c\right)^3+2\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)a^4-8\left(\cos\left(dx+c\right)^3-3\right.}{}$$

input
```
integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/6*(8*a^3*b*cos(d*x + c)^3 - 12*a^2*b^2*sin(d*x + c)^3 + 2*(sin(d*x + c)
^3 - 3*sin(d*x + c))*a^4 - 8*(cos(d*x + c)^3 - 3*cos(d*x + c))*a*b^3 + (2*
sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1) + 6*sin
(d*x + c))*b^4)/d
```

3.80.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.45

$$\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$
$$= \frac{3\,b^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,b^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(3\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-12}{}}{}$$

input
```
integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

output
```
1/3*(3*b^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*b^4*log(abs(tan(1/2*d*x
+ 1/2*c) - 1)) + 2*(3*a^4*tan(1/2*d*x + 1/2*c)^5 - 3*b^4*tan(1/2*d*x + 1/2
*c)^5 - 12*a^3*b*tan(1/2*d*x + 1/2*c)^4 + 2*a^4*tan(1/2*d*x + 1/2*c)^3 + 2
4*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 10*b^4*tan(1/2*d*x + 1/2*c)^3 - 24*a*b^
3*tan(1/2*d*x + 1/2*c)^2 + 3*a^4*tan(1/2*d*x + 1/2*c) - 3*b^4*tan(1/2*d*x
+ 1/2*c) - 4*a^3*b - 8*a*b^3)/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.80.9 Mupad [B] (verification not implemented)

Time = 26.10 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.27

$$\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \frac{2\,b^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$
$$-\frac{\frac{16\,a\,b^3}{3}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\left(2\,a^4-2\,b^4\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(\frac{4\,a^4}{3}+16\,a^2\,b^2-\frac{20\,b^4}{3}\right)+\frac{8\,a^3\,b}{3}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^4\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right.}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))^4/cos(c + d*x),x)
```

output
```
(2*b^4*atanh(tan(c/2 + (d*x)/2)))/d - ((16*a*b^3)/3 - tan(c/2 + (d*x)/2)^5
*(2*a^4 - 2*b^4) - tan(c/2 + (d*x)/2)^3*((4*a^4)/3 - (20*b^4)/3 + 16*a^2*b
^2) + (8*a^3*b)/3 - tan(c/2 + (d*x)/2)*(2*a^4 - 2*b^4) + 16*a*b^3*tan(c/2
+ (d*x)/2)^2 + 8*a^3*b*tan(c/2 + (d*x)/2)^4)/(d*(3*tan(c/2 + (d*x)/2)^2 +
3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 + 1))
```

3.80. $\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

3.81 $\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

3.81.1 Optimal result

Integrand size = 28, antiderivative size = 119

$$\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \frac{1}{2}\left(a^4+6a^2b^2-3b^4\right)x-\frac{4ab^3\log(\sin(c+dx))}{d}+\frac{4ab^3\log(\tan(c+dx))}{d}+\frac{\left(4ab\left(a^2-b^2\right)+\left(a^4-6a^2b^2+b^4\right)\cot(c+dx)\right)\sin^2(c+dx)}{2d}+\frac{b^4\tan(c+dx)}{d}$$

output
```
1/2*(a^4+6*a^2*b^2-3*b^4)*x-4*a*b^3*ln(sin(d*x+c))/d+4*a*b^3*ln(tan(d*x+c)
)/d+1/2*(4*a*b*(a^2-b^2)+(a^4-6*a^2*b^2+b^4)*cot(d*x+c))*sin(d*x+c)^2/d+b^
4*tan(d*x+c)/d
```

3.81.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 477 vs. 2(119) = 238.

Time = 6.35 (sec) , antiderivative size = 477, normalized size of antiderivative = 4.01

$$\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \frac{b^3\left(\frac{\cos^2(c+dx)(a+b\tan(c+dx))^5\left(b^2+ab\tan(c+dx)\right)}{2b^4\left(a^2+b^2\right)}-\frac{\left(-5a^2+3b^2\right)\left(\frac{1}{2}\left(4a(a-b)(a+b)+\frac{a^4-6a^2b^2+b^4}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)+\frac{1}{2}\left(4a(a\right.\right.}{}}{}$$

input
```
Integrate[Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

output
```
(b^3*((Cos[c + d*x]^2*(a + b*Tan[c + d*x])^5*(b^2 + a*b*Tan[c + d*x]))/(2*
b^4*(a^2 + b^2)) - ((-5*a^2 + 3*b^2)*(((4*a*(a - b)*(a + b) + (a^4 - 6*a^2
*b^2 + b^4)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]])/2 + ((4*a*(a - b
)*(a + b) - (a^4 - 6*a^2*b^2 + b^4)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c +
 d*x]])/2 + b*(6*a^2 - b^2)*Tan[c + d*x] + 2*a*b^2*Tan[c + d*x]^2 + (b^3*T
an[c + d*x]^3)/3) + 4*a*(((5*a^4 - 10*a^2*b^2 + b^4 + (a^5 - 10*a^3*b^2 +
5*a*b^4)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]])/2 + ((5*a^4 - 10*a^
2*b^2 + b^4 - (a^5 - 10*a^3*b^2 + 5*a*b^4)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*
Tan[c + d*x]])/2 + 5*a*b*(2*a^2 - b^2)*Tan[c + d*x] + (b^2*(10*a^2 - b^2)*
Tan[c + d*x]^2)/2 + (5*a*b^3*Tan[c + d*x]^3)/3 + (b^4*Tan[c + d*x]^4)/4))/
(2*b^2*(a^2 + b^2))))/d
```

3.81.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 3567, 532, 25, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(a\cos(c+dx)+b\sin(c+dx))^4}{\cos(c+dx)^2}dx$$

$$\Big\downarrow \ 3567$$

$$-\frac{\int \frac{(b+a\cot(c+dx))^4\tan^2(c+dx)}{(\cot^2(c+dx)+1)^2}d\cot(c+dx)}{d}$$

$$\Big\downarrow \ 532$$

$$-\frac{-\frac{1}{2}\int -\frac{(2b^4+8a\cot(c+dx)b^3+(a^4+6b^2a^2-b^4)\cot^2(c+dx))\tan^2(c+dx)}{\cot^2(c+dx)+1}d\cot(c+dx)-\frac{4ab(a^2-b^2)+(a^4-6a^2b^2+b^4)\cot(c+dx)}{2(\cot^2(c+dx)+1)}}{d}$$

$$\Big\downarrow \ 25$$

$$-\frac{\frac{1}{2}\int \frac{(2b^4+8a\cot(c+dx)b^3+(a^4+6b^2a^2-b^4)\cot^2(c+dx))\tan^2(c+dx)}{\cot^2(c+dx)+1}d\cot(c+dx)-\frac{4ab(a^2-b^2)+(a^4-6a^2b^2+b^4)\cot(c+dx)}{2(\cot^2(c+dx)+1)}}{d}$$

$\big\downarrow$ 2333

$$-\frac{\frac{1}{2}\int \left(2\tan^2(c+dx)b^4+8a\tan(c+dx)b^3+\frac{a^4+6b^2a^2-8b^3\cot(c+dx)a-3b^4}{\cot^2(c+dx)+1}\right)d\cot(c+dx)-\frac{4ab(a^2-b^2)+(a^4-6a^2b^2+b^4)\cot(c+dx)}{2(\cot^2(c+dx)+1)}}{d}$$

$\big\downarrow$ 2009

$$-\frac{\frac{1}{2}\left((a^4+6a^2b^2-3b^4)\arctan(\cot(c+dx))-4ab^3\log\left(\cot^2(c+dx)+1\right)+8ab^3\log(\cot(c+dx))-2b^4\tan(c+dx)\right)\ldots}{d}$$

input
```
Int[Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

output
```
-((-1/2*(4*a*b*(a^2 - b^2) + (a^4 - 6*a^2*b^2 + b^4)*Cot[c + d*x])/(1 + Co
t[c + d*x]^2) + ((a^4 + 6*a^2*b^2 - 3*b^4)*ArcTan[Cot[c + d*x]] + 8*a*b^3*
Log[Cot[c + d*x]] - 4*a*b^3*Log[1 + Cot[c + d*x]^2] - 2*b^4*Tan[c + d*x])/
2)/d)
```

3.81.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3567
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[x^m*((b
+ a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a,
b, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[
n, 0] && GtQ[m, 1])
```

3.81.4 Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.30

method	result
derivativedivides	$\frac{a^4\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-2\cos(dx+c)^2a^3b+6a^2b^2\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+4a\,b^3\left(-\frac{\sin(dx+c)^2}{2}-\ln(\cos\right.}{d}$
default	$\frac{a^4\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-2\cos(dx+c)^2a^3b+6a^2b^2\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+4a\,b^3\left(-\frac{\sin(dx+c)^2}{2}-\ln(\cos\right.}{d}$
parts	$\frac{a^4\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{b^4\left(\frac{\sin(dx+c)^5}{\cos(dx+c)}+\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)}{d}+\frac{4a\,b^3\left(-\sin\right.}{}$
parallelrisch	$\frac{32a\,b^3\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\cos(dx+c)-32a\,b^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)-32a\,b^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+(a}{8a}$
risch	$-\frac{i\mathrm{e}^{2i(dx+c)}a^4}{8d}+\frac{a^4x}{2}+3a^2b^2x-\frac{3b^4x}{2}-\frac{\mathrm{e}^{2i(dx+c)}a^3b}{2d}+\frac{\mathrm{e}^{2i(dx+c)}a\,b^3}{2d}+\frac{i\mathrm{e}^{-2i(dx+c)}b^4}{8d}+\frac{i\mathrm{e}^{-2i(dx+c)}a^4}{8d}$
norman	$\frac{(-\frac{1}{2}a^4-3a^2b^2+\frac{3}{2}b^4)x+(-\frac{3}{2}a^4-9a^2b^2+\frac{9}{2}b^4)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+(\frac{1}{2}a^4+3a^2b^2-\frac{3}{2}b^4)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+(\frac{3}{2}a^4+9a^2b^2-\frac{9}{2}b^4}{}$

input
```
int(sec(d*x+c)^2*(cos(d*x+c)*a+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^4*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)-2*cos(d*x+c)^2*a^3*b+6*
a^2*b^2*(-1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)+4*a*b^3*(-1/2*sin(d*x+c
)^2-ln(cos(d*x+c)))+b^4*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x
+c))*cos(d*x+c)-3/2*d*x-3/2*c))
```

3.81.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.14

$$\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = -\frac{8\,ab^3\cos(dx+c)\log(-\cos(dx+c))+4\,(a^3b-ab^3)\cos(dx+c)^3-(2\,a^3b-2\,ab^3+(a^4+6\,a^2b^2-3\,b}{2\,d\cos(dx+c)}$$

input
```
integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas"
)
```

output
```
-1/2*(8*a*b^3*cos(d*x + c)*log(-cos(d*x + c)) + 4*(a^3*b - a*b^3)*cos(d*x
+ c)^3 - (2*a^3*b - 2*a*b^3 + (a^4 + 6*a^2*b^2 - 3*b^4)*d*x)*cos(d*x + c)
- (2*b^4 + (a^4 - 6*a^2*b^2 + b^4)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*
x + c))
```

3.81.6 Sympy [F]

$$\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \int (a\cos(c+dx)+b\sin(c+dx))^4\sec^2(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**2*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

output
```
Integral((a*cos(c + d*x) + b*sin(c + d*x))**4*sec(c + d*x)**2, x)
```

3.81.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.13

$$\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$
$$= \frac{8\,a^3b\sin\left(dx+c\right)^2+(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))a^4+6\,(2\,dx+2\,c-\sin\left(2\,dx+2\,c\right))a^2b^2-8\,(\sin\left(dx+\right.}{4\,d}$$

input
```
integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima"
)
```

output
```
1/4*(8*a^3*b*sin(d*x + c)^2 + (2*d*x + 2*c + sin(2*d*x + 2*c))*a^4 + 6*(2*
d*x + 2*c - sin(2*d*x + 2*c))*a^2*b^2 - 8*(sin(d*x + c)^2 + log(sin(d*x +
c)^2 - 1))*a*b^3 - 2*(3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*
tan(d*x + c))*b^4)/d
```

3.81.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.08

$$\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$
$$= \frac{4\,ab^3\log\left(\tan\left(dx+c\right)^2+1\right)+2\,b^4\tan\left(dx+c\right)+(a^4+6\,a^2b^2-3\,b^4)(dx+c)-\frac{4\,ab^3\tan(dx+c)^2-a^4\tan(dx+c)+}{\tan(}}{2\,d}$$

input
```
integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

output
```
1/2*(4*a*b^3*log(tan(d*x + c)^2 + 1) + 2*b^4*tan(d*x + c) + (a^4 + 6*a^2*b
^2 - 3*b^4)*(d*x + c) - (4*a*b^3*tan(d*x + c)^2 - a^4*tan(d*x + c) + 6*a^2
*b^2*tan(d*x + c) - b^4*tan(d*x + c) + 4*a^3*b)/(tan(d*x + c)^2 + 1))/d
```

3.81.9 Mupad [B] (verification not implemented)

Time = 22.91 (sec) , antiderivative size = 255, normalized size of antiderivative = 2.14

$$\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$

$$= \frac{a^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)-3\,b^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)+4\,a\,b^3\,\ln\left(\frac{1}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}\right)+6\,a^2\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)-4\,a\,b}{d}$$
$$+\frac{\frac{a^4\,\sin(c+d\,x)}{8}+\frac{9\,b^4\,\sin(c+d\,x)}{8}+\frac{a^4\,\sin(3\,c+3\,d\,x)}{8}+\frac{b^4\,\sin(3\,c+3\,d\,x)}{8}+\frac{a\,b^3\,\cos(3\,c+3\,d\,x)}{2}-\frac{a^3\,b\,\cos(3\,c+3\,d\,x)}{2}-\frac{3\,a^2\,b^2\,\sin(c+}{4}}{d\,\cos\left(c+d\,x\right)}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))^4/cos(c + d*x)^2,x)
```

output
```
(a^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) - 3*b^4*atan(sin(c/2 + (d
*x)/2)/cos(c/2 + (d*x)/2)) + 4*a*b^3*log(1/cos(c/2 + (d*x)/2)^2) + 6*a^2*b
^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) - 4*a*b^3*log(cos(c + d*x)/
(cos(c + d*x) + 1)))/d + ((a^4*sin(c + d*x))/8 + (9*b^4*sin(c + d*x))/8 +
(a^4*sin(3*c + 3*d*x))/8 + (b^4*sin(3*c + 3*d*x))/8 + (a*b^3*cos(3*c + 3*d
*x))/2 - (a^3*b*cos(3*c + 3*d*x))/2 - (3*a^2*b^2*sin(c + d*x))/4 - (3*a^2*
b^2*sin(3*c + 3*d*x))/4)/(d*cos(c + d*x))
```

3.82 $\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

3.82.1 Optimal result

Integrand size = 28, antiderivative size = 151

$$\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \frac{6a^2b^2\text{arctanh}(\sin(c+dx))}{d}-\frac{3b^4\text{arctanh}(\sin(c+dx))}{2d}-\frac{4a^3b\cos(c+dx)}{d}+\frac{4ab^3\cos(c+dx)}{d}+\frac{4ab^3\sec(c+dx)}{d}+\frac{a^4\sin(c+dx)}{d}-\frac{6a^2b^2\sin(c+dx)}{d}+\frac{3b^4\sin(c+dx)}{2d}+\frac{b^4\sin(c+dx)\tan^2(c+dx)}{2d}$$

output
```
6*a^2*b^2*arctanh(sin(d*x+c))/d-3/2*b^4*arctanh(sin(d*x+c))/d-4*a^3*b*cos(
d*x+c)/d+4*a*b^3*cos(d*x+c)/d+4*a*b^3*sec(d*x+c)/d+a^4*sin(d*x+c)/d-6*a^2*
b^2*sin(d*x+c)/d+3/2*b^4*sin(d*x+c)/d+1/2*b^4*sin(d*x+c)*tan(d*x+c)^2/d
```

3.82.2 Mathematica [A] (verified)

Time = 3.55 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.77

$$\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \frac{16ab^3-16ab(a^2-b^2)\cos(c+dx)-24a^2b^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+6b^4\log\left(\cos\left(\frac{1}{2}(c+\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

output
```
(16*a*b^3 - 16*a*b*(a^2 - b^2)*Cos[c + d*x] - 24*a^2*b^2*Log[Cos[(c + d*x)
/2] - Sin[(c + d*x)/2]] + 6*b^4*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] +
 24*a^2*b^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 6*b^4*Log[Cos[(c +
d*x)/2] + Sin[(c + d*x)/2]] + b^4/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2
+ 32*a*b^3*Sec[c + d*x]*Sin[(c + d*x)/2]^2 - b^4/(Cos[(c + d*x)/2] + Sin[(
c + d*x)/2])^2 + 4*a^4*Sin[c + d*x] - 24*a^2*b^2*Sin[c + d*x] + 4*b^4*Sin[
c + d*x])/(4*d)
```

3.82.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\cos(c+dx)+b\sin(c+dx))^4}{\cos(c+dx)^3}dx$$

$$\Big\downarrow \text{3569}$$

$$\int \left(a^4\cos(c+dx)+4a^3b\sin(c+dx)+6a^2b^2\sin(c+dx)\tan(c+dx)+4ab^3\sin(c+dx)\tan^2(c+dx)+b^4\sin(c+\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^4\sin(c+dx)}{d}-\frac{4a^3b\cos(c+dx)}{d}+\frac{6a^2b^2\text{arctanh}(\sin(c+dx))}{d}-\frac{6a^2b^2\sin(c+dx)}{d}+\frac{4ab^3\cos(c+dx)}{d}+\frac{4ab^3\sec(c+dx)}{d}-\frac{3b^4\text{arctanh}(\sin(c+dx))}{2d}+\frac{3b^4\sin(c+dx)}{2d}+\frac{b^4\sin(c+dx)\tan^2(c+dx)}{2d}$$

input
```
Int[Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

output
```
(6*a^2*b^2*ArcTanh[Sin[c + d*x]])/d - (3*b^4*ArcTanh[Sin[c + d*x]])/(2*d)
- (4*a^3*b*Cos[c + d*x])/d + (4*a*b^3*Cos[c + d*x])/d + (4*a*b^3*Sec[c + d
*x])/d + (a^4*Sin[c + d*x])/d - (6*a^2*b^2*Sin[c + d*x])/d + (3*b^4*Sin[c
+ d*x])/(2*d) + (b^4*Sin[c + d*x]*Tan[c + d*x]^2)/(2*d)
```

3.82.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

3.82.4 Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.04

method	result
derivativedivides	$\frac{\sin(dx+c)a^4-4\cos(dx+c)a^3b+6a^2b^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+4a\,b^3\left(\frac{\sin(dx+c)^4}{\cos(dx+c)}+\left(2+\sin(dx+c)^2\right)c\right.}{d}$
default	$\frac{\sin(dx+c)a^4-4\cos(dx+c)a^3b+6a^2b^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+4a\,b^3\left(\frac{\sin(dx+c)^4}{\cos(dx+c)}+\left(2+\sin(dx+c)^2\right)c\right.}{d}$
parts	$\frac{a^4\sin(dx+c)}{d}+\frac{b^4\left(\frac{\sin(dx+c)^5}{2\cos(dx+c)^2}+\frac{\sin(dx+c)^3}{2}+\frac{3\sin(dx+c)}{2}-\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}-\frac{4a^3b\cos(dx+c)}{d}+\frac{4a\,b^3\left(\right.}{}$
parallelrisch	$\frac{-12\left(a+\frac{b}{2}\right)b^2\left(a-\frac{b}{2}\right)(1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+12\left(a+\frac{b}{2}\right)b^2\left(a-\frac{b}{2}\right)(1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{}$
risch	$-\frac{2\,e^{i(dx+c)}a^3b}{d}+\frac{2\,e^{i(dx+c)}a\,b^3}{d}-\frac{ie^{i(dx+c)}a^4}{2d}+\frac{3ie^{i(dx+c)}a^2b^2}{d}-\frac{ie^{i(dx+c)}b^4}{2d}-\frac{2\,e^{-i(dx+c)}a^3b}{d}+\frac{2\,e^{-i(dx+}}{d}$
norman	$\frac{\frac{8a^3b+16a\,b^3}{d}+\frac{16a^3b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{d}+\frac{\left(2a^4-12a^2b^2+3b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{\left(2a^4-12a^2b^2+3b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}}{d}+\frac{\left(2a^4-12a^2b^2+7b\right.}{d}}{}$

input
```
int(sec(d*x+c)^3*(cos(d*x+c)*a+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(sin(d*x+c)*a^4-4*cos(d*x+c)*a^3*b+6*a^2*b^2*(-sin(d*x+c)+ln(sec(d*x+c
)+tan(d*x+c)))+4*a*b^3*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c
))+b^4*(1/2*sin(d*x+c)^5/cos(d*x+c)^2+1/2*sin(d*x+c)^3+3/2*sin(d*x+c)-3/2*
ln(sec(d*x+c)+tan(d*x+c))))
```

3.82.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.01

$$\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$
$$= \frac{16\,ab^3\cos\left(dx+c\right)-16\left(a^3b-ab^3\right)\cos\left(dx+c\right)^3+3\left(4\,a^2b^2-b^4\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-3\,(}{4\,d\cos\,(}$$

input
```
integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas"
)
```

output
```
1/4*(16*a*b^3*cos(d*x + c) - 16*(a^3*b - a*b^3)*cos(d*x + c)^3 + 3*(4*a^2*
b^2 - b^4)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - 3*(4*a^2*b^2 - b^4)*cos(
d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(b^4 + 2*(a^4 - 6*a^2*b^2 + b^4)*cos
(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

3.82.6 Sympy [F(-1)]

Timed out.

$$\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**3*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

output
```
Timed out
```

3.82.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.94

$$\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx =$$

$$-\frac{b^4\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}+3\,\log\left(\sin\left(dx+c\right)+1\right)-3\,\log\left(\sin\left(dx+c\right)-1\right)-4\,\sin\left(dx+c\right)\right)-16\,ab^3\left(\frac{1}{\cos(dx+c)}\right.}{}$$

input
```
integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima"
)
```

output
```
-1/4*(b^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + 3*log(sin(d*x + c) + 1) -
 3*log(sin(d*x + c) - 1) - 4*sin(d*x + c)) - 16*a*b^3*(1/cos(d*x + c) + co
s(d*x + c)) - 12*a^2*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) -
2*sin(d*x + c)) + 16*a^3*b*cos(d*x + c) - 4*a^4*sin(d*x + c))/d
```

3.82.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.36

$$\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$

$$=\frac{3\left(4\,a^2b^2-b^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(4\,a^2b^2-b^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{4\left(a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right.}{}}{2\,d}$$

input
```
integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

output
```
1/2*(3*(4*a^2*b^2 - b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(4*a^2*b^2
 - b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 4*(a^4*tan(1/2*d*x + 1/2*c) -
 6*a^2*b^2*tan(1/2*d*x + 1/2*c) + b^4*tan(1/2*d*x + 1/2*c) - 4*a^3*b + 4*a
*b^3)/(tan(1/2*d*x + 1/2*c)^2 + 1) + 2*(b^4*tan(1/2*d*x + 1/2*c)^3 - 8*a*b
^3*tan(1/2*d*x + 1/2*c)^2 + b^4*tan(1/2*d*x + 1/2*c) + 8*a*b^3)/(tan(1/2*d
*x + 1/2*c)^2 - 1)^2)/d
```

3.82.9 Mupad [B] (verification not implemented)

Time = 24.83 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.46

$$\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$
$$= \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(4a^4-24a^2b^2+2b^4\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\left(2a^4-12a^2b^2+3b^4\right)-16ab^3+8a^3b-\tan\left(\frac{c}{2}+\right.}{d\left(-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+\mathrm{ta}\right.}$$
$$-\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(3b^4-12a^2b^2\right)}{d}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))^4/cos(c + d*x)^3,x)
```

output
```
(tan(c/2 + (d*x)/2)^3*(4*a^4 + 2*b^4 - 24*a^2*b^2) - tan(c/2 + (d*x)/2)^5*
(2*a^4 + 3*b^4 - 12*a^2*b^2) - 16*a*b^3 + 8*a^3*b - tan(c/2 + (d*x)/2)*(2*
a^4 + 3*b^4 - 12*a^2*b^2) + tan(c/2 + (d*x)/2)^2*(16*a*b^3 - 16*a^3*b) + 8
*a^3*b*tan(c/2 + (d*x)/2)^4)/(d*(tan(c/2 + (d*x)/2)^2 + tan(c/2 + (d*x)/2)
^4 - tan(c/2 + (d*x)/2)^6 - 1)) - (atanh(tan(c/2 + (d*x)/2))*(3*b^4 - 12*a
^2*b^2))/d
```

3.83 $\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

3.83.1 Optimal result

Integrand size = 28, antiderivative size = 103

$$\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \left(a^4-6a^2b^2+b^4\right)x-\frac{4ab(a^2-b^2)\log(\cos(c+dx))}{d} + \frac{b^2(3a^2-b^2)\tan(c+dx)}{d}+\frac{ab(a+b\tan(c+dx))^2}{d}+\frac{b(a+b\tan(c+dx))^3}{3d}$$

output
```
(a^4-6*a^2*b^2+b^4)*x-4*a*b*(a^2-b^2)*ln(cos(d*x+c))/d+b^2*(3*a^2-b^2)*tan
(d*x+c)/d+a*b*(a+b*tan(d*x+c))^2/d+1/3*b*(a+b*tan(d*x+c))^3/d
```

3.83.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.02

$$\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \frac{-3i(a+ib)^4\log(i-\tan(c+dx))+3i(a-ib)^4\log(i+\tan(c+dx))-6b^2(-6a^2+b^2)\tan(c+dx)+12ab}{6d}$$

input
```
Integrate[Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

output
```
((-3*I)*(a + I*b)^4*Log[I - Tan[c + d*x]] + (3*I)*(a - I*b)^4*Log[I + Tan[
c + d*x]] - 6*b^2*(-6*a^2 + b^2)*Tan[c + d*x] + 12*a*b^3*Tan[c + d*x]^2 +
2*b^4*Tan[c + d*x]^3)/(6*d)
```

3.83.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3565, 3042, 3963, 3042, 4011, 3042, 4008, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\cos(c+dx)+b\sin(c+dx))^4}{\cos(c+dx)^4}dx$$

$$\Big\downarrow \text{3565}$$

$$\int (a+b\tan(c+dx))^4dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(c+dx))^4dx$$

$$\Big\downarrow \text{3963}$$

$$\int (a+b\tan(c+dx))^2\left(a^2+2b\tan(c+dx)a-b^2\right)dx+\frac{b(a+b\tan(c+dx))^3}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(c+dx))^2\left(a^2+2b\tan(c+dx)a-b^2\right)dx+\frac{b(a+b\tan(c+dx))^3}{3d}$$

$$\Big\downarrow \text{4011}$$

$$\int (a+b\tan(c+dx))\left(a\left(a^2-3b^2\right)+b\left(3a^2-b^2\right)\tan(c+dx)\right)dx+\frac{b(a+b\tan(c+dx))^3}{3d}+\frac{ab(a+b\tan(c+dx))^2}{d}$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(c+dx))\left(a\left(a^2-3b^2\right)+b\left(3a^2-b^2\right)\tan(c+dx)\right)dx+\frac{b(a+b\tan(c+dx))^3}{3d}+\frac{ab(a+b\tan(c+dx))^2}{d}$$

$\downarrow$ 4008

$$4ab\left(a^2-b^2\right)\int \tan(c+dx)dx+\frac{b^2\left(3a^2-b^2\right)\tan(c+dx)}{d}+x\left(a^4-6a^2b^2+b^4\right)+\frac{b(a+b\tan(c+dx))^3}{3d}+\frac{ab(a+b\tan(c+dx))^2}{d}$$

$\downarrow$ 3042

$$4ab\left(a^2-b^2\right)\int \tan(c+dx)dx+\frac{b^2\left(3a^2-b^2\right)\tan(c+dx)}{d}+x\left(a^4-6a^2b^2+b^4\right)+\frac{b(a+b\tan(c+dx))^3}{3d}+\frac{ab(a+b\tan(c+dx))^2}{d}$$

$\downarrow$ 3956

$$\frac{b^2\left(3a^2-b^2\right)\tan(c+dx)}{d}-\frac{4ab\left(a^2-b^2\right)\log(\cos(c+dx))}{d}+x\left(a^4-6a^2b^2+b^4\right)+\frac{b(a+b\tan(c+dx))^3}{3d}+\frac{ab(a+b\tan(c+dx))^2}{d}$$

input
```
Int[Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

output
```
(a^4 - 6*a^2*b^2 + b^4)*x - (4*a*b*(a^2 - b^2)*Log[Cos[c + d*x]])/d + (b^2
*(3*a^2 - b^2)*Tan[c + d*x])/d + (a*b*(a + b*Tan[c + d*x])^2)/d + (b*(a +
b*Tan[c + d*x])^3)/(3*d)
```

3.83.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3565
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(a + b*Tan[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 3963
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d
*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2
+ b^2, 0] && GtQ[n, 1]
```

rule 4008
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Simp[b*d*(Tan[e + f*x]/f),
x] + Simp[(b*c + a*d)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

3.83.4 Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{a^4(dx+c)-4a^3b\ln(\cos(dx+c))+6a^2b^2(\tan(dx+c)-dx-c)+4a\,b^3\left(\frac{\tan(dx+c)^2}{2}+\ln(\cos(dx+c))\right)+b^4\left(\frac{\tan(dx+c)^3}{3}-\tan(\right.}{d}$
default	$\frac{a^4(dx+c)-4a^3b\ln(\cos(dx+c))+6a^2b^2(\tan(dx+c)-dx-c)+4a\,b^3\left(\frac{\tan(dx+c)^2}{2}+\ln(\cos(dx+c))\right)+b^4\left(\frac{\tan(dx+c)^3}{3}-\tan(\right.}{d}$
parts	$\frac{a^4(dx+c)}{d}+\frac{b^4\left(\frac{\tan(dx+c)^3}{3}-\tan(dx+c)+dx+c\right)}{d}+\frac{4a^3b\ln(\sec(dx+c))}{d}+\frac{4a\,b^3\left(\frac{\tan(dx+c)^2}{2}+\ln(\cos(dx+c))\right)}{d}+$
risch	$4ia^3bx-4ixa\,b^3+a^4x-6a^2b^2x+b^4x+\frac{8ia^3bc}{d}-\frac{8ia\,b^3c}{d}-\frac{4ib^2\left(-9a^2e^{4i(dx+c)}+3b^2e^{4i(dx+c)}+6ia\right.}{}$
parallelrisch	$\frac{36b\left(\frac{\cos(3dx+3c)}{3}+\cos(dx+c)\right)(a-b)a(a+b)\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)-36b\left(\frac{\cos(3dx+3c)}{3}+\cos(dx+c)\right)(a-b)a(a+b)\ln\left(\tan\left(\frac{dx}{2}\right.\right.}{}$
norman	$\frac{\frac{8a\,b^3}{d}+\left(-a^4+6a^2b^2-b^4\right)x+\frac{40a\,b^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{d}+\left(-3a^4+18a^2b^2-3b^4\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8+\left(-3a^4+18a^2b^2-3b^4\right)x\tan\left(\frac{dx}{2}\right.}{}$

input
```
int(sec(d*x+c)^4*(cos(d*x+c)*a+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^4*(d*x+c)-4*a^3*b*ln(cos(d*x+c))+6*a^2*b^2*(tan(d*x+c)-d*x-c)+4*a*b
^3*(1/2*tan(d*x+c)^2+ln(cos(d*x+c)))+b^4*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+
c))
```

3.83.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.16

$$\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$
$$= \frac{3\,(a^4-6\,a^2b^2+b^4)dx\cos{(dx+c)}^3+6\,ab^3\cos{(dx+c)}-12\,(a^3b-ab^3)\cos{(dx+c)}^3\log{(-\cos{(dx+c)})}}{3\,d\cos{(dx+c)}^3}$$

input
```
integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas"
)
```

output
```
1/3*(3*(a^4 - 6*a^2*b^2 + b^4)*d*x*cos(d*x + c)^3 + 6*a*b^3*cos(d*x + c) -
 12*(a^3*b - a*b^3)*cos(d*x + c)^3*log(-cos(d*x + c)) + (b^4 + 2*(9*a^2*b^
2 - 2*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

3.83.6 Sympy [F(-1)]

Timed out.

$$\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**4*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

output
```
Timed out
```

3.83.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.13

$$\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$
$$=\frac{3\,(dx+c)a^4-18\,(dx+c-\tan{(dx+c)})a^2b^2+\left(\tan{(dx+c)}^3+3\,dx+3\,c-3\,\tan{(dx+c)}\right)b^4-6\,ab^3\left(\right.}{3\,d}$$

input
```
integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima"
)
```

output
```
1/3*(3*(d*x + c)*a^4 - 18*(d*x + c - tan(d*x + c))*a^2*b^2 + (tan(d*x + c)
^3 + 3*d*x + 3*c - 3*tan(d*x + c))*b^4 - 6*a*b^3*(1/(sin(d*x + c)^2 - 1) -
 log(sin(d*x + c)^2 - 1)) - 6*a^3*b*log(-sin(d*x + c)^2 + 1))/d
```

3.83.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.01

$$\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$
$$=\frac{b^4\tan{(dx+c)}^3+6\,ab^3\tan{(dx+c)}^2+18\,a^2b^2\tan{(dx+c)}-3\,b^4\tan{(dx+c)}+3\,(a^4-6\,a^2b^2+b^4)(dx-}{3\,d}$$

input
```
integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

output
```
1/3*(b^4*tan(d*x + c)^3 + 6*a*b^3*tan(d*x + c)^2 + 18*a^2*b^2*tan(d*x + c)
 - 3*b^4*tan(d*x + c) + 3*(a^4 - 6*a^2*b^2 + b^4)*(d*x + c) + 6*(a^3*b - a
*b^3)*log(tan(d*x + c)^2 + 1))/d
```

3.83.9 Mupad [B] (verification not implemented)

Time = 23.65 (sec) , antiderivative size = 546, normalized size of antiderivative = 5.30

$$\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$
$$= \frac{\frac{3a^4\cos(c+dx)\operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{2}-\frac{b^4\sin(3c+3dx)}{3}+\frac{3b^4\cos(c+dx)\operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{2}-\frac{ab^3\cos(3c+3dx)}{2}+\frac{3a^2b^2\sin(c+dx)}{2}}{}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))^4/cos(c + d*x)^4,x)
```

output
```
((3*a^4*cos(c + d*x)*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/2 - (b^4
*sin(3*c + 3*d*x))/3 + (3*b^4*cos(c + d*x)*atan(sin(c/2 + (d*x)/2)/cos(c/2
 + (d*x)/2)))/2 - (a*b^3*cos(3*c + 3*d*x))/2 + (3*a^2*b^2*sin(c + d*x))/2
+ (a^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x))/2 + (
b^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x))/2 + (3*a
^2*b^2*sin(3*c + 3*d*x))/2 + (a*b^3*cos(c + d*x))/2 + 3*a*b^3*log(-cos(c +
 d*x)/cos(c/2 + (d*x)/2)^2)*cos(c + d*x) - 3*a^3*b*log(-cos(c + d*x)/cos(c
/2 + (d*x)/2)^2)*cos(c + d*x) - 3*a^2*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/2
+ (d*x)/2))*cos(3*c + 3*d*x) - 3*a*b^3*cos(c + d*x)*log(1/cos(c/2 + (d*x)/
2)^2) + 3*a^3*b*cos(c + d*x)*log(1/cos(c/2 + (d*x)/2)^2) + a*b^3*log(-cos(
c + d*x)/cos(c/2 + (d*x)/2)^2)*cos(3*c + 3*d*x) - a^3*b*log(-cos(c + d*x)/
cos(c/2 + (d*x)/2)^2)*cos(3*c + 3*d*x) - a*b^3*log(1/cos(c/2 + (d*x)/2)^2)
*cos(3*c + 3*d*x) + a^3*b*log(1/cos(c/2 + (d*x)/2)^2)*cos(3*c + 3*d*x) - 9
*a^2*b^2*cos(c + d*x)*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(d*((3*
cos(c + d*x))/4 + cos(3*c + 3*d*x)/4))
```

3.84 $\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

3.84.1 Optimal result

Integrand size = 28, antiderivative size = 168

$$\begin{aligned}\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx &= \frac{a^4\text{arctanh}(\sin(c+dx))}{d}-\frac{3a^2b^2\text{arctanh}(\sin(c+dx))}{d}\\ &\quad+\frac{3b^4\text{arctanh}(\sin(c+dx))}{8d}+\frac{4a^3b\sec(c+dx)}{d}-\frac{4ab^3\sec(c+dx)}{d}\\ &\quad+\frac{4ab^3\sec^3(c+dx)}{3d}+\frac{3a^2b^2\sec(c+dx)\tan(c+dx)}{d}\\ &\quad-\frac{3b^4\sec(c+dx)\tan(c+dx)}{8d}+\frac{b^4\sec(c+dx)\tan^3(c+dx)}{4d}\end{aligned}$$

output
```
a^4*arctanh(sin(d*x+c))/d-3*a^2*b^2*arctanh(sin(d*x+c))/d+3/8*b^4*arctanh(
sin(d*x+c))/d+4*a^3*b*sec(d*x+c)/d-4*a*b^3*sec(d*x+c)/d+4/3*a*b^3*sec(d*x+
c)^3/d+3*a^2*b^2*sec(d*x+c)*tan(d*x+c)/d-3/8*b^4*sec(d*x+c)*tan(d*x+c)/d+1
/4*b^4*sec(d*x+c)*tan(d*x+c)^3/d
```

3.84.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 936 vs. $2(168) = 336$.

Time = 7.52 (sec) , antiderivative size = 936, normalized size of antiderivative = 5.57

$$
\begin{aligned}
&\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx \\
&= \frac{2ab(6a^2-5b^2)\cos^4(c+dx)(a+b\tan(c+dx))^4}{3d(a\cos(c+dx)+b\sin(c+dx))^4} \\
&+ \frac{(-8a^4+24a^2b^2-3b^4)\cos^4(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^4}{8d(a\cos(c+dx)+b\sin(c+dx))^4} \\
&+ \frac{(8a^4-24a^2b^2+3b^4)\cos^4(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^4}{8d(a\cos(c+dx)+b\sin(c+dx))^4} \\
&+ \frac{b^4\cos^4(c+dx)(a+b\tan(c+dx))^4}{16d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^4(a\cos(c+dx)+b\sin(c+dx))^4} \\
&+ \frac{(72a^2b^2+16ab^3-15b^4)\cos^4(c+dx)(a+b\tan(c+dx))^4}{48d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2(a\cos(c+dx)+b\sin(c+dx))^4} \\
&+ \frac{2ab^3\cos^4(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^4}{3d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3(a\cos(c+dx)+b\sin(c+dx))^4} \\
&- \frac{b^4\cos^4(c+dx)(a+b\tan(c+dx))^4}{16d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4(a\cos(c+dx)+b\sin(c+dx))^4} \\
&- \frac{2ab^3\cos^4(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^4}{3d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3(a\cos(c+dx)+b\sin(c+dx))^4} \\
&+ \frac{(-72a^2b^2+16ab^3+15b^4)\cos^4(c+dx)(a+b\tan(c+dx))^4}{48d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2(a\cos(c+dx)+b\sin(c+dx))^4} \\
&+ \frac{2\cos^4(c+dx)\left(6a^3b\sin\left(\frac{1}{2}(c+dx)\right)-5ab^3\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^4}{3d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b\sin(c+dx))^4} \\
&- \frac{2\cos^4(c+dx)\left(6a^3b\sin\left(\frac{1}{2}(c+dx)\right)-5ab^3\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^4}{3d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b\sin(c+dx))^4}
\end{aligned}
$$

input
```
Integrate[Sec[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

output
```
(2*a*b*(6*a^2 - 5*b^2)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(3*d*(a*Cos[
c + d*x] + b*Sin[c + d*x])^4) + ((-8*a^4 + 24*a^2*b^2 - 3*b^4)*Cos[c + d*x
]^4*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^4)/(8*d*
(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + ((8*a^4 - 24*a^2*b^2 + 3*b^4)*Cos[c
 + d*x]^4*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^4)
/(8*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (b^4*Cos[c + d*x]^4*(a + b*Ta
n[c + d*x])^4)/(16*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^4*(a*Cos[c + d*
x] + b*Sin[c + d*x])^4) + ((72*a^2*b^2 + 16*a*b^3 - 15*b^4)*Cos[c + d*x]^4
*(a + b*Tan[c + d*x])^4)/(48*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2*(a*
Cos[c + d*x] + b*Sin[c + d*x])^4) + (2*a*b^3*Cos[c + d*x]^4*Sin[(c + d*x)/
2]*(a + b*Tan[c + d*x])^4)/(3*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*(a
*Cos[c + d*x] + b*Sin[c + d*x])^4) - (b^4*Cos[c + d*x]^4*(a + b*Tan[c + d*
x])^4)/(16*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*(a*Cos[c + d*x] + b*S
in[c + d*x])^4) - (2*a*b^3*Cos[c + d*x]^4*Sin[(c + d*x)/2]*(a + b*Tan[c +
d*x])^4)/(3*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(a*Cos[c + d*x] + b*
Sin[c + d*x])^4) + ((-72*a^2*b^2 + 16*a*b^3 + 15*b^4)*Cos[c + d*x]^4*(a +
b*Tan[c + d*x])^4)/(48*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(a*Cos[c
+ d*x] + b*Sin[c + d*x])^4) + (2*Cos[c + d*x]^4*(6*a^3*b*Sin[(c + d*x)/2]
- 5*a*b^3*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^4)/(3*d*(Cos[(c + d*x)/2]
 - Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) - (2*Cos[c + ...
```

3.84.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\cos(c+dx)+b\sin(c+dx))^4}{\cos(c+dx)^5}dx$$

$$\Big\downarrow \text{3569}$$

$$\int \left(a^4\sec(c+dx)+4a^3b\tan(c+dx)\sec(c+dx)+6a^2b^2\tan^2(c+dx)\sec(c+dx)+4ab^3\tan^3(c+dx)\sec(c+dx\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^4\text{arctanh}(\sin(c+dx))}{d}+\frac{4a^3b\sec(c+dx)}{d}-\frac{3a^2b^2\text{arctanh}(\sin(c+dx))}{d}+\frac{3a^2b^2\tan(c+dx)\sec(c+dx)}{d}+\frac{4ab^3\sec^3(c+dx)}{3d}-\frac{4ab^3\sec(c+dx)}{d}+\frac{3b^4\text{arctanh}(\sin(c+dx))}{8d}+\frac{b^4\tan^3(c+dx)\sec(c+dx)}{4d}-\frac{3b^4\tan(c+dx)\sec(c+dx)}{8d}$$

input
```
Int[Sec[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

output
```
(a^4*ArcTanh[Sin[c + d*x]])/d - (3*a^2*b^2*ArcTanh[Sin[c + d*x]])/d + (3*b
^4*ArcTanh[Sin[c + d*x]])/(8*d) + (4*a^3*b*Sec[c + d*x])/d - (4*a*b^3*Sec[
c + d*x])/d + (4*a*b^3*Sec[c + d*x]^3)/(3*d) + (3*a^2*b^2*Sec[c + d*x]*Tan
[c + d*x])/d - (3*b^4*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (b^4*Sec[c + d*x]
*Tan[c + d*x]^3)/(4*d)
```

3.84.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

3.84.4 Maple [A] (verified)

Time = 1.57 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.18

method	result
parts	$\frac{a^4\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{b^4\left(\frac{\sin(dx+c)^5}{4\cos(dx+c)^4}-\frac{\sin(dx+c)^5}{8\cos(dx+c)^2}-\frac{\sin(dx+c)^3}{8}-\frac{3\sin(dx+c)}{8}+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}$
derivativedivides	$a^4\ln(\sec(dx+c)+\tan(dx+c))+\frac{4a^3b}{\cos(dx+c)}+6a^2b^2\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+4a\,b^3\left(\frac{\sin(dx+c}{3\cos(dx+}\right.$
default	$a^4\ln(\sec(dx+c)+\tan(dx+c))+\frac{4a^3b}{\cos(dx+c)}+6a^2b^2\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+4a\,b^3\left(\frac{\sin(dx+c}{3\cos(dx+}\right.$
parallelrisch	$-4\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\cos(2dx+2c)\right)\left(a^4-3a^2b^2+\frac{3}{8}b^4\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+4\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\cos(2dx+2c)\right)(a^4-3a^2$
risch	$\frac{b\,e^{i(dx+c)}\left(72ib\,a^2+72ia^2b\,e^{2i(dx+c)}+96a^3e^{6i(dx+c)}-96a\,b^2e^{6i(dx+c)}-9ib^3e^{4i(dx+c)}+15ib^3e^{6i(dx+c)}+288a^3e^{4i(dx+c)}-\right.}{12d(e}$
norman	$\frac{24a^3b-16a\,b^3}{3d}-\frac{4\left(2a^3b+4a\,b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{d}+\frac{2\left(12a^3b-8a\,b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}}{d}-\frac{4\left(18a^3b-28a\,b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{3d}+\frac{8a^3b\tan\left(\frac{dx}{2}+\right.}{d}$

input
```
int(sec(d*x+c)^5*(cos(d*x+c)*a+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
a^4/d*ln(sec(d*x+c)+tan(d*x+c))+b^4/d*(1/4*sin(d*x+c)^5/cos(d*x+c)^4-1/8*s
in(d*x+c)^5/cos(d*x+c)^2-1/8*sin(d*x+c)^3-3/8*sin(d*x+c)+3/8*ln(sec(d*x+c)
+tan(d*x+c)))+4*a^3*b*sec(d*x+c)/d+4*a*b^3/d*(1/3*sec(d*x+c)^3-sec(d*x+c))
+6*a^2*b^2/d*(1/2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*sin(d*x+c)-1/2*ln(sec(d*x+
c)+tan(d*x+c)))
```

3.84.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.97

$$\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$
$$=\frac{3\left(8a^4-24a^2b^2+3b^4\right)\cos(dx+c)^4\log(\sin(dx+c)+1)-3\left(8a^4-24a^2b^2+3b^4\right)\cos(dx+c)^4\log(-}{}$$

input
```
integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas"
)
```

output
```
1/48*(3*(8*a^4 - 24*a^2*b^2 + 3*b^4)*cos(d*x + c)^4*log(sin(d*x + c) + 1)
- 3*(8*a^4 - 24*a^2*b^2 + 3*b^4)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 6
4*a*b^3*cos(d*x + c) + 192*(a^3*b - a*b^3)*cos(d*x + c)^3 + 6*(2*b^4 + (24
*a^2*b^2 - 5*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

3.84.6 Sympy [F(-1)]

Timed out.

$$\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**5*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

output
```
Timed out
```

3.84.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.14

$$\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \frac{3\,b^4\left(\frac{2\left(5\,\sin(dx+c)^3-3\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}+3\,\log\left(\sin\left(dx+c\right)+1\right)-3\,\log\left(\sin\left(dx+c\right)-1\right)\right)-72\,a^2b^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}\right.}{}$$

input
```
integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima"
)
```

output
```
1/48*(3*b^4*(2*(5*sin(d*x + c)^3 - 3*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin
(d*x + c)^2 + 1) + 3*log(sin(d*x + c) + 1) - 3*log(sin(d*x + c) - 1)) - 72
*a^2*b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - lo
g(sin(d*x + c) - 1)) + 24*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) -
1)) + 192*a^3*b/cos(d*x + c) - 64*(3*cos(d*x + c)^2 - 1)*a*b^3/cos(d*x + c
)^3)/d
```

3.84.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 325 vs. 2(160) = 320.

Time = 0.45 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.93

$$\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$

$$= \frac{3\left(8a^4-24a^2b^2+3b^4\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-3\left(8a^4-24a^2b^2+3b^4\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{}$$

input
```
integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

output
```
1/24*(3*(8*a^4 - 24*a^2*b^2 + 3*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) -
3*(8*a^4 - 24*a^2*b^2 + 3*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(72*
a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 9*b^4*tan(1/2*d*x + 1/2*c)^7 - 96*a^3*b*t
an(1/2*d*x + 1/2*c)^6 - 72*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 33*b^4*tan(1/2
*d*x + 1/2*c)^5 + 288*a^3*b*tan(1/2*d*x + 1/2*c)^4 - 192*a*b^3*tan(1/2*d*x
 + 1/2*c)^4 - 72*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 33*b^4*tan(1/2*d*x + 1/2
*c)^3 - 288*a^3*b*tan(1/2*d*x + 1/2*c)^2 + 256*a*b^3*tan(1/2*d*x + 1/2*c)^
2 + 72*a^2*b^2*tan(1/2*d*x + 1/2*c) - 9*b^4*tan(1/2*d*x + 1/2*c) + 96*a^3*
b - 64*a*b^3)/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.84.9 Mupad [B] (verification not implemented)

Time = 25.49 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.65

$$\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(2a^4-6a^2b^2+\frac{3b^4}{4}\right)}{d}$$

$$-\frac{\frac{16ab^3}{3}-8a^3b+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{3b^4}{4}-6a^2b^2\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7\left(\frac{3b^4}{4}-6a^2b^2\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{11b^4}{4}-6\right.}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-4\tan\right.}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))^4/cos(c + d*x)^5,x)
```

output
```
(atanh(tan(c/2 + (d*x)/2))*(2*a^4 + (3*b^4)/4 - 6*a^2*b^2))/d - ((16*a*b^3
)/3 - 8*a^3*b + tan(c/2 + (d*x)/2)*((3*b^4)/4 - 6*a^2*b^2) + tan(c/2 + (d*
x)/2)^7*((3*b^4)/4 - 6*a^2*b^2) - tan(c/2 + (d*x)/2)^3*((11*b^4)/4 - 6*a^2
*b^2) - tan(c/2 + (d*x)/2)^5*((11*b^4)/4 - 6*a^2*b^2) + tan(c/2 + (d*x)/2)
^4*(16*a*b^3 - 24*a^3*b) - tan(c/2 + (d*x)/2)^2*((64*a*b^3)/3 - 24*a^3*b)
+ 8*a^3*b*tan(c/2 + (d*x)/2)^6)/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (
d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

3.84. $\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

3.85 $\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

3.85.1 Optimal result

Integrand size = 28, antiderivative size = 30

$$\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx=\frac{(b+a\cot(c+dx))^5\tan^5(c+dx)}{5bd}$$

output
```
1/5*(b+a*cot(d*x+c))^5*tan(d*x+c)^5/b/d
```

3.85.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 73 vs. $2(30)=60$.

Time = 0.35 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.43

$$\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$
$$=\frac{\tan(c+dx)\left(5a^4+10a^3b\tan(c+dx)+10a^2b^2\tan^2(c+dx)+5ab^3\tan^3(c+dx)+b^4\tan^4(c+dx)\right)}{5d}$$

input
```
Integrate[Sec[c + d*x]^6*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

output
```
(Tan[c + d*x]*(5*a^4 + 10*a^3*b*Tan[c + d*x] + 10*a^2*b^2*Tan[c + d*x]^2 +
 5*a*b^3*Tan[c + d*x]^3 + b^4*Tan[c + d*x]^4))/(5*d)
```

3.85.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3042, 3567, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(a\cos(c+dx)+b\sin(c+dx))^4}{\cos(c+dx)^6}dx$$

$$\Big\downarrow \ 3567$$

$$-\frac{\int (b+a\cot(c+dx))^4\tan^6(c+dx)d\cot(c+dx)}{d}$$

$$\Big\downarrow \ 48$$

$$\frac{\tan^5(c+dx)(a\cot(c+dx)+b)^5}{5bd}$$

input
```
Int[Sec[c + d*x]^6*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

output
```
((b + a*Cot[c + d*x])^5*Tan[c + d*x]^5)/(5*b*d)
```

3.85.3.1 Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3567
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[x^m*((b
+ a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a,
b, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[
n, 0] && GtQ[m, 1])
```

3.85.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(28) = 56$.

Time = 1.31 (sec) , antiderivative size = 96, normalized size of antiderivative = 3.20

method	result
derivativedivides	$\frac{a^4\tan(dx+c)+\frac{2a^3b}{\cos(dx+c)^2}+\frac{2a^2b^2\sin(dx+c)^3}{\cos(dx+c)^3}+\frac{a\,b^3\sin(dx+c)^4}{\cos(dx+c)^4}+\frac{b^4\sin(dx+c)^5}{5\cos(dx+c)^5}}{d}$
default	$\frac{a^4\tan(dx+c)+\frac{2a^3b}{\cos(dx+c)^2}+\frac{2a^2b^2\sin(dx+c)^3}{\cos(dx+c)^3}+\frac{a\,b^3\sin(dx+c)^4}{\cos(dx+c)^4}+\frac{b^4\sin(dx+c)^5}{5\cos(dx+c)^5}}{d}$
parts	$\frac{a^4\tan(dx+c)}{d}+\frac{b^4\sin(dx+c)^5}{5d\cos(dx+c)^5}+\frac{2a^3b\sec(dx+c)^2}{d}+\frac{4a\,b^3\left(\frac{\sec(dx+c)^4}{4}-\frac{\sec(dx+c)^2}{2}\right)}{d}+\frac{2a^2b^2\sin(dx+c)^3}{d\cos(dx+c)^3}$
parallelrisch	$-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8a^4-4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7a^3b+\left(-4a^4+8a^2b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+\left(12a^3b-8a\,b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5+}{d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}$
risch	$\frac{2i\left(-60ia^3b\,e^{6i(dx+c)}+20ia\,b^3e^{8i(dx+c)}+5a^4e^{8i(dx+c)}-30a^2b^2e^{8i(dx+c)}+5b^4e^{8i(dx+c)}+20ia\,b^3e^{6i(dx+c)}+20ia\,b^3e^{2i(dx}\right.}{}$

input
```
int(sec(d*x+c)^6*(cos(d*x+c)*a+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^4*tan(d*x+c)+2*a^3*b/cos(d*x+c)^2+2*a^2*b^2*sin(d*x+c)^3/cos(d*x+c)
^3+a*b^3*sin(d*x+c)^4/cos(d*x+c)^4+1/5*b^4*sin(d*x+c)^5/cos(d*x+c)^5)
```

3.85.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(28) = 56$.

Time = 0.25 (sec) , antiderivative size = 109, normalized size of antiderivative = 3.63

$$\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$
$$=\frac{5\,ab^3\cos(dx+c)+10\,(a^3b-ab^3)\cos(dx+c)^3+\left((5\,a^4-10\,a^2b^2+b^4)\cos(dx+c)^4+b^4+2\,(5\,a^2b^2-b\right.}{5\,d\cos(dx+c)^5}$$

input
```
integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas"
)
```

output
```
1/5*(5*a*b^3*cos(d*x + c) + 10*(a^3*b - a*b^3)*cos(d*x + c)^3 + ((5*a^4 -
10*a^2*b^2 + b^4)*cos(d*x + c)^4 + b^4 + 2*(5*a^2*b^2 - b^4)*cos(d*x + c)^
2)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

3.85.6 Sympy [F(-1)]

Timed out.

$$\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**6*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

output
```
Timed out
```

3.85.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(28) = 56$.

Time = 0.24 (sec) , antiderivative size = 103, normalized size of antiderivative = 3.43

$$\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$
$$= \frac{b^4\tan(dx+c)^5 + 10\,a^2b^2\tan(dx+c)^3 + 5\,a^4\tan(dx+c) + \frac{5\left(2\,\sin(dx+c)^2-1\right)ab^3}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1} - \frac{10\,a^3b}{\sin(dx+c)^2-1}}{5\,d}$$

input
```
integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima"
)
```

output
```
1/5*(b^4*tan(d*x + c)^5 + 10*a^2*b^2*tan(d*x + c)^3 + 5*a^4*tan(d*x + c) +
 5*(2*sin(d*x + c)^2 - 1)*a*b^3/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) -
10*a^3*b/(sin(d*x + c)^2 - 1))/d
```

3.85.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(28) = 56.

Time = 0.44 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.43

$$\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$
$$=\frac{b^4\tan{(dx+c)}^5+5\,ab^3\tan{(dx+c)}^4+10\,a^2b^2\tan{(dx+c)}^3+10\,a^3b\tan{(dx+c)}^2+5\,a^4\tan{(dx+c)}}{5\,d}$$

input
```
integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

output
```
1/5*(b^4*tan(d*x + c)^5 + 5*a*b^3*tan(d*x + c)^4 + 10*a^2*b^2*tan(d*x + c)
^3 + 10*a^3*b*tan(d*x + c)^2 + 5*a^4*tan(d*x + c))/d
```

3.85.9 Mupad [B] (verification not implemented)

Time = 22.35 (sec) , antiderivative size = 139, normalized size of antiderivative = 4.63

$$\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$
$$=\frac{\frac{b^4\sin(c+d\,x)}{5}-\cos{(c+d\,x)}^3\,(2\,a\,b^3-2\,a^3\,b)-\cos{(c+d\,x)}^2\left(\frac{2\,b^4\sin(c+d\,x)}{5}-2\,a^2\,b^2\,\sin{(c+d\,x)}\right)+\cos{(c+}}{d\cos{(c+d\,x)}^5}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))^4/cos(c + d*x)^6,x)
```

output
```
((b^4*sin(c + d*x))/5 - cos(c + d*x)^3*(2*a*b^3 - 2*a^3*b) - cos(c + d*x)^
2*((2*b^4*sin(c + d*x))/5 - 2*a^2*b^2*sin(c + d*x)) + cos(c + d*x)^4*(a^4*
sin(c + d*x) + (b^4*sin(c + d*x))/5 - 2*a^2*b^2*sin(c + d*x)) + a*b^3*cos(
c + d*x))/(d*cos(c + d*x)^5)
```

3.86 $\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

3.86.1 Optimal result

Integrand size = 28, antiderivative size = 258

$$\begin{aligned}
&\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx \\
&= \frac{a^4\text{arctanh}(\sin(c+dx))}{2d} - \frac{3a^2b^2\text{arctanh}(\sin(c+dx))}{4d} + \frac{b^4\text{arctanh}(\sin(c+dx))}{16d} \\
&\quad + \frac{4a^3b\sec^3(c+dx)}{3d} - \frac{4ab^3\sec^3(c+dx)}{3d} + \frac{4ab^3\sec^5(c+dx)}{5d} \\
&\quad + \frac{a^4\sec(c+dx)\tan(c+dx)}{2d} - \frac{3a^2b^2\sec(c+dx)\tan(c+dx)}{4d} \\
&\quad + \frac{b^4\sec(c+dx)\tan(c+dx)}{16d} + \frac{3a^2b^2\sec^3(c+dx)\tan(c+dx)}{2d} \\
&\quad - \frac{b^4\sec^3(c+dx)\tan(c+dx)}{8d} + \frac{b^4\sec^3(c+dx)\tan^3(c+dx)}{6d}
\end{aligned}$$

output
```
1/2*a^4*arctanh(sin(d*x+c))/d-3/4*a^2*b^2*arctanh(sin(d*x+c))/d+1/16*b^4*a
rctanh(sin(d*x+c))/d+4/3*a^3*b*sec(d*x+c)^3/d-4/3*a*b^3*sec(d*x+c)^3/d+4/5
*a*b^3*sec(d*x+c)^5/d+1/2*a^4*sec(d*x+c)*tan(d*x+c)/d-3/4*a^2*b^2*sec(d*x+
c)*tan(d*x+c)/d+1/16*b^4*sec(d*x+c)*tan(d*x+c)/d+3/2*a^2*b^2*sec(d*x+c)^3*
tan(d*x+c)/d-1/8*b^4*sec(d*x+c)^3*tan(d*x+c)/d+1/6*b^4*sec(d*x+c)^3*tan(d*
x+c)^3/d
```

3.86.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1342 vs. $2(258) = 516$.

Time = 7.85 (sec) , antiderivative size = 1342, normalized size of antiderivative = 5.20

$$\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \text{Too large to display}$$

input
```
Integrate[Sec[c + d*x]^7*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

output
```
(a*b*(20*a^2 - 11*b^2)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(30*d*(a*Cos
[c + d*x] + b*Sin[c + d*x])^4) + ((-8*a^4 + 12*a^2*b^2 - b^4)*Cos[c + d*x]
^4*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^4)/(16*d*
(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + ((8*a^4 - 12*a^2*b^2 + b^4)*Cos[c +
 d*x]^4*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^4)/(
16*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (b^4*Cos[c + d*x]^4*(a + b*Tan
[c + d*x])^4)/(48*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^6*(a*Cos[c + d*x
] + b*Sin[c + d*x])^4) + ((30*a^2*b^2 + 8*a*b^3 - 5*b^4)*Cos[c + d*x]^4*(a
 + b*Tan[c + d*x])^4)/(80*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^4*(a*Cos
[c + d*x] + b*Sin[c + d*x])^4) + ((120*a^4 + 160*a^3*b - 180*a^2*b^2 - 88*
a*b^3 + 15*b^4)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(480*d*(Cos[(c + d*
x)/2] - Sin[(c + d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (a*b^3*
Cos[c + d*x]^4*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^4)/(5*d*(Cos[(c + d*x
)/2] - Sin[(c + d*x)/2])^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) - (b^4*Cos
[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(48*d*(Cos[(c + d*x)/2] + Sin[(c + d*x
)/2])^6*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) - (a*b^3*Cos[c + d*x]^4*Sin[(
c + d*x)/2]*(a + b*Tan[c + d*x])^4)/(5*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)
/2])^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + ((-30*a^2*b^2 + 8*a*b^3 + 5*
b^4)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(80*d*(Cos[(c + d*x)/2] + Sin[
(c + d*x)/2])^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + ((-120*a^4 + 160...
```

3.86.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\cos(c+dx)+b\sin(c+dx))^4}{\cos(c+dx)^7}dx$$

$$\Big\downarrow \text{3569}$$

$$\int \left(a^4\sec^3(c+dx)+4a^3b\tan(c+dx)\sec^3(c+dx)+6a^2b^2\tan^2(c+dx)\sec^3(c+dx)+4ab^3\tan^3(c+dx)\sec^3(c+\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^4\text{arctanh}(\sin(c+dx))}{2d}+\frac{a^4\tan(c+dx)\sec(c+dx)}{2d}+\frac{4a^3b\sec^3(c+dx)}{3d}-\frac{3a^2b^2\text{arctanh}(\sin(c+dx))}{4d}+\frac{3a^2b^2\tan(c+dx)\sec^3(c+dx)}{2d}-\frac{3a^2b^2\tan(c+dx)\sec(c+dx)}{4d}+\frac{4ab^3\sec^5(c+dx)}{5d}-\frac{4ab^3\sec^3(c+dx)}{3d}+\frac{b^4\text{arctanh}(\sin(c+dx))}{16d}+\frac{b^4\tan^3(c+dx)\sec^3(c+dx)}{6d}-\frac{b^4\tan(c+dx)\sec^3(c+dx)}{8d}+\frac{b^4\tan(c+dx)\sec(c+dx)}{16d}$$

input
```
Int[Sec[c + d*x]^7*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

output
```
(a^4*ArcTanh[Sin[c + d*x]])/(2*d) - (3*a^2*b^2*ArcTanh[Sin[c + d*x]])/(4*d
) + (b^4*ArcTanh[Sin[c + d*x]])/(16*d) + (4*a^3*b*Sec[c + d*x]^3)/(3*d) -
(4*a*b^3*Sec[c + d*x]^3)/(3*d) + (4*a*b^3*Sec[c + d*x]^5)/(5*d) + (a^4*Sec
[c + d*x]*Tan[c + d*x])/(2*d) - (3*a^2*b^2*Sec[c + d*x]*Tan[c + d*x])/(4*d
) + (b^4*Sec[c + d*x]*Tan[c + d*x])/(16*d) + (3*a^2*b^2*Sec[c + d*x]^3*Tan
[c + d*x])/(2*d) - (b^4*Sec[c + d*x]^3*Tan[c + d*x])/(8*d) + (b^4*Sec[c +
d*x]^3*Tan[c + d*x]^3)/(6*d)
```

3.86.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

3.86.4 Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.99

method	result
parts	$\frac{a^4\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}+\frac{b^4\left(\frac{\sin(dx+c)^5}{6\cos(dx+c)^6}+\frac{\sin(dx+c)^5}{24\cos(dx+c)^4}-\frac{\sin(dx+c)^5}{48\cos(dx+c)^2}-\frac{\sin(dx+c)^3}{48}-\sin(\cdots\right.}{d}$
derivativedivides	$\frac{a^4\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{4a^3b}{3\cos(dx+c)^3}+6a^2b^2\left(\frac{\sin(dx+c)^3}{4\cos(dx+c)^4}+\frac{\sin(dx+c)^3}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\ln(\sec(\cdots\right.}{}$
default	$\frac{a^4\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{4a^3b}{3\cos(dx+c)^3}+6a^2b^2\left(\frac{\sin(dx+c)^3}{4\cos(dx+c)^4}+\frac{\sin(dx+c)^3}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\ln(\sec(\cdots\right.}{}$
parallelrisch	$\frac{-120\left(a^4-\frac{3}{2}a^2b^2+\frac{1}{8}b^4\right)(\cos(6dx+6c)+6\cos(4dx+4c)+15\cos(2dx+2c)+10)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+120\left(a^4-\frac{3}{2}a^2b^2+\frac{1}{8}b^4\cdots\right.}{}$
risch	$-\frac{ie^{i(dx+c)}\left(-120a^4-15b^4+180a^2b^2+120a^4e^{10i(dx+c)}+235b^4e^{2i(dx+c)}+390b^4e^{6i(dx+c)}+15b^4e^{10i(dx+c)}-180a^2b^2e^{10i\cdots}\right.}{}$

input
```
int(sec(d*x+c)^7*(cos(d*x+c)*a+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
a^4/d*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+b^4/d*(1/6
*sin(d*x+c)^5/cos(d*x+c)^6+1/24*sin(d*x+c)^5/cos(d*x+c)^4-1/48*sin(d*x+c)^
5/cos(d*x+c)^2-1/48*sin(d*x+c)^3-1/16*sin(d*x+c)+1/16*ln(sec(d*x+c)+tan(d*
x+c)))+4/3*a^3*b*sec(d*x+c)^3/d+4*a*b^3/d*(1/5*sec(d*x+c)^5-1/3*sec(d*x+c)
^3)+6*a^2*b^2/d*(1/4*sin(d*x+c)^3/cos(d*x+c)^4+1/8*sin(d*x+c)^3/cos(d*x+c)
^2+1/8*sin(d*x+c)-1/8*ln(sec(d*x+c)+tan(d*x+c)))
```

3.86.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.72

$$\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$
$$= \frac{15\,(8\,a^4-12\,a^2b^2+b^4)\cos{(dx+c)}^6\log{(\sin{(dx+c)}+1)}-15\,(8\,a^4-12\,a^2b^2+b^4)\cos{(dx+c)}^6\log{(-\mathrm{s}}}{}$$

input
```
integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas"
)
```

output
```
1/480*(15*(8*a^4 - 12*a^2*b^2 + b^4)*cos(d*x + c)^6*log(sin(d*x + c) + 1)
- 15*(8*a^4 - 12*a^2*b^2 + b^4)*cos(d*x + c)^6*log(-sin(d*x + c) + 1) + 38
4*a*b^3*cos(d*x + c) + 640*(a^3*b - a*b^3)*cos(d*x + c)^3 + 10*(3*(8*a^4 -
 12*a^2*b^2 + b^4)*cos(d*x + c)^4 + 8*b^4 + 2*(36*a^2*b^2 - 7*b^4)*cos(d*x
 + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^6)
```

3.86.6 Sympy [F(-1)]

Timed out.

$$\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**7*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

output
```
Timed out
```

3.86.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.97

$$\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx =$$
$$-\frac{5\,b^4\left(\frac{2\left(3\,\sin(dx+c)^5+8\,\sin(dx+c)^3-3\,\sin(dx+c)\right)}{\sin(dx+c)^6-3\,\sin(dx+c)^4+3\,\sin(dx+c)^2-1}-3\,\log{(\sin{(dx+c)}+1)}+3\,\log{(\sin{(dx+c)}-1)}\right)-180\,a^{2}\cdots}{}$$

input
```
integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima"
)
```

output
```
-1/480*(5*b^4*(2*(3*sin(d*x + c)^5 + 8*sin(d*x + c)^3 - 3*sin(d*x + c))/(s
in(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 3*log(sin(d*x +
 c) + 1) + 3*log(sin(d*x + c) - 1)) - 180*a^2*b^2*(2*(sin(d*x + c)^3 + sin
(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - log(sin(d*x + c) + 1)
 + log(sin(d*x + c) - 1)) + 120*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) -
 log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 640*a^3*b/cos(d*x + c)^3
 + 128*(5*cos(d*x + c)^2 - 3)*a*b^3/cos(d*x + c)^5)/d
```

3.86.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 536 vs. $2(234) = 468$.

Time = 0.45 (sec) , antiderivative size = 536, normalized size of antiderivative = 2.08

$$\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$

$$= \frac{15\,(8\,a^4-12\,a^2b^2+b^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,(8\,a^4-12\,a^2b^2+b^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{}$$

input
```
integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

output
```
1/240*(15*(8*a^4 - 12*a^2*b^2 + b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) -
15*(8*a^4 - 12*a^2*b^2 + b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(120*
a^4*tan(1/2*d*x + 1/2*c)^11 + 180*a^2*b^2*tan(1/2*d*x + 1/2*c)^11 - 15*b^4
*tan(1/2*d*x + 1/2*c)^11 - 960*a^3*b*tan(1/2*d*x + 1/2*c)^10 - 360*a^4*tan
(1/2*d*x + 1/2*c)^9 + 900*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 + 85*b^4*tan(1/2*
d*x + 1/2*c)^9 + 2880*a^3*b*tan(1/2*d*x + 1/2*c)^8 - 1920*a*b^3*tan(1/2*d*
x + 1/2*c)^8 + 240*a^4*tan(1/2*d*x + 1/2*c)^7 - 1080*a^2*b^2*tan(1/2*d*x +
 1/2*c)^7 + 570*b^4*tan(1/2*d*x + 1/2*c)^7 - 3200*a^3*b*tan(1/2*d*x + 1/2*
c)^6 + 1280*a*b^3*tan(1/2*d*x + 1/2*c)^6 + 240*a^4*tan(1/2*d*x + 1/2*c)^5
- 1080*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 570*b^4*tan(1/2*d*x + 1/2*c)^5 + 1
920*a^3*b*tan(1/2*d*x + 1/2*c)^4 - 360*a^4*tan(1/2*d*x + 1/2*c)^3 + 900*a^
2*b^2*tan(1/2*d*x + 1/2*c)^3 + 85*b^4*tan(1/2*d*x + 1/2*c)^3 - 960*a^3*b*t
an(1/2*d*x + 1/2*c)^2 + 768*a*b^3*tan(1/2*d*x + 1/2*c)^2 + 120*a^4*tan(1/2
*d*x + 1/2*c) + 180*a^2*b^2*tan(1/2*d*x + 1/2*c) - 15*b^4*tan(1/2*d*x + 1/
2*c) + 320*a^3*b - 128*a*b^3)/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

3.86.9 Mupad [B] (verification not implemented)

Time = 26.55 (sec) , antiderivative size = 419, normalized size of antiderivative = 1.62

$$\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(a^4-\frac{3a^2b^2}{2}+\frac{b^4}{8}\right)}{d} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\left(2a^4-9a^2b^2+\frac{19b^4}{4}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7\left(2a^4-9a^2b^2+\frac{19b^4}{4}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(-3a^4+\right.}{}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))^4/cos(c + d*x)^7,x)
```

output
```
(atanh(tan(c/2 + (d*x)/2))*(a^4 + b^4/8 - (3*a^2*b^2)/2))/d + (tan(c/2 + (
d*x)/2)^5*(2*a^4 + (19*b^4)/4 - 9*a^2*b^2) + tan(c/2 + (d*x)/2)^7*(2*a^4 +
 (19*b^4)/4 - 9*a^2*b^2) + tan(c/2 + (d*x)/2)^3*((17*b^4)/24 - 3*a^4 + (15
*a^2*b^2)/2) + tan(c/2 + (d*x)/2)^9*((17*b^4)/24 - 3*a^4 + (15*a^2*b^2)/2)
 + tan(c/2 + (d*x)/2)*(a^4 - b^4/8 + (3*a^2*b^2)/2) - (16*a*b^3)/15 + (8*a
^3*b)/3 + tan(c/2 + (d*x)/2)^11*(a^4 - b^4/8 + (3*a^2*b^2)/2) + tan(c/2 +
(d*x)/2)^2*((32*a*b^3)/5 - 8*a^3*b) - tan(c/2 + (d*x)/2)^8*(16*a*b^3 - 24*
a^3*b) + tan(c/2 + (d*x)/2)^6*((32*a*b^3)/3 - (80*a^3*b)/3) + 16*a^3*b*tan
(c/2 + (d*x)/2)^4 - 8*a^3*b*tan(c/2 + (d*x)/2)^10)/(d*(15*tan(c/2 + (d*x)/
2)^4 - 6*tan(c/2 + (d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*
x)/2)^8 - 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1))
```

3.87 $\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

3.87.1 Optimal result

Integrand size = 28, antiderivative size = 143

$$\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \frac{a^4\tan(c+dx)}{d}+\frac{2a^3b\tan^2(c+dx)}{d}+\frac{a^2(a^2+6b^2)\tan^3(c+dx)}{3d}+\frac{ab(a^2+b^2)\tan^4(c+dx)}{d}+\frac{b^2(6a^2+b^2)\tan^5(c+dx)}{5d}+\frac{2ab^3\tan^6(c+dx)}{3d}+\frac{b^4\tan^7(c+dx)}{7d}$$

output

```
a^4*tan(d*x+c)/d+2*a^3*b*tan(d*x+c)^2/d+1/3*a^2*(a^2+6*b^2)*tan(d*x+c)^3/d
+a*b*(a^2+b^2)*tan(d*x+c)^4/d+1/5*b^2*(6*a^2+b^2)*tan(d*x+c)^5/d+2/3*a*b^3
*tan(d*x+c)^6/d+1/7*b^4*tan(d*x+c)^7/d
```

3.87.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.38

$$\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \frac{(a+b\tan(c+dx))^5\left(a^2+21b^2-5ab\tan(c+dx)+15b^2\tan^2(c+dx)\right)}{105b^3d}$$

input
```
Integrate[Sec[c + d*x]^8*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

output
```
((a + b*Tan[c + d*x])^5*(a^2 + 21*b^2 - 5*a*b*Tan[c + d*x] + 15*b^2*Tan[c
+ d*x]^2))/(105*b^3*d)
```

3.87.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3567, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\cos(c+dx)+b\sin(c+dx))^4}{\cos(c+dx)^8}dx$$

$$\Big\downarrow \text{3567}$$

$$-\frac{\int (b+a\cot(c+dx))^4\left(\cot^2(c+dx)+1\right)\tan^8(c+dx)d\cot(c+dx)}{d}$$

$$\Big\downarrow \text{522}$$

$$-\frac{\int \left(b^4\tan^8(c+dx)+4ab^3\tan^7(c+dx)+\left(b^4+6a^2b^2\right)\tan^6(c+dx)+4ab\left(a^2+b^2\right)\tan^5(c+dx)+\left(a^4+6b^2a^2\right)\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-a^4\tan(c+dx)-2a^3b\tan^2(c+dx)-\frac{1}{5}b^2\left(6a^2+b^2\right)\tan^5(c+dx)-ab\left(a^2+b^2\right)\tan^4(c+dx)-\frac{1}{3}a^2\left(a^2+6b^2\right)}{d}$$

input
```
Int[Sec[c + d*x]^8*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

output
```
-((-(a^4*Tan[c + d*x]) - 2*a^3*b*Tan[c + d*x]^2 - (a^2*(a^2 + 6*b^2)*Tan[c
 + d*x]^3)/3 - a*b*(a^2 + b^2)*Tan[c + d*x]^4 - (b^2*(6*a^2 + b^2)*Tan[c +
 d*x]^5)/5 - (2*a*b^3*Tan[c + d*x]^6)/3 - (b^4*Tan[c + d*x]^7)/7)/d)
```

3.87.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3567
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[x^m*((b
+ a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a,
b, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[
n, 0] && GtQ[m, 1])
```

3.87.4 Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.16

method	result
parts	$-\frac{a^4\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)}{d}+\frac{b^4\left(\frac{\sin(dx+c)^5}{7\cos(dx+c)^7}+\frac{2\sin(dx+c)^5}{35\cos(dx+c)^5}\right)}{d}+\frac{4ab^3\left(\frac{\sec(dx+c)^6}{6}-\frac{\sec(dx+c)^4}{4}\right)}{d}+ a^3 b s$
derivativedivides	$\frac{-a^4\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+\frac{a^3b}{\cos(dx+c)^4}+6a^2b^2\left(\frac{\sin(dx+c)^3}{5\cos(dx+c)^5}+\frac{2\sin(dx+c)^3}{15\cos(dx+c)^3}\right)+4ab^3\left(\frac{\sin(dx+c)^4}{6\cos(dx+c)^6}+\frac{\sin(dx+c)}{12\cos(dx+c}\right.}{d}$
default	$\frac{-a^4\left(-\frac{2}{3}-\frac{\sec(dx+c)^2}{3}\right)\tan(dx+c)+\frac{a^3b}{\cos(dx+c)^4}+6a^2b^2\left(\frac{\sin(dx+c)^3}{5\cos(dx+c)^5}+\frac{2\sin(dx+c)^3}{15\cos(dx+c)^3}\right)+4ab^3\left(\frac{\sin(dx+c)^4}{6\cos(dx+c)^6}+\frac{\sin(dx+c)}{12\cos(dx+c}\right.}{d}$
parallelrisch	$-\frac{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}a^4-4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}a^3b+\left(8a^2b^2-\frac{14}{3}a^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+\left(12a^3b-8ab^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9+\left(-\frac{64}{5}a^2b^2\right.\right.}{}$
risch	$\frac{4i\left(35a^4+3b^4-42a^2b^2+105a^4e^{10i(dx+c)}+21b^4e^{2i(dx+c)}+210b^4e^{6i(dx+c)}+105b^4e^{10i(dx+c)}+420iab^3e^{10i(dx+c)}-420ia^3\right.}{}$

input
```
int(sec(d*x+c)^8*(cos(d*x+c)*a+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
-a^4/d*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+b^4/d*(1/7*sin(d*x+c)^5/cos(d*x+
c)^7+2/35*sin(d*x+c)^5/cos(d*x+c)^5)+4*a*b^3/d*(1/6*sec(d*x+c)^6-1/4*sec(d
*x+c)^4)+a^3*b*sec(d*x+c)^4/d+6*a^2*b^2/d*(1/5*sin(d*x+c)^3/cos(d*x+c)^5+2
/15*sin(d*x+c)^3/cos(d*x+c)^3)
```

3.87.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.99

$$\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \frac{70\,ab^3\cos(dx+c)+105\,(a^3b-ab^3)\cos(dx+c)^3+\left(2\,(35\,a^4-42\,a^2b^2+3\,b^4)\cos(dx+c)^6+(35\,a^4-42}{105\,d\cos(dx+c)^7}$$

input
```
integrate(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas"
)
```

output
```
1/105*(70*a*b^3*cos(d*x + c) + 105*(a^3*b - a*b^3)*cos(d*x + c)^3 + (2*(35
*a^4 - 42*a^2*b^2 + 3*b^4)*cos(d*x + c)^6 + (35*a^4 - 42*a^2*b^2 + 3*b^4)*
cos(d*x + c)^4 + 15*b^4 + 6*(21*a^2*b^2 - 4*b^4)*cos(d*x + c)^2)*sin(d*x +
 c))/(d*cos(d*x + c)^7)
```

3.87.6 Sympy [F(-1)]

Timed out.

$$\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**8*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

output
```
Timed out
```

3.87.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.06

$$\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$

$$= \frac{35\left(\tan(dx+c)^3+3\tan(dx+c)\right)a^4+42\left(3\tan(dx+c)^5+5\tan(dx+c)^3\right)a^2b^2+3\left(5\tan(dx+c)^7+\right.}{105\,d}$$

input
```
integrate(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima"
)
```

output
```
1/105*(35*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^4 + 42*(3*tan(d*x + c)^5 + 5
*tan(d*x + c)^3)*a^2*b^2 + 3*(5*tan(d*x + c)^7 + 7*tan(d*x + c)^5)*b^4 - 3
5*(3*sin(d*x + c)^2 - 1)*a*b^3/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(
d*x + c)^2 - 1) + 105*a^3*b/(sin(d*x + c)^2 - 1)^2)/d
```

3.87.8 Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.01

$$\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$

$$= \frac{15\,b^4\tan(dx+c)^7+70\,ab^3\tan(dx+c)^6+126\,a^2b^2\tan(dx+c)^5+21\,b^4\tan(dx+c)^5+105\,a^3b\tan(dx+}{}$$

input
```
integrate(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

output
```
1/105*(15*b^4*tan(d*x + c)^7 + 70*a*b^3*tan(d*x + c)^6 + 126*a^2*b^2*tan(d
*x + c)^5 + 21*b^4*tan(d*x + c)^5 + 105*a^3*b*tan(d*x + c)^4 + 105*a*b^3*t
an(d*x + c)^4 + 35*a^4*tan(d*x + c)^3 + 210*a^2*b^2*tan(d*x + c)^3 + 210*a
^3*b*tan(d*x + c)^2 + 105*a^4*tan(d*x + c))/d
```

3.87.9 Mupad [B] (verification not implemented)

Time = 24.21 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.30

$$\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$
$$= \frac{\frac{b^4\sin(c+dx)}{7} - \cos(c+dx)^3\,(a\,b^3 - a^3\,b) - \cos(c+dx)^2\left(\frac{8\,b^4\sin(c+dx)}{35} - \frac{6\,a^2\,b^2\sin(c+dx)}{5}\right) + \cos(c+dx)^4\left(\frac{\mathrm{s}}{}\right.}{d}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))^4/cos(c + d*x)^8,x)
```

output
```
((b^4*sin(c + d*x))/7 - cos(c + d*x)^3*(a*b^3 - a^3*b) - cos(c + d*x)^2*((
8*b^4*sin(c + d*x))/35 - (6*a^2*b^2*sin(c + d*x))/5) + cos(c + d*x)^4*((a^
4*sin(c + d*x))/3 + (b^4*sin(c + d*x))/35 - (2*a^2*b^2*sin(c + d*x))/5) +
cos(c + d*x)^6*((2*a^4*sin(c + d*x))/3 + (2*b^4*sin(c + d*x))/35 - (4*a^2*
b^2*sin(c + d*x))/5) + (2*a*b^3*cos(c + d*x))/3)/(d*cos(c + d*x)^7)
```

3.88 $\int \sec^9(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

3.88.1 Optimal result

Integrand size = 28, antiderivative size = 330

$$\begin{aligned}
&\int \sec^9(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx \\
&= \frac{3a^4\operatorname{arctanh}(\sin(c+dx))}{8d} - \frac{3a^2b^2\operatorname{arctanh}(\sin(c+dx))}{8d} \\
&\quad + \frac{3b^4\operatorname{arctanh}(\sin(c+dx))}{128d} + \frac{4a^3b\sec^5(c+dx)}{5d} - \frac{4ab^3\sec^5(c+dx)}{5d} \\
&\quad + \frac{4ab^3\sec^7(c+dx)}{7d} + \frac{3a^4\sec(c+dx)\tan(c+dx)}{8d} \\
&\quad - \frac{3a^2b^2\sec(c+dx)\tan(c+dx)}{8d} + \frac{3b^4\sec(c+dx)\tan(c+dx)}{128d} \\
&\quad + \frac{a^4\sec^3(c+dx)\tan(c+dx)}{4d} - \frac{a^2b^2\sec^3(c+dx)\tan(c+dx)}{4d} \\
&\quad + \frac{b^4\sec^3(c+dx)\tan(c+dx)}{64d} + \frac{a^2b^2\sec^5(c+dx)\tan(c+dx)}{d} \\
&\quad - \frac{b^4\sec^5(c+dx)\tan(c+dx)}{16d} + \frac{b^4\sec^5(c+dx)\tan^3(c+dx)}{8d}
\end{aligned}$$

output
```
3/8*a^4*arctanh(sin(d*x+c))/d-3/8*a^2*b^2*arctanh(sin(d*x+c))/d+3/128*b^4*
arctanh(sin(d*x+c))/d+4/5*a^3*b*sec(d*x+c)^5/d-4/5*a*b^3*sec(d*x+c)^5/d+4/
7*a*b^3*sec(d*x+c)^7/d+3/8*a^4*sec(d*x+c)*tan(d*x+c)/d-3/8*a^2*b^2*sec(d*x
+c)*tan(d*x+c)/d+3/128*b^4*sec(d*x+c)*tan(d*x+c)/d+1/4*a^4*sec(d*x+c)^3*ta
n(d*x+c)/d-1/4*a^2*b^2*sec(d*x+c)^3*tan(d*x+c)/d+1/64*b^4*sec(d*x+c)^3*tan
(d*x+c)/d+a^2*b^2*sec(d*x+c)^5*tan(d*x+c)/d-1/16*b^4*sec(d*x+c)^5*tan(d*x+
c)/d+1/8*b^4*sec(d*x+c)^5*tan(d*x+c)^3/d
```

3.88.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1732 vs. $2(330) = 660$.

Time = 7.98 (sec) , antiderivative size = 1732, normalized size of antiderivative = 5.25

$$\int \sec^9(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \text{Too large to display}$$

input
```
Integrate[Sec[c + d*x]^9*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

output
```
(a*b*(42*a^2 - 17*b^2)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(140*d*(a*Co
s[c + d*x] + b*Sin[c + d*x])^4) - (3*(16*a^4 - 16*a^2*b^2 + b^4)*Cos[c + d
*x]^4*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^4)/(12
8*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (3*(16*a^4 - 16*a^2*b^2 + b^4)*
Cos[c + d*x]^4*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x
])^4)/(128*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (b^4*Cos[c + d*x]^4*(a
 + b*Tan[c + d*x])^4)/(128*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^8*(a*Co
s[c + d*x] + b*Sin[c + d*x])^4) + ((56*a^2*b^2 + 16*a*b^3 - 7*b^4)*Cos[c +
 d*x]^4*(a + b*Tan[c + d*x])^4)/(448*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2
])^6*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + ((560*a^4 + 896*a^3*b - 256*a*
b^3 - 35*b^4)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(8960*d*(Cos[(c + d*x
)/2] - Sin[(c + d*x)/2])^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + ((1680*a
^4 + 1344*a^3*b - 1680*a^2*b^2 - 544*a*b^3 + 105*b^4)*Cos[c + d*x]^4*(a +
b*Tan[c + d*x])^4)/(8960*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2*(a*Cos[
c + d*x] + b*Sin[c + d*x])^4) + (a*b^3*Cos[c + d*x]^4*Sin[(c + d*x)/2]*(a
+ b*Tan[c + d*x])^4)/(14*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^7*(a*Cos[
c + d*x] + b*Sin[c + d*x])^4) - (b^4*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4
)/(128*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8*(a*Cos[c + d*x] + b*Sin[c
 + d*x])^4) - (a*b^3*Cos[c + d*x]^4*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^
4)/(14*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^7*(a*Cos[c + d*x] + b*Si...
```

3.88.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^9(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\cos(c+dx)+b\sin(c+dx))^4}{\cos(c+dx)^9}dx$$

$$\Big\downarrow \text{3569}$$

$$\int \left(a^4\sec^5(c+dx)+4a^3b\tan(c+dx)\sec^5(c+dx)+6a^2b^2\tan^2(c+dx)\sec^5(c+dx)+4ab^3\tan^3(c+dx)\sec^5(c+\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{3a^4\text{arctanh}(\sin(c+dx))}{8d}+\frac{a^4\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3a^4\tan(c+dx)\sec(c+dx)}{8d}+\frac{4a^3b\sec^5(c+dx)}{5d}-\frac{3a^2b^2\text{arctanh}(\sin(c+dx))}{8d}+\frac{a^2b^2\tan(c+dx)\sec^5(c+dx)}{d}-\frac{a^2b^2\tan(c+dx)\sec^3(c+dx)}{4d}-\frac{3a^2b^2\tan(c+dx)\sec(c+dx)}{8d}+\frac{4ab^3\sec^7(c+dx)}{7d}-\frac{4ab^3\sec^5(c+dx)}{5d}+\frac{3b^4\text{arctanh}(\sin(c+dx))}{128d}+\frac{b^4\tan^3(c+dx)\sec^5(c+dx)}{8d}-\frac{b^4\tan(c+dx)\sec^5(c+dx)}{16d}+\frac{b^4\tan(c+dx)\sec^3(c+dx)}{64d}+\frac{3b^4\tan(c+dx)\sec(c+dx)}{128d}$$

input
```
Int[Sec[c + d*x]^9*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

output
```
(3*a^4*ArcTanh[Sin[c + d*x]])/(8*d) - (3*a^2*b^2*ArcTanh[Sin[c + d*x]])/(8
*d) + (3*b^4*ArcTanh[Sin[c + d*x]])/(128*d) + (4*a^3*b*Sec[c + d*x]^5)/(5*
d) - (4*a*b^3*Sec[c + d*x]^5)/(5*d) + (4*a*b^3*Sec[c + d*x]^7)/(7*d) + (3*
a^4*Sec[c + d*x]*Tan[c + d*x])/(8*d) - (3*a^2*b^2*Sec[c + d*x]*Tan[c + d*x
])/(8*d) + (3*b^4*Sec[c + d*x]*Tan[c + d*x])/(128*d) + (a^4*Sec[c + d*x]^3
*Tan[c + d*x])/(4*d) - (a^2*b^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (b^4*
Sec[c + d*x]^3*Tan[c + d*x])/(64*d) + (a^2*b^2*Sec[c + d*x]^5*Tan[c + d*x]
)/d - (b^4*Sec[c + d*x]^5*Tan[c + d*x])/(16*d) + (b^4*Sec[c + d*x]^5*Tan[c
 + d*x]^3)/(8*d)
```

3.88. $\int \sec^9(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

3.88.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

3.88.4 Maple [A] (verified)

Time = 1.95 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.92

method	result
parts	$\frac{a^4\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}+\frac{b^4\left(\frac{\sin(dx+c)^5}{8\cos(dx+c)^8}+\frac{\sin(dx+c)^5}{16\cos(dx+c)^6}+\ldots\right.}{\ldots}$
derivativedivides	$\frac{a^4\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{4a^3b}{5\cos(dx+c)^5}+6a^2b^2\left(\frac{\sin(dx+c)^3}{6\cos(dx+c)^6}+\ldots\right.}{\ldots}$
default	$\frac{a^4\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{4a^3b}{5\cos(dx+c)^5}+6a^2b^2\left(\frac{\sin(dx+c)^3}{6\cos(dx+c)^6}+\ldots\right.}{\ldots}$
parallelrisch	$\frac{-13440\left(a^4-a^2b^2+\frac{1}{16}b^4\right)\left(\frac{35}{8}+\frac{\cos(8dx+8c)}{8}+\cos(6dx+6c)+\frac{7\cos(4dx+4c)}{2}+7\cos(2dx+2c)\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+13440\ldots}{\ldots}$
risch	$-\frac{ie^{i(dx+c)}\left(-1680a^4-105b^4+1680a^2b^2+28560a^4e^{10i(dx+c)}-805b^4e^{2i(dx+c)}-23485b^4e^{6i(dx+c)}-11655b^4e^{10i(dx+c)}-\ldots\right.}{\ldots}$

input
```
int(sec(d*x+c)^9*(cos(d*x+c)*a+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
a^4/d*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+ta
n(d*x+c)))+b^4/d*(1/8*sin(d*x+c)^5/cos(d*x+c)^8+1/16*sin(d*x+c)^5/cos(d*x+
c)^6+1/64*sin(d*x+c)^5/cos(d*x+c)^4-1/128*sin(d*x+c)^5/cos(d*x+c)^2-1/128*
sin(d*x+c)^3-3/128*sin(d*x+c)+3/128*ln(sec(d*x+c)+tan(d*x+c)))+4*a*b^3/d*(
1/7*sec(d*x+c)^7-1/5*sec(d*x+c)^5)+4/5*a^3*b*sec(d*x+c)^5/d+6*a^2*b^2/d*(1
/6*sin(d*x+c)^3/cos(d*x+c)^6+1/8*sin(d*x+c)^3/cos(d*x+c)^4+1/16*sin(d*x+c)
^3/cos(d*x+c)^2+1/16*sin(d*x+c)-1/16*ln(sec(d*x+c)+tan(d*x+c)))
```

3.88.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.65

$$\int \sec^9(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$
$$=\frac{105\,(16\,a^4-16\,a^2b^2+b^4)\cos{(dx+c)}^8\log{(\sin{(dx+c)}+1)}-105\,(16\,a^4-16\,a^2b^2+b^4)\cos{(dx+c)}^8\log}{}$$

input
```
integrate(sec(d*x+c)^9*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas"
)
```

output
```
1/8960*(105*(16*a^4 - 16*a^2*b^2 + b^4)*cos(d*x + c)^8*log(sin(d*x + c) +
1) - 105*(16*a^4 - 16*a^2*b^2 + b^4)*cos(d*x + c)^8*log(-sin(d*x + c) + 1)
 + 5120*a*b^3*cos(d*x + c) + 7168*(a^3*b - a*b^3)*cos(d*x + c)^3 + 70*(3*(
16*a^4 - 16*a^2*b^2 + b^4)*cos(d*x + c)^6 + 2*(16*a^4 - 16*a^2*b^2 + b^4)*
cos(d*x + c)^4 + 16*b^4 + 8*(16*a^2*b^2 - 3*b^4)*cos(d*x + c)^2)*sin(d*x +
 c))/(d*cos(d*x + c)^8)
```

3.88.6 Sympy [F(-1)]

Timed out.

$$\int \sec^9(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx=\text{Timed out}$$

input
```
integrate(sec(d*x+c)**9*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

output
```
Timed out
```

3.88.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 322, normalized size of antiderivative = 0.98

$$\int \sec^9(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx=$$
$$-\frac{35\,b^4\left(\frac{2\left(3\,\sin(dx+c)^7-11\,\sin(dx+c)^5-11\,\sin(dx+c)^3+3\,\sin(dx+c)\right)}{\sin(dx+c)^8-4\,\sin(dx+c)^6+6\,\sin(dx+c)^4-4\,\sin(dx+c)^2+1}-3\,\log{(\sin{(dx+c)}+1)}+3\,\log{(\sin{(dx+c)}-}\right.}{}$$

input
```
integrate(sec(d*x+c)^9*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima"
)
```

output
```
-1/8960*(35*b^4*(2*(3*sin(d*x + c)^7 - 11*sin(d*x + c)^5 - 11*sin(d*x + c)
^3 + 3*sin(d*x + c))/(sin(d*x + c)^8 - 4*sin(d*x + c)^6 + 6*sin(d*x + c)^4
 - 4*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) -
1)) - 560*a^2*b^2*(2*(3*sin(d*x + c)^5 - 8*sin(d*x + c)^3 - 3*sin(d*x + c)
)/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 3*log(sin(d
*x + c) + 1) + 3*log(sin(d*x + c) - 1)) + 560*a^4*(2*(3*sin(d*x + c)^3 - 5
*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c
) + 1) + 3*log(sin(d*x + c) - 1)) - 7168*a^3*b/cos(d*x + c)^5 + 1024*(7*co
s(d*x + c)^2 - 5)*a*b^3/cos(d*x + c)^7)/d
```

3.88.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 706 vs. $2(302) = 604$.

Time = 0.47 (sec) , antiderivative size = 706, normalized size of antiderivative = 2.14

$$\int \sec^9(c + dx)(a\cos(c + dx) + b\sin(c + dx))^4 \, dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^9*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

output
```
1/4480*(105*(16*a^4 - 16*a^2*b^2 + b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1))
 - 105*(16*a^4 - 16*a^2*b^2 + b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*
(2800*a^4*tan(1/2*d*x + 1/2*c)^15 + 1680*a^2*b^2*tan(1/2*d*x + 1/2*c)^15 -
 105*b^4*tan(1/2*d*x + 1/2*c)^15 - 17920*a^3*b*tan(1/2*d*x + 1/2*c)^14 - 9
520*a^4*tan(1/2*d*x + 1/2*c)^13 + 22960*a^2*b^2*tan(1/2*d*x + 1/2*c)^13 +
805*b^4*tan(1/2*d*x + 1/2*c)^13 + 53760*a^3*b*tan(1/2*d*x + 1/2*c)^12 - 35
840*a*b^3*tan(1/2*d*x + 1/2*c)^12 + 11760*a^4*tan(1/2*d*x + 1/2*c)^11 - 72
80*a^2*b^2*tan(1/2*d*x + 1/2*c)^11 + 11655*b^4*tan(1/2*d*x + 1/2*c)^11 - 8
9600*a^3*b*tan(1/2*d*x + 1/2*c)^10 - 5040*a^4*tan(1/2*d*x + 1/2*c)^9 - 173
60*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 + 23485*b^4*tan(1/2*d*x + 1/2*c)^9 + 125
440*a^3*b*tan(1/2*d*x + 1/2*c)^8 - 35840*a*b^3*tan(1/2*d*x + 1/2*c)^8 - 50
40*a^4*tan(1/2*d*x + 1/2*c)^7 - 17360*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 + 234
85*b^4*tan(1/2*d*x + 1/2*c)^7 - 111104*a^3*b*tan(1/2*d*x + 1/2*c)^6 + 5734
4*a*b^3*tan(1/2*d*x + 1/2*c)^6 + 11760*a^4*tan(1/2*d*x + 1/2*c)^5 - 7280*a
^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 11655*b^4*tan(1/2*d*x + 1/2*c)^5 + 46592*a
^3*b*tan(1/2*d*x + 1/2*c)^4 + 7168*a*b^3*tan(1/2*d*x + 1/2*c)^4 - 9520*a^4
*tan(1/2*d*x + 1/2*c)^3 + 22960*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 805*b^4*t
an(1/2*d*x + 1/2*c)^3 - 10752*a^3*b*tan(1/2*d*x + 1/2*c)^2 + 8192*a*b^3*ta
n(1/2*d*x + 1/2*c)^2 + 2800*a^4*tan(1/2*d*x + 1/2*c) + 1680*a^2*b^2*tan(1/
2*d*x + 1/2*c) - 105*b^4*tan(1/2*d*x + 1/2*c) + 3584*a^3*b - 1024*a*b^3...
```

3.88.9 Mupad [B] (verification not implemented)

Time = 26.24 (sec) , antiderivative size = 566, normalized size of antiderivative = 1.72

$$\int \sec^9(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$

$$= \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}\left(\frac{5a^4}{4}+\frac{3a^2b^2}{4}-\frac{3b^4}{64}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}\left(-\frac{17a^4}{4}+\frac{41a^2b^2}{4}+\frac{23b^4}{64}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}\left(-\frac{17a^4}{4}+\right.}{}$$

$$+\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(\frac{3a^4}{4}-\frac{3a^2b^2}{4}+\frac{3b^4}{64}\right)}{d}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))^4/cos(c + d*x)^9,x)
```

output
```
(tan(c/2 + (d*x)/2)^15*((5*a^4)/4 - (3*b^4)/64 + (3*a^2*b^2)/4) + tan(c/2
+ (d*x)/2)^3*((23*b^4)/64 - (17*a^4)/4 + (41*a^2*b^2)/4) + tan(c/2 + (d*x)
/2)^13*((23*b^4)/64 - (17*a^4)/4 + (41*a^2*b^2)/4) + tan(c/2 + (d*x)/2)^5*
((21*a^4)/4 + (333*b^4)/64 - (13*a^2*b^2)/4) + tan(c/2 + (d*x)/2)^11*((21*
a^4)/4 + (333*b^4)/64 - (13*a^2*b^2)/4) - tan(c/2 + (d*x)/2)^7*((9*a^4)/4
- (671*b^4)/64 + (31*a^2*b^2)/4) - tan(c/2 + (d*x)/2)^9*((9*a^4)/4 - (671*
b^4)/64 + (31*a^2*b^2)/4) - (16*a*b^3)/35 + (8*a^3*b)/5 + tan(c/2 + (d*x)/
2)*((5*a^4)/4 - (3*b^4)/64 + (3*a^2*b^2)/4) - tan(c/2 + (d*x)/2)^12*(16*a*
b^3 - 24*a^3*b) - tan(c/2 + (d*x)/2)^8*(16*a*b^3 - 56*a^3*b) + tan(c/2 + (
d*x)/2)^4*((16*a*b^3)/5 + (104*a^3*b)/5) + tan(c/2 + (d*x)/2)^2*((128*a*b^
3)/35 - (24*a^3*b)/5) + tan(c/2 + (d*x)/2)^6*((128*a*b^3)/5 - (248*a^3*b)/
5) - 40*a^3*b*tan(c/2 + (d*x)/2)^10 - 8*a^3*b*tan(c/2 + (d*x)/2)^14)/(d*(2
8*tan(c/2 + (d*x)/2)^4 - 8*tan(c/2 + (d*x)/2)^2 - 56*tan(c/2 + (d*x)/2)^6
+ 70*tan(c/2 + (d*x)/2)^8 - 56*tan(c/2 + (d*x)/2)^10 + 28*tan(c/2 + (d*x)/
2)^12 - 8*tan(c/2 + (d*x)/2)^14 + tan(c/2 + (d*x)/2)^16 + 1)) + (atanh(tan
(c/2 + (d*x)/2))*((3*a^4)/4 + (3*b^4)/64 - (3*a^2*b^2)/4))/d
```

3.88. $\int \sec^9(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

3.89 $\int \sec^{10}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

3.89.1 Optimal result

Integrand size = 28, antiderivative size = 201

$$\begin{aligned}\int \sec^{10}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx &= \frac{a^4\tan(c+dx)}{d}+\frac{2a^3b\tan^2(c+dx)}{d}+\frac{2a^2(a^2+3b^2)\tan^3(c+dx)}{3d}\\ &+\frac{ab(2a^2+b^2)\tan^4(c+dx)}{d}+\frac{(a^4+12a^2b^2+b^4)\tan^5(c+dx)}{5d}\\ &+\frac{2ab(a^2+2b^2)\tan^6(c+dx)}{3d}+\frac{2b^2(3a^2+b^2)\tan^7(c+dx)}{7d}\\ &+\frac{ab^3\tan^8(c+dx)}{2d}+\frac{b^4\tan^9(c+dx)}{9d}\end{aligned}$$

output
```
a^4*tan(d*x+c)/d+2*a^3*b*tan(d*x+c)^2/d+2/3*a^2*(a^2+3*b^2)*tan(d*x+c)^3/d
+a*b*(2*a^2+b^2)*tan(d*x+c)^4/d+1/5*(a^4+12*a^2*b^2+b^4)*tan(d*x+c)^5/d+2/
3*a*b*(a^2+2*b^2)*tan(d*x+c)^6/d+2/7*b^2*(3*a^2+b^2)*tan(d*x+c)^7/d+1/2*a*
b^3*tan(d*x+c)^8/d+1/9*b^4*tan(d*x+c)^9/d
```

3.89.2 Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.57

$$\int \sec^{10}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$
$$=\frac{\frac{1}{5}(a^2+b^2)^2\,(a+b\tan(c+dx))^5-\frac{2}{3}a(a^2+b^2)\,(a+b\tan(c+dx))^6+\frac{2}{7}(3a^2+b^2)\,(a+b\tan(c+dx))^7-\frac{1}{2}\cdots}{b^5d}$$

input
```
Integrate[Sec[c + d*x]^10*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

output
```
(((a^2 + b^2)^2*(a + b*Tan[c + d*x])^5)/5 - (2*a*(a^2 + b^2)*(a + b*Tan[c
+ d*x])^6)/3 + (2*(3*a^2 + b^2)*(a + b*Tan[c + d*x])^7)/7 - (a*(a + b*Tan[
c + d*x])^8)/2 + (a + b*Tan[c + d*x])^9/9)/(b^5*d)
```

3.89.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3567, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{10}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a\cos(c+dx)+b\sin(c+dx))^4}{\cos(c+dx)^{10}}dx$$

$$\Big\downarrow \; 3567$$

$$-\frac{\int (b+a\cot(c+dx))^4\left(\cot^2(c+dx)+1\right)^2\tan^{10}(c+dx)d\cot(c+dx)}{d}$$

$$\Big\downarrow \; 522$$

$$-\frac{\int \left(b^4\tan^{10}(c+dx)+4ab^3\tan^9(c+dx)+2\left(b^4+3a^2b^2\right)\tan^8(c+dx)+4ab\left(a^2+2b^2\right)\tan^7(c+dx)+\left(a^4+12b\cdots\right.\right.}{}$$

$$\Big\downarrow \; 2009$$

$$-\frac{-a^4\tan(c+dx)-2a^3b\tan^2(c+dx)-\frac{2}{7}b^2(3a^2+b^2)\tan^7(c+dx)-\frac{2}{3}ab(a^2+2b^2)\tan^6(c+dx)-ab(2a^2+b^2)$$

input
```
Int[Sec[c + d*x]^10*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

output
```
-((-(a^4*Tan[c + d*x]) - 2*a^3*b*Tan[c + d*x]^2 - (2*a^2*(a^2 + 3*b^2)*Tan
[c + d*x]^3)/3 - a*b*(2*a^2 + b^2)*Tan[c + d*x]^4 - ((a^4 + 12*a^2*b^2 + b
^4)*Tan[c + d*x]^5)/5 - (2*a*b*(a^2 + 2*b^2)*Tan[c + d*x]^6)/3 - (2*b^2*(3
*a^2 + b^2)*Tan[c + d*x]^7)/7 - (a*b^3*Tan[c + d*x]^8)/2 - (b^4*Tan[c + d*
x]^9)/9)/d)
```

3.89.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3567
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[x^m*((b
+ a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a,
b, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[
n, 0] && GtQ[m, 1])
```

3.89.4 Maple [A] (verified)

Time = 1.90 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.06

method	result
parts	$-\frac{a^4\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)}{d}+\frac{b^4\left(\frac{\sin(dx+c)^5}{9\cos(dx+c)^9}+\frac{4\sin(dx+c)^5}{63\cos(dx+c)^7}+\frac{8\sin(dx+c)^5}{315\cos(dx+c)^5}\right)}{d}+\frac{4a\,b^3\left(\sec\right.}{}$
derivativedivides	$\frac{-a^4\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)+\frac{2a^3b}{3\cos(dx+c)^6}+6a^2b^2\left(\frac{\sin(dx+c)^3}{7\cos(dx+c)^7}+\frac{4\sin(dx+c)^3}{35\cos(dx+c)^5}+\frac{8\sin(dx+c)^3}{105\cos(dx+c)^3}\right)}{d}$
default	$\frac{-a^4\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)+\frac{2a^3b}{3\cos(dx+c)^6}+6a^2b^2\left(\frac{\sin(dx+c)^3}{7\cos(dx+c)^7}+\frac{4\sin(dx+c)^3}{35\cos(dx+c)^5}+\frac{8\sin(dx+c)^3}{105\cos(dx+c)^3}\right)}{d}$
risch	$16i\left(21a^4+b^4-18a^2b^2+945a^4\mathrm{e}^{10i(dx+c)}+9b^4\mathrm{e}^{2i(dx+c)}-126b^4\mathrm{e}^{6i(dx+c)}-315b^4\mathrm{e}^{10i(dx+c)}-1260a^2b^2\mathrm{e}^{12i(dx+c)}-2520ia\right.$
parallelrisch	$-\frac{2\left(a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{16}-4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{15}a^3b+8\left(a^2b^2-\frac{2}{3}a^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{14}+4\left(3a^3b-2a\,b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{13}+\frac{4\left(19a^4-1\right.}{}\right.}{}$

input
```
int(sec(d*x+c)^10*(cos(d*x+c)*a+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
-a^4/d*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+b^4/d*(1/9*si
n(d*x+c)^5/cos(d*x+c)^9+4/63*sin(d*x+c)^5/cos(d*x+c)^7+8/315*sin(d*x+c)^5/
cos(d*x+c)^5)+4*a*b^3/d*(1/8*sec(d*x+c)^8-1/6*sec(d*x+c)^6)+2/3*a^3*b/d*se
c(d*x+c)^6+6*a^2*b^2/d*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*sin(d*x+c)^3/co
s(d*x+c)^5+8/105*sin(d*x+c)^3/cos(d*x+c)^3)
```

3.89.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.83

$$\int \sec^{10}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$
$$=\frac{315\,ab^3\cos\left(dx+c\right)+420\left(a^3b-ab^3\right)\cos\left(dx+c\right)^3+2\left(8\left(21\,a^4-18\,a^2b^2+b^4\right)\cos\left(dx+c\right)^8+4\left(21\,a^4-\right.\right.}{6}$$

input
```
integrate(sec(d*x+c)^10*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas
")
```

output
```
1/630*(315*a*b^3*cos(d*x + c) + 420*(a^3*b - a*b^3)*cos(d*x + c)^3 + 2*(8*
(21*a^4 - 18*a^2*b^2 + b^4)*cos(d*x + c)^8 + 4*(21*a^4 - 18*a^2*b^2 + b^4)
*cos(d*x + c)^6 + 3*(21*a^4 - 18*a^2*b^2 + b^4)*cos(d*x + c)^4 + 35*b^4 +
10*(27*a^2*b^2 - 5*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^9)
```

3.89.6 Sympy [F(-1)]

Timed out.

$$\int \sec^{10}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**10*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

output
```
Timed out
```

3.89.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.96

$$\int \sec^{10}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$

$$= \frac{42\left(3\tan\left(dx+c\right)^{5}+10\tan\left(dx+c\right)^{3}+15\tan\left(dx+c\right)\right)a^{4}+36\left(15\tan\left(dx+c\right)^{7}+42\tan\left(dx+c\right)^{5}+\cdots\right.}{\cdots}$$

input
```
integrate(sec(d*x+c)^10*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima
")
```

output
```
1/630*(42*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^4 + 3
6*(15*tan(d*x + c)^7 + 42*tan(d*x + c)^5 + 35*tan(d*x + c)^3)*a^2*b^2 + 2*
(35*tan(d*x + c)^9 + 90*tan(d*x + c)^7 + 63*tan(d*x + c)^5)*b^4 + 105*(4*s
in(d*x + c)^2 - 1)*a*b^3/(sin(d*x + c)^8 - 4*sin(d*x + c)^6 + 6*sin(d*x +
c)^4 - 4*sin(d*x + c)^2 + 1) - 420*a^3*b/(sin(d*x + c)^2 - 1)^3)/d
```

3.89.8 Giac [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.06

$$\int \sec^{10}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$
$$=\frac{70\,b^4\tan{(dx+c)}^9+315\,ab^3\tan{(dx+c)}^8+540\,a^2b^2\tan{(dx+c)}^7+180\,b^4\tan{(dx+c)}^7+420\,a^3b\tan{(d}}{}$$

input
```
integrate(sec(d*x+c)^10*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

output
```
1/630*(70*b^4*tan(d*x + c)^9 + 315*a*b^3*tan(d*x + c)^8 + 540*a^2*b^2*tan(
d*x + c)^7 + 180*b^4*tan(d*x + c)^7 + 420*a^3*b*tan(d*x + c)^6 + 840*a*b^3
*tan(d*x + c)^6 + 126*a^4*tan(d*x + c)^5 + 1512*a^2*b^2*tan(d*x + c)^5 + 1
26*b^4*tan(d*x + c)^5 + 1260*a^3*b*tan(d*x + c)^4 + 630*a*b^3*tan(d*x + c)
^4 + 420*a^4*tan(d*x + c)^3 + 1260*a^2*b^2*tan(d*x + c)^3 + 1260*a^3*b*tan
(d*x + c)^2 + 630*a^4*tan(d*x + c))/d
```

3.89.9 Mupad [B] (verification not implemented)

Time = 26.00 (sec) , antiderivative size = 447, normalized size of antiderivative = 2.22

$$\int \sec^{10}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx =$$
$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\left(\frac{152\,a^4}{5}-\frac{96\,a^2\,b^2}{5}+\frac{32\,b^4}{5}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}\left(\frac{152\,a^4}{5}-\frac{96\,a^2\,b^2}{5}+\frac{32\,b^4}{5}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\left(-\frac{288}{5}\right.}{}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))^4/cos(c + d*x)^10,x)
```

output
```
-(tan(c/2 + (d*x)/2)^5*((152*a^4)/5 + (32*b^4)/5 - (96*a^2*b^2)/5) + tan(c
/2 + (d*x)/2)^13*((152*a^4)/5 + (32*b^4)/5 - (96*a^2*b^2)/5) + tan(c/2 + (
d*x)/2)^7*((384*b^4)/35 - (288*a^4)/5 + (1488*a^2*b^2)/35) + tan(c/2 + (d*
x)/2)^11*((384*b^4)/35 - (288*a^4)/5 + (1488*a^2*b^2)/35) + tan(c/2 + (d*x
)/2)^9*((1076*a^4)/15 + (6976*b^4)/315 - (2752*a^2*b^2)/35) + 2*a^4*tan(c/
2 + (d*x)/2)^17 - tan(c/2 + (d*x)/2)^3*((32*a^4)/3 - 16*a^2*b^2) - tan(c/2
 + (d*x)/2)^15*((32*a^4)/3 - 16*a^2*b^2) + 2*a^4*tan(c/2 + (d*x)/2) + tan(
c/2 + (d*x)/2)^4*(16*a*b^3 - 24*a^3*b) - tan(c/2 + (d*x)/2)^14*(16*a*b^3 -
 24*a^3*b) + tan(c/2 + (d*x)/2)^8*(32*a*b^3 - 88*a^3*b) - tan(c/2 + (d*x)/
2)^10*(32*a*b^3 - 88*a^3*b) + tan(c/2 + (d*x)/2)^6*((16*a*b^3)/3 + (152*a^
3*b)/3) - tan(c/2 + (d*x)/2)^12*((16*a*b^3)/3 + (152*a^3*b)/3) + 8*a^3*b*t
an(c/2 + (d*x)/2)^2 - 8*a^3*b*tan(c/2 + (d*x)/2)^16)/(d*(tan(c/2 + (d*x)/2
)^2 - 1)^9)
```

3.89. $\int \sec^{10}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

3.90 $\int \sec^{11}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

3.90.1 Optimal result

Integrand size = 28, antiderivative size = 408

$$\begin{aligned}
&\int \sec^{11}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx \\
&= \frac{5a^4\operatorname{arctanh}(\sin(c+dx))}{16d}-\frac{15a^2b^2\operatorname{arctanh}(\sin(c+dx))}{64d}+\frac{3b^4\operatorname{arctanh}(\sin(c+dx))}{256d} \\
&\quad+\frac{4a^3b\sec^7(c+dx)}{7d}-\frac{4ab^3\sec^7(c+dx)}{7d}+\frac{4ab^3\sec^9(c+dx)}{9d} \\
&\quad+\frac{5a^4\sec(c+dx)\tan(c+dx)}{16d}-\frac{15a^2b^2\sec(c+dx)\tan(c+dx)}{64d} \\
&\quad+\frac{3b^4\sec(c+dx)\tan(c+dx)}{256d}+\frac{5a^4\sec^3(c+dx)\tan(c+dx)}{24d} \\
&\quad-\frac{5a^2b^2\sec^3(c+dx)\tan(c+dx)}{32d}+\frac{b^4\sec^3(c+dx)\tan(c+dx)}{128d} \\
&\quad+\frac{a^4\sec^5(c+dx)\tan(c+dx)}{6d}-\frac{a^2b^2\sec^5(c+dx)\tan(c+dx)}{8d} \\
&\quad+\frac{b^4\sec^5(c+dx)\tan(c+dx)}{160d}+\frac{3a^2b^2\sec^7(c+dx)\tan(c+dx)}{4d} \\
&\quad-\frac{3b^4\sec^7(c+dx)\tan(c+dx)}{80d}+\frac{b^4\sec^7(c+dx)\tan^3(c+dx)}{10d}
\end{aligned}$$

output
```
5/16*a^4*arctanh(sin(d*x+c))/d-15/64*a^2*b^2*arctanh(sin(d*x+c))/d+3/256*b
^4*arctanh(sin(d*x+c))/d+4/7*a^3*b*sec(d*x+c)^7/d-4/7*a*b^3*sec(d*x+c)^7/d
+4/9*a*b^3*sec(d*x+c)^9/d+5/16*a^4*sec(d*x+c)*tan(d*x+c)/d-15/64*a^2*b^2*s
ec(d*x+c)*tan(d*x+c)/d+3/256*b^4*sec(d*x+c)*tan(d*x+c)/d+5/24*a^4*sec(d*x+
c)^3*tan(d*x+c)/d-5/32*a^2*b^2*sec(d*x+c)^3*tan(d*x+c)/d+1/128*b^4*sec(d*x
+c)^3*tan(d*x+c)/d+1/6*a^4*sec(d*x+c)^5*tan(d*x+c)/d-1/8*a^2*b^2*sec(d*x+c
)^5*tan(d*x+c)/d+1/160*b^4*sec(d*x+c)^5*tan(d*x+c)/d+3/4*a^2*b^2*sec(d*x+c
)^7*tan(d*x+c)/d-3/80*b^4*sec(d*x+c)^7*tan(d*x+c)/d+1/10*b^4*sec(d*x+c)^7*
tan(d*x+c)^3/d
```

3.90.2 Mathematica [A] (verified)

Time = 3.16 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.59

$$\int \sec^{11}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$
$$= \frac{-80640(80a^4-60a^2b^2+3b^4)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c\right.\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]^11*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

output
```
(-80640*(80*a^4 - 60*a^2*b^2 + 3*b^4)*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x
)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 3*Sec[c + d*x]^10*(983
040*a*b*(a^2 - b^2)*Cos[3*(c + d*x)] + 420*(1552*a^4 + 1908*a^2*b^2 - 505*
b^4)*Sin[3*(c + d*x)] + 7*(80*a^4 - 60*a^2*b^2 + 3*b^4)*(628*Sin[5*(c + d*
x)] + 145*Sin[7*(c + d*x)] + 15*Sin[9*(c + d*x)])) + 10*Sec[c + d*x]^9*(32
768*a*b*(27*a^2 + b^2) + 189*(592*a^4 + 1604*a^2*b^2 + 739*b^4)*Tan[c + d*
x]))/(20643840*d)
```

3.90.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{11}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a\cos(c+dx)+b\sin(c+dx))^4}{\cos(c+dx)^{11}}dx$$

$$\Big\downarrow \text{ 3569}$$

$$\int \left(a^4\sec^7(c+dx)+4a^3b\tan(c+dx)\sec^7(c+dx)+6a^2b^2\tan^2(c+dx)\sec^7(c+dx)+4ab^3\tan^3(c+dx)\sec^7(c+\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{5a^4\text{arctanh}(\sin(c+dx))}{16d}+\frac{a^4\tan(c+dx)\sec^5(c+dx)}{6d}+\frac{5a^4\tan(c+dx)\sec^3(c+dx)}{24d}+\frac{5a^4\tan(c+dx)\sec(c+dx)}{16d}+\frac{4a^3b\sec^7(c+dx)}{7d}-\frac{15a^2b^2\text{arctanh}(\sin(c+dx))}{64d}+\frac{3a^2b^2\tan(c+dx)\sec^7(c+dx)}{4d}-\frac{a^2b^2\tan(c+dx)\sec^5(c+dx)}{8d}-\frac{5a^2b^2\tan(c+dx)\sec^3(c+dx)}{32d}-\frac{15a^2b^2\tan(c+dx)\sec(c+dx)}{64d}+\frac{4ab^3\sec^9(c+dx)}{9d}-\frac{4ab^3\sec^7(c+dx)}{7d}+\frac{3b^4\text{arctanh}(\sin(c+dx))}{256d}+\frac{b^4\tan^3(c+dx)\sec^7(c+dx)}{10d}-\frac{3b^4\tan(c+dx)\sec^7(c+dx)}{80d}+\frac{b^4\tan(c+dx)\sec^5(c+dx)}{160d}+\frac{b^4\tan(c+dx)\sec^3(c+dx)}{128d}+\frac{3b^4\tan(c+dx)\sec(c+dx)}{256d}$$

input
```
Int[Sec[c + d*x]^11*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

output
```
(5*a^4*ArcTanh[Sin[c + d*x]])/(16*d) - (15*a^2*b^2*ArcTanh[Sin[c + d*x]])/
(64*d) + (3*b^4*ArcTanh[Sin[c + d*x]])/(256*d) + (4*a^3*b*Sec[c + d*x]^7)/
(7*d) - (4*a*b^3*Sec[c + d*x]^7)/(7*d) + (4*a*b^3*Sec[c + d*x]^9)/(9*d) +
(5*a^4*Sec[c + d*x]*Tan[c + d*x])/(16*d) - (15*a^2*b^2*Sec[c + d*x]*Tan[c
+ d*x])/(64*d) + (3*b^4*Sec[c + d*x]*Tan[c + d*x])/(256*d) + (5*a^4*Sec[c
+ d*x]^3*Tan[c + d*x])/(24*d) - (5*a^2*b^2*Sec[c + d*x]^3*Tan[c + d*x])/(3
2*d) + (b^4*Sec[c + d*x]^3*Tan[c + d*x])/(128*d) + (a^4*Sec[c + d*x]^5*Tan
[c + d*x])/(6*d) - (a^2*b^2*Sec[c + d*x]^5*Tan[c + d*x])/(8*d) + (b^4*Sec[
c + d*x]^5*Tan[c + d*x])/(160*d) + (3*a^2*b^2*Sec[c + d*x]^7*Tan[c + d*x])
/(4*d) - (3*b^4*Sec[c + d*x]^7*Tan[c + d*x])/(80*d) + (b^4*Sec[c + d*x]^7*
Tan[c + d*x]^3)/(10*d)
```

3.90. $\int \sec^{11}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

3.90.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

3.90.4 Maple [A] (verified)

Time = 2.51 (sec) , antiderivative size = 350, normalized size of antiderivative = 0.86

method	result
parts	$\frac{a^4\left(-\left(-\frac{\sec(dx+c)^5}{6}-\frac{5\sec(dx+c)^3}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)}{d}+\frac{b^4\left(\frac{\sin(dx+c)^5}{10\cos(dx+c)^{10}}+\cdots\right.}{\cdots}$
derivativedivides	$a^4\left(-\left(-\frac{\sec(dx+c)^5}{6}-\frac{5\sec(dx+c)^3}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)+\frac{4a^3b}{7\cos(dx+c)^7}+6a^2b^2\left(\cdots\right.$
default	$a^4\left(-\left(-\frac{\sec(dx+c)^5}{6}-\frac{5\sec(dx+c)^3}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)+\frac{4a^3b}{7\cos(dx+c)^7}+6a^2b^2\left(\cdots\right.$
parallelrisch	$-1134000\left(\frac{\cos(10dx+10c)}{45}+\frac{2\cos(8dx+8c)}{9}+\cos(6dx+6c)+\frac{8\cos(4dx+4c)}{3}+\frac{14\cos(2dx+2c)}{3}+\frac{14}{5}\right)\left(a^4-\frac{3}{4}a^2b^2+\frac{3}{80}b^4\right)\ln\left(\tan\cdots\right.$
risch	$-\frac{ie^{i(dx+c)}\left(-25200a^4-945b^4+18900a^2b^2+1118880a^4e^{10i(dx+c)}-9135b^4e^{2i(dx+c)}+636300b^4e^{6i(dx+c)}+1396710b^4e^{10\cdots}\right.}{\cdots}$

input
```
int(sec(d*x+c)^11*(cos(d*x+c)*a+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
a^4/d*(-(-1/6*sec(d*x+c)^5-5/24*sec(d*x+c)^3-5/16*sec(d*x+c))*tan(d*x+c)+5
/16*ln(sec(d*x+c)+tan(d*x+c)))+b^4/d*(1/10*sin(d*x+c)^5/cos(d*x+c)^10+1/16
*sin(d*x+c)^5/cos(d*x+c)^8+1/32*sin(d*x+c)^5/cos(d*x+c)^6+1/128*sin(d*x+c)
^5/cos(d*x+c)^4-1/256*sin(d*x+c)^5/cos(d*x+c)^2-1/256*sin(d*x+c)^3-3/256*s
in(d*x+c)+3/256*ln(sec(d*x+c)+tan(d*x+c)))+4*a*b^3/d*(1/9*sec(d*x+c)^9-1/7
*sec(d*x+c)^7)+4/7*a^3*b*sec(d*x+c)^7/d+6*a^2*b^2/d*(1/8*sin(d*x+c)^3/cos(
d*x+c)^8+5/48*sin(d*x+c)^3/cos(d*x+c)^6+5/64*sin(d*x+c)^3/cos(d*x+c)^4+5/1
28*sin(d*x+c)^3/cos(d*x+c)^2+5/128*sin(d*x+c)-5/128*ln(sec(d*x+c)+tan(d*x+
c)))
```

3.90.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.62

$$\int \sec^{11}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$
$$= \frac{315\left(80\,a^4-60\,a^2b^2+3\,b^4\right)\cos\left(dx+c\right)^{10}\log\left(\sin\left(dx+c\right)+1\right)-315\left(80\,a^4-60\,a^2b^2+3\,b^4\right)\cos\left(dx+c\right)}{}$$

input
```
integrate(sec(d*x+c)^11*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas
")
```

output
```
1/161280*(315*(80*a^4 - 60*a^2*b^2 + 3*b^4)*cos(d*x + c)^10*log(sin(d*x +
c) + 1) - 315*(80*a^4 - 60*a^2*b^2 + 3*b^4)*cos(d*x + c)^10*log(-sin(d*x +
 c) + 1) + 71680*a*b^3*cos(d*x + c) + 92160*(a^3*b - a*b^3)*cos(d*x + c)^3
 + 42*(15*(80*a^4 - 60*a^2*b^2 + 3*b^4)*cos(d*x + c)^8 + 10*(80*a^4 - 60*a
^2*b^2 + 3*b^4)*cos(d*x + c)^6 + 8*(80*a^4 - 60*a^2*b^2 + 3*b^4)*cos(d*x +
 c)^4 + 384*b^4 + 48*(60*a^2*b^2 - 11*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(
d*cos(d*x + c)^10)
```

3.90.6 Sympy [F(-1)]

Timed out.

$$\int \sec^{11}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**11*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

output
```
Timed out
```

3.90.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 382, normalized size of antiderivative = 0.94

$$\int \sec^{11}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx =$$

$$-\frac{63\,b^4\left(\frac{2\left(15\,\sin(dx+c)^9-70\,\sin(dx+c)^7+128\,\sin(dx+c)^5+70\,\sin(dx+c)^3-15\,\sin(dx+c)\right)}{\sin(dx+c)^{10}-5\,\sin(dx+c)^8+10\,\sin(dx+c)^6-10\,\sin(dx+c)^4+5\,\sin(dx+c)^2-1}-15\,\log\left(\sin\left(dx+c\right)+1\right)+15\,\mathrm{lo}\right.}{}$$

input
```
integrate(sec(d*x+c)^11*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima
")
```

output
```
-1/161280*(63*b^4*(2*(15*sin(d*x + c)^9 - 70*sin(d*x + c)^7 + 128*sin(d*x
+ c)^5 + 70*sin(d*x + c)^3 - 15*sin(d*x + c))/(sin(d*x + c)^10 - 5*sin(d*x
 + c)^8 + 10*sin(d*x + c)^6 - 10*sin(d*x + c)^4 + 5*sin(d*x + c)^2 - 1) -
15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) - 1260*a^2*b^2*(2*(15
*sin(d*x + c)^7 - 55*sin(d*x + c)^5 + 73*sin(d*x + c)^3 + 15*sin(d*x + c))
/(sin(d*x + c)^8 - 4*sin(d*x + c)^6 + 6*sin(d*x + c)^4 - 4*sin(d*x + c)^2
+ 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) + 1680*a^4*(2*
(15*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*sin(d*x + c))/(sin(d*x + c)^6
- 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15
*log(sin(d*x + c) - 1)) - 92160*a^3*b/cos(d*x + c)^7 + 10240*(9*cos(d*x +
c)^2 - 7)*a*b^3/cos(d*x + c)^9)/d
```

3.90.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 880 vs. 2(372) = 744.

Time = 0.50 (sec) , antiderivative size = 880, normalized size of antiderivative = 2.16

$$\int \sec^{11}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^11*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

output
```
1/80640*(315*(80*a^4 - 60*a^2*b^2 + 3*b^4)*log(abs(tan(1/2*d*x + 1/2*c) +
1)) - 315*(80*a^4 - 60*a^2*b^2 + 3*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1))
 + 2*(55440*a^4*tan(1/2*d*x + 1/2*c)^19 + 18900*a^2*b^2*tan(1/2*d*x + 1/2*
c)^19 - 945*b^4*tan(1/2*d*x + 1/2*c)^19 - 322560*a^3*b*tan(1/2*d*x + 1/2*c
)^18 - 213360*a^4*tan(1/2*d*x + 1/2*c)^17 + 462420*a^2*b^2*tan(1/2*d*x + 1
/2*c)^17 + 9135*b^4*tan(1/2*d*x + 1/2*c)^17 + 967680*a^3*b*tan(1/2*d*x + 1
/2*c)^16 - 645120*a*b^3*tan(1/2*d*x + 1/2*c)^16 + 450240*a^4*tan(1/2*d*x +
 1/2*c)^15 + 146160*a^2*b^2*tan(1/2*d*x + 1/2*c)^15 + 218484*b^4*tan(1/2*d
*x + 1/2*c)^15 - 2580480*a^3*b*tan(1/2*d*x + 1/2*c)^14 - 430080*a*b^3*tan(
1/2*d*x + 1/2*c)^14 - 624960*a^4*tan(1/2*d*x + 1/2*c)^13 + 468720*a^2*b^2*
tan(1/2*d*x + 1/2*c)^13 + 653940*b^4*tan(1/2*d*x + 1/2*c)^13 + 5160960*a^3
*b*tan(1/2*d*x + 1/2*c)^12 - 2150400*a*b^3*tan(1/2*d*x + 1/2*c)^12 + 33264
0*a^4*tan(1/2*d*x + 1/2*c)^11 - 1096200*a^2*b^2*tan(1/2*d*x + 1/2*c)^11 +
1183770*b^4*tan(1/2*d*x + 1/2*c)^11 - 5806080*a^3*b*tan(1/2*d*x + 1/2*c)^1
0 + 1290240*a*b^3*tan(1/2*d*x + 1/2*c)^10 + 332640*a^4*tan(1/2*d*x + 1/2*c
)^9 - 1096200*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 + 1183770*b^4*tan(1/2*d*x + 1
/2*c)^9 + 4515840*a^3*b*tan(1/2*d*x + 1/2*c)^8 - 624960*a^4*tan(1/2*d*x +
1/2*c)^7 + 468720*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 + 653940*b^4*tan(1/2*d*x
+ 1/2*c)^7 - 2949120*a^3*b*tan(1/2*d*x + 1/2*c)^6 + 1658880*a*b^3*tan(1/2*
d*x + 1/2*c)^6 + 450240*a^4*tan(1/2*d*x + 1/2*c)^5 + 146160*a^2*b^2*tan...
```

3.90.9 Mupad [B] (verification not implemented)

Time = 27.36 (sec) , antiderivative size = 703, normalized size of antiderivative = 1.72

$$\int \sec^{11}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \text{Too large to display}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))^4/cos(c + d*x)^11,x)
```

output
```
(atanh(tan(c/2 + (d*x)/2))*((5*a^4)/8 + (3*b^4)/128 - (15*a^2*b^2)/32))/d
+ (tan(c/2 + (d*x)/2)^19*((11*a^4)/8 - (3*b^4)/128 + (15*a^2*b^2)/32) + ta
n(c/2 + (d*x)/2)^7*((519*b^4)/32 - (31*a^4)/2 + (93*a^2*b^2)/8) + tan(c/2
+ (d*x)/2)^13*((519*b^4)/32 - (31*a^4)/2 + (93*a^2*b^2)/8) + tan(c/2 + (d*
x)/2)^3*((29*b^4)/128 - (127*a^4)/24 + (367*a^2*b^2)/32) + tan(c/2 + (d*x)
/2)^17*((29*b^4)/128 - (127*a^4)/24 + (367*a^2*b^2)/32) + tan(c/2 + (d*x)/
2)^5*((67*a^4)/6 + (867*b^4)/160 + (29*a^2*b^2)/8) + tan(c/2 + (d*x)/2)^15
*((67*a^4)/6 + (867*b^4)/160 + (29*a^2*b^2)/8) + tan(c/2 + (d*x)/2)^9*((33
*a^4)/4 + (1879*b^4)/64 - (435*a^2*b^2)/16) + tan(c/2 + (d*x)/2)^11*((33*a
^4)/4 + (1879*b^4)/64 - (435*a^2*b^2)/16) - (16*a*b^3)/63 + (8*a^3*b)/7 +
tan(c/2 + (d*x)/2)*((11*a^4)/8 - (3*b^4)/128 + (15*a^2*b^2)/32) - tan(c/2
+ (d*x)/2)^16*(16*a*b^3 - 24*a^3*b) - tan(c/2 + (d*x)/2)^14*((32*a*b^3)/3
+ 64*a^3*b) + tan(c/2 + (d*x)/2)^10*(32*a*b^3 - 144*a^3*b) + tan(c/2 + (d*
x)/2)^4*((32*a*b^3)/7 + (192*a^3*b)/7) + tan(c/2 + (d*x)/2)^2*((160*a*b^3)
/63 - (24*a^3*b)/7) - tan(c/2 + (d*x)/2)^12*((160*a*b^3)/3 - 128*a^3*b) +
tan(c/2 + (d*x)/2)^6*((288*a*b^3)/7 - (512*a^3*b)/7) + 112*a^3*b*tan(c/2 +
 (d*x)/2)^8 - 8*a^3*b*tan(c/2 + (d*x)/2)^18)/(d*(45*tan(c/2 + (d*x)/2)^4 -
 10*tan(c/2 + (d*x)/2)^2 - 120*tan(c/2 + (d*x)/2)^6 + 210*tan(c/2 + (d*x)/
2)^8 - 252*tan(c/2 + (d*x)/2)^10 + 210*tan(c/2 + (d*x)/2)^12 - 120*tan(c/2
 + (d*x)/2)^14 + 45*tan(c/2 + (d*x)/2)^16 - 10*tan(c/2 + (d*x)/2)^18 + ...
```

3.90. $\int \sec^{11}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

3.91 $\int \sec^{12}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

3.91.1 Optimal result

Integrand size = 28, antiderivative size = 254

$$\begin{aligned}\int \sec^{12}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx &= \frac{a^4\tan(c+dx)}{d}+\frac{2a^3b\tan^2(c+dx)}{d}+\frac{a^2(a^2+2b^2)\tan^3(c+dx)}{d}\\ &+\frac{ab(3a^2+b^2)\tan^4(c+dx)}{d}+\frac{(3a^4+18a^2b^2+b^4)\tan^5(c+dx)}{5d}\\ &+\frac{2ab(a^2+b^2)\tan^6(c+dx)}{d}+\frac{(a^4+18a^2b^2+3b^4)\tan^7(c+dx)}{7d}\\ &+\frac{ab(a^2+3b^2)\tan^8(c+dx)}{2d}+\frac{b^2(2a^2+b^2)\tan^9(c+dx)}{3d}\\ &+\frac{2ab^3\tan^{10}(c+dx)}{5d}+\frac{b^4\tan^{11}(c+dx)}{11d}\end{aligned}$$

output
```
a^4*tan(d*x+c)/d+2*a^3*b*tan(d*x+c)^2/d+a^2*(a^2+2*b^2)*tan(d*x+c)^3/d+a*b
*(3*a^2+b^2)*tan(d*x+c)^4/d+1/5*(3*a^4+18*a^2*b^2+b^4)*tan(d*x+c)^5/d+2*a*
b*(a^2+b^2)*tan(d*x+c)^6/d+1/7*(a^4+18*a^2*b^2+3*b^4)*tan(d*x+c)^7/d+1/2*a
*b*(a^2+3*b^2)*tan(d*x+c)^8/d+1/3*b^2*(2*a^2+b^2)*tan(d*x+c)^9/d+2/5*a*b^3
*tan(d*x+c)^10/d+1/11*b^4*tan(d*x+c)^11/d
```

3.91.2 Mathematica [A] (verified)

Time = 1.84 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.69

$$\int \sec^{12}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \frac{\frac{1}{5}(a^2+b^2)^3(a+b\tan(c+dx))^5-a(a^2+b^2)^2(a+b\tan(c+dx))^6+\frac{3}{7}(a^2+b^2)(5a^2+b^2)(a+b\tan(c+$$

input
```
Integrate[Sec[c + d*x]^12*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

output
```
(((a^2 + b^2)^3*(a + b*Tan[c + d*x])^5)/5 - a*(a^2 + b^2)^2*(a + b*Tan[c +
 d*x])^6 + (3*(a^2 + b^2)*(5*a^2 + b^2)*(a + b*Tan[c + d*x])^7)/7 - (a*(5*
a^2 + 3*b^2)*(a + b*Tan[c + d*x])^8)/2 + ((5*a^2 + b^2)*(a + b*Tan[c + d*x
])^9)/3 - (3*a*(a + b*Tan[c + d*x])^10)/5 + (a + b*Tan[c + d*x])^11/11)/(b
^7*d)
```

3.91.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3567, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{12}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\cos(c+dx)+b\sin(c+dx))^4}{\cos(c+dx)^{12}}dx$$

$$\Big\downarrow \text{3567}$$

$$-\frac{\int (b+a\cot(c+dx))^4\left(\cot^2(c+dx)+1\right)^3\tan^{12}(c+dx)d\cot(c+dx)}{d}$$

$$\Big\downarrow \text{522}$$

$$-\frac{\int \left(b^4\tan^{12}(c+dx)+4ab^3\tan^{11}(c+dx)+3\left(b^4+2a^2b^2\right)\tan^{10}(c+dx)+4ab\left(a^2+3b^2\right)\tan^9(c+dx)+\left(a^4+1\right.}{}$$

$$\downarrow \text{2009}$$

$$-\frac{-a^4\tan(c+dx)-2a^3b\tan^2(c+dx)-\frac{1}{3}b^2(2a^2+b^2)\tan^9(c+dx)-\frac{1}{2}ab(a^2+3b^2)\tan^8(c+dx)-2ab(a^2+b^2)}{}$$

input
```
Int[Sec[c + d*x]^12*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

output
```
-((-(a^4*Tan[c + d*x]) - 2*a^3*b*Tan[c + d*x]^2 - a^2*(a^2 + 2*b^2)*Tan[c
+ d*x]^3 - a*b*(3*a^2 + b^2)*Tan[c + d*x]^4 - ((3*a^4 + 18*a^2*b^2 + b^4)*
Tan[c + d*x]^5)/5 - 2*a*b*(a^2 + b^2)*Tan[c + d*x]^6 - ((a^4 + 18*a^2*b^2
+ 3*b^4)*Tan[c + d*x]^7)/7 - (a*b*(a^2 + 3*b^2)*Tan[c + d*x]^8)/2 - (b^2*(
2*a^2 + b^2)*Tan[c + d*x]^9)/3 - (2*a*b^3*Tan[c + d*x]^10)/5 - (b^4*Tan[c
+ d*x]^11)/11)/d)
```

3.91.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3567
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[x^m*((b
+ a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a,
b, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[
n, 0] && GtQ[m, 1])
```

3.91.4 Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.02

method	result
parts	$-\frac{a^4\left(-\frac{16}{35}-\frac{\sec(dx+c)^6}{7}-\frac{6\sec(dx+c)^4}{35}-\frac{8\sec(dx+c)^2}{35}\right)\tan(dx+c)}{d}+\frac{b^4\left(\frac{\sin(dx+c)^5}{11\cos(dx+c)^{11}}+\frac{2\sin(dx+c)^5}{33\cos(dx+c)^9}+\frac{8\sin(dx+c)^5}{231\cos(dx+c)^7}\right.}{d}$
derivativedivides	$\frac{-a^4\left(-\frac{16}{35}-\frac{\sec(dx+c)^6}{7}-\frac{6\sec(dx+c)^4}{35}-\frac{8\sec(dx+c)^2}{35}\right)\tan(dx+c)+\frac{a^3b}{2\cos(dx+c)^8}+6a^2b^2\left(\frac{\sin(dx+c)^3}{9\cos(dx+c)^9}+\frac{2\sin(dx+c)^3}{21\cos(dx+c)^7}+\frac{8}{10}\right.}{}$
default	$\frac{-a^4\left(-\frac{16}{35}-\frac{\sec(dx+c)^6}{7}-\frac{6\sec(dx+c)^4}{35}-\frac{8\sec(dx+c)^2}{35}\right)\tan(dx+c)+\frac{a^3b}{2\cos(dx+c)^8}+6a^2b^2\left(\frac{\sin(dx+c)^3}{9\cos(dx+c)^9}+\frac{2\sin(dx+c)^3}{21\cos(dx+c)^7}+\frac{8}{10}\right.}{}$
risch	$\frac{32i\left(33a^4+b^4-22a^2b^2+9933a^4e^{10i(dx+c)}+11b^4e^{2i(dx+c)}+165b^4e^{6i(dx+c)}+2541b^4e^{10i(dx+c)}-9702a^2b^2e^{12i(dx+c)}-693\right.}{}$
parallelrisch	$-\frac{2\left(a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{20}-4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{19}a^3b+2\left(-3a^4+4a^2b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{18}+4\left(3a^3b-2a\,b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{17}+\frac{(113a^4}{}\right.}{}$

input
```
int(sec(d*x+c)^12*(cos(d*x+c)*a+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
-a^4/d*(-16/35-1/7*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*tan(d
*x+c)+b^4/d*(1/11*sin(d*x+c)^5/cos(d*x+c)^11+2/33*sin(d*x+c)^5/cos(d*x+c)^
9+8/231*sin(d*x+c)^5/cos(d*x+c)^7+16/1155*sin(d*x+c)^5/cos(d*x+c)^5)+1/2*a
^3*b/d*sec(d*x+c)^8+4*a*b^3/d*(1/10*sec(d*x+c)^10-1/8*sec(d*x+c)^8)+6*a^2*
b^2/d*(1/9*sin(d*x+c)^3/cos(d*x+c)^9+2/21*sin(d*x+c)^3/cos(d*x+c)^7+8/105*
sin(d*x+c)^3/cos(d*x+c)^5+16/315*sin(d*x+c)^3/cos(d*x+c)^3)
```

3.91.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.76

$$\int \sec^{12}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$
$$=\frac{924\,ab^3\cos(dx+c)+1155\,(a^3b-ab^3)\cos(dx+c)^3+2\left(16\left(33\,a^4-22\,a^2b^2+b^4\right)\cos(dx+c)^{10}+8\,(33\,a\right.}{}$$

input
```
integrate(sec(d*x+c)^12*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas
")
```

output
```
1/2310*(924*a*b^3*cos(d*x + c) + 1155*(a^3*b - a*b^3)*cos(d*x + c)^3 + 2*(
16*(33*a^4 - 22*a^2*b^2 + b^4)*cos(d*x + c)^10 + 8*(33*a^4 - 22*a^2*b^2 +
b^4)*cos(d*x + c)^8 + 6*(33*a^4 - 22*a^2*b^2 + b^4)*cos(d*x + c)^6 + 5*(33
*a^4 - 22*a^2*b^2 + b^4)*cos(d*x + c)^4 + 105*b^4 + 70*(11*a^2*b^2 - 2*b^4
)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^11)
```

3.91.6 Sympy [F(-1)]

Timed out.

$$\int \sec^{12}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**12*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

output
```
Timed out
```

3.91.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.92

$$\int \sec^{12}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$

$$= \frac{66\left(5\,\tan\left(dx+c\right)^{7}+21\,\tan\left(dx+c\right)^{5}+35\,\tan\left(dx+c\right)^{3}+35\,\tan\left(dx+c\right)\right)a^{4}+44\left(35\,\tan\left(dx+c\right)^{9}+\right.}{}$$

input
```
integrate(sec(d*x+c)^12*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima
")
```

output
```
1/2310*(66*(5*tan(d*x + c)^7 + 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 35*
tan(d*x + c))*a^4 + 44*(35*tan(d*x + c)^9 + 135*tan(d*x + c)^7 + 189*tan(d
*x + c)^5 + 105*tan(d*x + c)^3)*a^2*b^2 + 2*(105*tan(d*x + c)^11 + 385*tan
(d*x + c)^9 + 495*tan(d*x + c)^7 + 231*tan(d*x + c)^5)*b^4 - 231*(5*sin(d*
x + c)^2 - 1)*a*b^3/(sin(d*x + c)^10 - 5*sin(d*x + c)^8 + 10*sin(d*x + c)^
6 - 10*sin(d*x + c)^4 + 5*sin(d*x + c)^2 - 1) + 1155*a^3*b/(sin(d*x + c)^2
 - 1)^4)/d
```

3.91.8 Giac [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.12

$$\int \sec^{12}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$$
$$=\frac{210\,b^4\tan\left(dx+c\right)^{11}+924\,ab^3\tan\left(dx+c\right)^{10}+1540\,a^2b^2\tan\left(dx+c\right)^9+770\,b^4\tan\left(dx+c\right)^9+1155\,a^3b\,\mathrm{t}}{}$$

input
```
integrate(sec(d*x+c)^12*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

output
```
1/2310*(210*b^4*tan(d*x + c)^11 + 924*a*b^3*tan(d*x + c)^10 + 1540*a^2*b^2
*tan(d*x + c)^9 + 770*b^4*tan(d*x + c)^9 + 1155*a^3*b*tan(d*x + c)^8 + 346
5*a*b^3*tan(d*x + c)^8 + 330*a^4*tan(d*x + c)^7 + 5940*a^2*b^2*tan(d*x + c
)^7 + 990*b^4*tan(d*x + c)^7 + 4620*a^3*b*tan(d*x + c)^6 + 4620*a*b^3*tan(
d*x + c)^6 + 1386*a^4*tan(d*x + c)^5 + 8316*a^2*b^2*tan(d*x + c)^5 + 462*b
^4*tan(d*x + c)^5 + 6930*a^3*b*tan(d*x + c)^4 + 2310*a*b^3*tan(d*x + c)^4
+ 2310*a^4*tan(d*x + c)^3 + 4620*a^2*b^2*tan(d*x + c)^3 + 4620*a^3*b*tan(d
*x + c)^2 + 2310*a^4*tan(d*x + c))/d
```

3.91.9 Mupad [B] (verification not implemented)

Time = 26.40 (sec) , antiderivative size = 560, normalized size of antiderivative = 2.20

$$\int \sec^{12}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx =$$
$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\left(\frac{226\,a^4}{5}-\frac{64\,a^2\,b^2}{5}+\frac{32\,b^4}{5}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}\left(\frac{226\,a^4}{5}-\frac{64\,a^2\,b^2}{5}+\frac{32\,b^4}{5}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9\left(\frac{1308\,a}{7}\right.}{}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))^4/cos(c + d*x)^12,x)
```

output
```
-(tan(c/2 + (d*x)/2)^5*((226*a^4)/5 + (32*b^4)/5 - (64*a^2*b^2)/5) + tan(c
/2 + (d*x)/2)^17*((226*a^4)/5 + (32*b^4)/5 - (64*a^2*b^2)/5) + tan(c/2 + (
d*x)/2)^9*((1308*a^4)/7 + (992*b^4)/21 - (3008*a^2*b^2)/21) + tan(c/2 + (d
*x)/2)^13*((1308*a^4)/7 + (992*b^4)/21 - (3008*a^2*b^2)/21) + tan(c/2 + (d
*x)/2)^7*((576*b^4)/35 - (3952*a^4)/35 + (3008*a^2*b^2)/35) + tan(c/2 + (d
*x)/2)^15*((576*b^4)/35 - (3952*a^4)/35 + (3008*a^2*b^2)/35) + tan(c/2 + (
d*x)/2)^11*((10624*b^4)/231 - (1528*a^4)/7 + (2272*a^2*b^2)/21) + 2*a^4*ta
n(c/2 + (d*x)/2)^21 - tan(c/2 + (d*x)/2)^3*(12*a^4 - 16*a^2*b^2) - tan(c/2
 + (d*x)/2)^19*(12*a^4 - 16*a^2*b^2) + 2*a^4*tan(c/2 + (d*x)/2) + tan(c/2
+ (d*x)/2)^4*(16*a*b^3 - 24*a^3*b) - tan(c/2 + (d*x)/2)^18*(16*a*b^3 - 24*
a^3*b) + tan(c/2 + (d*x)/2)^6*(16*a*b^3 + 80*a^3*b) - tan(c/2 + (d*x)/2)^1
6*(16*a*b^3 + 80*a^3*b) + tan(c/2 + (d*x)/2)^8*(80*a*b^3 - 176*a^3*b) - ta
n(c/2 + (d*x)/2)^14*(80*a*b^3 - 176*a^3*b) - tan(c/2 + (d*x)/2)^10*((112*a
*b^3)/5 - 224*a^3*b) + tan(c/2 + (d*x)/2)^12*((112*a*b^3)/5 - 224*a^3*b) +
 8*a^3*b*tan(c/2 + (d*x)/2)^2 - 8*a^3*b*tan(c/2 + (d*x)/2)^20)/(d*(tan(c/2
 + (d*x)/2)^2 - 1)^11)
```

3.91. $\int \sec^{12}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

3.92 $\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

3.92.1 Optimal result

Integrand size = 28, antiderivative size = 515

$$\begin{aligned}
&\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx \\
&= \frac{63a^5x}{256}+\frac{35}{128}a^3b^2x+\frac{15}{256}ab^4x-\frac{5a^2b^3\cos^8(c+dx)}{4d}-\frac{a^4b\cos^{10}(c+dx)}{2d} \\
&\quad+\frac{a^2b^3\cos^{10}(c+dx)}{d}+\frac{63a^5\cos(c+dx)\sin(c+dx)}{256d}+\frac{35a^3b^2\cos(c+dx)\sin(c+dx)}{128d} \\
&\quad+\frac{15ab^4\cos(c+dx)\sin(c+dx)}{256d}+\frac{21a^5\cos^3(c+dx)\sin(c+dx)}{128d} \\
&\quad+\frac{35a^3b^2\cos^3(c+dx)\sin(c+dx)}{192d}+\frac{5ab^4\cos^3(c+dx)\sin(c+dx)}{128d} \\
&\quad+\frac{21a^5\cos^5(c+dx)\sin(c+dx)}{160d}+\frac{7a^3b^2\cos^5(c+dx)\sin(c+dx)}{48d} \\
&\quad+\frac{ab^4\cos^5(c+dx)\sin(c+dx)}{32d}+\frac{9a^5\cos^7(c+dx)\sin(c+dx)}{80d} \\
&\quad+\frac{a^3b^2\cos^7(c+dx)\sin(c+dx)}{8d}-\frac{3ab^4\cos^7(c+dx)\sin(c+dx)}{16d} \\
&\quad+\frac{a^5\cos^9(c+dx)\sin(c+dx)}{10d}-\frac{a^3b^2\cos^9(c+dx)\sin(c+dx)}{d} \\
&\quad-\frac{ab^4\cos^7(c+dx)\sin^3(c+dx)}{2d}+\frac{b^5\sin^6(c+dx)}{6d}-\frac{b^5\sin^8(c+dx)}{4d}+\frac{b^5\sin^{10}(c+dx)}{10d}
\end{aligned}$$

output
```
63/256*a^5*x+35/128*a^3*b^2*x+15/256*a*b^4*x-5/4*a^2*b^3*cos(d*x+c)^8/d-1/
2*a^4*b*cos(d*x+c)^10/d+a^2*b^3*cos(d*x+c)^10/d+63/256*a^5*cos(d*x+c)*sin(
d*x+c)/d+35/128*a^3*b^2*cos(d*x+c)*sin(d*x+c)/d+15/256*a*b^4*cos(d*x+c)*si
n(d*x+c)/d+21/128*a^5*cos(d*x+c)^3*sin(d*x+c)/d+35/192*a^3*b^2*cos(d*x+c)^
3*sin(d*x+c)/d+5/128*a*b^4*cos(d*x+c)^3*sin(d*x+c)/d+21/160*a^5*cos(d*x+c)
^5*sin(d*x+c)/d+7/48*a^3*b^2*cos(d*x+c)^5*sin(d*x+c)/d+1/32*a*b^4*cos(d*x+
c)^5*sin(d*x+c)/d+9/80*a^5*cos(d*x+c)^7*sin(d*x+c)/d+1/8*a^3*b^2*cos(d*x+c
)^7*sin(d*x+c)/d-3/16*a*b^4*cos(d*x+c)^7*sin(d*x+c)/d+1/10*a^5*cos(d*x+c)^
9*sin(d*x+c)/d-a^3*b^2*cos(d*x+c)^9*sin(d*x+c)/d-1/2*a*b^4*cos(d*x+c)^7*si
n(d*x+c)^3/d+1/6*b^5*sin(d*x+c)^6/d-1/4*b^5*sin(d*x+c)^8/d+1/10*b^5*sin(d*
x+c)^10/d
```

3.92.2 Mathematica [A] (verified)

Time = 7.34 (sec) , antiderivative size = 307, normalized size of antiderivative = 0.60

$$\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$
$$= \frac{120a(63a^4+70a^2b^2+15b^4)(c+dx)-300b(21a^4+14a^2b^2+b^4)\cos(2(c+dx))-1200a^2b(3a^2+b^2)\cos(}{}$$

input
```
Integrate[Cos[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

output
```
(120*a*(63*a^4 + 70*a^2*b^2 + 15*b^4)*(c + d*x) - 300*b*(21*a^4 + 14*a^2*b
^2 + b^4)*Cos[2*(c + d*x)] - 1200*a^2*b*(3*a^2 + b^2)*Cos[4*(c + d*x)] + 5
0*b*(-27*a^4 + 6*a^2*b^2 + b^4)*Cos[6*(c + d*x)] - 300*a^2*b*(a^2 - b^2)*C
os[8*(c + d*x)] - 6*b*(5*a^4 - 10*a^2*b^2 + b^4)*Cos[10*(c + d*x)] + 300*a
*(21*a^4 + 14*a^2*b^2 + b^4)*Sin[2*(c + d*x)] + 600*a*(3*a^4 - 2*a^2*b^2 -
 b^4)*Sin[4*(c + d*x)] + 50*a*(9*a^4 - 26*a^2*b^2 - 3*b^4)*Sin[6*(c + d*x)
] + 75*a*(a^4 - 6*a^2*b^2 + b^4)*Sin[8*(c + d*x)] + 6*a*(a^4 - 10*a^2*b^2
+ 5*b^4)*Sin[10*(c + d*x)])/(30720*d)
```

3.92.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(c+dx)^5(a\cos(c+dx)+b\sin(c+dx))^5dx$$

$$\Big\downarrow \text{3569}$$

$$\int \left(a^5\cos^{10}(c+dx)+5a^4b\sin(c+dx)\cos^9(c+dx)+10a^3b^2\sin^2(c+dx)\cos^8(c+dx)+10a^2b^3\sin^3(c+dx)\cos^7(\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^5\sin(c+dx)\cos^9(c+dx)}{10d}+\frac{9a^5\sin(c+dx)\cos^7(c+dx)}{80d}+\frac{21a^5\sin(c+dx)\cos^5(c+dx)}{160d}+\frac{21a^5\sin(c+dx)\cos^3(c+dx)}{128d}+\frac{63a^5\sin(c+dx)\cos(c+dx)}{256d}+\frac{63a^5x}{256}-\frac{a^4b\cos^{10}(c+dx)}{2d}-\frac{a^3b^2\sin(c+dx)\cos^9(c+dx)}{d}+\frac{a^3b^2\sin(c+dx)\cos^7(c+dx)}{8d}+\frac{7a^3b^2\sin(c+dx)\cos^5(c+dx)}{48d}+\frac{35a^3b^2\sin(c+dx)\cos^3(c+dx)}{192d}+\frac{35a^3b^2\sin(c+dx)\cos(c+dx)}{128d}+\frac{35}{128}a^3b^2x+\frac{a^2b^3\cos^{10}(c+dx)}{d}-\frac{5a^2b^3\cos^8(c+dx)}{4d}-\frac{ab^4\sin^3(c+dx)\cos^7(c+dx)}{2d}-\frac{3ab^4\sin(c+dx)\cos^7(c+dx)}{16d}+\frac{ab^4\sin(c+dx)\cos^5(c+dx)}{32d}+\frac{5ab^4\sin(c+dx)\cos^3(c+dx)}{128d}+\frac{15ab^4\sin(c+dx)\cos(c+dx)}{256d}+\frac{15}{256}ab^4x+\frac{b^5\sin^{10}(c+dx)}{10d}-\frac{b^5\sin^8(c+dx)}{4d}+\frac{b^5\sin^6(c+dx)}{6d}$$

input
```
Int[Cos[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

output
```
(63*a^5*x)/256 + (35*a^3*b^2*x)/128 + (15*a*b^4*x)/256 - (5*a^2*b^3*Cos[c
+ d*x]^8)/(4*d) - (a^4*b*Cos[c + d*x]^10)/(2*d) + (a^2*b^3*Cos[c + d*x]^10
)/d + (63*a^5*Cos[c + d*x]*Sin[c + d*x])/(256*d) + (35*a^3*b^2*Cos[c + d*x
]*Sin[c + d*x])/(128*d) + (15*a*b^4*Cos[c + d*x]*Sin[c + d*x])/(256*d) + (
21*a^5*Cos[c + d*x]^3*Sin[c + d*x])/(128*d) + (35*a^3*b^2*Cos[c + d*x]^3*S
in[c + d*x])/(192*d) + (5*a*b^4*Cos[c + d*x]^3*Sin[c + d*x])/(128*d) + (21
*a^5*Cos[c + d*x]^5*Sin[c + d*x])/(160*d) + (7*a^3*b^2*Cos[c + d*x]^5*Sin[
c + d*x])/(48*d) + (a*b^4*Cos[c + d*x]^5*Sin[c + d*x])/(32*d) + (9*a^5*Cos
[c + d*x]^7*Sin[c + d*x])/(80*d) + (a^3*b^2*Cos[c + d*x]^7*Sin[c + d*x])/(
8*d) - (3*a*b^4*Cos[c + d*x]^7*Sin[c + d*x])/(16*d) + (a^5*Cos[c + d*x]^9*
Sin[c + d*x])/(10*d) - (a^3*b^2*Cos[c + d*x]^9*Sin[c + d*x])/d - (a*b^4*Co
s[c + d*x]^7*Sin[c + d*x]^3)/(2*d) + (b^5*Sin[c + d*x]^6)/(6*d) - (b^5*Sin
[c + d*x]^8)/(4*d) + (b^5*Sin[c + d*x]^10)/(10*d)
```

3.92.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

3.92.4 Maple [A] (verified)

Time = 2.05 (sec) , antiderivative size = 325, normalized size of antiderivative = 0.63

method	result
parts	$\frac{a^5\left(\frac{\left(\cos(dx+c)^9+\frac{9\cos(dx+c)^7}{8}+\frac{21\cos(dx+c)^5}{16}+\frac{105\cos(dx+c)^3}{64}+\frac{315\cos(dx+c)}{128}\right)\sin(dx+c)}{10}+\frac{63dx}{256}+\frac{63c}{256}\right)}{d}+\frac{b^5\left(\frac{\sin(dx+c)}{10}\right.}{}$
derivativedivides	$\frac{a^5\left(\frac{\left(\cos(dx+c)^9+\frac{9\cos(dx+c)^7}{8}+\frac{21\cos(dx+c)^5}{16}+\frac{105\cos(dx+c)^3}{64}+\frac{315\cos(dx+c)}{128}\right)\sin(dx+c)}{10}+\frac{63dx}{256}+\frac{63c}{256}\right)-\frac{a^4b\cos(dx+c)^{10}}{2}-}{}$
default	$\frac{a^5\left(\frac{\left(\cos(dx+c)^9+\frac{9\cos(dx+c)^7}{8}+\frac{21\cos(dx+c)^5}{16}+\frac{105\cos(dx+c)^3}{64}+\frac{315\cos(dx+c)}{128}\right)\sin(dx+c)}{10}+\frac{63dx}{256}+\frac{63c}{256}\right)-\frac{a^4b\cos(dx+c)^{10}}{2}-}{}$
parallelrisch	$\frac{6(-5a^4b+10a^2b^3-b^5)\cos(10dx+10c)+6(a^5-10a^3b^2+5ab^4)\sin(10dx+10c)+300(-21a^4b-14a^2b^3-b^5)\cos(2dx+2c)+}{}$
risch	$\frac{35a^3b^2x}{128}+\frac{15ab^4x}{256}+\frac{5b^5\cos(6dx+6c)}{3072d}-\frac{5b^5\cos(2dx+2c)}{512d}+\frac{105a^5\sin(2dx+2c)}{512d}-\frac{5a^3\sin(4dx+4c)b^2}{128d}-\frac{5a\sin}{}$

input
```
int(cos(d*x+c)^5*(cos(d*x+c)*a+b*sin(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
a^5/d*(1/10*(cos(d*x+c)^9+9/8*cos(d*x+c)^7+21/16*cos(d*x+c)^5+105/64*cos(d
*x+c)^3+315/128*cos(d*x+c))*sin(d*x+c)+63/256*d*x+63/256*c)+b^5/d*(1/10*si
n(d*x+c)^10-1/4*sin(d*x+c)^8+1/6*sin(d*x+c)^6)+5*a*b^4/d*(-1/10*sin(d*x+c)
^3*cos(d*x+c)^7-3/80*sin(d*x+c)*cos(d*x+c)^7+1/160*(cos(d*x+c)^5+5/4*cos(d
*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+3/256*d*x+3/256*c)-1/2*a^4*b*cos(d*x+c
)^10/d+10*a^3*b^2/d*(-1/10*sin(d*x+c)*cos(d*x+c)^9+1/80*(cos(d*x+c)^7+7/6*
cos(d*x+c)^5+35/24*cos(d*x+c)^3+35/16*cos(d*x+c))*sin(d*x+c)+7/256*d*x+7/2
56*c)+10*a^2*b^3/d*(1/10*cos(d*x+c)^10-1/8*cos(d*x+c)^8)
```

3.92.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.49

$$\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx =$$
$$-\frac{640\,b^5\cos(dx+c)^6+384\,(5\,a^4b-10\,a^2b^3+b^5)\cos(dx+c)^{10}+960\,(5\,a^2b^3-b^5)\cos(dx+c)^8-15\,(63}{}$$

input
```
integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas"
)
```

output
```
-1/3840*(640*b^5*cos(d*x + c)^6 + 384*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x
 + c)^10 + 960*(5*a^2*b^3 - b^5)*cos(d*x + c)^8 - 15*(63*a^5 + 70*a^3*b^2
+ 15*a*b^4)*d*x - (384*(a^5 - 10*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^9 + 48*(9
*a^5 + 10*a^3*b^2 - 55*a*b^4)*cos(d*x + c)^7 + 8*(63*a^5 + 70*a^3*b^2 + 15
*a*b^4)*cos(d*x + c)^5 + 10*(63*a^5 + 70*a^3*b^2 + 15*a*b^4)*cos(d*x + c)^
3 + 15*(63*a^5 + 70*a^3*b^2 + 15*a*b^4)*cos(d*x + c))*sin(d*x + c))/d
```

3.92.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1037 vs. $2(498) = 996$.

Time = 1.51 (sec) , antiderivative size = 1037, normalized size of antiderivative = 2.01

$$\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)**5*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

output
```
Piecewise((63*a**5*x*sin(c + d*x)**10/256 + 315*a**5*x*sin(c + d*x)**8*cos
(c + d*x)**2/256 + 315*a**5*x*sin(c + d*x)**6*cos(c + d*x)**4/128 + 315*a*
*5*x*sin(c + d*x)**4*cos(c + d*x)**6/128 + 315*a**5*x*sin(c + d*x)**2*cos(
c + d*x)**8/256 + 63*a**5*x*cos(c + d*x)**10/256 + 63*a**5*sin(c + d*x)**9
*cos(c + d*x)/(256*d) + 147*a**5*sin(c + d*x)**7*cos(c + d*x)**3/(128*d) +
 21*a**5*sin(c + d*x)**5*cos(c + d*x)**5/(10*d) + 237*a**5*sin(c + d*x)**3
*cos(c + d*x)**7/(128*d) + 193*a**5*sin(c + d*x)*cos(c + d*x)**9/(256*d) -
 a**4*b*cos(c + d*x)**10/(2*d) + 35*a**3*b**2*x*sin(c + d*x)**10/128 + 175
*a**3*b**2*x*sin(c + d*x)**8*cos(c + d*x)**2/128 + 175*a**3*b**2*x*sin(c +
 d*x)**6*cos(c + d*x)**4/64 + 175*a**3*b**2*x*sin(c + d*x)**4*cos(c + d*x)
**6/64 + 175*a**3*b**2*x*sin(c + d*x)**2*cos(c + d*x)**8/128 + 35*a**3*b**
2*x*cos(c + d*x)**10/128 + 35*a**3*b**2*sin(c + d*x)**9*cos(c + d*x)/(128*
d) + 245*a**3*b**2*sin(c + d*x)**7*cos(c + d*x)**3/(192*d) + 7*a**3*b**2*s
in(c + d*x)**5*cos(c + d*x)**5/(3*d) + 395*a**3*b**2*sin(c + d*x)**3*cos(c
 + d*x)**7/(192*d) - 35*a**3*b**2*sin(c + d*x)*cos(c + d*x)**9/(128*d) + a
**2*b**3*sin(c + d*x)**10/(4*d) + 5*a**2*b**3*sin(c + d*x)**8*cos(c + d*x)
**2/(4*d) + 5*a**2*b**3*sin(c + d*x)**6*cos(c + d*x)**4/(2*d) + 5*a**2*b**
3*sin(c + d*x)**4*cos(c + d*x)**6/(2*d) + 15*a*b**4*x*sin(c + d*x)**10/256
 + 75*a*b**4*x*sin(c + d*x)**8*cos(c + d*x)**2/256 + 75*a*b**4*x*sin(c + d
*x)**6*cos(c + d*x)**4/128 + 75*a*b**4*x*sin(c + d*x)**4*cos(c + d*x)**...
```

3.92.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.56

$$\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx =$$
$$-\frac{15360\,a^4b\cos\left(dx+c\right)^{10}-3\left(32\,\sin\left(2\,dx+2\,c\right)^5-640\,\sin\left(2\,dx+2\,c\right)^3+2520\,dx+2520\,c+25\,\sin\left(8\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima"
)
```

output
```
-1/30720*(15360*a^4*b*cos(d*x + c)^10 - 3*(32*sin(2*d*x + 2*c)^5 - 640*sin
(2*d*x + 2*c)^3 + 2520*d*x + 2520*c + 25*sin(8*d*x + 8*c) + 600*sin(4*d*x
+ 4*c) + 2560*sin(2*d*x + 2*c))*a^5 + 10*(96*sin(2*d*x + 2*c)^5 - 640*sin(
2*d*x + 2*c)^3 - 840*d*x - 840*c + 45*sin(8*d*x + 8*c) + 120*sin(4*d*x + 4
*c))*a^3*b^2 + 7680*(4*sin(d*x + c)^10 - 15*sin(d*x + c)^8 + 20*sin(d*x +
c)^6 - 10*sin(d*x + c)^4)*a^2*b^3 - 15*(32*sin(2*d*x + 2*c)^5 + 120*d*x +
120*c + 5*sin(8*d*x + 8*c) - 40*sin(4*d*x + 4*c))*a*b^4 - 512*(6*sin(d*x +
 c)^10 - 15*sin(d*x + c)^8 + 10*sin(d*x + c)^6)*b^5)/d
```

3.92.8 Giac [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 342, normalized size of antiderivative = 0.66

$$\begin{aligned}\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx &= \frac{1}{256}\left(63\,a^5+70\,a^3b^2+15\,ab^4\right)x-\frac{\left(5\,a^4b-10\,a^2b^3+b^5\right)\cos\left(10\,dx+10\,c\right)}{5120\,d}\\ &\quad-\frac{5\left(a^4b-a^2b^3\right)\cos\left(8\,dx+8\,c\right)}{512\,d}-\frac{5\left(27\,a^4b-6\,a^2b^3-b^5\right)\cos\left(6\,dx+6\,c\right)}{3072\,d}\\ &\quad-\frac{5\left(3\,a^4b+a^2b^3\right)\cos\left(4\,dx+4\,c\right)}{128\,d}-\frac{5\left(21\,a^4b+14\,a^2b^3+b^5\right)\cos\left(2\,dx+2\,c\right)}{512\,d}\\ &\quad+\frac{\left(a^5-10\,a^3b^2+5\,ab^4\right)\sin\left(10\,dx+10\,c\right)}{5120\,d}+\frac{5\left(a^5-6\,a^3b^2+ab^4\right)\sin\left(8\,dx+8\,c\right)}{2048\,d}\\ &\quad+\frac{5\left(9\,a^5-26\,a^3b^2-3\,ab^4\right)\sin\left(6\,dx+6\,c\right)}{3072\,d}\\ &\quad+\frac{5\left(3\,a^5-2\,a^3b^2-ab^4\right)\sin\left(4\,dx+4\,c\right)}{256\,d}+\frac{5\left(21\,a^5+14\,a^3b^2+ab^4\right)\sin\left(2\,dx+2\,c\right)}{512\,d}\end{aligned}$$

input
```
integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

output
```
1/256*(63*a^5 + 70*a^3*b^2 + 15*a*b^4)*x - 1/5120*(5*a^4*b - 10*a^2*b^3 +
b^5)*cos(10*d*x + 10*c)/d - 5/512*(a^4*b - a^2*b^3)*cos(8*d*x + 8*c)/d - 5
/3072*(27*a^4*b - 6*a^2*b^3 - b^5)*cos(6*d*x + 6*c)/d - 5/128*(3*a^4*b + a
^2*b^3)*cos(4*d*x + 4*c)/d - 5/512*(21*a^4*b + 14*a^2*b^3 + b^5)*cos(2*d*x
 + 2*c)/d + 1/5120*(a^5 - 10*a^3*b^2 + 5*a*b^4)*sin(10*d*x + 10*c)/d + 5/2
048*(a^5 - 6*a^3*b^2 + a*b^4)*sin(8*d*x + 8*c)/d + 5/3072*(9*a^5 - 26*a^3*
b^2 - 3*a*b^4)*sin(6*d*x + 6*c)/d + 5/256*(3*a^5 - 2*a^3*b^2 - a*b^4)*sin(
4*d*x + 4*c)/d + 5/512*(21*a^5 + 14*a^3*b^2 + a*b^4)*sin(2*d*x + 2*c)/d
```

3.92.9 Mupad [B] (verification not implemented)

Time = 24.16 (sec) , antiderivative size = 801, normalized size of antiderivative = 1.56

$$\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \text{Too large to display}$$

input
```
int(cos(c + d*x)^5*(a*cos(c + d*x) + b*sin(c + d*x))^5,x)
```

output
```
(tan(c/2 + (d*x)/2)^19*((15*a*b^4)/128 - (193*a^5)/128 + (35*a^3*b^2)/64)
- tan(c/2 + (d*x)/2)*((15*a*b^4)/128 - (193*a^5)/128 + (35*a^3*b^2)/64) +
tan(c/2 + (d*x)/2)^3*((159*a^5)/128 - (145*a*b^4)/128 + (4105*a^3*b^2)/192
) - tan(c/2 + (d*x)/2)^17*((159*a^5)/128 - (145*a*b^4)/128 + (4105*a^3*b^2
)/192) - tan(c/2 + (d*x)/2)^7*((2595*a*b^4)/32 + (147*a^5)/32 - (2905*a^3*
b^2)/16) + tan(c/2 + (d*x)/2)^13*((2595*a*b^4)/32 + (147*a^5)/32 - (2905*a
^3*b^2)/16) + tan(c/2 + (d*x)/2)^5*((867*a*b^4)/32 + (2847*a^5)/160 - (289
1*a^3*b^2)/48) - tan(c/2 + (d*x)/2)^15*((867*a*b^4)/32 + (2847*a^5)/160 -
(2891*a^3*b^2)/48) + tan(c/2 + (d*x)/2)^9*((9395*a*b^4)/64 + (1827*a^5)/64
 - (7945*a^3*b^2)/32) - tan(c/2 + (d*x)/2)^11*((9395*a*b^4)/64 + (1827*a^5
)/64 - (7945*a^3*b^2)/32) + tan(c/2 + (d*x)/2)^6*(120*a^4*b + (32*b^5)/3 -
 80*a^2*b^3) + tan(c/2 + (d*x)/2)^14*(120*a^4*b + (32*b^5)/3 - 80*a^2*b^3)
 + tan(c/2 + (d*x)/2)^10*(252*a^4*b + (192*b^5)/5 - 224*a^2*b^3) - tan(c/2
 + (d*x)/2)^8*((64*b^5)/3 - 280*a^2*b^3) - tan(c/2 + (d*x)/2)^12*((64*b^5)
/3 - 280*a^2*b^3) + 40*a^2*b^3*tan(c/2 + (d*x)/2)^4 + 40*a^2*b^3*tan(c/2 +
 (d*x)/2)^16 + 10*a^4*b*tan(c/2 + (d*x)/2)^2 + 10*a^4*b*tan(c/2 + (d*x)/2)
^18)/(d*(10*tan(c/2 + (d*x)/2)^2 + 45*tan(c/2 + (d*x)/2)^4 + 120*tan(c/2 +
 (d*x)/2)^6 + 210*tan(c/2 + (d*x)/2)^8 + 252*tan(c/2 + (d*x)/2)^10 + 210*t
an(c/2 + (d*x)/2)^12 + 120*tan(c/2 + (d*x)/2)^14 + 45*tan(c/2 + (d*x)/2)^1
6 + 10*tan(c/2 + (d*x)/2)^18 + tan(c/2 + (d*x)/2)^20 + 1)) + (a*atan((a...
```

3.93 $\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

3.93.1 Optimal result

Integrand size = 28, antiderivative size = 337

$$\begin{aligned}
&\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx \\
&= -\frac{b^5\cos^5(c+dx)}{5d} - \frac{10a^2b^3\cos^7(c+dx)}{7d} + \frac{2b^5\cos^7(c+dx)}{7d} - \frac{5a^4b\cos^9(c+dx)}{9d} \\
&\quad + \frac{10a^2b^3\cos^9(c+dx)}{9d} - \frac{b^5\cos^9(c+dx)}{9d} + \frac{a^5\sin(c+dx)}{d} - \frac{4a^5\sin^3(c+dx)}{3d} \\
&\quad + \frac{10a^3b^2\sin^3(c+dx)}{3d} + \frac{6a^5\sin^5(c+dx)}{5d} - \frac{6a^3b^2\sin^5(c+dx)}{d} \\
&\quad + \frac{ab^4\sin^5(c+dx)}{d} - \frac{4a^5\sin^7(c+dx)}{7d} + \frac{30a^3b^2\sin^7(c+dx)}{7d} - \frac{10ab^4\sin^7(c+dx)}{7d} \\
&\quad + \frac{a^5\sin^9(c+dx)}{9d} - \frac{10a^3b^2\sin^9(c+dx)}{9d} + \frac{5ab^4\sin^9(c+dx)}{9d}
\end{aligned}$$

output
```
-1/5*b^5*cos(d*x+c)^5/d-10/7*a^2*b^3*cos(d*x+c)^7/d+2/7*b^5*cos(d*x+c)^7/d
-5/9*a^4*b*cos(d*x+c)^9/d+10/9*a^2*b^3*cos(d*x+c)^9/d-1/9*b^5*cos(d*x+c)^9
/d+a^5*sin(d*x+c)/d-4/3*a^5*sin(d*x+c)^3/d+10/3*a^3*b^2*sin(d*x+c)^3/d+6/5
*a^5*sin(d*x+c)^5/d-6*a^3*b^2*sin(d*x+c)^5/d+a*b^4*sin(d*x+c)^5/d-4/7*a^5*
sin(d*x+c)^7/d+30/7*a^3*b^2*sin(d*x+c)^7/d-10/7*a*b^4*sin(d*x+c)^7/d+1/9*a
^5*sin(d*x+c)^9/d-10/9*a^3*b^2*sin(d*x+c)^9/d+5/9*a*b^4*sin(d*x+c)^9/d
```

3.93.2 Mathematica [A] (verified)

Time = 6.21 (sec) , antiderivative size = 542, normalized size of antiderivative = 1.61

$$\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = -\frac{5a^4b\cos^9(c+dx)}{9d}+\frac{a^5\sin(c+dx)}{d} -\frac{4a^5\sin^3(c+dx)}{3d}+\frac{6a^5\sin^5(c+dx)}{5d}-\frac{4a^5\sin^7(c+dx)}{7d}+\frac{a^5\sin^9(c+dx)}{9d} +\frac{2a^3b^2(105\sin^3(c+dx)-189\sin^5(c+dx)+135\sin^7(c+dx)-35\sin^9(c+dx))}{63d} +\frac{ab^4(63\sin^5(c+dx)-90\sin^7(c+dx)+35\sin^9(c+dx))}{63d} +\frac{b^5\cos(c+dx)\sin^8(c+dx)\left(8\csc^8(c+dx)-35\sqrt{1-\sin^2(c+dx)}+50\csc^2(c+dx)\sqrt{1-\sin^2(c+dx)}\right.}{315d\sqrt{}} +\frac{10a^2b^3\cos(c+dx)\sin^8(c+dx)\left(2\csc^8(c+dx)+7\sqrt{1-\sin^2(c+dx)}-19\csc^2(c+dx)\sqrt{1-\sin^2(c+\right.}{63d\sqrt{}}$$

input
```
Integrate[Cos[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

output
```
(-5*a^4*b*Cos[c + d*x]^9)/(9*d) + (a^5*Sin[c + d*x])/d - (4*a^5*Sin[c + d*
x]^3)/(3*d) + (6*a^5*Sin[c + d*x]^5)/(5*d) - (4*a^5*Sin[c + d*x]^7)/(7*d)
+ (a^5*Sin[c + d*x]^9)/(9*d) + (2*a^3*b^2*(105*Sin[c + d*x]^3 - 189*Sin[c
+ d*x]^5 + 135*Sin[c + d*x]^7 - 35*Sin[c + d*x]^9))/(63*d) + (a*b^4*(63*Si
n[c + d*x]^5 - 90*Sin[c + d*x]^7 + 35*Sin[c + d*x]^9))/(63*d) + (b^5*Cos[c
 + d*x]*Sin[c + d*x]^8*(8*Csc[c + d*x]^8 - 35*Sqrt[1 - Sin[c + d*x]^2] + 5
0*Csc[c + d*x]^2*Sqrt[1 - Sin[c + d*x]^2] - 3*Csc[c + d*x]^4*Sqrt[1 - Sin[
c + d*x]^2] - 4*Csc[c + d*x]^6*Sqrt[1 - Sin[c + d*x]^2] - 8*Csc[c + d*x]^8
*Sqrt[1 - Sin[c + d*x]^2]))/(315*d*Sqrt[Cos[c + d*x]^2]) + (10*a^2*b^3*Cos
[c + d*x]*Sin[c + d*x]^8*(2*Csc[c + d*x]^8 + 7*Sqrt[1 - Sin[c + d*x]^2] -
19*Csc[c + d*x]^2*Sqrt[1 - Sin[c + d*x]^2] + 15*Csc[c + d*x]^4*Sqrt[1 - Si
n[c + d*x]^2] - Csc[c + d*x]^6*Sqrt[1 - Sin[c + d*x]^2] - 2*Csc[c + d*x]^8
*Sqrt[1 - Sin[c + d*x]^2]))/(63*d*Sqrt[Cos[c + d*x]^2])
```

3.93.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \cos(c+dx)^4(a\cos(c+dx)+b\sin(c+dx))^5dx$$

$$\Big\downarrow \; 3569$$

$$\int \left(a^5\cos^9(c+dx)+5a^4b\sin(c+dx)\cos^8(c+dx)+10a^3b^2\sin^2(c+dx)\cos^7(c+dx)+10a^2b^3\sin^3(c+dx)\cos^6(c$$

$$\Big\downarrow \; 2009$$

$$\frac{a^5\sin^9(c+dx)}{9d}-\frac{4a^5\sin^7(c+dx)}{7d}+\frac{6a^5\sin^5(c+dx)}{5d}-\frac{4a^5\sin^3(c+dx)}{3d}+\frac{a^5\sin(c+dx)}{d}-\frac{5a^4b\cos^9(c+dx)}{9d}-\frac{10a^3b^2\sin^9(c+dx)}{9d}+\frac{30a^3b^2\sin^7(c+dx)}{7d}-\frac{6a^3b^2\sin^5(c+dx)}{d}+\frac{10a^3b^2\sin^3(c+dx)}{3d}+\frac{10a^2b^3\cos^9(c+dx)}{9d}-\frac{10a^2b^3\cos^7(c+dx)}{7d}+\frac{5ab^4\sin^9(c+dx)}{9d}-\frac{10ab^4\sin^7(c+dx)}{7d}+\frac{ab^4\sin^5(c+dx)}{d}-\frac{b^5\cos^9(c+dx)}{9d}+\frac{2b^5\cos^7(c+dx)}{7d}-\frac{b^5\cos^5(c+dx)}{5d}$$

input
```
Int[Cos[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

output
```
-1/5*(b^5*Cos[c + d*x]^5)/d - (10*a^2*b^3*Cos[c + d*x]^7)/(7*d) + (2*b^5*C
os[c + d*x]^7)/(7*d) - (5*a^4*b*Cos[c + d*x]^9)/(9*d) + (10*a^2*b^3*Cos[c
+ d*x]^9)/(9*d) - (b^5*Cos[c + d*x]^9)/(9*d) + (a^5*Sin[c + d*x])/d - (4*a
^5*Sin[c + d*x]^3)/(3*d) + (10*a^3*b^2*Sin[c + d*x]^3)/(3*d) + (6*a^5*Sin[
c + d*x]^5)/(5*d) - (6*a^3*b^2*Sin[c + d*x]^5)/d + (a*b^4*Sin[c + d*x]^5)/
d - (4*a^5*Sin[c + d*x]^7)/(7*d) + (30*a^3*b^2*Sin[c + d*x]^7)/(7*d) - (10
*a*b^4*Sin[c + d*x]^7)/(7*d) + (a^5*Sin[c + d*x]^9)/(9*d) - (10*a^3*b^2*Si
n[c + d*x]^9)/(9*d) + (5*a*b^4*Sin[c + d*x]^9)/(9*d)
```

3.93.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

3.93.4 Maple [A] (verified)

Time = 1.87 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.70

method	result
parts	$\frac{a^5\left(\frac{128}{35}+\cos(dx+c)^8+\frac{8\cos(dx+c)^6}{7}+\frac{48\cos(dx+c)^4}{35}+\frac{64\cos(dx+c)^2}{35}\right)\sin(dx+c)}{9d}-\frac{b^5\left(\frac{\cos(dx+c)^9}{9}-\frac{2\cos(dx+c)^7}{7}+\frac{\cos(dx}{5}\right.}{d}$
derivativedivides	$\frac{\frac{a^5\left(\frac{128}{35}+\cos(dx+c)^8+\frac{8\cos(dx+c)^6}{7}+\frac{48\cos(dx+c)^4}{35}+\frac{64\cos(dx+c)^2}{35}\right)\sin(dx+c)}{9}-\frac{5a^4b\cos(dx+c)^9}{9}+10a^3b^2\left(-\frac{\sin(dx+c)\cos(dx}{9}\right.}{}$
default	$\frac{\frac{a^5\left(\frac{128}{35}+\cos(dx+c)^8+\frac{8\cos(dx+c)^6}{7}+\frac{48\cos(dx+c)^4}{35}+\frac{64\cos(dx+c)^2}{35}\right)\sin(dx+c)}{9}-\frac{5a^4b\cos(dx+c)^9}{9}+10a^3b^2\left(-\frac{\sin(dx+c)\cos(dx}{9}\right.}{}$
parallelrisch	$\frac{630\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{17}a^5-3150\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{16}a^4b+(1680a^5+8400a^3b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{15}-12600\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{14}a^2b^3+(9576}{}$
risch	$-\frac{5b\cos(9dx+9c)a^4}{2304d}+\frac{5b^3\cos(9dx+9c)a^2}{1152d}-\frac{5a^3\sin(9dx+9c)b^2}{1152d}+\frac{5a\sin(9dx+9c)b^4}{2304d}-\frac{5b\cos(7dx+7c)a^4}{256d}+15$
norman	$\frac{-\frac{350a^4b+200a^2b^3+16b^5}{315d}+\frac{2a^5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2a^5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{17}}{d}-\frac{40a^2b^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{14}}{d}-\frac{10a^4b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{16}}{d}-\frac{2\left(100a^2b^3-8\right.}{}}{}$

input
```
int(cos(d*x+c)^4*(cos(d*x+c)*a+b*sin(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
1/9*a^5/d*(128/35+cos(d*x+c)^8+8/7*cos(d*x+c)^6+48/35*cos(d*x+c)^4+64/35*c
os(d*x+c)^2)*sin(d*x+c)-b^5/d*(1/9*cos(d*x+c)^9-2/7*cos(d*x+c)^7+1/5*cos(d
*x+c)^5)+10*a^2*b^3/d*(1/9*cos(d*x+c)^9-1/7*cos(d*x+c)^7)-5/9*a^4*b*cos(d*
x+c)^9/d-10*a^3*b^2/d*(1/9*sin(d*x+c)^9-3/7*sin(d*x+c)^7+3/5*sin(d*x+c)^5-
1/3*sin(d*x+c)^3)+5*a*b^4/d*(1/9*sin(d*x+c)^9-2/7*sin(d*x+c)^7+1/5*sin(d*x
+c)^5)
```

3.93.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.64

$$\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = -\frac{63\,b^5\cos\left(dx+c\right)^5+35\left(5\,a^4b-10\,a^2b^3+b^5\right)\cos\left(dx+c\right)^9+90\left(5\,a^2b^3-b^5\right)\cos\left(dx+c\right)^7-\left(35\left(a^5-\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas"
)
```

output
```
-1/315*(63*b^5*cos(d*x + c)^5 + 35*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x +
c)^9 + 90*(5*a^2*b^3 - b^5)*cos(d*x + c)^7 - (35*(a^5 - 10*a^3*b^2 + 5*a*b
^4)*cos(d*x + c)^8 + 10*(4*a^5 + 5*a^3*b^2 - 25*a*b^4)*cos(d*x + c)^6 + 12
8*a^5 + 160*a^3*b^2 + 40*a*b^4 + 3*(16*a^5 + 20*a^3*b^2 + 5*a*b^4)*cos(d*x
 + c)^4 + 4*(16*a^5 + 20*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^2)*sin(d*x + c))/
d
```

3.93.6 Sympy [A] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.31

$$\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \begin{cases} \frac{128a^5\sin^9(c+dx)}{315d}+\frac{64a^5\sin^7(c+dx)\cos^2(c+dx)}{35d}+\frac{16a^5\sin^5(c+dx)\cos^4(c+dx)}{5d}+\frac{8a^5\sin^3(c+dx)\cos^6(c+dx)}{3d}+\frac{a^5\sin(c+dx)\cos^8(}{d} \\ x(a\cos(c)+b\sin(c))^5\cos^4(c) \end{cases}$$

input
```
integrate(cos(d*x+c)**4*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

output
```
Piecewise((128*a**5*sin(c + d*x)**9/(315*d) + 64*a**5*sin(c + d*x)**7*cos(
c + d*x)**2/(35*d) + 16*a**5*sin(c + d*x)**5*cos(c + d*x)**4/(5*d) + 8*a**
5*sin(c + d*x)**3*cos(c + d*x)**6/(3*d) + a**5*sin(c + d*x)*cos(c + d*x)**
8/d - 5*a**4*b*cos(c + d*x)**9/(9*d) + 32*a**3*b**2*sin(c + d*x)**9/(63*d)
 + 16*a**3*b**2*sin(c + d*x)**7*cos(c + d*x)**2/(7*d) + 4*a**3*b**2*sin(c
+ d*x)**5*cos(c + d*x)**4/d + 10*a**3*b**2*sin(c + d*x)**3*cos(c + d*x)**6
/(3*d) - 10*a**2*b**3*sin(c + d*x)**2*cos(c + d*x)**7/(7*d) - 20*a**2*b**3
*cos(c + d*x)**9/(63*d) + 8*a*b**4*sin(c + d*x)**9/(63*d) + 4*a*b**4*sin(c
 + d*x)**7*cos(c + d*x)**2/(7*d) + a*b**4*sin(c + d*x)**5*cos(c + d*x)**4/
d - b**5*sin(c + d*x)**4*cos(c + d*x)**5/(5*d) - 4*b**5*sin(c + d*x)**2*co
s(c + d*x)**7/(35*d) - 8*b**5*cos(c + d*x)**9/(315*d), Ne(d, 0)), (x*(a*co
s(c) + b*sin(c))**5*cos(c)**4, True))
```

3.93.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.66

$$\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx =$$

$$-\frac{175\,a^4b\cos\left(dx+c\right)^9-\left(35\,\sin\left(dx+c\right)^9-180\,\sin\left(dx+c\right)^7+378\,\sin\left(dx+c\right)^5-420\,\sin\left(dx+c\right)^3+\right.}{}$$

input
```
integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima"
)
```

output
```
-1/315*(175*a^4*b*cos(d*x + c)^9 - (35*sin(d*x + c)^9 - 180*sin(d*x + c)^7
 + 378*sin(d*x + c)^5 - 420*sin(d*x + c)^3 + 315*sin(d*x + c))*a^5 + 10*(3
5*sin(d*x + c)^9 - 135*sin(d*x + c)^7 + 189*sin(d*x + c)^5 - 105*sin(d*x +
 c)^3)*a^3*b^2 - 50*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^7)*a^2*b^3 - 5*(35*
sin(d*x + c)^9 - 90*sin(d*x + c)^7 + 63*sin(d*x + c)^5)*a*b^4 + (35*cos(d*
x + c)^9 - 90*cos(d*x + c)^7 + 63*cos(d*x + c)^5)*b^5)/d
```

3.93.8 Giac [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.93

$$\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = -\frac{(5\,a^4b-10\,a^2b^3+b^5)\cos(9\,dx+9\,c)}{2304\,d}-\frac{(35\,a^4b-30\,a^2b^3-b^5)\cos(7\,dx+7\,c)}{1792\,d}-\frac{(25\,a^4b-b^5)\cos(5\,dx+5\,c)}{320\,d}-\frac{(35\,a^4b+20\,a^2b^3+b^5)\cos(3\,dx+3\,c)}{192\,d}-\frac{(35\,a^4b+30\,a^2b^3+3\,b^5)\cos(dx+c)}{128\,d}+\frac{(a^5-10\,a^3b^2+5\,ab^4)\sin(9\,dx+9\,c)}{2304\,d}+\frac{(9\,a^5-50\,a^3b^2+5\,ab^4)\sin(7\,dx+7\,c)}{1792\,d}+\frac{(9\,a^5-20\,a^3b^2-5\,ab^4)\sin(5\,dx+5\,c)}{320\,d}+\frac{(21\,a^5-5\,ab^4)\sin(3\,dx+3\,c)}{192\,d}+\frac{(63\,a^5+70\,a^3b^2+15\,ab^4)\sin(dx+c)}{128\,d}$$

input
```
integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

output
```
-1/2304*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(9*d*x + 9*c)/d - 1/1792*(35*a^4*b
 - 30*a^2*b^3 - b^5)*cos(7*d*x + 7*c)/d - 1/320*(25*a^4*b - b^5)*cos(5*d*x
 + 5*c)/d - 1/192*(35*a^4*b + 20*a^2*b^3 + b^5)*cos(3*d*x + 3*c)/d - 1/128
*(35*a^4*b + 30*a^2*b^3 + 3*b^5)*cos(d*x + c)/d + 1/2304*(a^5 - 10*a^3*b^2
 + 5*a*b^4)*sin(9*d*x + 9*c)/d + 1/1792*(9*a^5 - 50*a^3*b^2 + 5*a*b^4)*sin
(7*d*x + 7*c)/d + 1/320*(9*a^5 - 20*a^3*b^2 - 5*a*b^4)*sin(5*d*x + 5*c)/d
+ 1/192*(21*a^5 - 5*a*b^4)*sin(3*d*x + 3*c)/d + 1/128*(63*a^5 + 70*a^3*b^2
 + 15*a*b^4)*sin(d*x + c)/d
```

3.93.9 Mupad [B] (verification not implemented)

Time = 26.80 (sec) , antiderivative size = 495, normalized size of antiderivative = 1.47

$$\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \frac{2\,a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\left(\frac{152\,a^5}{5}-32\,a^3\,b^2+32\,a\,b^4\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}\left(\frac{152\,a^5}{5}-32\,a^3\,b^2+32\,a\,b\right.}{}$$

input
```
int(cos(c + d*x)^4*(a*cos(c + d*x) + b*sin(c + d*x))^5,x)
```

output
```
(2*a^5*tan(c/2 + (d*x)/2)^17 + tan(c/2 + (d*x)/2)^5*(32*a*b^4 + (152*a^5)/
5 - 32*a^3*b^2) + tan(c/2 + (d*x)/2)^13*(32*a*b^4 + (152*a^5)/5 - 32*a^3*b
^2) + tan(c/2 + (d*x)/2)^7*((1136*a^5)/35 - (384*a*b^4)/7 + (1264*a^3*b^2)
/7) + tan(c/2 + (d*x)/2)^11*((1136*a^5)/35 - (384*a*b^4)/7 + (1264*a^3*b^2
)/7) + tan(c/2 + (d*x)/2)^9*((6976*a*b^4)/63 + (21316*a^5)/315 - (5696*a^3
*b^2)/63) - tan(c/2 + (d*x)/2)^4*(40*a^4*b + (64*b^5)/35 - (120*a^2*b^3)/7
) - tan(c/2 + (d*x)/2)^8*(140*a^4*b + (112*b^5)/5 - 120*a^2*b^3) - tan(c/2
 + (d*x)/2)^12*((280*a^4*b)/3 + (32*b^5)/3 - (200*a^2*b^3)/3) - (10*a^4*b)
/9 - (16*b^5)/315 - (40*a^2*b^3)/63 + tan(c/2 + (d*x)/2)^3*((16*a^5)/3 + (
80*a^3*b^2)/3) + tan(c/2 + (d*x)/2)^15*((16*a^5)/3 + (80*a^3*b^2)/3) + 2*a
^5*tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)/2)^2*((16*b^5)/35 + (40*a^2*b^3)/7
) + tan(c/2 + (d*x)/2)^6*((32*b^5)/5 - 120*a^2*b^3) + tan(c/2 + (d*x)/2)^1
0*(16*b^5 - 200*a^2*b^3) - 40*a^2*b^3*tan(c/2 + (d*x)/2)^14 - 10*a^4*b*tan
(c/2 + (d*x)/2)^16)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^9)
```

3.93. $\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

3.94 $\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

3.94.1 Optimal result

Integrand size = 28, antiderivative size = 426

$$\begin{aligned}
&\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx \\
&= \frac{35a^5x}{128}+\frac{25}{64}a^3b^2x+\frac{15}{128}ab^4x-\frac{5a^2b^3\cos^6(c+dx)}{3d}-\frac{5a^4b\cos^8(c+dx)}{8d} \\
&\quad+\frac{5a^2b^3\cos^8(c+dx)}{4d}+\frac{35a^5\cos(c+dx)\sin(c+dx)}{128d} \\
&\quad+\frac{25a^3b^2\cos(c+dx)\sin(c+dx)}{64d}+\frac{15ab^4\cos(c+dx)\sin(c+dx)}{128d} \\
&\quad+\frac{35a^5\cos^3(c+dx)\sin(c+dx)}{192d}+\frac{25a^3b^2\cos^3(c+dx)\sin(c+dx)}{96d} \\
&\quad+\frac{5ab^4\cos^3(c+dx)\sin(c+dx)}{64d}+\frac{7a^5\cos^5(c+dx)\sin(c+dx)}{48d} \\
&\quad+\frac{5a^3b^2\cos^5(c+dx)\sin(c+dx)}{24d}-\frac{5ab^4\cos^5(c+dx)\sin(c+dx)}{16d} \\
&\quad+\frac{a^5\cos^7(c+dx)\sin(c+dx)}{8d}-\frac{5a^3b^2\cos^7(c+dx)\sin(c+dx)}{4d} \\
&\quad-\frac{5ab^4\cos^5(c+dx)\sin^3(c+dx)}{8d}+\frac{b^5\sin^6(c+dx)}{6d}-\frac{b^5\sin^8(c+dx)}{8d}
\end{aligned}$$

output
```
35/128*a^5*x+25/64*a^3*b^2*x+15/128*a*b^4*x-5/3*a^2*b^3*cos(d*x+c)^6/d-5/8
*a^4*b*cos(d*x+c)^8/d+5/4*a^2*b^3*cos(d*x+c)^8/d+35/128*a^5*cos(d*x+c)*sin
(d*x+c)/d+25/64*a^3*b^2*cos(d*x+c)*sin(d*x+c)/d+15/128*a*b^4*cos(d*x+c)*si
n(d*x+c)/d+35/192*a^5*cos(d*x+c)^3*sin(d*x+c)/d+25/96*a^3*b^2*cos(d*x+c)^3
*sin(d*x+c)/d+5/64*a*b^4*cos(d*x+c)^3*sin(d*x+c)/d+7/48*a^5*cos(d*x+c)^5*s
in(d*x+c)/d+5/24*a^3*b^2*cos(d*x+c)^5*sin(d*x+c)/d-5/16*a*b^4*cos(d*x+c)^5
*sin(d*x+c)/d+1/8*a^5*cos(d*x+c)^7*sin(d*x+c)/d-5/4*a^3*b^2*cos(d*x+c)^7*s
in(d*x+c)/d-5/8*a*b^4*cos(d*x+c)^5*sin(d*x+c)^3/d+1/6*b^5*sin(d*x+c)^6/d-1
/8*b^5*sin(d*x+c)^8/d
```

3.94.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.57 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.61

$$\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$
$$= \frac{120a(a-ib)(a+ib)\left(7a^2+3b^2\right)(c+dx)-24b(35a^4+30a^2b^2+3b^4)\cos(2(c+dx))+12b(-35a^4-10a^2}{}$$

input
```
Integrate[Cos[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

output
```
(120*a*(a - I*b)*(a + I*b)*(7*a^2 + 3*b^2)*(c + d*x) - 24*b*(35*a^4 + 30*a
^2*b^2 + 3*b^4)*Cos[2*(c + d*x)] + 12*b*(-35*a^4 - 10*a^2*b^2 + b^4)*Cos[4
*(c + d*x)] + 8*b*(-15*a^4 + 10*a^2*b^2 + b^4)*Cos[6*(c + d*x)] - 3*b*(5*a
^4 - 10*a^2*b^2 + b^4)*Cos[8*(c + d*x)] + 96*a^3*(7*a^2 + 5*b^2)*Sin[2*(c
+ d*x)] + 24*a*(7*a^4 - 10*a^2*b^2 - 5*b^4)*Sin[4*(c + d*x)] + 32*a^3*(a^2
 - 5*b^2)*Sin[6*(c + d*x)] + 3*a*(a^4 - 10*a^2*b^2 + 5*b^4)*Sin[8*(c + d*x
)])/(3072*d)
```

3.94.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(c+dx)^3(a\cos(c+dx)+b\sin(c+dx))^5dx$$

$$\Big\downarrow \text{3569}$$

$$\int \left(a^5\cos^8(c+dx)+5a^4b\sin(c+dx)\cos^7(c+dx)+10a^3b^2\sin^2(c+dx)\cos^6(c+dx)+10a^2b^3\sin^3(c+dx)\cos^5(c+dx)\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^5\sin(c+dx)\cos^7(c+dx)}{8d}+\frac{7a^5\sin(c+dx)\cos^5(c+dx)}{48d}+\frac{35a^5\sin(c+dx)\cos^3(c+dx)}{192d}+\frac{35a^5\sin(c+dx)\cos(c+dx)}{128d}+\frac{35a^5x}{128}-\frac{5a^4b\cos^8(c+dx)}{8d}-\frac{5a^3b^2\sin(c+dx)\cos^7(c+dx)}{4d}+\frac{5a^3b^2\sin(c+dx)\cos^5(c+dx)}{24d}+\frac{25a^3b^2\sin(c+dx)\cos^3(c+dx)}{96d}+\frac{25a^3b^2\sin(c+dx)\cos(c+dx)}{64d}+\frac{25}{64}a^3b^2x+\frac{5a^2b^3\cos^8(c+dx)}{4d}-\frac{5a^2b^3\cos^6(c+dx)}{3d}-\frac{5ab^4\sin^3(c+dx)\cos^5(c+dx)}{8d}-\frac{5ab^4\sin(c+dx)\cos^5(c+dx)}{16d}+\frac{5ab^4\sin(c+dx)\cos^3(c+dx)}{64d}+\frac{15ab^4\sin(c+dx)\cos(c+dx)}{128d}+\frac{15}{128}ab^4x-\frac{b^5\sin^8(c+dx)}{8d}+\frac{b^5\sin^6(c+dx)}{6d}$$

input
```
Int[Cos[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

output
```
(35*a^5*x)/128 + (25*a^3*b^2*x)/64 + (15*a*b^4*x)/128 - (5*a^2*b^3*Cos[c +
 d*x]^6)/(3*d) - (5*a^4*b*Cos[c + d*x]^8)/(8*d) + (5*a^2*b^3*Cos[c + d*x]^
8)/(4*d) + (35*a^5*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (25*a^3*b^2*Cos[c
+ d*x]*Sin[c + d*x])/(64*d) + (15*a*b^4*Cos[c + d*x]*Sin[c + d*x])/(128*d)
 + (35*a^5*Cos[c + d*x]^3*Sin[c + d*x])/(192*d) + (25*a^3*b^2*Cos[c + d*x]
^3*Sin[c + d*x])/(96*d) + (5*a*b^4*Cos[c + d*x]^3*Sin[c + d*x])/(64*d) + (
7*a^5*Cos[c + d*x]^5*Sin[c + d*x])/(48*d) + (5*a^3*b^2*Cos[c + d*x]^5*Sin[
c + d*x])/(24*d) - (5*a*b^4*Cos[c + d*x]^5*Sin[c + d*x])/(16*d) + (a^5*Cos
[c + d*x]^7*Sin[c + d*x])/(8*d) - (5*a^3*b^2*Cos[c + d*x]^7*Sin[c + d*x])/
(4*d) - (5*a*b^4*Cos[c + d*x]^5*Sin[c + d*x]^3)/(8*d) + (b^5*Sin[c + d*x]^
6)/(6*d) - (b^5*Sin[c + d*x]^8)/(8*d)
```

3.94. $\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

3.94.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

3.94.4 Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.65

method	result
parallelrisch	$\frac{24(-35a^4b-30a^2b^3-3b^5)\cos(2dx+2c)+12(-35a^4b-10a^2b^3+b^5)\cos(4dx+4c)+8(-15a^4b+10a^2b^3+b^5)\cos(6dx+6c)+}{}$
parts	$\frac{a^5\left(\frac{\left(\cos(dx+c)^7+\frac{7\cos(dx+c)^5}{6}+\frac{35\cos(dx+c)^3}{24}+\frac{35\cos(dx+c)}{16}\right)\sin(dx+c)}{8}+\frac{35dx}{128}+\frac{35c}{128}\right)}{d}+\frac{b^5\left(-\frac{\sin(dx+c)^8}{8}+\frac{\sin(dx+c)^6}{6}\right)}{d}$
derivativedivides	$\frac{a^5\left(\frac{\left(\cos(dx+c)^7+\frac{7\cos(dx+c)^5}{6}+\frac{35\cos(dx+c)^3}{24}+\frac{35\cos(dx+c)}{16}\right)\sin(dx+c)}{8}+\frac{35dx}{128}+\frac{35c}{128}\right)-\frac{5\cos(dx+c)^8a^4b}{8}+10a^3b^2\left(-\frac{\sin(d}{}\right.}{}$
default	$\frac{a^5\left(\frac{\left(\cos(dx+c)^7+\frac{7\cos(dx+c)^5}{6}+\frac{35\cos(dx+c)^3}{24}+\frac{35\cos(dx+c)}{16}\right)\sin(dx+c)}{8}+\frac{35dx}{128}+\frac{35c}{128}\right)-\frac{5\cos(dx+c)^8a^4b}{8}+10a^3b^2\left(-\frac{\sin(d}{}\right.}{}$
risch	$\frac{25a^3b^2x}{64}+\frac{15a\,b^4x}{128}+\frac{b^5\cos(6dx+6c)}{384d}-\frac{3b^5\cos(2dx+2c)}{128d}+\frac{7a^5\sin(2dx+2c)}{32d}-\frac{5a^3\sin(4dx+4c)b^2}{64d}-\frac{5a\sin(4}{12}$
norman	$\frac{(\frac{35}{128}a^5+\frac{25}{64}a^3b^2+\frac{15}{128}a\,b^4)x+(\frac{35}{16}a^5+\frac{25}{8}a^3b^2+\frac{15}{16}a\,b^4)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+(\frac{35}{16}a^5+\frac{25}{8}a^3b^2+\frac{15}{16}a\,b^4)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{14}+(\frac{35}{128}}{}$

input
```
int(cos(d*x+c)^3*(cos(d*x+c)*a+b*sin(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
1/3072*(24*(-35*a^4*b-30*a^2*b^3-3*b^5)*cos(2*d*x+2*c)+12*(-35*a^4*b-10*a^
2*b^3+b^5)*cos(4*d*x+4*c)+8*(-15*a^4*b+10*a^2*b^3+b^5)*cos(6*d*x+6*c)+3*(-
5*a^4*b+10*a^2*b^3-b^5)*cos(8*d*x+8*c)+24*(7*a^5-10*a^3*b^2-5*a*b^4)*sin(4
*d*x+4*c)+3*(a^5-10*a^3*b^2+5*a*b^4)*sin(8*d*x+8*c)+96*(7*a^5+5*a^3*b^2)*s
in(2*d*x+2*c)+32*(a^5-5*a^3*b^2)*sin(6*d*x+6*c)+840*a^5*d*x+1200*a^3*b^2*d
*x+360*a*b^4*d*x+1395*a^4*b+730*a^2*b^3+55*b^5)/d
```

3.94.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.52

$$\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx =$$

$$-\frac{96\,b^5\cos{(dx+c)}^4+48\,(5\,a^4b-10\,a^2b^3+b^5)\cos{(dx+c)}^8+128\,(5\,a^2b^3-b^5)\cos{(dx+c)}^6-15\,(7\,a^5-$$

input
```
integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas"
)
```

output
```
-1/384*(96*b^5*cos(d*x + c)^4 + 48*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x +
c)^8 + 128*(5*a^2*b^3 - b^5)*cos(d*x + c)^6 - 15*(7*a^5 + 10*a^3*b^2 + 3*a
*b^4)*d*x - (48*(a^5 - 10*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^7 + 8*(7*a^5 + 1
0*a^3*b^2 - 45*a*b^4)*cos(d*x + c)^5 + 10*(7*a^5 + 10*a^3*b^2 + 3*a*b^4)*c
os(d*x + c)^3 + 15*(7*a^5 + 10*a^3*b^2 + 3*a*b^4)*cos(d*x + c))*sin(d*x +
c))/d
```

3.94.6 Sympy [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 826, normalized size of antiderivative = 1.94

$$\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$

$$= \begin{cases} \frac{35a^5x\sin^8{(c+dx)}}{128} + \frac{35a^5x\sin^6{(c+dx)}\cos^2{(c+dx)}}{32} + \frac{105a^5x\sin^4{(c+dx)}\cos^4{(c+dx)}}{64} + \frac{35a^5x\sin^2{(c+dx)}\cos^6{(c+dx)}}{32} + \frac{35a^5x\cos^8{(}}{128} \\ x(a\cos{(c)}+b\sin{(c)})^5\cos^3{(c)} \end{cases}$$

input
```
integrate(cos(d*x+c)**3*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

output
```
Piecewise((35*a**5*x*sin(c + d*x)**8/128 + 35*a**5*x*sin(c + d*x)**6*cos(c
 + d*x)**2/32 + 105*a**5*x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 35*a**5*x*
sin(c + d*x)**2*cos(c + d*x)**6/32 + 35*a**5*x*cos(c + d*x)**8/128 + 35*a*
*5*sin(c + d*x)**7*cos(c + d*x)/(128*d) + 385*a**5*sin(c + d*x)**5*cos(c +
 d*x)**3/(384*d) + 511*a**5*sin(c + d*x)**3*cos(c + d*x)**5/(384*d) + 93*a
**5*sin(c + d*x)*cos(c + d*x)**7/(128*d) - 5*a**4*b*cos(c + d*x)**8/(8*d)
+ 25*a**3*b**2*x*sin(c + d*x)**8/64 + 25*a**3*b**2*x*sin(c + d*x)**6*cos(c
 + d*x)**2/16 + 75*a**3*b**2*x*sin(c + d*x)**4*cos(c + d*x)**4/32 + 25*a**
3*b**2*x*sin(c + d*x)**2*cos(c + d*x)**6/16 + 25*a**3*b**2*x*cos(c + d*x)*
*8/64 + 25*a**3*b**2*sin(c + d*x)**7*cos(c + d*x)/(64*d) + 275*a**3*b**2*s
in(c + d*x)**5*cos(c + d*x)**3/(192*d) + 365*a**3*b**2*sin(c + d*x)**3*cos
(c + d*x)**5/(192*d) - 25*a**3*b**2*sin(c + d*x)*cos(c + d*x)**7/(64*d) +
5*a**2*b**3*sin(c + d*x)**8/(12*d) + 5*a**2*b**3*sin(c + d*x)**6*cos(c + d
*x)**2/(3*d) + 5*a**2*b**3*sin(c + d*x)**4*cos(c + d*x)**4/(2*d) + 15*a*b*
*4*x*sin(c + d*x)**8/128 + 15*a*b**4*x*sin(c + d*x)**6*cos(c + d*x)**2/32
+ 45*a*b**4*x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 15*a*b**4*x*sin(c + d*x
)**2*cos(c + d*x)**6/32 + 15*a*b**4*x*cos(c + d*x)**8/128 + 15*a*b**4*sin(
c + d*x)**7*cos(c + d*x)/(128*d) + 55*a*b**4*sin(c + d*x)**5*cos(c + d*x)*
*3/(128*d) - 55*a*b**4*sin(c + d*x)**3*cos(c + d*x)**5/(128*d) - 15*a*b**4
*sin(c + d*x)*cos(c + d*x)**7/(128*d) + b**5*sin(c + d*x)**8/(24*d) + b...
```

3.94.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.54

$$\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx =$$

$$-\frac{1920\,a^4b\cos\left(dx+c\right)^8+\left(128\,\sin\left(2\,dx+2\,c\right)^3-840\,dx-840\,c-3\,\sin\left(8\,dx+8\,c\right)-168\,\sin\left(4\,dx+4\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima"
)
```

output
```
-1/3072*(1920*a^4*b*cos(d*x + c)^8 + (128*sin(2*d*x + 2*c)^3 - 840*d*x - 8
40*c - 3*sin(8*d*x + 8*c) - 168*sin(4*d*x + 4*c) - 768*sin(2*d*x + 2*c))*a
^5 - 10*(64*sin(2*d*x + 2*c)^3 + 120*d*x + 120*c - 3*sin(8*d*x + 8*c) - 24
*sin(4*d*x + 4*c))*a^3*b^2 - 1280*(3*sin(d*x + c)^8 - 8*sin(d*x + c)^6 + 6
*sin(d*x + c)^4)*a^2*b^3 - 15*(24*d*x + 24*c + sin(8*d*x + 8*c) - 8*sin(4*
d*x + 4*c))*a*b^4 + 128*(3*sin(d*x + c)^8 - 4*sin(d*x + c)^6)*b^5)/d
```

3.94.8 Giac [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.65

$$\begin{aligned}&\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx\\&=\frac{5}{128}\left(7a^5+10a^3b^2+3ab^4\right)x-\frac{(5a^4b-10a^2b^3+b^5)\cos(8dx+8c)}{1024d}\\&-\frac{(15a^4b-10a^2b^3-b^5)\cos(6dx+6c)}{384d}-\frac{(35a^4b+10a^2b^3-b^5)\cos(4dx+4c)}{256d}\\&-\frac{(35a^4b+30a^2b^3+3b^5)\cos(2dx+2c)}{128d}\\&+\frac{(a^5-10a^3b^2+5ab^4)\sin(8dx+8c)}{1024d}+\frac{(a^5-5a^3b^2)\sin(6dx+6c)}{96d}\\&+\frac{(7a^5-10a^3b^2-5ab^4)\sin(4dx+4c)}{128d}+\frac{(7a^5+5a^3b^2)\sin(2dx+2c)}{32d}\end{aligned}$$

input
```
integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

output
```
5/128*(7*a^5 + 10*a^3*b^2 + 3*a*b^4)*x - 1/1024*(5*a^4*b - 10*a^2*b^3 + b^
5)*cos(8*d*x + 8*c)/d - 1/384*(15*a^4*b - 10*a^2*b^3 - b^5)*cos(6*d*x + 6*
c)/d - 1/256*(35*a^4*b + 10*a^2*b^3 - b^5)*cos(4*d*x + 4*c)/d - 1/128*(35*
a^4*b + 30*a^2*b^3 + 3*b^5)*cos(2*d*x + 2*c)/d + 1/1024*(a^5 - 10*a^3*b^2
+ 5*a*b^4)*sin(8*d*x + 8*c)/d + 1/96*(a^5 - 5*a^3*b^2)*sin(6*d*x + 6*c)/d
+ 1/128*(7*a^5 - 10*a^3*b^2 - 5*a*b^4)*sin(4*d*x + 4*c)/d + 1/32*(7*a^5 +
5*a^3*b^2)*sin(2*d*x + 2*c)/d
```

3.94.9 Mupad [B] (verification not implemented)

Time = 26.06 (sec) , antiderivative size = 650, normalized size of antiderivative = 1.53

$$\begin{aligned}&\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx\\&=\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}\left(-\frac{93a^5}{64}+\frac{25a^3b^2}{32}+\frac{15ab^4}{64}\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(-\frac{93a^5}{64}+\frac{25a^3b^2}{32}+\frac{15ab^4}{64}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{91a^5}{192}\cdots\right.}{\cdots}\\&-\frac{5a\left(\operatorname{atan}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)-\frac{dx}{2}\right)\left(7a^4+10a^2b^2+3b^4\right)}{64d}\\&+\frac{5a\operatorname{atan}\left(\frac{5a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(7a^2+3b^2\right)\left(a^2+b^2\right)}{64\left(\frac{35a^5}{64}+\frac{25a^3b^2}{32}+\frac{15ab^4}{64}\right)}\right)\left(7a^2+3b^2\right)\left(a^2+b^2\right)}{64d}\end{aligned}$$

input
```
int(cos(c + d*x)^3*(a*cos(c + d*x) + b*sin(c + d*x))^5,x)
```

output
```
(tan(c/2 + (d*x)/2)^15*((15*a*b^4)/64 - (93*a^5)/64 + (25*a^3*b^2)/32) - t
an(c/2 + (d*x)/2)*((15*a*b^4)/64 - (93*a^5)/64 + (25*a^3*b^2)/32) + tan(c/
2 + (d*x)/2)^3*((91*a^5)/192 - (115*a*b^4)/64 + (1985*a^3*b^2)/96) - tan(c
/2 + (d*x)/2)^13*((91*a^5)/192 - (115*a*b^4)/64 + (1985*a^3*b^2)/96) + tan
(c/2 + (d*x)/2)^5*((1665*a*b^4)/64 + (1799*a^5)/192 - (4475*a^3*b^2)/96) -
 tan(c/2 + (d*x)/2)^11*((1665*a*b^4)/64 + (1799*a^5)/192 - (4475*a^3*b^2)/
96) - tan(c/2 + (d*x)/2)^7*((3355*a*b^4)/64 + (1085*a^5)/192 - (8825*a^3*b
^2)/96) + tan(c/2 + (d*x)/2)^9*((3355*a*b^4)/64 + (1085*a^5)/192 - (8825*a
^3*b^2)/96) + tan(c/2 + (d*x)/2)^6*(70*a^4*b + (32*b^5)/3 - (160*a^2*b^3)/
3) + tan(c/2 + (d*x)/2)^10*(70*a^4*b + (32*b^5)/3 - (160*a^2*b^3)/3) - tan
(c/2 + (d*x)/2)^8*((32*b^5)/3 - (400*a^2*b^3)/3) + 40*a^2*b^3*tan(c/2 + (d
*x)/2)^4 + 40*a^2*b^3*tan(c/2 + (d*x)/2)^12 + 10*a^4*b*tan(c/2 + (d*x)/2)^
2 + 10*a^4*b*tan(c/2 + (d*x)/2)^14)/(d*(8*tan(c/2 + (d*x)/2)^2 + 28*tan(c/
2 + (d*x)/2)^4 + 56*tan(c/2 + (d*x)/2)^6 + 70*tan(c/2 + (d*x)/2)^8 + 56*ta
n(c/2 + (d*x)/2)^10 + 28*tan(c/2 + (d*x)/2)^12 + 8*tan(c/2 + (d*x)/2)^14 +
 tan(c/2 + (d*x)/2)^16 + 1)) - (5*a*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2)*(
7*a^4 + 3*b^4 + 10*a^2*b^2))/(64*d) + (5*a*atan((5*a*tan(c/2 + (d*x)/2)*(7
*a^2 + 3*b^2)*(a^2 + b^2))/(64*((15*a*b^4)/64 + (35*a^5)/64 + (25*a^3*b^2)
/32)))*(7*a^2 + 3*b^2)*(a^2 + b^2))/(64*d)
```

3.94. $\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

3.95 $\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

3.95.1 Optimal result

Integrand size = 28, antiderivative size = 275

$$\begin{aligned}
&\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx \\
&= -\frac{b^5\cos^3(c+dx)}{3d}-\frac{2a^2b^3\cos^5(c+dx)}{d}+\frac{2b^5\cos^5(c+dx)}{5d}-\frac{5a^4b\cos^7(c+dx)}{7d} \\
&\quad+\frac{10a^2b^3\cos^7(c+dx)}{7d}-\frac{b^5\cos^7(c+dx)}{7d}+\frac{a^5\sin(c+dx)}{d}-\frac{a^5\sin^3(c+dx)}{d} \\
&\quad+\frac{10a^3b^2\sin^3(c+dx)}{3d}+\frac{3a^5\sin^5(c+dx)}{5d}-\frac{4a^3b^2\sin^5(c+dx)}{d} \\
&\quad+\frac{ab^4\sin^5(c+dx)}{d}-\frac{a^5\sin^7(c+dx)}{7d}+\frac{10a^3b^2\sin^7(c+dx)}{7d}-\frac{5ab^4\sin^7(c+dx)}{7d}
\end{aligned}$$

output
```
-1/3*b^5*cos(d*x+c)^3/d-2*a^2*b^3*cos(d*x+c)^5/d+2/5*b^5*cos(d*x+c)^5/d-5/
7*a^4*b*cos(d*x+c)^7/d+10/7*a^2*b^3*cos(d*x+c)^7/d-1/7*b^5*cos(d*x+c)^7/d+
a^5*sin(d*x+c)/d-a^5*sin(d*x+c)^3/d+10/3*a^3*b^2*sin(d*x+c)^3/d+3/5*a^5*si
n(d*x+c)^5/d-4*a^3*b^2*sin(d*x+c)^5/d+a*b^4*sin(d*x+c)^5/d-1/7*a^5*sin(d*x
+c)^7/d+10/7*a^3*b^2*sin(d*x+c)^7/d-5/7*a*b^4*sin(d*x+c)^7/d
```

3.95.2 Mathematica [A] (verified)

Time = 6.21 (sec) , antiderivative size = 450, normalized size of antiderivative = 1.64

$$\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$
$$= -\frac{5a^4b\cos^7(c+dx)}{7d} + \frac{a^5\sin(c+dx)}{d} - \frac{a^5\sin^3(c+dx)}{d} + \frac{3a^5\sin^5(c+dx)}{5d} - \frac{a^5\sin^7(c+dx)}{7d} + \frac{ab^4(7\sin^5(c+dx)-5\sin^7(c+dx))}{7d} + \frac{2a^3b^2(35\sin^3(c+dx)-42\sin^5(c+dx)+15\sin^7(c+dx))}{21d} + \frac{b^5\cos(c+dx)\sin^6(c+dx)\left(8\csc^6(c+dx)+15\sqrt{1-\sin^2(c+dx)}-3\csc^2(c+dx)\sqrt{1-\sin^2(c+dx)}\right.}{105d\sqrt{\cos^2(c+dx)}} + \frac{2a^2b^3\cos(c+dx)\sin^6(c+dx)\left(2\csc^6(c+dx)-5\sqrt{1-\sin^2(c+dx)}+8\csc^2(c+dx)\sqrt{1-\sin^2(c+dx}\right.}{7d\sqrt{\cos^2(c+dx)}}$$

input
```
Integrate[Cos[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

output
```
(-5*a^4*b*Cos[c + d*x]^7)/(7*d) + (a^5*Sin[c + d*x])/d - (a^5*Sin[c + d*x]
^3)/d + (3*a^5*Sin[c + d*x]^5)/(5*d) - (a^5*Sin[c + d*x]^7)/(7*d) + (a*b^4
*(7*Sin[c + d*x]^5 - 5*Sin[c + d*x]^7))/(7*d) + (2*a^3*b^2*(35*Sin[c + d*x
]^3 - 42*Sin[c + d*x]^5 + 15*Sin[c + d*x]^7))/(21*d) + (b^5*Cos[c + d*x]*S
in[c + d*x]^6*(8*Csc[c + d*x]^6 + 15*Sqrt[1 - Sin[c + d*x]^2] - 3*Csc[c +
d*x]^2*Sqrt[1 - Sin[c + d*x]^2] - 4*Csc[c + d*x]^4*Sqrt[1 - Sin[c + d*x]^2
] - 8*Csc[c + d*x]^6*Sqrt[1 - Sin[c + d*x]^2]))/(105*d*Sqrt[Cos[c + d*x]^2
]) + (2*a^2*b^3*Cos[c + d*x]*Sin[c + d*x]^6*(2*Csc[c + d*x]^6 - 5*Sqrt[1 -
 Sin[c + d*x]^2] + 8*Csc[c + d*x]^2*Sqrt[1 - Sin[c + d*x]^2] - Csc[c + d*x
]^4*Sqrt[1 - Sin[c + d*x]^2] - 2*Csc[c + d*x]^6*Sqrt[1 - Sin[c + d*x]^2]))
/(7*d*Sqrt[Cos[c + d*x]^2])
```

3.95.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))^5dx$$

$$\Big\downarrow \text{3569}$$

$$\int \left(a^5\cos^7(c+dx)+5a^4b\sin(c+dx)\cos^6(c+dx)+10a^3b^2\sin^2(c+dx)\cos^5(c+dx)+10a^2b^3\sin^3(c+dx)\cos^4(\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5\sin^7(c+dx)}{7d}+\frac{3a^5\sin^5(c+dx)}{5d}-\frac{a^5\sin^3(c+dx)}{d}+\frac{a^5\sin(c+dx)}{d}-\frac{5a^4b\cos^7(c+dx)}{7d}+\frac{10a^3b^2\sin^7(c+dx)}{7d}-\frac{4a^3b^2\sin^5(c+dx)}{d}+\frac{10a^3b^2\sin^3(c+dx)}{3d}+\frac{10a^2b^3\cos^7(c+dx)}{7d}-\frac{2a^2b^3\cos^5(c+dx)}{d}-\frac{5ab^4\sin^7(c+dx)}{7d}+\frac{ab^4\sin^5(c+dx)}{d}-\frac{b^5\cos^7(c+dx)}{7d}+\frac{2b^5\cos^5(c+dx)}{5d}-\frac{b^5\cos^3(c+dx)}{3d}$$

input
```
Int[Cos[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

output
```
-1/3*(b^5*Cos[c + d*x]^3)/d - (2*a^2*b^3*Cos[c + d*x]^5)/d + (2*b^5*Cos[c
+ d*x]^5)/(5*d) - (5*a^4*b*Cos[c + d*x]^7)/(7*d) + (10*a^2*b^3*Cos[c + d*x
]^7)/(7*d) - (b^5*Cos[c + d*x]^7)/(7*d) + (a^5*Sin[c + d*x])/d - (a^5*Sin[
c + d*x]^3)/d + (10*a^3*b^2*Sin[c + d*x]^3)/(3*d) + (3*a^5*Sin[c + d*x]^5)
/(5*d) - (4*a^3*b^2*Sin[c + d*x]^5)/d + (a*b^4*Sin[c + d*x]^5)/d - (a^5*Si
n[c + d*x]^7)/(7*d) + (10*a^3*b^2*Sin[c + d*x]^7)/(7*d) - (5*a*b^4*Sin[c +
 d*x]^7)/(7*d)
```

3.95.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

3.95.4 Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.75

method	result
parts	$\frac{a^5\left(\frac{16}{5}+\cos(dx+c)^6+\frac{6\cos(dx+c)^4}{5}+\frac{8\cos(dx+c)^2}{5}\right)\sin(dx+c)}{7d}-\frac{b^5\left(\frac{\cos(dx+c)^7}{7}-\frac{2\cos(dx+c)^5}{5}+\frac{\cos(dx+c)^3}{3}\right)}{d}+$ $10a^2b$
derivativedivides	$\frac{a^5\left(\frac{16}{5}+\cos(dx+c)^6+\frac{6\cos(dx+c)^4}{5}+\frac{8\cos(dx+c)^2}{5}\right)\sin(dx+c)}{7}-\frac{5a^4b\cos(dx+c)^7}{7}+10a^3b^2\Bigg(-\frac{\sin(dx+c)\cos(dx+c)^6}{7}+$ $\left(\frac{8}{3}+\cos(d\right.$
default	$\frac{a^5\left(\frac{16}{5}+\cos(dx+c)^6+\frac{6\cos(dx+c)^4}{5}+\frac{8\cos(dx+c)^2}{5}\right)\sin(dx+c)}{7}-\frac{5a^4b\cos(dx+c)^7}{7}+10a^3b^2\Bigg(-\frac{\sin(dx+c)\cos(dx+c)^6}{7}+$ $\left(\frac{8}{3}+\cos(d\right.$
parallelrisch	$210\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{13}a^5-1050\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}a^4b+(420a^5+2800a^3b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}-4200\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}a^2b^3+(1806a^5$
norman	$-\frac{150a^4b+120a^2b^3+16b^5}{105d}+\frac{2a^5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2a^5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{13}}{d}-\frac{40a^2b^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{d}-\frac{10a^4b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{d}-$ $(120a^2b^3+16$
risch	$-\frac{25a^4b\cos(dx+c)}{64d}-\frac{15a^2b^3\cos(dx+c)}{32d}-\frac{5b^5\cos(dx+c)}{64d}+\frac{35a^5\sin(dx+c)}{64d}+\frac{25a^3b^2\sin(dx+c)}{32d}+\frac{15ab^4\sin(d}{64d}$

input
```
int(cos(d*x+c)^2*(cos(d*x+c)*a+b*sin(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
1/7*a^5/d*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c)
-b^5/d*(1/7*cos(d*x+c)^7-2/5*cos(d*x+c)^5+1/3*cos(d*x+c)^3)+10*a^2*b^3/d*(
1/7*cos(d*x+c)^7-1/5*cos(d*x+c)^5)-5/7*a^4*b*cos(d*x+c)^7/d+10*a^3*b^2/d*(
1/7*sin(d*x+c)^7-2/5*sin(d*x+c)^5+1/3*sin(d*x+c)^3)+5*a*b^4/d*(-1/7*sin(d*
x+c)^7+1/5*sin(d*x+c)^5)
```

3.95.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.68

$$\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx =$$
$$-\frac{35\,b^5\cos\left(dx+c\right)^3+15\left(5\,a^4b-10\,a^2b^3+b^5\right)\cos\left(dx+c\right)^7+42\left(5\,a^2b^3-b^5\right)\cos\left(dx+c\right)^5-\left(15\left(a^5-\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas"
)
```

output
```
-1/105*(35*b^5*cos(d*x + c)^3 + 15*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x +
c)^7 + 42*(5*a^2*b^3 - b^5)*cos(d*x + c)^5 - (15*(a^5 - 10*a^3*b^2 + 5*a*b
^4)*cos(d*x + c)^6 + 48*a^5 + 80*a^3*b^2 + 30*a*b^4 + 6*(3*a^5 + 5*a^3*b^2
 - 20*a*b^4)*cos(d*x + c)^4 + (24*a^5 + 40*a^3*b^2 + 15*a*b^4)*cos(d*x + c
)^2)*sin(d*x + c))/d
```

3.95.6 Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.30

$$\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$
$$=\begin{cases}\frac{16a^5\sin^7(c+dx)}{35d}+\frac{8a^5\sin^5(c+dx)\cos^2(c+dx)}{5d}+\frac{2a^5\sin^3(c+dx)\cos^4(c+dx)}{d}+\frac{a^5\sin(c+dx)\cos^6(c+dx)}{d}-\frac{5a^4b\cos^7(c+dx)}{7d}+\frac{16a}{} \\ x(a\cos(c)+b\sin(c))^5\cos^2(c)\end{cases}$$

input
```
integrate(cos(d*x+c)**2*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

output
```
Piecewise((16*a**5*sin(c + d*x)**7/(35*d) + 8*a**5*sin(c + d*x)**5*cos(c +
 d*x)**2/(5*d) + 2*a**5*sin(c + d*x)**3*cos(c + d*x)**4/d + a**5*sin(c + d
*x)*cos(c + d*x)**6/d - 5*a**4*b*cos(c + d*x)**7/(7*d) + 16*a**3*b**2*sin(
c + d*x)**7/(21*d) + 8*a**3*b**2*sin(c + d*x)**5*cos(c + d*x)**2/(3*d) + 1
0*a**3*b**2*sin(c + d*x)**3*cos(c + d*x)**4/(3*d) - 2*a**2*b**3*sin(c + d*
x)**2*cos(c + d*x)**5/d - 4*a**2*b**3*cos(c + d*x)**7/(7*d) + 2*a*b**4*sin
(c + d*x)**7/(7*d) + a*b**4*sin(c + d*x)**5*cos(c + d*x)**2/d - b**5*sin(c
 + d*x)**4*cos(c + d*x)**3/(3*d) - 4*b**5*sin(c + d*x)**2*cos(c + d*x)**5/
(15*d) - 8*b**5*cos(c + d*x)**7/(105*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c
))**5*cos(c)**2, True))
```

3.95.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.71

$$\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx =$$

$$-\frac{75\,a^4b\cos\left(dx+c\right)^7+3\left(5\,\sin\left(dx+c\right)^7-21\,\sin\left(dx+c\right)^5+35\,\sin\left(dx+c\right)^3-35\,\sin\left(dx+c\right)\right)a^5-10}{}$$

input
```
integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima"
)
```

output
```
-1/105*(75*a^4*b*cos(d*x + c)^7 + 3*(5*sin(d*x + c)^7 - 21*sin(d*x + c)^5
+ 35*sin(d*x + c)^3 - 35*sin(d*x + c))*a^5 - 10*(15*sin(d*x + c)^7 - 42*si
n(d*x + c)^5 + 35*sin(d*x + c)^3)*a^3*b^2 - 30*(5*cos(d*x + c)^7 - 7*cos(d
*x + c)^5)*a^2*b^3 + 15*(5*sin(d*x + c)^7 - 7*sin(d*x + c)^5)*a*b^4 + (15*
cos(d*x + c)^7 - 42*cos(d*x + c)^5 + 35*cos(d*x + c)^3)*b^5)/d
```

3.95.8 Giac [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.94

$$\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$
$$= -\frac{(5\,a^4b-10\,a^2b^3+b^5)\cos(7\,dx+7\,c)}{448\,d} - \frac{(25\,a^4b-10\,a^2b^3-3\,b^5)\cos(5\,dx+5\,c)}{320\,d} - \frac{(45\,a^4b+30\,a^2b^3+b^5)\cos(3\,dx+3\,c)}{192\,d} - \frac{5\,(5\,a^4b+6\,a^2b^3+b^5)\cos(dx+c)}{64\,d} + \frac{(a^5-10\,a^3b^2+5\,ab^4)\sin(7\,dx+7\,c)}{448\,d} + \frac{(7\,a^5-30\,a^3b^2-5\,ab^4)\sin(5\,dx+5\,c)}{320\,d} + \frac{(21\,a^5-10\,a^3b^2-15\,ab^4)\sin(3\,dx+3\,c)}{192\,d} + \frac{5\,(7\,a^5+10\,a^3b^2+3\,ab^4)\sin(dx+c)}{64\,d}$$

input
```
integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

output
```
-1/448*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(7*d*x + 7*c)/d - 1/320*(25*a^4*b -
 10*a^2*b^3 - 3*b^5)*cos(5*d*x + 5*c)/d - 1/192*(45*a^4*b + 30*a^2*b^3 + b
^5)*cos(3*d*x + 3*c)/d - 5/64*(5*a^4*b + 6*a^2*b^3 + b^5)*cos(d*x + c)/d +
 1/448*(a^5 - 10*a^3*b^2 + 5*a*b^4)*sin(7*d*x + 7*c)/d + 1/320*(7*a^5 - 30
*a^3*b^2 - 5*a*b^4)*sin(5*d*x + 5*c)/d + 1/192*(21*a^5 - 10*a^3*b^2 - 15*a
*b^4)*sin(3*d*x + 3*c)/d + 5/64*(7*a^5 + 10*a^3*b^2 + 3*a*b^4)*sin(d*x + c
)/d
```

3.95.9 Mupad [B] (verification not implemented)

Time = 27.65 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.35

$$\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$
$$= \frac{2\,a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\left(\frac{86\,a^5}{5}-\frac{64\,a^3\,b^2}{3}+32\,a\,b^4\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9\left(\frac{86\,a^5}{5}-\frac{64\,a^3\,b^2}{3}+32\,a\,b^4\right)+}{}$$

input
```
int(cos(c + d*x)^2*(a*cos(c + d*x) + b*sin(c + d*x))^5,x)
```

output
```
(2*a^5*tan(c/2 + (d*x)/2)^13 + tan(c/2 + (d*x)/2)^5*(32*a*b^4 + (86*a^5)/5
 - (64*a^3*b^2)/3) + tan(c/2 + (d*x)/2)^9*(32*a*b^4 + (86*a^5)/5 - (64*a^3
*b^2)/3) + tan(c/2 + (d*x)/2)^7*((424*a^5)/35 - (192*a*b^4)/7 + (608*a^3*b
^2)/7) - tan(c/2 + (d*x)/2)^4*(30*a^4*b + (16*b^5)/5 - 16*a^2*b^3) - tan(c
/2 + (d*x)/2)^8*(50*a^4*b + (32*b^5)/3 - 40*a^2*b^3) - (10*a^4*b)/7 - (16*
b^5)/105 - (8*a^2*b^3)/7 + tan(c/2 + (d*x)/2)^3*(4*a^5 + (80*a^3*b^2)/3) +
 tan(c/2 + (d*x)/2)^11*(4*a^5 + (80*a^3*b^2)/3) + 2*a^5*tan(c/2 + (d*x)/2)
 - tan(c/2 + (d*x)/2)^2*((16*b^5)/15 + 8*a^2*b^3) + tan(c/2 + (d*x)/2)^6*(
(16*b^5)/3 - 80*a^2*b^3) - 40*a^2*b^3*tan(c/2 + (d*x)/2)^10 - 10*a^4*b*tan
(c/2 + (d*x)/2)^12)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^7)
```

3.95. $\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

3.96 $\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

3.96.1 Optimal result

Integrand size = 26, antiderivative size = 126

$$\begin{aligned}\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx &= \frac{5}{16}a\left(a^2+b^2\right)^2x+\frac{5a(a^2+b^2)\left(b+a\cot(c+dx)\right)(a-b\cot(c+dx))\sin^2(c+dx)}{16d}\\&\quad+\frac{5a(b+a\cot(c+dx))^3(a-b\cot(c+dx))\sin^4(c+dx)}{24d}\\&\quad+\frac{(b+a\cot(c+dx))^5\sin^6(c+dx)}{6d}\end{aligned}$$

output
```
5/16*a*(a^2+b^2)^2*x+5/16*a*(a^2+b^2)*(b+a*cot(d*x+c))*(a-b*cot(d*x+c))*si
n(d*x+c)^2/d+5/24*a*(b+a*cot(d*x+c))^3*(a-b*cot(d*x+c))*sin(d*x+c)^4/d+1/6
*(b+a*cot(d*x+c))^5*sin(d*x+c)^6/d
```

3.96.2 Mathematica [A] (verified)

Time = 3.82 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.49

$$\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$

$$= \frac{60a(a^2+b^2)^2(c+dx)-15b(5a^4+6a^2b^2+b^4)\cos(2(c+dx))+6b(-5a^4+b^4)\cos(4(c+dx))-b(5a^4-}{}$$

input
```
Integrate[Cos[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

output
```
(60*a*(a^2 + b^2)^2*(c + d*x) - 15*b*(5*a^4 + 6*a^2*b^2 + b^4)*Cos[2*(c +
d*x)] + 6*b*(-5*a^4 + b^4)*Cos[4*(c + d*x)] - b*(5*a^4 - 10*a^2*b^2 + b^4)
*Cos[6*(c + d*x)] + 15*a*(3*a^4 + 2*a^2*b^2 - b^4)*Sin[2*(c + d*x)] + 3*a*
(3*a^4 - 10*a^2*b^2 - 5*b^4)*Sin[4*(c + d*x)] + a*(a^4 - 10*a^2*b^2 + 5*b^
4)*Sin[6*(c + d*x)])/(192*d)
```

3.96.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {3042, 3567, 531, 27, 487, 487, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5dx$$

$$\Big\downarrow \text{3567}$$

$$-\frac{\int \frac{\cot(c+dx)(b+a\cot(c+dx))^5}{(\cot^2(c+dx)+1)^4}d\cot(c+dx)}{d}$$

$$\Big\downarrow \text{531}$$

$$-\frac{-\frac{1}{6}\int -\frac{5a(b+a\cot(c+dx))^4}{(\cot^2(c+dx)+1)^3}d\cot(c+dx)-\frac{(a\cot(c+dx)+b)^5}{6(\cot^2(c+dx)+1)^3}}{d}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{5}{6}a\int \frac{(b+a\cot(c+dx))^4}{(\cot^2(c+dx)+1)^3}d\cot(c+dx)-\frac{(a\cot(c+dx)+b)^5}{6(\cot^2(c+dx)+1)^3}}{d}$$

$$\Big\downarrow \text{487}$$

$$-\frac{\frac{5}{6}a\left(\frac{3}{4}(a^2+b^2)\int \frac{(b+a\cot(c+dx))^2}{(\cot^2(c+dx)+1)^2}d\cot(c+dx)-\frac{(a\cot(c+dx)+b)^3(a-b\cot(c+dx))}{4(\cot^2(c+dx)+1)^2}\right)-\frac{(a\cot(c+dx)+b)^5}{6(\cot^2(c+dx)+1)^3}}{d}$$

$$\Big\downarrow \text{487}$$

$$-\frac{\frac{5}{6}a\left(\frac{3}{4}\left(a^2+b^2\right)\left(\frac{1}{2}\left(a^2+b^2\right)\int \frac{1}{\cot^2(c+dx)+1}d\cot(c+dx)-\frac{(a\cot(c+dx)+b)(a-b\cot(c+dx))}{2(\cot^2(c+dx)+1)}\right)-\frac{(a\cot(c+dx)+b)^3(a-b\cot(c+dx))}{4(\cot^2(c+dx)+1)^2}}{d}$$

$\big\downarrow$ 216

$$-\frac{\frac{5}{6}a\left(\frac{3}{4}\left(a^2+b^2\right)\left(\frac{1}{2}\left(a^2+b^2\right)\arctan(\cot(c+dx))-\frac{(a\cot(c+dx)+b)(a-b\cot(c+dx))}{2(\cot^2(c+dx)+1)}\right)-\frac{(a\cot(c+dx)+b)^3(a-b\cot(c+dx))}{4(\cot^2(c+dx)+1)^2}\right)-}{d}$$

input
```
Int[Cos[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

output
```
-((-1/6*(b + a*Cot[c + d*x])^5/(1 + Cot[c + d*x]^2)^3 + (5*a*(-1/4*((b + a
*Cot[c + d*x])^3*(a - b*Cot[c + d*x]))/(1 + Cot[c + d*x]^2)^2 + (3*(a^2 +
b^2)*(((a^2 + b^2)*ArcTan[Cot[c + d*x]])/2 - ((b + a*Cot[c + d*x])*(a - b*
Cot[c + d*x]))/(2*(1 + Cot[c + d*x]^2))))/4))/6)/d)
```

3.96.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 487
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n - 1)*(a*d - b*c*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] +
 Simp[(2*p + 3)*((b*c^2 + a*d^2)/(2*a*b*(p + 1)))   Int[(c + d*x)^(n - 2)*(
a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[n + 2*p + 2, 0]
 && LtQ[p, -1]
```

rule 531
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symb
ol] :> With[{Qx = PolynomialQuotient[x^m, a + b*x^2, x], e = Coeff[Polynomi
alRemainder[x^m, a + b*x^2, x], x, 0], f = Coeff[PolynomialRemainder[x^m, a
 + b*x^2, x], x, 1]}, Simp[(c + d*x)^n*(a*f - b*e*x)*((a + b*x^2)^(p + 1)/(
2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b
*x^2)^(p + 1)*ExpandToSum[2*a*b*(p + 1)*(c + d*x)*Qx - a*d*f*n + b*c*e*(2*p
 + 3) + b*d*e*(n + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGt
Q[n, 0] && IGtQ[m, 0] && LtQ[p, -1] && GtQ[n, 1] && IntegerQ[2*p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3567
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[x^m*((b
+ a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a,
b, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[
n, 0] && GtQ[m, 1])
```

3.96.4 Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.75

method	result
parallelrisch	$\frac{(-75a^4b-90a^2b^3-15b^5)\cos(2dx+2c)+(-5a^4b+10a^2b^3-b^5)\cos(6dx+6c)+(45a^5+30a^3b^2-15ab^4)\sin(2dx+2c)+(9a^5}{}$
derivativedivides	$\frac{a^5\left(\frac{\left(\cos(dx+c)^5+\frac{5\cos(dx+c)^3}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)-\frac{5\cos(dx+c)^6a^4b}{6}+10a^3b^2\left(-\frac{\cos(dx+c)^5\sin(dx+c)}{6}+\right.}{}$
default	$\frac{a^5\left(\frac{\left(\cos(dx+c)^5+\frac{5\cos(dx+c)^3}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)-\frac{5\cos(dx+c)^6a^4b}{6}+10a^3b^2\left(-\frac{\cos(dx+c)^5\sin(dx+c)}{6}+\right.}{}$
parts	$\frac{a^5\left(\frac{\left(\cos(dx+c)^5+\frac{5\cos(dx+c)^3}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)}{d}+\frac{b^5\sin(dx+c)^6}{6d}+\frac{5ab^4\left(-\frac{\cos(dx+c)^3\sin(dx+c)}{6}\right.}{}$
risch	$\frac{5a^5x}{16}+\frac{5a^3b^2x}{8}+\frac{5ab^4x}{16}-\frac{5b\cos(6dx+6c)a^4}{192d}+\frac{5b^3\cos(6dx+6c)a^2}{96d}-\frac{b^5\cos(6dx+6c)}{192d}+\frac{a^5\sin(6dx+6c)}{192d}-$
norman	$\frac{(\frac{5}{16}a^5+\frac{5}{8}a^3b^2+\frac{5}{16}ab^4)x+(\frac{5}{16}a^5+\frac{5}{8}a^3b^2+\frac{5}{16}ab^4)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}+(\frac{15}{8}a^5+\frac{15}{4}a^3b^2+\frac{15}{8}ab^4)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+(\frac{15}{8}a^5+}{}$

input
```
int(cos(d*x+c)*(cos(d*x+c)*a+b*sin(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
1/192*((-75*a^4*b-90*a^2*b^3-15*b^5)*cos(2*d*x+2*c)+(-5*a^4*b+10*a^2*b^3-b
^5)*cos(6*d*x+6*c)+(45*a^5+30*a^3*b^2-15*a*b^4)*sin(2*d*x+2*c)+(9*a^5-30*a
^3*b^2-15*a*b^4)*sin(4*d*x+4*c)+a*(a^4-10*a^2*b^2+5*b^4)*sin(6*d*x+6*c)+(-
30*a^4*b+6*b^5)*cos(4*d*x+4*c)+60*a^5*d*x+120*a^3*b^2*d*x+60*a*b^4*d*x+110
*a^4*b+80*a^2*b^3+10*b^5)/d
```

3.96.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.44

$$\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = -\frac{24\,b^5\cos\left(dx+c\right)^2+8\left(5\,a^4b-10\,a^2b^3+b^5\right)\cos\left(dx+c\right)^6+24\left(5\,a^2b^3-b^5\right)\cos\left(dx+c\right)^4-15\left(a^5+2\,a\right.}{}$$

input
```
integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas")
```

output
```
-1/48*(24*b^5*cos(d*x + c)^2 + 8*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c)
^6 + 24*(5*a^2*b^3 - b^5)*cos(d*x + c)^4 - 15*(a^5 + 2*a^3*b^2 + a*b^4)*d*
x - (8*(a^5 - 10*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^5 + 10*(a^5 + 2*a^3*b^2 -
 7*a*b^4)*cos(d*x + c)^3 + 15*(a^5 + 2*a^3*b^2 + a*b^4)*cos(d*x + c))*sin(
d*x + c))/d
```

3.96.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 609 vs. $2(117) = 234$.

Time = 0.42 (sec) , antiderivative size = 609, normalized size of antiderivative = 4.83

$$\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \begin{cases} \frac{5a^5x\sin^6(c+dx)}{16} + \frac{15a^5x\sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{15a^5x\sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{5a^5x\cos^6(c+dx)}{16} + \frac{5a^5\sin^5(c+dx)\cos(c+dx)}{16d} \\ x(a\cos(c)+b\sin(c))^5\cos(c) \end{cases}$$

input
```
integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

output
```
Piecewise((5*a**5*x*sin(c + d*x)**6/16 + 15*a**5*x*sin(c + d*x)**4*cos(c +
 d*x)**2/16 + 15*a**5*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*a**5*x*cos(
c + d*x)**6/16 + 5*a**5*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 5*a**5*sin(c
 + d*x)**3*cos(c + d*x)**3/(6*d) + 11*a**5*sin(c + d*x)*cos(c + d*x)**5/(1
6*d) - 5*a**4*b*cos(c + d*x)**6/(6*d) + 5*a**3*b**2*x*sin(c + d*x)**6/8 +
15*a**3*b**2*x*sin(c + d*x)**4*cos(c + d*x)**2/8 + 15*a**3*b**2*x*sin(c +
d*x)**2*cos(c + d*x)**4/8 + 5*a**3*b**2*x*cos(c + d*x)**6/8 + 5*a**3*b**2*
sin(c + d*x)**5*cos(c + d*x)/(8*d) + 5*a**3*b**2*sin(c + d*x)**3*cos(c + d
*x)**3/(3*d) - 5*a**3*b**2*sin(c + d*x)*cos(c + d*x)**5/(8*d) + 5*a**2*b**
3*sin(c + d*x)**6/(6*d) + 5*a**2*b**3*sin(c + d*x)**4*cos(c + d*x)**2/(2*d
) + 5*a*b**4*x*sin(c + d*x)**6/16 + 15*a*b**4*x*sin(c + d*x)**4*cos(c + d*
x)**2/16 + 15*a*b**4*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*a*b**4*x*cos
(c + d*x)**6/16 + 5*a*b**4*sin(c + d*x)**5*cos(c + d*x)/(16*d) - 5*a*b**4*
sin(c + d*x)**3*cos(c + d*x)**3/(6*d) - 5*a*b**4*sin(c + d*x)*cos(c + d*x)
**5/(16*d) + b**5*sin(c + d*x)**6/(6*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c
))**5*cos(c), True))
```

3.96.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.48

$$\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx =$$

$$-\frac{160\,a^4b\cos{(dx+c)}^6 - 32\,b^5\sin{(dx+c)}^6 + \left(4\,\sin{(2\,dx+2\,c)}^3 - 60\,dx - 60\,c - 9\,\sin{(4\,dx+4\,c)} - 48$$

input
```
integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima")
```

output
```
-1/192*(160*a^4*b*cos(d*x + c)^6 - 32*b^5*sin(d*x + c)^6 + (4*sin(2*d*x +
2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a^5 - 1
0*(4*sin(2*d*x + 2*c)^3 + 12*d*x + 12*c - 3*sin(4*d*x + 4*c))*a^3*b^2 + 16
0*(2*sin(d*x + c)^6 - 3*sin(d*x + c)^4)*a^2*b^3 + 5*(4*sin(2*d*x + 2*c)^3
- 12*d*x - 12*c + 3*sin(4*d*x + 4*c))*a*b^4)/d
```

3.96.8 Giac [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.67

$$\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$
$$= \frac{5}{16}\left(a^5+2\,a^3b^2+ab^4\right)x-\frac{\left(5\,a^4b-10\,a^2b^3+b^5\right)\cos\left(6\,dx+6\,c\right)}{192\,d}$$
$$-\frac{\left(5\,a^4b-b^5\right)\cos\left(4\,dx+4\,c\right)}{32\,d}-\frac{5\left(5\,a^4b+6\,a^2b^3+b^5\right)\cos\left(2\,dx+2\,c\right)}{64\,d}$$
$$+\frac{\left(a^5-10\,a^3b^2+5\,ab^4\right)\sin\left(6\,dx+6\,c\right)}{192\,d}+\frac{\left(3\,a^5-10\,a^3b^2-5\,ab^4\right)\sin\left(4\,dx+4\,c\right)}{64\,d}$$
$$+\frac{5\left(3\,a^5+2\,a^3b^2-ab^4\right)\sin\left(2\,dx+2\,c\right)}{64\,d}$$

input
```
integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

output
```
5/16*(a^5 + 2*a^3*b^2 + a*b^4)*x - 1/192*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(
6*d*x + 6*c)/d - 1/32*(5*a^4*b - b^5)*cos(4*d*x + 4*c)/d - 5/64*(5*a^4*b +
 6*a^2*b^3 + b^5)*cos(2*d*x + 2*c)/d + 1/192*(a^5 - 10*a^3*b^2 + 5*a*b^4)*
sin(6*d*x + 6*c)/d + 1/64*(3*a^5 - 10*a^3*b^2 - 5*a*b^4)*sin(4*d*x + 4*c)/
d + 5/64*(3*a^5 + 2*a^3*b^2 - a*b^4)*sin(2*d*x + 2*c)/d
```

3.96.9 Mupad [B] (verification not implemented)

Time = 24.99 (sec) , antiderivative size = 472, normalized size of antiderivative = 3.75

$$\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$
$$= \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}\left(-\frac{11\,a^5}{8}+\frac{5\,a^3\,b^2}{4}+\frac{5\,a\,b^4}{8}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(-\frac{11\,a^5}{8}+\frac{5\,a^3\,b^2}{4}+\frac{5\,a\,b^4}{8}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\left(\frac{15\,a^5}{4}-\right.}{}$$
$$+\frac{5\,a\,\mathrm{atan}\left(\frac{5\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,{\left(a^2+b^2\right)}^2}{8\,\left(\frac{5\,a^5}{8}+\frac{5\,a^3\,b^2}{4}+\frac{5\,a\,b^4}{8}\right)}\right)\,{\left(a^2+b^2\right)}^2}{8\,d}-\frac{5\,a\,\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)\,{\left(a^2+b^2\right)}^2}{8\,d}$$

input
```
int(cos(c + d*x)*(a*cos(c + d*x) + b*sin(c + d*x))^5,x)
```

output
```
(tan(c/2 + (d*x)/2)^11*((5*a*b^4)/8 - (11*a^5)/8 + (5*a^3*b^2)/4) - tan(c/
2 + (d*x)/2)*((5*a*b^4)/8 - (11*a^5)/8 + (5*a^3*b^2)/4) + tan(c/2 + (d*x)/
2)^5*((95*a*b^4)/4 + (15*a^5)/4 - (65*a^3*b^2)/2) - tan(c/2 + (d*x)/2)^7*(
(95*a*b^4)/4 + (15*a^5)/4 - (65*a^3*b^2)/2) - tan(c/2 + (d*x)/2)^3*((85*a*
b^4)/24 + (5*a^5)/24 - (235*a^3*b^2)/12) + tan(c/2 + (d*x)/2)^9*((85*a*b^4
)/24 + (5*a^5)/24 - (235*a^3*b^2)/12) + tan(c/2 + (d*x)/2)^6*((100*a^4*b)/
3 + (32*b^5)/3 - (80*a^2*b^3)/3) + 40*a^2*b^3*tan(c/2 + (d*x)/2)^4 + 40*a^
2*b^3*tan(c/2 + (d*x)/2)^8 + 10*a^4*b*tan(c/2 + (d*x)/2)^2 + 10*a^4*b*tan(
c/2 + (d*x)/2)^10)/(d*(6*tan(c/2 + (d*x)/2)^2 + 15*tan(c/2 + (d*x)/2)^4 +
20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 + 6*tan(c/2 + (d*x)/2)^1
0 + tan(c/2 + (d*x)/2)^12 + 1)) + (5*a*atan((5*a*tan(c/2 + (d*x)/2)*(a^2 +
 b^2)^2)/(8*((5*a*b^4)/8 + (5*a^5)/8 + (5*a^3*b^2)/4)))*(a^2 + b^2)^2)/(8*
d) - (5*a*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2)*(a^2 + b^2)^2)/(8*d)
```

3.96. $\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

3.97 $\int (a\cos(c+dx)+b\sin(c+dx))^5\,dx$

3.97.1 Optimal result

Integrand size = 19, antiderivative size = 94

$$\int (a\cos(c+dx)+b\sin(c+dx))^5\,dx=-\frac{(a^2+b^2)^2(b\cos(c+dx)-a\sin(c+dx))}{d}+\frac{2(a^2+b^2)(b\cos(c+dx)-a\sin(c+dx))^3}{3d}-\frac{(b\cos(c+dx)-a\sin(c+dx))^5}{5d}$$

output
```
-(a^2+b^2)^2*(b*cos(d*x+c)-a*sin(d*x+c))/d+2/3*(a^2+b^2)*(b*cos(d*x+c)-a*s
in(d*x+c))^3/d-1/5*(b*cos(d*x+c)-a*sin(d*x+c))^5/d
```

3.97.2 Mathematica [A] (verified)

Time = 1.74 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.66

$$\int (a\cos(c+dx)+b\sin(c+dx))^5\,dx$$
$$=\frac{-150b(a^2+b^2)^2\cos(c+dx)+25b(-3a^4-2a^2b^2+b^4)\cos(3(c+dx))-3b(5a^4-10a^2b^2+b^4)\cos(5(c+\ldots}{}$$

input
```
Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

output
```
(-150*b*(a^2 + b^2)^2*Cos[c + d*x] + 25*b*(-3*a^4 - 2*a^2*b^2 + b^4)*Cos[3
*(c + d*x)] - 3*b*(5*a^4 - 10*a^2*b^2 + b^4)*Cos[5*(c + d*x)] + 150*a*(a^2
 + b^2)^2*Sin[c + d*x] + 25*a*(a^4 - 2*a^2*b^2 - 3*b^4)*Sin[3*(c + d*x)] +
 3*a*(a^4 - 10*a^2*b^2 + 5*b^4)*Sin[5*(c + d*x)])/(240*d)
```

3.97.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 3551, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(c+dx)+b\sin(c+dx))^5\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\cos(c+dx)+b\sin(c+dx))^5dx$$

$$\Big\downarrow \text{3551}$$

$$-\frac{\int \left(a^2+b^2-(b\cos(c+dx)-a\sin(c+dx))^2\right)^2 d(b\cos(c+dx)-a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{210}$$

$$-\frac{\int \left(\left(\frac{b^4+2a^2b^2}{a^4}+1\right)a^4-2\left(\frac{b^2}{a^2}+1\right)(b\cos(c+dx)-a\sin(c+dx))^2a^2+(b\cos(c+dx)-a\sin(c+dx))^4\right) d(b\cos(}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{2}{3}\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))^3+\left(a^2+b^2\right)^2(b\cos(c+dx)-a\sin(c+dx))+\frac{1}{5}(b\cos(c+dx)-a\,s}{d}$$

input
```
Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

output
```
-(((a^2 + b^2)^2*(b*Cos[c + d*x] - a*Sin[c + d*x]) - (2*(a^2 + b^2)*(b*Cos
[c + d*x] - a*Sin[c + d*x])^3)/3 + (b*Cos[c + d*x] - a*Sin[c + d*x])^5/5)/
d)
```

3.97.3.1 Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3551
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[(a^2 + b^2 - x^2)^((n - 1)/2), x], x,
b*Cos[c + d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
 b^2, 0] && IGtQ[(n - 1)/2, 0]
```

3.97.4 Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.78

method	result
parts	$\frac{a^5\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5d}-\frac{b^5\left(\frac{8}{3}+\sin(dx+c)^4+\frac{4\sin(dx+c)^2}{3}\right)\cos(dx+c)}{5d}+\frac{a\,b^4\sin(dx+c)^5}{d}+$
derivativedivides	$\frac{\frac{a^5\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}-a^4b\cos(dx+c)^5+10a^3b^2\left(-\frac{\sin(dx+c)\cos(dx+c)^4}{5}+\frac{\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{15}\right.}{d}$
default	$\frac{\frac{a^5\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}-a^4b\cos(dx+c)^5+10a^3b^2\left(-\frac{\sin(dx+c)\cos(dx+c)^4}{5}+\frac{\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{15}\right.}{d}$
parallelrisch	$2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9a^5-10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8a^4b+\frac{8\left(a^5+10a^3b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{3}-40\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6a^2b^3+\frac{4\left(29a^5-40a^3b^2+120a\,b^4\right)\text{ta}}{15}$
norman	$-\frac{30a^4b+40a^2b^3+16b^5}{15d}+\frac{2a^5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2a^5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{d}-\frac{40a^2b^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{d}-\frac{10a^4b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{d}-\frac{\left(40a^2b^3+16b^5\right)\tan}{3d}$
risch	$-\frac{5a^4b\cos(dx+c)}{8d}-\frac{5a^2b^3\cos(dx+c)}{4d}-\frac{5b^5\cos(dx+c)}{8d}+\frac{5a^5\sin(dx+c)}{8d}+\frac{5a^3b^2\sin(dx+c)}{4d}+\frac{5a\,b^4\sin(dx+c)}{8d}$

input
```
int((cos(d*x+c)*a+b*sin(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
1/5*a^5/d*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)-1/5*b^5/d*(8/3+si
n(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)+a*b^4*sin(d*x+c)^5/d+10*a^2*b^3/d*
(1/5*cos(d*x+c)^5-1/3*cos(d*x+c)^3)+10*a^3*b^2/d*(-1/5*sin(d*x+c)^5+1/3*si
n(d*x+c)^3)-a^4*b/d*cos(d*x+c)^5
```

3.97.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.65

$$\int (a\cos(c+dx)+b\sin(c+dx))^5\,dx = -\frac{15\,b^5\cos\left(dx+c\right)+3\left(5\,a^4b-10\,a^2b^3+b^5\right)\cos\left(dx+c\right)^5+10\left(5\,a^2b^3-b^5\right)\cos\left(dx+c\right)^3-\left(8\,a^5+20\,a^{3}\right.}{1}$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas")
```

output
```
-1/15*(15*b^5*cos(d*x + c) + 3*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c)^5
 + 10*(5*a^2*b^3 - b^5)*cos(d*x + c)^3 - (8*a^5 + 20*a^3*b^2 + 15*a*b^4 +
3*(a^5 - 10*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^4 + 2*(2*a^5 + 5*a^3*b^2 - 15*
a*b^4)*cos(d*x + c)^2)*sin(d*x + c))/d
```

3.97.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. 2(82) = 164.

Time = 0.29 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.84

$$\int (a\cos(c+dx)+b\sin(c+dx))^5\,dx = \begin{cases} \frac{8a^5\sin^5\left(c+dx\right)}{15d}+\frac{4a^5\sin^3\left(c+dx\right)\cos^2\left(c+dx\right)}{3d}+\frac{a^5\sin\left(c+dx\right)\cos^4\left(c+dx\right)}{d}-\frac{a^4b\cos^5\left(c+dx\right)}{d}+\frac{4a^3b^2\sin^5\left(c+dx\right)}{3d}+\frac{10a^3b^2\sin^3\left(c+\right.}{3} \\ x\left(a\cos\left(c\right)+b\sin\left(c\right)\right)^5 \end{cases}$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

output
```
Piecewise((8*a**5*sin(c + d*x)**5/(15*d) + 4*a**5*sin(c + d*x)**3*cos(c +
d*x)**2/(3*d) + a**5*sin(c + d*x)*cos(c + d*x)**4/d - a**4*b*cos(c + d*x)*
*5/d + 4*a**3*b**2*sin(c + d*x)**5/(3*d) + 10*a**3*b**2*sin(c + d*x)**3*co
s(c + d*x)**2/(3*d) - 10*a**2*b**3*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) -
 4*a**2*b**3*cos(c + d*x)**5/(3*d) + a*b**4*sin(c + d*x)**5/d - b**5*sin(c
 + d*x)**4*cos(c + d*x)/d - 4*b**5*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) -
 8*b**5*cos(c + d*x)**5/(15*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**5, Tr
ue))
```

3.97.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.83

$$
\begin{aligned}
&\int (a\cos(c+dx)+b\sin(c+dx))^5\,dx \\
&= -\frac{a^4b\cos(dx+c)^5}{d}+\frac{ab^4\sin(dx+c)^5}{d} \\
&\quad+\frac{(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c))a^5}{15\,d} \\
&\quad-\frac{2\left(3\sin(dx+c)^5-5\sin(dx+c)^3\right)a^3b^2}{3\,d}+\frac{2\left(3\cos(dx+c)^5-5\cos(dx+c)^3\right)a^2b^3}{3\,d} \\
&\quad-\frac{(3\cos(dx+c)^5-10\cos(dx+c)^3+15\cos(dx+c))b^5}{15\,d}
\end{aligned}
$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima")
```

output
```
-a^4*b*cos(d*x + c)^5/d + a*b^4*sin(d*x + c)^5/d + 1/15*(3*sin(d*x + c)^5
- 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^5/d - 2/3*(3*sin(d*x + c)^5 - 5*s
in(d*x + c)^3)*a^3*b^2/d + 2/3*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a^2*b
^3/d - 1/15*(3*cos(d*x + c)^5 - 10*cos(d*x + c)^3 + 15*cos(d*x + c))*b^5/d
```

3.97.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(90) = 180$.

Time = 0.36 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.99

$$\int (a\cos(c+dx)+b\sin(c+dx))^5\,dx = -\frac{(5\,a^4b-10\,a^2b^3+b^5)\cos(5\,dx+5\,c)}{80\,d} - \frac{5\,(3\,a^4b+2\,a^2b^3-b^5)\cos(3\,dx+3\,c)}{48\,d} - \frac{5\,(a^4b+2\,a^2b^3+b^5)\cos(dx+c)}{8\,d} + \frac{(a^5-10\,a^3b^2+5\,ab^4)\sin(5\,dx+5\,c)}{80\,d} + \frac{5\,(a^5-2\,a^3b^2-3\,ab^4)\sin(3\,dx+3\,c)}{48\,d} + \frac{5\,(a^5+2\,a^3b^2+ab^4)\sin(dx+c)}{8\,d}$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

output
```
-1/80*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(5*d*x + 5*c)/d - 5/48*(3*a^4*b + 2*
a^2*b^3 - b^5)*cos(3*d*x + 3*c)/d - 5/8*(a^4*b + 2*a^2*b^3 + b^5)*cos(d*x
+ c)/d + 1/80*(a^5 - 10*a^3*b^2 + 5*a*b^4)*sin(5*d*x + 5*c)/d + 5/48*(a^5
- 2*a^3*b^2 - 3*a*b^4)*sin(3*d*x + 3*c)/d + 5/8*(a^5 + 2*a^3*b^2 + a*b^4)*
sin(d*x + c)/d
```

3.97.9 Mupad [B] (verification not implemented)

Time = 23.20 (sec) , antiderivative size = 248, normalized size of antiderivative = 2.64

$$\int (a\cos(c+dx)+b\sin(c+dx))^5\,dx = \frac{2\left(\frac{3\sin(c+d\,x)\,a^5\cos(c+d\,x)^4}{2}+2\,\sin(c+d\,x)\,a^5\cos(c+d\,x)^2+4\,\sin(c+d\,x)\,a^5-\frac{15\,a^4\,b\cos(c+d\,x)^5}{2}-15\,\sin(c\right.}{}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))^5,x)
```

output
```
(2*(4*a^5*sin(c + d*x) - (15*b^5*cos(c + d*x))/2 + 5*b^5*cos(c + d*x)^3 -
(3*b^5*cos(c + d*x)^5)/2 - (15*a^4*b*cos(c + d*x)^5)/2 + 2*a^5*cos(c + d*x
)^2*sin(c + d*x) + (3*a^5*cos(c + d*x)^4*sin(c + d*x))/2 + 10*a^3*b^2*sin(
c + d*x) - 25*a^2*b^3*cos(c + d*x)^3 + 15*a^2*b^3*cos(c + d*x)^5 + (15*a*b
^4*sin(c + d*x))/2 + 5*a^3*b^2*cos(c + d*x)^2*sin(c + d*x) - 15*a^3*b^2*co
s(c + d*x)^4*sin(c + d*x) - 15*a*b^4*cos(c + d*x)^2*sin(c + d*x) + (15*a*b
^4*cos(c + d*x)^4*sin(c + d*x))/2))/(15*d)
```

3.97. $\int (a\cos(c+dx)+b\sin(c+dx))^5\,dx$

3.98 $\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

3.98.1 Optimal result

Integrand size = 26, antiderivative size = 170

$$\begin{aligned}&\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx\\&=\frac{1}{8}a\left(3a^4+10a^2b^2+15b^4\right)x-\frac{b^5\log(\sin(c+dx))}{d}+\frac{b^5\log(\tan(c+dx))}{d}\\&\quad+\frac{\left(4b(5a^4-b^4)+5a(a^2-3b^2)\left(a^2+b^2\right)\cot(c+dx)\right)\sin^2(c+dx)}{8d}\\&\quad-\frac{\left(b(5a^4-10a^2b^2+b^4)+a(a^4-10a^2b^2+5b^4)\cot(c+dx)\right)\sin^4(c+dx)}{4d}\end{aligned}$$

output
```
1/8*a*(3*a^4+10*a^2*b^2+15*b^4)*x-b^5*ln(sin(d*x+c))/d+b^5*ln(tan(d*x+c))/
d+1/8*(4*b*(5*a^4-b^4)+5*a*(a^2-3*b^2)*(a^2+b^2)*cot(d*x+c))*sin(d*x+c)^2/
d-1/4*(b*(5*a^4-10*a^2*b^2+b^4)+a*(a^4-10*a^2*b^2+5*b^4)*cot(d*x+c))*sin(d
*x+c)^4/d
```

3.98.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 711 vs. 2(170) = 340.

Time = 6.51 (sec) , antiderivative size = 711, normalized size of antiderivative = 4.18

$$\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$
$$= \frac{b^5\left(\frac{\cos^4(c+dx)(a+b\tan(c+dx))^6(b^2+ab\tan(c+dx))}{4b^6(a^2+b^2)} - \frac{\frac{\cos^2(c+dx)(a+b\tan(c+dx))^6\left(-3a^2b^2+b^2\left(-3a^2+2b^2\right)+b\left(3ab^2+a\left(-3a^2+2b^2\right)\right)\tan(c+dx)\right)}{2b^4\left(a^2+b^2\right)}}{}\right.}{}$$

input
```
Integrate[Sec[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

output
```
(b^5*((Cos[c + d*x]^4*(a + b*Tan[c + d*x])^6*(b^2 + a*b*Tan[c + d*x]))/(4*
b^6*(a^2 + b^2)) - ((Cos[c + d*x]^2*(a + b*Tan[c + d*x])^6*(-3*a^2*b^2 + b
^2*(-3*a^2 + 2*b^2) + b*(3*a*b^2 + a*(-3*a^2 + 2*b^2))*Tan[c + d*x]))/(2*b
^4*(a^2 + b^2)) - ((3*a^4 - 29*a^2*b^2 + 8*b^4 + 5*a^2*(3*a^2 - 5*b^2))*((
(5*a^4 - 10*a^2*b^2 + b^4 + (a^5 - 10*a^3*b^2 + 5*a*b^4)/Sqrt[-b^2])*Log[S
qrt[-b^2] - b*Tan[c + d*x]])/2 + ((5*a^4 - 10*a^2*b^2 + b^4 - (a^5 - 10*a^
3*b^2 + 5*a*b^4)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]])/2 + 5*a*b*(
2*a^2 - b^2)*Tan[c + d*x] + (b^2*(10*a^2 - b^2)*Tan[c + d*x]^2)/2 + (5*a*b
^3*Tan[c + d*x]^3)/3 + (b^4*Tan[c + d*x]^4)/4) - 5*a*(3*a^2 - 5*b^2)*(((6*
a^5 - 20*a^3*b^2 + 6*a*b^4 + (a^6 - 15*a^4*b^2 + 15*a^2*b^4 - b^6)/Sqrt[-b
^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]])/2 + ((6*a^5 - 20*a^3*b^2 + 6*a*b^4
- (a^6 - 15*a^4*b^2 + 15*a^2*b^4 - b^6)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan
[c + d*x]])/2 + b*(15*a^4 - 15*a^2*b^2 + b^4)*Tan[c + d*x] + a*b^2*(10*a^2
 - 3*b^2)*Tan[c + d*x]^2 + (b^3*(15*a^2 - b^2)*Tan[c + d*x]^3)/3 + (3*a*b^
4*Tan[c + d*x]^4)/2 + (b^5*Tan[c + d*x]^5)/5))/(2*b^2*(a^2 + b^2)))/(4*b^2
*(a^2 + b^2))))/d
```

3.98.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 3567, 532, 25, 2336, 25, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\cos(c+dx)+b\sin(c+dx))^5}{\cos(c+dx)}dx$$

$$\Big\downarrow \text{3567}$$

$$-\frac{\int \frac{(b+a\cot(c+dx))^5\tan(c+dx)}{(\cot^2(c+dx)+1)^3}d\cot(c+dx)}{d}$$

$$\Big\downarrow \text{532}$$

$$-\frac{\frac{a(a^4-10a^2b^2+5b^4)\cot(c+dx)+b(5a^4-10a^2b^2+b^4)}{4(\cot^2(c+dx)+1)^2}-\frac{1}{4}\int -\frac{(4\cot^3(c+dx)a^5+20b\cot^2(c+dx)a^4-(a^4-10b^2a^2-15b^4)\cot(c+dx)a+4b^5)\tan(}{(\cot^2(c+dx)+1)^2}}{d}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{1}{4}\int \frac{(4\cot^3(c+dx)a^5+20b\cot^2(c+dx)a^4-(a^4-10b^2a^2-15b^4)\cot(c+dx)a+4b^5)\tan(c+dx)}{(\cot^2(c+dx)+1)^2}d\cot(c+dx)+\frac{a(a^4-10a^2b^2+5b^4)\cot(c+dx)}{4(\cot^2(c+dx)}}{d}$$

$$\Big\downarrow \text{2336}$$

$$-\frac{\frac{1}{4}\left(-\frac{1}{2}\int -\frac{(8b^5+a(3a^4+10b^2a^2+15b^4)\cot(c+dx))\tan(c+dx)}{\cot^2(c+dx)+1}d\cot(c+dx)-\frac{4b(5a^4-b^4)+5a(a^2-3b^2)(a^2+b^2)\cot(c+dx)}{2(\cot^2(c+dx)+1)}\right)+\frac{a(a^4-}{}}{d}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{1}{4}\left(\frac{1}{2}\int \frac{(8b^5+a(3a^4+10b^2a^2+15b^4)\cot(c+dx))\tan(c+dx)}{\cot^2(c+dx)+1}d\cot(c+dx)-\frac{4b(5a^4-b^4)+5a(a^2-3b^2)(a^2+b^2)\cot(c+dx)}{2(\cot^2(c+dx)+1)}\right)+\frac{a(a^4-10a^2}{}}{d}$$

$$\Big\downarrow \text{523}$$

$$-\frac{\frac{1}{4}\left(\frac{1}{2}\int \left(8\tan(c+dx)b^5+\frac{3a^5+10b^2a^3+15b^4a-8b^5\cot(c+dx)}{\cot^2(c+dx)+1}\right)d\cot(c+dx)-\frac{4b(5a^4-b^4)+5a(a^2-3b^2)(a^2+b^2)\cot(c+dx)}{2(\cot^2(c+dx)+1)}\right)+}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{1}{4}\left(\frac{1}{2}\left(a(3a^4+10a^2b^2+15b^4)\arctan(\cot(c+dx))-4b^5\log\left(\cot^2(c+dx)+1\right)+8b^5\log(\cot(c+dx))\right)-\frac{4b(5a^4-}{}}{d}$$

input

```
Int[Sec[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

3.98. $\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

output
```
-(((b*(5*a^4 - 10*a^2*b^2 + b^4) + a*(a^4 - 10*a^2*b^2 + 5*b^4)*Cot[c + d*
x])/(4*(1 + Cot[c + d*x]^2)^2) + (-1/2*(4*b*(5*a^4 - b^4) + 5*a*(a^2 - 3*b
^2)*(a^2 + b^2)*Cot[c + d*x])/(1 + Cot[c + d*x]^2) + (a*(3*a^4 + 10*a^2*b^
2 + 15*b^4)*ArcTan[Cot[c + d*x]] + 8*b^5*Log[Cot[c + d*x]] - 4*b^5*Log[1 +
 Cot[c + d*x]^2])/2)/4)/d)
```

3.98.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3567
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[x^m*((b
+ a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a,
b, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[
n, 0] && GtQ[m, 1])
```

3.98.4 Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.13

method	result
derivativedivides	$\frac{a^5\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)-\frac{5\cos(dx+c)^4a^4b}{4}+10a^3b^2\left(-\frac{\sin(dx+c)\cos(dx+c)^3}{4}+\frac{\sin(dx+c)\cos(dx}{8}\right.}{}$
default	$\frac{a^5\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)-\frac{5\cos(dx+c)^4a^4b}{4}+10a^3b^2\left(-\frac{\sin(dx+c)\cos(dx+c)^3}{4}+\frac{\sin(dx+c)\cos(dx}{8}\right.}{}$
parts	$\frac{a^5\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}+\frac{b^5\left(-\frac{\sin(dx+c)^4}{4}-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)}{d}-\frac{5a^4b}{4d\sec(dx+}$
parallelrisch	$\frac{32b^5\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)-32b^5\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-32b^5\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+4(-5a^4b-10a^2b^3+3b^5)\cos(2dx+2c)+}{}$
risch	$ixb^5+\frac{3a^5x}{8}+\frac{5a^3b^2x}{4}+\frac{15ab^4x}{8}-\frac{5e^{2i(dx+c)}a^4b}{16d}-\frac{5e^{2i(dx+c)}a^2b^3}{8d}+\frac{3e^{2i(dx+c)}b^5}{16d}+\frac{ie^{-2i(dx+c)}a^5}{8d}+$
norman	$\frac{(\frac{3}{8}a^5+\frac{5}{4}a^3b^2+\frac{15}{8}ab^4)x+(\frac{3}{8}a^5+\frac{5}{4}a^3b^2+\frac{15}{8}ab^4)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+(\frac{15}{4}a^5+\frac{25}{2}a^3b^2+\frac{75}{4}ab^4)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+(\frac{15}{4}a^5+\frac{25}{2}}{}$

input
```
int(sec(d*x+c)*(cos(d*x+c)*a+b*sin(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^5*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)-5/4*
cos(d*x+c)^4*a^4*b+10*a^3*b^2*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*sin(d*x+c)
*cos(d*x+c)+1/8*d*x+1/8*c)+5/2*a^2*b^3*sin(d*x+c)^4+5*a*b^4*(-1/4*(sin(d*x
+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*c)+b^5*(-1/4*sin(d*x+c)^4-1/2
*sin(d*x+c)^2-ln(cos(d*x+c))))
```

3.98.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.94

$$\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx =$$
$$-\frac{8\,b^5\log\left(-\cos\left(dx+c\right)\right)+2\left(5\,a^4b-10\,a^2b^3+b^5\right)\cos\left(dx+c\right)^4-(3\,a^5+10\,a^3b^2+15\,ab^4)dx+8\left(5\,a^2b\right.}{}$$

input
```
integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas")
```

output
```
-1/8*(8*b^5*log(-cos(d*x + c)) + 2*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x +
c)^4 - (3*a^5 + 10*a^3*b^2 + 15*a*b^4)*d*x + 8*(5*a^2*b^3 - b^5)*cos(d*x +
 c)^2 - (2*(a^5 - 10*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^3 + (3*a^5 + 10*a^3*b
^2 - 25*a*b^4)*cos(d*x + c))*sin(d*x + c))/d
```

3.98.6 Sympy [F]

$$\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$
$$=\int\left(a\cos\left(c+dx\right)+b\sin\left(c+dx\right)\right)^5\sec\left(c+dx\right)dx$$

input
```
integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

output
```
Integral((a*cos(c + d*x) + b*sin(c + d*x))**5*sec(c + d*x), x)
```

3.98.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00

$$\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$
$$=\frac{80\,a^2b^3\sin\left(dx+c\right)^4-40\left(\sin\left(dx+c\right)^2-1\right)^2a^4b+\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)a^5}{}$$

input
```
integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima")
```

output
```
1/32*(80*a^2*b^3*sin(d*x + c)^4 - 40*(sin(d*x + c)^2 - 1)^2*a^4*b + (12*d*
x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^5 + 10*(4*d*x + 4*c -
sin(4*d*x + 4*c))*a^3*b^2 + 5*(12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*
d*x + 2*c))*a*b^4 - 8*(sin(d*x + c)^4 + 2*sin(d*x + c)^2 + 2*log(sin(d*x +
 c)^2 - 1))*b^5)/d
```

3.98.8 Giac [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.17

$$\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \frac{4\,b^5\log\left(\tan\left(dx+c\right)^2+1\right)+\left(3\,a^5+10\,a^3b^2+15\,ab^4\right)(dx+c)-\frac{6\,b^5\tan(dx+c)^4-3\,a^5\tan(dx+c)^3-10\,a^3b^2\tan(dx+c)^3}{}}{8\,d}$$

input
```
integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

output
```
1/8*(4*b^5*log(tan(d*x + c)^2 + 1) + (3*a^5 + 10*a^3*b^2 + 15*a*b^4)*(d*x
+ c) - (6*b^5*tan(d*x + c)^4 - 3*a^5*tan(d*x + c)^3 - 10*a^3*b^2*tan(d*x +
 c)^3 + 25*a*b^4*tan(d*x + c)^3 + 40*a^2*b^3*tan(d*x + c)^2 + 4*b^5*tan(d*
x + c)^2 - 5*a^5*tan(d*x + c) + 10*a^3*b^2*tan(d*x + c) + 15*a*b^4*tan(d*x
 + c) + 10*a^4*b + 20*a^2*b^3)/(tan(d*x + c)^2 + 1)^2)/d
```

3.98.9 Mupad [B] (verification not implemented)

Time = 24.93 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.75

$$\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \frac{4\,b^5\,\ln\left(\frac{1}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}\right)-4\,b^5\,\ln\left(\frac{\cos(c+d\,x)}{\cos(c+d\,x)+1}\right)+3\,a^5\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)+\frac{3\,b^5\,\cos(2\,c+2\,d\,x)}{2}-\frac{b^5\,\cos(4\,c+4\,d\,x)}{8}+}{}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))^5/cos(c + d*x),x)
```

output
```
(4*b^5*log(1/cos(c/2 + (d*x)/2)^2) - 4*b^5*log(cos(c + d*x)/(cos(c + d*x)
+ 1)) + 3*a^5*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + (3*b^5*cos(2*c
 + 2*d*x))/2 - (b^5*cos(4*c + 4*d*x))/8 + a^5*sin(2*c + 2*d*x) + (a^5*sin(
4*c + 4*d*x))/8 + 15*a*b^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) - (
5*a^4*b*cos(2*c + 2*d*x))/2 - (5*a^4*b*cos(4*c + 4*d*x))/8 - 5*a*b^4*sin(2
*c + 2*d*x) + (5*a*b^4*sin(4*c + 4*d*x))/8 + 10*a^3*b^2*atan(sin(c/2 + (d*
x)/2)/cos(c/2 + (d*x)/2)) - 5*a^2*b^3*cos(2*c + 2*d*x) + (5*a^2*b^3*cos(4*
c + 4*d*x))/4 - (5*a^3*b^2*sin(4*c + 4*d*x))/4)/(4*d)
```

3.98. $\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

3.99 $\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

3.99.1 Optimal result

Integrand size = 28, antiderivative size = 205

$$\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \frac{5ab^4\text{arctanh}(\sin(c+dx))}{d} - \frac{10a^2b^3\cos(c+dx)}{d} + \frac{2b^5\cos(c+dx)}{d} - \frac{5a^4b\cos^3(c+dx)}{3d} + \frac{10a^2b^3\cos^3(c+dx)}{3d} - \frac{b^5\cos^3(c+dx)}{3d} + \frac{b^5\sec(c+dx)}{d} + \frac{a^5\sin(c+dx)}{d} - \frac{5ab^4\sin(c+dx)}{d} - \frac{a^5\sin^3(c+dx)}{3d} + \frac{10a^3b^2\sin^3(c+dx)}{3d} - \frac{5ab^4\sin^3(c+dx)}{3d}$$

output
```
5*a*b^4*arctanh(sin(d*x+c))/d-10*a^2*b^3*cos(d*x+c)/d+2*b^5*cos(d*x+c)/d-5
/3*a^4*b*cos(d*x+c)^3/d+10/3*a^2*b^3*cos(d*x+c)^3/d-1/3*b^5*cos(d*x+c)^3/d
+b^5*sec(d*x+c)/d+a^5*sin(d*x+c)/d-5*a*b^4*sin(d*x+c)/d-1/3*a^5*sin(d*x+c)
^3/d+10/3*a^3*b^2*sin(d*x+c)^3/d-5/3*a*b^4*sin(d*x+c)^3/d
```

3.99.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 632 vs. 2(205) = 410.

Time = 8.36 (sec) , antiderivative size = 632, normalized size of antiderivative = 3.08

$$\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$
$$= \frac{b^5\cos^5(c+dx)(a+b\tan(c+dx))^5}{d(a\cos(c+dx)+b\sin(c+dx))^5}$$
$$- \frac{b(5a^4+30a^2b^2-7b^4)\cos^6(c+dx)(a+b\tan(c+dx))^5}{4d(a\cos(c+dx)+b\sin(c+dx))^5}$$
$$- \frac{b(5a^4-10a^2b^2+b^4)\cos^5(c+dx)\cos(3(c+dx))(a+b\tan(c+dx))^5}{12d(a\cos(c+dx)+b\sin(c+dx))^5}$$
$$- \frac{5ab^4\cos^5(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^5}{d(a\cos(c+dx)+b\sin(c+dx))^5}$$
$$+ \frac{5ab^4\cos^5(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^5}{d(a\cos(c+dx)+b\sin(c+dx))^5}$$
$$+ \frac{b^5\cos^5(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^5}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b\sin(c+dx))^5}$$
$$- \frac{b^5\cos^5(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^5}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b\sin(c+dx))^5}$$
$$+ \frac{a(3a^4+10a^2b^2-25b^4)\cos^5(c+dx)\sin(c+dx)(a+b\tan(c+dx))^5}{4d(a\cos(c+dx)+b\sin(c+dx))^5}$$
$$+ \frac{a(a^4-10a^2b^2+5b^4)\cos^5(c+dx)\sin(3(c+dx))(a+b\tan(c+dx))^5}{12d(a\cos(c+dx)+b\sin(c+dx))^5}$$

input
```
Integrate[Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

3.99. $\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

output
```
(b^5*Cos[c + d*x]^5*(a + b*Tan[c + d*x])^5)/(d*(a*Cos[c + d*x] + b*Sin[c +
 d*x])^5) - (b*(5*a^4 + 30*a^2*b^2 - 7*b^4)*Cos[c + d*x]^6*(a + b*Tan[c +
d*x])^5)/(4*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) - (b*(5*a^4 - 10*a^2*b^
2 + b^4)*Cos[c + d*x]^5*Cos[3*(c + d*x)]*(a + b*Tan[c + d*x])^5)/(12*d*(a*
Cos[c + d*x] + b*Sin[c + d*x])^5) - (5*a*b^4*Cos[c + d*x]^5*Log[Cos[(c + d
*x)/2] - Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^5)/(d*(a*Cos[c + d*x] + b*
Sin[c + d*x])^5) + (5*a*b^4*Cos[c + d*x]^5*Log[Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2]]*(a + b*Tan[c + d*x])^5)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^5)
+ (b^5*Cos[c + d*x]^5*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^5)/(d*(Cos[(c
+ d*x)/2] - Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) - (b^5*
Cos[c + d*x]^5*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^5)/(d*(Cos[(c + d*x)/
2] + Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + (a*(3*a^4 +
10*a^2*b^2 - 25*b^4)*Cos[c + d*x]^5*Sin[c + d*x]*(a + b*Tan[c + d*x])^5)/(
4*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + (a*(a^4 - 10*a^2*b^2 + 5*b^4)*C
os[c + d*x]^5*Sin[3*(c + d*x)]*(a + b*Tan[c + d*x])^5)/(12*d*(a*Cos[c + d*
x] + b*Sin[c + d*x])^5)
```

3.99.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\cos(c+dx)+b\sin(c+dx))^5}{\cos(c+dx)^2}dx$$

$$\Big\downarrow \text{3569}$$

$$\int \left(a^5\cos^3(c+dx)+5a^4b\sin(c+dx)\cos^2(c+dx)+10a^3b^2\sin^2(c+dx)\cos(c+dx)+10a^2b^3\sin^3(c+dx)+5ab^4\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5\sin^3(c+dx)}{3d}+\frac{a^5\sin(c+dx)}{d}-\frac{5a^4b\cos^3(c+dx)}{3d}+\frac{10a^3b^2\sin^3(c+dx)}{3d}+\frac{10a^2b^3\cos^3(c+dx)}{3d}-\frac{10a^2b^3\cos(c+dx)}{d}+\frac{5ab^4\text{arctanh}(\sin(c+dx))}{d}-\frac{5ab^4\sin^3(c+dx)}{3d}-\frac{5ab^4\sin(c+dx)}{d}-\frac{b^5\cos^3(c+dx)}{3d}+\frac{2b^5\cos(c+dx)}{d}+\frac{b^5\sec(c+dx)}{d}$$

input
```
Int[Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

output
```
(5*a*b^4*ArcTanh[Sin[c + d*x]])/d - (10*a^2*b^3*Cos[c + d*x])/d + (2*b^5*C
os[c + d*x])/d - (5*a^4*b*Cos[c + d*x]^3)/(3*d) + (10*a^2*b^3*Cos[c + d*x]
^3)/(3*d) - (b^5*Cos[c + d*x]^3)/(3*d) + (b^5*Sec[c + d*x])/d + (a^5*Sin[c
 + d*x])/d - (5*a*b^4*Sin[c + d*x])/d - (a^5*Sin[c + d*x]^3)/(3*d) + (10*a
^3*b^2*Sin[c + d*x]^3)/(3*d) - (5*a*b^4*Sin[c + d*x]^3)/(3*d)
```

3.99.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

3.99.4 Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{\frac{a^5(2+\cos(dx+c)^2)\sin(dx+c)}{3}-\frac{5a^4b\cos(dx+c)^3}{3}+\frac{10a^3b^2\sin(dx+c)^3}{3}-\frac{10a^2b^3(2+\sin(dx+c)^2)\cos(dx+c)}{3}+5a\,b^4\left(-\frac{\sin(dx+c)^3}{3}-\right.}{d}$
default	$\frac{\frac{a^5(2+\cos(dx+c)^2)\sin(dx+c)}{3}-\frac{5a^4b\cos(dx+c)^3}{3}+\frac{10a^3b^2\sin(dx+c)^3}{3}-\frac{10a^2b^3(2+\sin(dx+c)^2)\cos(dx+c)}{3}+5a\,b^4\left(-\frac{\sin(dx+c)^3}{3}-\right.}{d}$
parts	$\frac{a^5(2+\cos(dx+c)^2)\sin(dx+c)}{3d}+\frac{b^5\left(\frac{\sin(dx+c)^6}{\cos(dx+c)}+\left(\frac{8}{3}+\sin(dx+c)^4+\frac{4\sin(dx+c)^2}{3}\right)\cos(dx+c)\right)}{d}+\frac{10a^3b^2\sin(dx+c)^3}{3d}$
parallelrisch	$-120a\,b^4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\cos(dx+c)+120a\,b^4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\cos(dx+c)+20(-a^4b-4a^2b^3+b^5)\cos(2dx+2c)+$
risch	$-\frac{5\,e^{i(dx+c)}a^4b}{8d}-\frac{15\,e^{i(dx+c)}a^2b^3}{4d}+\frac{7\,e^{i(dx+c)}b^5}{8d}+\frac{3ie^{-i(dx+c)}a^5}{8d}-\frac{25ie^{-i(dx+c)}a\,b^4}{8d}+\frac{25ie^{i(dx+c)}a\,b^4}{8d}-\frac{5}{}$
norman	$\frac{\frac{10a^4b+40a^2b^3-16b^5}{3d}-\frac{10a^4b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{d}-\frac{5(2a^4b+8a^2b^3)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{d}+\frac{5(4a^4b+16a^2b^3-16b^5)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{3d}+\frac{2(5a^4b+80}{}}{}$

input
```
int(sec(d*x+c)^2*(cos(d*x+c)*a+b*sin(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/3*a^5*(2+cos(d*x+c)^2)*sin(d*x+c)-5/3*a^4*b*cos(d*x+c)^3+10/3*a^3*b
^2*sin(d*x+c)^3-10/3*a^2*b^3*(2+sin(d*x+c)^2)*cos(d*x+c)+5*a*b^4*(-1/3*sin
(d*x+c)^3-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+b^5*(sin(d*x+c)^6/cos(d*x+
c)+(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)))
```

3.99.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.86

$$\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$
$$=\frac{15\,ab^4\cos(dx+c)\log(\sin(dx+c)+1)-15\,ab^4\cos(dx+c)\log(-\sin(dx+c)+1)+6\,b^5-2\,(5\,a^4b-1}{}$$

input
```
integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas"
)
```

output
```
1/6*(15*a*b^4*cos(d*x + c)*log(sin(d*x + c) + 1) - 15*a*b^4*cos(d*x + c)*l
og(-sin(d*x + c) + 1) + 6*b^5 - 2*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c
)^4 - 12*(5*a^2*b^3 - b^5)*cos(d*x + c)^2 + 2*((a^5 - 10*a^3*b^2 + 5*a*b^4
)*cos(d*x + c)^3 + 2*(a^5 + 5*a^3*b^2 - 10*a*b^4)*cos(d*x + c))*sin(d*x +
c))/(d*cos(d*x + c))
```

3.99.6 Sympy [F(-1)]

Timed out.

$$\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**2*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

output
```
Timed out
```

3.99.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.79

$$\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx =$$

$$-\frac{10\,a^4b\cos(dx+c)^3-20\,a^3b^2\sin(dx+c)^3+2\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)a^5-20\left(\cos(dx+c)^3-\right.}{}$$

input
```
integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima"
)
```

output
```
-1/6*(10*a^4*b*cos(d*x + c)^3 - 20*a^3*b^2*sin(d*x + c)^3 + 2*(sin(d*x + c
)^3 - 3*sin(d*x + c))*a^5 - 20*(cos(d*x + c)^3 - 3*cos(d*x + c))*a^2*b^3 +
 5*(2*sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1) +
 6*sin(d*x + c))*a*b^4 + 2*(cos(d*x + c)^3 - 3/cos(d*x + c) - 6*cos(d*x +
c))*b^5)/d
```

3.99.8 Giac [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.38

$$\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$
$$= \frac{15\,ab^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,ab^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{6\,b^5}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}+\frac{2\left(3\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}{}}{}$$

input
```
integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

output
```
1/3*(15*a*b^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*a*b^4*log(abs(tan(1/
2*d*x + 1/2*c) - 1)) - 6*b^5/(tan(1/2*d*x + 1/2*c)^2 - 1) + 2*(3*a^5*tan(1
/2*d*x + 1/2*c)^5 - 15*a*b^4*tan(1/2*d*x + 1/2*c)^5 - 15*a^4*b*tan(1/2*d*x
 + 1/2*c)^4 + 3*b^5*tan(1/2*d*x + 1/2*c)^4 + 2*a^5*tan(1/2*d*x + 1/2*c)^3
+ 40*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 50*a*b^4*tan(1/2*d*x + 1/2*c)^3 - 60
*a^2*b^3*tan(1/2*d*x + 1/2*c)^2 + 12*b^5*tan(1/2*d*x + 1/2*c)^2 + 3*a^5*ta
n(1/2*d*x + 1/2*c) - 15*a*b^4*tan(1/2*d*x + 1/2*c) - 5*a^4*b - 20*a^2*b^3
+ 5*b^5)/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.99.9 Mupad [B] (verification not implemented)

Time = 26.78 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.35

$$\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \frac{10\,a\,b^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$
$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(10\,a\,b^4-2\,a^5\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(10\,a^4\,b-40\,a^2\,b^3\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{2\,a^5}{3}-\frac{80\,a^3\,b^2}{3}+\frac{70\,a\,b^4}{3}\right.}{d}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))^5/cos(c + d*x)^2,x)
```

output
```
(10*a*b^4*atanh(tan(c/2 + (d*x)/2)))/d - (tan(c/2 + (d*x)/2)*(10*a*b^4 - 2
*a^5) + tan(c/2 + (d*x)/2)^4*(10*a^4*b - 40*a^2*b^3) + tan(c/2 + (d*x)/2)^
3*((70*a*b^4)/3 + (2*a^5)/3 - (80*a^3*b^2)/3) - tan(c/2 + (d*x)/2)^5*((70*
a*b^4)/3 + (2*a^5)/3 - (80*a^3*b^2)/3) - tan(c/2 + (d*x)/2)^2*((10*a^4*b)/
3 + (32*b^5)/3 - (80*a^2*b^3)/3) + (10*a^4*b)/3 - tan(c/2 + (d*x)/2)^7*(10
*a*b^4 - 2*a^5) - (16*b^5)/3 + (40*a^2*b^3)/3 - 10*a^4*b*tan(c/2 + (d*x)/2
)^6)/(d*(2*tan(c/2 + (d*x)/2)^2 - 2*tan(c/2 + (d*x)/2)^6 - tan(c/2 + (d*x)
/2)^8 + 1))
```

3.99. $\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

3.100 $\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

3.100.1 Optimal result

Integrand size = 28, antiderivative size = 169

$$\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$
$$= \frac{1}{2}a\left(a^4+10a^2b^2-15b^4\right)x-\frac{2b^3(5a^2-b^2)\log(\sin(c+dx))}{d}$$
$$+\frac{2b^3(5a^2-b^2)\log(\tan(c+dx))}{d}$$
$$+\frac{(b(5a^4-10a^2b^2+b^4)+a(a^4-10a^2b^2+5b^4)\cot(c+dx))\sin^2(c+dx)}{2d}$$
$$+\frac{5ab^4\tan(c+dx)}{d}+\frac{b^5\tan^2(c+dx)}{2d}$$

output
```
1/2*a*(a^4+10*a^2*b^2-15*b^4)*x-2*b^3*(5*a^2-b^2)*ln(sin(d*x+c))/d+2*b^3*(
5*a^2-b^2)*ln(tan(d*x+c))/d+1/2*(b*(5*a^4-10*a^2*b^2+b^4)+a*(a^4-10*a^2*b^
2+5*b^4)*cot(d*x+c))*sin(d*x+c)^2/d+5*a*b^4*tan(d*x+c)/d+1/2*b^5*tan(d*x+c
)^2/d
```

3.100.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 571 vs. $2(169) = 338$.

Time = 6.36 (sec) , antiderivative size = 571, normalized size of antiderivative = 3.38

$$\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$

$$= \frac{b^3\left(\frac{\cos^2(c+dx)(a+b\tan(c+dx))^6(b^2+ab\tan(c+dx))}{2b^4(a^2+b^2)} - \frac{(-6a^2+4b^2)\left(\frac{1}{2}\left(5a^4-10a^2b^2+b^4+\frac{a^5-10a^3b^2+5ab^4}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)+\frac{1}{2}\left(\right.\right.}{}\right.}{}$$

input
```
Integrate[Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

output
```
(b^3*((Cos[c + d*x]^2*(a + b*Tan[c + d*x])^6*(b^2 + a*b*Tan[c + d*x]))/(2*
b^4*(a^2 + b^2)) - ((-6*a^2 + 4*b^2)*(((5*a^4 - 10*a^2*b^2 + b^4 + (a^5 -
10*a^3*b^2 + 5*a*b^4)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]])/2 + ((
5*a^4 - 10*a^2*b^2 + b^4 - (a^5 - 10*a^3*b^2 + 5*a*b^4)/Sqrt[-b^2])*Log[Sq
rt[-b^2] + b*Tan[c + d*x]])/2 + 5*a*b*(2*a^2 - b^2)*Tan[c + d*x] + (b^2*(1
0*a^2 - b^2)*Tan[c + d*x]^2)/2 + (5*a*b^3*Tan[c + d*x]^3)/3 + (b^4*Tan[c +
 d*x]^4)/4) + 5*a*(((6*a^5 - 20*a^3*b^2 + 6*a*b^4 + (a^6 - 15*a^4*b^2 + 15
*a^2*b^4 - b^6)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]])/2 + ((6*a^5
- 20*a^3*b^2 + 6*a*b^4 - (a^6 - 15*a^4*b^2 + 15*a^2*b^4 - b^6)/Sqrt[-b^2])
*Log[Sqrt[-b^2] + b*Tan[c + d*x]])/2 + b*(15*a^4 - 15*a^2*b^2 + b^4)*Tan[c
 + d*x] + a*b^2*(10*a^2 - 3*b^2)*Tan[c + d*x]^2 + (b^3*(15*a^2 - b^2)*Tan[
c + d*x]^3)/3 + (3*a*b^4*Tan[c + d*x]^4)/2 + (b^5*Tan[c + d*x]^5)/5))/(2*b
^2*(a^2 + b^2))))/d
```

3.100.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 3567, 532, 25, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a\cos(c+dx)+b\sin(c+dx))^5}{\cos(c+dx)^3}dx$$

$\Big\downarrow$ 3567

$$-\frac{\int \frac{(b+a\cot(c+dx))^5\tan^3(c+dx)}{(\cot^2(c+dx)+1)^2}d\cot(c+dx)}{d}$$

$\Big\downarrow$ 532

$$-\frac{-\frac{1}{2}\int -\frac{(2b^5+10a\cot(c+dx)b^4+2(10a^2-b^2)\cot^2(c+dx)b^3+a(a^4+10b^2a^2-5b^4)\cot^3(c+dx))\tan^3(c+dx)}{\cot^2(c+dx)+1}d\cot(c+dx)-\frac{a(a^4-10a^2b^2+}{}}{d}$$

$\Big\downarrow$ 25

$$-\frac{\frac{1}{2}\int \frac{(2b^5+10a\cot(c+dx)b^4+2(10a^2-b^2)\cot^2(c+dx)b^3+a(a^4+10b^2a^2-5b^4)\cot^3(c+dx))\tan^3(c+dx)}{\cot^2(c+dx)+1}d\cot(c+dx)-\frac{a(a^4-10a^2b^2+5b^4)}{2(c}}{d}$$

$\Big\downarrow$ 2333

$$-\frac{\frac{1}{2}\int \left(2\tan^3(c+dx)b^5+10a\tan^2(c+dx)b^4+4(5a^2b^3-b^5)\tan(c+dx)+\frac{a(a^4+10b^2a^2-15b^4)-4b^3(5a^2-b^2)\cot(c+dx)}{\cot^2(c+dx)+1}\right.}{d}$$

$\Big\downarrow$ 2009

$$-\frac{\frac{1}{2}\left(-2b^3(5a^2-b^2)\log\left(\cot^2(c+dx)+1\right)+4b^3(5a^2-b^2)\log(\cot(c+dx))+a(a^4+10a^2b^2-15b^4)\arctan(\cot(c\right.}{d}$$

input
```
Int[Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

output
```
-((-1/2*(b*(5*a^4 - 10*a^2*b^2 + b^4) + a*(a^4 - 10*a^2*b^2 + 5*b^4)*Cot[c
 + d*x])/(1 + Cot[c + d*x]^2) + (a*(a^4 + 10*a^2*b^2 - 15*b^4)*ArcTan[Cot[
c + d*x]] + 4*b^3*(5*a^2 - b^2)*Log[Cot[c + d*x]] - 2*b^3*(5*a^2 - b^2)*Lo
g[1 + Cot[c + d*x]^2] - 10*a*b^4*Tan[c + d*x] - b^5*Tan[c + d*x]^2)/2)/d)
```

3.100. $\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

3.100.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3567
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[x^m*((b
+ a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a,
b, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[
n, 0] && GtQ[m, 1])
```

3.100.4 Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.24

method	result
derivativedivides	$\frac{a^5\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-\frac{5\cos(dx+c)^2a^4b}{2}+10a^3b^2\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+10a^2b^3\left(-\frac{\sin(dx+c)^2}{2}-\ln(c\right.}{}$
default	$\frac{a^5\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-\frac{5\cos(dx+c)^2a^4b}{2}+10a^3b^2\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+10a^2b^3\left(-\frac{\sin(dx+c)^2}{2}-\ln(c\right.}{}$
parts	$\frac{a^5\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{b^5\left(\frac{\sin(dx+c)^6}{2\cos(dx+c)^2}+\frac{\sin(dx+c)^4}{2}+\sin(dx+c)^2+2\ln(\cos(dx+c))\right)}{d}+\frac{10a^3b^2\left(-\frac{\sin(dx\right.}{}}{}$
parallelrisch	$\frac{80b^3\left(a^2-\frac{b^2}{5}\right)(1+\cos(2dx+2c))\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)-80b^3\left(a^2-\frac{b^2}{5}\right)(1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-80b^3\left(a^2-\frac{b}{5}\right.}{}$
risch	$-\frac{4ib^5c}{d}-\frac{5ie^{2i(dx+c)}a\,b^4}{8d}+\frac{a^5x}{2}+5a^3b^2x-\frac{15a\,b^4x}{2}-\frac{5\,e^{2i(dx+c)}a^4b}{8d}+\frac{5\,e^{2i(dx+c)}a^2b^3}{4d}-\frac{e^{2i(dx+c)}b^5}{8d}-$
norman	$\frac{\left(\frac{1}{2}a^5+5a^3b^2-\frac{15}{2}a\,b^4\right)x+\left(-\frac{5}{2}a^5-25a^3b^2+\frac{75}{2}a\,b^4\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+\left(-\frac{5}{2}a^5-25a^3b^2+\frac{75}{2}a\,b^4\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8+\left(\frac{1}{2}a^5+5a\right.}{}$

input
```
int(sec(d*x+c)^3*(cos(d*x+c)*a+b*sin(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^5*(1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)-5/2*cos(d*x+c)^2*a^4*b+
10*a^3*b^2*(-1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)+10*a^2*b^3*(-1/2*sin
(d*x+c)^2-ln(cos(d*x+c)))+5*a*b^4*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3
/2*sin(d*x+c))*cos(d*x+c)-3/2*d*x-3/2*c)+b^5*(1/2*sin(d*x+c)^6/cos(d*x+c)^
2+1/2*sin(d*x+c)^4+sin(d*x+c)^2+2*ln(cos(d*x+c))))
```

3.100.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.05

$$\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$
$$=\frac{2\,b^5-2\,(5\,a^4b-10\,a^2b^3+b^5)\cos{(dx+c)}^4-8\,(5\,a^2b^3-b^5)\cos{(dx+c)}^2\log{(-\cos{(dx+c)})}+(5\,a^4b-1}{}$$

input
```
integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas"
)
```

output
```
1/4*(2*b^5 - 2*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c)^4 - 8*(5*a^2*b^3
- b^5)*cos(d*x + c)^2*log(-cos(d*x + c)) + (5*a^4*b - 10*a^2*b^3 + b^5 + 2
*(a^5 + 10*a^3*b^2 - 15*a*b^4)*d*x)*cos(d*x + c)^2 + 2*(10*a*b^4*cos(d*x +
 c) + (a^5 - 10*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*
x + c)^2)
```

3.100.6 Sympy [F(-1)]

Timed out.

$$\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**3*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

output
```
Timed out
```

3.100.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.06

$$\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$
$$= \frac{10\,a^4b\sin\left(dx+c\right)^2+\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a^5+10\left(2\,dx+2\,c-\sin\left(2\,dx+2\,c\right)\right)a^3b^2-20\left(\sin\left(d\right.\right.}{}$$

input
```
integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima"
)
```

output
```
1/4*(10*a^4*b*sin(d*x + c)^2 + (2*d*x + 2*c + sin(2*d*x + 2*c))*a^5 + 10*(
2*d*x + 2*c - sin(2*d*x + 2*c))*a^3*b^2 - 20*(sin(d*x + c)^2 + log(sin(d*x
 + c)^2 - 1))*a^2*b^3 - 10*(3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1
) - 2*tan(d*x + c))*a*b^4 + 2*(sin(d*x + c)^2 - 1/(sin(d*x + c)^2 - 1) + 2
*log(sin(d*x + c)^2 - 1))*b^5)/d
```

3.100.8 Giac [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.02

$$\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \frac{b^5\tan(dx+c)^2+10\,ab^4\tan(dx+c)+(a^5+10\,a^3b^2-15\,ab^4)(dx+c)+2\,(5\,a^2b^3-b^5)\log(\tan(dx+c)}{2\,d}$$

input
```
integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

output
```
1/2*(b^5*tan(d*x + c)^2 + 10*a*b^4*tan(d*x + c) + (a^5 + 10*a^3*b^2 - 15*a
*b^4)*(d*x + c) + 2*(5*a^2*b^3 - b^5)*log(tan(d*x + c)^2 + 1) - (10*a^2*b^
3*tan(d*x + c)^2 - 2*b^5*tan(d*x + c)^2 - a^5*tan(d*x + c) + 10*a^3*b^2*ta
n(d*x + c) - 5*a*b^4*tan(d*x + c) + 5*a^4*b - b^5)/(tan(d*x + c)^2 + 1))/d
```

3.100.9 Mupad [B] (verification not implemented)

Time = 25.01 (sec) , antiderivative size = 354, normalized size of antiderivative = 2.09

$$\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \frac{2\left(b^5\,\ln\left(\frac{\cos(c+d\,x)}{\cos(c+d\,x)+1}\right)-b^5\,\ln\left(\frac{1}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}\right)+\frac{a^5\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{2}-5\,a^2\,b^3\,\ln\left(\frac{\cos(c+d\,x)}{\cos(c+d\,x)+1}\right)-\frac{15\,a\,b^4\,\mathrm{atan}\left(\frac{\sin}{\cos}\right.}{2}\right.}{d} + \frac{\frac{5\,a^4\,b}{16}+\frac{9\,b^5}{16}-\frac{5\,a^2\,b^3}{8}-\frac{b^5\,\cos(4\,c+4\,d\,x)}{16}+\frac{a^5\,\sin(2\,c+2\,d\,x)}{8}+\frac{a^5\,\sin(4\,c+4\,d\,x)}{16}-\frac{5\,a^4\,b\,\cos(4\,c+4\,d\,x)}{16}+\frac{25\,a\,b^4\,\sin(2\,c+2\,d\,x)}{8}}{d\,\left(\frac{\cos(2\,c+2\,d\,x)}{2}+\frac{1}{2}\right)}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))^5/cos(c + d*x)^3,x)
```

output
```
(2*(b^5*log(cos(c + d*x)/(cos(c + d*x) + 1)) - b^5*log(1/cos(c/2 + (d*x)/2
)^2) + (a^5*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/2 - 5*a^2*b^3*log
(cos(c + d*x)/(cos(c + d*x) + 1)) - (15*a*b^4*atan(sin(c/2 + (d*x)/2)/cos(
c/2 + (d*x)/2)))/2 + 5*a^2*b^3*log(1/cos(c/2 + (d*x)/2)^2) + 5*a^3*b^2*ata
n(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))))/d + ((5*a^4*b)/16 + (9*b^5)/16
- (5*a^2*b^3)/8 - (b^5*cos(4*c + 4*d*x))/16 + (a^5*sin(2*c + 2*d*x))/8 + (
a^5*sin(4*c + 4*d*x))/16 - (5*a^4*b*cos(4*c + 4*d*x))/16 + (25*a*b^4*sin(2
*c + 2*d*x))/8 + (5*a*b^4*sin(4*c + 4*d*x))/16 + (5*a^2*b^3*cos(4*c + 4*d*
x))/8 - (5*a^3*b^2*sin(2*c + 2*d*x))/4 - (5*a^3*b^2*sin(4*c + 4*d*x))/8)/(
d*(cos(2*c + 2*d*x)/2 + 1/2))
```

3.100. $\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

3.101 $\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

3.101.1 Optimal result

Integrand size = 28, antiderivative size = 204

$$\begin{aligned}
&\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx \\
&= \frac{10a^3b^2\operatorname{arctanh}(\sin(c+dx))}{d}-\frac{15ab^4\operatorname{arctanh}(\sin(c+dx))}{2d} \\
&\quad -\frac{5a^4b\cos(c+dx)}{d}+\frac{10a^2b^3\cos(c+dx)}{d}-\frac{b^5\cos(c+dx)}{d} \\
&\quad +\frac{10a^2b^3\sec(c+dx)}{d}-\frac{2b^5\sec(c+dx)}{d}+\frac{b^5\sec^3(c+dx)}{3d}+\frac{a^5\sin(c+dx)}{d} \\
&\quad -\frac{10a^3b^2\sin(c+dx)}{d}+\frac{15ab^4\sin(c+dx)}{2d}+\frac{5ab^4\sin(c+dx)\tan^2(c+dx)}{2d}
\end{aligned}$$

output
```
10*a^3*b^2*arctanh(sin(d*x+c))/d-15/2*a*b^4*arctanh(sin(d*x+c))/d-5*a^4*b*
cos(d*x+c)/d+10*a^2*b^3*cos(d*x+c)/d-b^5*cos(d*x+c)/d+10*a^2*b^3*sec(d*x+c
)/d-2*b^5*sec(d*x+c)/d+1/3*b^5*sec(d*x+c)^3/d+a^5*sin(d*x+c)/d-10*a^3*b^2*
sin(d*x+c)/d+15/2*a*b^4*sin(d*x+c)/d+5/2*a*b^4*sin(d*x+c)*tan(d*x+c)^2/d
```

3.101.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 892 vs. $2(204) = 408$.

Time = 7.69 (sec) , antiderivative size = 892, normalized size of antiderivative = 4.37

$$\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$
$$= -\frac{b^3(-60a^2+11b^2)\cos^5(c+dx)(a+b\tan(c+dx))^5}{6d(a\cos(c+dx)+b\sin(c+dx))^5} - \frac{b(5a^4-10a^2b^2+b^4)\cos^6(c+dx)(a+b\tan(c+dx))^5}{d(a\cos(c+dx)+b\sin(c+dx))^5} - \frac{5(4a^3b^2-3ab^4)\cos^5(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^5}{2d(a\cos(c+dx)+b\sin(c+dx))^5} + \frac{5(4a^3b^2-3ab^4)\cos^5(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^5}{2d(a\cos(c+dx)+b\sin(c+dx))^5} + \frac{(15ab^4+b^5)\cos^5(c+dx)(a+b\tan(c+dx))^5}{12d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2(a\cos(c+dx)+b\sin(c+dx))^5} + \frac{b^5\cos^5(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^5}{6d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3(a\cos(c+dx)+b\sin(c+dx))^5} - \frac{b^5\cos^5(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^5}{6d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3(a\cos(c+dx)+b\sin(c+dx))^5} + \frac{(-15ab^4+b^5)\cos^5(c+dx)(a+b\tan(c+dx))^5}{12d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2(a\cos(c+dx)+b\sin(c+dx))^5} + \frac{\cos^5(c+dx)\left(60a^2b^3\sin\left(\frac{1}{2}(c+dx)\right)-11b^5\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^5}{6d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b\sin(c+dx))^5} + \frac{\cos^5(c+dx)\left(-60a^2b^3\sin\left(\frac{1}{2}(c+dx)\right)+11b^5\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^5}{6d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b\sin(c+dx))^5} + \frac{a(a^4-10a^2b^2+5b^4)\cos^5(c+dx)\sin(c+dx)(a+b\tan(c+dx))^5}{d(a\cos(c+dx)+b\sin(c+dx))^5}$$

input
```
Integrate[Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

output
```
-1/6*(b^3*(-60*a^2 + 11*b^2)*Cos[c + d*x]^5*(a + b*Tan[c + d*x])^5)/(d*(a*
Cos[c + d*x] + b*Sin[c + d*x])^5) - (b*(5*a^4 - 10*a^2*b^2 + b^4)*Cos[c +
d*x]^6*(a + b*Tan[c + d*x])^5)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) - (
5*(4*a^3*b^2 - 3*a*b^4)*Cos[c + d*x]^5*Log[Cos[(c + d*x)/2] - Sin[(c + d*x
)/2]]*(a + b*Tan[c + d*x])^5)/(2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) +
(5*(4*a^3*b^2 - 3*a*b^4)*Cos[c + d*x]^5*Log[Cos[(c + d*x)/2] + Sin[(c + d*
x)/2]]*(a + b*Tan[c + d*x])^5)/(2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) +
 ((15*a*b^4 + b^5)*Cos[c + d*x]^5*(a + b*Tan[c + d*x])^5)/(12*d*(Cos[(c +
d*x)/2] - Sin[(c + d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + (b^5*
Cos[c + d*x]^5*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^5)/(6*d*(Cos[(c + d*x
)/2] - Sin[(c + d*x)/2])^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) - (b^5*Cos
[c + d*x]^5*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^5)/(6*d*(Cos[(c + d*x)/2
] + Sin[(c + d*x)/2])^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + ((-15*a*b^4
 + b^5)*Cos[c + d*x]^5*(a + b*Tan[c + d*x])^5)/(12*d*(Cos[(c + d*x)/2] + S
in[(c + d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + (Cos[c + d*x]^5*
(60*a^2*b^3*Sin[(c + d*x)/2] - 11*b^5*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x
])^5)/(6*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c
 + d*x])^5) + (Cos[c + d*x]^5*(-60*a^2*b^3*Sin[(c + d*x)/2] + 11*b^5*Sin[(
c + d*x)/2])*(a + b*Tan[c + d*x])^5)/(6*d*(Cos[(c + d*x)/2] + Sin[(c + d*x
)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + (a*(a^4 - 10*a^2*b^2 + 5*b...
```

3.101.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\cos(c+dx)+b\sin(c+dx))^5}{\cos(c+dx)^4}dx$$

$$\Big\downarrow \text{3569}$$

$$\int \left(a^5\cos(c+dx)+5a^4b\sin(c+dx)+10a^3b^2\sin(c+dx)\tan(c+dx)+10a^2b^3\sin(c+dx)\tan^2(c+dx)+5ab^4\text{si}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^5\sin(c+dx)}{d}-\frac{5a^4b\cos(c+dx)}{d}+\frac{10a^3b^2\text{arctanh}(\sin(c+dx))}{d}-\frac{10a^3b^2\sin(c+dx)}{d}+\frac{10a^2b^3\cos(c+dx)}{d}+\frac{10a^2b^3\sec(c+dx)}{d}-\frac{15ab^4\text{arctanh}(\sin(c+dx))}{2d}+\frac{15ab^4\sin(c+dx)}{2d}+\frac{5ab^4\sin(c+dx)\tan^2(c+dx)}{2d}-\frac{b^5\cos(c+dx)}{d}+\frac{b^5\sec^3(c+dx)}{3d}-\frac{2b^5\sec(c+dx)}{d}$$

input
```
Int[Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

output
```
(10*a^3*b^2*ArcTanh[Sin[c + d*x]])/d - (15*a*b^4*ArcTanh[Sin[c + d*x]])/(2
*d) - (5*a^4*b*Cos[c + d*x])/d + (10*a^2*b^3*Cos[c + d*x])/d - (b^5*Cos[c
+ d*x])/d + (10*a^2*b^3*Sec[c + d*x])/d - (2*b^5*Sec[c + d*x])/d + (b^5*Se
c[c + d*x]^3)/(3*d) + (a^5*Sin[c + d*x])/d - (10*a^3*b^2*Sin[c + d*x])/d +
 (15*a*b^4*Sin[c + d*x])/(2*d) + (5*a*b^4*Sin[c + d*x]*Tan[c + d*x]^2)/(2*
d)
```

3.101.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

3.101.4 Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.13

method	result
derivativedivides	$\frac{a^5 \sin(dx+c)-5\cos(dx+c)a^4b+10a^3b^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+10a^2b^3\left(\frac{\sin(dx+c)^4}{\cos(dx+c)}+\left(2+\sin(dx+c)\right.\right.}{}$
default	$\frac{a^5 \sin(dx+c)-5\cos(dx+c)a^4b+10a^3b^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+10a^2b^3\left(\frac{\sin(dx+c)^4}{\cos(dx+c)}+\left(2+\sin(dx+c)\right.\right.}{}$
parts	$\frac{a^5\sin(dx+c)}{d}+\frac{b^5\left(\frac{\sin(dx+c)^6}{3\cos(dx+c)^3}-\frac{\sin(dx+c)^6}{\cos(dx+c)}-\left(\frac{8}{3}+\sin(dx+c)^4+\frac{4\sin(dx+c)^2}{3}\right)\cos(dx+c)\right)}{d}+\frac{10a^3b^2(-\sin(dx+c)+}{}$
parallelrisch	$\frac{-180b^2\left(\frac{\cos(3dx+3c)}{3}+\cos(dx+c)\right)a\left(a^2-\frac{3b^2}{4}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+180b^2\left(\frac{\cos(3dx+3c)}{3}+\cos(dx+c)\right)a\left(a^2-\frac{3b^2}{4}\right)\ln\left(\mathrm{ta}\right.}{}$
risch	$-\frac{5\,e^{i(dx+c)}a^4b}{2d}+\frac{5\,e^{i(dx+c)}a^2b^3}{d}-\frac{e^{i(dx+c)}b^5}{2d}+\frac{5ie^{-i(dx+c)}a\,b^4}{2d}-\frac{5ie^{-i(dx+c)}a^3b^2}{d}+\frac{5ie^{i(dx+c)}a^3b^2}{d}-\frac{5\,e^{-}}{}$
norman	$\frac{\frac{a\left(2a^4-20a^2b^2+15b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{15}}{d}+\frac{a\left(2a^4-20a^2b^2+35b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{13}}{d}+\frac{a\left(6a^4-60a^2b^2+5b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{d}+\frac{30a^4b-120a}{3a}}{}$

input
```
int(sec(d*x+c)^4*(cos(d*x+c)*a+b*sin(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^5*sin(d*x+c)-5*cos(d*x+c)*a^4*b+10*a^3*b^2*(-sin(d*x+c)+ln(sec(d*x+
c)+tan(d*x+c)))+10*a^2*b^3*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d
*x+c))+5*a*b^4*(1/2*sin(d*x+c)^5/cos(d*x+c)^2+1/2*sin(d*x+c)^3+3/2*sin(d*x
+c)-3/2*ln(sec(d*x+c)+tan(d*x+c)))+b^5*(1/3*sin(d*x+c)^6/cos(d*x+c)^3-sin(
d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)))
```

3.101.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.93

$$\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$
$$=\frac{4\,b^5-12\left(5\,a^4b-10\,a^2b^3+b^5\right)\cos\left(dx+c\right)^4+15\left(4\,a^3b^2-3\,ab^4\right)\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-15}{}$$

input
```
integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas"
)
```

output
```
1/12*(4*b^5 - 12*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c)^4 + 15*(4*a^3*b
^2 - 3*a*b^4)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 15*(4*a^3*b^2 - 3*a*b
^4)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 24*(5*a^2*b^3 - b^5)*cos(d*x +
 c)^2 + 6*(5*a*b^4*cos(d*x + c) + 2*(a^5 - 10*a^3*b^2 + 5*a*b^4)*cos(d*x +
 c)^3)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

3.101.6 Sympy [F(-1)]

Timed out.

$$\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**4*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

output
```
Timed out
```

3.101.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.89

$$\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx =$$
$$-\frac{15\,ab^4\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}+3\,\log\left(\sin\left(dx+c\right)+1\right)-3\,\log\left(\sin\left(dx+c\right)-1\right)-4\,\sin\left(dx+c\right)\right)-120\,a^2b^3\left(\frac{}{\text{co}}\right.}{}$$

input
```
integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima"
)
```

output
```
-1/12*(15*a*b^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + 3*log(sin(d*x + c)
+ 1) - 3*log(sin(d*x + c) - 1) - 4*sin(d*x + c)) - 120*a^2*b^3*(1/cos(d*x
+ c) + cos(d*x + c)) + 4*b^5*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*co
s(d*x + c)) - 60*a^3*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) -
2*sin(d*x + c)) + 60*a^4*b*cos(d*x + c) - 12*a^5*sin(d*x + c))/d
```

3.101.8 Giac [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.38

$$\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$

$$= \frac{15\,(4\,a^3b^2-3\,ab^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,(4\,a^3b^2-3\,ab^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{12\,(a^5\tan\cdots}{\cdots}}{\cdots}$$

input
```
integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

output
```
1/6*(15*(4*a^3*b^2 - 3*a*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(4*a
^3*b^2 - 3*a*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 12*(a^5*tan(1/2*d*x
 + 1/2*c) - 10*a^3*b^2*tan(1/2*d*x + 1/2*c) + 5*a*b^4*tan(1/2*d*x + 1/2*c)
 - 5*a^4*b + 10*a^2*b^3 - b^5)/(tan(1/2*d*x + 1/2*c)^2 + 1) + 2*(15*a*b^4*
tan(1/2*d*x + 1/2*c)^5 - 60*a^2*b^3*tan(1/2*d*x + 1/2*c)^4 + 6*b^5*tan(1/2
*d*x + 1/2*c)^4 + 120*a^2*b^3*tan(1/2*d*x + 1/2*c)^2 - 24*b^5*tan(1/2*d*x
+ 1/2*c)^2 - 15*a*b^4*tan(1/2*d*x + 1/2*c) - 60*a^2*b^3 + 10*b^5)/(tan(1/2
*d*x + 1/2*c)^2 - 1)^3)/d
```

3.101.9 Mupad [B] (verification not implemented)

Time = 26.85 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.48

$$\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = -\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(15\,a\,b^4-20\,a^3\,b^2\right)}{d}$$

$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^5-20\,a^3\,b^2+15\,a\,b^4\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(30\,a^4\,b-40\,a^2\,b^3\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7\,\left(2\,a^5-20\,a^3\cdots\right.}{\cdots}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))^5/cos(c + d*x)^4,x)
```

output
```
- (atanh(tan(c/2 + (d*x)/2))*(15*a*b^4 - 20*a^3*b^2))/d - (tan(c/2 + (d*x)
/2)*(15*a*b^4 + 2*a^5 - 20*a^3*b^2) - tan(c/2 + (d*x)/2)^4*(30*a^4*b - 40*
a^2*b^3) - tan(c/2 + (d*x)/2)^7*(15*a*b^4 + 2*a^5 - 20*a^3*b^2) - tan(c/2
+ (d*x)/2)^3*(25*a*b^4 + 6*a^5 - 60*a^3*b^2) + tan(c/2 + (d*x)/2)^5*(25*a*
b^4 + 6*a^5 - 60*a^3*b^2) + tan(c/2 + (d*x)/2)^2*(30*a^4*b + (32*b^5)/3 -
80*a^2*b^3) - 10*a^4*b - (16*b^5)/3 + 40*a^2*b^3 + 10*a^4*b*tan(c/2 + (d*x
)/2)^6)/(d*(2*tan(c/2 + (d*x)/2)^2 - 2*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d
*x)/2)^8 - 1))
```

3.101. $\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

3.102 $\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

3.102.1 Optimal result

Integrand size = 28, antiderivative size = 147

$$\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = a(a^4-10a^2b^2+5b^4)\,x-\frac{b(5a^4-10a^2b^2+b^4)\log(\cos(c+dx))}{d} + \frac{4ab^2(a^2-b^2)\tan(c+dx)}{d}+\frac{b(3a^2-b^2)\,(a+b\tan(c+dx))^2}{2d} + \frac{2ab(a+b\tan(c+dx))^3}{3d}+\frac{b(a+b\tan(c+dx))^4}{4d}$$

output
```
a*(a^4-10*a^2*b^2+5*b^4)*x-b*(5*a^4-10*a^2*b^2+b^4)*ln(cos(d*x+c))/d+4*a*b
^2*(a^2-b^2)*tan(d*x+c)/d+1/2*b*(3*a^2-b^2)*(a+b*tan(d*x+c))^2/d+2/3*a*b*(
a+b*tan(d*x+c))^3/d+1/4*b*(a+b*tan(d*x+c))^4/d
```

3.102.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.84 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.86

$$\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \frac{6(-ia+b)^5\log(i-\tan(c+dx))+6(ia+b)^5\log(i+\tan(c+dx))+60ab^2(2a^2-b^2)\tan(c+dx)-6b^3(-}{12d}$$

input
```
Integrate[Sec[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

output
```
(6*((-I)*a + b)^5*Log[I - Tan[c + d*x]] + 6*(I*a + b)^5*Log[I + Tan[c + d*
x]] + 60*a*b^2*(2*a^2 - b^2)*Tan[c + d*x] - 6*b^3*(-10*a^2 + b^2)*Tan[c +
d*x]^2 + 20*a*b^4*Tan[c + d*x]^3 + 3*b^5*Tan[c + d*x]^4)/(12*d)
```

3.102.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3565, 3042, 3963, 3042, 4011, 3042, 4011, 3042, 4008, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\cos(c+dx)+b\sin(c+dx))^5}{\cos(c+dx)^5}dx$$

$$\Big\downarrow \text{3565}$$

$$\int (a+b\tan(c+dx))^5dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(c+dx))^5dx$$

$$\Big\downarrow \text{3963}$$

$$\int (a+b\tan(c+dx))^3\left(a^2+2b\tan(c+dx)a-b^2\right)dx+\frac{b(a+b\tan(c+dx))^4}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(c+dx))^3\left(a^2+2b\tan(c+dx)a-b^2\right)dx+\frac{b(a+b\tan(c+dx))^4}{4d}$$

$$\Big\downarrow \text{4011}$$

$$\int (a+b\tan(c+dx))^2\left(a\left(a^2-3b^2\right)+b\left(3a^2-b^2\right)\tan(c+dx)\right)dx+\frac{b(a+b\tan(c+dx))^4}{4d}+\frac{2ab(a+b\tan(c+dx))^3}{3d}$$

$$\downarrow 3042$$

$$\int (a+b\tan(c+dx))^2\left(a\left(a^2-3b^2\right)+b\left(3a^2-b^2\right)\tan(c+dx)\right)dx+\frac{b(a+b\tan(c+dx))^4}{4d}+\frac{2ab(a+b\tan(c+dx))^3}{3d}$$

$$\downarrow 4011$$

$$\int (a+b\tan(c+dx))\left(a^4-6b^2a^2+4b\left(a^2-b^2\right)\tan(c+dx)a+b^4\right)dx+\frac{b\left(3a^2-b^2\right)(a+b\tan(c+dx))^2}{2d}+\frac{b(a+b\tan(c+dx))^4}{4d}+\frac{2ab(a+b\tan(c+dx))^3}{3d}$$

$$\downarrow 3042$$

$$\int (a+b\tan(c+dx))\left(a^4-6b^2a^2+4b\left(a^2-b^2\right)\tan(c+dx)a+b^4\right)dx+\frac{b\left(3a^2-b^2\right)(a+b\tan(c+dx))^2}{2d}+\frac{b(a+b\tan(c+dx))^4}{4d}+\frac{2ab(a+b\tan(c+dx))^3}{3d}$$

$$\downarrow 4008$$

$$b\left(5a^4-10a^2b^2+b^4\right)\int \tan(c+dx)dx+\frac{b\left(3a^2-b^2\right)(a+b\tan(c+dx))^2}{2d}+\frac{4ab^2\left(a^2-b^2\right)\tan(c+dx)}{d}+ax\left(a^4-10a^2b^2+5b^4\right)+\frac{b(a+b\tan(c+dx))^4}{4d}+\frac{2ab(a+b\tan(c+dx))^3}{3d}$$

$$\downarrow 3042$$

$$b\left(5a^4-10a^2b^2+b^4\right)\int \tan(c+dx)dx+\frac{b\left(3a^2-b^2\right)(a+b\tan(c+dx))^2}{2d}+\frac{4ab^2\left(a^2-b^2\right)\tan(c+dx)}{d}+ax\left(a^4-10a^2b^2+5b^4\right)+\frac{b(a+b\tan(c+dx))^4}{4d}+\frac{2ab(a+b\tan(c+dx))^3}{3d}$$

$$\downarrow 3956$$

$$\frac{b\left(3a^2-b^2\right)(a+b\tan(c+dx))^2}{2d}+\frac{4ab^2\left(a^2-b^2\right)\tan(c+dx)}{d}-\frac{b\left(5a^4-10a^2b^2+b^4\right)\log(\cos(c+dx))}{d}+ax\left(a^4-10a^2b^2+5b^4\right)+\frac{b(a+b\tan(c+dx))^4}{4d}+\frac{2ab(a+b\tan(c+dx))^3}{3d}$$

input
```
Int[Sec[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

3.102. $\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

output
```
a*(a^4 - 10*a^2*b^2 + 5*b^4)*x - (b*(5*a^4 - 10*a^2*b^2 + b^4)*Log[Cos[c +
 d*x]])/d + (4*a*b^2*(a^2 - b^2)*Tan[c + d*x])/d + (b*(3*a^2 - b^2)*(a + b
*Tan[c + d*x])^2)/(2*d) + (2*a*b*(a + b*Tan[c + d*x])^3)/(3*d) + (b*(a + b
*Tan[c + d*x])^4)/(4*d)
```

3.102.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3565
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(a + b*Tan[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 3963
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d
*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2
+ b^2, 0] && GtQ[n, 1]
```

rule 4008
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Simp[b*d*(Tan[e + f*x]/f),
x] + Simp[(b*c + a*d)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

3.102.4 Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{a^5(dx+c)-5a^4b\ln(\cos(dx+c))+10a^3b^2(\tan(dx+c)-dx-c)+10a^2b^3\left(\frac{\tan(dx+c)^2}{2}+\ln(\cos(dx+c))\right)+5a\,b^4\left(\frac{\tan(dx+c)^3}{3}-\right.}{d}$
default	$\frac{a^5(dx+c)-5a^4b\ln(\cos(dx+c))+10a^3b^2(\tan(dx+c)-dx-c)+10a^2b^3\left(\frac{\tan(dx+c)^2}{2}+\ln(\cos(dx+c))\right)+5a\,b^4\left(\frac{\tan(dx+c)^3}{3}-\right.}{d}$
parts	$\frac{a^5(dx+c)}{d}+\frac{b^5\left(\frac{\tan(dx+c)^4}{4}-\frac{\tan(dx+c)^2}{2}-\ln(\cos(dx+c))\right)}{d}+\frac{10a^3b^2(\tan(dx+c)-dx-c)}{d}+\frac{5a\,b^4\left(\frac{\tan(dx+c)^3}{3}-\tan\right.}{d}$
risch	$ix\,b^5+\frac{2ib^5c}{d}+5ia^4bx+a^5x-10a^3b^2x+5a\,b^4x-10ix\,a^2b^3-\frac{20ib^3a^2c}{d}+\frac{10ib\,a^4c}{d}-\frac{4b^2(-1}{}$
parallelrisch	$240b\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\cos(2dx+2c)\right)\left(a^4-2a^2b^2+\frac{1}{5}b^4\right)\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)-240b\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\cos(2dx+2c)\right)(a^4-2$

input
```
int(sec(d*x+c)^5*(cos(d*x+c)*a+b*sin(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^5*(d*x+c)-5*a^4*b*ln(cos(d*x+c))+10*a^3*b^2*(tan(d*x+c)-d*x-c)+10*a
^2*b^3*(1/2*tan(d*x+c)^2+ln(cos(d*x+c)))+5*a*b^4*(1/3*tan(d*x+c)^3-tan(d*x
+c)+d*x+c)+b^5*(1/4*tan(d*x+c)^4-1/2*tan(d*x+c)^2-ln(cos(d*x+c))))
```

3.102.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.05

$$\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$
$$=\frac{12\,(a^5-10\,a^3b^2+5\,ab^4)dx\cos{(dx+c)}^4-12\,(5\,a^4b-10\,a^2b^3+b^5)\cos{(dx+c)}^4\log{(-\cos{(dx+c)})}+3\,\ldots}{12\,d\cos{(d\ldots}$$

input
```
integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas"
)
```

output
```
1/12*(12*(a^5 - 10*a^3*b^2 + 5*a*b^4)*d*x*cos(d*x + c)^4 - 12*(5*a^4*b - 1
0*a^2*b^3 + b^5)*cos(d*x + c)^4*log(-cos(d*x + c)) + 3*b^5 + 12*(5*a^2*b^3
 - b^5)*cos(d*x + c)^2 + 20*(a*b^4*cos(d*x + c) + 2*(3*a^3*b^2 - 2*a*b^4)*
cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

3.102.6 Sympy [F(-1)]

Timed out.

$$\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**5*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

output
```
Timed out
```

3.102.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.18

$$\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$
$$= \frac{12\,(dx+c)a^5 - 120\,(dx+c-\tan(dx+c))a^3b^2 + 20\,\left(\tan(dx+c)^3 + 3\,dx + 3\,c - 3\,\tan(dx+c)\right)ab^4 + 3}{}$$

input
```
integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima"
)
```

output
```
1/12*(12*(d*x + c)*a^5 - 120*(d*x + c - tan(d*x + c))*a^3*b^2 + 20*(tan(d*
x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a*b^4 + 3*b^5*((4*sin(d*x + c)^2
- 3)/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 2*log(sin(d*x + c)^2 - 1))
- 60*a^2*b^3*(1/(sin(d*x + c)^2 - 1) - log(sin(d*x + c)^2 - 1)) - 30*a^4*b
*log(-sin(d*x + c)^2 + 1))/d
```

3.102.8 Giac [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.98

$$\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$
$$= \frac{3\,b^5\tan(dx+c)^4 + 20\,ab^4\tan(dx+c)^3 + 60\,a^2b^3\tan(dx+c)^2 - 6\,b^5\tan(dx+c)^2 + 120\,a^3b^2\tan(dx+c}{1}$$

input
```
integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

output
```
1/12*(3*b^5*tan(d*x + c)^4 + 20*a*b^4*tan(d*x + c)^3 + 60*a^2*b^3*tan(d*x
+ c)^2 - 6*b^5*tan(d*x + c)^2 + 120*a^3*b^2*tan(d*x + c) - 60*a*b^4*tan(d*
x + c) + 12*(a^5 - 10*a^3*b^2 + 5*a*b^4)*(d*x + c) + 6*(5*a^4*b - 10*a^2*b
^3 + b^5)*log(tan(d*x + c)^2 + 1))/d
```

3.102.9 Mupad [B] (verification not implemented)

Time = 26.66 (sec) , antiderivative size = 971, normalized size of antiderivative = 6.61

$$\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \text{Too large to display}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))^5/cos(c + d*x)^5,x)
```

output
```
((3*b^5*log(1/cos(c/2 + (d*x)/2)^2))/8 - (3*b^5*log(-cos(c + d*x)/cos(c/2
+ (d*x)/2)^2))/8 + b^5/32 + (3*a^5*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)
/2)))/4 + (5*a^2*b^3)/8 - (b^5*cos(2*c + 2*d*x))/8 + (3*b^5*cos(4*c + 4*d*
x))/32 + (15*a^4*b*log(1/cos(c/2 + (d*x)/2)^2))/8 + (15*a^2*b^3*log(-cos(c
 + d*x)/cos(c/2 + (d*x)/2)^2))/4 - (b^5*log(-cos(c + d*x)/cos(c/2 + (d*x)/
2)^2)*cos(2*c + 2*d*x))/2 - (b^5*log(-cos(c + d*x)/cos(c/2 + (d*x)/2)^2)*c
os(4*c + 4*d*x))/8 + (15*a*b^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))
)/4 - (5*a*b^4*sin(2*c + 2*d*x))/6 - (5*a*b^4*sin(4*c + 4*d*x))/6 - (15*a^
2*b^3*log(1/cos(c/2 + (d*x)/2)^2))/4 + (b^5*log(1/cos(c/2 + (d*x)/2)^2)*co
s(2*c + 2*d*x))/2 + (b^5*log(1/cos(c/2 + (d*x)/2)^2)*cos(4*c + 4*d*x))/8 +
 a^5*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x) + (a^5*a
tan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(4*c + 4*d*x))/4 - (15*a^3*b
^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/2 - (5*a^2*b^3*cos(4*c + 4
*d*x))/8 + (5*a^3*b^2*sin(2*c + 2*d*x))/2 + (5*a^3*b^2*sin(4*c + 4*d*x))/4
 - (15*a^4*b*log(-cos(c + d*x)/cos(c/2 + (d*x)/2)^2))/8 - 5*a^2*b^3*log(1/
cos(c/2 + (d*x)/2)^2)*cos(2*c + 2*d*x) - (5*a^2*b^3*log(1/cos(c/2 + (d*x)/
2)^2)*cos(4*c + 4*d*x))/4 - 10*a^3*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (
d*x)/2))*cos(2*c + 2*d*x) - (5*a^3*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (
d*x)/2))*cos(4*c + 4*d*x))/2 - (5*a^4*b*log(-cos(c + d*x)/cos(c/2 + (d*x)/
2)^2)*cos(2*c + 2*d*x))/2 - (5*a^4*b*log(-cos(c + d*x)/cos(c/2 + (d*x)/...
```

3.103 $\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

3.103.1 Optimal result

Integrand size = 28, antiderivative size = 224

$$\begin{aligned}\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx &= \frac{a^5\text{arctanh}(\sin(c+dx))}{d}-\frac{5a^3b^2\text{arctanh}(\sin(c+dx))}{d}+\frac{15ab^4\text{arctanh}(\sin(c+dx))}{8d}\\&+\frac{5a^4b\sec(c+dx)}{d}-\frac{10a^2b^3\sec(c+dx)}{d}+\frac{b^5\sec(c+dx)}{d}+\frac{10a^2b^3\sec^3(c+dx)}{3d}\\&-\frac{2b^5\sec^3(c+dx)}{3d}+\frac{b^5\sec^5(c+dx)}{5d}+\frac{5a^3b^2\sec(c+dx)\tan(c+dx)}{d}\\&-\frac{15ab^4\sec(c+dx)\tan(c+dx)}{8d}+\frac{5ab^4\sec(c+dx)\tan^3(c+dx)}{4d}\end{aligned}$$

output
```
a^5*arctanh(sin(d*x+c))/d-5*a^3*b^2*arctanh(sin(d*x+c))/d+15/8*a*b^4*arcta
nh(sin(d*x+c))/d+5*a^4*b*sec(d*x+c)/d-10*a^2*b^3*sec(d*x+c)/d+b^5*sec(d*x+
c)/d+10/3*a^2*b^3*sec(d*x+c)^3/d-2/3*b^5*sec(d*x+c)^3/d+1/5*b^5*sec(d*x+c)
^5/d+5*a^3*b^2*sec(d*x+c)*tan(d*x+c)/d-15/8*a*b^4*sec(d*x+c)*tan(d*x+c)/d+
5/4*a*b^4*sec(d*x+c)*tan(d*x+c)^3/d
```

3.103.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1219 vs. $2(224) = 448$.

Time = 8.12 (sec) , antiderivative size = 1219, normalized size of antiderivative = 5.44

$$\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \text{Too large to display}$$

input
```
Integrate[Sec[c + d*x]^6*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

output
```
(b*(600*a^4 - 1000*a^2*b^2 + 89*b^4)*Cos[c + d*x]^5*(a + b*Tan[c + d*x])^5
)/(120*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + ((-8*a^5 + 40*a^3*b^2 - 15
*a*b^4)*Cos[c + d*x]^5*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a + b*Tan
[c + d*x])^5)/(8*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + ((8*a^5 - 40*a^3
*b^2 + 15*a*b^4)*Cos[c + d*x]^5*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(
a + b*Tan[c + d*x])^5)/(8*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + ((25*a*
b^4 + 2*b^5)*Cos[c + d*x]^5*(a + b*Tan[c + d*x])^5)/(80*d*(Cos[(c + d*x)/2
] - Sin[(c + d*x)/2])^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + ((600*a^3*b
^2 + 200*a^2*b^3 - 375*a*b^4 - 31*b^5)*Cos[c + d*x]^5*(a + b*Tan[c + d*x])
^5)/(240*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2*(a*Cos[c + d*x] + b*Sin
[c + d*x])^5) + (b^5*Cos[c + d*x]^5*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^
5)/(20*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^5*(a*Cos[c + d*x] + b*Sin[c
 + d*x])^5) - (b^5*Cos[c + d*x]^5*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^5)
/(20*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5*(a*Cos[c + d*x] + b*Sin[c +
 d*x])^5) + ((-25*a*b^4 + 2*b^5)*Cos[c + d*x]^5*(a + b*Tan[c + d*x])^5)/(8
0*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*(a*Cos[c + d*x] + b*Sin[c + d*
x])^5) + ((-600*a^3*b^2 + 200*a^2*b^3 + 375*a*b^4 - 31*b^5)*Cos[c + d*x]^5
*(a + b*Tan[c + d*x])^5)/(240*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(a
*Cos[c + d*x] + b*Sin[c + d*x])^5) + (Cos[c + d*x]^5*(-600*a^4*b*Sin[(c +
d*x)/2] + 1000*a^2*b^3*Sin[(c + d*x)/2] - 89*b^5*Sin[(c + d*x)/2])*(a +...
```

3.103.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\cos(c+dx)+b\sin(c+dx))^5}{\cos(c+dx)^6}dx$$

$$\Big\downarrow \text{3569}$$

$$\int \left(a^5\sec(c+dx)+5a^4b\tan(c+dx)\sec(c+dx)+10a^3b^2\tan^2(c+dx)\sec(c+dx)+10a^2b^3\tan^3(c+dx)\sec(c+\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^5\text{arctanh}(\sin(c+dx))}{d}+\frac{5a^4b\sec(c+dx)}{d}-\frac{5a^3b^2\text{arctanh}(\sin(c+dx))}{d}+\frac{5a^3b^2\tan(c+dx)\sec(c+dx)}{d}+\frac{10a^2b^3\sec^3(c+dx)}{3d}-\frac{10a^2b^3\sec(c+dx)}{d}+\frac{15ab^4\text{arctanh}(\sin(c+dx))}{8d}+\frac{5ab^4\tan^3(c+dx)\sec(c+dx)}{4d}-\frac{15ab^4\tan(c+dx)\sec(c+dx)}{8d}+\frac{b^5\sec^5(c+dx)}{5d}-\frac{2b^5\sec^3(c+dx)}{3d}+\frac{b^5\sec(c+dx)}{d}$$

input
```
Int[Sec[c + d*x]^6*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

output
```
(a^5*ArcTanh[Sin[c + d*x]])/d - (5*a^3*b^2*ArcTanh[Sin[c + d*x]])/d + (15*
a*b^4*ArcTanh[Sin[c + d*x]])/(8*d) + (5*a^4*b*Sec[c + d*x])/d - (10*a^2*b^
3*Sec[c + d*x])/d + (b^5*Sec[c + d*x])/d + (10*a^2*b^3*Sec[c + d*x]^3)/(3*
d) - (2*b^5*Sec[c + d*x]^3)/(3*d) + (b^5*Sec[c + d*x]^5)/(5*d) + (5*a^3*b^
2*Sec[c + d*x]*Tan[c + d*x])/d - (15*a*b^4*Sec[c + d*x]*Tan[c + d*x])/(8*d
) + (5*a*b^4*Sec[c + d*x]*Tan[c + d*x]^3)/(4*d)
```

3.103.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

3.103.4 Maple [A] (verified)

Time = 1.98 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.05

method	result
parts	$\frac{a^5 \ln(\sec(dx+c)+\tan(dx+c))}{d} + \frac{b^5\left(\frac{\sec(dx+c)^5}{5} - \frac{2\sec(dx+c)^3}{3} + \sec(dx+c)\right)}{d} + \frac{10a^3b^2\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2} + \frac{\sin(dx+c)}{2} - \ln(\mathrm{s}\right.}{d}$
derivativedivides	$\frac{a^5 \ln(\sec(dx+c)+\tan(dx+c)) + \frac{5a^4b}{\cos(dx+c)} + 10a^3b^2\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2} + \frac{\sin(dx+c)}{2} - \frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right) + 10a^2b^3\left(\frac{\sin(d}{3\cos(d}\right.}{}$
default	$\frac{a^5 \ln(\sec(dx+c)+\tan(dx+c)) + \frac{5a^4b}{\cos(dx+c)} + 10a^3b^2\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2} + \frac{\sin(dx+c)}{2} - \frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right) + 10a^2b^3\left(\frac{\sin(d}{3\cos(d}\right.}{}$
parallelrisch	$\frac{-a(\cos(5dx+5c)+5\cos(3dx+3c)+10\cos(dx+c))\left(a^4-5a^2b^2+\frac{15}{8}b^4\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+a(\cos(5dx+5c)+5\cos(3dx+3}{}$
risch	$\frac{b\,e^{i(dx+c)}\left(1200ia^3b\,e^{2i(dx+c)}+600ia^3b+600a^4e^{8i(dx+c)}-1200a^2b^2e^{8i(dx+c)}+120b^4e^{8i(dx+c)}-1200ia^3b\,e^{6i(dx+c)}-150\right.}{}$

input
```
int(sec(d*x+c)^6*(cos(d*x+c)*a+b*sin(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
a^5/d*ln(sec(d*x+c)+tan(d*x+c))+b^5/d*(1/5*sec(d*x+c)^5-2/3*sec(d*x+c)^3+s
ec(d*x+c))+10*a^3*b^2/d*(1/2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*sin(d*x+c)-1/2*
ln(sec(d*x+c)+tan(d*x+c)))+5*a*b^4/d*(1/4*sin(d*x+c)^5/cos(d*x+c)^4-1/8*si
n(d*x+c)^5/cos(d*x+c)^2-1/8*sin(d*x+c)^3-3/8*sin(d*x+c)+3/8*ln(sec(d*x+c)+
tan(d*x+c)))+10*a^2*b^3/d*(1/3*sec(d*x+c)^3-sec(d*x+c))+5*a^4*b*sec(d*x+c)
/d
```

3.103.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.88

$$\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$
$$= \frac{15\,(8\,a^5-40\,a^3b^2+15\,ab^4)\cos{(dx+c)}^5\log{(\sin{(dx+c)}+1)}-15\,(8\,a^5-40\,a^3b^2+15\,ab^4)\cos{(dx+c)}^5}{}$$

input
```
integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas"
)
```

output
```
1/240*(15*(8*a^5 - 40*a^3*b^2 + 15*a*b^4)*cos(d*x + c)^5*log(sin(d*x + c)
+ 1) - 15*(8*a^5 - 40*a^3*b^2 + 15*a*b^4)*cos(d*x + c)^5*log(-sin(d*x + c)
 + 1) + 48*b^5 + 240*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c)^4 + 160*(5*
a^2*b^3 - b^5)*cos(d*x + c)^2 + 150*(2*a*b^4*cos(d*x + c) + (8*a^3*b^2 - 5
*a*b^4)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

3.103.6 Sympy [F(-1)]

Timed out.

$$\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**6*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

output
```
Timed out
```

3.103.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.03

$$\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$
$$= \frac{75\,ab^4\left(\frac{2\left(5\,\sin(dx+c)^3-3\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}+3\,\log{(\sin{(dx+c)}+1)}-3\,\log{(\sin{(dx+c)}-1)}\right)-600\,a^3b^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2}\right.}{}$$

input
```
integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima"
)
```

output
```
1/240*(75*a*b^4*(2*(5*sin(d*x + c)^3 - 3*sin(d*x + c))/(sin(d*x + c)^4 - 2
*sin(d*x + c)^2 + 1) + 3*log(sin(d*x + c) + 1) - 3*log(sin(d*x + c) - 1))
- 600*a^3*b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1)
 - log(sin(d*x + c) - 1)) + 120*a^5*(log(sin(d*x + c) + 1) - log(sin(d*x +
 c) - 1)) + 1200*a^4*b/cos(d*x + c) - 800*(3*cos(d*x + c)^2 - 1)*a^2*b^3/c
os(d*x + c)^3 + 16*(15*cos(d*x + c)^4 - 10*cos(d*x + c)^2 + 3)*b^5/cos(d*x
 + c)^5)/d
```

3.103.8 Giac [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.83

$$\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$

$$= \frac{15\,(8\,a^5-40\,a^3b^2+15\,ab^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,(8\,a^5-40\,a^3b^2+15\,ab^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.\right.}{}$$

input
```
integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

output
```
1/120*(15*(8*a^5 - 40*a^3*b^2 + 15*a*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1
)) - 15*(8*a^5 - 40*a^3*b^2 + 15*a*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1))
 + 2*(600*a^3*b^2*tan(1/2*d*x + 1/2*c)^9 - 225*a*b^4*tan(1/2*d*x + 1/2*c)^
9 - 600*a^4*b*tan(1/2*d*x + 1/2*c)^8 - 1200*a^3*b^2*tan(1/2*d*x + 1/2*c)^7
 + 1050*a*b^4*tan(1/2*d*x + 1/2*c)^7 + 2400*a^4*b*tan(1/2*d*x + 1/2*c)^6 -
 2400*a^2*b^3*tan(1/2*d*x + 1/2*c)^6 - 3600*a^4*b*tan(1/2*d*x + 1/2*c)^4 +
 5600*a^2*b^3*tan(1/2*d*x + 1/2*c)^4 - 640*b^5*tan(1/2*d*x + 1/2*c)^4 + 12
00*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 1050*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 24
00*a^4*b*tan(1/2*d*x + 1/2*c)^2 - 4000*a^2*b^3*tan(1/2*d*x + 1/2*c)^2 + 32
0*b^5*tan(1/2*d*x + 1/2*c)^2 - 600*a^3*b^2*tan(1/2*d*x + 1/2*c) + 225*a*b^
4*tan(1/2*d*x + 1/2*c) - 600*a^4*b + 800*a^2*b^3 - 64*b^5)/(tan(1/2*d*x +
1/2*c)^2 - 1)^5)/d
```

3.103.9 Mupad [B] (verification not implemented)

Time = 27.41 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.54

$$\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$
$$=\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(2a^5-10a^3b^2+\frac{15ab^4}{4}\right)}{d}$$
$$-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9\left(\frac{15ab^4}{4}-10a^3b^2\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{35ab^4}{2}-20a^3b^2\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7\left(\frac{35ab^4}{2}-20a^3b^2\right)}{}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))^5/cos(c + d*x)^6,x)
```

output
```
(atanh(tan(c/2 + (d*x)/2))*((15*a*b^4)/4 + 2*a^5 - 10*a^3*b^2))/d - (tan(c
/2 + (d*x)/2)^9*((15*a*b^4)/4 - 10*a^3*b^2) + tan(c/2 + (d*x)/2)^3*((35*a*
b^4)/2 - 20*a^3*b^2) - tan(c/2 + (d*x)/2)^7*((35*a*b^4)/2 - 20*a^3*b^2) -
tan(c/2 + (d*x)/2)^6*(40*a^4*b - 40*a^2*b^3) - tan(c/2 + (d*x)/2)^2*(40*a^
4*b + (16*b^5)/3 - (200*a^2*b^3)/3) + tan(c/2 + (d*x)/2)^4*(60*a^4*b + (32
*b^5)/3 - (280*a^2*b^3)/3) + 10*a^4*b + (16*b^5)/15 - (40*a^2*b^3)/3 - tan
(c/2 + (d*x)/2)*((15*a*b^4)/4 - 10*a^3*b^2) + 10*a^4*b*tan(c/2 + (d*x)/2)^
8)/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*
x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

3.104 $\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

3.104.1 Optimal result

Integrand size = 28, antiderivative size = 30

$$\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx=\frac{(b+a\cot(c+dx))^6\tan^6(c+dx)}{6bd}$$

output
```
1/6*(b+a*cot(d*x+c))^6*tan(d*x+c)^6/b/d
```

3.104.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 89 vs. 2(30) = 60.

Time = 0.54 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.97

$$\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$
$$=\frac{\tan(c+dx)\left(6a^5+15a^4b\tan(c+dx)+20a^3b^2\tan^2(c+dx)+15a^2b^3\tan^3(c+dx)+6ab^4\tan^4(c+dx)+\right.}{6d}$$

input
```
Integrate[Sec[c + d*x]^7*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

output
```
(Tan[c + d*x]*(6*a^5 + 15*a^4*b*Tan[c + d*x] + 20*a^3*b^2*Tan[c + d*x]^2 +
 15*a^2*b^3*Tan[c + d*x]^3 + 6*a*b^4*Tan[c + d*x]^4 + b^5*Tan[c + d*x]^5))
/(6*d)
```

3.104.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3042, 3567, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\cos(c+dx)+b\sin(c+dx))^5}{\cos(c+dx)^7}dx$$

$$\Big\downarrow \text{3567}$$

$$-\frac{\int (b+a\cot(c+dx))^5\tan^7(c+dx)d\cot(c+dx)}{d}$$

$$\Big\downarrow \text{48}$$

$$\frac{\tan^6(c+dx)(a\cot(c+dx)+b)^6}{6bd}$$

input
```
Int[Sec[c + d*x]^7*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

output
```
((b + a*Cot[c + d*x])^6*Tan[c + d*x]^6)/(6*b*d)
```

3.104.3.1 Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3567
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[x^m*((b
+ a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a,
b, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[
n, 0] && GtQ[m, 1])
```

3.104.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(28) = 56$.

Time = 1.60 (sec) , antiderivative size = 120, normalized size of antiderivative = 4.00

method	result
derivativedivides	$\frac{a^5\tan(dx+c)+\frac{5a^4b}{2\cos(dx+c)^2}+\frac{10a^3b^2\sin(dx+c)^3}{3\cos(dx+c)^3}+\frac{5a^2b^3\sin(dx+c)^4}{2\cos(dx+c)^4}+\frac{a\,b^4\sin(dx+c)^5}{\cos(dx+c)^5}+\frac{b^5\sin(dx+c)^6}{6\cos(dx+c)^6}}{d}$
default	$\frac{a^5\tan(dx+c)+\frac{5a^4b}{2\cos(dx+c)^2}+\frac{10a^3b^2\sin(dx+c)^3}{3\cos(dx+c)^3}+\frac{5a^2b^3\sin(dx+c)^4}{2\cos(dx+c)^4}+\frac{a\,b^4\sin(dx+c)^5}{\cos(dx+c)^5}+\frac{b^5\sin(dx+c)^6}{6\cos(dx+c)^6}}{d}$
parts	$\frac{a^5\tan(dx+c)}{d}+\frac{b^5\left(\frac{\sec(dx+c)^6}{6}-\frac{\sec(dx+c)^4}{2}+\frac{\sec(dx+c)^2}{2}\right)}{d}+\frac{10a^3b^2\sin(dx+c)^3}{3d\cos(dx+c)^3}+\frac{a\,b^4\sin(dx+c)^5}{d\cos(dx+c)^5}+\frac{10a^2b^3\left(\frac{\sec}{}\right.}{}$
parallelrisch	$\frac{2\left(\left(5a^5-\frac{10}{3}a^3b^2-3a\,b^4\right)\cos(3dx+3c)+\left(a^5-\frac{10}{3}a^3b^2+a\,b^4\right)\cos(5dx+5c)+\frac{5\left(3a^4b+a^2b^3-\frac{1}{3}b^5\right)\sin(3dx+3c)}{2}+\frac{5b\left(a^4-a^2b^2+\right.}{}\right.}{d(\cos(6dx+6c)+6\cos(4dx+4c)+15\cos}$
risch	$-\frac{200ia^3b^2e^{6i(dx+c)}}{3}+2ia\,b^4e^{2i(dx+c)}-20ia^3b^2e^{10i(dx+c)}+10ia\,b^4e^{10i(dx+c)}-60ia^3b^2e^{8i(dx+c)}+10ia\,b^4e^{8i(dx+c)}-40ia^3$

input
```
int(sec(d*x+c)^7*(cos(d*x+c)*a+b*sin(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^5*tan(d*x+c)+5/2*a^4*b/cos(d*x+c)^2+10/3*a^3*b^2*sin(d*x+c)^3/cos(d
*x+c)^3+5/2*a^2*b^3*sin(d*x+c)^4/cos(d*x+c)^4+a*b^4*sin(d*x+c)^5/cos(d*x+c
)^5+1/6*b^5*sin(d*x+c)^6/cos(d*x+c)^6)
```

3.104.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(28) = 56.

Time = 0.26 (sec) , antiderivative size = 144, normalized size of antiderivative = 4.80

$$\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$
$$=\frac{b^5+3\left(5a^4b-10a^2b^3+b^5\right)\cos(dx+c)^4+3\left(5a^2b^3-b^5\right)\cos(dx+c)^2+2\left(3ab^4\cos(dx+c)+\left(3a^5-1\right.\right.}{6d\cos(dx+c)^6}$$

input
```
integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas"
)
```

output
```
1/6*(b^5 + 3*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c)^4 + 3*(5*a^2*b^3 -
b^5)*cos(d*x + c)^2 + 2*(3*a*b^4*cos(d*x + c) + (3*a^5 - 10*a^3*b^2 + 3*a*
b^4)*cos(d*x + c)^5 + 2*(5*a^3*b^2 - 3*a*b^4)*cos(d*x + c)^3)*sin(d*x + c)
)/(d*cos(d*x + c)^6)
```

3.104.6 Sympy [F(-1)]

Timed out.

$$\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx=\text{Timed out}$$

input
```
integrate(sec(d*x+c)**7*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

output
```
Timed out
```

3.104.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. 2(28) = 56.

Time = 0.22 (sec) , antiderivative size = 166, normalized size of antiderivative = 5.53

$$\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$
$$=\frac{6ab^4\tan(dx+c)^5+20a^3b^2\tan(dx+c)^3+6a^5\tan(dx+c)+\frac{15\left(2\sin(dx+c)^2-1\right)a^2b^3}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-\frac{\left(3\sin(dx+c)^4-3\right.}{\sin(dx+c)^6-3\sin(dx+}}{6d}$$

input
```
integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima"
)
```

output
```
1/6*(6*a*b^4*tan(d*x + c)^5 + 20*a^3*b^2*tan(d*x + c)^3 + 6*a^5*tan(d*x +
c) + 15*(2*sin(d*x + c)^2 - 1)*a^2*b^3/(sin(d*x + c)^4 - 2*sin(d*x + c)^2
+ 1) - (3*sin(d*x + c)^4 - 3*sin(d*x + c)^2 + 1)*b^5/(sin(d*x + c)^6 - 3*s
in(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15*a^4*b/(sin(d*x + c)^2 - 1))/d
```

3.104.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(28) = 56$.

Time = 0.58 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.97

$$\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$
$$= \frac{b^5\tan(dx+c)^6 + 6\,ab^4\tan(dx+c)^5 + 15\,a^2b^3\tan(dx+c)^4 + 20\,a^3b^2\tan(dx+c)^3 + 15\,a^4b\tan(dx+c)^2}{6\,d}$$

input
```
integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

output
```
1/6*(b^5*tan(d*x + c)^6 + 6*a*b^4*tan(d*x + c)^5 + 15*a^2*b^3*tan(d*x + c)
^4 + 20*a^3*b^2*tan(d*x + c)^3 + 15*a^4*b*tan(d*x + c)^2 + 6*a^5*tan(d*x +
 c))/d
```

3.104.9 Mupad [B] (verification not implemented)

Time = 23.90 (sec) , antiderivative size = 169, normalized size of antiderivative = 5.63

$$\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$
$$= \frac{\cos(c+d\,x)^4\left(\frac{5\,a^4\,b}{2} - 5\,a^2\,b^3 + \frac{b^5}{2}\right) + \cos(c+d\,x)^5\left(\sin(c+d\,x)\;a^5 - \frac{10\,\sin(c+d\,x)\,a^3\,b^2}{3} + \sin(c+d\,x)\;a\,b^4\right)}{}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))^5/cos(c + d*x)^7,x)
```

output
```
(cos(c + d*x)^4*((5*a^4*b)/2 + b^5/2 - 5*a^2*b^3) + cos(c + d*x)^5*(a^5*si
n(c + d*x) - (10*a^3*b^2*sin(c + d*x))/3 + a*b^4*sin(c + d*x)) - cos(c + d
*x)^2*(b^5/2 - (5*a^2*b^3)/2) + b^5/6 + cos(c + d*x)^3*((10*a^3*b^2*sin(c
+ d*x))/3 - 2*a*b^4*sin(c + d*x)) + a*b^4*cos(c + d*x)*sin(c + d*x))/(d*co
s(c + d*x)^6)
```

3.104. $\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

3.105 $\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

3.105.1 Optimal result

Integrand size = 28, antiderivative size = 318

$$\begin{aligned}
&\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx \\
&= \frac{a^5\text{arctanh}(\sin(c+dx))}{2d} - \frac{5a^3b^2\text{arctanh}(\sin(c+dx))}{4d} + \frac{5ab^4\text{arctanh}(\sin(c+dx))}{16d} \\
&\quad + \frac{5a^4b\sec^3(c+dx)}{3d} - \frac{10a^2b^3\sec^3(c+dx)}{3d} + \frac{b^5\sec^3(c+dx)}{3d} \\
&\quad + \frac{2a^2b^3\sec^5(c+dx)}{d} - \frac{2b^5\sec^5(c+dx)}{5d} + \frac{b^5\sec^7(c+dx)}{7d} \\
&\quad + \frac{a^5\sec(c+dx)\tan(c+dx)}{2d} - \frac{5a^3b^2\sec(c+dx)\tan(c+dx)}{4d} \\
&\quad + \frac{5ab^4\sec(c+dx)\tan(c+dx)}{16d} + \frac{5a^3b^2\sec^3(c+dx)\tan(c+dx)}{2d} \\
&\quad - \frac{5ab^4\sec^3(c+dx)\tan(c+dx)}{8d} + \frac{5ab^4\sec^3(c+dx)\tan^3(c+dx)}{6d}
\end{aligned}$$

output
```
1/2*a^5*arctanh(sin(d*x+c))/d-5/4*a^3*b^2*arctanh(sin(d*x+c))/d+5/16*a*b^4
*arctanh(sin(d*x+c))/d+5/3*a^4*b*sec(d*x+c)^3/d-10/3*a^2*b^3*sec(d*x+c)^3/
d+1/3*b^5*sec(d*x+c)^3/d+2*a^2*b^3*sec(d*x+c)^5/d-2/5*b^5*sec(d*x+c)^5/d+1
/7*b^5*sec(d*x+c)^7/d+1/2*a^5*sec(d*x+c)*tan(d*x+c)/d-5/4*a^3*b^2*sec(d*x+
c)*tan(d*x+c)/d+5/16*a*b^4*sec(d*x+c)*tan(d*x+c)/d+5/2*a^3*b^2*sec(d*x+c)^
3*tan(d*x+c)/d-5/8*a*b^4*sec(d*x+c)^3*tan(d*x+c)/d+5/6*a*b^4*sec(d*x+c)^3*
tan(d*x+c)^3/d
```

3.105.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1677 vs. $2(318) = 636$.

Time = 8.72 (sec) , antiderivative size = 1677, normalized size of antiderivative = 5.27

$$\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \text{Too large to display}$$

input
```
Integrate[Sec[c + d*x]^8*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

output
```
(b*(1400*a^4 - 1540*a^2*b^2 + 103*b^4)*Cos[c + d*x]^5*(a + b*Tan[c + d*x])
^5)/(1680*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + ((-8*a^5 + 20*a^3*b^2 -
 5*a*b^4)*Cos[c + d*x]^5*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a + b*T
an[c + d*x])^5)/(16*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + ((8*a^5 - 20*
a^3*b^2 + 5*a*b^4)*Cos[c + d*x]^5*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]
*(a + b*Tan[c + d*x])^5)/(16*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + ((35
*a*b^4 + 3*b^5)*Cos[c + d*x]^5*(a + b*Tan[c + d*x])^5)/(336*d*(Cos[(c + d*
x)/2] - Sin[(c + d*x)/2])^6*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + ((350*a
^3*b^2 + 140*a^2*b^3 - 175*a*b^4 - 18*b^5)*Cos[c + d*x]^5*(a + b*Tan[c + d
*x])^5)/(560*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^4*(a*Cos[c + d*x] + b
*Sin[c + d*x])^5) + ((840*a^5 + 1400*a^4*b - 2100*a^3*b^2 - 1540*a^2*b^3 +
 525*a*b^4 + 103*b^5)*Cos[c + d*x]^5*(a + b*Tan[c + d*x])^5)/(3360*d*(Cos[
(c + d*x)/2] - Sin[(c + d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) +
(b^5*Cos[c + d*x]^5*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^5)/(56*d*(Cos[(c
 + d*x)/2] - Sin[(c + d*x)/2])^7*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) - (b
^5*Cos[c + d*x]^5*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^5)/(56*d*(Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2])^7*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + ((-3
5*a*b^4 + 3*b^5)*Cos[c + d*x]^5*(a + b*Tan[c + d*x])^5)/(336*d*(Cos[(c + d
*x)/2] + Sin[(c + d*x)/2])^6*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + ((-350
*a^3*b^2 + 140*a^2*b^3 + 175*a*b^4 - 18*b^5)*Cos[c + d*x]^5*(a + b*Tan[...
```

3.105.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\cos(c+dx)+b\sin(c+dx))^5}{\cos(c+dx)^8}dx$$

$$\Big\downarrow \text{3569}$$

$$\int \left(a^5\sec^3(c+dx)+5a^4b\tan(c+dx)\sec^3(c+dx)+10a^3b^2\tan^2(c+dx)\sec^3(c+dx)+10a^2b^3\tan^3(c+dx)\sec^3\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^5\text{arctanh}(\sin(c+dx))}{2d}+\frac{a^5\tan(c+dx)\sec(c+dx)}{2d}+\frac{5a^4b\sec^3(c+dx)}{3d}-\frac{5a^3b^2\text{arctanh}(\sin(c+dx))}{4d}+\frac{5a^3b^2\tan(c+dx)\sec^3(c+dx)}{2d}-\frac{5a^3b^2\tan(c+dx)\sec(c+dx)}{4d}+\frac{2a^2b^3\sec^5(c+dx)}{d}-\frac{10a^2b^3\sec^3(c+dx)}{3d}+\frac{5ab^4\text{arctanh}(\sin(c+dx))}{16d}+\frac{5ab^4\tan^3(c+dx)\sec^3(c+dx)}{6d}-\frac{5ab^4\tan(c+dx)\sec^3(c+dx)}{8d}+\frac{5ab^4\tan(c+dx)\sec(c+dx)}{16d}+\frac{b^5\sec^7(c+dx)}{7d}-\frac{2b^5\sec^5(c+dx)}{5d}+\frac{b^5\sec^3(c+dx)}{3d}$$

input
```
Int[Sec[c + d*x]^8*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

output
```
(a^5*ArcTanh[Sin[c + d*x]])/(2*d) - (5*a^3*b^2*ArcTanh[Sin[c + d*x]])/(4*d
) + (5*a*b^4*ArcTanh[Sin[c + d*x]])/(16*d) + (5*a^4*b*Sec[c + d*x]^3)/(3*d
) - (10*a^2*b^3*Sec[c + d*x]^3)/(3*d) + (b^5*Sec[c + d*x]^3)/(3*d) + (2*a^
2*b^3*Sec[c + d*x]^5)/d - (2*b^5*Sec[c + d*x]^5)/(5*d) + (b^5*Sec[c + d*x]
^7)/(7*d) + (a^5*Sec[c + d*x]*Tan[c + d*x])/(2*d) - (5*a^3*b^2*Sec[c + d*x
]*Tan[c + d*x])/(4*d) + (5*a*b^4*Sec[c + d*x]*Tan[c + d*x])/(16*d) + (5*a^
3*b^2*Sec[c + d*x]^3*Tan[c + d*x])/(2*d) - (5*a*b^4*Sec[c + d*x]^3*Tan[c +
 d*x])/(8*d) + (5*a*b^4*Sec[c + d*x]^3*Tan[c + d*x]^3)/(6*d)
```

3.105. $\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

3.105.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

3.105.4 Maple [A] (verified)

Time = 2.39 (sec) , antiderivative size = 297, normalized size of antiderivative = 0.93

method	result
parts	$\frac{a^5\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}+\frac{b^5\left(\frac{\sec(dx+c)^7}{7}-\frac{2\sec(dx+c)^5}{5}+\frac{\sec(dx+c)^3}{3}\right)}{d}+\frac{10a^3b^2\left(\frac{\sin(dx+}{4\cos(dx-}\right.}{}$
derivativedivides	$\frac{a^5\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{5a^4b}{3\cos(dx+c)^3}+10a^3b^2\left(\frac{\sin(dx+c)^3}{4\cos(dx+c)^4}+\frac{\sin(dx+c)^3}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\mathrm{sec}}{}\right.}{}$
default	$\frac{a^5\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{5a^4b}{3\cos(dx+c)^3}+10a^3b^2\left(\frac{\sin(dx+c)^3}{4\cos(dx+c)^4}+\frac{\sin(dx+c)^3}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\mathrm{sec}}{}\right.}{}$
parallelrisch	$\frac{-5880a\left(\frac{\cos(7dx+7c)}{7}+\cos(5dx+5c)+3\cos(3dx+3c)+5\cos(dx+c)\right)\left(a^4-\frac{5}{2}a^2b^2+\frac{5}{8}b^4\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+5880a\left(\frac{\cos(}{}\right.}{}$
risch	$\frac{e^{i(dx+c)}\left(-7700iab^4e^{2i(dx+c)}-8400ia^3b^2e^{10i(dx+c)}+7700iab^4e^{10i(dx+c)}-23100ia^3b^2e^{8i(dx+c)}-5425iab^4e^{8i(dx+c)}+2\right.}{}$

input
```
int(sec(d*x+c)^8*(cos(d*x+c)*a+b*sin(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
a^5/d*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+b^5/d*(1/7
*sec(d*x+c)^7-2/5*sec(d*x+c)^5+1/3*sec(d*x+c)^3)+10*a^3*b^2/d*(1/4*sin(d*x
+c)^3/cos(d*x+c)^4+1/8*sin(d*x+c)^3/cos(d*x+c)^2+1/8*sin(d*x+c)-1/8*ln(sec
(d*x+c)+tan(d*x+c)))+5*a*b^4/d*(1/6*sin(d*x+c)^5/cos(d*x+c)^6+1/24*sin(d*x
+c)^5/cos(d*x+c)^4-1/48*sin(d*x+c)^5/cos(d*x+c)^2-1/48*sin(d*x+c)^3-1/16*s
in(d*x+c)+1/16*ln(sec(d*x+c)+tan(d*x+c)))+10*a^2*b^3/d*(1/5*sec(d*x+c)^5-1
/3*sec(d*x+c)^3)+5/3*a^4*b*sec(d*x+c)^3/d
```

3.105.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.71

$$\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$
$$= \frac{105\,(8\,a^5-20\,a^3b^2+5\,ab^4)\cos{(dx+c)}^7\log{(\sin{(dx+c)}+1)}-105\,(8\,a^5-20\,a^3b^2+5\,ab^4)\cos{(dx+c)}^7}{}$$

input
```
integrate(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas"
)
```

output
```
1/3360*(105*(8*a^5 - 20*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^7*log(sin(d*x + c)
 + 1) - 105*(8*a^5 - 20*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^7*log(-sin(d*x + c
) + 1) + 480*b^5 + 1120*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c)^4 + 1344
*(5*a^2*b^3 - b^5)*cos(d*x + c)^2 + 70*(40*a*b^4*cos(d*x + c) + 3*(8*a^5 -
 20*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^5 + 10*(12*a^3*b^2 - 7*a*b^4)*cos(d*x
+ c)^3)*sin(d*x + c))/(d*cos(d*x + c)^7)
```

3.105.6 Sympy [F(-1)]

Timed out.

$$\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**8*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

output
```
Timed out
```

3.105.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.91

$$\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx =$$

$$-\frac{175\,ab^4\left(\frac{2\left(3\,\sin(dx+c)^5+8\,\sin(dx+c)^3-3\,\sin(dx+c)\right)}{\sin(dx+c)^6-3\,\sin(dx+c)^4+3\,\sin(dx+c)^2-1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)-210}{}$$

input
```
integrate(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima"
)
```

output
```
-1/3360*(175*a*b^4*(2*(3*sin(d*x + c)^5 + 8*sin(d*x + c)^3 - 3*sin(d*x + c
))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 3*log(sin(
d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 2100*a^3*b^2*(2*(sin(d*x + c)^3
 + sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - log(sin(d*x + c
) + 1) + log(sin(d*x + c) - 1)) + 840*a^5*(2*sin(d*x + c)/(sin(d*x + c)^2
- 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 5600*a^4*b/cos(d*x
 + c)^3 + 2240*(5*cos(d*x + c)^2 - 3)*a^2*b^3/cos(d*x + c)^5 - 32*(35*cos(
d*x + c)^4 - 42*cos(d*x + c)^2 + 15)*b^5/cos(d*x + c)^7)/d
```

3.105.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 680 vs. 2(290) = 580.

Time = 0.57 (sec) , antiderivative size = 680, normalized size of antiderivative = 2.14

$$\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

output
```
1/1680*(105*(8*a^5 - 20*a^3*b^2 + 5*a*b^4)*log(abs(tan(1/2*d*x + 1/2*c) +
1)) - 105*(8*a^5 - 20*a^3*b^2 + 5*a*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)
) + 2*(840*a^5*tan(1/2*d*x + 1/2*c)^13 + 2100*a^3*b^2*tan(1/2*d*x + 1/2*c)
^13 - 525*a*b^4*tan(1/2*d*x + 1/2*c)^13 - 8400*a^4*b*tan(1/2*d*x + 1/2*c)^
12 - 3360*a^5*tan(1/2*d*x + 1/2*c)^11 + 8400*a^3*b^2*tan(1/2*d*x + 1/2*c)^
11 + 3500*a*b^4*tan(1/2*d*x + 1/2*c)^11 + 33600*a^4*b*tan(1/2*d*x + 1/2*c)
^10 - 33600*a^2*b^3*tan(1/2*d*x + 1/2*c)^10 + 4200*a^5*tan(1/2*d*x + 1/2*c
)^9 - 23100*a^3*b^2*tan(1/2*d*x + 1/2*c)^9 + 16975*a*b^4*tan(1/2*d*x + 1/2
*c)^9 - 53200*a^4*b*tan(1/2*d*x + 1/2*c)^8 + 56000*a^2*b^3*tan(1/2*d*x + 1
/2*c)^8 - 8960*b^5*tan(1/2*d*x + 1/2*c)^8 + 44800*a^4*b*tan(1/2*d*x + 1/2*
c)^6 - 22400*a^2*b^3*tan(1/2*d*x + 1/2*c)^6 - 4480*b^5*tan(1/2*d*x + 1/2*c
)^6 - 4200*a^5*tan(1/2*d*x + 1/2*c)^5 + 23100*a^3*b^2*tan(1/2*d*x + 1/2*c)
^5 - 16975*a*b^4*tan(1/2*d*x + 1/2*c)^5 - 25200*a^4*b*tan(1/2*d*x + 1/2*c)
^4 + 13440*a^2*b^3*tan(1/2*d*x + 1/2*c)^4 - 2688*b^5*tan(1/2*d*x + 1/2*c)^
4 + 3360*a^5*tan(1/2*d*x + 1/2*c)^3 - 8400*a^3*b^2*tan(1/2*d*x + 1/2*c)^3
- 3500*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 11200*a^4*b*tan(1/2*d*x + 1/2*c)^2 -
 15680*a^2*b^3*tan(1/2*d*x + 1/2*c)^2 + 896*b^5*tan(1/2*d*x + 1/2*c)^2 - 8
40*a^5*tan(1/2*d*x + 1/2*c) - 2100*a^3*b^2*tan(1/2*d*x + 1/2*c) + 525*a*b^
4*tan(1/2*d*x + 1/2*c) - 2800*a^4*b + 2240*a^2*b^3 - 128*b^5)/(tan(1/2*d*x
 + 1/2*c)^2 - 1)^7)/d
```

3.105.9 Mupad [B] (verification not implemented)

Time = 27.40 (sec) , antiderivative size = 514, normalized size of antiderivative = 1.62

$$\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(a^5-\frac{5a^3b^2}{2}+\frac{5ab^4}{8}\right)}{d}$$
$$-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(-4a^5+10a^3b^2+\frac{25ab^4}{6}\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\left(40a^4b-40a^2b^3\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}\left(a^5+5\right.}{}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))^5/cos(c + d*x)^8,x)
```

output
```
(atanh(tan(c/2 + (d*x)/2))*((5*a*b^4)/8 + a^5 - (5*a^3*b^2)/2))/d - (tan(c
/2 + (d*x)/2)^3*((25*a*b^4)/6 - 4*a^5 + 10*a^3*b^2) - tan(c/2 + (d*x)/2)^1
0*(40*a^4*b - 40*a^2*b^3) - tan(c/2 + (d*x)/2)^13*(a^5 - (5*a*b^4)/8 + (5*
a^3*b^2)/2) - tan(c/2 + (d*x)/2)^11*((25*a*b^4)/6 - 4*a^5 + 10*a^3*b^2) +
tan(c/2 + (d*x)/2)^5*((485*a*b^4)/24 + 5*a^5 - (55*a^3*b^2)/2) - tan(c/2 +
 (d*x)/2)^9*((485*a*b^4)/24 + 5*a^5 - (55*a^3*b^2)/2) + tan(c/2 + (d*x)/2)
^4*(30*a^4*b + (16*b^5)/5 - 16*a^2*b^3) - tan(c/2 + (d*x)/2)^2*((40*a^4*b)
/3 + (16*b^5)/15 - (56*a^2*b^3)/3) + tan(c/2 + (d*x)/2)^6*((16*b^5)/3 - (1
60*a^4*b)/3 + (80*a^2*b^3)/3) + tan(c/2 + (d*x)/2)^8*((190*a^4*b)/3 + (32*
b^5)/3 - (200*a^2*b^3)/3) + (10*a^4*b)/3 + tan(c/2 + (d*x)/2)*(a^5 - (5*a*
b^4)/8 + (5*a^3*b^2)/2) + (16*b^5)/105 - (8*a^2*b^3)/3 + 10*a^4*b*tan(c/2
+ (d*x)/2)^12)/(d*(7*tan(c/2 + (d*x)/2)^2 - 21*tan(c/2 + (d*x)/2)^4 + 35*t
an(c/2 + (d*x)/2)^6 - 35*tan(c/2 + (d*x)/2)^8 + 21*tan(c/2 + (d*x)/2)^10 -
 7*tan(c/2 + (d*x)/2)^12 + tan(c/2 + (d*x)/2)^14 - 1))
```

3.105. $\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

3.106 $\int \sec^9(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

3.106.1 Optimal result

Integrand size = 28, antiderivative size = 177

$$\begin{aligned}&\int \sec^9(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx\\&=\frac{a^5\tan(c+dx)}{d}+\frac{5a^4b\tan^2(c+dx)}{2d}+\frac{a^3(a^2+10b^2)\tan^3(c+dx)}{3d}\\&\quad+\frac{5a^2b(a^2+2b^2)\tan^4(c+dx)}{4d}+\frac{ab^2(2a^2+b^2)\tan^5(c+dx)}{d}\\&\quad+\frac{b^3(10a^2+b^2)\tan^6(c+dx)}{6d}+\frac{5ab^4\tan^7(c+dx)}{7d}+\frac{b^5\tan^8(c+dx)}{8d}\end{aligned}$$

output
```
a^5*tan(d*x+c)/d+5/2*a^4*b*tan(d*x+c)^2/d+1/3*a^3*(a^2+10*b^2)*tan(d*x+c)^
3/d+5/4*a^2*b*(a^2+2*b^2)*tan(d*x+c)^4/d+a*b^2*(2*a^2+b^2)*tan(d*x+c)^5/d+
1/6*b^3*(10*a^2+b^2)*tan(d*x+c)^6/d+5/7*a*b^4*tan(d*x+c)^7/d+1/8*b^5*tan(d
*x+c)^8/d
```

3.106.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.31

$$\begin{aligned}&\int \sec^9(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx\\&=\frac{(a+b\tan(c+dx))^6\,(a^2+28b^2-6ab\tan(c+dx)+21b^2\tan^2(c+dx))}{168b^3d}\end{aligned}$$

input
```
Integrate[Sec[c + d*x]^9*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

output
```
((a + b*Tan[c + d*x])^6*(a^2 + 28*b^2 - 6*a*b*Tan[c + d*x] + 21*b^2*Tan[c
+ d*x]^2))/(168*b^3*d)
```

3.106.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3567, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^9(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\cos(c+dx)+b\sin(c+dx))^5}{\cos(c+dx)^9}dx$$

$$\Big\downarrow \text{3567}$$

$$-\frac{\int (b+a\cot(c+dx))^5\left(\cot^2(c+dx)+1\right)\tan^9(c+dx)d\cot(c+dx)}{d}$$

$$\Big\downarrow \text{522}$$

$$-\frac{\int \left(b^5\tan^9(c+dx)+5ab^4\tan^8(c+dx)+\left(b^5+10a^2b^3\right)\tan^7(c+dx)+5ab^2\left(2a^2+b^2\right)\tan^6(c+dx)+5a^2b(a^2+\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-a^5\tan(c+dx)-\frac{5}{2}a^4b\tan^2(c+dx)-ab^2\left(2a^2+b^2\right)\tan^5(c+dx)-\frac{5}{4}a^2b\left(a^2+2b^2\right)\tan^4(c+dx)-\frac{1}{6}b^3\left(10a^2+\right.}{d}$$

input
```
Int[Sec[c + d*x]^9*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

output
```
-((-(a^5*Tan[c + d*x]) - (5*a^4*b*Tan[c + d*x]^2)/2 - (a^3*(a^2 + 10*b^2)*
Tan[c + d*x]^3)/3 - (5*a^2*b*(a^2 + 2*b^2)*Tan[c + d*x]^4)/4 - a*b^2*(2*a^
2 + b^2)*Tan[c + d*x]^5 - (b^3*(10*a^2 + b^2)*Tan[c + d*x]^6)/6 - (5*a*b^4
*Tan[c + d*x]^7)/7 - (b^5*Tan[c + d*x]^8)/8)/d)
```

3.106.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3567
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[x^m*((b
+ a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a,
b, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[
n, 0] && GtQ[m, 1])
```

3.106.4 Maple [A] (verified)

Time = 2.19 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.18

method	result
parts	$-\frac{a^{5}\left(-\frac{2}{3}-\frac{\sec\left(dx+c\right)^{2}}{3}\right)\tan\left(dx+c\right)}{d}+\frac{b^{5}\left(\frac{\sec\left(dx+c\right)^{8}}{8}-\frac{\sec\left(dx+c\right)^{6}}{3}+\frac{\sec\left(dx+c\right)^{4}}{4}\right)}{d}+\frac{10a^{3}b^{2}\left(\frac{\sin\left(dx+c\right)^{3}}{5\cos\left(dx+c\right)^{5}}+\frac{2\sin(dx+\cdots}{15\cos(dx+\cdots}\right.}{d}$
derivativedivides	$\frac{-a^{5}\left(-\frac{2}{3}-\frac{\sec\left(dx+c\right)^{2}}{3}\right)\tan\left(dx+c\right)+\frac{5a^{4}b}{4\cos\left(dx+c\right)^{4}}+10a^{3}b^{2}\left(\frac{\sin\left(dx+c\right)^{3}}{5\cos\left(dx+c\right)^{5}}+\frac{2\sin\left(dx+c\right)^{3}}{15\cos\left(dx+c\right)^{3}}\right)+10a^{2}b^{3}\left(\frac{\sin\left(dx+c\right)^{4}}{6\cos\left(dx+c\right)^{6}}+\frac{\sin(d\cdots}{12\cos(\cdots}\right.}{d}$
default	$\frac{-a^{5}\left(-\frac{2}{3}-\frac{\sec\left(dx+c\right)^{2}}{3}\right)\tan\left(dx+c\right)+\frac{5a^{4}b}{4\cos\left(dx+c\right)^{4}}+10a^{3}b^{2}\left(\frac{\sin\left(dx+c\right)^{3}}{5\cos\left(dx+c\right)^{5}}+\frac{2\sin\left(dx+c\right)^{3}}{15\cos\left(dx+c\right)^{3}}\right)+10a^{2}b^{3}\left(\frac{\sin\left(dx+c\right)^{4}}{6\cos\left(dx+c\right)^{6}}+\frac{\sin(d\cdots}{12\cos(\cdots}\right.}{d}$
risch	$\frac{-40a^{2}b^{3}{\mathrm e}^{12i\left(dx+c\right)}+20a^{4}b\,{\mathrm e}^{12i\left(dx+c\right)}-\frac{128ia^{3}b^{2}{\mathrm e}^{6i\left(dx+c\right)}}{3}+\frac{32ia\,b^{4}{\mathrm e}^{2i\left(dx+c\right)}}{7}-\frac{320ia^{3}b^{2}{\mathrm e}^{10i\left(dx+c\right)}}{3}+4b^{5}{\mathrm e}^{12i\left(dx+c\right)}-\frac{280ia^{3}\cdots}{}}{}$
parallelrisch	$-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(301\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}a^{5}-455\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}a^{5}+455\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}a^{5}-112\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}b^{5}-301\tan\left(\frac{dx}{2}\cdots\right.\right.}{}$

input
```
int(sec(d*x+c)^9*(cos(d*x+c)*a+b*sin(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
-a^5/d*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+b^5/d*(1/8*sec(d*x+c)^8-1/3*sec(
d*x+c)^6+1/4*sec(d*x+c)^4)+10*a^3*b^2/d*(1/5*sin(d*x+c)^3/cos(d*x+c)^5+2/1
5*sin(d*x+c)^3/cos(d*x+c)^3)+5*a*b^4/d*(1/7*sin(d*x+c)^5/cos(d*x+c)^7+2/35
*sin(d*x+c)^5/cos(d*x+c)^5)+10*a^2*b^3/d*(1/6*sec(d*x+c)^6-1/4*sec(d*x+c)^
4)+5/4*a^4*b*sec(d*x+c)^4/d
```

3.106.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.99

$$\int \sec^9(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$
$$= \frac{21\,b^5+42\,(5\,a^4b-10\,a^2b^3+b^5)\cos{(dx+c)}^4+56\,(5\,a^2b^3-b^5)\cos{(dx+c)}^2+8\left(2\,(7\,a^5-14\,a^3b^2+3\,ab\right.}{1}$$

input
```
integrate(sec(d*x+c)^9*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas"
)
```

output
```
1/168*(21*b^5 + 42*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c)^4 + 56*(5*a^2
*b^3 - b^5)*cos(d*x + c)^2 + 8*(2*(7*a^5 - 14*a^3*b^2 + 3*a*b^4)*cos(d*x +
 c)^7 + 15*a*b^4*cos(d*x + c) + (7*a^5 - 14*a^3*b^2 + 3*a*b^4)*cos(d*x + c
)^5 + 6*(7*a^3*b^2 - 4*a*b^4)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c
)^8)
```

3.106.6 Sympy [F(-1)]

Timed out.

$$\int \sec^9(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**9*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

output
```
Timed out
```

3.106.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.26

$$\int \sec^9(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$

$$= \frac{56\left(\tan\left(dx+c\right)^3+3\tan\left(dx+c\right)\right)a^5+112\left(3\tan\left(dx+c\right)^5+5\tan\left(dx+c\right)^3\right)a^3b^2+24\left(5\tan\left(dx+c\right)^7\right.}{}$$

input
```
integrate(sec(d*x+c)^9*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima"
)
```

output
```
1/168*(56*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^5 + 112*(3*tan(d*x + c)^5 +
5*tan(d*x + c)^3)*a^3*b^2 + 24*(5*tan(d*x + c)^7 + 7*tan(d*x + c)^5)*a*b^4
 - 140*(3*sin(d*x + c)^2 - 1)*a^2*b^3/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 +
 3*sin(d*x + c)^2 - 1) + 7*(6*sin(d*x + c)^4 - 4*sin(d*x + c)^2 + 1)*b^5/(
sin(d*x + c)^8 - 4*sin(d*x + c)^6 + 6*sin(d*x + c)^4 - 4*sin(d*x + c)^2 +
1) + 210*a^4*b/(sin(d*x + c)^2 - 1)^2)/d
```

3.106.8 Giac [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.99

$$\int \sec^9(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$

$$= \frac{21\,b^5\tan\left(dx+c\right)^8+120\,ab^4\tan\left(dx+c\right)^7+280\,a^2b^3\tan\left(dx+c\right)^6+28\,b^5\tan\left(dx+c\right)^6+336\,a^3b^2\tan\left(d\right.}{}$$

input
```
integrate(sec(d*x+c)^9*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

output
```
1/168*(21*b^5*tan(d*x + c)^8 + 120*a*b^4*tan(d*x + c)^7 + 280*a^2*b^3*tan(
d*x + c)^6 + 28*b^5*tan(d*x + c)^6 + 336*a^3*b^2*tan(d*x + c)^5 + 168*a*b^
4*tan(d*x + c)^5 + 210*a^4*b*tan(d*x + c)^4 + 420*a^2*b^3*tan(d*x + c)^4 +
 56*a^5*tan(d*x + c)^3 + 560*a^3*b^2*tan(d*x + c)^3 + 420*a^4*b*tan(d*x +
c)^2 + 168*a^5*tan(d*x + c))/d
```

3.106.9 Mupad [B] (verification not implemented)

Time = 27.56 (sec) , antiderivative size = 419, normalized size of antiderivative = 2.37

$$\int \sec^9(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$
$$= \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\left(\frac{86\,a^5}{3}-\frac{208\,a^3\,b^2}{3}+32\,a\,b^4\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(40\,a^4\,b-40\,a^2\,b^3\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}\left(40\,a^4\,b-40\right.}{}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))^5/cos(c + d*x)^9,x)
```

output
```
(tan(c/2 + (d*x)/2)^5*(32*a*b^4 + (86*a^5)/3 - (208*a^3*b^2)/3) - tan(c/2
+ (d*x)/2)^4*(40*a^4*b - 40*a^2*b^3) - tan(c/2 + (d*x)/2)^12*(40*a^4*b - 4
0*a^2*b^3) - 2*a^5*tan(c/2 + (d*x)/2)^15 - tan(c/2 + (d*x)/2)^11*(32*a*b^4
 + (86*a^5)/3 - (208*a^3*b^2)/3) - tan(c/2 + (d*x)/2)^7*((32*a*b^4)/7 + (1
30*a^5)/3 - (224*a^3*b^2)/3) + tan(c/2 + (d*x)/2)^9*((32*a*b^4)/7 + (130*a
^5)/3 - (224*a^3*b^2)/3) + tan(c/2 + (d*x)/2)^8*((32*b^5)/3 - 80*a^4*b + (
80*a^2*b^3)/3) + tan(c/2 + (d*x)/2)^6*(70*a^4*b + (32*b^5)/3 - (160*a^2*b^
3)/3) + tan(c/2 + (d*x)/2)^10*(70*a^4*b + (32*b^5)/3 - (160*a^2*b^3)/3) -
tan(c/2 + (d*x)/2)^3*((34*a^5)/3 - (80*a^3*b^2)/3) + tan(c/2 + (d*x)/2)^13
*((34*a^5)/3 - (80*a^3*b^2)/3) + 2*a^5*tan(c/2 + (d*x)/2) + 10*a^4*b*tan(c
/2 + (d*x)/2)^2 + 10*a^4*b*tan(c/2 + (d*x)/2)^14)/(d*(tan(c/2 + (d*x)/2)^2
 - 1)^8)
```

3.107 $\int \sec^{10}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

3.107.1 Optimal result

Integrand size = 28, antiderivative size = 391

$$\begin{aligned}\int \sec^{10}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx &= \frac{3a^5\operatorname{arctanh}(\sin(c+dx))}{8d}-\frac{5a^3b^2\operatorname{arctanh}(\sin(c+dx))}{8d}+\frac{15ab^4\operatorname{arctanh}(\sin(c+dx))}{128d}\\
&+\frac{a^4b\sec^5(c+dx)}{d}-\frac{2a^2b^3\sec^5(c+dx)}{d}+\frac{b^5\sec^5(c+dx)}{5d}+\frac{10a^2b^3\sec^7(c+dx)}{7d}\\
&-\frac{2b^5\sec^7(c+dx)}{7d}+\frac{b^5\sec^9(c+dx)}{9d}+\frac{3a^5\sec(c+dx)\tan(c+dx)}{8d}\\
&-\frac{5a^3b^2\sec(c+dx)\tan(c+dx)}{8d}+\frac{15ab^4\sec(c+dx)\tan(c+dx)}{128d}\\
&+\frac{a^5\sec^3(c+dx)\tan(c+dx)}{4d}-\frac{5a^3b^2\sec^3(c+dx)\tan(c+dx)}{12d}\\
&+\frac{5ab^4\sec^3(c+dx)\tan(c+dx)}{64d}+\frac{5a^3b^2\sec^5(c+dx)\tan(c+dx)}{3d}\\
&-\frac{5ab^4\sec^5(c+dx)\tan(c+dx)}{16d}+\frac{5ab^4\sec^5(c+dx)\tan^3(c+dx)}{8d}\end{aligned}$$

output
```
3/8*a^5*arctanh(sin(d*x+c))/d-5/8*a^3*b^2*arctanh(sin(d*x+c))/d+15/128*a*b
^4*arctanh(sin(d*x+c))/d+a^4*b*sec(d*x+c)^5/d-2*a^2*b^3*sec(d*x+c)^5/d+1/5
*b^5*sec(d*x+c)^5/d+10/7*a^2*b^3*sec(d*x+c)^7/d-2/7*b^5*sec(d*x+c)^7/d+1/9
*b^5*sec(d*x+c)^9/d+3/8*a^5*sec(d*x+c)*tan(d*x+c)/d-5/8*a^3*b^2*sec(d*x+c)
*tan(d*x+c)/d+15/128*a*b^4*sec(d*x+c)*tan(d*x+c)/d+1/4*a^5*sec(d*x+c)^3*ta
n(d*x+c)/d-5/12*a^3*b^2*sec(d*x+c)^3*tan(d*x+c)/d+5/64*a*b^4*sec(d*x+c)^3*
tan(d*x+c)/d+5/3*a^3*b^2*sec(d*x+c)^5*tan(d*x+c)/d-5/16*a*b^4*sec(d*x+c)^5
*tan(d*x+c)/d+5/8*a*b^4*sec(d*x+c)^5*tan(d*x+c)^3/d
```

3.107.2 Mathematica [A] (verified)

Time = 4.74 (sec) , antiderivative size = 331, normalized size of antiderivative = 0.85

$$\int \sec^{10}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$
$$= \frac{-40320a(48a^4-80a^2b^2+15b^4)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(\right.\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]^10*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

output
```
(-40320*a*(48*a^4 - 80*a^2*b^2 + 15*b^4)*(Log[Cos[(c + d*x)/2] - Sin[(c +
d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Sec[c + d*x]^9*(193
5360*a^4*b - 184320*a^2*b^3 + 223232*b^5 + 73728*(35*a^4*b - 20*a^2*b^3 -
3*b^5)*Cos[2*(c + d*x)] + 129024*(5*a^4*b - 10*a^2*b^3 + b^5)*Cos[4*(c + d
*x)] + 372960*a^5*Sin[4*(c + d*x)] + 453600*a^3*b^2*Sin[4*(c + d*x)] - 488
250*a*b^4*Sin[4*(c + d*x)] + 131040*a^5*Sin[6*(c + d*x)] - 218400*a^3*b^2*
Sin[6*(c + d*x)] + 40950*a*b^4*Sin[6*(c + d*x)] + 15120*a^5*Sin[8*(c + d*x
)] - 25200*a^3*b^2*Sin[8*(c + d*x)] + 4725*a*b^4*Sin[8*(c + d*x)]) + 1260*
a*(656*a^4 + 2320*a^2*b^2 + 845*b^4)*Sec[c + d*x]^7*Tan[c + d*x])/(5160960
*d)
```

3.107.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{10}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{(a\cos(c+dx)+b\sin(c+dx))^5}{\cos(c+dx)^{10}}dx$$
$$\Big\downarrow \; 3569$$
$$\int \left(a^5\sec^5(c+dx)+5a^4b\tan(c+dx)\sec^5(c+dx)+10a^3b^2\tan^2(c+dx)\sec^5(c+dx)+10a^2b^3\tan^3(c+dx)\sec^5\right.$$

$$\downarrow \text{2009}$$

$$\frac{3a^5\text{arctanh}(\sin(c+dx))}{8d}+\frac{a^5\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3a^5\tan(c+dx)\sec(c+dx)}{8d}+\frac{a^4b\sec^5(c+dx)}{d}-\frac{5a^3b^2\text{arctanh}(\sin(c+dx))}{8d}+\frac{5a^3b^2\tan(c+dx)\sec^5(c+dx)}{3d}-\frac{5a^3b^2\tan(c+dx)\sec^3(c+dx)}{12d}-\frac{5a^3b^2\tan(c+dx)\sec(c+dx)}{8d}+\frac{10a^2b^3\sec^7(c+dx)}{7d}-\frac{2a^2b^3\sec^5(c+dx)}{d}+\frac{15ab^4\text{arctanh}(\sin(c+dx))}{128d}+\frac{5ab^4\tan^3(c+dx)\sec^5(c+dx)}{8d}-\frac{5ab^4\tan(c+dx)\sec^5(c+dx)}{16d}+\frac{5ab^4\tan(c+dx)\sec^3(c+dx)}{64d}+\frac{15ab^4\tan(c+dx)\sec(c+dx)}{128d}+\frac{b^5\sec^9(c+dx)}{9d}-\frac{2b^5\sec^7(c+dx)}{7d}+\frac{b^5\sec^5(c+dx)}{5d}$$

input
```
Int[Sec[c + d*x]^10*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

output
```
(3*a^5*ArcTanh[Sin[c + d*x]])/(8*d) - (5*a^3*b^2*ArcTanh[Sin[c + d*x]])/(8
*d) + (15*a*b^4*ArcTanh[Sin[c + d*x]])/(128*d) + (a^4*b*Sec[c + d*x]^5)/d
- (2*a^2*b^3*Sec[c + d*x]^5)/d + (b^5*Sec[c + d*x]^5)/(5*d) + (10*a^2*b^3*
Sec[c + d*x]^7)/(7*d) - (2*b^5*Sec[c + d*x]^7)/(7*d) + (b^5*Sec[c + d*x]^9
)/(9*d) + (3*a^5*Sec[c + d*x]*Tan[c + d*x])/(8*d) - (5*a^3*b^2*Sec[c + d*x
]*Tan[c + d*x])/(8*d) + (15*a*b^4*Sec[c + d*x]*Tan[c + d*x])/(128*d) + (a^
5*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) - (5*a^3*b^2*Sec[c + d*x]^3*Tan[c + d
*x])/(12*d) + (5*a*b^4*Sec[c + d*x]^3*Tan[c + d*x])/(64*d) + (5*a^3*b^2*Se
c[c + d*x]^5*Tan[c + d*x])/(3*d) - (5*a*b^4*Sec[c + d*x]^5*Tan[c + d*x])/(
16*d) + (5*a*b^4*Sec[c + d*x]^5*Tan[c + d*x]^3)/(8*d)
```

3.107.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

3.107.4 Maple [A] (verified)

Time = 2.93 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.88

method	result
parts	$\frac{a^5\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)}{d}+\frac{b^5\left(\frac{\sec(dx+c)^9}{9}-\frac{2\sec(dx+c)^7}{7}+\frac{\sec(dx+c)}{5}\right.}{d}$
derivativedivides	$\frac{a^5\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{a^4b}{\cos(dx+c)^5}+10a^3b^2\left(\frac{\sin(dx+c)^3}{6\cos(dx+c)^6}+\frac{\sin(dx-}{8\cos(dx}\right.}{}$
default	$\frac{a^5\left(-\left(-\frac{\sec(dx+c)^3}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{a^4b}{\cos(dx+c)^5}+10a^3b^2\left(\frac{\sin(dx+c)^3}{6\cos(dx+c)^6}+\frac{\sin(dx-}{8\cos(dx}\right.}{}$
parallelrisch	$\frac{-544320\left(a^4-\frac{5}{3}a^2b^2+\frac{5}{16}b^4\right)a\left(\frac{\cos(9dx+9c)}{36}+\frac{\cos(7dx+7c)}{4}+\cos(5dx+5c)+\frac{7\cos(3dx+3c)}{3}+\frac{7\cos(dx+c)}{2}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right.}{}$
risch	$\frac{e^{i(dx+c)}\left(-1290240a^2b^3e^{12i(dx+c)}+645120a^4b\,e^{12i(dx+c)}-15120ia^5e^{16i(dx+c)}-131040ia^5e^{14i(dx+c)}+1461600ia^3b^2e^{6i}\right.}{}$

input
```
int(sec(d*x+c)^10*(cos(d*x+c)*a+b*sin(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
a^5/d*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+ta
n(d*x+c)))+b^5/d*(1/9*sec(d*x+c)^9-2/7*sec(d*x+c)^7+1/5*sec(d*x+c)^5)+10*a
^3*b^2/d*(1/6*sin(d*x+c)^3/cos(d*x+c)^6+1/8*sin(d*x+c)^3/cos(d*x+c)^4+1/16
*sin(d*x+c)^3/cos(d*x+c)^2+1/16*sin(d*x+c)-1/16*ln(sec(d*x+c)+tan(d*x+c)))
+5*a*b^4/d*(1/8*sin(d*x+c)^5/cos(d*x+c)^8+1/16*sin(d*x+c)^5/cos(d*x+c)^6+1
/64*sin(d*x+c)^5/cos(d*x+c)^4-1/128*sin(d*x+c)^5/cos(d*x+c)^2-1/128*sin(d*
x+c)^3-3/128*sin(d*x+c)+3/128*ln(sec(d*x+c)+tan(d*x+c)))+10*a^2*b^3/d*(1/7
*sec(d*x+c)^7-1/5*sec(d*x+c)^5)+a^4*b*sec(d*x+c)^5/d
```

3.107.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.66

$$\int \sec^{10}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$
$$=\frac{315\left(48a^5-80a^3b^2+15ab^4\right)\cos(dx+c)^9\log(\sin(dx+c)+1)-315\left(48a^5-80a^3b^2+15ab^4\right)\cos(dx+\right.}{}$$

input
```
integrate(sec(d*x+c)^10*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas
")
```

output
```
1/80640*(315*(48*a^5 - 80*a^3*b^2 + 15*a*b^4)*cos(d*x + c)^9*log(sin(d*x +
 c) + 1) - 315*(48*a^5 - 80*a^3*b^2 + 15*a*b^4)*cos(d*x + c)^9*log(-sin(d*
x + c) + 1) + 8960*b^5 + 16128*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c)^4
 + 23040*(5*a^2*b^3 - b^5)*cos(d*x + c)^2 + 210*(3*(48*a^5 - 80*a^3*b^2 +
15*a*b^4)*cos(d*x + c)^7 + 240*a*b^4*cos(d*x + c) + 2*(48*a^5 - 80*a^3*b^2
 + 15*a*b^4)*cos(d*x + c)^5 + 40*(16*a^3*b^2 - 9*a*b^4)*cos(d*x + c)^3)*si
n(d*x + c))/(d*cos(d*x + c)^9)
```

3.107.6 Sympy [F(-1)]

Timed out.

$$\int \sec^{10}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**10*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

output
```
Timed out
```

3.107.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 360, normalized size of antiderivative = 0.92

$$\int \sec^{10}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx =$$
$$-\frac{1575\,ab^4\left(\frac{2\left(3\,\sin(dx+c)^7-11\,\sin(dx+c)^5-11\,\sin(dx+c)^3+3\,\sin(dx+c)\right)}{\sin(dx+c)^8-4\,\sin(dx+c)^6+6\,\sin(dx+c)^4-4\,\sin(dx+c)^2+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)\right.\right.}{}$$

input
```
integrate(sec(d*x+c)^10*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima
")
```

output
```
-1/80640*(1575*a*b^4*(2*(3*sin(d*x + c)^7 - 11*sin(d*x + c)^5 - 11*sin(d*x
 + c)^3 + 3*sin(d*x + c))/(sin(d*x + c)^8 - 4*sin(d*x + c)^6 + 6*sin(d*x +
 c)^4 - 4*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x +
c) - 1)) - 8400*a^3*b^2*(2*(3*sin(d*x + c)^5 - 8*sin(d*x + c)^3 - 3*sin(d*
x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 3*log
(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) + 5040*a^5*(2*(3*sin(d*x + c
)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(
d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 80640*a^4*b/cos(d*x + c)^5 + 23
040*(7*cos(d*x + c)^2 - 5)*a^2*b^3/cos(d*x + c)^7 - 256*(63*cos(d*x + c)^4
 - 90*cos(d*x + c)^2 + 35)*b^5/cos(d*x + c)^9)/d
```

3.107.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 888 vs. $2(359) = 718$.

Time = 0.62 (sec) , antiderivative size = 888, normalized size of antiderivative = 2.27

$$\int \sec^{10}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^10*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

output
```
1/40320*(315*(48*a^5 - 80*a^3*b^2 + 15*a*b^4)*log(abs(tan(1/2*d*x + 1/2*c)
 + 1)) - 315*(48*a^5 - 80*a^3*b^2 + 15*a*b^4)*log(abs(tan(1/2*d*x + 1/2*c)
 - 1)) + 2*(25200*a^5*tan(1/2*d*x + 1/2*c)^17 + 25200*a^3*b^2*tan(1/2*d*x
+ 1/2*c)^17 - 4725*a*b^4*tan(1/2*d*x + 1/2*c)^17 - 201600*a^4*b*tan(1/2*d*
x + 1/2*c)^16 - 110880*a^5*tan(1/2*d*x + 1/2*c)^15 + 319200*a^3*b^2*tan(1/
2*d*x + 1/2*c)^15 + 40950*a*b^4*tan(1/2*d*x + 1/2*c)^15 + 806400*a^4*b*tan
(1/2*d*x + 1/2*c)^14 - 806400*a^2*b^3*tan(1/2*d*x + 1/2*c)^14 + 191520*a^5
*tan(1/2*d*x + 1/2*c)^13 - 453600*a^3*b^2*tan(1/2*d*x + 1/2*c)^13 + 488250
*a*b^4*tan(1/2*d*x + 1/2*c)^13 - 1612800*a^4*b*tan(1/2*d*x + 1/2*c)^12 + 8
06400*a^2*b^3*tan(1/2*d*x + 1/2*c)^12 - 215040*b^5*tan(1/2*d*x + 1/2*c)^12
 - 151200*a^5*tan(1/2*d*x + 1/2*c)^11 - 151200*a^3*b^2*tan(1/2*d*x + 1/2*c
)^11 + 532350*a*b^4*tan(1/2*d*x + 1/2*c)^11 + 2419200*a^4*b*tan(1/2*d*x +
1/2*c)^10 - 806400*a^2*b^3*tan(1/2*d*x + 1/2*c)^10 - 322560*b^5*tan(1/2*d*
x + 1/2*c)^10 - 2661120*a^4*b*tan(1/2*d*x + 1/2*c)^8 + 2096640*a^2*b^3*tan
(1/2*d*x + 1/2*c)^8 - 451584*b^5*tan(1/2*d*x + 1/2*c)^8 + 151200*a^5*tan(1
/2*d*x + 1/2*c)^7 + 151200*a^3*b^2*tan(1/2*d*x + 1/2*c)^7 - 532350*a*b^4*t
an(1/2*d*x + 1/2*c)^7 + 1774080*a^4*b*tan(1/2*d*x + 1/2*c)^6 - 1128960*a^2
*b^3*tan(1/2*d*x + 1/2*c)^6 - 129024*b^5*tan(1/2*d*x + 1/2*c)^6 - 191520*a
^5*tan(1/2*d*x + 1/2*c)^5 + 453600*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 - 488250
*a*b^4*tan(1/2*d*x + 1/2*c)^5 - 645120*a^4*b*tan(1/2*d*x + 1/2*c)^4 + 2...
```

3.107.9 Mupad [B] (verification not implemented)

Time = 27.63 (sec) , antiderivative size = 675, normalized size of antiderivative = 1.73

$$\int \sec^{10}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$
$$= \frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(\frac{3a^5}{4}-\frac{5a^3b^2}{4}+\frac{15ab^4}{64}\right)}{d}$$
$$-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{5a^5}{4}+\frac{5a^3b^2}{4}-\frac{15ab^4}{64}\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}(40a^4b-40a^2b^3)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{17}\left(\frac{5a^5}{4}+\frac{5a^3b^2}{4}\right.}{}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))^5/cos(c + d*x)^10,x)
```

output
```
(atanh(tan(c/2 + (d*x)/2))*((15*a*b^4)/64 + (3*a^5)/4 - (5*a^3*b^2)/4))/d
- (tan(c/2 + (d*x)/2)*((5*a^5)/4 - (15*a*b^4)/64 + (5*a^3*b^2)/4) - tan(c/
2 + (d*x)/2)^14*(40*a^4*b - 40*a^2*b^3) - tan(c/2 + (d*x)/2)^17*((5*a^5)/4
 - (15*a*b^4)/64 + (5*a^3*b^2)/4) + tan(c/2 + (d*x)/2)^3*((65*a*b^4)/32 -
(11*a^5)/2 + (95*a^3*b^2)/6) - tan(c/2 + (d*x)/2)^15*((65*a*b^4)/32 - (11*
a^5)/2 + (95*a^3*b^2)/6) + tan(c/2 + (d*x)/2)^5*((775*a*b^4)/32 + (19*a^5)
/2 - (45*a^3*b^2)/2) - tan(c/2 + (d*x)/2)^13*((775*a*b^4)/32 + (19*a^5)/2
- (45*a^3*b^2)/2) - tan(c/2 + (d*x)/2)^7*((15*a^5)/2 - (845*a*b^4)/32 + (1
5*a^3*b^2)/2) + tan(c/2 + (d*x)/2)^11*((15*a^5)/2 - (845*a*b^4)/32 + (15*a
^3*b^2)/2) - tan(c/2 + (d*x)/2)^2*(8*a^4*b + (16*b^5)/35 - (72*a^2*b^3)/7)
 + tan(c/2 + (d*x)/2)^4*(32*a^4*b + (64*b^5)/35 - (8*a^2*b^3)/7) + tan(c/2
 + (d*x)/2)^12*(80*a^4*b + (32*b^5)/3 - 40*a^2*b^3) + tan(c/2 + (d*x)/2)^1
0*(16*b^5 - 120*a^4*b + 40*a^2*b^3) + tan(c/2 + (d*x)/2)^6*((32*b^5)/5 - 8
8*a^4*b + 56*a^2*b^3) + tan(c/2 + (d*x)/2)^8*(132*a^4*b + (112*b^5)/5 - 10
4*a^2*b^3) + 2*a^4*b + (16*b^5)/315 - (8*a^2*b^3)/7 + 10*a^4*b*tan(c/2 + (
d*x)/2)^16)/(d*(9*tan(c/2 + (d*x)/2)^2 - 36*tan(c/2 + (d*x)/2)^4 + 84*tan(
c/2 + (d*x)/2)^6 - 126*tan(c/2 + (d*x)/2)^8 + 126*tan(c/2 + (d*x)/2)^10 -
84*tan(c/2 + (d*x)/2)^12 + 36*tan(c/2 + (d*x)/2)^14 - 9*tan(c/2 + (d*x)/2)
^16 + tan(c/2 + (d*x)/2)^18 - 1))
```

3.107. $\int \sec^{10}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

3.108 $\int \sec^{11}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

3.108.1 Optimal result

Integrand size = 28, antiderivative size = 242

$$\begin{aligned}
&\int \sec^{11}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx \\
&= \frac{a^5\tan(c+dx)}{d}+\frac{5a^4b\tan^2(c+dx)}{2d}+\frac{2a^3(a^2+5b^2)\tan^3(c+dx)}{3d} \\
&\quad+\frac{5a^2b(a^2+b^2)\tan^4(c+dx)}{2d}+\frac{a(a^4+20a^2b^2+5b^4)\tan^5(c+dx)}{5d} \\
&\quad+\frac{b(5a^4+20a^2b^2+b^4)\tan^6(c+dx)}{6d}+\frac{10ab^2(a^2+b^2)\tan^7(c+dx)}{7d} \\
&\quad+\frac{b^3(5a^2+b^2)\tan^8(c+dx)}{4d}+\frac{5ab^4\tan^9(c+dx)}{9d}+\frac{b^5\tan^{10}(c+dx)}{10d}
\end{aligned}$$

output
```
a^5*tan(d*x+c)/d+5/2*a^4*b*tan(d*x+c)^2/d+2/3*a^3*(a^2+5*b^2)*tan(d*x+c)^3
/d+5/2*a^2*b*(a^2+b^2)*tan(d*x+c)^4/d+1/5*a*(a^4+20*a^2*b^2+5*b^4)*tan(d*x
+c)^5/d+1/6*b*(5*a^4+20*a^2*b^2+b^4)*tan(d*x+c)^6/d+10/7*a*b^2*(a^2+b^2)*t
an(d*x+c)^7/d+1/4*b^3*(5*a^2+b^2)*tan(d*x+c)^8/d+5/9*a*b^4*tan(d*x+c)^9/d+
1/10*b^5*tan(d*x+c)^10/d
```

3.108.2 Mathematica [A] (verified)

Time = 1.26 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.48

$$\int \sec^{11}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$
$$=\frac{\frac{1}{6}(a^2+b^2)^2(a+b\tan(c+dx))^6-\frac{4}{7}a(a^2+b^2)(a+b\tan(c+dx))^7+\frac{1}{4}(3a^2+b^2)(a+b\tan(c+dx))^8-\frac{4}{9}\ldots}{b^5d}$$

input
```
Integrate[Sec[c + d*x]^11*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

output
```
(((a^2 + b^2)^2*(a + b*Tan[c + d*x])^6)/6 - (4*a*(a^2 + b^2)*(a + b*Tan[c
+ d*x])^7)/7 + ((3*a^2 + b^2)*(a + b*Tan[c + d*x])^8)/4 - (4*a*(a + b*Tan[
c + d*x])^9)/9 + (a + b*Tan[c + d*x])^10/10)/(b^5*d)
```

3.108.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3567, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{11}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a\cos(c+dx)+b\sin(c+dx))^5}{\cos(c+dx)^{11}}dx$$
$$\Big\downarrow \text{3567}$$
$$-\frac{\int (b+a\cot(c+dx))^5\left(\cot^2(c+dx)+1\right)^2\tan^{11}(c+dx)d\cot(c+dx)}{d}$$
$$\Big\downarrow \text{522}$$
$$-\frac{\int \left(b^5\tan^{11}(c+dx)+5ab^4\tan^{10}(c+dx)+2\left(b^5+5a^2b^3\right)\tan^9(c+dx)+10ab^2\left(a^2+b^2\right)\tan^8(c+dx)+\left(b^5+20\ldots\right.}{\ldots}$$
$$\Big\downarrow \text{2009}$$

$$-\frac{-a^5\tan(c+dx)-\frac{5}{2}a^4b\tan^2(c+dx)-\frac{10}{7}ab^2(a^2+b^2)\tan^7(c+dx)-\frac{5}{2}a^2b(a^2+b^2)\tan^4(c+dx)-\frac{1}{4}b^3(5a^2+$$

input
```
Int[Sec[c + d*x]^11*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

output
```
-((-(a^5*Tan[c + d*x]) - (5*a^4*b*Tan[c + d*x]^2)/2 - (2*a^3*(a^2 + 5*b^2)
*Tan[c + d*x]^3)/3 - (5*a^2*b*(a^2 + b^2)*Tan[c + d*x]^4)/2 - (a*(a^4 + 20
*a^2*b^2 + 5*b^4)*Tan[c + d*x]^5)/5 - (b*(5*a^4 + 20*a^2*b^2 + b^4)*Tan[c
+ d*x]^6)/6 - (10*a*b^2*(a^2 + b^2)*Tan[c + d*x]^7)/7 - (b^3*(5*a^2 + b^2)
*Tan[c + d*x]^8)/4 - (5*a*b^4*Tan[c + d*x]^9)/9 - (b^5*Tan[c + d*x]^10)/10
)/d)
```

3.108.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3567
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[x^m*((b
+ a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a,
b, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[
n, 0] && GtQ[m, 1])
```

3.108.4 Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.05

method	result
parts	$-\frac{a^5\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)}{d}+\frac{b^5\left(\frac{\sec(dx+c)^{10}}{10}-\frac{\sec(dx+c)^8}{4}+\frac{\sec(dx+c)^6}{6}\right)}{d}+\frac{10a^3b^2\left(\frac{\sin(dx+}{7\cos(dx}\right.}{}$
derivativedivides	$\frac{-a^5\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)+\frac{5a^4b}{6\cos(dx+c)^6}+10a^3b^2\left(\frac{\sin(dx+c)^3}{7\cos(dx+c)^7}+\frac{4\sin(dx+c)^3}{35\cos(dx+c)^5}+\frac{8\sin(dx+c)^3}{105\cos(dx+c)^3}\right)}{}$
default	$\frac{-a^5\left(-\frac{8}{15}-\frac{\sec(dx+c)^4}{5}-\frac{4\sec(dx+c)^2}{15}\right)\tan(dx+c)+\frac{5a^4b}{6\cos(dx+c)^6}+10a^3b^2\left(\frac{\sin(dx+c)^3}{7\cos(dx+c)^7}+\frac{4\sin(dx+c)^3}{35\cos(dx+c)^5}+\frac{8\sin(dx+c)^3}{105\cos(dx+c)^3}\right)}{}$
parallelrisch	$-\frac{2\left(a^5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{18}-5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{17}a^4b+\frac{\left(-19a^5+40a^3b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{16}}{3}+20\left(a^4b-a^2b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{15}+4\left(-\frac{22}{3}a^3\right.\right.}{}$
risch	$\frac{-\frac{320a^2b^3e^{12i(dx+c)}}{3}+\frac{640a^4b\,e^{12i(dx+c)}}{3}+\frac{32ia^5e^{14i(dx+c)}}{3}-\frac{1600ia^3b^2e^{6i(dx+c)}}{21}+\frac{160ia\,b^4e^{2i(dx+c)}}{63}-192ia^3b^2e^{10i(dx+c)}-}{}$

input
```
int(sec(d*x+c)^11*(cos(d*x+c)*a+b*sin(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
-a^5/d*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+b^5/d*(1/10*s
ec(d*x+c)^10-1/4*sec(d*x+c)^8+1/6*sec(d*x+c)^6)+10*a^3*b^2/d*(1/7*sin(d*x+
c)^3/cos(d*x+c)^7+4/35*sin(d*x+c)^3/cos(d*x+c)^5+8/105*sin(d*x+c)^3/cos(d*
x+c)^3)+5*a*b^4/d*(1/9*sin(d*x+c)^5/cos(d*x+c)^9+4/63*sin(d*x+c)^5/cos(d*x
+c)^7+8/315*sin(d*x+c)^5/cos(d*x+c)^5)+5/6*a^4*b/d*sec(d*x+c)^6+10*a^2*b^3
/d*(1/8*sec(d*x+c)^8-1/6*sec(d*x+c)^6)
```

3.108.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.86

$$\int \sec^{11}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$
$$=\frac{126\,b^5+210\,(5\,a^4b-10\,a^2b^3+b^5)\cos(dx+c)^4+315\,(5\,a^2b^3-b^5)\cos(dx+c)^2+4\,\big(8\,(21\,a^5-30\,a^3b^2+}{}$$

input
```
integrate(sec(d*x+c)^11*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas
")
```

output
```
1/1260*(126*b^5 + 210*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c)^4 + 315*(5
*a^2*b^3 - b^5)*cos(d*x + c)^2 + 4*(8*(21*a^5 - 30*a^3*b^2 + 5*a*b^4)*cos(
d*x + c)^9 + 4*(21*a^5 - 30*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^7 + 175*a*b^4*
cos(d*x + c) + 3*(21*a^5 - 30*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^5 + 50*(9*a^
3*b^2 - 5*a*b^4)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^10)
```

3.108.6 Sympy [F(-1)]

Timed out.

$$\int \sec^{11}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**11*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

output
```
Timed out
```

3.108.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.14

$$\int \sec^{11}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$

$$= \frac{84\left(3\tan\left(dx+c\right)^5+10\tan\left(dx+c\right)^3+15\tan\left(dx+c\right)\right)a^5+120\left(15\tan\left(dx+c\right)^7+42\tan\left(dx+c\right)^5+\right.}{}$$

input
```
integrate(sec(d*x+c)^11*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima
")
```

output
```
1/1260*(84*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^5 +
120*(15*tan(d*x + c)^7 + 42*tan(d*x + c)^5 + 35*tan(d*x + c)^3)*a^3*b^2 +
20*(35*tan(d*x + c)^9 + 90*tan(d*x + c)^7 + 63*tan(d*x + c)^5)*a*b^4 + 525
*(4*sin(d*x + c)^2 - 1)*a^2*b^3/(sin(d*x + c)^8 - 4*sin(d*x + c)^6 + 6*sin
(d*x + c)^4 - 4*sin(d*x + c)^2 + 1) - 21*(10*sin(d*x + c)^4 - 5*sin(d*x +
c)^2 + 1)*b^5/(sin(d*x + c)^10 - 5*sin(d*x + c)^8 + 10*sin(d*x + c)^6 - 10
*sin(d*x + c)^4 + 5*sin(d*x + c)^2 - 1) - 1050*a^4*b/(sin(d*x + c)^2 - 1)^
3)/d
```

3.108.8 Giac [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.08

$$\int \sec^{11}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$
$$=\frac{126\,b^5\tan(dx+c)^{10}+700\,ab^4\tan(dx+c)^9+1575\,a^2b^3\tan(dx+c)^8+315\,b^5\tan(dx+c)^8+1800\,a^3b^2\,\mathrm{t}}{}$$

input
```
integrate(sec(d*x+c)^11*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

output
```
1/1260*(126*b^5*tan(d*x + c)^10 + 700*a*b^4*tan(d*x + c)^9 + 1575*a^2*b^3*
tan(d*x + c)^8 + 315*b^5*tan(d*x + c)^8 + 1800*a^3*b^2*tan(d*x + c)^7 + 18
00*a*b^4*tan(d*x + c)^7 + 1050*a^4*b*tan(d*x + c)^6 + 4200*a^2*b^3*tan(d*x
 + c)^6 + 210*b^5*tan(d*x + c)^6 + 252*a^5*tan(d*x + c)^5 + 5040*a^3*b^2*t
an(d*x + c)^5 + 1260*a*b^4*tan(d*x + c)^5 + 3150*a^4*b*tan(d*x + c)^4 + 31
50*a^2*b^3*tan(d*x + c)^4 + 840*a^5*tan(d*x + c)^3 + 4200*a^3*b^2*tan(d*x
+ c)^3 + 3150*a^4*b*tan(d*x + c)^2 + 1260*a^5*tan(d*x + c))/d
```

3.108.9 Mupad [B] (verification not implemented)

Time = 27.27 (sec) , antiderivative size = 548, normalized size of antiderivative = 2.26

$$\int \sec^{11}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$
$$=\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\left(\frac{616\,a^5}{15}-\frac{176\,a^3\,b^2}{3}+32\,a\,b^4\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(40\,a^4\,b-40\,a^2\,b^3\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}\left(40\,a^4\,b-4\right.}{}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))^5/cos(c + d*x)^11,x)
```

output
```
(tan(c/2 + (d*x)/2)^5*(32*a*b^4 + (616*a^5)/15 - (176*a^3*b^2)/3) - tan(c/
2 + (d*x)/2)^4*(40*a^4*b - 40*a^2*b^3) - tan(c/2 + (d*x)/2)^16*(40*a^4*b -
 40*a^2*b^3) - 2*a^5*tan(c/2 + (d*x)/2)^19 - tan(c/2 + (d*x)/2)^15*(32*a*b
^4 + (616*a^5)/15 - (176*a^3*b^2)/3) + tan(c/2 + (d*x)/2)^7*((160*a*b^4)/7
 - 88*a^5 + (720*a^3*b^2)/7) - tan(c/2 + (d*x)/2)^13*((160*a*b^4)/7 - 88*a
^5 + (720*a^3*b^2)/7) + tan(c/2 + (d*x)/2)^9*((3520*a*b^4)/63 + (388*a^5)/
3 - (4240*a^3*b^2)/21) - tan(c/2 + (d*x)/2)^11*((3520*a*b^4)/63 + (388*a^5
)/3 - (4240*a^3*b^2)/21) + tan(c/2 + (d*x)/2)^6*((280*a^4*b)/3 + (32*b^5)/
3 - (80*a^2*b^3)/3) + tan(c/2 + (d*x)/2)^14*((280*a^4*b)/3 + (32*b^5)/3 -
(80*a^2*b^3)/3) + tan(c/2 + (d*x)/2)^10*(220*a^4*b + (192*b^5)/5 - 160*a^2
*b^3) + tan(c/2 + (d*x)/2)^8*((64*b^5)/3 - (520*a^4*b)/3 + (200*a^2*b^3)/3
) + tan(c/2 + (d*x)/2)^12*((64*b^5)/3 - (520*a^4*b)/3 + (200*a^2*b^3)/3) -
 tan(c/2 + (d*x)/2)^3*((38*a^5)/3 - (80*a^3*b^2)/3) + tan(c/2 + (d*x)/2)^1
7*((38*a^5)/3 - (80*a^3*b^2)/3) + 2*a^5*tan(c/2 + (d*x)/2) + 10*a^4*b*tan(
c/2 + (d*x)/2)^2 + 10*a^4*b*tan(c/2 + (d*x)/2)^18)/(d*(tan(c/2 + (d*x)/2)^
2 - 1)^10)
```

3.108. $\int \sec^{11}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

3.109 $\int \sec^{12}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

3.109.1 Optimal result

Integrand size = 28, antiderivative size = 472

$$\begin{aligned}
&\int \sec^{12}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx \\
&= \frac{5a^5\text{arctanh}(\sin(c+dx))}{16d}-\frac{25a^3b^2\text{arctanh}(\sin(c+dx))}{64d} \\
&\quad+\frac{15ab^4\text{arctanh}(\sin(c+dx))}{256d}+\frac{5a^4b\sec^7(c+dx)}{7d}-\frac{10a^2b^3\sec^7(c+dx)}{7d} \\
&\quad+\frac{b^5\sec^7(c+dx)}{7d}+\frac{10a^2b^3\sec^9(c+dx)}{9d}-\frac{2b^5\sec^9(c+dx)}{9d}+\frac{b^5\sec^{11}(c+dx)}{11d} \\
&\quad+\frac{5a^5\sec(c+dx)\tan(c+dx)}{16d}-\frac{25a^3b^2\sec(c+dx)\tan(c+dx)}{64d} \\
&\quad+\frac{15ab^4\sec(c+dx)\tan(c+dx)}{256d}+\frac{5a^5\sec^3(c+dx)\tan(c+dx)}{24d} \\
&\quad-\frac{25a^3b^2\sec^3(c+dx)\tan(c+dx)}{96d}+\frac{5ab^4\sec^3(c+dx)\tan(c+dx)}{128d} \\
&\quad+\frac{a^5\sec^5(c+dx)\tan(c+dx)}{6d}-\frac{5a^3b^2\sec^5(c+dx)\tan(c+dx)}{24d} \\
&\quad+\frac{ab^4\sec^5(c+dx)\tan(c+dx)}{32d}+\frac{5a^3b^2\sec^7(c+dx)\tan(c+dx)}{4d} \\
&\quad-\frac{3ab^4\sec^7(c+dx)\tan(c+dx)}{16d}+\frac{ab^4\sec^7(c+dx)\tan^3(c+dx)}{2d}
\end{aligned}$$

output
```
5/16*a^5*arctanh(sin(d*x+c))/d-25/64*a^3*b^2*arctanh(sin(d*x+c))/d+15/256*
a*b^4*arctanh(sin(d*x+c))/d+5/7*a^4*b*sec(d*x+c)^7/d-10/7*a^2*b^3*sec(d*x+
c)^7/d+1/7*b^5*sec(d*x+c)^7/d+10/9*a^2*b^3*sec(d*x+c)^9/d-2/9*b^5*sec(d*x+
c)^9/d+1/11*b^5*sec(d*x+c)^11/d+5/16*a^5*sec(d*x+c)*tan(d*x+c)/d-25/64*a^3
*b^2*sec(d*x+c)*tan(d*x+c)/d+15/256*a*b^4*sec(d*x+c)*tan(d*x+c)/d+5/24*a^5
*sec(d*x+c)^3*tan(d*x+c)/d-25/96*a^3*b^2*sec(d*x+c)^3*tan(d*x+c)/d+5/128*a
*b^4*sec(d*x+c)^3*tan(d*x+c)/d+1/6*a^5*sec(d*x+c)^5*tan(d*x+c)/d-5/24*a^3*
b^2*sec(d*x+c)^5*tan(d*x+c)/d+1/32*a*b^4*sec(d*x+c)^5*tan(d*x+c)/d+5/4*a^3
*b^2*sec(d*x+c)^7*tan(d*x+c)/d-3/16*a*b^4*sec(d*x+c)^7*tan(d*x+c)/d+1/2*a*
b^4*sec(d*x+c)^7*tan(d*x+c)^3/d
```

3.109.2 Mathematica [A] (verified)

Time = 4.52 (sec) , antiderivative size = 374, normalized size of antiderivative = 0.79

$$\int \sec^{12}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$
$$= \frac{-1774080a(16a^4-20a^2b^2+3b^4)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}\right.\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]^12*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

output
```
(-1774080*a*(16*a^4 - 20*a^2*b^2 + 3*b^4)*(Log[Cos[(c + d*x)/2] - Sin[(c +
 d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Sec[c + d*x]^11*(2
4330240*a^4*b + 1802240*a^2*b^3 + 3031040*b^5 + 3604480*(9*a^4*b - 4*a^2*b
^3 - b^5)*Cos[2*(c + d*x)] + 1622016*(5*a^4*b - 10*a^2*b^3 + b^5)*Cos[4*(c
 + d*x)] + 6623232*a^5*Sin[4*(c + d*x)] + 5913600*a^3*b^2*Sin[4*(c + d*x)]
 - 6564096*a*b^4*Sin[4*(c + d*x)] + 2857008*a^5*Sin[6*(c + d*x)] - 3571260
*a^3*b^2*Sin[6*(c + d*x)] + 535689*a*b^4*Sin[6*(c + d*x)] + 591360*a^5*Sin
[8*(c + d*x)] - 739200*a^3*b^2*Sin[8*(c + d*x)] + 110880*a*b^4*Sin[8*(c +
d*x)] + 55440*a^5*Sin[10*(c + d*x)] - 69300*a^3*b^2*Sin[10*(c + d*x)] + 10
395*a*b^4*Sin[10*(c + d*x)]) + 13860*a*(976*a^4 + 2876*a^2*b^2 + 1207*b^4)
*Sec[c + d*x]^9*Tan[c + d*x])/(90832896*d)
```

3.109.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 472, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^{12}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\cos(c+dx)+b\sin(c+dx))^5}{\cos(c+dx)^{12}}dx$$

$$\Big\downarrow \text{3569}$$

$$\int \left(a^5\sec^7(c+dx)+5a^4b\tan(c+dx)\sec^7(c+dx)+10a^3b^2\tan^2(c+dx)\sec^7(c+dx)+10a^2b^3\tan^3(c+dx)\sec^7\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{5a^5\text{arctanh}(\sin(c+dx))}{16d}+\frac{a^5\tan(c+dx)\sec^5(c+dx)}{6d}+\frac{5a^5\tan(c+dx)\sec^3(c+dx)}{24d}+\frac{5a^5\tan(c+dx)\sec(c+dx)}{16d}+\frac{5a^4b\sec^7(c+dx)}{7d}-\frac{25a^3b^2\text{arctanh}(\sin(c+dx))}{64d}+\frac{5a^3b^2\tan(c+dx)\sec^7(c+dx)}{4d}-\frac{5a^3b^2\tan(c+dx)\sec^5(c+dx)}{24d}-\frac{25a^3b^2\tan(c+dx)\sec^3(c+dx)}{96d}-\frac{25a^3b^2\tan(c+dx)\sec(c+dx)}{64d}+\frac{10a^2b^3\sec^9(c+dx)}{9d}-\frac{10a^2b^3\sec^7(c+dx)}{7d}+\frac{15ab^4\text{arctanh}(\sin(c+dx))}{256d}+\frac{ab^4\tan^3(c+dx)\sec^7(c+dx)}{2d}-\frac{3ab^4\tan(c+dx)\sec^7(c+dx)}{16d}+\frac{ab^4\tan(c+dx)\sec^5(c+dx)}{32d}+\frac{5ab^4\tan(c+dx)\sec^3(c+dx)}{128d}+\frac{15ab^4\tan(c+dx)\sec(c+dx)}{256d}+\frac{b^5\sec^{11}(c+dx)}{11d}-\frac{2b^5\sec^9(c+dx)}{9d}+\frac{b^5\sec^7(c+dx)}{7d}$$

input
```
Int[Sec[c + d*x]^12*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

output
```
(5*a^5*ArcTanh[Sin[c + d*x]])/(16*d) - (25*a^3*b^2*ArcTanh[Sin[c + d*x]])/
(64*d) + (15*a*b^4*ArcTanh[Sin[c + d*x]])/(256*d) + (5*a^4*b*Sec[c + d*x]^
7)/(7*d) - (10*a^2*b^3*Sec[c + d*x]^7)/(7*d) + (b^5*Sec[c + d*x]^7)/(7*d)
+ (10*a^2*b^3*Sec[c + d*x]^9)/(9*d) - (2*b^5*Sec[c + d*x]^9)/(9*d) + (b^5*
Sec[c + d*x]^11)/(11*d) + (5*a^5*Sec[c + d*x]*Tan[c + d*x])/(16*d) - (25*a
^3*b^2*Sec[c + d*x]*Tan[c + d*x])/(64*d) + (15*a*b^4*Sec[c + d*x]*Tan[c +
d*x])/(256*d) + (5*a^5*Sec[c + d*x]^3*Tan[c + d*x])/(24*d) - (25*a^3*b^2*S
ec[c + d*x]^3*Tan[c + d*x])/(96*d) + (5*a*b^4*Sec[c + d*x]^3*Tan[c + d*x])
/(128*d) + (a^5*Sec[c + d*x]^5*Tan[c + d*x])/(6*d) - (5*a^3*b^2*Sec[c + d*
x]^5*Tan[c + d*x])/(24*d) + (a*b^4*Sec[c + d*x]^5*Tan[c + d*x])/(32*d) + (
5*a^3*b^2*Sec[c + d*x]^7*Tan[c + d*x])/(4*d) - (3*a*b^4*Sec[c + d*x]^7*Tan
[c + d*x])/(16*d) + (a*b^4*Sec[c + d*x]^7*Tan[c + d*x]^3)/(2*d)
```

3.109.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

3.109.4 Maple [A] (verified)

Time = 3.30 (sec) , antiderivative size = 392, normalized size of antiderivative = 0.83

method	result
parts	$\frac{a^5\left(-\left(-\frac{\sec(dx+c)^5}{6}-\frac{5\sec(dx+c)^3}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)}{d}+\frac{b^5\left(\frac{\sec(dx+c)^{11}}{11}-\frac{2\sec(dx+}{9}\right.}{d}$
derivativedivides	$a^5\left(-\left(-\frac{\sec(dx+c)^5}{6}-\frac{5\sec(dx+c)^3}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)+\frac{5a^4b}{7\cos(dx+c)^7}+10a^3b^2\left(\frac{\sin(d}{8\cos(c}\right.$
default	$a^5\left(-\left(-\frac{\sec(dx+c)^5}{6}-\frac{5\sec(dx+c)^3}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)+\frac{5a^4b}{7\cos(dx+c)^7}+10a^3b^2\left(\frac{\sin(d}{8\cos(c}\right.$
parallelrisch	$-9147600\left(\frac{\cos(11dx+11c)}{165}+\frac{\cos(9dx+9c)}{15}+\frac{\cos(7dx+7c)}{3}+\cos(5dx+5c)+2\cos(3dx+3c)+\frac{14\cos(dx+c)}{5}\right)a\left(a^4-\frac{5}{4}a^2b^2+\frac{3}{16}b^4\right)$
risch	Expression too large to display

input
```
int(sec(d*x+c)^12*(cos(d*x+c)*a+b*sin(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
a^5/d*(-(-1/6*sec(d*x+c)^5-5/24*sec(d*x+c)^3-5/16*sec(d*x+c))*tan(d*x+c)+5
/16*ln(sec(d*x+c)+tan(d*x+c)))+b^5/d*(1/11*sec(d*x+c)^11-2/9*sec(d*x+c)^9+
1/7*sec(d*x+c)^7)+5/7*a^4*b*sec(d*x+c)^7/d+10*a^3*b^2/d*(1/8*sin(d*x+c)^3/
cos(d*x+c)^8+5/48*sin(d*x+c)^3/cos(d*x+c)^6+5/64*sin(d*x+c)^3/cos(d*x+c)^4
+5/128*sin(d*x+c)^3/cos(d*x+c)^2+5/128*sin(d*x+c)-5/128*ln(sec(d*x+c)+tan(
d*x+c)))+5*a*b^4/d*(1/10*sin(d*x+c)^5/cos(d*x+c)^10+1/16*sin(d*x+c)^5/cos(
d*x+c)^8+1/32*sin(d*x+c)^5/cos(d*x+c)^6+1/128*sin(d*x+c)^5/cos(d*x+c)^4-1/
256*sin(d*x+c)^5/cos(d*x+c)^2-1/256*sin(d*x+c)^3-3/256*sin(d*x+c)+3/256*ln
(sec(d*x+c)+tan(d*x+c)))+10*a^2*b^3/d*(1/9*sec(d*x+c)^9-1/7*sec(d*x+c)^7)
```

3.109.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.61

$$\int \sec^{12}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$$
$$=\frac{3465\,(16\,a^5-20\,a^3b^2+3\,ab^4)\cos{(dx+c)}^{11}\log{(\sin{(dx+c)}+1)}-3465\,(16\,a^5-20\,a^3b^2+3\,ab^4)\cos{(dx}}{}$$

input
```
integrate(sec(d*x+c)^12*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas
")
```

output
```
1/354816*(3465*(16*a^5 - 20*a^3*b^2 + 3*a*b^4)*cos(d*x + c)^11*log(sin(d*x
 + c) + 1) - 3465*(16*a^5 - 20*a^3*b^2 + 3*a*b^4)*cos(d*x + c)^11*log(-sin
(d*x + c) + 1) + 32256*b^5 + 50688*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x +
c)^4 + 78848*(5*a^2*b^3 - b^5)*cos(d*x + c)^2 + 462*(15*(16*a^5 - 20*a^3*b
^2 + 3*a*b^4)*cos(d*x + c)^9 + 10*(16*a^5 - 20*a^3*b^2 + 3*a*b^4)*cos(d*x
+ c)^7 + 384*a*b^4*cos(d*x + c) + 8*(16*a^5 - 20*a^3*b^2 + 3*a*b^4)*cos(d*
x + c)^5 + 48*(20*a^3*b^2 - 11*a*b^4)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos
(d*x + c)^11)
```

3.109.6 Sympy [F(-1)]

Timed out.

$$\int \sec^{12}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \text{Timed out}$$

input
```
integrate(sec(d*x+c)**12*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

output
```
Timed out
```

3.109.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 420, normalized size of antiderivative = 0.89

$$\int \sec^{12}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx =$$

$$-\frac{693\,ab^4\left(\frac{2\left(15\,\sin(dx+c)^9-70\,\sin(dx+c)^7+128\,\sin(dx+c)^5+70\,\sin(dx+c)^3-15\,\sin(dx+c)\right)}{\sin(dx+c)^{10}-5\,\sin(dx+c)^8+10\,\sin(dx+c)^6-10\,\sin(dx+c)^4+5\,\sin(dx+c)^2-1}-15\,\log\left(\sin\left(dx+c\right)+1\right)+15}{}$$

input
```
integrate(sec(d*x+c)^12*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima
")
```

output
```
-1/354816*(693*a*b^4*(2*(15*sin(d*x + c)^9 - 70*sin(d*x + c)^7 + 128*sin(d
*x + c)^5 + 70*sin(d*x + c)^3 - 15*sin(d*x + c))/(sin(d*x + c)^10 - 5*sin(
d*x + c)^8 + 10*sin(d*x + c)^6 - 10*sin(d*x + c)^4 + 5*sin(d*x + c)^2 - 1)
 - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) - 4620*a^3*b^2*(2*
(15*sin(d*x + c)^7 - 55*sin(d*x + c)^5 + 73*sin(d*x + c)^3 + 15*sin(d*x +
c))/(sin(d*x + c)^8 - 4*sin(d*x + c)^6 + 6*sin(d*x + c)^4 - 4*sin(d*x + c)
^2 + 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) + 3696*a^5*
(2*(15*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*sin(d*x + c))/(sin(d*x + c)
^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) +
 15*log(sin(d*x + c) - 1)) - 253440*a^4*b/cos(d*x + c)^7 + 56320*(9*cos(d*
x + c)^2 - 7)*a^2*b^3/cos(d*x + c)^9 - 512*(99*cos(d*x + c)^4 - 154*cos(d*
x + c)^2 + 63)*b^5/cos(d*x + c)^11)/d
```

3.109.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1096 vs. $2(430) = 860$.

Time = 0.65 (sec) , antiderivative size = 1096, normalized size of antiderivative = 2.32

$$\int \sec^{12}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^12*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

output
```
1/177408*(3465*(16*a^5 - 20*a^3*b^2 + 3*a*b^4)*log(abs(tan(1/2*d*x + 1/2*c
) + 1)) - 3465*(16*a^5 - 20*a^3*b^2 + 3*a*b^4)*log(abs(tan(1/2*d*x + 1/2*c
) - 1)) + 2*(121968*a^5*tan(1/2*d*x + 1/2*c)^21 + 69300*a^3*b^2*tan(1/2*d*
x + 1/2*c)^21 - 10395*a*b^4*tan(1/2*d*x + 1/2*c)^21 - 887040*a^4*b*tan(1/2
*d*x + 1/2*c)^20 - 591360*a^5*tan(1/2*d*x + 1/2*c)^19 + 1626240*a^3*b^2*ta
n(1/2*d*x + 1/2*c)^19 + 110880*a*b^4*tan(1/2*d*x + 1/2*c)^19 + 3548160*a^4
*b*tan(1/2*d*x + 1/2*c)^18 - 3548160*a^2*b^3*tan(1/2*d*x + 1/2*c)^18 + 145
9920*a^5*tan(1/2*d*x + 1/2*c)^17 - 1159620*a^3*b^2*tan(1/2*d*x + 1/2*c)^17
 + 2302839*a*b^4*tan(1/2*d*x + 1/2*c)^17 - 9757440*a^4*b*tan(1/2*d*x + 1/2
*c)^16 + 1182720*a^2*b^3*tan(1/2*d*x + 1/2*c)^16 - 946176*b^5*tan(1/2*d*x
+ 1/2*c)^16 - 2365440*a^5*tan(1/2*d*x + 1/2*c)^15 + 1182720*a^3*b^2*tan(1/
2*d*x + 1/2*c)^15 + 4790016*a*b^4*tan(1/2*d*x + 1/2*c)^15 + 21288960*a^4*b
*tan(1/2*d*x + 1/2*c)^14 - 9461760*a^2*b^3*tan(1/2*d*x + 1/2*c)^14 - 23654
40*b^5*tan(1/2*d*x + 1/2*c)^14 + 2106720*a^5*tan(1/2*d*x + 1/2*c)^13 - 573
8040*a^3*b^2*tan(1/2*d*x + 1/2*c)^13 + 5828130*a*b^4*tan(1/2*d*x + 1/2*c)^
13 - 30159360*a^4*b*tan(1/2*d*x + 1/2*c)^12 + 18923520*a^2*b^3*tan(1/2*d*x
 + 1/2*c)^12 - 5203968*b^5*tan(1/2*d*x + 1/2*c)^12 + 28385280*a^4*b*tan(1/
2*d*x + 1/2*c)^10 - 7096320*a^2*b^3*tan(1/2*d*x + 1/2*c)^10 - 4257792*b^5*
tan(1/2*d*x + 1/2*c)^10 - 2106720*a^5*tan(1/2*d*x + 1/2*c)^9 + 5738040*a^3
*b^2*tan(1/2*d*x + 1/2*c)^9 - 5828130*a*b^4*tan(1/2*d*x + 1/2*c)^9 - 20...
```

3.109.9 Mupad [B] (verification not implemented)

Time = 29.27 (sec) , antiderivative size = 831, normalized size of antiderivative = 1.76

$$\int \sec^{12}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \text{Too large to display}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))^5/cos(c + d*x)^12,x)
```

output
```
(5*a*atanh(tan(c/2 + (d*x)/2))*(16*a^4 + 3*b^4 - 20*a^2*b^2))/(128*d) - (t
an(c/2 + (d*x)/2)*((11*a^5)/8 - (15*a*b^4)/128 + (25*a^3*b^2)/32) - tan(c/
2 + (d*x)/2)^18*(40*a^4*b - 40*a^2*b^3) + tan(c/2 + (d*x)/2)^3*((5*a*b^4)/
4 - (20*a^5)/3 + (55*a^3*b^2)/3) - tan(c/2 + (d*x)/2)^19*((5*a*b^4)/4 - (2
0*a^5)/3 + (55*a^3*b^2)/3) + tan(c/2 + (d*x)/2)^7*(54*a*b^4 - (80*a^5)/3 +
 (40*a^3*b^2)/3) - tan(c/2 + (d*x)/2)^15*(54*a*b^4 - (80*a^5)/3 + (40*a^3*
b^2)/3) - tan(c/2 + (d*x)/2)^21*((11*a^5)/8 - (15*a*b^4)/128 + (25*a^3*b^2
)/32) + tan(c/2 + (d*x)/2)^5*((3323*a*b^4)/128 + (395*a^5)/24 - (1255*a^3*
b^2)/96) - tan(c/2 + (d*x)/2)^17*((3323*a*b^4)/128 + (395*a^5)/24 - (1255*
a^3*b^2)/96) + tan(c/2 + (d*x)/2)^9*((4205*a*b^4)/64 + (95*a^5)/4 - (1035*
a^3*b^2)/16) - tan(c/2 + (d*x)/2)^13*((4205*a*b^4)/64 + (95*a^5)/4 - (1035
*a^3*b^2)/16) + tan(c/2 + (d*x)/2)^16*(110*a^4*b + (32*b^5)/3 - (40*a^2*b^
3)/3) + tan(c/2 + (d*x)/2)^10*(48*b^5 - 320*a^4*b + 80*a^2*b^3) - tan(c/2
+ (d*x)/2)^2*((40*a^4*b)/7 + (16*b^5)/63 - (440*a^2*b^3)/63) + tan(c/2 + (
d*x)/2)^14*((80*b^5)/3 - 240*a^4*b + (320*a^2*b^3)/3) + tan(c/2 + (d*x)/2)
^4*((270*a^4*b)/7 + (80*b^5)/63 + (320*a^2*b^3)/63) + tan(c/2 + (d*x)/2)^1
2*(340*a^4*b + (176*b^5)/3 - (640*a^2*b^3)/3) + tan(c/2 + (d*x)/2)^6*((48*
b^5)/7 - (880*a^4*b)/7 + (640*a^2*b^3)/7) + tan(c/2 + (d*x)/2)^8*((1620*a^
4*b)/7 + (240*b^5)/7 - (720*a^2*b^3)/7) + (10*a^4*b)/7 + (16*b^5)/693 - (4
0*a^2*b^3)/63 + 10*a^4*b*tan(c/2 + (d*x)/2)^20)/(d*(11*tan(c/2 + (d*x)/...
```

3.109. $\int \sec^{12}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

3.110 $\int \frac{\cos^5(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

3.110.1 Optimal result

Integrand size = 28, antiderivative size = 227

$$\begin{aligned}\int \frac{\cos^5(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx &= \frac{ab^4x}{(a^2+b^2)^3}+\frac{ab^2x}{2(a^2+b^2)^2}+\frac{3ax}{8(a^2+b^2)}\\ &+\frac{b^3\cos^2(c+dx)}{2(a^2+b^2)^2 d}+\frac{b\cos^4(c+dx)}{4(a^2+b^2)d}\\ &+\frac{b^5\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^3 d}\\ &+\frac{ab^2\cos(c+dx)\sin(c+dx)}{2(a^2+b^2)^2 d}\\ &+\frac{3a\cos(c+dx)\sin(c+dx)}{8(a^2+b^2)d}\\ &+\frac{a\cos^3(c+dx)\sin(c+dx)}{4(a^2+b^2)d}\end{aligned}$$

output
```
a*b^4*x/(a^2+b^2)^3+1/2*a*b^2*x/(a^2+b^2)^2+3/8*a*x/(a^2+b^2)+1/2*b^3*cos(
d*x+c)^2/(a^2+b^2)^2/d+1/4*b*cos(d*x+c)^4/(a^2+b^2)/d+b^5*ln(a*cos(d*x+c)+
b*sin(d*x+c))/(a^2+b^2)^3/d+1/2*a*b^2*cos(d*x+c)*sin(d*x+c)/(a^2+b^2)^2/d+
3/8*a*cos(d*x+c)*sin(d*x+c)/(a^2+b^2)/d+1/4*a*cos(d*x+c)^3*sin(d*x+c)/(a^2
+b^2)/d
```

3.110.2 Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.96

$$\int \frac{\cos^5(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$$
$$= \frac{12a^5c+40a^3b^2c+60ab^4c+12a^5dx+40a^3b^2dx+60ab^4dx+4b(a^4+4a^2b^2+3b^4)\cos(2(c+dx))+b(a^2}{}$$

input
```
Integrate[Cos[c + d*x]^5/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

output
```
(12*a^5*c + 40*a^3*b^2*c + 60*a*b^4*c + 12*a^5*d*x + 40*a^3*b^2*d*x + 60*a
*b^4*d*x + 4*b*(a^4 + 4*a^2*b^2 + 3*b^4)*Cos[2*(c + d*x)] + b*(a^2 + b^2)^
2*Cos[4*(c + d*x)] + 32*b^5*Log[a*Cos[c + d*x] + b*Sin[c + d*x]] + 8*a^5*S
in[2*(c + d*x)] + 24*a^3*b^2*Sin[2*(c + d*x)] + 16*a*b^4*Sin[2*(c + d*x)]
+ a^5*Sin[4*(c + d*x)] + 2*a^3*b^2*Sin[4*(c + d*x)] + a*b^4*Sin[4*(c + d*x
)])/(32*(a^2 + b^2)^3*d)
```

3.110.3 Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.97, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3579, 3042, 3115, 3042, 3115, 24, 3579, 3042, 3115, 24, 3577, 3042, 3612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^5(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos(c+dx)^5}{a\cos(c+dx)+b\sin(c+dx)}dx$$
$$\Big\downarrow \text{3579}$$
$$\frac{a\int \cos^4(c+dx)dx}{a^2+b^2}+\frac{b^2\int \frac{\cos^3(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}+\frac{b\cos^4(c+dx)}{4d\,(a^2+b^2)}$$
$$\Big\downarrow \text{3042}$$

$$\frac{a\int \sin\left(c+dx+\frac{\pi}{2}\right)^4dx}{a^2+b^2}+\frac{b^2\int \frac{\cos(c+dx)^3}{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}+\frac{b\cos^4(c+dx)}{4d\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{3115}$$

$$\frac{a\left(\frac{3}{4}\int \cos^2(c+dx)dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{a^2+b^2}+\frac{b^2\int \frac{\cos(c+dx)^3}{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}+\frac{b\cos^4(c+dx)}{4d\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\left(\frac{3}{4}\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{a^2+b^2}+\frac{b^2\int \frac{\cos(c+dx)^3}{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}+\frac{b\cos^4(c+dx)}{4d\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{3115}$$

$$\frac{a\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\sin(c+dx)\cos^3(c+dx)}{4d}\right)}{a^2+b^2}+\frac{b^2\int \frac{\cos(c+dx)^3}{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}+\frac{b\cos^4(c+dx)}{4d\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{24}$$

$$\frac{b^2\int \frac{\cos(c+dx)^3}{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}+\frac{b\cos^4(c+dx)}{4d\left(a^2+b^2\right)}+\frac{a\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)}{a^2+b^2}$$

$$\Big\downarrow \text{3579}$$

$$\frac{b^2\left(\frac{a\int \cos^2(c+dx)dx}{a^2+b^2}+\frac{b^2\int \frac{\cos(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}+\frac{b\cos^2(c+dx)}{2d(a^2+b^2)}\right)}{a^2+b^2}+\frac{b\cos^4(c+dx)}{4d\left(a^2+b^2\right)}+\frac{a\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2\left(\frac{a\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx}{a^2+b^2}+\frac{b^2\int \frac{\cos(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}+\frac{b\cos^2(c+dx)}{2d(a^2+b^2)}\right)}{a^2+b^2}+\frac{b\cos^4(c+dx)}{4d\left(a^2+b^2\right)}+\frac{a\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)}{a^2+b^2}$$

$$\Big\downarrow \text{3115}$$

3.110. $\int \frac{\cos^5(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

$$\frac{b^2\left(\frac{b^2\int\frac{\cos(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}+\frac{a\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{a^2+b^2}+\frac{b\cos^2(c+dx)}{2d(a^2+b^2)}\right)}{a^2+b^2}+\frac{b\cos^4(c+dx)}{4d\left(a^2+b^2\right)}+\frac{a\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)}{a^2+b^2}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{b^2\left(\frac{b^2\int\frac{\cos(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}+\frac{b\cos^2(c+dx)}{2d(a^2+b^2)}+\frac{a\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\cos^4(c+dx)}{4d\left(a^2+b^2\right)}+\frac{a\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)}{a^2+b^2}$$

$$\Big\downarrow \text{ 3577}$$

$$\frac{b^2\left(\frac{b^2\left(\frac{b\int\frac{b\cos(c+dx)-a\sin(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}+\frac{ax}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\cos^2(c+dx)}{2d(a^2+b^2)}+\frac{a\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\cos^4(c+dx)}{4d\left(a^2+b^2\right)}+\frac{a\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)}{a^2+b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b^2\left(\frac{b^2\left(\frac{b\int\frac{b\cos(c+dx)-a\sin(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}+\frac{ax}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\cos^2(c+dx)}{2d(a^2+b^2)}+\frac{a\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\cos^4(c+dx)}{4d\left(a^2+b^2\right)}+\frac{a\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)}{a^2+b^2}$$

$$\Big\downarrow \text{ 3612}$$

$$\frac{b\cos^4(c+dx)}{4d\left(a^2+b^2\right)}+\frac{a\left(\frac{\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3}{4}\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)\right)}{a^2+b^2}+\frac{b^2\left(\frac{b\cos^2(c+dx)}{2d(a^2+b^2)}+\frac{a\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)}{a^2+b^2}+\frac{b^2\left(\frac{b\log(a\cos(c+dx)+b\sin(c+dx))}{d\left(a^2+b^2\right)}+\frac{ax}{a^2+b^2}\right)}{a^2+b^2}\right)}{a^2+b^2}$$

3.110. $\int\frac{\cos^5(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

input
```
Int[Cos[c + d*x]^5/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

output
```
(b*Cos[c + d*x]^4)/(4*(a^2 + b^2)*d) + (a*((Cos[c + d*x]^3*Sin[c + d*x])/(
4*d) + (3*(x/2 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/4))/(a^2 + b^2) + (b^
2*((b*Cos[c + d*x]^2)/(2*(a^2 + b^2)*d) + (b^2*((a*x)/(a^2 + b^2) + (b*Log
[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)*d)))/(a^2 + b^2) + (a*(x/2
 + (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/(a^2 + b^2)))/(a^2 + b^2)
```

3.110.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3577
```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[a*(x/(a^2 + b^2)), x] + Simp[b/(a^2 + b
^2)   Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x
]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3579
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(Cos[c + d*x]^(m - 1)/(d*(a^2 +
b^2)*(m - 1))), x] + (Simp[a/(a^2 + b^2)   Int[Cos[c + d*x]^(m - 1), x], x]
 + Simp[b^2/(a^2 + b^2)   Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[
c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m,
 1]
```

rule 3612
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

3.110.4 Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.87

method	result
derivativedivides	$\frac{\frac{\frac{\left(\frac{3}{8}a^5+\frac{5}{4}a^3b^2+\frac{7}{8}ab^4\right)\tan(dx+c)^3+\left(\frac{1}{2}a^2b^3+\frac{1}{2}b^5\right)\tan(dx+c)^2+\left(\frac{7}{4}a^3b^2+\frac{9}{8}ab^4+\frac{5}{8}a^5\right)\tan(dx+c)+\frac{a^4b}{4}+a^2b^3+\frac{3b^5}{4}}{\left(1+\tan(dx+c)^2\right)^2}-\frac{b^5\ln\left(1+\tan\cdots\right.}{2\cdots}}{\left(a^2+b^2\right)^3}}{d}$
default	$\frac{\frac{\frac{\left(\frac{3}{8}a^5+\frac{5}{4}a^3b^2+\frac{7}{8}ab^4\right)\tan(dx+c)^3+\left(\frac{1}{2}a^2b^3+\frac{1}{2}b^5\right)\tan(dx+c)^2+\left(\frac{7}{4}a^3b^2+\frac{9}{8}ab^4+\frac{5}{8}a^5\right)\tan(dx+c)+\frac{a^4b}{4}+a^2b^3+\frac{3b^5}{4}}{\left(1+\tan(dx+c)^2\right)^2}-\frac{b^5\ln\left(1+\tan\cdots\right.}{2\cdots}}{\left(a^2+b^2\right)^3}}{d}$
parallelrisch	$\frac{32b^5\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)-32b^5\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)+4\left(a^4b+4a^2b^3+3b^5\right)\cos(2dx+2c)+b\left(a^2+b^2\right)^2c\cdots}{32d\cdots}$
risch	$\frac{9xab}{8ia^3-24iab^2+24a^2b-8b^3}+\frac{3ixa^2}{8ia^3-24iab^2+24a^2b-8b^3}-\frac{8ixb^2}{8ia^3-24iab^2+24a^2b-8b^3}-\frac{3e^{2i(dx+c)}b}{16(-2iba+a^2-b^2)d}-\frac{\cdots}{8(\cdots}$
norman	$\frac{\left(-2a^2b-4b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d\left(a^4+2a^2b^2+b^4\right)}+\frac{\left(-2a^2b-4b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{d\left(a^4+2a^2b^2+b^4\right)}+\frac{a\left(3a^4+10a^2b^2+15b^4\right)x}{8a^6+24a^4b^2+24a^2b^4+8b^6}+\frac{2\left(-a^2b-4b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{d\left(a^4+2a^2b^2+b^4\right)}+\frac{2\left(-a^2b-\cdots\right.}{d\left(a\cdots\right.}$

input
```
int(cos(d*x+c)^5/(cos(d*x+c)*a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/(a^2+b^2)^3*(((3/8*a^5+5/4*a^3*b^2+7/8*a*b^4)*tan(d*x+c)^3+(1/2*a^2
*b^3+1/2*b^5)*tan(d*x+c)^2+(7/4*a^3*b^2+9/8*a*b^4+5/8*a^5)*tan(d*x+c)+1/4*
a^4*b+a^2*b^3+3/4*b^5)/(1+tan(d*x+c)^2)^2-1/2*b^5*ln(1+tan(d*x+c)^2)+1/8*(
3*a^5+10*a^3*b^2+15*a*b^4)*arctan(tan(d*x+c)))+b^5/(a^2+b^2)^3*ln(a+b*tan(
d*x+c)))
```

3.110.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.92

$$\int \frac{\cos^5(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \frac{4\,b^5\log\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)+2\left(a^4b+2\,a^2b^3+b^5\right)\cos\left(dx+c\right)^4}{}$$

input
```
integrate(cos(d*x+c)^5/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/8*(4*b^5*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^
2 + b^2) + 2*(a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c)^4 + (3*a^5 + 10*a^3*b^
2 + 15*a*b^4)*d*x + 4*(a^2*b^3 + b^5)*cos(d*x + c)^2 + (2*(a^5 + 2*a^3*b^2
 + a*b^4)*cos(d*x + c)^3 + (3*a^5 + 10*a^3*b^2 + 7*a*b^4)*cos(d*x + c))*si
n(d*x + c))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d)
```

3.110.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^5(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**5/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

output
```
Timed out
```

3.110.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 564 vs. 2(213) = 426.

Time = 0.34 (sec) , antiderivative size = 564, normalized size of antiderivative = 2.48

$$\int \frac{\cos^5(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \frac{\frac{4\,b^5\log\left(-a-\frac{2\,b\sin(dx+c)}{\cos(dx+c)+1}+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{4\,b^5\log\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{\left(3\,a^5+10\,a^3b^2+15\,ab^4\right)\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{16\,b^3\sin(dx+}{(\cos(dx+c)+}}{}$$

input
```
integrate(cos(d*x+c)^5/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
1/4*(4*b^5*log(-a - 2*b*sin(d*x + c)/(cos(d*x + c) + 1) + a*sin(d*x + c)^2
/(cos(d*x + c) + 1)^2)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 4*b^5*log(sin
(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
+ (3*a^5 + 10*a^3*b^2 + 15*a*b^4)*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/
(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (16*b^3*sin(d*x + c)^4/(cos(d*x + c)
 + 1)^4 - (5*a^3 + 9*a*b^2)*sin(d*x + c)/(cos(d*x + c) + 1) + 8*(a^2*b + 2
*b^3)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + (3*a^3 - a*b^2)*sin(d*x + c)^3
/(cos(d*x + c) + 1)^3 - (3*a^3 - a*b^2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^
5 + 8*(a^2*b + 2*b^3)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + (5*a^3 + 9*a*b
^2)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/(a^4 + 2*a^2*b^2 + b^4 + 4*(a^4 +
 2*a^2*b^2 + b^4)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*(a^4 + 2*a^2*b^2
 + b^4)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*si
n(d*x + c)^6/(cos(d*x + c) + 1)^6 + (a^4 + 2*a^2*b^2 + b^4)*sin(d*x + c)^8
/(cos(d*x + c) + 1)^8))/d
```

3.110.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.42

$$\int \frac{\cos^5(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \frac{\frac{8\,b^6\log(|b\tan(dx+c)+a|)}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7} - \frac{4\,b^5\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{(3\,a^5+10\,a^3b^2+15\,ab^4)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{6\,b^5\tan(dx+c)^4+3\,a^5\tan(dx+c)^3+10\,a^3b^2\tan(dx}{}}{8\,d}$$

input
```
integrate(cos(d*x+c)^5/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
1/8*(8*b^6*log(abs(b*tan(d*x + c) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b
^7) - 4*b^5*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) +
(3*a^5 + 10*a^3*b^2 + 15*a*b^4)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b
^6) + (6*b^5*tan(d*x + c)^4 + 3*a^5*tan(d*x + c)^3 + 10*a^3*b^2*tan(d*x +
c)^3 + 7*a*b^4*tan(d*x + c)^3 + 4*a^2*b^3*tan(d*x + c)^2 + 16*b^5*tan(d*x
+ c)^2 + 5*a^5*tan(d*x + c) + 14*a^3*b^2*tan(d*x + c) + 9*a*b^4*tan(d*x +
c) + 2*a^4*b + 8*a^2*b^3 + 12*b^5)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*(t
an(d*x + c)^2 + 1)^2))/d
```

3.110.9 Mupad [B] (verification not implemented)

Time = 35.84 (sec) , antiderivative size = 6099, normalized size of antiderivative = 26.87

$$\int \frac{\cos^5(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(cos(c + d*x)^5/(a*cos(c + d*x) + b*sin(c + d*x)),x)
```

output
```
(b^5*log(a + 2*b*tan(c/2 + (d*x)/2) - a*tan(c/2 + (d*x)/2)^2))/(d*(a^6 + b
^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (64*b^5*log(1/(cos(c + d*x) + 1)))/(d*(64*a
^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2)) - ((4*b^3*tan(c/2 + (d*x)/2)^4)/
(a^4 + b^4 + 2*a^2*b^2) - (tan(c/2 + (d*x)/2)*(9*a*b^2 + 5*a^3))/(4*(a^4 +
 b^4 + 2*a^2*b^2)) - (tan(c/2 + (d*x)/2)^3*(a*b^2 - 3*a^3))/(4*(a^4 + b^4
+ 2*a^2*b^2)) + (tan(c/2 + (d*x)/2)^5*(a*b^2 - 3*a^3))/(4*(a^4 + b^4 + 2*a
^2*b^2)) + (tan(c/2 + (d*x)/2)^7*(9*a*b^2 + 5*a^3))/(4*(a^4 + b^4 + 2*a^2*
b^2)) + (2*b*tan(c/2 + (d*x)/2)^2*(a^2 + 2*b^2))/(a^4 + b^4 + 2*a^2*b^2) +
 (2*b*tan(c/2 + (d*x)/2)^6*(a^2 + 2*b^2))/(a^4 + b^4 + 2*a^2*b^2))/(d*(4*t
an(c/2 + (d*x)/2)^2 + 6*tan(c/2 + (d*x)/2)^4 + 4*tan(c/2 + (d*x)/2)^6 + ta
n(c/2 + (d*x)/2)^8 + 1)) - (a*atan((tan(c/2 + (d*x)/2)*((((64*b^5*((a*((64
*a*b^15 + 48*a^15*b + 624*a^3*b^13 + 2016*a^5*b^11 + 3152*a^7*b^9 + 2688*a
^9*b^7 + 1296*a^11*b^5 + 352*a^13*b^3)/(2*(a^12 + b^12 + 6*a^2*b^10 + 15*a
^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)) - (32*b^5*(192*a*b^16 + 13
44*a^3*b^14 + 4032*a^5*b^12 + 6720*a^7*b^10 + 6720*a^9*b^8 + 4032*a^11*b^6
 + 1344*a^13*b^4 + 192*a^15*b^2))/((64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^
4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 +
6*a^10*b^2)))*(3*a^4 + 15*b^4 + 10*a^2*b^2))/(8*(a^6 + b^6 + 3*a^2*b^4 + 3
*a^4*b^2)) - (4*a*b^5*(3*a^4 + 15*b^4 + 10*a^2*b^2)*(192*a*b^16 + 1344*a^3
*b^14 + 4032*a^5*b^12 + 6720*a^7*b^10 + 6720*a^9*b^8 + 4032*a^11*b^6 + ...
```

3.111 $\int \frac{\cos^4(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

3.111.1 Optimal result

Integrand size = 28, antiderivative size = 166

$$\int \frac{\cos^4(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = -\frac{b^4\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}\,d} + \frac{b^3\cos(c+dx)}{(a^2+b^2)^2\,d} + \frac{b\cos^3(c+dx)}{3\,(a^2+b^2)\,d} + \frac{ab^2\sin(c+dx)}{(a^2+b^2)^2\,d} + \frac{a\sin(c+dx)}{(a^2+b^2)\,d} - \frac{a\sin^3(c+dx)}{3\,(a^2+b^2)\,d}$$

output
```
-b^4*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2)/
d+b^3*cos(d*x+c)/(a^2+b^2)^2/d+1/3*b*cos(d*x+c)^3/(a^2+b^2)/d+a*b^2*sin(d*
x+c)/(a^2+b^2)^2/d+a*sin(d*x+c)/(a^2+b^2)/d-1/3*a*sin(d*x+c)^3/(a^2+b^2)/d
```

3.111.2 Mathematica [A] (verified)

Time = 1.47 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.83

$$\int \frac{\cos^4(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \frac{24b^4\operatorname{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right) + \sqrt{a^2+b^2}(3b(a^2+5b^2)\cos(c+dx) + b(a^2+b^2)\cos(3(c+dx)) + 2a(5a^2}{12\,(a^2+b^2)^{5/2}\,d}$$

input
```
Integrate[Cos[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

output
```
(24*b^4*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] + Sqrt[a^2 + b^
2]*(3*b*(a^2 + 5*b^2)*Cos[c + d*x] + b*(a^2 + b^2)*Cos[3*(c + d*x)] + 2*a*
(5*a^2 + 11*b^2 + (a^2 + b^2)*Cos[2*(c + d*x)])*Sin[c + d*x]))/(12*(a^2 +
b^2)^(5/2)*d)
```

3.111.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3579, 3042, 3113, 2009, 3579, 3042, 3117, 3553, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(c+dx)^4}{a\cos(c+dx)+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{3579}$$

$$\frac{a\int \cos^3(c+dx)dx}{a^2+b^2}+\frac{b^2\int \frac{\cos^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}+\frac{b\cos^3(c+dx)}{3d\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \sin\left(c+dx+\frac{\pi}{2}\right)^3dx}{a^2+b^2}+\frac{b^2\int \frac{\cos(c+dx)^2}{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}+\frac{b\cos^3(c+dx)}{3d\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{3113}$$

$$-\frac{a\int \left(1-\sin^2(c+dx)\right)d(-\sin(c+dx))}{d\left(a^2+b^2\right)}+\frac{b^2\int \frac{\cos(c+dx)^2}{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}+\frac{b\cos^3(c+dx)}{3d\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^2\int \frac{\cos(c+dx)^2}{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d\left(a^2+b^2\right)}+\frac{b\cos^3(c+dx)}{3d\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{3579}$$

$$\frac{b^2\left(\frac{a\int \cos(c+dx)dx}{a^2+b^2}+\frac{b^2\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}+\frac{b\cos(c+dx)}{d(a^2+b^2)}\right)}{a^2+b^2}-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d\left(a^2+b^2\right)}+\frac{b\cos^3(c+dx)}{3d\left(a^2+b^2\right)}$$

$\big\downarrow$ 3042

$$\frac{b^2\left(\frac{a\int \sin\left(c+dx+\frac{\pi}{2}\right)dx}{a^2+b^2}+\frac{b^2\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}+\frac{b\cos(c+dx)}{d(a^2+b^2)}\right)}{a^2+b^2}-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d\left(a^2+b^2\right)}+\frac{b\cos^3(c+dx)}{3d\left(a^2+b^2\right)}$$

$\big\downarrow$ 3117

$$\frac{b^2\left(\frac{b^2\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}+\frac{a\sin(c+dx)}{d(a^2+b^2)}+\frac{b\cos(c+dx)}{d(a^2+b^2)}\right)}{a^2+b^2}-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d\left(a^2+b^2\right)}+\frac{b\cos^3(c+dx)}{3d\left(a^2+b^2\right)}$$

$\big\downarrow$ 3553

$$\frac{b^2\left(-\frac{b^2\int \frac{1}{a^2+b^2-(b\cos(c+dx)-a\sin(c+dx))^2}d(b\cos(c+dx)-a\sin(c+dx))}{d(a^2+b^2)}+\frac{a\sin(c+dx)}{d(a^2+b^2)}+\frac{b\cos(c+dx)}{d(a^2+b^2)}\right)}{a^2+b^2}-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d\left(a^2+b^2\right)}+\frac{b\cos^3(c+dx)}{3d\left(a^2+b^2\right)}$$

$\big\downarrow$ 219

$$\frac{b^2\left(-\frac{b^2\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{d(a^2+b^2)^{3/2}}+\frac{a\sin(c+dx)}{d(a^2+b^2)}+\frac{b\cos(c+dx)}{d(a^2+b^2)}\right)}{a^2+b^2}-\frac{a\left(\frac{1}{3}\sin^3(c+dx)-\sin(c+dx)\right)}{d\left(a^2+b^2\right)}+\frac{b\cos^3(c+dx)}{3d\left(a^2+b^2\right)}$$

input
```
Int[Cos[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

output
```
(b*Cos[c + d*x]^3)/(3*(a^2 + b^2)*d) + (b^2*(-((b^2*ArcTanh[(b*Cos[c + d*x
] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/((a^2 + b^2)^(3/2)*d)) + (b*Cos[c +
d*x])/((a^2 + b^2)*d) + (a*Sin[c + d*x])/((a^2 + b^2)*d)))/(a^2 + b^2) - (
a*(-Sin[c + d*x] + Sin[c + d*x]^3/3))/((a^2 + b^2)*d)
```

3.111. $\int \frac{\cos^4(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

3.111.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3579
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(Cos[c + d*x]^(m - 1)/(d*(a^2 +
b^2)*(m - 1))), x] + (Simp[a/(a^2 + b^2)   Int[Cos[c + d*x]^(m - 1), x], x]
 + Simp[b^2/(a^2 + b^2)   Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[
c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m,
 1]
```

3.111. $\int \frac{\cos^4(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

3.111.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.33

method	result
derivativedivides	$\frac{-\frac{2\left(\left(-a^3-2a\,b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5+\left(-a^2b-2b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\left(-\frac{2}{3}a^3-\frac{8}{3}a\,b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b^3+\left(-a^3-2a\,b^2\right)\tan}{\left(a^4+2a^2b^2+b^4\right)\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}}{d}$
default	$\frac{-\frac{2\left(\left(-a^3-2a\,b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5+\left(-a^2b-2b^3\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\left(-\frac{2}{3}a^3-\frac{8}{3}a\,b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b^3+\left(-a^3-2a\,b^2\right)\tan}{\left(a^4+2a^2b^2+b^4\right)\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}}{d}$
risch	$-\frac{5\,e^{i(dx+c)}b}{8(-2iba+a^2-b^2)d}-\frac{3ie^{i(dx+c)}a}{8(-2iba+a^2-b^2)d}-\frac{5\,e^{-i(dx+c)}b}{8(ib+a)^2d}+\frac{3ie^{-i(dx+c)}a}{8(ib+a)^2d}+\frac{b^4\ln\left(e^{i(dx+c)}+\frac{ia^5+2ia^3b^2+ia\,b^4-}{\left(a^2+b^2\right.}\right.}{\left(a^2+b^2\right)^{\frac{5}{2}}d}$

input
```
int(cos(d*x+c)^4/(cos(d*x+c)*a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/(a^4+2*a^2*b^2+b^4)*((-a^3-2*a*b^2)*tan(1/2*d*x+1/2*c)^5+(-a^2*b-2
*b^3)*tan(1/2*d*x+1/2*c)^4+(-2/3*a^3-8/3*a*b^2)*tan(1/2*d*x+1/2*c)^3-2*tan
(1/2*d*x+1/2*c)^2*b^3+(-a^3-2*a*b^2)*tan(1/2*d*x+1/2*c)-1/3*a^2*b-4/3*b^3)
/(1+tan(1/2*d*x+1/2*c)^2)^3+2*b^4/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)^(1/2)*arct
anh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

3.111.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.58

$$\int \frac{\cos^4(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \frac{3\sqrt{a^2+b^2}b^4\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2}\right)+2\,(a^4b+2\,a}{6\,(a^6}$$

input
```
integrate(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/6*(3*sqrt(a^2 + b^2)*b^4*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 -
b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*
sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)
^2 + b^2)) + 2*(a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c)^3 + 6*(a^2*b^3 + b^5
)*cos(d*x + c) + 2*(2*a^5 + 7*a^3*b^2 + 5*a*b^4 + (a^5 + 2*a^3*b^2 + a*b^4
)*cos(d*x + c)^2)*sin(d*x + c))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d)
```

3.111.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**4/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

output
```
Timed out
```

3.111.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(158) = 316$.

Time = 0.32 (sec) , antiderivative size = 379, normalized size of antiderivative = 2.28

$$\int \frac{\cos^4(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = -\frac{\frac{3\,b^4\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}} - \frac{2\left(a^2b+4\,b^3+\frac{6\,b^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\left(a^3+2\,ab^2\right)\sin(dx+c)}{\cos(dx+c)+1}+\frac{2\left(a^3+4\,ab^2\right)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\left(a^2b+2\,b^3\right)\sin(dx+c}{(\cos(dx+c)+1)^4}\cdots\right.}{a^4+2\,a^2b^2+b^4+\frac{3\left(a^4+2\,a^2b^2+b^4\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\left(a^4+2\,a^2b^2+b^4\right)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\left(a^4+2\,a^2b^2\cdots\right.}{(\cos(\cdots}}}{3\,d}$$

input
```
integrate(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/3*(3*b^4*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/
(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b
^2 + b^4)*sqrt(a^2 + b^2)) - 2*(a^2*b + 4*b^3 + 6*b^3*sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 + 3*(a^3 + 2*a*b^2)*sin(d*x + c)/(cos(d*x + c) + 1) + 2*(a
^3 + 4*a*b^2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*(a^2*b + 2*b^3)*sin(
d*x + c)^4/(cos(d*x + c) + 1)^4 + 3*(a^3 + 2*a*b^2)*sin(d*x + c)^5/(cos(d*
x + c) + 1)^5)/(a^4 + 2*a^2*b^2 + b^4 + 3*(a^4 + 2*a^2*b^2 + b^4)*sin(d*x
+ c)^2/(cos(d*x + c) + 1)^2 + 3*(a^4 + 2*a^2*b^2 + b^4)*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 + (a^4 + 2*a^2*b^2 + b^4)*sin(d*x + c)^6/(cos(d*x + c) +
 1)^6))/d
```

3.111.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.72

$$\int \frac{\cos^4(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx =$$

$$-\frac{\frac{3\,b^4\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}}-\frac{2\left(3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+6\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4 \cdots}{(a^4 \cdots}}{3\,d}$$

input
```
integrate(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
-1/3*(3*b^4*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/ab
s(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 +
 b^4)*sqrt(a^2 + b^2)) - 2*(3*a^3*tan(1/2*d*x + 1/2*c)^5 + 6*a*b^2*tan(1/2
*d*x + 1/2*c)^5 + 3*a^2*b*tan(1/2*d*x + 1/2*c)^4 + 6*b^3*tan(1/2*d*x + 1/2
*c)^4 + 2*a^3*tan(1/2*d*x + 1/2*c)^3 + 8*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*
b^3*tan(1/2*d*x + 1/2*c)^2 + 3*a^3*tan(1/2*d*x + 1/2*c) + 6*a*b^2*tan(1/2*
d*x + 1/2*c) + a^2*b + 4*b^3)/((a^4 + 2*a^2*b^2 + b^4)*(tan(1/2*d*x + 1/2*
c)^2 + 1)^3))/d
```

3.111.9 Mupad [B] (verification not implemented)

Time = 26.02 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.06

$$\int \frac{\cos^4(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$$
$$= \frac{\frac{\frac{2a^2b}{3}+\frac{8b^3}{3}}{a^4+2a^2b^2+b^4}+\frac{4b^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{a^4+2a^2b^2+b^4}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5(2a^3+4ab^2)}{a^4+2a^2b^2+b^4}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{4a^3}{3}+\frac{16ab^2}{3}\right)}{a^4+2a^2b^2+b^4}+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(a^3+2ab^2)}{a^4+2a^2b^2+b^4}+\frac{2b\,\mathrm{t}\cdots}{\cdots}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}$$
$$-\frac{2b^4\,\mathrm{atanh}\left(\frac{a^4b+b^5+2a^2b^3-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(a^4+2a^2b^2+b^4)}{(a^2+b^2)^{5/2}}\right)}{d\,(a^2+b^2)^{5/2}}$$

input
```
int(cos(c + d*x)^4/(a*cos(c + d*x) + b*sin(c + d*x)),x)
```

output
```
(((2*a^2*b)/3 + (8*b^3)/3)/(a^4 + b^4 + 2*a^2*b^2) + (4*b^3*tan(c/2 + (d*x
)/2)^2)/(a^4 + b^4 + 2*a^2*b^2) + (tan(c/2 + (d*x)/2)^5*(4*a*b^2 + 2*a^3))
/(a^4 + b^4 + 2*a^2*b^2) + (tan(c/2 + (d*x)/2)^3*((16*a*b^2)/3 + (4*a^3)/3
))/(a^4 + b^4 + 2*a^2*b^2) + (2*tan(c/2 + (d*x)/2)*(2*a*b^2 + a^3))/(a^4 +
 b^4 + 2*a^2*b^2) + (2*b*tan(c/2 + (d*x)/2)^4*(a^2 + 2*b^2))/(a^4 + b^4 +
2*a^2*b^2))/(d*(3*tan(c/2 + (d*x)/2)^2 + 3*tan(c/2 + (d*x)/2)^4 + tan(c/2
+ (d*x)/2)^6 + 1)) - (2*b^4*atanh((a^4*b + b^5 + 2*a^2*b^3 - a*tan(c/2 + (
d*x)/2)*(a^4 + b^4 + 2*a^2*b^2))/(a^2 + b^2)^(5/2)))/(d*(a^2 + b^2)^(5/2))
```

3.112 $\int \frac{\cos^3(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

3.112.1 Optimal result

Integrand size = 28, antiderivative size = 119

$$\int \frac{\cos^3(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \frac{ab^2x}{(a^2+b^2)^2}+\frac{ax}{2(a^2+b^2)}+\frac{b\cos^2(c+dx)}{2(a^2+b^2)d} + \frac{b^3\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^2d} + \frac{a\cos(c+dx)\sin(c+dx)}{2(a^2+b^2)d}$$

output
```
a*b^2*x/(a^2+b^2)^2+1/2*a*x/(a^2+b^2)+1/2*b*cos(d*x+c)^2/(a^2+b^2)/d+b^3*l
n(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^2/d+1/2*a*cos(d*x+c)*sin(d*x+c)/(a^
2+b^2)/d
```

3.112.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.20

$$\int \frac{\cos^3(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \frac{2a^3c+6ab^2c+4ib^3c+2a^3dx+6ab^2dx+4ib^3dx-4ib^3\arctan(\tan(c+dx))+b(a^2+b^2)\cos(2(c+dx)) -}{4(a^2+b^2)^2d}$$

input
```
Integrate[Cos[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

output
```
(2*a^3*c + 6*a*b^2*c + (4*I)*b^3*c + 2*a^3*d*x + 6*a*b^2*d*x + (4*I)*b^3*d
*x - (4*I)*b^3*ArcTan[Tan[c + d*x]] + b*(a^2 + b^2)*Cos[2*(c + d*x)] + 2*b
^3*Log[(a*Cos[c + d*x] + b*Sin[c + d*x])^2] + a^3*Sin[2*(c + d*x)] + a*b^2
*Sin[2*(c + d*x)])/(4*(a^2 + b^2)^2*d)
```

3.112.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3579, 3042, 3115, 24, 3577, 3042, 3612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(c+dx)^3}{a\cos(c+dx)+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{3579}$$

$$\frac{a\int \cos^2(c+dx)dx}{a^2+b^2}+\frac{b^2\int \frac{\cos(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}+\frac{b\cos^2(c+dx)}{2d\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \sin\left(c+dx+\frac{\pi}{2}\right)^2dx}{a^2+b^2}+\frac{b^2\int \frac{\cos(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}+\frac{b\cos^2(c+dx)}{2d\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{3115}$$

$$\frac{b^2\int \frac{\cos(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}+\frac{a\left(\frac{\int 1dx}{2}+\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)}{a^2+b^2}+\frac{b\cos^2(c+dx)}{2d\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{24}$$

$$\frac{b^2\int \frac{\cos(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}+\frac{b\cos^2(c+dx)}{2d\left(a^2+b^2\right)}+\frac{a\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{3577}$$

$$\frac{b^2\left(\frac{b\int \frac{b\cos(c+dx)-a\sin(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}+\frac{ax}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\cos^2(c+dx)}{2d\,(a^2+b^2)}+\frac{a\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b^2\left(\frac{b\int \frac{b\cos(c+dx)-a\sin(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}+\frac{ax}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\cos^2(c+dx)}{2d\,(a^2+b^2)}+\frac{a\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{ 3612}$$

$$\frac{b\cos^2(c+dx)}{2d\,(a^2+b^2)}+\frac{a\left(\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{x}{2}\right)}{a^2+b^2}+\frac{b^2\left(\frac{b\log(a\cos(c+dx)+b\sin(c+dx))}{d(a^2+b^2)}+\frac{ax}{a^2+b^2}\right)}{a^2+b^2}$$

input
```
Int[Cos[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

output
```
(b*Cos[c + d*x]^2)/(2*(a^2 + b^2)*d) + (b^2*((a*x)/(a^2 + b^2) + (b*Log[a*
Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)*d)))/(a^2 + b^2) + (a*(x/2 +
(Cos[c + d*x]*Sin[c + d*x])/(2*d)))/(a^2 + b^2)
```

3.112.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3577
```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[a*(x/(a^2 + b^2)), x] + Simp[b/(a^2 + b
^2)   Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x
]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

3.112. $\int \frac{\cos^3(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

rule 3579
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(Cos[c + d*x]^(m - 1)/(d*(a^2 +
b^2)*(m - 1))), x] + (Simp[a/(a^2 + b^2)   Int[Cos[c + d*x]^(m - 1), x], x]
 + Simp[b^2/(a^2 + b^2)   Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[
c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m,
 1]
```

rule 3612
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

3.112.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{\frac{b^3\ln(a+b\tan(dx+c))}{(a^2+b^2)^2}+\frac{\frac{\left(\frac{1}{2}a^3+\frac{1}{2}a\,b^2\right)\tan(dx+c)+\frac{a^2b}{2}+\frac{b^3}{2}}{1+\tan(dx+c)^2}-\frac{b^3\ln\left(1+\tan(dx+c)^2\right)}{2}+\frac{\left(a^3+3a\,b^2\right)\arctan(\tan(dx+c))}{2}}{(a^2+b^2)^2}}{d}$
default	$\frac{\frac{b^3\ln(a+b\tan(dx+c))}{(a^2+b^2)^2}+\frac{\frac{\left(\frac{1}{2}a^3+\frac{1}{2}a\,b^2\right)\tan(dx+c)+\frac{a^2b}{2}+\frac{b^3}{2}}{1+\tan(dx+c)^2}-\frac{b^3\ln\left(1+\tan(dx+c)^2\right)}{2}+\frac{\left(a^3+3a\,b^2\right)\arctan(\tan(dx+c))}{2}}{(a^2+b^2)^2}}{d}$
parallelrisch	$\frac{4b^3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)-4b^3\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)+(a^2b+b^3)\cos(2dx+2c)+(a^3+a\,b^2)\sin(2dx+2c)}{4(a^2+b^2)^2d}$
risch	$\frac{2ixb}{4iba-2a^2+2b^2}-\frac{xa}{4iba-2a^2+2b^2}-\frac{i\mathrm{e}^{2i(dx+c)}}{8(-ib+a)d}+\frac{i\mathrm{e}^{-2i(dx+c)}}{8(ib+a)d}-\frac{2ib^3x}{a^4+2a^2b^2+b^4}-\frac{2ib^3c}{d(a^4+2a^2b^2+b^4)}+$
norman	$\frac{\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d(a^2+b^2)}-\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d(a^2+b^2)}-\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{d(a^2+b^2)}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{d(a^2+b^2)}+\frac{a(a^2+3b^2)x}{2a^4+4a^2b^2+2b^4}+\frac{3(a^2+3b^2)ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2(a^4+2a^2b^2+b^4)}+}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}$

input
```
int(cos(d*x+c)^3/(cos(d*x+c)*a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^3/(a^2+b^2)^2*ln(a+b*tan(d*x+c))+1/(a^2+b^2)^2*(((1/2*a^3+1/2*a*b^2
)*tan(d*x+c)+1/2*a^2*b+1/2*b^3)/(1+tan(d*x+c)^2)-1/2*b^3*ln(1+tan(d*x+c)^2
)+1/2*(a^3+3*a*b^2)*arctan(tan(d*x+c))))
```

3.112.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00

$$\int \frac{\cos^3(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$$
$$= \frac{b^3\log\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)+\left(a^3+3\,ab^2\right)dx+\left(a^2b+b^3\right)\cos\left(dx+\right.}{2\left(a^4+2\,a^2b^2+b^4\right)d}$$

input
```
integrate(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(b^3*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2
+ b^2) + (a^3 + 3*a*b^2)*d*x + (a^2*b + b^3)*cos(d*x + c)^2 + (a^3 + a*b^2
)*cos(d*x + c)*sin(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d)
```

3.112.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

output
```
Timed out
```

3.112.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(113) = 226$.

Time = 0.34 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.39

$$\int \frac{\cos^3(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$$
$$= \frac{\frac{b^3\log\left(-a-\frac{2\,b\sin(dx+c)}{\cos(dx+c)+1}+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^4+2\,a^2b^2+b^4}-\frac{b^3\log\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{(a^3+3\,ab^2)\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4+2\,a^2b^2+b^4}+\frac{\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\frac{2\,b\sin(dx+}{(\cos(dx+c)-}}{a^2+b^2+\frac{2\left(a^2+b^2\right)\sin(dx+c}{(\cos(dx+c)+1)^2}}}{d}$$

input
```
integrate(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
(b^3*log(-a - 2*b*sin(d*x + c)/(cos(d*x + c) + 1) + a*sin(d*x + c)^2/(cos(
d*x + c) + 1)^2)/(a^4 + 2*a^2*b^2 + b^4) - b^3*log(sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + (a^3 + 3*a*b^2)*arctan(sin(d*x
 + c)/(cos(d*x + c) + 1))/(a^4 + 2*a^2*b^2 + b^4) + (a*sin(d*x + c)/(cos(d
*x + c) + 1) - 2*b*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - a*sin(d*x + c)^3/
(cos(d*x + c) + 1)^3)/(a^2 + b^2 + 2*(a^2 + b^2)*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + (a^2 + b^2)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4))/d
```

3.112.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.53

$$\int \frac{\cos^3(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$$
$$= \frac{\frac{2\,b^4\log(|b\tan(dx+c)+a|)}{a^4b+2\,a^2b^3+b^5}-\frac{b^3\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{(a^3+3\,ab^2)(dx+c)}{a^4+2\,a^2b^2+b^4}+\frac{b^3\tan(dx+c)^2+a^3\tan(dx+c)+ab^2\tan(dx+c)+a^2b+2\,b^3}{(a^4+2\,a^2b^2+b^4)\left(\tan(dx+c)^2+1\right)}}{2\,d}$$

input
```
integrate(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(2*b^4*log(abs(b*tan(d*x + c) + a))/(a^4*b + 2*a^2*b^3 + b^5) - b^3*lo
g(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + (a^3 + 3*a*b^2)*(d*x + c)/
(a^4 + 2*a^2*b^2 + b^4) + (b^3*tan(d*x + c)^2 + a^3*tan(d*x + c) + a*b^2*t
an(d*x + c) + a^2*b + 2*b^3)/((a^4 + 2*a^2*b^2 + b^4)*(tan(d*x + c)^2 + 1)
))/d
```

3.112.9 Mupad [B] (verification not implemented)

Time = 29.78 (sec) , antiderivative size = 3572, normalized size of antiderivative = 30.02

$$\int \frac{\cos^3(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(cos(c + d*x)^3/(a*cos(c + d*x) + b*sin(c + d*x)),x)
```

output
```
(b^3*log(a + 2*b*tan(c/2 + (d*x)/2) - a*tan(c/2 + (d*x)/2)^2))/(d*(a^4 + b
^4 + 2*a^2*b^2)) - (4*b^3*log(1/(cos(c + d*x) + 1)))/(d*(4*a^4 + 4*b^4 + 8
*a^2*b^2)) - ((a*tan(c/2 + (d*x)/2)^3)/(a^2 + b^2) + (2*b*tan(c/2 + (d*x)/
2)^2)/(a^2 + b^2) - (a*tan(c/2 + (d*x)/2))/(a^2 + b^2))/(d*(2*tan(c/2 + (d
*x)/2)^2 + tan(c/2 + (d*x)/2)^4 + 1)) - (a*atan((tan(c/2 + (d*x)/2)*((((4*
b^3*((a*((8*(4*a*b^9 + 4*a^9*b + 28*a^3*b^7 + 48*a^5*b^5 + 28*a^7*b^3))/(a
^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (32*b^3*(12*a*b^10 + 48*a^3*b^8 + 72*a
^5*b^6 + 48*a^7*b^4 + 12*a^9*b^2))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^6 + b^6
 + 3*a^2*b^4 + 3*a^4*b^2)))*(a^2 + 3*b^2))/(2*(a^4 + b^4 + 2*a^2*b^2)) - (
16*a*b^3*(a^2 + 3*b^2)*(12*a*b^10 + 48*a^3*b^8 + 72*a^5*b^6 + 48*a^7*b^4 +
 12*a^9*b^2))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^4 + b^4 + 2*a^2*b^2)*(a^6 +
b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(4*a^4 + 4*b^4 + 8*a^2*b^2) - (a*((8*(a^9
- 12*a*b^8 - 6*a^3*b^6 + 13*a^5*b^4 + 8*a^7*b^2))/(a^6 + b^6 + 3*a^2*b^4 +
 3*a^4*b^2) - (4*b^3*((8*(4*a*b^9 + 4*a^9*b + 28*a^3*b^7 + 48*a^5*b^5 + 28
*a^7*b^3))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (32*b^3*(12*a*b^10 + 48*a
^3*b^8 + 72*a^5*b^6 + 48*a^7*b^4 + 12*a^9*b^2))/((4*a^4 + 4*b^4 + 8*a^2*b^
2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(4*a^4 + 4*b^4 + 8*a^2*b^2))*(a^
2 + 3*b^2))/(2*(a^4 + b^4 + 2*a^2*b^2)) + (a^3*(a^2 + 3*b^2)^3*(12*a*b^10
+ 48*a^3*b^8 + 72*a^5*b^6 + 48*a^7*b^4 + 12*a^9*b^2))/((a^4 + b^4 + 2*a^2*
b^2)^3*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))*(a^8 + 16*b^8 - 73*a^2*b^6...
```

3.113 $\int \frac{\cos^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

3.113.1 Optimal result

Integrand size = 28, antiderivative size = 91

$$\int \frac{\cos^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = -\frac{b^2\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d} + \frac{b\cos(c+dx)}{(a^2+b^2)\,d} + \frac{a\sin(c+dx)}{(a^2+b^2)\,d}$$

output
```
-b^2*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)/
d+b*cos(d*x+c)/(a^2+b^2)/d+a*sin(d*x+c)/(a^2+b^2)/d
```

3.113.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.87

$$\int \frac{\cos^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \frac{2b^2\operatorname{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)+\sqrt{a^2+b^2}\,(b\cos(c+dx)+a\sin(c+dx))}{(a^2+b^2)^{3/2}\,d}$$

input
```
Integrate[Cos[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

output
```
(2*b^2*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] + Sqrt[a^2 + b^2
]*(b*Cos[c + d*x] + a*Sin[c + d*x]))/((a^2 + b^2)^(3/2)*d)
```

3.113.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 3579, 3042, 3117, 3553, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(c+dx)^2}{a\cos(c+dx)+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{3579}$$

$$\frac{a\int \cos(c+dx)dx}{a^2+b^2}+\frac{b^2\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}+\frac{b\cos(c+dx)}{d\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \sin\left(c+dx+\frac{\pi}{2}\right)dx}{a^2+b^2}+\frac{b^2\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}+\frac{b\cos(c+dx)}{d\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{3117}$$

$$\frac{b^2\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}+\frac{a\sin(c+dx)}{d\left(a^2+b^2\right)}+\frac{b\cos(c+dx)}{d\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{3553}$$

$$-\frac{b^2\int \frac{1}{a^2+b^2-(b\cos(c+dx)-a\sin(c+dx))^2}d(b\cos(c+dx)-a\sin(c+dx))}{d\left(a^2+b^2\right)}+\frac{a\sin(c+dx)}{d\left(a^2+b^2\right)}+\frac{b\cos(c+dx)}{d\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{219}$$

$$-\frac{b^2\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{3/2}}+\frac{a\sin(c+dx)}{d\left(a^2+b^2\right)}+\frac{b\cos(c+dx)}{d\left(a^2+b^2\right)}$$

input
```
Int[Cos[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

output
```
-((b^2*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/((a^2 +
 b^2)^(3/2)*d)) + (b*Cos[c + d*x])/((a^2 + b^2)*d) + (a*Sin[c + d*x])/((a^
2 + b^2)*d)
```

3.113. $\int \frac{\cos^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx$

3.113.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3579
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(Cos[c + d*x]^(m - 1)/(d*(a^2 +
b^2)*(m - 1))), x] + (Simp[a/(a^2 + b^2)   Int[Cos[c + d*x]^(m - 1), x], x]
 + Simp[b^2/(a^2 + b^2)   Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[
c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m,
 1]
```

3.113.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.99

method	result	siz
derivativedivides	$\frac{-\frac{2\left(-a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-b\right)}{\left(a^2+b^2\right)\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}+\frac{2b^2\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}}{d}$	90
default	$\frac{-\frac{2\left(-a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-b\right)}{\left(a^2+b^2\right)\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}+\frac{2b^2\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}}{d}$	90
risch	$-\frac{i\,\mathrm{e}^{i(dx+c)}}{2(-ib+a)d}+\frac{i\,\mathrm{e}^{-i(dx+c)}}{2(ib+a)d}+\frac{b^2\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia^3+ia\,b^2-a^2b-b^3}{\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}d}-\frac{b^2\ln\left(\mathrm{e}^{i(dx+c)}-\frac{ia^3+ia\,b^2-a^2b-b^3}{\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}d}$	17

input
```
int(cos(d*x+c)^2/(cos(d*x+c)*a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/(a^2+b^2)*(-a*tan(1/2*d*x+1/2*c)-b)/(1+tan(1/2*d*x+1/2*c)^2)+2*b^2
/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))
)
```

3.113.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(87) = 174$.

Time = 0.28 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.05

$$\int \frac{\cos^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$$
$$= \frac{\sqrt{a^2+b^2}b^2\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2}\right)+2\,(a^2b+b^3)\,c}{2\,(a^4+2\,a^2b^2+b^4)d}$$

input
```
integrate(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(sqrt(a^2 + b^2)*b^2*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^
2)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*si
n(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2
 + b^2)) + 2*(a^2*b + b^3)*cos(d*x + c) + 2*(a^3 + a*b^2)*sin(d*x + c))/((
a^4 + 2*a^2*b^2 + b^4)*d)
```

3.113.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 115.44 (sec) , antiderivative size = 1034, normalized size of antiderivative = 11.36

$$\int \frac{\cos^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)**2/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

output
```
Piecewise((zoo*x*cos(c)**2/sin(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((log(
tan(c/2 + d*x/2))*tan(c/2 + d*x/2)**2/(d*tan(c/2 + d*x/2)**2 + d) + log(ta
n(c/2 + d*x/2))/(d*tan(c/2 + d*x/2)**2 + d) + 2/(d*tan(c/2 + d*x/2)**2 + d
))/b, Eq(a, 0)), (-2*sin(c + d*x)**2/(3*I*b*d*sin(c + d*x) + 3*b*d*cos(c +
 d*x)) + 2*I*sin(c + d*x)*cos(c + d*x)/(3*I*b*d*sin(c + d*x) + 3*b*d*cos(c
 + d*x)) - cos(c + d*x)**2/(3*I*b*d*sin(c + d*x) + 3*b*d*cos(c + d*x)), Eq
(a, -I*b)), (-2*sin(c + d*x)**2/(-3*I*b*d*sin(c + d*x) + 3*b*d*cos(c + d*x
)) - 2*I*sin(c + d*x)*cos(c + d*x)/(-3*I*b*d*sin(c + d*x) + 3*b*d*cos(c +
d*x)) - cos(c + d*x)**2/(-3*I*b*d*sin(c + d*x) + 3*b*d*cos(c + d*x)), Eq(a
, I*b)), (x*cos(c)**2/(a*cos(c) + b*sin(c)), Eq(d, 0)), (2*a*sqrt(a**2 + b
**2)*tan(c/2 + d*x/2)/(a**2*d*sqrt(a**2 + b**2)*tan(c/2 + d*x/2)**2 + a**2
*d*sqrt(a**2 + b**2) + b**2*d*sqrt(a**2 + b**2)*tan(c/2 + d*x/2)**2 + b**2
*d*sqrt(a**2 + b**2)) - b**2*log(tan(c/2 + d*x/2) - b/a - sqrt(a**2 + b**2
)/a)*tan(c/2 + d*x/2)**2/(a**2*d*sqrt(a**2 + b**2)*tan(c/2 + d*x/2)**2 + a
**2*d*sqrt(a**2 + b**2) + b**2*d*sqrt(a**2 + b**2)*tan(c/2 + d*x/2)**2 + b
**2*d*sqrt(a**2 + b**2)) - b**2*log(tan(c/2 + d*x/2) - b/a - sqrt(a**2 + b
**2)/a)/(a**2*d*sqrt(a**2 + b**2)*tan(c/2 + d*x/2)**2 + a**2*d*sqrt(a**2 +
 b**2) + b**2*d*sqrt(a**2 + b**2)*tan(c/2 + d*x/2)**2 + b**2*d*sqrt(a**2 +
 b**2)) + b**2*log(tan(c/2 + d*x/2) - b/a + sqrt(a**2 + b**2)/a)*tan(c/2 +
 d*x/2)**2/(a**2*d*sqrt(a**2 + b**2)*tan(c/2 + d*x/2)**2 + a**2*d*sqrt(...
```

3.113.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.56

$$\int \frac{\cos^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = -\frac{\frac{b^2\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}}-\frac{2\left(b+\frac{a\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2+b^2+\frac{(a^2+b^2)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}}{d}$$

input
```
integrate(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-(b^2*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a
*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2
*(b + a*sin(d*x + c)/(cos(d*x + c) + 1))/(a^2 + b^2 + (a^2 + b^2)*sin(d*x
+ c)^2/(cos(d*x + c) + 1)^2))/d
```

3.113.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.30

$$\int \frac{\cos^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = -\frac{\frac{b^2\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^2+b^2)^{\frac{3}{2}}} - \frac{2\,\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b\right)}{(a^2+b^2)\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)}}{d}$$

input
```
integrate(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
-(b^2*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*
tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(a*
tan(1/2*d*x + 1/2*c) + b)/((a^2 + b^2)*(tan(1/2*d*x + 1/2*c)^2 + 1)))/d
```

3.113.9 Mupad [B] (verification not implemented)

Time = 22.62 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.21

$$\int \frac{\cos^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \frac{\frac{2\,b}{a^2+b^2}+\frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^2+b^2}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)} - \frac{2\,b^2\,\mathrm{atanh}\left(\frac{a^2\,b+b^3-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^2+b^2\right)}{{\left(a^2+b^2\right)}^{3/2}}\right)}{d\,{\left(a^2+b^2\right)}^{3/2}}$$

input
```
int(cos(c + d*x)^2/(a*cos(c + d*x) + b*sin(c + d*x)),x)
```

output
```
((2*b)/(a^2 + b^2) + (2*a*tan(c/2 + (d*x)/2))/(a^2 + b^2))/(d*(tan(c/2 + (
d*x)/2)^2 + 1)) - (2*b^2*atanh((a^2*b + b^3 - a*tan(c/2 + (d*x)/2)*(a^2 +
b^2))/(a^2 + b^2)^(3/2)))/(d*(a^2 + b^2)^(3/2))
```

3.113. $\int \frac{\cos^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

3.114 $\int \frac{\cos(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

3.114.1 Optimal result

Integrand size = 26, antiderivative size = 45

$$\int \frac{\cos(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx=\frac{ax}{a^2+b^2}+\frac{b\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)\,d}$$

output
```
a*x/(a^2+b^2)+b*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)/d
```

3.114.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \frac{\cos(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx=\frac{a(c+dx)+b\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)\,d}$$

input
```
Integrate[Cos[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

output
```
(a*(c + d*x) + b*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)*d)
```

3.114.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3042, 3577, 3042, 3612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\cos(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx$$
$$\Big\downarrow \text{ 3577}$$
$$\frac{b\int \frac{b\cos(c+dx)-a\sin(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}+\frac{ax}{a^2+b^2}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{b\int \frac{b\cos(c+dx)-a\sin(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}+\frac{ax}{a^2+b^2}$$
$$\Big\downarrow \text{ 3612}$$
$$\frac{b\log(a\cos(c+dx)+b\sin(c+dx))}{d\,(a^2+b^2)}+\frac{ax}{a^2+b^2}$$

input
```
Int[Cos[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

output
```
(a*x)/(a^2 + b^2) + (b*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)*
d)
```

3.114.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3577
```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[a*(x/(a^2 + b^2)), x] + Simp[b/(a^2 + b
^2)   Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x
]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3612
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

3.114.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.38

method	result	size
derivativedivides	$\frac{\frac{-\frac{b\ln\left(1+\tan(dx+c)^2\right)}{2}+a\arctan(\tan(dx+c))}{a^2+b^2}+\frac{b\ln(a+b\tan(dx+c))}{a^2+b^2}}{d}$	62
default	$\frac{\frac{-\frac{b\ln\left(1+\tan(dx+c)^2\right)}{2}+a\arctan(\tan(dx+c))}{a^2+b^2}+\frac{b\ln(a+b\tan(dx+c))}{a^2+b^2}}{d}$	62
parallelrisch	$\frac{axd-b\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)+b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{d(a^2+b^2)}$	66
risch	$-\frac{x}{ib-a}-\frac{2ibx}{a^2+b^2}-\frac{2ibc}{d(a^2+b^2)}+\frac{b\ln\left(e^{2i(dx+c)}-\frac{ib+a}{ib-a}\right)}{d(a^2+b^2)}$	89
norman	$\frac{\frac{ax}{a^2+b^2}+\frac{ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{a^2+b^2}}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{d(a^2+b^2)}-\frac{b\ln\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}{d(a^2+b^2)}$	127

input
```
int(cos(d*x+c)/(cos(d*x+c)*a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/(a^2+b^2)*(-1/2*b*ln(1+tan(d*x+c)^2)+a*arctan(tan(d*x+c)))+b/(a^2+b
^2)*ln(a+b*tan(d*x+c)))
```

3.114.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.36

$$\int \frac{\cos(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \frac{2\,adx+b\log\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)}{2\,(a^2+b^2)d}$$

input
```
integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(2*a*d*x + b*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x
 + c)^2 + b^2))/((a^2 + b^2)*d)
```

3.114.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.07 (sec) , antiderivative size = 296, normalized size of antiderivative = 6.58

$$\int \frac{\cos(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \begin{cases} \frac{\tilde{\infty}x\cos\left(c\right)}{\sin\left(c\right)} & \text{for}\ a=0\wedge b=0\wedge \\ \frac{\log\left(\sin\left(c+dx\right)\right)}{bd} & \text{for}\ a=0 \\ -\frac{dx\sin\left(c+dx\right)}{2ibd\sin\left(c+dx\right)+2bd\cos\left(c+dx\right)}+\frac{idx\cos\left(c+dx\right)}{2ibd\sin\left(c+dx\right)+2bd\cos\left(c+dx\right)}-\frac{\cos\left(c+dx\right)}{2ibd\sin\left(c+dx\right)+2bd\cos\left(c+dx\right)} & \text{for}\ a=-ib \\ -\frac{dx\sin\left(c+dx\right)}{-2ibd\sin\left(c+dx\right)+2bd\cos\left(c+dx\right)}-\frac{idx\cos\left(c+dx\right)}{-2ibd\sin\left(c+dx\right)+2bd\cos\left(c+dx\right)}-\frac{\cos\left(c+dx\right)}{-2ibd\sin\left(c+dx\right)+2bd\cos\left(c+dx\right)} & \text{for}\ a=ib \\ \frac{x\cos\left(c\right)}{a\cos\left(c\right)+b\sin\left(c\right)} & \text{for}\ d=0 \\ \frac{adx}{a^2d+b^2d}+\frac{b\log\left(\cos\left(c+dx\right)+\frac{b\sin\left(c+dx\right)}{a}\right)}{a^2d+b^2d} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

output
```
Piecewise((zoo*x*cos(c)/sin(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (log(sin(
c + d*x))/(b*d), Eq(a, 0)), (-d*x*sin(c + d*x)/(2*I*b*d*sin(c + d*x) + 2*b
*d*cos(c + d*x)) + I*d*x*cos(c + d*x)/(2*I*b*d*sin(c + d*x) + 2*b*d*cos(c
+ d*x)) - cos(c + d*x)/(2*I*b*d*sin(c + d*x) + 2*b*d*cos(c + d*x)), Eq(a,
-I*b)), (-d*x*sin(c + d*x)/(-2*I*b*d*sin(c + d*x) + 2*b*d*cos(c + d*x)) -
I*d*x*cos(c + d*x)/(-2*I*b*d*sin(c + d*x) + 2*b*d*cos(c + d*x)) - cos(c +
d*x)/(-2*I*b*d*sin(c + d*x) + 2*b*d*cos(c + d*x)), Eq(a, I*b)), (x*cos(c)/
(a*cos(c) + b*sin(c)), Eq(d, 0)), (a*d*x/(a**2*d + b**2*d) + b*log(cos(c +
 d*x) + b*sin(c + d*x)/a)/(a**2*d + b**2*d), True))
```

3.114.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(45) = 90$.

Time = 0.30 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.76

$$\int \frac{\cos(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \frac{\frac{2\,a\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2+b^2}+\frac{b\log\left(-a-\frac{2\,b\sin(dx+c)}{\cos(dx+c)+1}+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^2+b^2}-\frac{b\log\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)}{a^2+b^2}}{d}$$

input
```
integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
(2*a*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/(a^2 + b^2) + b*log(-a - 2*b*
sin(d*x + c)/(cos(d*x + c) + 1) + a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/(
a^2 + b^2) - b*log(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)/(a^2 + b^2))/d
```

3.114.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.64

$$\int \frac{\cos(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \frac{\frac{2\,b^2\log(|b\tan(dx+c)+a|)}{a^2b+b^3}+\frac{2\,(dx+c)a}{a^2+b^2}-\frac{b\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}}{2\,d}$$

input
```
integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(2*b^2*log(abs(b*tan(d*x + c) + a))/(a^2*b + b^3) + 2*(d*x + c)*a/(a^2
 + b^2) - b*log(tan(d*x + c)^2 + 1)/(a^2 + b^2))/d
```

3.114.9 Mupad [B] (verification not implemented)

Time = 23.01 (sec) , antiderivative size = 1069, normalized size of antiderivative = 23.76

$$\int \frac{\cos(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \frac{b\ln\left(-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)}{d\,(a^2+b^2)} - \frac{2a\,\mathrm{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{(a^4-13a^2b^2+4b^4)\left(\frac{a^3\,(96a^3b^2+96ab^4)}{(a^2+b^2)^3}+\frac{a\left(96ab^2-32a^3+\frac{b\left(32ab^3+128a^3b-\frac{b\,(96a^3b^2+96ab^4)}{a^2+b^2}\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\left(\frac{a\left(32\cdots\right)}{\cdots}\cdots\right)}{\cdots}\right)}{(a^4+5a^2b^2+4b^4)^2}\cdots\right)}{\cdots}\right)}{\cdots} - \frac{b\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{d\,(a^2+b^2)}$$

input
```
int(cos(c + d*x)/(a*cos(c + d*x) + b*sin(c + d*x)),x)
```

output
```
(b*log(a + 2*b*tan(c/2 + (d*x)/2) - a*tan(c/2 + (d*x)/2)^2))/(d*(a^2 + b^2
)) - (2*a*atan((tan(c/2 + (d*x)/2)*(((a^4 + 4*b^4 - 13*a^2*b^2)*((a^3*(96*
a*b^4 + 96*a^3*b^2))/(a^2 + b^2)^3 + (a*(96*a*b^2 - 32*a^3 + (b*(32*a*b^3
+ 128*a^3*b - (b*(96*a*b^4 + 96*a^3*b^2))/(a^2 + b^2)))/(a^2 + b^2)))/(a^2
 + b^2) + (b*((a*(32*a*b^3 + 128*a^3*b - (b*(96*a*b^4 + 96*a^3*b^2))/(a^2
+ b^2)))/(a^2 + b^2) - (a*b*(96*a*b^4 + 96*a^3*b^2))/(a^2 + b^2)^2))/(a^2
+ b^2)))/(a^4 + 4*b^4 + 5*a^2*b^2)^2 - (6*a*b*(a^2 - 2*b^2)*(32*a*b - (b*(
96*a*b^2 - 32*a^3 + (b*(32*a*b^3 + 128*a^3*b - (b*(96*a*b^4 + 96*a^3*b^2))
/(a^2 + b^2)))/(a^2 + b^2)))/(a^2 + b^2) + (a*((a*(32*a*b^3 + 128*a^3*b -
(b*(96*a*b^4 + 96*a^3*b^2))/(a^2 + b^2)))/(a^2 + b^2) - (a*b*(96*a*b^4 + 9
6*a^3*b^2))/(a^2 + b^2)^2))/(a^2 + b^2) - (a^2*b*(96*a*b^4 + 96*a^3*b^2))/
(a^2 + b^2)^3))/(a^4 + 4*b^4 + 5*a^2*b^2)^2)*(a^4 + b^4 + 2*a^2*b^2))/(32*
a^2) + ((a^4 + 4*b^4 - 13*a^2*b^2)*((a*(32*a^2*b - (b*(64*a^2*b^2 - 32*a^4
 + (b*(96*a^4*b + 96*a^2*b^3))/(a^2 + b^2)))/(a^2 + b^2)))/(a^2 + b^2) + (
a^3*(96*a^4*b + 96*a^2*b^3))/(a^2 + b^2)^3 - (b*((a*(64*a^2*b^2 - 32*a^4 +
 (b*(96*a^4*b + 96*a^2*b^3))/(a^2 + b^2)))/(a^2 + b^2) + (a*b*(96*a^4*b +
96*a^2*b^3))/(a^2 + b^2)^2))/(a^2 + b^2))*(a^4 + b^4 + 2*a^2*b^2))/(32*a^2
*(a^4 + 4*b^4 + 5*a^2*b^2)^2) + (3*b*(a^2 - 2*b^2)*((b*(32*a^2*b - (b*(64*
a^2*b^2 - 32*a^4 + (b*(96*a^4*b + 96*a^2*b^3))/(a^2 + b^2)))/(a^2 + b^2)))
/(a^2 + b^2) + (a*((a*(64*a^2*b^2 - 32*a^4 + (b*(96*a^4*b + 96*a^2*b^3)...
```

3.114. $\int \frac{\cos(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

3.115 $\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

3.115.1 Optimal result

Integrand size = 19, antiderivative size = 47

$$\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}\,dx = -\frac{\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}d}$$

output
```
-arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/d/(a^2+b^2)^(1/2)
```

3.115.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96

$$\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \frac{2\operatorname{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}d}$$

input
```
Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-1),x]
```

output
```
(2*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(Sqrt[a^2 + b^2]*d)
```

3.115.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3042, 3553, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx$$

$$\Big\downarrow \; 3553$$

$$-\frac{\int \frac{1}{a^2+b^2-(b\cos(c+dx)-a\sin(c+dx))^2}d(b\cos(c+dx)-a\sin(c+dx))}{d}$$

$$\Big\downarrow \; 219$$

$$-\frac{\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$$

input
```
Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-1),x]
```

output
```
-(ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]]/(Sqrt[a^2 + b
^2]*d))
```

3.115.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

3.115.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{2\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$	43
default	$\frac{2\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$	43
risch	$\frac{\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia-b}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,d}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}-\frac{ia-b}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,d}$	88

input
```
int(1/(cos(d*x+c)*a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2/d/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/
2))
```

3.115.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(43) = 86.

Time = 0.25 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.79

$$\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \frac{\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2}\right)}{2\,\sqrt{a^2+b^2}d}$$

input
```
integrate(1/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2
*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*c
os(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2))/(sqrt(a^2 +
b^2)*d)
```

3.115.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.90 (sec) , antiderivative size = 163, normalized size of antiderivative = 3.47

$$\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \begin{cases} \frac{\tilde{\infty}x}{\sin(c)} & \text{for } a=0 \wedge b=0 \wedge d=0 \\ \frac{\log\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{bd} & \text{for } a=0 \\ \frac{x}{a\cos(c)+b\sin(c)} & \text{for } d=0 \\ -\frac{1}{ibd\sin(c+dx)+bd\cos(c+dx)} & \text{for } a=-ib \\ -\frac{1}{-ibd\sin(c+dx)+bd\cos(c+dx)} & \text{for } a=ib \\ -\frac{\log\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-\frac{b}{a}-\frac{\sqrt{a^2+b^2}}{a}\right)}{d\sqrt{a^2+b^2}}+\frac{\log\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-\frac{b}{a}+\frac{\sqrt{a^2+b^2}}{a}\right)}{d\sqrt{a^2+b^2}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

output
```
Piecewise((zoo*x/sin(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (log(tan(c/2 + d
*x/2))/(b*d), Eq(a, 0)), (x/(a*cos(c) + b*sin(c)), Eq(d, 0)), (-1/(I*b*d*s
in(c + d*x) + b*d*cos(c + d*x)), Eq(a, -I*b)), (-1/(-I*b*d*sin(c + d*x) +
b*d*cos(c + d*x)), Eq(a, I*b)), (-log(tan(c/2 + d*x/2) - b/a - sqrt(a**2 +
 b**2)/a)/(d*sqrt(a**2 + b**2)) + log(tan(c/2 + d*x/2) - b/a + sqrt(a**2 +
 b**2)/a)/(d*sqrt(a**2 + b**2)), True))
```

3.115.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.70

$$\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}\,dx = -\frac{\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}d}$$

input
```
integrate(1/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*sin(
d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*d)
```

3.115.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.57

$$\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}\,dx = -\frac{\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}d}$$

input
```
integrate(1/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
-log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1
/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*d)
```

3.115.9 Mupad [B] (verification not implemented)

Time = 22.56 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{b-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{d\,\sqrt{a^2+b^2}}$$

input
```
int(1/(a*cos(c + d*x) + b*sin(c + d*x)),x)
```

output
```
-(2*atanh((b - a*tan(c/2 + (d*x)/2))/(a^2 + b^2)^(1/2)))/(d*(a^2 + b^2)^(1
/2))
```

3.116 $\int \frac{\sec(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

3.116.1 Optimal result

Integrand size = 26, antiderivative size = 41

$$\int \frac{\sec(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx=-\frac{\log(\cos(c+dx))}{bd}+\frac{\log(a\cos(c+dx)+b\sin(c+dx))}{bd}$$

output
```
-ln(cos(d*x+c))/b/d+ln(a*cos(d*x+c)+b*sin(d*x+c))/b/d
```

3.116.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.44

$$\int \frac{\sec(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx=\frac{\log(a+b\tan(c+dx))}{bd}$$

input
```
Integrate[Sec[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

output
```
Log[a + b*Tan[c + d*x]]/(b*d)
```

3.116.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3042, 3581, 3042, 3612, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))}dx$$

$$\Big\downarrow \text{3581}$$

$$\frac{\int \frac{b\cos(c+dx)-a\sin(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx}{b}+\frac{\int \tan(c+dx)dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{b\cos(c+dx)-a\sin(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx}{b}+\frac{\int \tan(c+dx)dx}{b}$$

$$\Big\downarrow \text{3612}$$

$$\frac{\int \tan(c+dx)dx}{b}+\frac{\log(a\cos(c+dx)+b\sin(c+dx))}{bd}$$

$$\Big\downarrow \text{3956}$$

$$\frac{\log(a\cos(c+dx)+b\sin(c+dx))}{bd}-\frac{\log(\cos(c+dx))}{bd}$$

input
```
Int[Sec[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

output
```
-(Log[Cos[c + d*x]]/(b*d)) + Log[a*Cos[c + d*x] + b*Sin[c + d*x]]/(b*d)
```

3.116.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3581
```
Int[1/(cos[(c_.) + (d_.)*(x_)]*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(
c_.) + (d_.)*(x_)])), x_Symbol] :> Simp[1/b   Int[Tan[c + d*x], x], x] + Si
mp[1/b   Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c +
d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3612
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

3.116.4 Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.46

method	result	size
derivativedivides	$\frac{\ln(a+b\tan(dx+c))}{db}$	19
default	$\frac{\ln(a+b\tan(dx+c))}{db}$	19
risch	$-\frac{\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{bd}+\frac{\ln\left(\mathrm{e}^{2i(dx+c)}-\frac{ib+a}{ib-a}\right)}{bd}$	58
parallelrisch	$\frac{-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{bd}$	67
norman	$\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{bd}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{bd}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{bd}$	79

input
```
int(sec(d*x+c)/(cos(d*x+c)*a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d/b*ln(a+b*tan(d*x+c))
```

3.116.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.44

$$\int \frac{\sec(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \frac{\log\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)-\log\left(\cos\left(dx+c\right)^2\right)}{2\,bd}$$

input
```
integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^
2) - log(cos(d*x + c)^2))/(b*d)
```

3.116.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \int \frac{\sec\left(c+dx\right)}{a\cos\left(c+dx\right)+b\sin\left(c+dx\right)}\,dx$$

input
```
integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)/(a*cos(c + d*x) + b*sin(c + d*x)), x)
```

3.116.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(41) = 82.

Time = 0.23 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.51

$$\int \frac{\sec(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \frac{\frac{\log\left(-a-\frac{2\,b\sin(dx+c)}{\cos(dx+c)+1}+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{b}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{b}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{b}}{d}$$

input
```
integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
(log(-a - 2*b*sin(d*x + c)/(cos(d*x + c) + 1) + a*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2)/b - log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/b - log(sin(d*x
+ c)/(cos(d*x + c) + 1) - 1)/b)/d
```

3.116.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.46

$$\int \frac{\sec(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx=\frac{\log\left(\left|b\tan\left(dx+c\right)+a\right|\right)}{bd}$$

input
```
integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
log(abs(b*tan(d*x + c) + a))/(b*d)
```

3.116.9 Mupad [B] (verification not implemented)

Time = 23.46 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.51

$$\int \frac{\sec(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx=-\frac{2\,\mathrm{atanh}\left(\frac{b\,\left(b\,\cos(c+d\,x)-a\,\sin(c+d\,x)\right)}{2\,\cos(c+d\,x)\,a^2+\sin(c+d\,x)\,a\,b+\cos(c+d\,x)\,b^2}\right)}{b\,d}$$

input
```
int(1/(cos(c + d*x)*(a*cos(c + d*x) + b*sin(c + d*x))),x)
```

output
```
-(2*atanh((b*(b*cos(c + d*x) - a*sin(c + d*x)))/(2*a^2*cos(c + d*x) + b^2*
cos(c + d*x) + a*b*sin(c + d*x))))/(b*d)
```

3.117 $\int \frac{\sec^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

3.117.1 Optimal result

Integrand size = 28, antiderivative size = 80

$$\int \frac{\sec^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx=-\frac{a\text{arctanh}(\sin(c+dx))}{b^2d}-\frac{\sqrt{a^2+b^2}\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d}+\frac{\sec(c+dx)}{bd}$$

output
```
-a*arctanh(sin(d*x+c))/b^2/d+sec(d*x+c)/b/d-arctanh((b*cos(d*x+c)-a*sin(d*
x+c))/(a^2+b^2)^(1/2))*(a^2+b^2)^(1/2)/b^2/d
```

3.117.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.36

$$\int \frac{\sec^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \frac{2\sqrt{a^2+b^2}\text{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)+a\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right.\right.}{b^2d}$$

input
```
Integrate[Sec[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

output
```
(2*Sqrt[a^2 + b^2]*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] + a*
(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2]]) + b*Sec[c + d*x])/(b^2*d)
```

3.117.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 3583, 3042, 3553, 219, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))}dx$$

$$\Big\downarrow \ 3583$$

$$\frac{(a^2+b^2)\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{b^2}-\frac{a\int \sec(c+dx)dx}{b^2}+\frac{\sec(c+dx)}{bd}$$

$$\Big\downarrow \ 3042$$

$$\frac{(a^2+b^2)\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}+\frac{\sec(c+dx)}{bd}$$

$$\Big\downarrow \ 3553$$

$$-\frac{(a^2+b^2)\int \frac{1}{a^2+b^2-(b\cos(c+dx)-a\sin(c+dx))^2}d(b\cos(c+dx)-a\sin(c+dx))}{b^2d}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}+\frac{\sec(c+dx)}{bd}$$

$$\Big\downarrow \ 219$$

$$-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{\sqrt{a^2+b^2}\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d}+\frac{\sec(c+dx)}{bd}$$

$$\Big\downarrow \ 4257$$

$$-\frac{\sqrt{a^2+b^2}\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d}-\frac{a\text{arctanh}(\sin(c+dx))}{b^2d}+\frac{\sec(c+dx)}{bd}$$

---

input
```
Int[Sec[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

output
```
-((a*ArcTanh[Sin[c + d*x]])/(b^2*d)) - (Sqrt[a^2 + b^2]*ArcTanh[(b*Cos[c +
 d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(b^2*d) + Sec[c + d*x]/(b*d)
```

3.117.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3583
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-Cos[c + d*x]^(m + 1)/(b*d*(m + 1)
), x] + (-Simp[a/b^2   Int[Cos[c + d*x]^(m + 1), x], x] + Simp[(a^2 + b^2)/
b^2   Int[Cos[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.117.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.61

method	result
derivativedivides	$\frac{\frac{1}{b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^2}-\frac{2\left(-a^2-b^2\right)\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}-\frac{1}{b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{b^2}}{d}$
default	$\frac{\frac{1}{b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^2}-\frac{2\left(-a^2-b^2\right)\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}-\frac{1}{b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{b^2}}{d}$
risch	$\frac{2\,e^{i(dx+c)}}{db\left(e^{2i(dx+c)}+1\right)}+\frac{a\ln\left(e^{i(dx+c)}-i\right)}{b^2d}-\frac{a\ln\left(i+e^{i(dx+c)}\right)}{b^2d}+\frac{\sqrt{a^2+b^2}\,\ln\left(e^{i(dx+c)}+\frac{ia-b}{\sqrt{a^2+b^2}}\right)}{d\,b^2}-\frac{\sqrt{a^2+b^2}\,\ln\left(e^{i(dx\right.}}{d\,b}$

input
```
int(sec(d*x+c)^2/(cos(d*x+c)*a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/b/(tan(1/2*d*x+1/2*c)+1)-a/b^2*ln(tan(1/2*d*x+1/2*c)+1)-2/b^2*(-a^2
-b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(
1/2))-1/b/(tan(1/2*d*x+1/2*c)-1)+a/b^2*ln(tan(1/2*d*x+1/2*c)-1))
```

3.117.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. 2(76) = 152.

Time = 0.28 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.39

$$\int \frac{\sec^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx =$$
$$-\frac{a\cos(dx+c)\log(\sin(dx+c)+1)-a\cos(dx+c)\log(-\sin(dx+c)+1)-\sqrt{a^2+b^2}\cos(dx+c)\log}{2\,b^2d\cos(dx+c)}$$

input
```
integrate(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2*(a*cos(d*x + c)*log(sin(d*x + c) + 1) - a*cos(d*x + c)*log(-sin(d*x +
 c) + 1) - sqrt(a^2 + b^2)*cos(d*x + c)*log(-(2*a*b*cos(d*x + c)*sin(d*x +
 c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(
d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)
*cos(d*x + c)^2 + b^2)) - 2*b)/(b^2*d*cos(d*x + c))
```

3.117.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx=\int \frac{\sec^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**2/(a*cos(c + d*x) + b*sin(c + d*x)), x)
```

3.117.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(76) = 152$.

Time = 0.32 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.04

$$\int \frac{\sec^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$$
$$=-\frac{\frac{a\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{b^2}-\frac{a\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{b^2}+\frac{\sqrt{a^2+b^2}\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{b^2}-\frac{2}{b-\frac{b\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}}{d}$$

input
```
integrate(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-(a*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/b^2 - a*log(sin(d*x + c)/(cos
(d*x + c) + 1) - 1)/b^2 + sqrt(a^2 + b^2)*log((b - a*sin(d*x + c)/(cos(d*x
 + c) + 1) + sqrt(a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqr
t(a^2 + b^2)))/b^2 - 2/(b - b*sin(d*x + c)^2/(cos(d*x + c) + 1)^2))/d
```

3.117.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.70

$$\int \frac{\sec^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx=$$
$$-\frac{\frac{a\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{b^2}-\frac{a\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{b^2}+\frac{\sqrt{a^2+b^2}\log\left(\frac{\left|2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2b-2\sqrt{a^2+b^2}\right|}{\left|2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2b+2\sqrt{a^2+b^2}\right|}\right)}{b^2}+\frac{2}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)b}}{d}$$

input
```
integrate(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
-(a*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^2 - a*log(abs(tan(1/2*d*x + 1/2*c
) - 1))/b^2 + sqrt(a^2 + b^2)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*s
qrt(a^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/b
^2 + 2/((tan(1/2*d*x + 1/2*c)^2 - 1)*b))/d
```

3.117.9 Mupad [B] (verification not implemented)

Time = 22.73 (sec) , antiderivative size = 310, normalized size of antiderivative = 3.88

$$\int \frac{\sec^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$$
$$= \frac{2\,\mathrm{atanh}\left(\frac{64\,a^2\,\sqrt{a^2+b^2}}{64\,a^2\,b+\frac{64\,a^4}{b}+128\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+128\,a\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{128\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{a^2+b^2}}{64\,a^2+\frac{64\,a^4}{b^2}+\frac{128\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{b}+128\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{\cdots}{64\,a^4+128\cdots}\cdots}{b^2\,d}$$
$$-\frac{2\,a\,\mathrm{atanh}\left(\frac{64\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64\,a^2+\frac{64\,a^4}{b^2}}+\frac{64\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64\,a^4+64\,a^2\,b^2}\right)}{b^2\,d}-\frac{2}{b\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

input
```
int(1/(cos(c + d*x)^2*(a*cos(c + d*x) + b*sin(c + d*x))),x)
```

output
```
(2*atanh((64*a^2*(a^2 + b^2)^(1/2))/(64*a^2*b + (64*a^4)/b + 128*a^3*tan(c
/2 + (d*x)/2) + 128*a*b^2*tan(c/2 + (d*x)/2)) + (128*a*tan(c/2 + (d*x)/2)*
(a^2 + b^2)^(1/2))/(64*a^2 + (64*a^4)/b^2 + (128*a^3*tan(c/2 + (d*x)/2))/b
 + 128*a*b*tan(c/2 + (d*x)/2)) + (64*a^3*tan(c/2 + (d*x)/2)*(a^2 + b^2)^(1
/2))/(64*a^4 + 64*a^2*b^2 + 128*a*b^3*tan(c/2 + (d*x)/2) + 128*a^3*b*tan(c
/2 + (d*x)/2)))*(a^2 + b^2)^(1/2))/(b^2*d) - (2*a*atanh((64*a^2*tan(c/2 +
(d*x)/2))/(64*a^2 + (64*a^4)/b^2) + (64*a^4*tan(c/2 + (d*x)/2))/(64*a^4 +
64*a^2*b^2)))/(b^2*d) - 2/(b*d*(tan(c/2 + (d*x)/2)^2 - 1))
```

3.118 $\int \frac{\sec^3(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

3.118.1 Optimal result

Integrand size = 28, antiderivative size = 88

$$\int \frac{\sec^3(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = -\frac{(a^2+b^2)\log(\cos(c+dx))}{b^3d} + \frac{(a^2+b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{b^3d} + \frac{\sec^2(c+dx)}{2bd} - \frac{a\tan(c+dx)}{b^2d}$$

output
```
-(a^2+b^2)*ln(cos(d*x+c))/b^3/d+(a^2+b^2)*ln(a*cos(d*x+c)+b*sin(d*x+c))/b^
3/d+1/2*sec(d*x+c)^2/b/d-a*tan(d*x+c)/b^2/d
```

3.118.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.59

$$\int \frac{\sec^3(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \frac{(a^2+b^2)\log(a+b\tan(c+dx)) - ab\tan(c+dx) + \frac{1}{2}b^2\tan^2(c+dx)}{b^3d}$$

input
```
Integrate[Sec[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

output
```
((a^2 + b^2)*Log[a + b*Tan[c + d*x]] - a*b*Tan[c + d*x] + (b^2*Tan[c + d*x
]^2)/2)/(b^3*d)
```

3.118.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {3042, 3583, 3042, 3581, 3042, 3612, 3956, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(c+dx)^3(a\cos(c+dx)+b\sin(c+dx))}dx$$

$$\Big\downarrow \text{3583}$$

$$\frac{(a^2+b^2)\int \frac{\sec(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx}{b^2}-\frac{a\int \sec^2(c+dx)dx}{b^2}+\frac{\sec^2(c+dx)}{2bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a^2+b^2)\int \frac{1}{\cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))}dx}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx}{b^2}+\frac{\sec^2(c+dx)}{2bd}$$

$$\Big\downarrow \text{3581}$$

$$\frac{(a^2+b^2)\left(\frac{\int \frac{b\cos(c+dx)-a\sin(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx}{b}+\frac{\int \tan(c+dx)dx}{b}\right)}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx}{b^2}+\frac{\sec^2(c+dx)}{2bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a^2+b^2)\left(\frac{\int \frac{b\cos(c+dx)-a\sin(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx}{b}+\frac{\int \tan(c+dx)dx}{b}\right)}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx}{b^2}+\frac{\sec^2(c+dx)}{2bd}$$

$$\Big\downarrow \text{3612}$$

$$\frac{(a^2+b^2)\left(\frac{\int \tan(c+dx)dx}{b}+\frac{\log(a\cos(c+dx)+b\sin(c+dx))}{bd}\right)}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx}{b^2}+\frac{\sec^2(c+dx)}{2bd}$$

$$\Big\downarrow \text{3956}$$

$$-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx}{b^2}+\frac{(a^2+b^2)\left(\frac{\log(a\cos(c+dx)+b\sin(c+dx))}{bd}-\frac{\log(\cos(c+dx))}{bd}\right)}{b^2}+\frac{\sec^2(c+dx)}{2bd}$$

$$\Big\downarrow \text{4254}$$

$$\frac{a\int 1d(-\tan(c+dx))}{b^2d}+\frac{(a^2+b^2)\left(\frac{\log(a\cos(c+dx)+b\sin(c+dx))}{bd}-\frac{\log(\cos(c+dx))}{bd}\right)}{b^2}+\frac{\sec^2(c+dx)}{2bd}$$

$$\Big\downarrow \text{24}$$

$$\frac{(a^2+b^2)\left(\frac{\log(a\cos(c+dx)+b\sin(c+dx))}{bd}-\frac{\log(\cos(c+dx))}{bd}\right)}{b^2}-\frac{a\tan(c+dx)}{b^2d}+\frac{\sec^2(c+dx)}{2bd}$$

input
```
Int[Sec[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

output
```
((a^2 + b^2)*(-(Log[Cos[c + d*x]]/(b*d)) + Log[a*Cos[c + d*x] + b*Sin[c +
d*x]]/(b*d)))/b^2 + Sec[c + d*x]^2/(2*b*d) - (a*Tan[c + d*x])/(b^2*d)
```

3.118.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3581
```
Int[1/(cos[(c_.) + (d_.)*(x_)]*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(
c_.) + (d_.)*(x_)])), x_Symbol] :> Simp[1/b   Int[Tan[c + d*x], x], x] + Si
mp[1/b   Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c +
d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3583
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-Cos[c + d*x]^(m + 1)/(b*d*(m + 1)
), x] + (-Simp[a/b^2   Int[Cos[c + d*x]^(m + 1), x], x] + Simp[(a^2 + b^2)/
b^2   Int[Cos[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

rule 3612
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.118.4 Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.60

method	result
derivativedivides	$\frac{-\frac{-\frac{b\tan(dx+c)^2}{2}+a\tan(dx+c)}{b^2}+\frac{(a^2+b^2)\ln(a+b\tan(dx+c))}{b^3}}{d}$
default	$\frac{-\frac{-\frac{b\tan(dx+c)^2}{2}+a\tan(dx+c)}{b^2}+\frac{(a^2+b^2)\ln(a+b\tan(dx+c))}{b^3}}{d}$
parallelrisch	$\frac{2(a^2+b^2)(1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)-2(a^2+b^2)(1+\cos(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2b^3d(1+\cos(2dx+2c))}$
risch	$\frac{-2ia\,e^{2i(dx+c)}+2b\,e^{2i(dx+c)}-2ia}{b^2d(e^{2i(dx+c)}+1)^2}-\frac{\ln(e^{2i(dx+c)}+1)a^2}{b^3d}-\frac{\ln(e^{2i(dx+c)}+1)}{bd}+\frac{\ln\left(e^{2i(dx+c)}-\frac{ib+a}{ib-a}\right)a^2}{b^3d}+\frac{\ln\left(e^{2i(d\right.}}{}$
norman	$\frac{-\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{b^2d}+\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{b^2d}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{bd}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)^2}+\frac{(a^2+b^2)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{b^3d}-\frac{(a^2+b^2)\,l}{}$

input
```
int(sec(d*x+c)^3/(cos(d*x+c)*a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/b^2*(-1/2*b*tan(d*x+c)^2+a*tan(d*x+c))+(a^2+b^2)/b^3*ln(a+b*tan(d*
x+c)))
```

3.118.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.33

$$\int \frac{\sec^3(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$$
$$= \frac{(a^2+b^2)\cos{(dx+c)}^2\log{(2\,ab\cos{(dx+c)}\sin{(dx+c)}+(a^2-b^2)\cos{(dx+c)}^2+b^2)}-(a^2+b^2)\cos{(dx}}{2\,b^3d\cos{(dx+c)}^2}$$

input
```
integrate(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*((a^2 + b^2)*cos(d*x + c)^2*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2
 - b^2)*cos(d*x + c)^2 + b^2) - (a^2 + b^2)*cos(d*x + c)^2*log(cos(d*x + c
)^2) - 2*a*b*cos(d*x + c)*sin(d*x + c) + b^2)/(b^3*d*cos(d*x + c)^2)
```

3.118.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \int \frac{\sec^3{(c+dx)}}{a\cos{(c+dx)}+b\sin{(c+dx)}}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**3/(a*cos(c + d*x) + b*sin(c + d*x)), x)
```

3.118.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(86) = 172.

Time = 0.24 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.70

$$\int \frac{\sec^3(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx =$$
$$-\frac{\frac{2\left(\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\frac{b\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{b^2-\frac{2\,b^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{b^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{(a^2+b^2)\log\left(-a-\frac{2\,b\sin(dx+c)}{\cos(dx+c)+1}+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{b^3}+\frac{(a^2+b^2)\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{b^3}}{d}$$

input
```
integrate(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-(2*(a*sin(d*x + c)/(cos(d*x + c) + 1) - b*sin(d*x + c)^2/(cos(d*x + c) +
1)^2 - a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/(b^2 - 2*b^2*sin(d*x + c)^2/
(cos(d*x + c) + 1)^2 + b^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) - (a^2 + b
^2)*log(-a - 2*b*sin(d*x + c)/(cos(d*x + c) + 1) + a*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2)/b^3 + (a^2 + b^2)*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)
/b^3 + (a^2 + b^2)*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/b^3)/d
```

3.118.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.61

$$\int \frac{\sec^3(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \frac{\frac{b\tan(dx+c)^2-2\,a\tan(dx+c)}{b^2}+\frac{2\,(a^2+b^2)\log(|b\tan(dx+c)+a|)}{b^3}}{2\,d}$$

input
```
integrate(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
1/2*((b*tan(d*x + c)^2 - 2*a*tan(d*x + c))/b^2 + 2*(a^2 + b^2)*log(abs(b*t
an(d*x + c) + a))/b^3)/d
```

3.118.9 Mupad [B] (verification not implemented)

Time = 23.13 (sec) , antiderivative size = 300, normalized size of antiderivative = 3.41

$$\int \frac{\sec^3(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$$
$$= \frac{2\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-2\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(b^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\,b^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+b^3\right)}$$
$$-\frac{a^2\,\mathrm{atan}\left(\frac{b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,1{}\mathrm{i}-b^2\,1{}\mathrm{i}+2{}\mathrm{i}\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{-2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,a^2+2\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+b^2}\right)\,2{}\mathrm{i}+b^2\,\mathrm{atan}\left(\frac{b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,1{}\mathrm{i}-b^2\,1{}\mathrm{i}+ \cdots}{-2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,a^2+2\,a\,b\tan(\cdots}\right.}{b^3\,d}$$

input
```
int(1/(cos(c + d*x)^3*(a*cos(c + d*x) + b*sin(c + d*x))),x)
```

output
```
(2*b^2*tan(c/2 + (d*x)/2)^2 + 2*a*b*tan(c/2 + (d*x)/2)^3 - 2*a*b*tan(c/2 +
 (d*x)/2))/(d*(b^3*tan(c/2 + (d*x)/2)^4 - 2*b^3*tan(c/2 + (d*x)/2)^2 + b^3
)) - (a^2*atan((b^2*tan(c/2 + (d*x)/2)^2*1i - b^2*1i + a*b*tan(c/2 + (d*x)
/2)*2i)/(2*a^2 - b^2*tan(c/2 + (d*x)/2)^2 - 2*a^2*tan(c/2 + (d*x)/2)^2 + b
^2 + 2*a*b*tan(c/2 + (d*x)/2)))*2i + b^2*atan((b^2*tan(c/2 + (d*x)/2)^2*1i
 - b^2*1i + a*b*tan(c/2 + (d*x)/2)*2i)/(2*a^2 - b^2*tan(c/2 + (d*x)/2)^2 -
 2*a^2*tan(c/2 + (d*x)/2)^2 + b^2 + 2*a*b*tan(c/2 + (d*x)/2)))*2i)/(b^3*d)
```

3.118. $\int \frac{\sec^3(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

3.119 $\int \frac{\sec^4(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

3.119.1 Optimal result

Integrand size = 28, antiderivative size = 153

$$\begin{aligned}\int \frac{\sec^4(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx =& -\frac{a\text{arctanh}(\sin(c+dx))}{2b^2d} \\ &- \frac{a(a^2+b^2)\,\text{arctanh}(\sin(c+dx))}{b^4d} \\ &- \frac{(a^2+b^2)^{3/2}\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^4d} \\ &+ \frac{(a^2+b^2)\sec(c+dx)}{b^3d} + \frac{\sec^3(c+dx)}{3bd} \\ &- \frac{a\sec(c+dx)\tan(c+dx)}{2b^2d}\end{aligned}$$

output
```
-1/2*a*arctanh(sin(d*x+c))/b^2/d-a*(a^2+b^2)*arctanh(sin(d*x+c))/b^4/d-(a^
2+b^2)^(3/2)*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/b^4/d+(a
^2+b^2)*sec(d*x+c)/b^3/d+1/3*sec(d*x+c)^3/b/d-1/2*a*sec(d*x+c)*tan(d*x+c)/
b^2/d
```

3.119.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 321 vs. $2(153) = 306$.

Time = 2.15 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.10

$$\int \frac{\sec^4(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$$
$$= \frac{48(a^2+b^2)^{3/2}\operatorname{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)+\sec^3(c+dx)\left(12a^2b+20b^3+12b(a^2+b^2)\cos(2(c+dx))+6a\right.}{}$$

input
```
Integrate[Sec[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

output
```
(48*(a^2 + b^2)^(3/2)*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] +
 Sec[c + d*x]^3*(12*a^2*b + 20*b^3 + 12*b*(a^2 + b^2)*Cos[2*(c + d*x)] + 6
*a^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 9*a*b^2*C
os[3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 9*a*(2*a^2 + 3*
b^2)*Cos[c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]]) - 6*a^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2
] + Sin[(c + d*x)/2]] - 9*a*b^2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]] - 6*a*b^2*Sin[2*(c + d*x)]))/(24*b^4*d)
```

3.119.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3583, 3042, 3583, 3042, 3553, 219, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\cos(c+dx)^4(a\cos(c+dx)+b\sin(c+dx))}dx$$
$$\Big\downarrow \; 3583$$
$$\frac{(a^2+b^2)\int \frac{\sec^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx}{b^2}-\frac{a\int \sec^3(c+dx)dx}{b^2}+\frac{\sec^3(c+dx)}{3bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{\left(a^2+b^2\right)\int \frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))}dx}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{b^2}+\frac{\sec^3(c+dx)}{3bd}$$

$$\Big\downarrow \; 3583$$

$$\frac{\left(a^2+b^2\right)\left(\frac{\left(a^2+b^2\right)\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{b^2}-\frac{a\int \sec(c+dx)dx}{b^2}+\frac{\sec(c+dx)}{bd}\right)}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{b^2}+\frac{\sec^3(c+dx)}{3bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{\left(a^2+b^2\right)\left(\frac{\left(a^2+b^2\right)\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}+\frac{\sec(c+dx)}{bd}\right)}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{b^2}+\frac{\sec^3(c+dx)}{3bd}$$

$$\Big\downarrow \; 3553$$

$$\frac{\left(a^2+b^2\right)\left(-\frac{\left(a^2+b^2\right)\int \frac{1}{a^2+b^2-(b\cos(c+dx)-a\sin(c+dx))^2}d(b\cos(c+dx)-a\sin(c+dx))}{b^2d}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}+\frac{\sec(c+dx)}{bd}\right)}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{b^2}+\frac{\sec^3(c+dx)}{3bd}$$

$$\Big\downarrow \; 219$$

$$\frac{\left(a^2+b^2\right)\left(-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{\sqrt{a^2+b^2}\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d}+\frac{\sec(c+dx)}{bd}\right)}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{b^2}+\frac{\sec^3(c+dx)}{3bd}$$

$$\Big\downarrow \; 4255$$

$$\frac{\left(a^2+b^2\right)\left(-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{\sqrt{a^2+b^2}\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d}+\frac{\sec(c+dx)}{bd}\right)}{b^2}-\frac{a\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{b^2}+\frac{\sec^3(c+dx)}{3bd}$$

$$\Big\downarrow \; 3042$$

3.119. $\int \frac{\sec^4(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx$

$$\frac{(a^2+b^2)\left(-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d}+\frac{\sec(c+dx)}{bd}\right)}{b^2}-\frac{a\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{b^2}+\frac{\sec^3(c+dx)}{3bd}$$

$$\Big\downarrow \text{4257}$$

$$\frac{(a^2+b^2)\left(-\frac{\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d}-\frac{a\operatorname{arctanh}(\sin(c+dx))}{b^2d}+\frac{\sec(c+dx)}{bd}\right)}{b^2}-\frac{a\left(\frac{\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{b^2}+\frac{\sec^3(c+dx)}{3bd}$$

input
```
Int[Sec[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

output
```
Sec[c + d*x]^3/(3*b*d) + ((a^2 + b^2)*(-((a*ArcTanh[Sin[c + d*x]])/(b^2*d)
) - (Sqrt[a^2 + b^2]*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 +
b^2]])/(b^2*d) + Sec[c + d*x]/(b*d)))/b^2 - (a*(ArcTanh[Sin[c + d*x]]/(2*d
) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/b^2
```

3.119.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3583
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-Cos[c + d*x]^(m + 1)/(b*d*(m + 1)
), x] + (-Simp[a/b^2   Int[Cos[c + d*x]^(m + 1), x], x] + Simp[(a^2 + b^2)/
b^2   Int[Cos[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.119.4 Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.76

method	result
derivativedivides	$\frac{-\frac{1}{3b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{a+b}{2b^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{2a^{2}+ab+3b^{2}}{2b^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\left(2a^{2}+3b^{2}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2b^{4}}+\frac{1}{3b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$
default	$\frac{-\frac{1}{3b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{a+b}{2b^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{2a^{2}+ab+3b^{2}}{2b^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\left(2a^{2}+3b^{2}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2b^{4}}+\frac{1}{3b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$
risch	$\frac{e^{i(dx+c)}\left(3iab\,e^{4i(dx+c)}+6a^{2}e^{4i(dx+c)}+6b^{2}e^{4i(dx+c)}+12a^{2}e^{2i(dx+c)}+20b^{2}e^{2i(dx+c)}-3iba+6a^{2}+6b^{2}\right)}{3d\,b^{3}\left(e^{2i(dx+c)}+1\right)^{3}}+\frac{a^{3}\ln\left(e^{i(dx+c}\right.}{b^{4}d}$

input
```
int(sec(d*x+c)^4/(cos(d*x+c)*a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/3/b/(tan(1/2*d*x+1/2*c)-1)^3-1/2*(a+b)/b^2/(tan(1/2*d*x+1/2*c)-1)^
2-1/2*(2*a^2+a*b+3*b^2)/b^3/(tan(1/2*d*x+1/2*c)-1)+1/2*a*(2*a^2+3*b^2)/b^4
*ln(tan(1/2*d*x+1/2*c)-1)+1/3/b/(tan(1/2*d*x+1/2*c)+1)^3-1/2*(-a+b)/b^2/(t
an(1/2*d*x+1/2*c)+1)^2-1/2*(-2*a^2+a*b-3*b^2)/b^3/(tan(1/2*d*x+1/2*c)+1)-1
/2*a*(2*a^2+3*b^2)/b^4*ln(tan(1/2*d*x+1/2*c)+1)-2/b^4*(-a^4-2*a^2*b^2-b^4)
/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))
)
```

3.119.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.69

$$\int \frac{\sec^4(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \frac{6\left(a^2+b^2\right)^{\frac{3}{2}}\cos\left(dx+c\right)^3\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2-2\,a^2-b^2+2\sqrt{a^2+b^2}(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2}\right) -}{}$$

input
```
integrate(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/12*(6*(a^2 + b^2)^(3/2)*cos(d*x + c)^3*log(-(2*a*b*cos(d*x + c)*sin(d*x
+ c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos
(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2
)*cos(d*x + c)^2 + b^2)) - 3*(2*a^3 + 3*a*b^2)*cos(d*x + c)^3*log(sin(d*x
+ c) + 1) + 3*(2*a^3 + 3*a*b^2)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) - 6*
a*b^2*cos(d*x + c)*sin(d*x + c) + 4*b^3 + 12*(a^2*b + b^3)*cos(d*x + c)^2)
/(b^4*d*cos(d*x + c)^3)
```

3.119.6 Sympy [F]

$$\int \frac{\sec^4(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \int \frac{\sec^4\left(c+dx\right)}{a\cos\left(c+dx\right)+b\sin\left(c+dx\right)}\,dx$$

input
```
integrate(sec(d*x+c)**4/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**4/(a*cos(c + d*x) + b*sin(c + d*x)), x)
```

3.119.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 361 vs. $2(143) = 286$.

Time = 0.32 (sec) , antiderivative size = 361, normalized size of antiderivative = 2.36

$$\int \frac{\sec^4(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \frac{\frac{2\left(6a^2+8b^2-\frac{3ab\sin(dx+c)}{\cos(dx+c)+1}+\frac{3ab\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{12(a^2+b^2)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6(a^2+2b^2)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)}{b^3-\frac{3b^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3b^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{b^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}-\frac{3(2a^3+3ab^2)\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{b^4}+\frac{3(2a}{}}{6d}$$

input
```
integrate(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
1/6*(2*(6*a^2 + 8*b^2 - 3*a*b*sin(d*x + c)/(cos(d*x + c) + 1) + 3*a*b*sin(
d*x + c)^5/(cos(d*x + c) + 1)^5 - 12*(a^2 + b^2)*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 6*(a^2 + 2*b^2)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)/(b^3 - 3
*b^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*b^3*sin(d*x + c)^4/(cos(d*x +
 c) + 1)^4 - b^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) - 3*(2*a^3 + 3*a*b^2
)*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/b^4 + 3*(2*a^3 + 3*a*b^2)*log(s
in(d*x + c)/(cos(d*x + c) + 1) - 1)/b^4 - 6*(a^4 + 2*a^2*b^2 + b^4)*log((b
 - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*sin(d*x + c
)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^4))/d
```

3.119.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.82

$$\int \frac{\sec^4(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = -\frac{\frac{3(2a^3+3ab^2)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{b^4}-\frac{3(2a^3+3ab^2)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{b^4}+\frac{6(a^4+2a^2b^2+b^4)\log\left(\frac{\left|2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2b-2\sqrt{a^2}\right.}{\left|2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2b+2\sqrt{a^2}\right.}\right)}{\sqrt{a^2+b^2}b^4}}{}$$

input
```
integrate(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
-1/6*(3*(2*a^3 + 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^4 - 3*(2*a^
3 + 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^4 + 6*(a^4 + 2*a^2*b^2 +
 b^4)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*
tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^4) + 2
*(3*a*b*tan(1/2*d*x + 1/2*c)^5 + 6*a^2*tan(1/2*d*x + 1/2*c)^4 + 12*b^2*tan
(1/2*d*x + 1/2*c)^4 - 12*a^2*tan(1/2*d*x + 1/2*c)^2 - 12*b^2*tan(1/2*d*x +
 1/2*c)^2 - 3*a*b*tan(1/2*d*x + 1/2*c) + 6*a^2 + 8*b^2)/((tan(1/2*d*x + 1/
2*c)^2 - 1)^3*b^3))/d
```

3.119.9 Mupad [B] (verification not implemented)

Time = 24.30 (sec) , antiderivative size = 724, normalized size of antiderivative = 4.73

$$\int \frac{\sec^4(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \frac{b^3\left(\cos\left(c+d\,x\right)+\frac{\cos(2\,c+2\,d\,x)}{2}+\frac{\cos(3\,c+3\,d\,x)}{3}+\frac{5}{6}\right)-b^2\left(\frac{a\,\sin(2\,c+2\,d\,x)}{4}+\frac{3\,a\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,\cos(3\,c+3\,d\,x)}{4}+\cdots\right.}{\cdots}$$

input
```
int(1/(cos(c + d*x)^4*(a*cos(c + d*x) + b*sin(c + d*x))),x)
```

output
```
(b^3*(cos(c + d*x) + cos(2*c + 2*d*x)/2 + cos(3*c + 3*d*x)/3 + 5/6) - b^2*
((a*sin(2*c + 2*d*x))/4 + (3*a*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)
)*cos(3*c + 3*d*x))/4 + (9*a*cos(c + d*x)*atanh(sin(c/2 + (d*x)/2)/cos(c/2
 + (d*x)/2)))/4) + b*((3*a^2*cos(c + d*x))/4 + a^2/2 + (a^2*cos(2*c + 2*d*
x))/2 + (a^2*cos(3*c + 3*d*x))/4) + (atanh((a^2*sin(c/2 + (d*x)/2)*(a^6 +
b^6 + 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 2*b^2*sin(c/2 + (d*x)/2)*(a^6 + b^6 +
 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + a*b*cos(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2*
b^4 + 3*a^4*b^2)^(1/2))/(a^5*cos(c/2 + (d*x)/2) + 2*b^5*sin(c/2 + (d*x)/2)
 + a*b^4*cos(c/2 + (d*x)/2) + 2*a^4*b*sin(c/2 + (d*x)/2) + 2*a^3*b^2*cos(c
/2 + (d*x)/2) + 4*a^2*b^3*sin(c/2 + (d*x)/2)))*cos(3*c + 3*d*x)*((a^2 + b^
2)^3)^(1/2))/2 - (3*a^3*cos(c + d*x)*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d
*x)/2)))/2 - (a^3*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(3*c + 3
*d*x))/2 + (3*cos(c + d*x)*atanh((a^2*sin(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^
2*b^4 + 3*a^4*b^2)^(1/2) + 2*b^2*sin(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2*b^4
 + 3*a^4*b^2)^(1/2) + a*b*cos(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^
4*b^2)^(1/2))/(a^5*cos(c/2 + (d*x)/2) + 2*b^5*sin(c/2 + (d*x)/2) + a*b^4*c
os(c/2 + (d*x)/2) + 2*a^4*b*sin(c/2 + (d*x)/2) + 2*a^3*b^2*cos(c/2 + (d*x)
/2) + 4*a^2*b^3*sin(c/2 + (d*x)/2)))*((a^2 + b^2)^3)^(1/2))/2)/(b^4*d*((3*
cos(c + d*x))/4 + cos(3*c + 3*d*x)/4))
```

3.119. $\int \frac{\sec^4(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

3.120 $\int \frac{\sec^5(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

3.120.1 Optimal result

Integrand size = 28, antiderivative size = 158

$$\int \frac{\sec^5(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = -\frac{(a^2+b^2)^2\log(\cos(c+dx))}{b^5d} + \frac{(a^2+b^2)^2\log(a\cos(c+dx)+b\sin(c+dx))}{b^5d} + \frac{(a^2+b^2)\sec^2(c+dx)}{2b^3d} + \frac{\sec^4(c+dx)}{4bd} - \frac{a\tan(c+dx)}{b^2d} - \frac{a(a^2+b^2)\tan(c+dx)}{b^4d} - \frac{a\tan^3(c+dx)}{3b^2d}$$

output
```
-(a^2+b^2)^2*ln(cos(d*x+c))/b^5/d+(a^2+b^2)^2*ln(a*cos(d*x+c)+b*sin(d*x+c)
)/b^5/d+1/2*(a^2+b^2)*sec(d*x+c)^2/b^3/d+1/4*sec(d*x+c)^4/b/d-a*tan(d*x+c)
/b^2/d-a*(a^2+b^2)*tan(d*x+c)/b^4/d-1/3*a*tan(d*x+c)^3/b^2/d
```

3.120.2 Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.63

$$\int \frac{\sec^5(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$$
$$=\frac{12(a^2+b^2)^2\log(a+b\tan(c+dx))+3b^4\sec^4(c+dx)-12ab(a^2+2b^2)\tan(c+dx)+6b^2(a^2+b^2)\tan^2(c}{12b^5d}$$

input
```
Integrate[Sec[c + d*x]^5/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

output
```
(12*(a^2 + b^2)^2*Log[a + b*Tan[c + d*x]] + 3*b^4*Sec[c + d*x]^4 - 12*a*b*
(a^2 + 2*b^2)*Tan[c + d*x] + 6*b^2*(a^2 + b^2)*Tan[c + d*x]^2 - 4*a*b^3*Ta
n[c + d*x]^3)/(12*b^5*d)
```

3.120.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.92, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3583, 3042, 3583, 3042, 3581, 3042, 3612, 3956, 4254, 24, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(c+dx)^5(a\cos(c+dx)+b\sin(c+dx))}dx$$

$$\Big\downarrow \text{3583}$$

$$\frac{(a^2+b^2)\int \frac{\sec^3(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx}{b^2}-\frac{a\int \sec^4(c+dx)dx}{b^2}+\frac{\sec^4(c+dx)}{4bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a^2+b^2)\int \frac{1}{\cos(c+dx)^3(a\cos(c+dx)+b\sin(c+dx))}dx}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx}{b^2}+\frac{\sec^4(c+dx)}{4bd}$$

$$\Big\downarrow \text{3583}$$

$$\frac{(a^2+b^2)\left(\frac{(a^2+b^2)\int \frac{\sec(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx}{b^2}-\frac{a\int \sec^2(c+dx)dx}{b^2}+\frac{\sec^2(c+dx)}{2bd}\right)}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx}{b^2}+\frac{\sec^4(c+dx)}{4bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a^2+b^2)\left(\frac{(a^2+b^2)\int \frac{1}{\cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))}dx}{b^2}-\frac{a\int \csc(c+dx+\frac{\pi}{2})^2dx}{b^2}+\frac{\sec^2(c+dx)}{2bd}\right)}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx}{b^2}+\frac{\sec^4(c+dx)}{4bd}$$

$$\Big\downarrow \text{3581}$$

$$\frac{(a^2+b^2)\left(\frac{(a^2+b^2)\left(\frac{\int \frac{b\cos(c+dx)-a\sin(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx}{b}+\frac{\int \tan(c+dx)dx}{b}\right)}{b^2}-\frac{a\int \csc(c+dx+\frac{\pi}{2})^2dx}{b^2}+\frac{\sec^2(c+dx)}{2bd}\right)}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx}{b^2}+\frac{\sec^4(c+dx)}{4bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a^2+b^2)\left(\frac{(a^2+b^2)\left(\frac{\int \frac{b\cos(c+dx)-a\sin(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx}{b}+\frac{\int \tan(c+dx)dx}{b}\right)}{b^2}-\frac{a\int \csc(c+dx+\frac{\pi}{2})^2dx}{b^2}+\frac{\sec^2(c+dx)}{2bd}\right)}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx}{b^2}+\frac{\sec^4(c+dx)}{4bd}$$

$$\Big\downarrow \text{3612}$$

$$\frac{(a^2+b^2)\left(\frac{(a^2+b^2)\left(\frac{\int \tan(c+dx)dx}{b}+\frac{\log(a\cos(c+dx)+b\sin(c+dx))}{bd}\right)}{b^2}-\frac{a\int \csc(c+dx+\frac{\pi}{2})^2dx}{b^2}+\frac{\sec^2(c+dx)}{2bd}\right)}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx}{b^2}+\frac{\sec^4(c+dx)}{4bd}$$

$$\Big\downarrow \text{3956}$$

$$\frac{(a^2+b^2)\left(-\frac{a\int \csc(c+dx+\frac{\pi}{2})^2dx}{b^2}+\frac{(a^2+b^2)\left(\frac{\log(a\cos(c+dx)+b\sin(c+dx))}{bd}-\frac{\log(\cos(c+dx))}{bd}\right)}{b^2}+\frac{\sec^2(c+dx)}{2bd}\right)}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^4dx}{b^2}+\frac{\sec^4(c+dx)}{4bd}$$

3.120. $\int \frac{\sec^5(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx$

$$\Big\downarrow \text{4254}$$

$$\frac{(a^2+b^2)\left(\frac{a\int 1d(-\tan(c+dx))}{b^2d}+\frac{(a^2+b^2)\left(\frac{\log(a\cos(c+dx)+b\sin(c+dx))}{bd}-\frac{\log(\cos(c+dx))}{bd}\right)}{b^2}+\frac{\sec^2(c+dx)}{2bd}\right)}{b^2}+\frac{a\int\left(\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{b^2d}+\frac{\sec^4(c+dx)}{4bd}$$

$$\Big\downarrow \text{24}$$

$$\frac{a\int\left(\tan^2(c+dx)+1\right)d(-\tan(c+dx))}{b^2d}+\frac{(a^2+b^2)\left(\frac{(a^2+b^2)\left(\frac{\log(a\cos(c+dx)+b\sin(c+dx))}{bd}-\frac{\log(\cos(c+dx))}{bd}\right)}{b^2}-\frac{a\tan(c+dx)}{b^2d}+\frac{\sec^2(c+dx)}{2bd}\right)}{b^2}+\frac{\sec^4(c+dx)}{4bd}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a^2+b^2)\left(\frac{(a^2+b^2)\left(\frac{\log(a\cos(c+dx)+b\sin(c+dx))}{bd}-\frac{\log(\cos(c+dx))}{bd}\right)}{b^2}-\frac{a\tan(c+dx)}{b^2d}+\frac{\sec^2(c+dx)}{2bd}\right)}{b^2}+\frac{a\left(-\frac{1}{3}\tan^3(c+dx)-\tan(c+dx)\right)}{b^2d}+\frac{\sec^4(c+dx)}{4bd}$$

input
```
Int[Sec[c + d*x]^5/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

output
```
Sec[c + d*x]^4/(4*b*d) + ((a^2 + b^2)*(((a^2 + b^2)*(-(Log[Cos[c + d*x]]/(
b*d)) + Log[a*Cos[c + d*x] + b*Sin[c + d*x]]/(b*d)))/b^2 + Sec[c + d*x]^2/
(2*b*d) - (a*Tan[c + d*x])/(b^2*d)))/b^2 + (a*(-Tan[c + d*x] - Tan[c + d*x
]^3/3))/(b^2*d)
```

3.120.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.120. $\int \frac{\sec^5(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

rule 3581
```
Int[1/(cos[(c_.) + (d_.)*(x_)]*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(
c_.) + (d_.)*(x_)])), x_Symbol] :> Simp[1/b   Int[Tan[c + d*x], x], x] + Si
mp[1/b   Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c +
d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3583
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-Cos[c + d*x]^(m + 1)/(b*d*(m + 1)
), x] + (-Simp[a/b^2   Int[Cos[c + d*x]^(m + 1), x], x] + Simp[(a^2 + b^2)/
b^2   Int[Cos[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

rule 3612
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.120.4 Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.67

method	result
derivativedivides	$\frac{-\frac{-\frac{\tan(dx+c)^4 b^3}{4}+\frac{a\tan(dx+c)^3 b^2}{3}-\frac{(a^2+2b^2)\tan(dx+c)^2 b}{2}+\tan(dx+c)a(a^2+2b^2)}{b^4}+\frac{(a^4+2a^2b^2+b^4)\ln(a+b\tan(dx+c))}{b^5}}{d}$
default	$\frac{-\frac{-\frac{\tan(dx+c)^4 b^3}{4}+\frac{a\tan(dx+c)^3 b^2}{3}-\frac{(a^2+2b^2)\tan(dx+c)^2 b}{2}+\tan(dx+c)a(a^2+2b^2)}{b^4}+\frac{(a^4+2a^2b^2+b^4)\ln(a+b\tan(dx+c))}{b^5}}{d}$
parallelrisch	$\frac{48(a^2+b^2)^2\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\cos(2dx+2c)\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)-48(a^2+b^2)^2\left(\frac{3}{4}+\frac{\cos(4dx+4c)}{4}+\text{c}\right.}{}$
norman	$\frac{-\frac{2(2a^2+2b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{b^3 d}+\frac{2(a^2+2b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{b^3 d}+\frac{2(a^2+2b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{b^3 d}-\frac{2a(a^2+2b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{b^4 d}+\frac{2a(a^2+2b^2)\text{ta}}{b^4 d}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)^4}$
risch	$\frac{-2ia^3 e^{6i(dx+c)}-2iab^2 e^{6i(dx+c)}+2a^2 b e^{6i(dx+c)}+2b^3 e^{6i(dx+c)}-6ia^3 e^{4i(dx+c)}-10iab^2 e^{4i(dx+c)}+4a^2 b e^{4i(dx+c)}+8b^3 e}{b^4 d(e^{2i(dx+c)}+1)^4}$

input
```
int(sec(d*x+c)^5/(cos(d*x+c)*a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/b^4*(-1/4*tan(d*x+c)^4*b^3+1/3*a*tan(d*x+c)^3*b^2-1/2*(a^2+2*b^2)*
tan(d*x+c)^2*b+tan(d*x+c)*a*(a^2+2*b^2))+(a^4+2*a^2*b^2+b^4)/b^5*ln(a+b*ta
n(d*x+c)))
```

3.120.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.16

$$\int \frac{\sec^5(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$$
$$= \frac{6\,(a^4+2\,a^2b^2+b^4)\cos(dx+c)^4\log\left(2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2\right)-6\,(a^4}{}$$

input
```
integrate(sec(d*x+c)^5/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/12*(6*(a^4 + 2*a^2*b^2 + b^4)*cos(d*x + c)^4*log(2*a*b*cos(d*x + c)*sin(
d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 6*(a^4 + 2*a^2*b^2 + b^4)*c
os(d*x + c)^4*log(cos(d*x + c)^2) + 3*b^4 + 6*(a^2*b^2 + b^4)*cos(d*x + c)
^2 - 4*(a*b^3*cos(d*x + c) + (3*a^3*b + 5*a*b^3)*cos(d*x + c)^3)*sin(d*x +
 c))/(b^5*d*cos(d*x + c)^4)
```

3.120.6 Sympy [F]

$$\int \frac{\sec^5(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \int \frac{\sec^5(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**5/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**5/(a*cos(c + d*x) + b*sin(c + d*x)), x)
```

3.120.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 462 vs. $2(152) = 304$.

Time = 0.24 (sec) , antiderivative size = 462, normalized size of antiderivative = 2.92

$$\int \frac{\sec^5(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = -\frac{\frac{2\left(\frac{3(a^3+2ab^2)\sin(dx+c)}{\cos(dx+c)+1}-\frac{3(a^2b+2b^3)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{(9a^3+14ab^2)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{6(a^2b+b^3)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{(9a^3+14ab^2)\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{3(a^2b+2b^3)\sin\cdots}{(\cos(dx+c)\cdots}\cdots}{b^4-\frac{4b^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6b^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4b^4\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{b^4\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}\cdots}{\cdots}$$

input
```
integrate(sec(d*x+c)^5/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/3*(2*(3*(a^3 + 2*a*b^2)*sin(d*x + c)/(cos(d*x + c) + 1) - 3*(a^2*b + 2*
b^3)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - (9*a^3 + 14*a*b^2)*sin(d*x + c)
^3/(cos(d*x + c) + 1)^3 + 6*(a^2*b + b^3)*sin(d*x + c)^4/(cos(d*x + c) + 1
)^4 + (9*a^3 + 14*a*b^2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 3*(a^2*b +
2*b^3)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 3*(a^3 + 2*a*b^2)*sin(d*x + c
)^7/(cos(d*x + c) + 1)^7)/(b^4 - 4*b^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
 + 6*b^4*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 4*b^4*sin(d*x + c)^6/(cos(d
*x + c) + 1)^6 + b^4*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) - 3*(a^4 + 2*a^2
*b^2 + b^4)*log(-a - 2*b*sin(d*x + c)/(cos(d*x + c) + 1) + a*sin(d*x + c)^
2/(cos(d*x + c) + 1)^2)/b^5 + 3*(a^4 + 2*a^2*b^2 + b^4)*log(sin(d*x + c)/(
cos(d*x + c) + 1) + 1)/b^5 + 3*(a^4 + 2*a^2*b^2 + b^4)*log(sin(d*x + c)/(c
os(d*x + c) + 1) - 1)/b^5)/d
```

3.120.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.76

$$\int \frac{\sec^5(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$$
$$= \frac{\frac{3\,b^3\tan(dx+c)^4-4\,ab^2\tan(dx+c)^3+6\,a^2b\tan(dx+c)^2+12\,b^3\tan(dx+c)^2-12\,a^3\tan(dx+c)-24\,ab^2\tan(dx+c)}{b^4}+\frac{12\,(a^4+2\,a^2b^2+b^4)\log(|b\tan(}{b^5}}{12\,d}$$

input
```
integrate(sec(d*x+c)^5/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
1/12*((3*b^3*tan(d*x + c)^4 - 4*a*b^2*tan(d*x + c)^3 + 6*a^2*b*tan(d*x + c
)^2 + 12*b^3*tan(d*x + c)^2 - 12*a^3*tan(d*x + c) - 24*a*b^2*tan(d*x + c))
/b^4 + 12*(a^4 + 2*a^2*b^2 + b^4)*log(abs(b*tan(d*x + c) + a))/b^5)/d
```

3.120.9 Mupad [B] (verification not implemented)

Time = 25.69 (sec) , antiderivative size = 575, normalized size of antiderivative = 3.64

$$\int \frac{\sec^5(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$$
$$= \frac{(6\,a^3\,b+12\,a\,b^3)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+(6\,a^2\,b^2+12\,b^4)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+(-18\,a^3\,b-28\,a\,b^3)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+(-}{d\,\left(3\,b^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-12\,b^5\tan(\right.}$$
$$-\frac{a^4\,\mathrm{atan}\left(\frac{b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,1\mathrm{i}-b^2\,1\mathrm{i}+2\mathrm{i}\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{-2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,a^2+2\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+b^2}\right)\,2\mathrm{i}+b^4\,\mathrm{atan}\left(\frac{b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,1\mathrm{i}-b^2\,1\mathrm{i}+}{-2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,a^2+2\,a\,b\tan(}\right.}{b^5\,d}$$

input
```
int(1/(cos(c + d*x)^5*(a*cos(c + d*x) + b*sin(c + d*x))),x)
```

output
```
(tan(c/2 + (d*x)/2)^2*(12*b^4 + 6*a^2*b^2) - tan(c/2 + (d*x)/2)*(12*a*b^3
+ 6*a^3*b) + tan(c/2 + (d*x)/2)^6*(12*b^4 + 6*a^2*b^2) - tan(c/2 + (d*x)/2
)^4*(12*b^4 + 12*a^2*b^2) + tan(c/2 + (d*x)/2)^7*(12*a*b^3 + 6*a^3*b) + ta
n(c/2 + (d*x)/2)^3*(28*a*b^3 + 18*a^3*b) - tan(c/2 + (d*x)/2)^5*(28*a*b^3
+ 18*a^3*b))/(d*(18*b^5*tan(c/2 + (d*x)/2)^4 - 12*b^5*tan(c/2 + (d*x)/2)^2
 - 12*b^5*tan(c/2 + (d*x)/2)^6 + 3*b^5*tan(c/2 + (d*x)/2)^8 + 3*b^5)) - (a
^4*atan((b^2*tan(c/2 + (d*x)/2)^2*1i - b^2*1i + a*b*tan(c/2 + (d*x)/2)*2i)
/(2*a^2 - b^2*tan(c/2 + (d*x)/2)^2 - 2*a^2*tan(c/2 + (d*x)/2)^2 + b^2 + 2*
a*b*tan(c/2 + (d*x)/2)))*2i + b^4*atan((b^2*tan(c/2 + (d*x)/2)^2*1i - b^2*
1i + a*b*tan(c/2 + (d*x)/2)*2i)/(2*a^2 - b^2*tan(c/2 + (d*x)/2)^2 - 2*a^2*
tan(c/2 + (d*x)/2)^2 + b^2 + 2*a*b*tan(c/2 + (d*x)/2)))*2i + a^2*b^2*atan(
(b^2*tan(c/2 + (d*x)/2)^2*1i - b^2*1i + a*b*tan(c/2 + (d*x)/2)*2i)/(2*a^2
- b^2*tan(c/2 + (d*x)/2)^2 - 2*a^2*tan(c/2 + (d*x)/2)^2 + b^2 + 2*a*b*tan(
c/2 + (d*x)/2)))*4i)/(b^5*d)
```

3.120. $\int \frac{\sec^5(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

3.121 $\int \frac{\sec^6(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

3.121.1 Optimal result

Integrand size = 28, antiderivative size = 262

$$\begin{aligned}\int \frac{\sec^6(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = &-\frac{3a\text{arctanh}(\sin(c+dx))}{8b^2d}\\ &-\frac{a(a^2+b^2)\,\text{arctanh}(\sin(c+dx))}{2b^4d}\\ &-\frac{a(a^2+b^2)^2\,\text{arctanh}(\sin(c+dx))}{b^6d}\\ &-\frac{(a^2+b^2)^{5/2}\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^6d}\\ &+\frac{(a^2+b^2)^2\sec(c+dx)}{b^5d}+\frac{(a^2+b^2)\sec^3(c+dx)}{3b^3d}\\ &+\frac{\sec^5(c+dx)}{5bd}-\frac{3a\sec(c+dx)\tan(c+dx)}{8b^2d}\\ &-\frac{a(a^2+b^2)\sec(c+dx)\tan(c+dx)}{2b^4d}\\ &-\frac{a\sec^3(c+dx)\tan(c+dx)}{4b^2d}\end{aligned}$$

output
```
-3/8*a*arctanh(sin(d*x+c))/b^2/d-1/2*a*(a^2+b^2)*arctanh(sin(d*x+c))/b^4/d
-a*(a^2+b^2)^2*arctanh(sin(d*x+c))/b^6/d-(a^2+b^2)^(5/2)*arctanh((b*cos(d*
x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/b^6/d+(a^2+b^2)^2*sec(d*x+c)/b^5/d+1/3
*(a^2+b^2)*sec(d*x+c)^3/b^3/d+1/5*sec(d*x+c)^5/b/d-3/8*a*sec(d*x+c)*tan(d*
x+c)/b^2/d-1/2*a*(a^2+b^2)*sec(d*x+c)*tan(d*x+c)/b^4/d-1/4*a*sec(d*x+c)^3*
tan(d*x+c)/b^2/d
```

3.121.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 661 vs. $2(262) = 524$.

Time = 6.27 (sec) , antiderivative size = 661, normalized size of antiderivative = 2.52

$$\int \frac{\sec^6(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$$

$$= \frac{\sec(c+dx)\left(240a^4b+520a^2b^3+298b^5+480(a^2+b^2)^{5/2}\operatorname{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)+30a(8a^4+20a^2b^2\right.}{}$$

input
```
Integrate[Sec[c + d*x]^6/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

output
```
(Sec[c + d*x]*(240*a^4*b + 520*a^2*b^3 + 298*b^5 + 480*(a^2 + b^2)^(5/2)*A
rcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] + 30*a*(8*a^4 + 20*a^2*b
^2 + 15*b^4)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 30*a*(8*a^4 + 20*a
^2*b^2 + 15*b^4)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (3*b^4*(-5*a +
 2*b))/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^4 + (b^2*(-60*a^3 + 20*a^2*b
- 105*a*b^2 + 29*b^3))/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (12*b^5*S
in[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^5 + (2*b^3*(20*a^2
+ 29*b^2)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3 + (2*b
*(120*a^4 + 260*a^2*b^2 + 149*b^4)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - S
in[(c + d*x)/2]) - (12*b^5*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c +
d*x)/2])^5 + (3*b^4*(5*a + 2*b))/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 -
 (2*b^3*(20*a^2 + 29*b^2)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d
*x)/2])^3 + (b^2*(60*a^3 + 20*a^2*b + 105*a*b^2 + 29*b^3))/(Cos[(c + d*x)/
2] + Sin[(c + d*x)/2])^2 - (2*b*(120*a^4 + 260*a^2*b^2 + 149*b^4)*Sin[(c +
 d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))*(a*Cos[c + d*x] + b*Sin[c
 + d*x]))/(240*b^6*d*(a + b*Tan[c + d*x]))
```

3.121.3 Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.94, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3583, 3042, 3583, 3042, 3583, 3042, 3553, 219, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^6(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx$$

$\big\downarrow$ 3042

$$\int \frac{1}{\cos(c+dx)^6(a\cos(c+dx)+b\sin(c+dx))}dx$$

$\big\downarrow$ 3583

$$\frac{(a^2+b^2)\int \frac{\sec^4(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx}{b^2}-\frac{a\int \sec^5(c+dx)dx}{b^2}+\frac{\sec^5(c+dx)}{5bd}$$

$\big\downarrow$ 3042

$$\frac{(a^2+b^2)\int \frac{1}{\cos(c+dx)^4(a\cos(c+dx)+b\sin(c+dx))}dx}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx}{b^2}+\frac{\sec^5(c+dx)}{5bd}$$

$\big\downarrow$ 3583

$$\frac{(a^2+b^2)\left(\frac{(a^2+b^2)\int \frac{\sec^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx}{b^2}-\frac{a\int \sec^3(c+dx)dx}{b^2}+\frac{\sec^3(c+dx)}{3bd}\right)}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx}{b^2}+\frac{\sec^5(c+dx)}{5bd}$$

$\big\downarrow$ 3042

$$\frac{(a^2+b^2)\left(\frac{(a^2+b^2)\int \frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))}dx}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{b^2}+\frac{\sec^3(c+dx)}{3bd}\right)}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx}{b^2}+\frac{\sec^5(c+dx)}{5bd}$$

$\big\downarrow$ 3583

$$\frac{(a^2+b^2)\left(\frac{(a^2+b^2)\left(\frac{(a^2+b^2)\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{b^2}-\frac{a\int \sec(c+dx)dx}{b^2}+\frac{\sec(c+dx)}{bd}\right)}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{b^2}+\frac{\sec^3(c+dx)}{3bd}\right)}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx}{b^2}+\frac{\sec^5(c+dx)}{5bd}$$

$\big\downarrow$ 3042

3.121. $\int \frac{\sec^6(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx$

$$\frac{\left(a^2+b^2\right)\left(\frac{\left(a^2+b^2\right)\left(\frac{\left(a^2+b^2\right)\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}+\frac{\sec(c+dx)}{bd}\right)}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{b^2}+\frac{\sec^3(c+dx)}{3bd}\right)}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx}{b^2}+\frac{\sec^5(c+dx)}{5bd}$$

$\big\downarrow$ 3553

$$\frac{\left(a^2+b^2\right)\left(\frac{\left(a^2+b^2\right)\left(-\frac{\left(a^2+b^2\right)\int \frac{1}{a^2+b^2-(b\cos(c+dx)-a\sin(c+dx))^2}d(b\cos(c+dx)-a\sin(c+dx))}{b^2d}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}+\frac{\sec(c+dx)}{bd}\right)}{b^2}-\frac{a\int \csc\left(c+\right.}{b}\right.}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx}{b^2}+\frac{\sec^5(c+dx)}{5bd}$$

$\big\downarrow$ 219

$$\frac{\left(a^2+b^2\right)\left(\frac{\left(a^2+b^2\right)\left(-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{\sqrt{a^2+b^2}\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d}+\frac{\sec(c+dx)}{bd}\right)}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{b^2}+\frac{\sec^3(c+d}{3bd}\right.}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^5dx}{b^2}+\frac{\sec^5(c+dx)}{5bd}$$

$\big\downarrow$ 4255

$$\frac{\left(a^2+b^2\right)\left(\frac{\left(a^2+b^2\right)\left(-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{\sqrt{a^2+b^2}\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d}+\frac{\sec(c+dx)}{bd}\right)}{b^2}-\frac{a\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c}{2d}\right.}{b^2}\right.}{b^2}-\frac{a\left(\frac{3}{4}\int \sec^3(c+dx)dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{b^2}+\frac{\sec^5(c+dx)}{5bd}$$

$\big\downarrow$ 3042

3.121. $\int \frac{\sec^6(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

$$\frac{(a^2+b^2)\left(\frac{(a^2+b^2)\left(-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{\sqrt{a^2+b^2}\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d}+\frac{\sec(c+dx)}{bd}\right)}{b^2}-\frac{a\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)}{2\ldots}\right.}{b^2}\right.}{b^2}$$

$$\frac{a\left(\frac{3}{4}\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{b^2}+\frac{\sec^5(c+dx)}{5bd}$$

$\Big\downarrow$ 4255

$$\frac{(a^2+b^2)\left(\frac{(a^2+b^2)\left(-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{\sqrt{a^2+b^2}\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d}+\frac{\sec(c+dx)}{bd}\right)}{b^2}-\frac{a\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)}{2\ldots}\right.}{b^2}\right.}{b^2}$$

$$\frac{a\left(\frac{3}{4}\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{b^2}+\frac{\sec^5(c+dx)}{5bd}$$

$\Big\downarrow$ 3042

$$\frac{(a^2+b^2)\left(\frac{(a^2+b^2)\left(-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{\sqrt{a^2+b^2}\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d}+\frac{\sec(c+dx)}{bd}\right)}{b^2}-\frac{a\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)}{2\ldots}\right.}{b^2}\right.}{b^2}$$

$$\frac{a\left(\frac{3}{4}\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{b^2}+\frac{\sec^5(c+dx)}{5bd}$$

$\Big\downarrow$ 4257

$$\frac{(a^2+b^2)\left(\frac{(a^2+b^2)\left(-\frac{\sqrt{a^2+b^2}\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d}-\frac{a\text{arctanh}(\sin(c+dx))}{b^2d}+\frac{\sec(c+dx)}{bd}\right)}{b^2}-\frac{a\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c\ldots}{}\right.}{b^2}\right.}{b^2}$$

$$\frac{a\left(\frac{3}{4}\left(\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{\tan(c+dx)\sec^3(c+dx)}{4d}\right)}{b^2}+\frac{\sec^5(c+dx)}{5bd}$$

input
```
Int[Sec[c + d*x]^6/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

3.121. $\int \frac{\sec^6(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

output
```
Sec[c + d*x]^5/(5*b*d) - (a*((Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*(Arc
Tanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/4))/b^2 + (
(a^2 + b^2)*(Sec[c + d*x]^3/(3*b*d) + ((a^2 + b^2)*(-((a*ArcTanh[Sin[c + d
*x]])/(b^2*d)) - (Sqrt[a^2 + b^2]*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x]
)/Sqrt[a^2 + b^2]])/(b^2*d) + Sec[c + d*x]/(b*d)))/b^2 - (a*(ArcTanh[Sin[c
 + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/b^2))/b^2
```

3.121.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3583
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-Cos[c + d*x]^(m + 1)/(b*d*(m + 1)
), x] + (-Simp[a/b^2   Int[Cos[c + d*x]^(m + 1), x], x] + Simp[(a^2 + b^2)/
b^2   Int[Cos[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.121. $\int \frac{\sec^6(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

3.121.4 Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.83

method	result
derivativedivides	$-\frac{1}{5b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5}-\frac{a+2b}{4b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}-\frac{4a^2+6ab+13b^2}{12b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{4a^3+4a^2b+11a\,b^2+9b^3}{8b^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{8a^4+4a^3b+20a^2b^2+9a\,b^3+}{8b^5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$
default	$-\frac{1}{5b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5}-\frac{a+2b}{4b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}-\frac{4a^2+6ab+13b^2}{12b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{4a^3+4a^2b+11a\,b^2+9b^3}{8b^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{8a^4+4a^3b+20a^2b^2+9a\,b^3+}{8b^5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$
risch	$e^{i(dx+c)}\left(-330ia\,b^3e^{2i(dx+c)}+330ia\,b^3e^{6i(dx+c)}+120a^4e^{8i(dx+c)}+240a^2b^2e^{8i(dx+c)}+120b^4e^{8i(dx+c)}-60ia^3b+60ia^3b\,e\right.$

input
```
int(sec(d*x+c)^6/(cos(d*x+c)*a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/5/b/(tan(1/2*d*x+1/2*c)-1)^5-1/4*(a+2*b)/b^2/(tan(1/2*d*x+1/2*c)-1
)^4-1/12*(4*a^2+6*a*b+13*b^2)/b^3/(tan(1/2*d*x+1/2*c)-1)^3-1/8*(4*a^3+4*a^
2*b+11*a*b^2+9*b^3)/b^4/(tan(1/2*d*x+1/2*c)-1)^2-1/8*(8*a^4+4*a^3*b+20*a^2
*b^2+9*a*b^3+15*b^4)/b^5/(tan(1/2*d*x+1/2*c)-1)+1/8*a*(8*a^4+20*a^2*b^2+15
*b^4)/b^6*ln(tan(1/2*d*x+1/2*c)-1)+1/5/b/(tan(1/2*d*x+1/2*c)+1)^5-1/4*(2*b
-a)/b^2/(tan(1/2*d*x+1/2*c)+1)^4-1/12*(-4*a^2+6*a*b-13*b^2)/b^3/(tan(1/2*d
*x+1/2*c)+1)^3-1/8*(-4*a^3+4*a^2*b-11*a*b^2+9*b^3)/b^4/(tan(1/2*d*x+1/2*c)
+1)^2-1/8*(-8*a^4+4*a^3*b-20*a^2*b^2+9*a*b^3-15*b^4)/b^5/(tan(1/2*d*x+1/2*
c)+1)-1/8*a*(8*a^4+20*a^2*b^2+15*b^4)/b^6*ln(tan(1/2*d*x+1/2*c)+1)-2/b^6*(
-a^6-3*a^4*b^2-3*a^2*b^4-b^6)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x
+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

3.121.5 Fricas [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.32

$$\int \frac{\sec^6(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$$
$$= \frac{120\,(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}\cos(dx+c)^5\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}(b\cos}{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2}\right.}{}$$

input
```
integrate(sec(d*x+c)^6/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/240*(120*(a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)*cos(d*x + c)^5*log(-(2*
a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 +
 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*
sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) - 15*(8*a^5 + 20*a^3*b^2
 + 15*a*b^4)*cos(d*x + c)^5*log(sin(d*x + c) + 1) + 15*(8*a^5 + 20*a^3*b^2
 + 15*a*b^4)*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 48*b^5 + 240*(a^4*b +
 2*a^2*b^3 + b^5)*cos(d*x + c)^4 + 80*(a^2*b^3 + b^5)*cos(d*x + c)^2 - 30*
(2*a*b^4*cos(d*x + c) + (4*a^3*b^2 + 7*a*b^4)*cos(d*x + c)^3)*sin(d*x + c)
)/(b^6*d*cos(d*x + c)^5)
```

3.121.6 Sympy [F]

$$\int \frac{\sec^6(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx=\int \frac{\sec^6\left(c+dx\right)}{a\cos\left(c+dx\right)+b\sin\left(c+dx\right)}\,dx$$

input
```
integrate(sec(d*x+c)**6/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**6/(a*cos(c + d*x) + b*sin(c + d*x)), x)
```

3.121.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 625 vs. 2(244) = 488.

Time = 0.33 (sec) , antiderivative size = 625, normalized size of antiderivative = 2.39

$$\int \frac{\sec^6(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$$
$$=\frac{\frac{2\left(120\,a^4+280\,a^2b^2+184\,b^4-\frac{15\left(4\,a^3b+9\,ab^3\right)\sin(dx+c)}{\cos(dx+c)+1}-\frac{80\left(6\,a^4+13\,a^2b^2+7\,b^4\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{30\left(4\,a^3b+5\,ab^3\right)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{80\left(9\,a^4+20\,a^2b^2+14\,b^4\right)\sin\cdots}{(\cos(dx+c)+1)^4}\cdots\right.}{b^5-\frac{5\,b^5\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10\,b^5\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{10\,b^5\sin(dx\cdots}{(\cos(dx+c)\cdots}\cdots}}{\cdots}$$

input
```
integrate(sec(d*x+c)^6/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
1/120*(2*(120*a^4 + 280*a^2*b^2 + 184*b^4 - 15*(4*a^3*b + 9*a*b^3)*sin(d*x
 + c)/(cos(d*x + c) + 1) - 80*(6*a^4 + 13*a^2*b^2 + 7*b^4)*sin(d*x + c)^2/
(cos(d*x + c) + 1)^2 + 30*(4*a^3*b + 5*a*b^3)*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 + 80*(9*a^4 + 20*a^2*b^2 + 14*b^4)*sin(d*x + c)^4/(cos(d*x + c) +
1)^4 - 240*(2*a^4 + 5*a^2*b^2 + 3*b^4)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6
 - 30*(4*a^3*b + 5*a*b^3)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 120*(a^4 +
 3*a^2*b^2 + 3*b^4)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 15*(4*a^3*b + 9*
a*b^3)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/(b^5 - 5*b^5*sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 + 10*b^5*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 10*b^5*s
in(d*x + c)^6/(cos(d*x + c) + 1)^6 + 5*b^5*sin(d*x + c)^8/(cos(d*x + c) +
1)^8 - b^5*sin(d*x + c)^10/(cos(d*x + c) + 1)^10) - 15*(8*a^5 + 20*a^3*b^2
 + 15*a*b^4)*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/b^6 + 15*(8*a^5 + 20
*a^3*b^2 + 15*a*b^4)*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/b^6 - 120*(a
^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*log((b - a*sin(d*x + c)/(cos(d*x + c) +
1) + sqrt(a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 +
b^2)))/(sqrt(a^2 + b^2)*b^6))/d
```

3.121.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 554 vs. $2(244) = 488$.

Time = 0.36 (sec) , antiderivative size = 554, normalized size of antiderivative = 2.11

$$\int \frac{\sec^6(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx =$$

$$-\frac{\frac{15\,(8\,a^5+20\,a^3b^2+15\,ab^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^6}-\frac{15\,(8\,a^5+20\,a^3b^2+15\,ab^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^6}+\frac{120\,(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\,l}{}}{}$$

input
```
integrate(sec(d*x+c)^6/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
-1/120*(15*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*log(abs(tan(1/2*d*x + 1/2*c) +
1))/b^6 - 15*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*log(abs(tan(1/2*d*x + 1/2*c)
- 1))/b^6 + 120*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*log(abs(2*a*tan(1/2*d*
x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b +
 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^6) + 2*(60*a^3*b*tan(1/2*d*x + 1/2
*c)^9 + 135*a*b^3*tan(1/2*d*x + 1/2*c)^9 + 120*a^4*tan(1/2*d*x + 1/2*c)^8
+ 360*a^2*b^2*tan(1/2*d*x + 1/2*c)^8 + 360*b^4*tan(1/2*d*x + 1/2*c)^8 - 12
0*a^3*b*tan(1/2*d*x + 1/2*c)^7 - 150*a*b^3*tan(1/2*d*x + 1/2*c)^7 - 480*a^
4*tan(1/2*d*x + 1/2*c)^6 - 1200*a^2*b^2*tan(1/2*d*x + 1/2*c)^6 - 720*b^4*t
an(1/2*d*x + 1/2*c)^6 + 720*a^4*tan(1/2*d*x + 1/2*c)^4 + 1600*a^2*b^2*tan(
1/2*d*x + 1/2*c)^4 + 1120*b^4*tan(1/2*d*x + 1/2*c)^4 + 120*a^3*b*tan(1/2*d
*x + 1/2*c)^3 + 150*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 480*a^4*tan(1/2*d*x + 1
/2*c)^2 - 1040*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 - 560*b^4*tan(1/2*d*x + 1/2*
c)^2 - 60*a^3*b*tan(1/2*d*x + 1/2*c) - 135*a*b^3*tan(1/2*d*x + 1/2*c) + 12
0*a^4 + 280*a^2*b^2 + 184*b^4)/((tan(1/2*d*x + 1/2*c)^2 - 1)^5*b^5))/d
```

3.121.9 Mupad [B] (verification not implemented)

Time = 25.10 (sec) , antiderivative size = 2979, normalized size of antiderivative = 11.37

$$\int \frac{\sec^6(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(c + d*x)^6*(a*cos(c + d*x) + b*sin(c + d*x))),x)
```

output
```
(atan(((((a^2 + b^2)^5)^(1/2)*(((225*a^4*b^13)/2 + 300*a^6*b^11 + 320*a^8*
b^9 + 160*a^10*b^7 + 32*a^12*b^5)/b^14 + (tan(c/2 + (d*x)/2)*(64*a*b^17 +
834*a^3*b^15 + 2385*a^5*b^13 + 3160*a^7*b^11 + 2240*a^9*b^9 + 832*a^11*b^7
 + 128*a^13*b^5))/(2*b^15) - (((a^2 + b^2)^5)^(1/2)*((28*a^2*b^16 + 44*a^4
*b^14 + 16*a^6*b^12)/b^14 - (tan(c/2 + (d*x)/2)*(128*a*b^18 + 384*a^3*b^16
 + 384*a^5*b^14 + 128*a^7*b^12))/(2*b^15) + (((a^2 + b^2)^5)^(1/2)*(32*a^2
*b^3 + (tan(c/2 + (d*x)/2)*(192*a*b^19 + 128*a^3*b^17))/(2*b^15)))/b^6))/b
^6)*1i)/b^6 + (((a^2 + b^2)^5)^(1/2)*(((225*a^4*b^13)/2 + 300*a^6*b^11 + 3
20*a^8*b^9 + 160*a^10*b^7 + 32*a^12*b^5)/b^14 + (tan(c/2 + (d*x)/2)*(64*a*
b^17 + 834*a^3*b^15 + 2385*a^5*b^13 + 3160*a^7*b^11 + 2240*a^9*b^9 + 832*a
^11*b^7 + 128*a^13*b^5))/(2*b^15) - (((a^2 + b^2)^5)^(1/2)*((tan(c/2 + (d*
x)/2)*(128*a*b^18 + 384*a^3*b^16 + 384*a^5*b^14 + 128*a^7*b^12))/(2*b^15)
- (28*a^2*b^16 + 44*a^4*b^14 + 16*a^6*b^12)/b^14 + (((a^2 + b^2)^5)^(1/2)*
(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(192*a*b^19 + 128*a^3*b^17))/(2*b^15)))/
b^6))/b^6)*1i)/b^6)/((32*a^16 + 120*a^2*b^14 + 655*a^4*b^12 + 1549*a^6*b^1
0 + 2069*a^8*b^8 + 1695*a^10*b^6 + 856*a^12*b^4 + 248*a^14*b^2)/b^14 + (((
a^2 + b^2)^5)^(1/2)*(((225*a^4*b^13)/2 + 300*a^6*b^11 + 320*a^8*b^9 + 160*
a^10*b^7 + 32*a^12*b^5)/b^14 + (tan(c/2 + (d*x)/2)*(64*a*b^17 + 834*a^3*b^
15 + 2385*a^5*b^13 + 3160*a^7*b^11 + 2240*a^9*b^9 + 832*a^11*b^7 + 128*a^1
3*b^5))/(2*b^15) - (((a^2 + b^2)^5)^(1/2)*((28*a^2*b^16 + 44*a^4*b^14 +...
```

3.121. $\int \frac{\sec^6(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

3.122 $\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$

3.122.1 Optimal result

Integrand size = 28, antiderivative size = 145

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = \frac{(a^4+6a^2b^2-3b^4)\,x}{2\,(a^2+b^2)^3} + \frac{b^4}{a\,(a^2+b^2)^2\,d(b+a\cot(c+dx))} + \frac{4ab^3\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^3\,d} - \frac{(2ab-(a^2-b^2)\cot(c+dx))\sin^2(c+dx)}{2\,(a^2+b^2)^2\,d}$$

output
```
1/2*(a^4+6*a^2*b^2-3*b^4)*x/(a^2+b^2)^3+b^4/a/(a^2+b^2)^2/d/(b+a*cot(d*x+c
))+4*a*b^3*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^3/d-1/2*(2*a*b-(a^2-b^2
)*cot(d*x+c))*sin(d*x+c)^2/(a^2+b^2)^2/d
```

3.122.2 Mathematica [A] (verified)

Time = 1.60 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.03

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = \frac{2(a^4+6a^2b^2-3b^4)\,(c+dx)+2ab(a^2+b^2)\cos(2(c+dx))+16ab^3\log(a\cos(c+dx)+b\sin(c+dx))+\frac{}{a}}{4\,(a^2+b^2)^3\,d}$$

input
```
Integrate[Cos[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

output
```
(2*(a^4 + 6*a^2*b^2 - 3*b^4)*(c + d*x) + 2*a*b*(a^2 + b^2)*Cos[2*(c + d*x)
] + 16*a*b^3*Log[a*Cos[c + d*x] + b*Sin[c + d*x]] + (4*b^4*(a^2 + b^2)*Sin
[c + d*x])/(a*(a*Cos[c + d*x] + b*Sin[c + d*x])) + (a^2 - b^2)*(a^2 + b^2)
*Sin[2*(c + d*x)])/(4*(a^2 + b^2)^3*d)
```

3.122.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.19, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 3567, 601, 25, 2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(c+dx)^4}{(a\cos(c+dx)+b\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{3567}$$

$$-\frac{\int \frac{\cot^4(c+dx)}{(b+a\cot(c+dx))^2(\cot^2(c+dx)+1)^2}d\cot(c+dx)}{d}$$

$$\Big\downarrow \text{601}$$

$$-\frac{\frac{2ab-(a^2-b^2)\cot(c+dx)}{2(a^2+b^2)^2(\cot^2(c+dx)+1)}-\frac{1}{2}\int -\frac{\frac{(a^2-b^2)b^2}{(a^2+b^2)^2}+\frac{2a\cot(c+dx)b}{a^2+b^2}+\frac{(a^4+5b^2a^2+2b^4)\cot^2(c+dx)}{(a^2+b^2)^2}}{(b+a\cot(c+dx))^2(\cot^2(c+dx)+1)}d\cot(c+dx)}{d}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{1}{2}\int \frac{\frac{(a^2-b^2)b^2}{(a^2+b^2)^2}+\frac{2a\cot(c+dx)b}{a^2+b^2}+\frac{(a^4+5b^2a^2+2b^4)\cot^2(c+dx)}{(a^2+b^2)^2}}{(b+a\cot(c+dx))^2(\cot^2(c+dx)+1)}d\cot(c+dx)+\frac{2ab-(a^2-b^2)\cot(c+dx)}{2(a^2+b^2)^2(\cot^2(c+dx)+1)}}{d}$$

$$\Big\downarrow \text{2160}$$

$$-\frac{\frac{1}{2}\int \left(\frac{2b^4}{(a^2+b^2)^2(b+a\cot(c+dx))^2}-\frac{8a^2b^3}{(a^2+b^2)^3(b+a\cot(c+dx))}+\frac{a^4+6b^2a^2+8b^3\cot(c+dx)a-3b^4}{(a^2+b^2)^3(\cot^2(c+dx)+1)}\right)d\cot(c+dx)+\frac{2ab-(a^2-b^2)\cot(}{2(a^2+b^2)^2(\cot^2(c+}}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{2ab-(a^2-b^2)\cot(c+dx)}{2(a^2+b^2)^2(\cot^2(c+dx)+1)}+\frac{1}{2}\left(-\frac{2b^4}{a(a^2+b^2)^2(a\cot(c+dx)+b)}+\frac{4ab^3\log(\cot^2(c+dx)+1)}{(a^2+b^2)^3}-\frac{8ab^3\log(a\cot(c+dx)+b)}{(a^2+b^2)^3}+\frac{(a^4+6a^2b^2-3b^4}{(a^2}\right.}{d}$$

input
```
Int[Cos[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

output
```
-(((2*a*b - (a^2 - b^2)*Cot[c + d*x])/(2*(a^2 + b^2)^2*(1 + Cot[c + d*x]^2
)) + (((a^4 + 6*a^2*b^2 - 3*b^4)*ArcTan[Cot[c + d*x]])/(a^2 + b^2)^3 - (2*
b^4)/(a*(a^2 + b^2)^2*(b + a*Cot[c + d*x])) - (8*a*b^3*Log[b + a*Cot[c + d
*x]])/(a^2 + b^2)^3 + (4*a*b^3*Log[1 + Cot[c + d*x]^2])/(a^2 + b^2)^3)/2)/
d)
```

3.122.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 601
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c
+ d*x)^n*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*(p + 1)*Qx)/(c + d*x)^n + (e*
(2*p + 3))/(c + d*x)^n, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 1]
 && LtQ[p, -1] && ILtQ[n, 0] && NeQ[b*c^2 + a*d^2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.122. $\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3567
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[x^m*((b
+ a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a,
b, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[
n, 0] && GtQ[m, 1])
```

3.122.4 Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{\frac{\frac{\left(\frac{a^4}{2}-\frac{b^4}{2}\right)\tan(dx+c)+a^3b+a\,b^3}{1+\tan(dx+c)^2}-2a\,b^3\ln\left(1+\tan(dx+c)^2\right)+\frac{\left(a^4+6a^2b^2-3b^4\right)\arctan(\tan(dx+c))}{2}}{\left(a^2+b^2\right)^3}-\frac{b^3}{\left(a^2+b^2\right)^2(a+b\tan(dx+c))}+}{d}$
default	$\frac{\frac{\frac{\left(\frac{a^4}{2}-\frac{b^4}{2}\right)\tan(dx+c)+a^3b+a\,b^3}{1+\tan(dx+c)^2}-2a\,b^3\ln\left(1+\tan(dx+c)^2\right)+\frac{\left(a^4+6a^2b^2-3b^4\right)\arctan(\tan(dx+c))}{2}}{\left(a^2+b^2\right)^3}-\frac{b^3}{\left(a^2+b^2\right)^2(a+b\tan(dx+c))}+}{d}$
parallelrisch	$\frac{32\left(a^3b^3\cos(dx+c)+a^2b^4\sin(dx+c)\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)+32\left(-a^3b^3\cos(dx+c)-a^2b^4\sin(dx+c)\right)}{}$
risch	$\frac{3ixb}{6ib\,a^2-2ib^3-2a^3+6a\,b^2}-\frac{xa}{6ib\,a^2-2ib^3-2a^3+6a\,b^2}-\frac{ie^{2i(dx+c)}}{8(-2iba+a^2-b^2)d}+\frac{ie^{-2i(dx+c)}}{8(2iba+a^2-b^2)d}-\frac{8ia\,b^3x}{a^6+3a^4b^2+3a^2b^4}$
norman	Expression too large to display

input
```
int(cos(d*x+c)^4/(cos(d*x+c)*a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/(a^2+b^2)^3*((((1/2*a^4-1/2*b^4)*tan(d*x+c)+a^3*b+a*b^3)/(1+tan(d*x+
c)^2)-2*a*b^3*ln(1+tan(d*x+c)^2)+1/2*(a^4+6*a^2*b^2-3*b^4)*arctan(tan(d*x+
c)))-b^3/(a^2+b^2)^2/(a+b*tan(d*x+c))+4*b^3/(a^2+b^2)^3*a*ln(a+b*tan(d*x+c
)))
```

3.122.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.92

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = \frac{(a^4b+2\,a^2b^3+b^5)\cos(dx+c)^3-(a^2b^3+3\,b^5-(a^5+6\,a^3b^2-3\,ab^4)dx)\cos(dx+c)+4\,(a^2b^3\cos(dx+$$
$$2\,((a^7+3$$

input
```
integrate(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas"
)
```

output
```
1/2*((a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c)^3 - (a^2*b^3 + 3*b^5 - (a^5 +
6*a^3*b^2 - 3*a*b^4)*d*x)*cos(d*x + c) + 4*(a^2*b^3*cos(d*x + c) + a*b^4*s
in(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c
)^2 + b^2) - (a^3*b^2 - a*b^4 - (a^4*b + 6*a^2*b^3 - 3*b^5)*d*x - (a^5 + 2
*a^3*b^2 + a*b^4)*cos(d*x + c)^2)*sin(d*x + c))/((a^7 + 3*a^5*b^2 + 3*a^3*
b^4 + a*b^6)*d*cos(d*x + c) + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*d*sin(
d*x + c))
```

3.122.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**4/(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

output
```
Timed out
```

3.122.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.94

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = \frac{\frac{8\,ab^3\log(b\tan(dx+c)+a)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} - \frac{4\,ab^3\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{(a^4+6\,a^2b^2-3\,b^4)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{2\,a^2b-2\,b^3+(a^2b-3\,b^3)\tan(\ldots}{a^5+2\,a^3b^2+ab^4+(a^4b+2\,a^2b^3+b^5)\tan(dx+c)^3+(a^5+\ldots}}{2\,d}$$

input
```
integrate(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima"
)
```

output
```
1/2*(8*a*b^3*log(b*tan(d*x + c) + a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) -
 4*a*b^3*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (a^
4 + 6*a^2*b^2 - 3*b^4)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (2*
a^2*b - 2*b^3 + (a^2*b - 3*b^3)*tan(d*x + c)^2 + (a^3 + a*b^2)*tan(d*x + c
))/(a^5 + 2*a^3*b^2 + a*b^4 + (a^4*b + 2*a^2*b^3 + b^5)*tan(d*x + c)^3 + (
a^5 + 2*a^3*b^2 + a*b^4)*tan(d*x + c)^2 + (a^4*b + 2*a^2*b^3 + b^5)*tan(d*
x + c)))/d
```

3.122.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.72

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = \frac{\frac{8\,ab^4\log(|b\tan(dx+c)+a|)}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7} - \frac{4\,ab^3\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{(a^4+6\,a^2b^2-3\,b^4)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{a^2b\tan(dx+c)^2-3\,b^3\tan(dx+c)^2+a^3\tan(dx+c)+ab\ldots}{(a^4+2\,a^2b^2+b^4)\left(b\tan(dx+c)^3+a\tan(dx+c)^2\ldots}}{2\,d}$$

input
```
integrate(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
1/2*(8*a*b^4*log(abs(b*tan(d*x + c) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 +
 b^7) - 4*a*b^3*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6
) + (a^4 + 6*a^2*b^2 - 3*b^4)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6
) + (a^2*b*tan(d*x + c)^2 - 3*b^3*tan(d*x + c)^2 + a^3*tan(d*x + c) + a*b^
2*tan(d*x + c) + 2*a^2*b - 2*b^3)/((a^4 + 2*a^2*b^2 + b^4)*(b*tan(d*x + c)
^3 + a*tan(d*x + c)^2 + b*tan(d*x + c) + a))/d
```

3.122.9 Mupad [B] (verification not implemented)

Time = 36.48 (sec) , antiderivative size = 6604, normalized size of antiderivative = 45.54

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(cos(c + d*x)^4/(a*cos(c + d*x) + b*sin(c + d*x))^2,x)
```

output
```
((2*b*tan(c/2 + (d*x)/2)^4)/(a^2 + b^2) - (2*b*tan(c/2 + (d*x)/2)^2)/(a^2
+ b^2) + (tan(c/2 + (d*x)/2)*(a^4 + 2*b^4 - a^2*b^2))/(a*(a^2 + b^2)^2) +
(tan(c/2 + (d*x)/2)^5*(a^4 + 2*b^4 - a^2*b^2))/(a*(a^4 + b^4 + 2*a^2*b^2))
 - (2*tan(c/2 + (d*x)/2)^3*(a^4 - 2*b^4 + 3*a^2*b^2))/(a*(a^2 + b^2)^2))/(
d*(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2 - a*tan(c/2 + (d*x)
/2)^4 - a*tan(c/2 + (d*x)/2)^6 + 4*b*tan(c/2 + (d*x)/2)^3 + 2*b*tan(c/2 +
(d*x)/2)^5)) - (atan((tan(c/2 + (d*x)/2)*(((((a^4 - 3*b^4 + 6*a^2*b^2)^3*(
12*a*b^16 + 84*a^3*b^14 + 252*a^5*b^12 + 420*a^7*b^10 + 420*a^9*b^8 + 252*
a^11*b^6 + 84*a^13*b^4 + 12*a^15*b^2))/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2
)^3*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a
^10*b^2)) - (((8*(18*a*b^12 + a^13 - 141*a^3*b^10 - 327*a^5*b^8 - 146*a^7*
b^6 + 36*a^9*b^4 + 15*a^11*b^2))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 +
20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) - (16*a*b^3*((8*(4*a^14*b + 4*a^2*b^
13 + 72*a^4*b^11 + 252*a^6*b^9 + 368*a^8*b^7 + 252*a^10*b^5 + 72*a^12*b^3)
)/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^1
0*b^2) - (128*a*b^3*(12*a*b^16 + 84*a^3*b^14 + 252*a^5*b^12 + 420*a^7*b^10
 + 420*a^9*b^8 + 252*a^11*b^6 + 84*a^13*b^4 + 12*a^15*b^2))/((4*a^6 + 4*b^
6 + 12*a^2*b^4 + 12*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a
^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))))/(4*a^6 + 4*b^6 + 12*a^2*b^4 + 12*a^4*
b^2))*(a^4 - 3*b^4 + 6*a^2*b^2))/(2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)...
```

3.123 $\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$

3.123.1 Optimal result

Integrand size = 28, antiderivative size = 138

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = -\frac{3ab^2\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}\,d} + \frac{2ab\cos(c+dx)}{(a^2+b^2)^2\,d} + \frac{(a^2-b^2)\sin(c+dx)}{(a^2+b^2)^2\,d} - \frac{b^3}{(a^2+b^2)^2\,d(a\cos(c+dx)+b\sin(c+dx))}$$

output
```
-3*a*b^2*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5
/2)/d+2*a*b*cos(d*x+c)/(a^2+b^2)^2/d+(a^2-b^2)*sin(d*x+c)/(a^2+b^2)^2/d-b^
3/(a^2+b^2)^2/d/(a*cos(d*x+c)+b*sin(d*x+c))
```

3.123.2 Mathematica [A] (verified)

Time = 1.31 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.94

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = \frac{\frac{12ab^2\operatorname{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} + \frac{3b(a^2-b^2)+b(a^2+b^2)\cos(2(c+dx))+a(a^2+b^2)\sin(2(c+dx))}{(a^2+b^2)^2(a\cos(c+dx)+b\sin(c+dx))}}{2d}$$

input
```
Integrate[Cos[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

output
```
((12*a*b^2*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)
^(5/2) + (3*b*(a^2 - b^2) + b*(a^2 + b^2)*Cos[2*(c + d*x)] + a*(a^2 + b^2)
*Sin[2*(c + d*x)])/((a^2 + b^2)^2*(a*Cos[c + d*x] + b*Sin[c + d*x])))/(2*d
)
```

3.123.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.61, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 4902, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(c+dx)^3}{(a\cos(c+dx)+b\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{4902}$$

$$\frac{2\int \frac{\left(1-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^3}{\left(\tan^2\left(\frac{1}{2}(c+dx)\right)+1\right)^2\left(-a\tan^2\left(\frac{1}{2}(c+dx)\right)+2b\tan\left(\frac{1}{2}(c+dx)\right)+a\right)^2}d\tan\left(\frac{1}{2}(c+dx)\right)}{d}$$

$$\Big\downarrow \text{7293}$$

$$\frac{2\int \left(-\frac{2\tan\left(\frac{1}{2}(c+dx)\right)b^3}{a(a^2+b^2)\left(a\tan^2\left(\frac{1}{2}(c+dx)\right)-2b\tan\left(\frac{1}{2}(c+dx)\right)-a\right)^2}-\frac{(3a^2+b^2)b^2}{a(a^2+b^2)^2\left(a\tan^2\left(\frac{1}{2}(c+dx)\right)-2b\tan\left(\frac{1}{2}(c+dx)\right)-a\right)}+\frac{b^2-a^2}{(a^2+b^2)^2\left(\tan^2\left(\frac{1}{2}(c+dx\right.\right.} \cdots}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(-\frac{b^2(3a^2+b^2)\text{arctanh}\left(\frac{b-a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a(a^2+b^2)^{5/2}}+\frac{b^4\text{arctanh}\left(\frac{b-a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a(a^2+b^2)^{5/2}}+\frac{(a^2-b^2)\tan\left(\frac{1}{2}(c+dx)\right)+2ab}{(a^2+b^2)^2\left(\tan^2\left(\frac{1}{2}(c+dx)\right)+1\right)}-\frac{b^3(\cdots}{a(a^2+b^2)^2(-a\tan \cdots}\right.}{d}$$

input
```
Int[Cos[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

output
```
(2*((b^4*ArcTanh[(b - a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a*(a^2 + b^2)
^(5/2)) - (b^2*(3*a^2 + b^2)*ArcTanh[(b - a*Tan[(c + d*x)/2])/Sqrt[a^2 + b
^2]])/(a*(a^2 + b^2)^(5/2)) + (2*a*b + (a^2 - b^2)*Tan[(c + d*x)/2])/((a^2
 + b^2)^2*(1 + Tan[(c + d*x)/2]^2)) - (b^3*(a + b*Tan[(c + d*x)/2]))/(a*(a
^2 + b^2)^2*(a + 2*b*Tan[(c + d*x)/2] - a*Tan[(c + d*x)/2]^2))))/d
```

3.123.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4902
```
Int[u_, x_Symbol] :> With[{w = Block[{$ShowSteps = False, $StepCounter = Nu
ll}, Int[SubstFor[1/(1 + FreeFactors[Tan[FunctionOfTrig[u, x]/2], x]^2*x^2)
, Tan[FunctionOfTrig[u, x]/2]/FreeFactors[Tan[FunctionOfTrig[u, x]/2], x],
u, x], x]]}, Module[{v = FunctionOfTrig[u, x], d}, Simp[d = FreeFactors[Tan
[v/2], x]; 2*(d/Coefficient[v, x, 1])   Subst[Int[SubstFor[1/(1 + d^2*x^2),
 Tan[v/2]/d, u, x], x], x, Tan[v/2]/d], x]] /; CalculusFreeQ[w, x]] /; Inve
rseFunctionFreeQ[u, x] &&  !FalseQ[FunctionOfTrig[u, x]]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.123.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.25

method	result
derivativedivides	$\frac{-\frac{2\left(\left(-a^2+b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2ab\right)}{\left(a^4+2a^2b^2+b^4\right)\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}-\frac{2b^2\left(\frac{-\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}-b}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a}-\frac{3a\,\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^2}}{d}$
default	$\frac{-\frac{2\left(\left(-a^2+b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2ab\right)}{\left(a^4+2a^2b^2+b^4\right)\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}-\frac{2b^2\left(\frac{-\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}-b}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a}-\frac{3a\,\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^2}}{d}$
risch	$-\frac{i e^{i(dx+c)}}{2(-2iba+a^2-b^2)d}+\frac{i e^{-i(dx+c)}}{2(2iba+a^2-b^2)d}-\frac{2ib^3 e^{i(dx+c)}}{(-ia+b)^2 d(ia+b)^2\left(b\,e^{2i(dx+c)}+ia\,e^{2i(dx+c)}-b+ia\right)}+\frac{3b^2a\ln\left(e^{i(dx+\cdots\right.}{\cdots}$

input
```
int(cos(d*x+c)^3/(cos(d*x+c)*a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/(a^4+2*a^2*b^2+b^4)*((-a^2+b^2)*tan(1/2*d*x+1/2*c)-2*a*b)/(1+tan(1
/2*d*x+1/2*c)^2)-2*b^2/(a^2+b^2)^2*((-b^2/a*tan(1/2*d*x+1/2*c)-b)/(tan(1/2
*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)-3*a/(a^2+b^2)^(1/2)*arctanh(1/2*
(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))))
```

3.123.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 302 vs. 2(134) = 268.

Time = 0.28 (sec) , antiderivative size = 302, normalized size of antiderivative = 2.19

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$$
$$= \frac{2\,a^4b-2\,a^2b^3-4\,b^5+2\left(a^4b+2\,a^2b^3+b^5\right)\cos\left(dx+c\right)^2+2\left(a^5+2\,a^3b^2+ab^4\right)\cos\left(dx+c\right)\sin\left(dx+c\right)\cdots}{2\left(\left(a^7+3\,a^5b^2+3\,a^3b^4+ab^6\right)d\,c\cdots\right.}$$

input
```
integrate(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas"
)
```

output
```
1/2*(2*a^4*b - 2*a^2*b^3 - 4*b^5 + 2*(a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c
)^2 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*cos(d*x + c)*sin(d*x + c) + 3*(a^2*b^2*c
os(d*x + c) + a*b^3*sin(d*x + c))*sqrt(a^2 + b^2)*log(-(2*a*b*cos(d*x + c)
*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^
2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (
a^2 - b^2)*cos(d*x + c)^2 + b^2)))/((a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*
d*cos(d*x + c) + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*d*sin(d*x + c))
```

3.123.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3/(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

output
```
Timed out
```

3.123.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(134) = 268$.

Time = 0.32 (sec) , antiderivative size = 348, normalized size of antiderivative = 2.52

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = -\frac{\frac{3\,ab^2\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}} - \frac{2\left(2\,a^3b-ab^3-\frac{3\,ab^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{(a^4+3\,a^2b^2-b^4)\sin(dx+c)}{\cos(dx+c)+1}-\frac{(a^4-a^2b^2+b^4)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^6+2\,a^4b^2+a^2b^4+\frac{2\,(a^5b+2\,a^3b^3+ab^5)\sin(dx+c)}{\cos(dx+c)+1}+\frac{2\,(a^5b+2\,a^3b^3+ab^5)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{(a^6+2\,a^4b^2+a^2b^4)}{(\cos(dx+c)+}}}{d}$$

input
```
integrate(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima"
)
```

output
```
-(3*a*b^2*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b
 - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2
 + b^4)*sqrt(a^2 + b^2)) - 2*(2*a^3*b - a*b^3 - 3*a*b^3*sin(d*x + c)^2/(co
s(d*x + c) + 1)^2 + (a^4 + 3*a^2*b^2 - b^4)*sin(d*x + c)/(cos(d*x + c) + 1
) - (a^4 - a^2*b^2 + b^4)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/(a^6 + 2*a^
4*b^2 + a^2*b^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*sin(d*x + c)/(cos(d*x + c)
 + 1) + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3
- (a^6 + 2*a^4*b^2 + a^2*b^4)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4))/d
```

3.123.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(134) = 268$.

Time = 0.36 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.07

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx =$$

$$-\frac{\frac{3\,ab^2\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}} - \frac{2\left(a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\right.}{(a^5+2\,a^3b^2+ab^4)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-2\,b\tan\left(\frac{1}{2}\,d\right.\right.}}{d}$$

input
```
integrate(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
-(3*a*b^2*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(
2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b
^4)*sqrt(a^2 + b^2)) - 2*(a^4*tan(1/2*d*x + 1/2*c)^3 - a^2*b^2*tan(1/2*d*x
 + 1/2*c)^3 + b^4*tan(1/2*d*x + 1/2*c)^3 + 3*a*b^3*tan(1/2*d*x + 1/2*c)^2
- a^4*tan(1/2*d*x + 1/2*c) - 3*a^2*b^2*tan(1/2*d*x + 1/2*c) + b^4*tan(1/2*
d*x + 1/2*c) - 2*a^3*b + a*b^3)/((a^5 + 2*a^3*b^2 + a*b^4)*(a*tan(1/2*d*x
+ 1/2*c)^4 - 2*b*tan(1/2*d*x + 1/2*c)^3 - 2*b*tan(1/2*d*x + 1/2*c) - a)))/
d
```

3.123.9 Mupad [B] (verification not implemented)

Time = 26.06 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.07

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = \frac{\frac{4a^2b-2b^3}{a^4+2a^2b^2+b^4}-\frac{6b^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{a^4+2a^2b^2+b^4}+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(a^4+3a^2b^2-b^4)}{a(a^4+2a^2b^2+b^4)}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(2a^4-2a^2b^2+2b^4)}{a(a^4+2a^2b^2+b^4)}}{d\left(-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)} - \frac{6ab^2\,\mathrm{atanh}\left(\frac{a^4b+b^5+2a^2b^3-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(a^4+2a^2b^2+b^4)}{(a^2+b^2)^{5/2}}\right)}{d(a^2+b^2)^{5/2}}$$

input
```
int(cos(c + d*x)^3/(a*cos(c + d*x) + b*sin(c + d*x))^2,x)
```

output
```
((4*a^2*b - 2*b^3)/(a^4 + b^4 + 2*a^2*b^2) - (6*b^3*tan(c/2 + (d*x)/2)^2)/
(a^4 + b^4 + 2*a^2*b^2) + (2*tan(c/2 + (d*x)/2)*(a^4 - b^4 + 3*a^2*b^2))/(
a*(a^4 + b^4 + 2*a^2*b^2)) - (tan(c/2 + (d*x)/2)^3*(2*a^4 + 2*b^4 - 2*a^2*
b^2))/(a*(a^4 + b^4 + 2*a^2*b^2)))/(d*(a + 2*b*tan(c/2 + (d*x)/2) - a*tan(
c/2 + (d*x)/2)^4 + 2*b*tan(c/2 + (d*x)/2)^3)) - (6*a*b^2*atanh((a^4*b + b^
5 + 2*a^2*b^3 - a*tan(c/2 + (d*x)/2)*(a^4 + b^4 + 2*a^2*b^2))/(a^2 + b^2)^
(5/2)))/(d*(a^2 + b^2)^(5/2))
```

3.124 $\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$

3.124.1 Optimal result

Integrand size = 28, antiderivative size = 82

$$\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = \frac{(a^2-b^2)\,x}{(a^2+b^2)^2}+\frac{2ab\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^2\,d} - \frac{b}{(a^2+b^2)\,d(a+b\tan(c+dx))}$$

output
```
(a^2-b^2)*x/(a^2+b^2)^2+2*a*b*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^2/d-
b/(a^2+b^2)/d/(a+b*tan(d*x+c))
```

3.124.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.88 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.34

$$\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = \frac{a^2\cos(c+dx)\left((a+ib)^2(c+dx)+ab\log\left((a\cos(c+dx)+b\sin(c+dx))^2\right)\right)+b((a+ib)\left(-ib^2+ab(1+i\right.}{a\left(a^2\right.}$$

input
```
Integrate[Cos[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

output
```
(a^2*Cos[c + d*x]*((a + I*b)^2*(c + d*x) + a*b*Log[(a*Cos[c + d*x] + b*Sin
[c + d*x])^2]) + b*((a + I*b)*((-I)*b^2 + a*b*(1 + I*c + I*d*x) + a^2*(c +
 d*x)) + a^2*b*Log[(a*Cos[c + d*x] + b*Sin[c + d*x])^2])*Sin[c + d*x] - (2
*I)*a^2*b*ArcTan[Tan[c + d*x]]*(a*Cos[c + d*x] + b*Sin[c + d*x]))/(a*(a^2
+ b^2)^2*d*(a*Cos[c + d*x] + b*Sin[c + d*x]))
```

3.124.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3565, 3042, 3964, 3042, 4014, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(c+dx)^2}{(a\cos(c+dx)+b\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{ 3565}$$

$$\int \frac{1}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{(a+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{ 3964}$$

$$\frac{\int \frac{a-b\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}-\frac{b}{d(a^2+b^2)(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{a-b\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}-\frac{b}{d(a^2+b^2)(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{ 4014}$$

$$\frac{\frac{2ab\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{x(a^2-b^2)}{a^2+b^2}}{a^2+b^2}-\frac{b}{d(a^2+b^2)(a+b\tan(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2ab\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{x(a^2-b^2)}{a^2+b^2}}{a^2+b^2}-\frac{b}{d\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)}$$

$$\Big\downarrow \text{4013}$$

$$\frac{\frac{2ab\log(a\cos(c+dx)+b\sin(c+dx))}{d(a^2+b^2)}+\frac{x(a^2-b^2)}{a^2+b^2}}{a^2+b^2}-\frac{b}{d\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)}$$

input
```
Int[Cos[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

output
```
(((a^2 - b^2)*x)/(a^2 + b^2) + (2*a*b*Log[a*Cos[c + d*x] + b*Sin[c + d*x]]
)/((a^2 + b^2)*d))/(a^2 + b^2) - b/((a^2 + b^2)*d*(a + b*Tan[c + d*x]))
```

3.124.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3565
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(a + b*Tan[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

rule 3964
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)
 Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

3.124. $\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

3.124.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.18

method	result
derivativedivides	$\frac{-\frac{b}{\left(a^2+b^2\right)\left(a+b\tan(dx+c)\right)}+\frac{2ab\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)^2}+\frac{-ab\ln\left(1+\tan(dx+c)^2\right)+\left(a^2-b^2\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^2}}{d}$
default	$\frac{-\frac{b}{\left(a^2+b^2\right)\left(a+b\tan(dx+c)\right)}+\frac{2ab\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)^2}+\frac{-ab\ln\left(1+\tan(dx+c)^2\right)+\left(a^2-b^2\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^2}}{d}$
risch	$-\frac{x}{2iba-a^2+b^2}-\frac{4iabx}{a^4+2a^2b^2+b^4}-\frac{4iabc}{d(a^4+2a^2b^2+b^4)}-\frac{2ib^2}{(-ia+b)d(ia+b)^2\left(b\,e^{2i(dx+c)}+ia\,e^{2i(dx+c)}-b+ia\right)}+\frac{2ab}{}$
parallelrisch	$\frac{2ba\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)-2ba\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2b\tan\left(\frac{dx}{2}+\right.\right.}{(a^2+b^2)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}$
norman	$\frac{\frac{\left(a^2-b^2\right)ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{\left(a^2+b^2\right)^2}+\frac{\left(a^2-b^2\right)ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{\left(a^2+b^2\right)^2}-\frac{\left(a^2-b^2\right)ax}{\left(a^2+b^2\right)^2}-\frac{2b\left(a^2-b^2\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2+b^2\right)^2}-\frac{4b\left(a^2-b^2\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{\left(a^2+b^2\right)^2}-\frac{2b}{}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a\right.}$

input
```
int(cos(d*x+c)^2/(cos(d*x+c)*a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-b/(a^2+b^2)/(a+b*tan(d*x+c))+2*a*b/(a^2+b^2)^2*ln(a+b*tan(d*x+c))+1/
(a^2+b^2)^2*(-a*b*ln(1+tan(d*x+c)^2)+(a^2-b^2)*arctan(tan(d*x+c))))
```

3.124.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(82) = 164.

Time = 0.27 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.11

$$\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = -\frac{(b^3-(a^3-ab^2)dx)\cos(dx+c)-(a^2b\cos(dx+c)+ab^2\sin(dx+c))\log\left(2\,ab\cos(dx+c)\sin(dx+c\right.}{(a^5+2\,a^3b^2+ab^4)d\cos(dx+c)+(a^4b+2\,a^2b^3+}$$

input
```
integrate(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas"
)
```

output
```
-((b^3 - (a^3 - a*b^2)*d*x)*cos(d*x + c) - (a^2*b*cos(d*x + c) + a*b^2*sin
(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^
2 + b^2) - (a*b^2 + (a^2*b - b^3)*d*x)*sin(d*x + c))/((a^5 + 2*a^3*b^2 + a
*b^4)*d*cos(d*x + c) + (a^4*b + 2*a^2*b^3 + b^5)*d*sin(d*x + c))
```

3.124.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.20 (sec) , antiderivative size = 1545, normalized size of antiderivative = 18.84

$$\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)**2/(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

output
```
Piecewise((zoo*x*cos(c)**2/sin(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((-
x - cos(c + d*x)/(d*sin(c + d*x)))/b**2, Eq(a, 0)), (2*d*x*sin(c + d*x)**2
/(-8*b**2*d*sin(c + d*x)**2 + 16*I*b**2*d*sin(c + d*x)*cos(c + d*x) + 8*b*
*2*d*cos(c + d*x)**2) - 4*I*d*x*sin(c + d*x)*cos(c + d*x)/(-8*b**2*d*sin(c
 + d*x)**2 + 16*I*b**2*d*sin(c + d*x)*cos(c + d*x) + 8*b**2*d*cos(c + d*x)
**2) - 2*d*x*cos(c + d*x)**2/(-8*b**2*d*sin(c + d*x)**2 + 16*I*b**2*d*sin(
c + d*x)*cos(c + d*x) + 8*b**2*d*cos(c + d*x)**2) - I*sin(c + d*x)**2/(-8*
b**2*d*sin(c + d*x)**2 + 16*I*b**2*d*sin(c + d*x)*cos(c + d*x) + 8*b**2*d*
cos(c + d*x)**2) - 3*I*cos(c + d*x)**2/(-8*b**2*d*sin(c + d*x)**2 + 16*I*b
**2*d*sin(c + d*x)*cos(c + d*x) + 8*b**2*d*cos(c + d*x)**2), Eq(a, -I*b)),
 (2*d*x*sin(c + d*x)**2/(-8*b**2*d*sin(c + d*x)**2 - 16*I*b**2*d*sin(c + d
*x)*cos(c + d*x) + 8*b**2*d*cos(c + d*x)**2) + 4*I*d*x*sin(c + d*x)*cos(c
+ d*x)/(-8*b**2*d*sin(c + d*x)**2 - 16*I*b**2*d*sin(c + d*x)*cos(c + d*x)
+ 8*b**2*d*cos(c + d*x)**2) - 2*d*x*cos(c + d*x)**2/(-8*b**2*d*sin(c + d*x
)**2 - 16*I*b**2*d*sin(c + d*x)*cos(c + d*x) + 8*b**2*d*cos(c + d*x)**2) +
 I*sin(c + d*x)**2/(-8*b**2*d*sin(c + d*x)**2 - 16*I*b**2*d*sin(c + d*x)*c
os(c + d*x) + 8*b**2*d*cos(c + d*x)**2) + 3*I*cos(c + d*x)**2/(-8*b**2*d*s
in(c + d*x)**2 - 16*I*b**2*d*sin(c + d*x)*cos(c + d*x) + 8*b**2*d*cos(c +
d*x)**2), Eq(a, I*b)), (x*cos(c)**2/(a*cos(c) + b*sin(c))**2, Eq(d, 0)), (
a**3*d*x*cos(c + d*x)/(a**5*d*cos(c + d*x) + a**4*b*d*sin(c + d*x) + 2*...
```

3.124.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.60

$$\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = \frac{\frac{2\,ab\log(b\tan(dx+c)+a)}{a^4+2\,a^2b^2+b^4}-\frac{ab\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{(a^2-b^2)(dx+c)}{a^4+2\,a^2b^2+b^4}-\frac{b}{a^3+ab^2+(a^2b+b^3)\tan(dx+c)}}{d}$$

input
```
integrate(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima"
)
```

output
```
(2*a*b*log(b*tan(d*x + c) + a)/(a^4 + 2*a^2*b^2 + b^4) - a*b*log(tan(d*x +
 c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + (a^2 - b^2)*(d*x + c)/(a^4 + 2*a^2*b^
2 + b^4) - b/(a^3 + a*b^2 + (a^2*b + b^3)*tan(d*x + c)))/d
```

3.124.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.94

$$\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$$
$$= \frac{\frac{2ab^2\log(|b\tan(dx+c)+a|)}{a^4b+2a^2b^3+b^5} - \frac{ab\log\left(\tan(dx+c)^2+1\right)}{a^4+2a^2b^2+b^4} + \frac{(a^2-b^2)(dx+c)}{a^4+2a^2b^2+b^4} - \frac{2ab^2\tan(dx+c)+3a^2b+b^3}{(a^4+2a^2b^2+b^4)(b\tan(dx+c)+a)}}{d}$$

input
```
integrate(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
(2*a*b^2*log(abs(b*tan(d*x + c) + a))/(a^4*b + 2*a^2*b^3 + b^5) - a*b*log(
tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + (a^2 - b^2)*(d*x + c)/(a^4 +
 2*a^2*b^2 + b^4) - (2*a*b^2*tan(d*x + c) + 3*a^2*b + b^3)/((a^4 + 2*a^2*b
^2 + b^4)*(b*tan(d*x + c) + a)))/d
```

3.124.9 Mupad [B] (verification not implemented)

Time = 27.38 (sec) , antiderivative size = 3114, normalized size of antiderivative = 37.98

$$\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(cos(c + d*x)^2/(a*cos(c + d*x) + b*sin(c + d*x))^2,x)
```

output
```
(2*a*b*log(a + 2*b*tan(c/2 + (d*x)/2) - a*tan(c/2 + (d*x)/2)^2))/(d*(a^4 +
 b^4 + 2*a^2*b^2)) - (2*a*b*log(1/(cos(c + d*x) + 1)))/(d*(a^4 + b^4 + 2*a
^2*b^2)) + (2*atan((tan(c/2 + (d*x)/2)*((((2*a*b*((((32*(6*a^8*b + 6*a^4*b
^5 + 12*a^6*b^3))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (64*a*b*(3*a*b^10
+ 12*a^3*b^8 + 18*a^5*b^6 + 12*a^7*b^4 + 3*a^9*b^2))/((a^4 + b^4 + 2*a^2*b
^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))*(a + b)*(a - b))/(a^4 + b^4 + 2*
a^2*b^2) - (64*a*b*(a + b)*(a - b)*(3*a*b^10 + 12*a^3*b^8 + 18*a^5*b^6 + 1
2*a^7*b^4 + 3*a^9*b^2))/((a^4 + b^4 + 2*a^2*b^2)^2*(a^6 + b^6 + 3*a^2*b^4
+ 3*a^4*b^2))))/(a^4 + b^4 + 2*a^2*b^2) - ((a + b)*((32*(2*a*b^6 + a^7 - 7
*a^3*b^4 - 8*a^5*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (2*a*b*((32*(
6*a^8*b + 6*a^4*b^5 + 12*a^6*b^3))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (
64*a*b*(3*a*b^10 + 12*a^3*b^8 + 18*a^5*b^6 + 12*a^7*b^4 + 3*a^9*b^2))/((a^
4 + b^4 + 2*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(a^4 + b^4 + 2
*a^2*b^2))*(a - b))/(a^4 + b^4 + 2*a^2*b^2) + (32*(a + b)^3*(a - b)^3*(3*a
*b^10 + 12*a^3*b^8 + 18*a^5*b^6 + 12*a^7*b^4 + 3*a^9*b^2))/((a^4 + b^4 + 2
*a^2*b^2)^3*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))*(a^6 - b^6 + 35*a^2*b^4
- 35*a^4*b^2))/(a^6 + b^6 + 15*a^2*b^4 + 15*a^4*b^2)^2 - (2*a*b*(5*a^4 + 5
*b^4 - 26*a^2*b^2)*((32*(2*a^4*b + 4*a^2*b^3))/(a^6 + b^6 + 3*a^2*b^4 + 3*
a^4*b^2) + ((a + b)*(a - b)*((((32*(6*a^8*b + 6*a^4*b^5 + 12*a^6*b^3))/(a^
6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (64*a*b*(3*a*b^10 + 12*a^3*b^8 + 18*...
```

3.124. $\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$

3.125 $\int \frac{\cos(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$

3.125.1 Optimal result

Integrand size = 26, antiderivative size = 83

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = -\frac{a\,\mathrm{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d} - \frac{b}{(a^2+b^2)\,d(a\cos(c+dx)+b\sin(c+dx))}$$

output
```
-a*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)/d-
b/(a^2+b^2)/d/(a*cos(d*x+c)+b*sin(d*x+c))
```

3.125.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.95

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = \frac{\frac{2a\,\mathrm{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}} - \frac{b}{(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))}}{d}$$

input
```
Integrate[Cos[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

output
```
((2*a*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2
) - b/((a^2 + b^2)*(a*Cos[c + d*x] + b*Sin[c + d*x])))/d
```

3.125.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3042, 3634, 3042, 3553, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{3634}$$

$$\frac{a\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}-\frac{b}{d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}-\frac{b}{d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))}$$

$$\Big\downarrow \text{3553}$$

$$-\frac{a\int \frac{1}{a^2+b^2-(b\cos(c+dx)-a\sin(c+dx))^2}d(b\cos(c+dx)-a\sin(c+dx))}{d\left(a^2+b^2\right)}-\frac{b}{d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))}$$

$$\Big\downarrow \text{219}$$

$$-\frac{a\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{3/2}}-\frac{b}{d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))}$$

input
```
Int[Cos[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

output
```
-((a*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/((a^2 + b
^2)^(3/2)*d)) - b/((a^2 + b^2)*d*(a*Cos[c + d*x] + b*Sin[c + d*x]))
```

3.125.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3634
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[(c*B + c*A*Co
s[d + e*x] + (a*B - b*A)*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])), x] + Simp[(a*A - b*B)/(a^2 - b^2 - c^2)   Int[1/(a
 + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B},
 x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - b*B, 0]
```

3.125.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.42

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2+b^2\right)a}-\frac{b}{a^2+b^2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a}+\frac{2a\,\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}}{d}$
default	$\frac{-\frac{2\left(-\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2+b^2\right)a}-\frac{b}{a^2+b^2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a}+\frac{2a\,\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}}{d}$
risch	$-\frac{2ib\,\mathrm{e}^{i(dx+c)}}{(-ia+b)d(ia+b)\left(b\,\mathrm{e}^{2i(dx+c)}+ia\,\mathrm{e}^{2i(dx+c)}-b+ia\right)}+\frac{a\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia^3+ia\,b^2-a^2b-b^3}{\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}d}-\frac{a\ln\left(\mathrm{e}^{i(dx+c)}-\frac{ia^3+i}{(a}\right)}{(a^2+b^2)^{\frac{3}{2}}}$

input
```
int(cos(d*x+c)/(cos(d*x+c)*a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*(-b^2/(a^2+b^2)/a*tan(1/2*d*x+1/2*c)-b/(a^2+b^2))/(tan(1/2*d*x+1/2
*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)+2*a/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tan
(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

3.125.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(79) = 158$.

Time = 0.26 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.59

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx =$$
$$-\frac{2\,a^2b+2\,b^3-\left(a^2\cos\left(dx+c\right)+ab\sin\left(dx+c\right)\right)\sqrt{a^2+b^2}\log\Big(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2-2\,a^2}{2\,ab\cos(dx+c)\sin(dx+c)+(a^2}}{2\left((a^5+2\,a^3b^2+ab^4)d\cos\left(dx+c\right)+(a^4b+2\,a^2b^3+b^5)d\sin\left(dx+c\right)\right.}$$

input
```
integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/2*(2*a^2*b + 2*b^3 - (a^2*cos(d*x + c) + a*b*sin(d*x + c))*sqrt(a^2 + b
^2)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2
*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*c
os(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)))/((a^5 + 2*a
^3*b^2 + a*b^4)*d*cos(d*x + c) + (a^4*b + 2*a^2*b^3 + b^5)*d*sin(d*x + c))
```

3.125.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

output
```
Exception raised: AttributeError >> 'NoneType' object has no attribute 'pr
imitive'
```

3.125.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(79) = 158$.

Time = 0.31 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.19

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$$
$$= -\frac{\frac{a\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}}+\frac{2\left(ab+\frac{b^2\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4+a^2b^2+\frac{2\left(a^3b+ab^3\right)\sin(dx+c)}{\cos(dx+c)+1}-\frac{\left(a^4+a^2b^2\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}}{d}$$

input
```
integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
-(a*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*s
in(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) + 2*(
a*b + b^2*sin(d*x + c)/(cos(d*x + c) + 1))/(a^4 + a^2*b^2 + 2*(a^3*b + a*b
^3)*sin(d*x + c)/(cos(d*x + c) + 1) - (a^4 + a^2*b^2)*sin(d*x + c)^2/(cos(
d*x + c) + 1)^2))/d
```

3.125.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.66

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$$
$$= -\frac{\frac{a\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^2+b^2)^{\frac{3}{2}}}-\frac{2\left(b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+ab\right)}{(a^3+ab^2)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)}}{d}$$

input
```
integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
-(a*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*ta
n(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(b^2*
tan(1/2*d*x + 1/2*c) + a*b)/((a^3 + a*b^2)*(a*tan(1/2*d*x + 1/2*c)^2 - 2*b
*tan(1/2*d*x + 1/2*c) - a)))/d
```

3.125.9 Mupad [B] (verification not implemented)

Time = 22.66 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.64

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = -\frac{\frac{2b}{a^2+b^2}+\frac{2b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a(a^2+b^2)}}{d\left(-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)} + \frac{a\,\mathrm{atan}\left(\frac{a^2\,b\,1\mathrm{i}+b^3\,1\mathrm{i}-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(a^2+b^2)\,1\mathrm{i}}{(a^2+b^2)^{3/2}}\right)2\mathrm{i}}{d\,(a^2+b^2)^{3/2}}$$

input
```
int(cos(c + d*x)/(a*cos(c + d*x) + b*sin(c + d*x))^2,x)
```

output
```
(a*atan((a^2*b*1i + b^3*1i - a*tan(c/2 + (d*x)/2)*(a^2 + b^2)*1i)/(a^2 + b
^2)^(3/2))*2i)/(d*(a^2 + b^2)^(3/2)) - ((2*b)/(a^2 + b^2) + (2*b^2*tan(c/2
 + (d*x)/2))/(a*(a^2 + b^2)))/(d*(a + 2*b*tan(c/2 + (d*x)/2) - a*tan(c/2 +
 (d*x)/2)^2))
```

3.126 $\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$

3.126.1 Optimal result

Integrand size = 19, antiderivative size = 32

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = \frac{\sin(c+dx)}{ad(a\cos(c+dx)+b\sin(c+dx))}$$

output
```
sin(d*x+c)/a/d/(a*cos(d*x+c)+b*sin(d*x+c))
```

3.126.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = \frac{\sin(c+dx)}{ad(a\cos(c+dx)+b\sin(c+dx))}$$

input
```
Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-2),x]
```

output
```
Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x]))
```

3.126.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3042, 3554}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^2}dx$$
$$\Big\downarrow \; 3554$$
$$\frac{\sin(c+dx)}{ad(a\cos(c+dx)+b\sin(c+dx))}$$

input
```
Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-2),x]
```

output
```
Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x]))
```

3.126.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3554
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

3.126.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.66

method	result	size
derivativedivides	$-\frac{1}{db(a+b\tan(dx+c))}$	21
default	$-\frac{1}{db(a+b\tan(dx+c))}$	21
risch	$\frac{2i}{d(-ib+a)\left(-ib\,\mathrm{e}^{2i(dx+c)}+\mathrm{e}^{2i(dx+c)}a+ib+a\right)}$	47
parallelrisch	$\frac{1-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{bd\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}$	54
norman	$\frac{\frac{1}{bd}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{bd}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a}$	60

input
```
int(1/(cos(d*x+c)*a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/d/b/(a+b*tan(d*x+c))
```

3.126.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.78

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = -\frac{b\cos(dx+c)-a\sin(dx+c)}{(a^3+ab^2)d\cos(dx+c)+(a^2b+b^3)d\sin(dx+c)}$$

input
```
integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
-(b*cos(d*x + c) - a*sin(d*x + c))/((a^3 + a*b^2)*d*cos(d*x + c) + (a^2*b
+ b^3)*d*sin(d*x + c))
```

3.126.6 Sympy [F]

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = \int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$$

input
```
integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

output
```
Integral((a*cos(c + d*x) + b*sin(c + d*x))**(-2), x)
```

3.126.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.66

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = -\frac{1}{(b^2\tan(dx+c)+ab)d}$$

input
```
integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/((b^2*tan(d*x + c) + a*b)*d)
```

3.126.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.62

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = -\frac{1}{(b\tan(dx+c)+a)bd}$$

input
```
integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/((b*tan(d*x + c) + a)*b*d)
```

3.126.9 Mupad [B] (verification not implemented)

Time = 22.38 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.47

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = \frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a\,d\,\left(-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)}$$

input
```
int(1/(a*cos(c + d*x) + b*sin(c + d*x))^2,x)
```

output
```
(2*tan(c/2 + (d*x)/2))/(a*d*(a + 2*b*tan(c/2 + (d*x)/2) - a*tan(c/2 + (d*x
)/2)^2))
```

3.127 $\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$

3.127.1 Optimal result

Integrand size = 26, antiderivative size = 92

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{b^2d} + \frac{a\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}\,d} - \frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}$$

output
```
arctanh(sin(d*x+c))/b^2/d-1/b/d/(a*cos(d*x+c)+b*sin(d*x+c))+a*arctanh((b*c
os(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/b^2/d/(a^2+b^2)^(1/2)
```

3.127.2 Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.30

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = -\frac{\frac{2a\operatorname{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} + \log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) - \log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c\right.\right.}{b^2d}$$

input
```
Integrate[Sec[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

output
```
-(((2*a*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2
] + Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[
(c + d*x)/2]] + (b*Sec[c + d*x])/(a + b*Tan[c + d*x]))/(b^2*d))
```

3.127.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 3573, 3042, 3553, 219, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{3573}$$

$$-\frac{a\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{b^2}+\frac{\int \sec(c+dx)dx}{b^2}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{b^2}+\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}$$

$$\Big\downarrow \text{3553}$$

$$\frac{a\int \frac{1}{a^2+b^2-(b\cos(c+dx)-a\sin(c+dx))^2}d(b\cos(c+dx)-a\sin(c+dx))}{b^2d}+\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}$$

$$\Big\downarrow \text{219}$$

$$\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}+\frac{a\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}$$

$$\Big\downarrow \text{4257}$$

$$\frac{a\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}+\frac{\operatorname{arctanh}(\sin(c+dx))}{b^2d}$$

input
```
Int[Sec[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

output
```
ArcTanh[Sin[c + d*x]]/(b^2*d) + (a*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x
])/Sqrt[a^2 + b^2]])/(b^2*Sqrt[a^2 + b^2]*d) - 1/(b*d*(a*Cos[c + d*x] + b*
Sin[c + d*x]))
```

3.127.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3573
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_)/co
s[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(a*Cos[c + d*x] + b*Sin[c + d*x])^
(n + 1)/(b*d*(n + 1)), x] + (Simp[1/b^2   Int[(a*Cos[c + d*x] + b*Sin[c + d
*x])^(n + 2)/Cos[c + d*x], x], x] - Simp[a/b^2   Int[(a*Cos[c + d*x] + b*Si
n[c + d*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
 && LtQ[n, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.127.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.47

method	result
derivativedivides	$\frac{\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^2}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^2}+\frac{\frac{2\left(\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}+b\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a}-\frac{2a\,\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}}{b^2}}{d}$
default	$\frac{\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^2}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^2}+\frac{\frac{2\left(\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}+b\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a}-\frac{2a\,\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}}{b^2}}{d}$
risch	$-\frac{2\,\mathrm{e}^{i(dx+c)}}{db\left(-ib\,\mathrm{e}^{2i(dx+c)}+\mathrm{e}^{2i(dx+c)}a+ib+a\right)}+\frac{a\ln\left(\mathrm{e}^{i(dx+c)}-\frac{ia-b}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,d\,b^2}-\frac{a\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia-b}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,d\,b^2}-\frac{\ln(\mathrm{e}^{i(dx+c)}-i)}{b^2d}$

input
```
int(sec(d*x+c)/(cos(d*x+c)*a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/b^2*ln(tan(1/2*d*x+1/2*c)+1)-1/b^2*ln(tan(1/2*d*x+1/2*c)-1)+2/b^2*(
(b^2/a*tan(1/2*d*x+1/2*c)+b)/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c
)-a)-a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^
(1/2))))
```

3.127.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. 2(88) = 176.

Time = 0.29 (sec) , antiderivative size = 293, normalized size of antiderivative = 3.18

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx =$$
$$-\frac{2\,a^2b+2\,b^3-\left(a^2\cos\left(dx+c\right)+ab\sin\left(dx+c\right)\right)\sqrt{a^2+b^2}\log\left(\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2-2\,a^2-b}{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b}\right.}{}$$

input
```
integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/2*(2*a^2*b + 2*b^3 - (a^2*cos(d*x + c) + a*b*sin(d*x + c))*sqrt(a^2 + b
^2)*log((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*
a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*co
s(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) - ((a^3 + a*b
^2)*cos(d*x + c) + (a^2*b + b^3)*sin(d*x + c))*log(sin(d*x + c) + 1) + ((a
^3 + a*b^2)*cos(d*x + c) + (a^2*b + b^3)*sin(d*x + c))*log(-sin(d*x + c) +
 1))/((a^3*b^2 + a*b^4)*d*cos(d*x + c) + (a^2*b^3 + b^5)*d*sin(d*x + c))
```

3.127.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = \int \frac{\sec{(c+dx)}}{(a\cos{(c+dx)}+b\sin{(c+dx)})^2}\,dx$$

input
```
integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)/(a*cos(c + d*x) + b*sin(c + d*x))**2, x)
```

3.127.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(88) = 176$.

Time = 0.31 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.30

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = -\frac{\frac{2\left(a+\frac{b\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2b+\frac{2ab^2\sin(dx+c)}{\cos(dx+c)+1}-\frac{a^2b\sin(dx+c)^2}{(\cos(dx+c)+1)^2}} - \frac{a\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,b^2} - \frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{b^2} + \frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{b^2}}{d}$$

input
```
integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
-(2*(a + b*sin(d*x + c)/(cos(d*x + c) + 1))/(a^2*b + 2*a*b^2*sin(d*x + c)/
(cos(d*x + c) + 1) - a^2*b*sin(d*x + c)^2/(cos(d*x + c) + 1)^2) - a*log((b
 - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*sin(d*x + c
)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^2) - log(sin(d
*x + c)/(cos(d*x + c) + 1) + 1)/b^2 + log(sin(d*x + c)/(cos(d*x + c) + 1)
- 1)/b^2)/d
```

3.127.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.80

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$$
$$= \frac{\frac{a\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}b^2}+\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^2}-\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^2}+\frac{2\left(b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right.}}{d}$$

input
```
integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
(a*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan
(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^2) + log(
abs(tan(1/2*d*x + 1/2*c) + 1))/b^2 - log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^
2 + 2*(b*tan(1/2*d*x + 1/2*c) + a)/((a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/
2*d*x + 1/2*c) - a)*a*b))/d
```

3.127.9 Mupad [B] (verification not implemented)

Time = 23.04 (sec) , antiderivative size = 383, normalized size of antiderivative = 4.16

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx =$$
$$-\frac{b^2\sin\left(c+d\,x\right)-\frac{2\left(a^2\cos(c+d\,x)\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\sqrt{a^2+b^2}+a^3\,\mathrm{atan}\left(\frac{1\mathrm{i}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^2+1\mathrm{i}\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)a\,b+2\mathrm{i}\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)b^2}{a\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{a^2+b^2}+2\,b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{a^2+b^2}}\right)\mathrm{co}}{\sqrt{a^2+b^2}}}{a\,b^2\,d}$$

input
```
int(1/(cos(c + d*x)*(a*cos(c + d*x) + b*sin(c + d*x))^2),x)
```

output
```
-(b^2*sin(c + d*x) - (2*(a^3*atan((a^2*sin(c/2 + (d*x)/2)*1i + b^2*sin(c/2
 + (d*x)/2)*2i + a*b*cos(c/2 + (d*x)/2)*1i)/(a*cos(c/2 + (d*x)/2)*(a^2 + b
^2)^(1/2) + 2*b*sin(c/2 + (d*x)/2)*(a^2 + b^2)^(1/2)))*cos(c + d*x)*1i + a
^2*cos(c + d*x)*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*(a^2 + b^2)^(
1/2)))/(a^2 + b^2)^(1/2) + (2*b*((a*(a^2 + b^2)^(1/2))/2 + (a*cos(c + d*x)
*(a^2 + b^2)^(1/2))/2 - a^2*atan((a^2*sin(c/2 + (d*x)/2)*1i + b^2*sin(c/2
+ (d*x)/2)*2i + a*b*cos(c/2 + (d*x)/2)*1i)/(a*cos(c/2 + (d*x)/2)*(a^2 + b^
2)^(1/2) + 2*b*sin(c/2 + (d*x)/2)*(a^2 + b^2)^(1/2)))*sin(c + d*x)*1i - a*
sin(c + d*x)*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*(a^2 + b^2)^(1/2
)))/(a^2 + b^2)^(1/2))/(a*b^2*d*(a*cos(c + d*x) + b*sin(c + d*x)))
```

3.128 $\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$

3.128.1 Optimal result

Integrand size = 28, antiderivative size = 75

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = \frac{\frac{1}{a}+\frac{a}{b^2}}{d(b+a\cot(c+dx))} - \frac{2a\log(b+a\cot(c+dx))}{b^3d} - \frac{2a\log(\tan(c+dx))}{b^3d} + \frac{\tan(c+dx)}{b^2d}$$

output
```
(1/a+a/b^2)/d/(b+a*cot(d*x+c))-2*a*ln(b+a*cot(d*x+c))/b^3/d-2*a*ln(tan(d*x
+c))/b^3/d+tan(d*x+c)/b^2/d
```

3.128.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.68

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = \frac{-2a\log(a+b\tan(c+dx))+b\tan(c+dx)-\frac{a^2+b^2}{a+b\tan(c+dx)}}{b^3d}$$

input
```
Integrate[Sec[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

output
```
(-2*a*Log[a + b*Tan[c + d*x]] + b*Tan[c + d*x] - (a^2 + b^2)/(a + b*Tan[c
+ d*x]))/(b^3*d)
```

3.128.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3567, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{3567}$$

$$-\frac{\int \frac{(\cot^2(c+dx)+1)\tan^2(c+dx)}{(b+a\cot(c+dx))^2}d\cot(c+dx)}{d}$$

$$\Big\downarrow \text{522}$$

$$-\frac{\int \left(\frac{2a^2}{b^3(b+a\cot(c+dx))}-\frac{2\tan(c+dx)a}{b^3}+\frac{\tan^2(c+dx)}{b^2}+\frac{a^2+b^2}{b^2(b+a\cot(c+dx))^2}\right)d\cot(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{2a\log(\cot(c+dx))}{b^3}+\frac{2a\log(a\cot(c+dx)+b)}{b^3}-\frac{\frac{a}{b^2}+\frac{1}{a}}{a\cot(c+dx)+b}-\frac{\tan(c+dx)}{b^2}}{d}$$

input
```
Int[Sec[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

output
```
-((-((a^(-1) + a/b^2)/(b + a*Cot[c + d*x])) - (2*a*Log[Cot[c + d*x]])/b^3
+ (2*a*Log[b + a*Cot[c + d*x]])/b^3 - Tan[c + d*x]/b^2)/d)
```

3.128.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3567
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[x^m*((b
+ a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a,
b, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[
n, 0] && GtQ[m, 1])
```

3.128.4 Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.76

method	result
derivativedivides	$\frac{\frac{\tan(dx+c)}{b^2}-\frac{a^2+b^2}{b^3(a+b\tan(dx+c))}-\frac{2a\ln(a+b\tan(dx+c))}{b^3}}{d}$
default	$\frac{\frac{\tan(dx+c)}{b^2}-\frac{a^2+b^2}{b^3(a+b\tan(dx+c))}-\frac{2a\ln(a+b\tan(dx+c))}{b^3}}{d}$
risch	$-\frac{4i\left(-ia\,e^{2i(dx+c)}+b-ia\right)}{\left(e^{2i(dx+c)}+1\right)\left(b\,e^{2i(dx+c)}+ia\,e^{2i(dx+c)}-b+ia\right)b^2d}+\frac{2a\ln\left(e^{2i(dx+c)}+1\right)}{b^3d}-\frac{2a\ln\left(e^{2i(dx+c)}-\frac{ib+a}{ib-a}\right)}{b^3d}$
parallelrisch	$\frac{-2a(a\cos(2dx+2c)+a+b\sin(2dx+2c))\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)+2a(a\cos(2dx+2c)+a+b\sin(2dx+2c))}{b^3d(a\cos(2dx+2c)+a}$
norman	$\frac{\frac{\left(4a^2+6b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{b^3d}-\frac{4a^2+2b^2}{2b^3d}-\frac{\left(4a^2+2b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{2b^3d}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}+\frac{2a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^3d}+\frac{2a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^3d}-$

input
```
int(sec(d*x+c)^2/(cos(d*x+c)*a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(tan(d*x+c)/b^2-1/b^3*(a^2+b^2)/(a+b*tan(d*x+c))-2*a/b^3*ln(a+b*tan(d*
x+c)))
```

3.128.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(75) = 150$.

Time = 0.27 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.37

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = -\frac{2\,b^2\cos\left(dx+c\right)^2 - 2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right) - b^2 + \left(a^2\cos\left(dx+c\right)^2 + ab\cos\left(dx+c\right)\sin\left(dx+c\right)\right)}{ab^3 d\cos}$$

input
```
integrate(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas"
)
```

output
```
-(2*b^2*cos(d*x + c)^2 - 2*a*b*cos(d*x + c)*sin(d*x + c) - b^2 + (a^2*cos(
d*x + c)^2 + a*b*cos(d*x + c)*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x
 + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - (a^2*cos(d*x + c)^2 + a*b*cos(
d*x + c)*sin(d*x + c))*log(cos(d*x + c)^2))/(a*b^3*d*cos(d*x + c)^2 + b^4*
d*cos(d*x + c)*sin(d*x + c))
```

3.128.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = \int \frac{\sec^2\left(c+dx\right)}{\left(a\cos\left(c+dx\right)+b\sin\left(c+dx\right)\right)^2}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)**2/(a*cos(c + d*x) + b*sin(c + d*x))**2, x)
```

3.128.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.80

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = -\frac{\frac{a^2+b^2}{b^4\tan(dx+c)+ab^3}+\frac{2a\log(b\tan(dx+c)+a)}{b^3}-\frac{\tan(dx+c)}{b^2}}{d}$$

input
```
integrate(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima"
)
```

output
```
-((a^2 + b^2)/(b^4*tan(d*x + c) + a*b^3) + 2*a*log(b*tan(d*x + c) + a)/b^3
 - tan(d*x + c)/b^2)/d
```

3.128.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.95

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = -\frac{\frac{2a\log(|b\tan(dx+c)+a|)}{b^3}-\frac{\tan(dx+c)}{b^2}-\frac{2ab\tan(dx+c)+a^2-b^2}{(b\tan(dx+c)+a)b^3}}{d}$$

input
```
integrate(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
-(2*a*log(abs(b*tan(d*x + c) + a))/b^3 - tan(d*x + c)/b^2 - (2*a*b*tan(d*x
 + c) + a^2 - b^2)/((b*tan(d*x + c) + a)*b^3))/d
```

3.128.9 Mupad [B] (verification not implemented)

Time = 24.40 (sec) , antiderivative size = 382, normalized size of antiderivative = 5.09

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$$
$$= \frac{\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{b}-\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(2a^2+b^2)}{ab^2}+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(2a^2+b^2)}{ab^2}}{d\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3-2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)}$$
$$-\frac{4a\,\mathrm{atanh}\left(\frac{64a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{64a^3-64a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+\frac{128a^5}{b^2}-\frac{128a^5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{b^2}+\frac{128a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{b}}-\frac{64a^3}{64a^3-64a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+\frac{128a^5}{b^2}-\frac{128a^5\,\mathrm{ta}}{}}\right.}{b^3d}$$

input
```
int(1/(cos(c + d*x)^2*(a*cos(c + d*x) + b*sin(c + d*x))^2),x)
```

output
```
((4*tan(c/2 + (d*x)/2)^2)/b - (2*tan(c/2 + (d*x)/2)^3*(2*a^2 + b^2))/(a*b^
2) + (2*tan(c/2 + (d*x)/2)*(2*a^2 + b^2))/(a*b^2))/(d*(a + 2*b*tan(c/2 + (
d*x)/2) - 2*a*tan(c/2 + (d*x)/2)^2 + a*tan(c/2 + (d*x)/2)^4 - 2*b*tan(c/2
+ (d*x)/2)^3)) - (4*a*atanh((64*a^3*tan(c/2 + (d*x)/2)^2)/(64*a^3 - 64*a^3
*tan(c/2 + (d*x)/2)^2 + (128*a^5)/b^2 - (128*a^5*tan(c/2 + (d*x)/2)^2)/b^2
 + (128*a^4*tan(c/2 + (d*x)/2))/b) - (64*a^3)/(64*a^3 - 64*a^3*tan(c/2 + (
d*x)/2)^2 + (128*a^5)/b^2 - (128*a^5*tan(c/2 + (d*x)/2)^2)/b^2 + (128*a^4*
tan(c/2 + (d*x)/2))/b) + (128*a^4*tan(c/2 + (d*x)/2))/(64*a^3*b + (128*a^5
)/b + 128*a^4*tan(c/2 + (d*x)/2) - (128*a^5*tan(c/2 + (d*x)/2)^2)/b - 64*a
^3*b*tan(c/2 + (d*x)/2)^2)))/(b^3*d)
```

3.128. $\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$

3.129 $\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$

3.129.1 Optimal result

Integrand size = 28, antiderivative size = 179

$$\begin{aligned}\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx &= \frac{2a^2\text{arctanh}(\sin(c+dx))}{b^4d}+\frac{\text{arctanh}(\sin(c+dx))}{2b^2d}\\ &+\frac{(a^2+b^2)\,\text{arctanh}(\sin(c+dx))}{b^4d}\\ &+\frac{3a\sqrt{a^2+b^2}\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^4d}\\ &-\frac{2a\sec(c+dx)}{b^3d}\\ &-\frac{a^2+b^2}{b^3d(a\cos(c+dx)+b\sin(c+dx))}\\ &+\frac{\sec(c+dx)\tan(c+dx)}{2b^2d}\end{aligned}$$

output
```
2*a^2*arctanh(sin(d*x+c))/b^4/d+1/2*arctanh(sin(d*x+c))/b^2/d+(a^2+b^2)*ar
ctanh(sin(d*x+c))/b^4/d-2*a*sec(d*x+c)/b^3/d+(-a^2-b^2)/b^3/d/(a*cos(d*x+c
)+b*sin(d*x+c))+3*a*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))*(
a^2+b^2)^(1/2)/b^4/d+1/2*sec(d*x+c)*tan(d*x+c)/b^2/d
```

3.129.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.08 (sec) , antiderivative size = 709, normalized size of antiderivative = 3.96

$$\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$$
$$= -\frac{(a-ib)(a+ib)\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))}{b^3d(a+b\tan(c+dx))^2} - \frac{2a\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}{b^3d(a+b\tan(c+dx))^2} - \frac{6a\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{\sqrt{a^2+b^2}\left(-b\cos\left(\frac{1}{2}(c+dx)\right)+a\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^2\cos\left(\frac{1}{2}(c+dx)\right)+b^2\cos\left(\frac{1}{2}(c+dx)\right)}\right)\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}{b^4d(a+b\tan(c+dx))^2} - \frac{3(2a^2+b^2)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}{2b^4d(a+b\tan(c+dx))^2} + \frac{3(2a^2+b^2)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}{2b^4d(a+b\tan(c+dx))^2} + \frac{\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}{4b^2d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2(a+b\tan(c+dx))^2} - \frac{2a\sec^2(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)(a\cos(c+dx)+b\sin(c+dx))^2}{b^3d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^2} - \frac{\sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}{4b^2d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2(a+b\tan(c+dx))^2} + \frac{2a\sec^2(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)(a\cos(c+dx)+b\sin(c+dx))^2}{b^3d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^2}$$

input
```
Integrate[Sec[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

output
```
-(((a - I*b)*(a + I*b)*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x]))/(
b^3*d*(a + b*Tan[c + d*x])^2)) - (2*a*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*S
in[c + d*x])^2)/(b^3*d*(a + b*Tan[c + d*x])^2) - (6*a*Sqrt[a^2 + b^2]*ArcT
anh[(Sqrt[a^2 + b^2]*(-(b*Cos[(c + d*x)/2]) + a*Sin[(c + d*x)/2]))/(a^2*Co
s[(c + d*x)/2] + b^2*Cos[(c + d*x)/2])]*Sec[c + d*x]^2*(a*Cos[c + d*x] + b
*Sin[c + d*x])^2)/(b^4*d*(a + b*Tan[c + d*x])^2) - (3*(2*a^2 + b^2)*Log[Co
s[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[
c + d*x])^2)/(2*b^4*d*(a + b*Tan[c + d*x])^2) + (3*(2*a^2 + b^2)*Log[Cos[(
c + d*x)/2] + Sin[(c + d*x)/2]]*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c +
 d*x])^2)/(2*b^4*d*(a + b*Tan[c + d*x])^2) + (Sec[c + d*x]^2*(a*Cos[c + d*
x] + b*Sin[c + d*x])^2)/(4*b^2*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2*(
a + b*Tan[c + d*x])^2) - (2*a*Sec[c + d*x]^2*Sin[(c + d*x)/2]*(a*Cos[c + d
*x] + b*Sin[c + d*x])^2)/(b^3*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(a +
 b*Tan[c + d*x])^2) - (Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)
/(4*b^2*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(a + b*Tan[c + d*x])^2)
+ (2*a*Sec[c + d*x]^2*Sin[(c + d*x)/2]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2
)/(b^3*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^2)
```

3.129.3 Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.27, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3585, 3042, 3573, 3042, 3553, 219, 3583, 3042, 3553, 219, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(c+dx)^3(a\cos(c+dx)+b\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{3585}$$

$$\frac{(a^2+b^2)\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}-\frac{2a\int \frac{\sec^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx}{b^2}+\frac{\int \sec^3(c+dx)dx}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a^2+b^2)\int \frac{1}{\cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}-\frac{2a\int \frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))}dx}{b^2}+\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{b^2}$$

$$\Big\downarrow \text{ 3573}$$

$$\frac{(a^2+b^2)\left(-\frac{a\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{b^2}+\frac{\int \sec(c+dx)dx}{b^2}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}\right)}{b^2}-\frac{2a\int \frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))}dx}{b^2}+\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(a^2+b^2)\left(-\frac{a\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{b^2}+\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}\right)}{b^2}-\frac{2a\int \frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))}dx}{b^2}+\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{b^2}$$

$$\Big\downarrow \text{ 3553}$$

$$\frac{(a^2+b^2)\left(\frac{a\int \frac{1}{a^2+b^2-(b\cos(c+dx)-a\sin(c+dx))^2}d(b\cos(c+dx)-a\sin(c+dx))}{b^2d}+\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}\right)}{b^2}-\frac{2a\int \frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))}dx}{b^2}+\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{b^2}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{(a^2+b^2)\left(\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}+\frac{a\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}\right)}{b^2}-\frac{2a\int \frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))}dx}{b^2}+\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{b^2}$$

$$\Big\downarrow \text{ 3583}$$

$$\frac{(a^2+b^2)\left(\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}+\frac{a\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}\right)}{b^2}-\frac{2a\left(\frac{(a^2+b^2)\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{b^2}-\frac{a\int \sec(c+dx)dx}{b^2}+\frac{\sec(c+dx)}{bd}\right)}{b^2}+\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{b^2}$$

$$\Big\downarrow \text{ 3042}$$

3.129. $\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$

$$\frac{(a^2+b^2)\left(\frac{\int \csc(c+dx+\frac{\pi}{2})dx}{b^2}+\frac{a\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}\right)}{b^2}-\frac{2a\left(\frac{(a^2+b^2)\int\frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{b^2}-\frac{a\int\csc(c+dx+\frac{\pi}{2})dx}{b^2}+\frac{\sec(c+dx)}{bd}\right)}{b^2}+\frac{\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{b^2}$$

$\Big\downarrow$ 3553

$$\frac{(a^2+b^2)\left(\frac{\int \csc(c+dx+\frac{\pi}{2})dx}{b^2}+\frac{a\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}\right)}{b^2}-\frac{2a\left(-\frac{(a^2+b^2)\int\frac{1}{a^2+b^2-(b\cos(c+dx)-a\sin(c+dx))^2}d(b\cos(c+dx)-a\sin(c+dx))}{b^2d}-\frac{a\int\csc(c+dx+\frac{\pi}{2})dx}{b^2}+\frac{\sec(c+dx)}{bd}\right)}{b^2}+\frac{\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{b^2}$$

$\Big\downarrow$ 219

$$\frac{(a^2+b^2)\left(\frac{\int \csc(c+dx+\frac{\pi}{2})dx}{b^2}+\frac{a\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}\right)}{b^2}-\frac{2a\left(-\frac{a\int\csc(c+dx+\frac{\pi}{2})dx}{b^2}-\frac{\sqrt{a^2+b^2}\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d}+\frac{\sec(c+dx)}{bd}\right)}{b^2}+\frac{\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{b^2}$$

$\Big\downarrow$ 4255

$$\frac{(a^2+b^2)\left(\frac{\int \csc(c+dx+\frac{\pi}{2})dx}{b^2}+\frac{a\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}\right)}{b^2}-\frac{2a\left(-\frac{a\int\csc(c+dx+\frac{\pi}{2})dx}{b^2}-\frac{\sqrt{a^2+b^2}\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d}+\frac{\sec(c+dx)}{bd}\right)}{b^2}+\frac{\frac{1}{2}\int\sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}}{b^2}$$

$\Big\downarrow$ 3042

3.129. $\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$

$$\frac{(a^2+b^2)\left(\frac{\int \csc(c+dx+\frac{\pi}{2})dx}{b^2}+\frac{a\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}\right)}{b^2}-\frac{2a\left(-\frac{a\int \csc(c+dx+\frac{\pi}{2})dx}{b^2}-\frac{\sqrt{a^2+b^2}\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d}+\frac{\sec(c+dx)}{bd}\right)}{b^2}+\frac{\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}}{b^2}$$

$$\Big\downarrow \; 4257$$

$$\frac{(a^2+b^2)\left(\frac{a\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}+\frac{\text{arctanh}(\sin(c+dx))}{b^2d}\right)}{b^2}-\frac{2a\left(-\frac{\sqrt{a^2+b^2}\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d}-\frac{a\text{arctanh}(\sin(c+dx))}{b^2d}+\frac{\sec(c+dx)}{bd}\right)}{b^2}+\frac{\frac{\text{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}}{b^2}$$

input
```
Int[Sec[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

output
```
(-2*a*(-((a*ArcTanh[Sin[c + d*x]])/(b^2*d)) - (Sqrt[a^2 + b^2]*ArcTanh[(b*
Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(b^2*d) + Sec[c + d*x]/(b
*d)))/b^2 + ((a^2 + b^2)*(ArcTanh[Sin[c + d*x]]/(b^2*d) + (a*ArcTanh[(b*Co
s[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(b^2*Sqrt[a^2 + b^2]*d) - 1
/(b*d*(a*Cos[c + d*x] + b*Sin[c + d*x]))))/b^2 + (ArcTanh[Sin[c + d*x]]/(2
*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d))/b^2
```

3.129.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.129. $\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3573
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_)/co
s[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(a*Cos[c + d*x] + b*Sin[c + d*x])^
(n + 1)/(b*d*(n + 1)), x] + (Simp[1/b^2   Int[(a*Cos[c + d*x] + b*Sin[c + d
*x])^(n + 2)/Cos[c + d*x], x], x] - Simp[a/b^2   Int[(a*Cos[c + d*x] + b*Si
n[c + d*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
 && LtQ[n, -1]
```

rule 3583
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-Cos[c + d*x]^(m + 1)/(b*d*(m + 1)
), x] + (-Simp[a/b^2   Int[Cos[c + d*x]^(m + 1), x], x] + Simp[(a^2 + b^2)/
b^2   Int[Cos[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

rule 3585
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a^2 + b^2)/b^2   Int[Cos[c +
 d*x]^(m + 2)*(a*Cos[c + d*x] + b*Sin[c + d*x])^n, x], x] + (Simp[1/b^2   I
nt[Cos[c + d*x]^m*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] - Simp[
2*(a/b^2)   Int[Cos[c + d*x]^(m + 1)*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n +
 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] &
& LtQ[m, -1]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.129. $\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$

3.129.4 Maple [A] (verified)

Time = 1.39 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.45

method	result
derivativedivides	$\frac{-\frac{1}{2b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{4a-b}{2b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\left(6a^2+3b^2\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2b^4}+\frac{2\left(\frac{\frac{\left(a^2+b^2\right)b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}+b\left(a^2+b^2\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a}-6}{b^4}}{d}$
default	$\frac{-\frac{1}{2b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{4a-b}{2b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\left(6a^2+3b^2\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2b^4}+\frac{2\left(\frac{\frac{\left(a^2+b^2\right)b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}+b\left(a^2+b^2\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a}-6}{b^4}}{d}$
risch	$-\frac{-3iab\,e^{5i(dx+c)}+6a^2e^{5i(dx+c)}+3b^2e^{5i(dx+c)}+12a^2e^{3i(dx+c)}+2b^2e^{3i(dx+c)}+3iab\,e^{i(dx+c)}+6a^2e^{i(dx+c)}+3b^2e^{i(dx+c)}}{\left(e^{2i(dx+c)}+1\right)^2\left(-ib\,e^{2i(dx+c)}+e^{2i(dx+c)}a+ib+a\right)b^3d}$

input
```
int(sec(d*x+c)^3/(cos(d*x+c)*a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2/b^2/(tan(1/2*d*x+1/2*c)+1)^2-1/2*(4*a-b)/b^3/(tan(1/2*d*x+1/2*c)
+1)+1/2/b^4*(6*a^2+3*b^2)*ln(tan(1/2*d*x+1/2*c)+1)+2/b^4*(((a^2+b^2)*b^2/a
*tan(1/2*d*x+1/2*c)+b*(a^2+b^2))/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1
/2*c)-a)-3*(a^2+b^2)^(1/2)*a*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2
+b^2)^(1/2)))+1/2/b^2/(tan(1/2*d*x+1/2*c)-1)^2-1/2*(-4*a-b)/b^3/(tan(1/2*d
*x+1/2*c)-1)+1/2/b^4*(-6*a^2-3*b^2)*ln(tan(1/2*d*x+1/2*c)-1))
```

3.129.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 355 vs. 2(171) = 342.

Time = 0.31 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.98

$$\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx =$$
$$-\frac{6\,ab^2\cos\left(dx+c\right)\sin\left(dx+c\right)-2\,b^3+6\left(2\,a^2b+b^3\right)\cos\left(dx+c\right)^2-6\left(a^2\cos\left(dx+c\right)^3+ab\cos\left(dx+c\right.\right.}{}$$

input
```
integrate(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas"
)
```

output
```
-1/4*(6*a*b^2*cos(d*x + c)*sin(d*x + c) - 2*b^3 + 6*(2*a^2*b + b^3)*cos(d*
x + c)^2 - 6*(a^2*cos(d*x + c)^3 + a*b*cos(d*x + c)^2*sin(d*x + c))*sqrt(a
^2 + b^2)*log((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^
2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*
a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) - 3*((2
*a^3 + a*b^2)*cos(d*x + c)^3 + (2*a^2*b + b^3)*cos(d*x + c)^2*sin(d*x + c)
)*log(sin(d*x + c) + 1) + 3*((2*a^3 + a*b^2)*cos(d*x + c)^3 + (2*a^2*b + b
^3)*cos(d*x + c)^2*sin(d*x + c))*log(-sin(d*x + c) + 1))/(a*b^4*d*cos(d*x
+ c)^3 + b^5*d*cos(d*x + c)^2*sin(d*x + c))
```

3.129.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = \int \frac{\sec^3\left(c+dx\right)}{\left(a\cos\left(c+dx\right)+b\sin\left(c+dx\right)\right)^2}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)**3/(a*cos(c + d*x) + b*sin(c + d*x))**2, x)
```

3.129.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 471 vs. $2(171) = 342$.

Time = 0.34 (sec) , antiderivative size = 471, normalized size of antiderivative = 2.63

$$\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx =$$

$$-\frac{\frac{2\left(6a^3+2ab^2+\frac{6a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\left(9a^2b+2b^3\right)\sin(dx+c)}{\cos(dx+c)+1}-\frac{6\left(2a^3+ab^2\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{4\left(3a^2b+b^3\right)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\left(3a^2b+2b^3\right)\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^2b^3+\frac{2ab^4\sin(dx+c)}{\cos(dx+c)+1}-\frac{3a^2b^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{4ab^4\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3a^2b^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{2ab^4\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{a^2b^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}} - \frac{6\sqrt{a^2}}{}}{2d}$$

input
```
integrate(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima"
)
```

output
```
-1/2*(2*(6*a^3 + 2*a*b^2 + 6*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + (9*
a^2*b + 2*b^3)*sin(d*x + c)/(cos(d*x + c) + 1) - 6*(2*a^3 + a*b^2)*sin(d*x
 + c)^2/(cos(d*x + c) + 1)^2 - 4*(3*a^2*b + b^3)*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + (3*a^2*b + 2*b^3)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^2*b
^3 + 2*a*b^4*sin(d*x + c)/(cos(d*x + c) + 1) - 3*a^2*b^3*sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 - 4*a*b^4*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*a^2*b
^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 2*a*b^4*sin(d*x + c)^5/(cos(d*x +
 c) + 1)^5 - a^2*b^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) - 6*sqrt(a^2 + b
^2)*a*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a
*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/b^4 - 3*(2*a^2 + b^2)
*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/b^4 + 3*(2*a^2 + b^2)*log(sin(d*
x + c)/(cos(d*x + c) + 1) - 1)/b^4)/d
```

3.129.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.56

$$\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$$

$$= \frac{\frac{3\,(2\,a^2+b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^4} - \frac{3\,(2\,a^2+b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^4} + \frac{6\,(a^3+ab^2)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,b^4} + 2\left(b\right.}{2\,d}$$

input
```
integrate(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
1/2*(3*(2*a^2 + b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^4 - 3*(2*a^2 + b
^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^4 + 6*(a^3 + a*b^2)*log(abs(2*a*t
an(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c
) - 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^4) + 2*(b*tan(1/2*d*x + 1
/2*c)^3 + 4*a*tan(1/2*d*x + 1/2*c)^2 + b*tan(1/2*d*x + 1/2*c) - 4*a)/((tan
(1/2*d*x + 1/2*c)^2 - 1)^2*b^3) + 4*(a^2*b*tan(1/2*d*x + 1/2*c) + b^3*tan(
1/2*d*x + 1/2*c) + a^3 + a*b^2)/((a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d
*x + 1/2*c) - a)*a*b^3))/d
```

3.129.9 Mupad [B] (verification not implemented)

Time = 25.12 (sec) , antiderivative size = 585, normalized size of antiderivative = 3.27

$$\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$$

$$= \frac{\operatorname{atanh}\left(\frac{648\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{216\,a\,b^2+648\,a^3+\frac{432\,a^5}{b^2}}+\frac{432\,a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{432\,a^5+648\,a^3\,b^2+216\,a\,b^4}+\frac{216\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{216\,a+\frac{648\,a^3}{b^2}+\frac{432\,a^5}{b^4}}\right)\left(6\,a^2+3\,b^2\right)}{b^4\,d}$$

$$-\frac{\frac{2\,(3\,a^2+b^2)}{b^3}+\frac{6\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{b^3}-\frac{6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2(2\,a^2+b^2)}{b^3}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)(9\,a^2+2\,b^2)}{a\,b^2}-\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3(3\,a^2+b^2)}{a\,b^2}+\tan}{d\left(-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\right.}$$

$$-\frac{6\,a\operatorname{atanh}\left(\frac{432\,a^3\sqrt{a^2+b^2}}{432\,a^3\,b+\frac{432\,a^5}{b}+864\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+864\,a^2\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{864\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{a^2+b^2}}{432\,a^3+\frac{432\,a^5}{b^2}+864\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{864\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{b}}-\right.}{b^4\,d}$$

input
```
int(1/(cos(c + d*x)^3*(a*cos(c + d*x) + b*sin(c + d*x))^2),x)
```

output
```
(atanh((648*a^3*tan(c/2 + (d*x)/2))/(216*a*b^2 + 648*a^3 + (432*a^5)/b^2)
+ (432*a^5*tan(c/2 + (d*x)/2))/(216*a*b^4 + 432*a^5 + 648*a^3*b^2) + (216*
a*tan(c/2 + (d*x)/2))/(216*a + (648*a^3)/b^2 + (432*a^5)/b^4))*(6*a^2 + 3*
b^2))/(b^4*d) - ((2*(3*a^2 + b^2))/b^3 + (6*a^2*tan(c/2 + (d*x)/2)^4)/b^3
- (6*tan(c/2 + (d*x)/2)^2*(2*a^2 + b^2))/b^3 + (tan(c/2 + (d*x)/2)*(9*a^2
+ 2*b^2))/(a*b^2) - (4*tan(c/2 + (d*x)/2)^3*(3*a^2 + b^2))/(a*b^2) + (tan(
c/2 + (d*x)/2)^5*(3*a^2 + 2*b^2))/(a*b^2))/(d*(a + 2*b*tan(c/2 + (d*x)/2)
- 3*a*tan(c/2 + (d*x)/2)^2 + 3*a*tan(c/2 + (d*x)/2)^4 - a*tan(c/2 + (d*x)/
2)^6 - 4*b*tan(c/2 + (d*x)/2)^3 + 2*b*tan(c/2 + (d*x)/2)^5)) - (6*a*atanh(
(432*a^3*(a^2 + b^2)^(1/2))/(432*a^3*b + (432*a^5)/b + 864*a^4*tan(c/2 + (
d*x)/2) + 864*a^2*b^2*tan(c/2 + (d*x)/2)) + (864*a^2*tan(c/2 + (d*x)/2)*(a
^2 + b^2)^(1/2))/(432*a^3 + (432*a^5)/b^2 + 864*a^2*b*tan(c/2 + (d*x)/2) +
 (864*a^4*tan(c/2 + (d*x)/2))/b) + (432*a^4*tan(c/2 + (d*x)/2)*(a^2 + b^2)
^(1/2))/(432*a^5 + 432*a^3*b^2 + 864*a^4*b*tan(c/2 + (d*x)/2) + 864*a^2*b^
3*tan(c/2 + (d*x)/2)))*(a^2 + b^2)^(1/2))/(b^4*d)
```

3.130 $\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$

3.130.1 Optimal result

Integrand size = 28, antiderivative size = 141

$$\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = \frac{(a^2+b^2)^2}{ab^4d(b+a\cot(c+dx))} - \frac{4a(a^2+b^2)\log(b+a\cot(c+dx))}{b^5d} - \frac{4a(a^2+b^2)\log(\tan(c+dx))}{b^5d} + \frac{(3a^2+2b^2)\tan(c+dx)}{b^4d} - \frac{a\tan^2(c+dx)}{b^3d} + \frac{\tan^3(c+dx)}{3b^2d}$$

output
```
(a^2+b^2)^2/a/b^4/d/(b+a*cot(d*x+c))-4*a*(a^2+b^2)*ln(b+a*cot(d*x+c))/b^5/
d-4*a*(a^2+b^2)*ln(tan(d*x+c))/b^5/d+(3*a^2+2*b^2)*tan(d*x+c)/b^4/d-a*tan(
d*x+c)^2/b^3/d+1/3*tan(d*x+c)^3/b^2/d
```

3.130.2 Mathematica [A] (verified)

Time = 6.02 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.99

$$\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$$
$$= \frac{\frac{b^4\sec^4(c+dx)}{3(a+b\tan(c+dx))}+\frac{4}{3}\left(-a\left(\left(a^2+b^2\right)\log(a+b\tan(c+dx))-ab\tan(c+dx)+\frac{1}{2}b^2\tan^2(c+dx)\right)+\left(a^2+b^2\right)\right.}{b^5d}$$

input
```
Integrate[Sec[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

output
```
((b^4*Sec[c + d*x]^4)/(3*(a + b*Tan[c + d*x])) + (4*(-(a*((a^2 + b^2)*Log[
a + b*Tan[c + d*x]] - a*b*Tan[c + d*x] + (b^2*Tan[c + d*x]^2)/2)) + (a^2 +
 b^2)*(-2*a*Log[a + b*Tan[c + d*x]] + b*Tan[c + d*x] - (a^2 + b^2)/(a + b*
Tan[c + d*x]))))/3)/(b^5*d)
```

3.130.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3567, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(c+dx)^4(a\cos(c+dx)+b\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{3567}$$

$$-\frac{\int \frac{\left(\cot^2(c+dx)+1\right)^2\tan^4(c+dx)}{(b+a\cot(c+dx))^2}d\cot(c+dx)}{d}$$

$$\Big\downarrow \text{522}$$

$$-\frac{\int \left(\frac{\tan^4(c+dx)}{b^2}-\frac{2a\tan^3(c+dx)}{b^3}+\frac{\left(3a^2+2b^2\right)\tan^2(c+dx)}{b^4}-\frac{4a\left(a^2+b^2\right)\tan(c+dx)}{b^5}+\frac{4a^2\left(a^2+b^2\right)}{b^5(b+a\cot(c+dx))}+\frac{\left(a^2+b^2\right)^2}{b^4(b+a\cot(c+dx))^2}\right)dc}{d}$$

$$\downarrow \text{2009}$$

$$-\frac{-\frac{4a(a^2+b^2)\log(\cot(c+dx))}{b^5}+\frac{4a(a^2+b^2)\log(a\cot(c+dx)+b)}{b^5}-\frac{(3a^2+2b^2)\tan(c+dx)}{b^4}-\frac{(a^2+b^2)^2}{ab^4(a\cot(c+dx)+b)}+\frac{a\tan^2(c+dx)}{b^3}-\frac{\tan^3(c}{3b}}{d}$$

input
```
Int[Sec[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

output
```
-((-((a^2 + b^2)^2/(a*b^4*(b + a*Cot[c + d*x]))) - (4*a*(a^2 + b^2)*Log[Co
t[c + d*x]])/b^5 + (4*a*(a^2 + b^2)*Log[b + a*Cot[c + d*x]])/b^5 - ((3*a^2
 + 2*b^2)*Tan[c + d*x])/b^4 + (a*Tan[c + d*x]^2)/b^3 - Tan[c + d*x]^3/(3*b
^2))/d)
```

3.130.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3567
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[x^m*((b
 + a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a,
b, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[
n, 0] && GtQ[m, 1])
```

3.130.4 Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.81

method	result
derivativedivides	$\frac{\frac{\frac{b^2 \tan(dx+c)^3}{3}-ab\tan(dx+c)^2+3\tan(dx+c)a^2+2\tan(dx+c)b^2}{b^4}-\frac{a^4+2a^2b^2+b^4}{b^5(a+b\tan(dx+c))}-\frac{4a\left(a^2+b^2\right)\ln(a+b\tan(dx+c))}{b^5}}{d}$
default	$\frac{\frac{\frac{b^2 \tan(dx+c)^3}{3}-ab\tan(dx+c)^2+3\tan(dx+c)a^2+2\tan(dx+c)b^2}{b^4}-\frac{a^4+2a^2b^2+b^4}{b^5(a+b\tan(dx+c))}-\frac{4a\left(a^2+b^2\right)\ln(a+b\tan(dx+c))}{b^5}}{d}$
risch	$-\frac{8i(-2ia\,b^2+3a^2b+2b^3-3ia^3-3ia\,b^2e^{6i(dx+c)}-6ia\,b^2e^{4i(dx+c)}-5ia\,b^2e^{2i(dx+c)}+3a^2b\,e^{4i(dx+c)}+6a^2b\,e^{2i(dx+c)}-3ia}{3\left(e^{2i(dx+c)}+1\right)^3\left(b\,e^{2i(dx+c)}+ia\,e^{2i(dx+c)}-b+ia\right)b^4d}$
norman	$\frac{-\frac{2\left(24a^2+16b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{3b^3d}+\frac{4\left(2a^2+2b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{b^3d}+\frac{4\left(2a^2+2b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{b^3d}-\frac{2\left(36a^4+44a^2b^2+9b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3ad\,b^4}+}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2\right.}$
parallelrisch	$\frac{-16(a^2+b^2)a\left(a\cos(2dx+2c)+\frac{3a}{4}+\frac{b\sin(2dx+2c)}{2}+\frac{a\cos(4dx+4c)}{4}+\frac{b\sin(4dx+4c)}{4}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right.}{}$

input
```
int(sec(d*x+c)^4/(cos(d*x+c)*a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/b^4*(1/3*b^2*tan(d*x+c)^3-a*b*tan(d*x+c)^2+3*tan(d*x+c)*a^2+2*tan(d
*x+c)*b^2)-1/b^5*(a^4+2*a^2*b^2+b^4)/(a+b*tan(d*x+c))-4*a/b^5*(a^2+b^2)*ln
(a+b*tan(d*x+c)))
```

3.130.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 281 vs. 2(139) = 278.

Time = 0.29 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.99

$$\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = -\frac{4\left(3\,a^2b^2+2\,b^4\right)\cos\left(dx+c\right)^4-b^4-2\left(3\,a^2b^2+2\,b^4\right)\cos\left(dx+c\right)^2+6\left(\left(a^4+a^2b^2\right)\cos\left(dx+c\right)^4+\left(a^3b\right.\right.}{}$$

input
```
integrate(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas"
)
```

output
```
-1/3*(4*(3*a^2*b^2 + 2*b^4)*cos(d*x + c)^4 - b^4 - 2*(3*a^2*b^2 + 2*b^4)*c
os(d*x + c)^2 + 6*((a^4 + a^2*b^2)*cos(d*x + c)^4 + (a^3*b + a*b^3)*cos(d*
x + c)^3*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*c
os(d*x + c)^2 + b^2) - 6*((a^4 + a^2*b^2)*cos(d*x + c)^4 + (a^3*b + a*b^3)
*cos(d*x + c)^3*sin(d*x + c))*log(cos(d*x + c)^2) + 2*(a*b^3*cos(d*x + c)
- 2*(3*a^3*b + 2*a*b^3)*cos(d*x + c)^3)*sin(d*x + c))/(a*b^5*d*cos(d*x + c
)^4 + b^6*d*cos(d*x + c)^3*sin(d*x + c))
```

3.130.6 Sympy [F]

$$\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = \int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$$

input
```
integrate(sec(d*x+c)**4/(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)**4/(a*cos(c + d*x) + b*sin(c + d*x))**2, x)
```

3.130.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.82

$$\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = -\frac{\frac{3\,(a^4+2\,a^2b^2+b^4)}{b^6\tan(dx+c)+ab^5} - \frac{b^2\tan(dx+c)^3-3\,ab\tan(dx+c)^2+3\,(3\,a^2+2\,b^2)\tan(dx+c)}{b^4} + \frac{12\,(a^3+ab^2)\log(b\tan(dx+c)+a)}{b^5}}{3\,d}$$

input
```
integrate(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima"
)
```

output
```
-1/3*(3*(a^4 + 2*a^2*b^2 + b^4)/(b^6*tan(d*x + c) + a*b^5) - (b^2*tan(d*x
+ c)^3 - 3*a*b*tan(d*x + c)^2 + 3*(3*a^2 + 2*b^2)*tan(d*x + c))/b^4 + 12*(
a^3 + a*b^2)*log(b*tan(d*x + c) + a)/b^5)/d
```

3.130.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.06

$$\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx =$$
$$-\frac{\frac{12\,(a^3+ab^2)\log(|b\tan(dx+c)+a|)}{b^5}-\frac{b^4\tan(dx+c)^3-3\,ab^3\tan(dx+c)^2+9\,a^2b^2\tan(dx+c)+6\,b^4\tan(dx+c)}{b^6}-\frac{3\,(4\,a^3b\tan(dx+c)+4\,ab^3\tan}{(b\tan(dx+}}{3\,d}$$

input
```
integrate(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/3*(12*(a^3 + a*b^2)*log(abs(b*tan(d*x + c) + a))/b^5 - (b^4*tan(d*x + c
)^3 - 3*a*b^3*tan(d*x + c)^2 + 9*a^2*b^2*tan(d*x + c) + 6*b^4*tan(d*x + c)
)/b^6 - 3*(4*a^3*b*tan(d*x + c) + 4*a*b^3*tan(d*x + c) + 3*a^4 + 2*a^2*b^2
 - b^4)/((b*tan(d*x + c) + a)*b^5))/d
```

3.130.9 Mupad [B] (verification not implemented)

Time = 26.12 (sec) , antiderivative size = 1132, normalized size of antiderivative = 8.03

$$\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(c + d*x)^4*(a*cos(c + d*x) + b*sin(c + d*x))^2),x)
```

output
```
((8*tan(c/2 + (d*x)/2)^2*(a^2 + b^2))/b^3 + (8*tan(c/2 + (d*x)/2)^6*(a^2 +
 b^2))/b^3 - (16*tan(c/2 + (d*x)/2)^4*(3*a^2 + 2*b^2))/(3*b^3) - (2*tan(c/
2 + (d*x)/2)^7*(4*a^4 + b^4 + 4*a^2*b^2))/(a*b^4) - (2*tan(c/2 + (d*x)/2)^
3*(36*a^4 + 9*b^4 + 44*a^2*b^2))/(3*a*b^4) + (2*tan(c/2 + (d*x)/2)^5*(36*a
^4 + 9*b^4 + 44*a^2*b^2))/(3*a*b^4) + (2*tan(c/2 + (d*x)/2)*(4*a^4 + b^4 +
 4*a^2*b^2))/(a*b^4))/(d*(a + 2*b*tan(c/2 + (d*x)/2) - 4*a*tan(c/2 + (d*x)
/2)^2 + 6*a*tan(c/2 + (d*x)/2)^4 - 4*a*tan(c/2 + (d*x)/2)^6 + a*tan(c/2 +
(d*x)/2)^8 - 6*b*tan(c/2 + (d*x)/2)^3 + 6*b*tan(c/2 + (d*x)/2)^5 - 2*b*tan
(c/2 + (d*x)/2)^7)) + (a*atan(((a*(a^2 + b^2)*((16*tan(c/2 + (d*x)/2)*(4*a
^5 + 4*a^3*b^2))/b^4 - (4*(8*a^2*b^7 + 8*a^4*b^5))/b^8 + (4*tan(c/2 + (d*x
)/2)^2*(8*a^2*b^7 + 8*a^4*b^5))/b^8 + (4*a*(a^2 + b^2)*((4*(a*b^10 + 4*a^3
*b^8))/b^8 - (4*tan(c/2 + (d*x)/2)^2*(3*a*b^10 + 4*a^3*b^8))/b^8 + 16*a^2*
b*tan(c/2 + (d*x)/2)))/b^5)*4i)/b^5 - (a*(a^2 + b^2)*((4*(8*a^2*b^7 + 8*a^
4*b^5))/b^8 - (16*tan(c/2 + (d*x)/2)*(4*a^5 + 4*a^3*b^2))/b^4 - (4*tan(c/2
 + (d*x)/2)^2*(8*a^2*b^7 + 8*a^4*b^5))/b^8 + (4*a*(a^2 + b^2)*((4*(a*b^10
+ 4*a^3*b^8))/b^8 - (4*tan(c/2 + (d*x)/2)^2*(3*a*b^10 + 4*a^3*b^8))/b^8 +
16*a^2*b*tan(c/2 + (d*x)/2)))/b^5)*4i)/b^5)/((8*(16*a^7 + 16*a^3*b^4 + 32*
a^5*b^2))/b^8 + (8*tan(c/2 + (d*x)/2)^2*(16*a^7 + 16*a^3*b^4 + 32*a^5*b^2)
)/b^8 + (4*a*(a^2 + b^2)*((16*tan(c/2 + (d*x)/2)*(4*a^5 + 4*a^3*b^2))/b^4
- (4*(8*a^2*b^7 + 8*a^4*b^5))/b^8 + (4*tan(c/2 + (d*x)/2)^2*(8*a^2*b^7 ...
```

3.130. $\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$

3.131 $\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

3.131.1 Optimal result

Integrand size = 28, antiderivative size = 216

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = -\frac{3b^2(4a^2-b^2)\operatorname{arctanh}\left(\frac{b-a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}\,d} + \frac{b(3a^2-b^2)\cos(c+dx)}{(a^2+b^2)^3\,d} + \frac{a(a^2-3b^2)\sin(c+dx)}{(a^2+b^2)^3\,d} + \frac{b^4\sin(c+dx)}{2a\,(a^2+b^2)^2\,d(a\cos(c+dx)+b\sin(c+dx))^2} - \frac{b^3(8a^2+b^2)}{2a\,(a^2+b^2)^3\,d(a\cos(c+dx)+b\sin(c+dx))}$$

output
```
-3*b^2*(4*a^2-b^2)*arctanh((b-a*tan(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/(a^2+
b^2)^(7/2)/d+b*(3*a^2-b^2)*cos(d*x+c)/(a^2+b^2)^3/d+a*(a^2-3*b^2)*sin(d*x+
c)/(a^2+b^2)^3/d+1/2*b^4*sin(d*x+c)/a/(a^2+b^2)^2/d/(a*cos(d*x+c)+b*sin(d*
x+c))^2-1/2*b^3*(8*a^2+b^2)/a/(a^2+b^2)^3/d/(a*cos(d*x+c)+b*sin(d*x+c))
```

3.131.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.97 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.98

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = \frac{-\frac{6b^2(-4a^2+b^2)\operatorname{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}}-\frac{2b(-3a^2+b^2)\cos(c+dx)}{(a^2+b^2)^3}+\frac{2a(a^2-3b^2)\sin(c+dx)}{(a^2+b^2)^3}+\frac{b^4\sin(c+dx)}{a(a-ib)^2(a+ib)^2(a\cos(c+dx)+}}{2d}$$

input
```
Integrate[Cos[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

output
```
((-6*b^2*(-4*a^2 + b^2)*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]]
)/(a^2 + b^2)^(7/2) - (2*b*(-3*a^2 + b^2)*Cos[c + d*x])/(a^2 + b^2)^3 + (2
*a*(a^2 - 3*b^2)*Sin[c + d*x])/(a^2 + b^2)^3 + (b^4*Sin[c + d*x])/(a*(a -
I*b)^2*(a + I*b)^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2) - (b^3*(8*a^2 + b^
2))/(a*(a^2 + b^2)^3*(a*Cos[c + d*x] + b*Sin[c + d*x])))/(2*d)
```

3.131.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 478 vs. 2(216) = 432.

Time = 1.19 (sec) , antiderivative size = 478, normalized size of antiderivative = 2.21, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 4902, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(c+dx)^4}{(a\cos(c+dx)+b\sin(c+dx))^3}dx$$

$$\Big\downarrow \text{4902}$$

$$\frac{2\int \frac{\left(1-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^4}{\left(\tan^2\left(\frac{1}{2}(c+dx)\right)+1\right)^2\left(-a\tan^2\left(\frac{1}{2}(c+dx)\right)+2b\tan\left(\frac{1}{2}(c+dx)\right)+a\right)^3}d\tan\left(\frac{1}{2}(c+dx)\right)}{d}$$

$$\downarrow 7293$$

$$\frac{2\int \left(-\frac{4\left(a+2b\tan\left(\frac{1}{2}(c+dx)\right)\right)b^4}{a^3(a^2+b^2)\left(a\tan^2\left(\frac{1}{2}(c+dx)\right)-2b\tan\left(\frac{1}{2}(c+dx)\right)-a\right)^3}+\frac{4\left(-b(a^2+b^2)-a(2a^2+b^2)\tan\left(\frac{1}{2}(c+dx)\right)\right)b^3}{a^3(a^2+b^2)^2\left(-a\tan^2\left(\frac{1}{2}(c+dx)\right)+2b\tan\left(\frac{1}{2}(c+dx)\right)+a\right)^2}-\frac{(6}{a^2(a^2+b^2)^3(a\tan^2}}{d}$$

$$\downarrow 2009$$

$$2\left(-\frac{3b^4(a^2+2b^2)\text{arctanh}\left(\frac{b-a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{2a^2(a^2+b^2)^{7/2}}+\frac{2b^4(3a^2+2b^2)\text{arctanh}\left(\frac{b-a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{7/2}}+\frac{a(a^2-3b^2)\tan\left(\frac{1}{2}(c+dx)\right)+b(3a^2-b^2)}{(a^2+b^2)^3\left(\tan^2\left(\frac{1}{2}(c+dx)\right)+1\right)}\right.$$

input
```
Int[Cos[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

output
```
(2*((-3*b^4*(a^2 + 2*b^2)*ArcTanh[(b - a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]
])/(2*a^2*(a^2 + b^2)^(7/2)) + (2*b^4*(3*a^2 + 2*b^2)*ArcTanh[(b - a*Tan[(
c + d*x)/2])/Sqrt[a^2 + b^2]])/(a^2*(a^2 + b^2)^(7/2)) - (b^2*(6*a^4 + 3*a
^2*b^2 + b^4)*ArcTanh[(b - a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a^2*(a^2
 + b^2)^(7/2)) + (b*(3*a^2 - b^2) + a*(a^2 - 3*b^2)*Tan[(c + d*x)/2])/((a^
2 + b^2)^3*(1 + Tan[(c + d*x)/2]^2)) + (b^4*(a*b + (a^2 + 2*b^2)*Tan[(c +
d*x)/2]))/(a^3*(a^2 + b^2)^2*(a + 2*b*Tan[(c + d*x)/2] - a*Tan[(c + d*x)/2
]^2)^2) - (3*b^4*(a^2 + 2*b^2)*(b - a*Tan[(c + d*x)/2]))/(2*a^3*(a^2 + b^2
)^3*(a + 2*b*Tan[(c + d*x)/2] - a*Tan[(c + d*x)/2]^2)) - (2*b^3*(2*a^4 - b
^4 + a*b*(3*a^2 + 2*b^2)*Tan[(c + d*x)/2]))/(a^3*(a^2 + b^2)^3*(a + 2*b*Ta
n[(c + d*x)/2] - a*Tan[(c + d*x)/2]^2))))/d
```

3.131.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.131. $\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

rule 4902
```
Int[u_, x_Symbol] :> With[{w = Block[{$ShowSteps = False, $StepCounter = Nu
ll}, Int[SubstFor[1/(1 + FreeFactors[Tan[FunctionOfTrig[u, x]/2], x]^2*x^2)
, Tan[FunctionOfTrig[u, x]/2]/FreeFactors[Tan[FunctionOfTrig[u, x]/2], x],
u, x], x]]}, Module[{v = FunctionOfTrig[u, x], d}, Simp[d = FreeFactors[Tan
[v/2], x]; 2*(d/Coefficient[v, x, 1])   Subst[Int[SubstFor[1/(1 + d^2*x^2),
 Tan[v/2]/d, u, x], x], x, Tan[v/2]/d], x]] /; CalculusFreeQ[w, x]] /; Inve
rseFunctionFreeQ[u, x] &&  !FalseQ[FunctionOfTrig[u, x]]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.131.4 Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.31

method	result
derivativedivides	$-\frac{\frac{2b^2\left(\frac{-\frac{b^2(9a^2+2b^2)\tan(\frac{dx}{2}+\frac{c}{2})^3}{2a}-\frac{b(8a^4-15a^2b^2-2b^4)\tan(\frac{dx}{2}+\frac{c}{2})^2}{2a^2}+\frac{b^2(23a^2+2b^2)\tan(\frac{dx}{2}+\frac{c}{2})}{2a}+4a^2b+\frac{b^3}{2}}{\left(\tan(\frac{dx}{2}+\frac{c}{2})^2a-2b\tan(\frac{dx}{2}+\frac{c}{2})-a\right)^2}-\frac{3(4a^2-b^2)}{}\right.}{(a^2+b^2)^3}}{d}$
default	$-\frac{\frac{2b^2\left(\frac{-\frac{b^2(9a^2+2b^2)\tan(\frac{dx}{2}+\frac{c}{2})^3}{2a}-\frac{b(8a^4-15a^2b^2-2b^4)\tan(\frac{dx}{2}+\frac{c}{2})^2}{2a^2}+\frac{b^2(23a^2+2b^2)\tan(\frac{dx}{2}+\frac{c}{2})}{2a}+4a^2b+\frac{b^3}{2}}{\left(\tan(\frac{dx}{2}+\frac{c}{2})^2a-2b\tan(\frac{dx}{2}+\frac{c}{2})-a\right)^2}-\frac{3(4a^2-b^2)}{}\right.}{(a^2+b^2)^3}}{d}$
risch	$-\frac{ie^{i(dx+c)}}{2(-3iba^2+ib^3+a^3-3ab^2)d}+\frac{ie^{-i(dx+c)}}{2(3iba^2-ib^3+a^3-3ab^2)d}+\frac{b^3e^{i(dx+c)}\left(-7iab\,e^{2i(dx+c)}+8a^2e^{2i(dx+c)}+b^2e^{2i(dx+c)}\right.}{(-ia+b)^3\left(b\,e^{2i(dx+c)}+ia\,e^{2i(dx+c)}-b+ia\right)^2}$

input
```
int(cos(d*x+c)^4/(cos(d*x+c)*a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*b^2/(a^2+b^2)^3*((-1/2*b^2*(9*a^2+2*b^2)/a*tan(1/2*d*x+1/2*c)^3-1/
2*b*(8*a^4-15*a^2*b^2-2*b^4)/a^2*tan(1/2*d*x+1/2*c)^2+1/2*b^2*(23*a^2+2*b^
2)/a*tan(1/2*d*x+1/2*c)+4*a^2*b+1/2*b^3)/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1
/2*d*x+1/2*c)-a)^2-3/2*(4*a^2-b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/
2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))-2/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*((-a^3
+3*a*b^2)*tan(1/2*d*x+1/2*c)-3*a^2*b+b^3)/(1+tan(1/2*d*x+1/2*c)^2))
```

3.131.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 480 vs. $2(209) = 418$.

Time = 0.29 (sec) , antiderivative size = 480, normalized size of antiderivative = 2.22

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$$
$$= \frac{4\left(a^6b+3\,a^4b^3+3\,a^2b^5+b^7\right)\cos\left(dx+c\right)^3-3\left(4\,a^2b^4-b^6+\left(4\,a^4b^2-5\,a^2b^4+b^6\right)\cos\left(dx+c\right)^2+2\left(4\,a^3\right.\right.}{4\left(\left(a^{10}+\right.\right.}$$

input
```
integrate(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas"
)
```

output
```
1/4*(4*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cos(d*x + c)^3 - 3*(4*a^2*b^4
 - b^6 + (4*a^4*b^2 - 5*a^2*b^4 + b^6)*cos(d*x + c)^2 + 2*(4*a^3*b^3 - a*b
^5)*cos(d*x + c)*sin(d*x + c))*sqrt(a^2 + b^2)*log((2*a*b*cos(d*x + c)*sin
(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(
b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2
- b^2)*cos(d*x + c)^2 + b^2)) + 2*(4*a^6*b - 10*a^4*b^3 - 17*a^2*b^5 - 3*b
^7)*cos(d*x + c) + 2*(2*a^5*b^2 - 11*a^3*b^4 - 13*a*b^6 + 2*(a^7 + 3*a^5*b
^2 + 3*a^3*b^4 + a*b^6)*cos(d*x + c)^2)*sin(d*x + c))/((a^10 + 3*a^8*b^2 +
 2*a^6*b^4 - 2*a^4*b^6 - 3*a^2*b^8 - b^10)*d*cos(d*x + c)^2 + 2*(a^9*b + 4
*a^7*b^3 + 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*d*cos(d*x + c)*sin(d*x + c) + (a
^8*b^2 + 4*a^6*b^4 + 6*a^4*b^6 + 4*a^2*b^8 + b^10)*d)
```

3.131.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**4/(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

output
```
Timed out
```

3.131.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 658 vs. $2(209) = 418$.

Time = 0.33 (sec) , antiderivative size = 658, normalized size of antiderivative = 3.05

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = -\frac{\frac{3(4a^2b^2-b^4)\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{(a^6+3a^4b^2+3a^2b^4+b^6)\sqrt{a^2+b^2}} - \frac{2\left(6a^6b-10a^4b^3-a^2b^5+\frac{(2a^7+18a^5b^2-31a^3b^4-2ab^6)\sin(dx+c)}{\cos(dx+c)+1}-\frac{2(2a^6b-2a^4b^3+\cdots}{(\cos(\cdots}\cdots\right.}{a^{10}+3a^8b^2+3a^6b^4+a^4b^6+\frac{4(a^9b+3a^7b^3+3a^5b^5+a^3b^7)\sin(dx+c)}{\cos(dx+c)+1}-\frac{(a^{10}-a^8b^2-9a^6b^4-11a^4\cdots}{(\cos(dx+\cdots}\cdots}}{\cdots}$$

input
```
integrate(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima"
)
```

output
```
-1/2*(3*(4*a^2*b^2 - b^4)*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqr
t(a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(
(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) - 2*(6*a^6*b - 10*a^4
*b^3 - a^2*b^5 + (2*a^7 + 18*a^5*b^2 - 31*a^3*b^4 - 2*a*b^6)*sin(d*x + c)/
(cos(d*x + c) + 1) - 2*(2*a^6*b - 2*a^4*b^3 + 12*a^2*b^5 + b^7)*sin(d*x +
c)^2/(cos(d*x + c) + 1)^2 - 2*(2*a^7 + 2*a^5*b^2 + 15*a^3*b^4)*sin(d*x + c
)^3/(cos(d*x + c) + 1)^3 - (2*a^6*b - 30*a^4*b^3 + 15*a^2*b^5 + 2*b^7)*sin
(d*x + c)^4/(cos(d*x + c) + 1)^4 + (2*a^7 - 6*a^5*b^2 + 9*a^3*b^4 + 2*a*b^
6)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^10 + 3*a^8*b^2 + 3*a^6*b^4 + a^
4*b^6 + 4*(a^9*b + 3*a^7*b^3 + 3*a^5*b^5 + a^3*b^7)*sin(d*x + c)/(cos(d*x
+ c) + 1) - (a^10 - a^8*b^2 - 9*a^6*b^4 - 11*a^4*b^6 - 4*a^2*b^8)*sin(d*x
+ c)^2/(cos(d*x + c) + 1)^2 - (a^10 - a^8*b^2 - 9*a^6*b^4 - 11*a^4*b^6 - 4
*a^2*b^8)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 4*(a^9*b + 3*a^7*b^3 + 3*a
^5*b^5 + a^3*b^7)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + (a^10 + 3*a^8*b^2
+ 3*a^6*b^4 + a^4*b^6)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6))/d
```

3.131.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.85

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = -\frac{\frac{3(4a^2b^2-b^4)\log\left(\frac{\left|2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2b-2\sqrt{a^2+b^2}\right|}{\left|2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2b+2\sqrt{a^2+b^2}\right|}\right)}{(a^6+3a^4b^2+3a^2b^4+b^6)\sqrt{a^2+b^2}} - \frac{4\left(a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-3ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3a^2b-b^3\right)}{(a^6+3a^4b^2+3a^2b^4+b^6)\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)} - \frac{2\left(9a^3b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}\cdots\right.\right.}{\cdots}}{\cdots}$$

input
```
integrate(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/2*(3*(4*a^2*b^2 - b^4)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(
a^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^6
 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) - 4*(a^3*tan(1/2*d*x + 1/
2*c) - 3*a*b^2*tan(1/2*d*x + 1/2*c) + 3*a^2*b - b^3)/((a^6 + 3*a^4*b^2 + 3
*a^2*b^4 + b^6)*(tan(1/2*d*x + 1/2*c)^2 + 1)) - 2*(9*a^3*b^4*tan(1/2*d*x +
 1/2*c)^3 + 2*a*b^6*tan(1/2*d*x + 1/2*c)^3 + 8*a^4*b^3*tan(1/2*d*x + 1/2*c
)^2 - 15*a^2*b^5*tan(1/2*d*x + 1/2*c)^2 - 2*b^7*tan(1/2*d*x + 1/2*c)^2 - 2
3*a^3*b^4*tan(1/2*d*x + 1/2*c) - 2*a*b^6*tan(1/2*d*x + 1/2*c) - 8*a^4*b^3
- a^2*b^5)/((a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*(a*tan(1/2*d*x + 1/2*c
)^2 - 2*b*tan(1/2*d*x + 1/2*c) - a)^2))/d
```

3.131.9 Mupad [B] (verification not implemented)

Time = 26.66 (sec) , antiderivative size = 610, normalized size of antiderivative = 2.82

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx =$$

$$-\frac{\frac{-6\,a^4\,b+10\,a^2\,b^3+b^5}{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(2\,a^5+2\,a^3\,b^2+15\,a\,b^4\right)}{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(2\,a^6\,b-30\,a^4\,b^3+15\,a^2\,b^5+2\,b^7\right)}{a^2\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^6\right.}{a\,\left(a^6+3\,a\right.}}{d\,\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+a^2-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(a^2-4\,b^2\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right.}$$
$$-\frac{\mathrm{atan}\left(\frac{-1\mathrm{i}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^7+a^6\,b\,1\mathrm{i}-3\mathrm{i}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^5\,b^2+a^4\,b^3\,3\mathrm{i}-3\mathrm{i}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^3\,b^4+a^2\,b^5\,3\mathrm{i}-1\mathrm{i}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b^6+b^7\,1\mathrm{i}}{\left(a^2+b^2\right)^{7/2}}\right)\,\left(3\,b^4-\right.}{d\,\left(a^2+b^2\right)^{7/2}}$$

input
```
int(cos(c + d*x)^4/(a*cos(c + d*x) + b*sin(c + d*x))^3,x)
```

output
```
- ((b^5 - 6*a^4*b + 10*a^2*b^3)/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (2*t
an(c/2 + (d*x)/2)^3*(15*a*b^4 + 2*a^5 + 2*a^3*b^2))/(a^6 + b^6 + 3*a^2*b^4
 + 3*a^4*b^2) + (tan(c/2 + (d*x)/2)^4*(2*a^6*b + 2*b^7 + 15*a^2*b^5 - 30*a
^4*b^3))/(a^2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (tan(c/2 + (d*x)/2)*(
2*a^6 - 2*b^6 - 31*a^2*b^4 + 18*a^4*b^2))/(a*(a^6 + b^6 + 3*a^2*b^4 + 3*a^
4*b^2)) - (tan(c/2 + (d*x)/2)^5*(2*a^6 + 2*b^6 + 9*a^2*b^4 - 6*a^4*b^2))/(
a*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (2*tan(c/2 + (d*x)/2)^2*(2*a^6*b
+ b^7 + 12*a^2*b^5 - 2*a^4*b^3))/(a^2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))
)/(d*(a^2*tan(c/2 + (d*x)/2)^6 + a^2 - tan(c/2 + (d*x)/2)^2*(a^2 - 4*b^2)
- tan(c/2 + (d*x)/2)^4*(a^2 - 4*b^2) - 4*a*b*tan(c/2 + (d*x)/2)^5 + 4*a*b*
tan(c/2 + (d*x)/2))) - (atan((a^6*b*1i + b^7*1i + a^2*b^5*3i + a^4*b^3*3i
- a^7*tan(c/2 + (d*x)/2)*1i - a*b^6*tan(c/2 + (d*x)/2)*1i - a^3*b^4*tan(c/
2 + (d*x)/2)*3i - a^5*b^2*tan(c/2 + (d*x)/2)*3i)/(a^2 + b^2)^(7/2))*(3*b^4
 - 12*a^2*b^2)*1i)/(d*(a^2 + b^2)^(7/2))
```

3.131. $\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

3.132 $\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

3.132.1 Optimal result

Integrand size = 28, antiderivative size = 122

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = \frac{a(a^2-3b^2)\,x}{(a^2+b^2)^3} + \frac{b(3a^2-b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^3 d} - \frac{b}{2(a^2+b^2)\,d(a+b\tan(c+dx))^2} - \frac{2ab}{(a^2+b^2)^2 d(a+b\tan(c+dx))}$$

output
```
a*(a^2-3*b^2)*x/(a^2+b^2)^3+b*(3*a^2-b^2)*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a
^2+b^2)^3/d-1/2*b/(a^2+b^2)/d/(a+b*tan(d*x+c))^2-2*a*b/(a^2+b^2)^2/d/(a+b*
tan(d*x+c))
```

3.132.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.97 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.26

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = \frac{\frac{2a(a^2-3b^2)(c+dx)}{(a^2+b^2)^3} - \frac{2b(-3a^2+b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^3} - \frac{b^3}{(a-ib)^2(a+ib)^2(a\cos(c+dx)+b\sin(c+dx))^2} + \frac{6b^2\sin(c+\cdots}{(a^2+b^2)^2(a\cos(c+dx)\cdots}}{2d}$$

input
```
Integrate[Cos[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

output
```
((2*a*(a^2 - 3*b^2)*(c + d*x))/(a^2 + b^2)^3 - (2*b*(-3*a^2 + b^2)*Log[a*C
os[c + d*x] + b*Sin[c + d*x]])/(a^2 + b^2)^3 - b^3/((a - I*b)^2*(a + I*b)^
2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2) + (6*b^2*Sin[c + d*x])/((a^2 + b^2)
^2*(a*Cos[c + d*x] + b*Sin[c + d*x])))/(2*d)
```

3.132.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.18, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3565, 3042, 3964, 3042, 4012, 3042, 4014, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(c+dx)^3}{(a\cos(c+dx)+b\sin(c+dx))^3}dx$$

$$\Big\downarrow \text{3565}$$

$$\int \frac{1}{(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3964}$$

$$\frac{\int \frac{a-b\tan(c+dx)}{(a+b\tan(c+dx))^2}dx}{a^2+b^2}-\frac{b}{2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a-b\tan(c+dx)}{(a+b\tan(c+dx))^2}dx}{a^2+b^2}-\frac{b}{2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}$$

$$\Big\downarrow \text{4012}$$

$$\frac{\frac{\int \frac{a^2-2b\tan(c+dx)a-b^2}{a+b\tan(c+dx)}dx}{a^2+b^2}-\frac{2ab}{d(a^2+b^2)(a+b\tan(c+dx))}}{a^2+b^2}-\frac{b}{2d\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{\int \frac{a^2-2b\tan(c+dx)a-b^2}{a+b\tan(c+dx)}dx}{a^2+b^2}-\frac{2ab}{d(a^2+b^2)(a+b\tan(c+dx))}}{a^2+b^2}-\frac{b}{2d\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)^2}$$

$$\Big\downarrow \text{ 4014}$$

$$\frac{\frac{\frac{b\left(3a^2-b^2\right)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{ax\left(a^2-3b^2\right)}{a^2+b^2}}{a^2+b^2}-\frac{2ab}{d(a^2+b^2)(a+b\tan(c+dx))}}{a^2+b^2}-\frac{b}{2d\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{\frac{b\left(3a^2-b^2\right)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{ax\left(a^2-3b^2\right)}{a^2+b^2}}{a^2+b^2}-\frac{2ab}{d(a^2+b^2)(a+b\tan(c+dx))}}{a^2+b^2}-\frac{b}{2d\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)^2}$$

$$\Big\downarrow \text{ 4013}$$

$$\frac{\frac{\frac{b\left(3a^2-b^2\right)\log(a\cos(c+dx)+b\sin(c+dx))}{d\left(a^2+b^2\right)}+\frac{ax\left(a^2-3b^2\right)}{a^2+b^2}}{a^2+b^2}-\frac{2ab}{d(a^2+b^2)(a+b\tan(c+dx))}}{a^2+b^2}-\frac{b}{2d\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)^2}$$

input
```
Int[Cos[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

output
```
-1/2*b/((a^2 + b^2)*d*(a + b*Tan[c + d*x])^2) + (((a*(a^2 - 3*b^2)*x)/(a^2
 + b^2) + (b*(3*a^2 - b^2)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b
^2)*d))/(a^2 + b^2) - (2*a*b)/((a^2 + b^2)*d*(a + b*Tan[c + d*x])))/(a^2 +
 b^2)
```

3.132. $\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

3.132.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3565
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(a + b*Tan[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

rule 3964
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)
 Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

3.132. $\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

3.132.4 Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.15

method	result
derivativedivides	$\frac{\frac{\frac{\left(-3a^2b+b^3\right)\ln\left(1+\tan(dx+c)^2\right)}{2}+\left(a^3-3a\,b^2\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^3}-\frac{b}{2\left(a^2+b^2\right)(a+b\tan(dx+c))^2}+\frac{b\left(3a^2-b^2\right)\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)^3}-}{d}$
default	$\frac{\frac{\frac{\left(-3a^2b+b^3\right)\ln\left(1+\tan(dx+c)^2\right)}{2}+\left(a^3-3a\,b^2\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^3}-\frac{b}{2\left(a^2+b^2\right)(a+b\tan(dx+c))^2}+\frac{b\left(3a^2-b^2\right)\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)^3}-}{d}$
parallelrisch	$6\left(\left(a^2-b^2\right)\cos(2dx+2c)+2ab\sin(2dx+2c)+a^2+b^2\right)b\left(a^2-2b^2\right)\left(a^2-\frac{b^2}{3}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)-6((a$
risch	$-\frac{x}{3ib\,a^2-ib^3-a^3+3a\,b^2}-\frac{6ib\,a^2x}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{2ib^3x}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{6ib\,a^2c}{d(a^6+3a^4b^2+3a^2b^4+b^6)}+\frac{}{d(a^6+3a}$
norman	Expression too large to display

input
```
int(cos(d*x+c)^3/(cos(d*x+c)*a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/(a^2+b^2)^3*(1/2*(-3*a^2*b+b^3)*ln(1+tan(d*x+c)^2)+(a^3-3*a*b^2)*ar
ctan(tan(d*x+c)))-1/2*b/(a^2+b^2)/(a+b*tan(d*x+c))^2+b*(3*a^2-b^2)/(a^2+b^
2)^3*ln(a+b*tan(d*x+c))-2*a*b/(a^2+b^2)^2/(a+b*tan(d*x+c)))
```

3.132.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. 2(120) = 240.

Time = 0.29 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.80

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$$
$$=\frac{5\,a^2b^3-b^5+2\,(a^3b^2-3\,ab^4)dx-2\,(6\,a^2b^3-(a^5-4\,a^3b^2+3\,ab^4)dx)\cos{(dx+c)}^2+2\,(3\,a^3b^2-3\,ab^4+2}{2\,((a^8+2\,a^6b^2-2\,a}$$

input
```
integrate(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas"
)
```

output
```
1/2*(5*a^2*b^3 - b^5 + 2*(a^3*b^2 - 3*a*b^4)*d*x - 2*(6*a^2*b^3 - (a^5 - 4
*a^3*b^2 + 3*a*b^4)*d*x)*cos(d*x + c)^2 + 2*(3*a^3*b^2 - 3*a*b^4 + 2*(a^4*
b - 3*a^2*b^3)*d*x)*cos(d*x + c)*sin(d*x + c) + (3*a^2*b^3 - b^5 + (3*a^4*
b - 4*a^2*b^3 + b^5)*cos(d*x + c)^2 + 2*(3*a^3*b^2 - a*b^4)*cos(d*x + c)*s
in(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c
)^2 + b^2))/((a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*d*cos(d*x + c)^2 + 2*(a^7
*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*cos(d*x + c)*sin(d*x + c) + (a^6*b^2
 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*d)
```

3.132.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(cos(d*x+c)**3/(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

output
```
Exception raised: AttributeError >> 'NoneType' object has no attribute 'pr
imitive'
```

3.132.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 481 vs. $2(120) = 240$.

Time = 0.33 (sec) , antiderivative size = 481, normalized size of antiderivative = 3.94

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$$
$$= \frac{\frac{2\,(a^3-3\,ab^2)\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{(3\,a^2b-b^3)\log\left(-a-\frac{2\,b\sin(dx+c)}{\cos(dx+c)+1}+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} - \frac{(3\,a^2b-b^3)\log\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{}{a^8+2\,a}}{d}$$

input
```
integrate(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima"
)
```

output
```
(2*(a^3 - 3*a*b^2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/(a^6 + 3*a^4*b^
2 + 3*a^2*b^4 + b^6) + (3*a^2*b - b^3)*log(-a - 2*b*sin(d*x + c)/(cos(d*x
+ c) + 1) + a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/(a^6 + 3*a^4*b^2 + 3*a^
2*b^4 + b^6) - (3*a^2*b - b^3)*log(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1
)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 2*((3*a^3*b^2 + a*b^4)*sin(d*x + c
)/(cos(d*x + c) + 1) + (5*a^2*b^3 + b^5)*sin(d*x + c)^2/(cos(d*x + c) + 1)
^2 - (3*a^3*b^2 + a*b^4)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/(a^8 + 2*a^6
*b^2 + a^4*b^4 + 4*(a^7*b + 2*a^5*b^3 + a^3*b^5)*sin(d*x + c)/(cos(d*x + c
) + 1) - 2*(a^8 - 3*a^4*b^4 - 2*a^2*b^6)*sin(d*x + c)^2/(cos(d*x + c) + 1)
^2 - 4*(a^7*b + 2*a^5*b^3 + a^3*b^5)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 +
 (a^8 + 2*a^6*b^2 + a^4*b^4)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4))/d
```

3.132.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(120) = 240$.

Time = 0.39 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.17

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$$
$$= \frac{\frac{2\,(a^3-3\,ab^2)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} - \frac{(3\,a^2b-b^3)\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{2\,(3\,a^2b^2-b^4)\log(|b\tan(dx+c)+a|)}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7} - \frac{9\,a^2b^3\tan(dx+c)^2-3\,b^5\tan(dx+c)^2+22}{(a^6+3\,a^4b^2+3}}{2\,d}$$

input
```
integrate(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
1/2*(2*(a^3 - 3*a*b^2)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (3*
a^2*b - b^3)*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) +
 2*(3*a^2*b^2 - b^4)*log(abs(b*tan(d*x + c) + a))/(a^6*b + 3*a^4*b^3 + 3*a
^2*b^5 + b^7) - (9*a^2*b^3*tan(d*x + c)^2 - 3*b^5*tan(d*x + c)^2 + 22*a^3*
b^2*tan(d*x + c) - 2*a*b^4*tan(d*x + c) + 14*a^4*b + 3*a^2*b^3 + b^5)/((a^
6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*(b*tan(d*x + c) + a)^2))/d
```

3.132.9 Mupad [B] (verification not implemented)

Time = 32.44 (sec) , antiderivative size = 6190, normalized size of antiderivative = 50.74

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(cos(c + d*x)^3/(a*cos(c + d*x) + b*sin(c + d*x))^3,x)
```

output
```
((2*tan(c/2 + (d*x)/2)^2*(b^5 + 5*a^2*b^3))/(a^2*(a^4 + b^4 + 2*a^2*b^2))
+ (2*b*tan(c/2 + (d*x)/2)*(3*a^2*b + b^3))/(a*(a^4 + b^4 + 2*a^2*b^2)) - (
2*b*tan(c/2 + (d*x)/2)^3*(3*a^2*b + b^3))/(a*(a^4 + b^4 + 2*a^2*b^2)))/(d*
(a^2*tan(c/2 + (d*x)/2)^4 - tan(c/2 + (d*x)/2)^2*(2*a^2 - 4*b^2) + a^2 - 4
*a*b*tan(c/2 + (d*x)/2)^3 + 4*a*b*tan(c/2 + (d*x)/2))) - (log((((-(a^2*(a^
2 - 3*b^2)^2)/(a^2 + b^2)^6)^(1/2) + (3*a^2*b - b^3)/(a^2 + b^2)^3)*(((-(a
^2*(a^2 - 3*b^2)^2)/(a^2 + b^2)^6)^(1/2) + (3*a^2*b - b^3)/(a^2 + b^2)^3)*
((32*a*b*tan(c/2 + (d*x)/2)*(b^4 - 8*a^4 + 5*a^2*b^2))/(a^2 + b^2)^2 - (32
*a^2*(a^4 + 4*b^4 - 7*a^2*b^2))/(a^2 + b^2)^2 + 96*a*b*(a + b*tan(c/2 + (d
*x)/2))*((-(a^2*(a^2 - 3*b^2)^2)/(a^2 + b^2)^6)^(1/2) + (3*a^2*b - b^3)/(a
^2 + b^2)^3)*(a^2 + b^2)) - (32*a^2*b*(5*a^2 - 3*b^2))/(a^2 + b^2)^3 + (32
*a*tan(c/2 + (d*x)/2)*(a^6 - 3*b^6 + 27*a^2*b^4 - 17*a^4*b^2))/(a^2 + b^2)
^4) - (64*a^2*b^2*(3*a^4 + b^4 - 4*a^2*b^2))/(a^2 + b^2)^6 + (32*a*b*tan(c
/2 + (d*x)/2)*(3*a^6 - b^6 - 3*a^2*b^4 + 17*a^4*b^2))/(a^2 + b^2)^6)*(((-(
a^2*(a^2 - 3*b^2)^2)/(a^2 + b^2)^6)^(1/2) - (3*a^2*b - b^3)/(a^2 + b^2)^3)
*(((-(a^2*(a^2 - 3*b^2)^2)/(a^2 + b^2)^6)^(1/2) - (3*a^2*b - b^3)/(a^2 + b
^2)^3)*((32*a^2*(a^4 + 4*b^4 - 7*a^2*b^2))/(a^2 + b^2)^2 - (32*a*b*tan(c/2
 + (d*x)/2)*(b^4 - 8*a^4 + 5*a^2*b^2))/(a^2 + b^2)^2 + 96*a*b*(a + b*tan(c
/2 + (d*x)/2))*((-(a^2*(a^2 - 3*b^2)^2)/(a^2 + b^2)^6)^(1/2) - (3*a^2*b -
b^3)/(a^2 + b^2)^3)*(a^2 + b^2)) - (32*a^2*b*(5*a^2 - 3*b^2))/(a^2 + b^...
```

3.133 $\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

3.133.1 Optimal result

Integrand size = 28, antiderivative size = 119

$$\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = \frac{(2a^2-b^2)\,\text{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}\,d} - \frac{b((4a^2+b^2)\cos(c+dx)+3ab\sin(c+dx))}{2\,(a^2+b^2)^2\,d(a\cos(c+dx)+b\sin(c+dx))^2}$$

output
```
(2*a^2-b^2)*arctanh((-b+a*tan(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(
5/2)/d-1/2*b*((4*a^2+b^2)*cos(d*x+c)+3*a*b*sin(d*x+c))/(a^2+b^2)^2/d/(a*co
s(d*x+c)+b*sin(d*x+c))^2
```

3.133.2 Mathematica [A] (verified)

Time = 1.28 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00

$$\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = \frac{\frac{2(2a^2-b^2)\text{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} - \frac{b((4a^2+b^2)\cos(c+dx)+3ab\sin(c+dx))}{(a^2+b^2)^2(a\cos(c+dx)+b\sin(c+dx))^2}}{2d}$$

input
```
Integrate[Cos[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

output
```
((2*(2*a^2 - b^2)*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a^2
 + b^2)^(5/2) - (b*((4*a^2 + b^2)*Cos[c + d*x] + 3*a*b*Sin[c + d*x]))/((a^
2 + b^2)^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2))/(2*d)
```

3.133.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 4902, 2191, 27, 2191, 27, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(c+dx)^2}{(a\cos(c+dx)+b\sin(c+dx))^3}dx$$

$$\Big\downarrow \text{4902}$$

$$\frac{2\int \frac{\left(1-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^2}{\left(-a\tan^2\left(\frac{1}{2}(c+dx)\right)+2b\tan\left(\frac{1}{2}(c+dx)\right)+a\right)^3}d\tan\left(\frac{1}{2}(c+dx)\right)}{d}$$

$$\Big\downarrow \text{2191}$$

$$\frac{2\left(\frac{b^2\left(\left(a^2+2b^2\right)\tan\left(\frac{1}{2}(c+dx)\right)+ab\right)}{a^3(a^2+b^2)\left(-a\tan^2\left(\frac{1}{2}(c+dx)\right)+a+2b\tan\left(\frac{1}{2}(c+dx)\right)\right)^2}-\frac{\int -\frac{8\left(-\left(\left(\frac{b^2}{a}+a\right)\tan^2\left(\frac{1}{2}(c+dx)\right)\right)-2b\left(\frac{b^2}{a^2}+1\right)\tan\left(\frac{1}{2}(c+dx)\right)+\frac{a^4+2b^4}{a^3}\right)}{\left(-a\tan^2\left(\frac{1}{2}(c+dx)\right)+2b\tan\left(\frac{1}{2}(c+dx)\right)+a\right)^2}d\tan\left(\frac{1}{2}(c+\right.}{8(a^2+b^2)}\right.}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\left(\frac{\int \frac{\frac{2b^4}{a^3}-2\left(\frac{b^2}{a^2}+1\right)\tan\left(\frac{1}{2}(c+dx)\right)b-\frac{\left(a^2+b^2\right)\tan^2\left(\frac{1}{2}(c+dx)\right)}{a}+a}{\left(-a\tan^2\left(\frac{1}{2}(c+dx)\right)+2b\tan\left(\frac{1}{2}(c+dx)\right)+a\right)^2}d\tan\left(\frac{1}{2}(c+dx)\right)}{a^2+b^2}+\frac{b^2\left(\left(a^2+2b^2\right)\tan\left(\frac{1}{2}(c+dx)\right)+ab\right)}{a^3(a^2+b^2)\left(-a\tan^2\left(\frac{1}{2}(c+dx)\right)+a+2b\tan\left(\frac{1}{2}(c+dx)\right)\right)^2}\right)}{d}$$

$$\Big\downarrow \text{2191}$$

$$2\left(\frac{-\frac{\int -\frac{2(2a^2-b^2)}{-a\tan^2\left(\frac{1}{2}(c+dx)\right)+2b\tan\left(\frac{1}{2}(c+dx)\right)+a}d\tan\left(\frac{1}{2}(c+dx)\right)}{4(a^2+b^2)}-\frac{b\left(\frac{2b^4}{a^3}+b\left(\frac{2b^2}{a^2}+5\right)\tan\left(\frac{1}{2}(c+dx)\right)+\frac{3b^2}{a}+4a\right)}{2(a^2+b^2)\left(-a\tan^2\left(\frac{1}{2}(c+dx)\right)+a+2b\tan\left(\frac{1}{2}(c+dx)\right)\right)}}{a^2+b^2}+\frac{b^2\left((a^2+2b^2)\tan\right.}{a^3(a^2+b^2)\left(-a\tan^2\left(\frac{1}{2}(c+dx\right.\right.}\right.}{d}$$

$\downarrow$ 27

$$2\left(\frac{\frac{(2a^2-b^2)\int \frac{1}{-a\tan^2\left(\frac{1}{2}(c+dx)\right)+2b\tan\left(\frac{1}{2}(c+dx)\right)+a}d\tan\left(\frac{1}{2}(c+dx)\right)}{2(a^2+b^2)}-\frac{b\left(\frac{2b^4}{a^3}+b\left(\frac{2b^2}{a^2}+5\right)\tan\left(\frac{1}{2}(c+dx)\right)+\frac{3b^2}{a}+4a\right)}{2(a^2+b^2)\left(-a\tan^2\left(\frac{1}{2}(c+dx)\right)+a+2b\tan\left(\frac{1}{2}(c+dx)\right)\right)}}{a^2+b^2}+\frac{b^2\left((a^2+2b\right.}{a^3(a^2+b^2)\left(-a\tan^2\left(\frac{1}{2}\right.\right.}\right.}{d}$$

$\downarrow$ 1083

$$2\left(\frac{-\frac{(2a^2-b^2)\int \frac{1}{4(a^2+b^2)-\left(2b-2a\tan\left(\frac{1}{2}(c+dx)\right)\right)^2}d\left(2b-2a\tan\left(\frac{1}{2}(c+dx)\right)\right)}{a^2+b^2}-\frac{b\left(\frac{2b^4}{a^3}+b\left(\frac{2b^2}{a^2}+5\right)\tan\left(\frac{1}{2}(c+dx)\right)+\frac{3b^2}{a}+4a\right)}{2(a^2+b^2)\left(-a\tan^2\left(\frac{1}{2}(c+dx)\right)+a+2b\tan\left(\frac{1}{2}(c+dx)\right)\right)}}{a^2+b^2}+\frac{b^2\left((\right.}{a^3(a^2+b^2)(-a}\right.}{d}$$

$\downarrow$ 219

$$2\left(\frac{-\frac{(2a^2-b^2)\text{arctanh}\left(\frac{2b-2a\tan\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{3/2}}-\frac{b\left(\frac{2b^4}{a^3}+b\left(\frac{2b^2}{a^2}+5\right)\tan\left(\frac{1}{2}(c+dx)\right)+\frac{3b^2}{a}+4a\right)}{2(a^2+b^2)\left(-a\tan^2\left(\frac{1}{2}(c+dx)\right)+a+2b\tan\left(\frac{1}{2}(c+dx)\right)\right)}}{a^2+b^2}+\frac{b^2\left((a^2+2b^2)\tan\left(\frac{1}{2}(c+dx)\right)+\right.}{a^3(a^2+b^2)\left(-a\tan^2\left(\frac{1}{2}(c+dx)\right)+a+2b\tan\right.}\right.}{d}$$

input
```
Int[Cos[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

output
```
(2*((b^2*(a*b + (a^2 + 2*b^2)*Tan[(c + d*x)/2]))/(a^3*(a^2 + b^2)*(a + 2*b
*Tan[(c + d*x)/2] - a*Tan[(c + d*x)/2]^2)^2) + (-1/2*((2*a^2 - b^2)*ArcTan
h[(2*b - 2*a*Tan[(c + d*x)/2])/(2*Sqrt[a^2 + b^2])])/(a^2 + b^2)^(3/2) - (
b*(4*a + (3*b^2)/a + (2*b^4)/a^3 + b*(5 + (2*b^2)/a^2)*Tan[(c + d*x)/2]))/
(2*(a^2 + b^2)*(a + 2*b*Tan[(c + d*x)/2] - a*Tan[(c + d*x)/2]^2)))/(a^2 +
b^2)))/d
```

3.133. $\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

3.133.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4902
```
Int[u_, x_Symbol] :> With[{w = Block[{$ShowSteps = False, $StepCounter = Nu
ll}, Int[SubstFor[1/(1 + FreeFactors[Tan[FunctionOfTrig[u, x]/2], x]^2*x^2)
, Tan[FunctionOfTrig[u, x]/2]/FreeFactors[Tan[FunctionOfTrig[u, x]/2], x],
u, x], x]]}, Module[{v = FunctionOfTrig[u, x], d}, Simp[d = FreeFactors[Tan
[v/2], x]; 2*(d/Coefficient[v, x, 1])   Subst[Int[SubstFor[1/(1 + d^2*x^2),
 Tan[v/2]/d, u, x], x], x, Tan[v/2]/d], x]] /; CalculusFreeQ[w, x]] /; Inve
rseFunctionFreeQ[u, x] &&  !FalseQ[FunctionOfTrig[u, x]]
```

3.133. $\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

3.133.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(112) = 224$.

Time = 0.88 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.35

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{b^2(5a^2+2b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2a(a^4+2a^2b^2+b^4)}-\frac{b(4a^4-7a^2b^2-2b^4)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2(a^4+2a^2b^2+b^4)a^2}+\frac{b^2(11a^2+2b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a^4+2a^2b^2+b^4)a}+\frac{b(4a^2+b^2)}{2a^4+4a^2b^2+2b^4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^2}+\frac{(2a^2 \cdots}{\cdots}}{d}$
default	$\frac{-\frac{2\left(-\frac{b^2(5a^2+2b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2a(a^4+2a^2b^2+b^4)}-\frac{b(4a^4-7a^2b^2-2b^4)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2(a^4+2a^2b^2+b^4)a^2}+\frac{b^2(11a^2+2b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a^4+2a^2b^2+b^4)a}+\frac{b(4a^2+b^2)}{2a^4+4a^2b^2+2b^4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^2}+\frac{(2a^2 \cdots}{\cdots}}{d}$
risch	$\frac{b\,e^{i(dx+c)}\left(-3iab\,e^{2i(dx+c)}+4a^2e^{2i(dx+c)}+b^2e^{2i(dx+c)}+3iba+4a^2+b^2\right)}{(-ia+b)^2\left(b\,e^{2i(dx+c)}+ia\,e^{2i(dx+c)}-b+ia\right)^2d(ia+b)^2}+\frac{\ln\left(e^{i(dx+c)}+\frac{ia^5+2ia^3b^2+ia\,b^4-a^4b-2a^2b^3 \cdots}{(a^2+b^2)^{\frac{5}{2}}}\right) \cdots}{(a^2+b^2)^{\frac{5}{2}}d}$

input
```
int(cos(d*x+c)^2/(cos(d*x+c)*a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*(-1/2*b^2*(5*a^2+2*b^2)/a/(a^4+2*a^2*b^2+b^4)*tan(1/2*d*x+1/2*c)^3
-1/2*b*(4*a^4-7*a^2*b^2-2*b^4)/(a^4+2*a^2*b^2+b^4)/a^2*tan(1/2*d*x+1/2*c)^
2+1/2*b^2*(11*a^2+2*b^2)/(a^4+2*a^2*b^2+b^4)/a*tan(1/2*d*x+1/2*c)+1/2*b*(4
*a^2+b^2)/(a^4+2*a^2*b^2+b^4))/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2
*c)-a)^2+(2*a^2-b^2)/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*
tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

3.133.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(112) = 224$.

Time = 0.27 (sec) , antiderivative size = 352, normalized size of antiderivative = 2.96

$$\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx =$$

$$-\frac{\left(2\,a^2b^2-b^4+\left(2\,a^4-3\,a^2b^2+b^4\right)\cos\left(dx+c\right)^2+2\left(2\,a^3b-ab^3\right)\cos\left(dx+c\right)\sin\left(dx+c\right)\right)\sqrt{a^2+b^2}\log \cdots}{4\left(\left(a^8+2\,a^6b^2-2\,a^2b^6-b^8\right)d\cos\left(dx+c\right)^2+2\,(\cdots\right.}$$

input
```
integrate(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas"
)
```

output
```
-1/4*((2*a^2*b^2 - b^4 + (2*a^4 - 3*a^2*b^2 + b^4)*cos(d*x + c)^2 + 2*(2*a
^3*b - a*b^3)*cos(d*x + c)*sin(d*x + c))*sqrt(a^2 + b^2)*log((2*a*b*cos(d*
x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^
2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x +
c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) + 2*(4*a^4*b + 5*a^2*b^3 + b^5)*co
s(d*x + c) + 6*(a^3*b^2 + a*b^4)*sin(d*x + c))/((a^8 + 2*a^6*b^2 - 2*a^2*b
^6 - b^8)*d*cos(d*x + c)^2 + 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*c
os(d*x + c)*sin(d*x + c) + (a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*d)
```

3.133.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2/(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

output
```
Timed out
```

3.133.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(112) = 224$.

Time = 0.33 (sec) , antiderivative size = 412, normalized size of antiderivative = 3.46

$$\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = -\frac{\frac{(2a^2-b^2)\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{(a^4+2a^2b^2+b^4)\sqrt{a^2+b^2}} + \frac{2\left(4a^4b+a^2b^3+\frac{(11a^3b^2+2ab^4)\sin(dx+c)}{\cos(dx+c)+1}-\frac{(4a^4b-7a^2b^3-2b^5)\sin(dx+c)^2}{(\cos(dx+c)+1)^2} \cdots\right.}{a^8+2a^6b^2+a^4b^4+\frac{4(a^7b+2a^5b^3+a^3b^5)\sin(dx+c)}{\cos(dx+c)+1}-\frac{2(a^8-3a^4b^4-2a^2b^6)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{4(a^7b+2a \cdots}{(c \cdots}}}{2d}$$

input
```
integrate(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima"
)
```

output
```
-1/2*((2*a^2 - b^2)*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2
+ b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/((a^4 +
 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) + 2*(4*a^4*b + a^2*b^3 + (11*a^3*b^2 +
2*a*b^4)*sin(d*x + c)/(cos(d*x + c) + 1) - (4*a^4*b - 7*a^2*b^3 - 2*b^5)*s
in(d*x + c)^2/(cos(d*x + c) + 1)^2 - (5*a^3*b^2 + 2*a*b^4)*sin(d*x + c)^3/
(cos(d*x + c) + 1)^3)/(a^8 + 2*a^6*b^2 + a^4*b^4 + 4*(a^7*b + 2*a^5*b^3 +
a^3*b^5)*sin(d*x + c)/(cos(d*x + c) + 1) - 2*(a^8 - 3*a^4*b^4 - 2*a^2*b^6)
*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 4*(a^7*b + 2*a^5*b^3 + a^3*b^5)*sin
(d*x + c)^3/(cos(d*x + c) + 1)^3 + (a^8 + 2*a^6*b^2 + a^4*b^4)*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4))/d
```

3.133.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(112) = 224$.

Time = 0.41 (sec) , antiderivative size = 293, normalized size of antiderivative = 2.46

$$\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = -\frac{\frac{(2\,a^2-b^2)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}} - \frac{2\left(5\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-7\,a^2b^3\tan(\right.}{(a^6+2\,a^4b^2+a^2b^4)\left(a\tan(\right.}}{2\,d}$$

input
```
integrate(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/2*((2*a^2 - b^2)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 +
b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a
^2*b^2 + b^4)*sqrt(a^2 + b^2)) - 2*(5*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + 2*a
*b^4*tan(1/2*d*x + 1/2*c)^3 + 4*a^4*b*tan(1/2*d*x + 1/2*c)^2 - 7*a^2*b^3*t
an(1/2*d*x + 1/2*c)^2 - 2*b^5*tan(1/2*d*x + 1/2*c)^2 - 11*a^3*b^2*tan(1/2*
d*x + 1/2*c) - 2*a*b^4*tan(1/2*d*x + 1/2*c) - 4*a^4*b - a^2*b^3)/((a^6 + 2
*a^4*b^2 + a^2*b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c) -
 a)^2))/d
```

3.133.9 Mupad [B] (verification not implemented)

Time = 23.98 (sec) , antiderivative size = 443, normalized size of antiderivative = 3.72

$$\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = \frac{\ln\left(\left(a^2+b^2\right)^{5/2}-a^4\,b-b^5-2\,a^2\,b^3+a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\,b^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+2\,a^3\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(a^2-\frac{b^2}{2}\right)}{d\,\left(a^2+b^2\right)^{5/2}} - \frac{\ln\left(\left(a^2+b^2\right)^{5/2}+a^4\,b+b^5+2\,a^2\,b^3-a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-a\,b^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2\,a^3\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(2\,a^2-b^2\right)}{2\,d\,\left(a^2+b^2\right)^{5/2}} - \frac{\frac{4\,a^2\,b+b^3}{a^4+2\,a^2\,b^2+b^4}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(a^2-2\,b^2\right)\,\left(4\,a^2\,b+b^3\right)}{a^2\,\left(a^4+2\,a^2\,b^2+b^4\right)}+\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(11\,a^2\,b+2\,b^3\right)}{a\,\left(a^4+2\,a^2\,b^2+b^4\right)}-\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(5\,a^2\,b+2\,b^3\right)}{a\,\left(a^4+2\,a^2\,b^2+b^4\right)}}{d\,\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(2\,a^2-4\,b^2\right)+a^2-4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}$$

input
```
int(cos(c + d*x)^2/(a*cos(c + d*x) + b*sin(c + d*x))^3,x)
```

output
```
(log((a^2 + b^2)^(5/2) - a^4*b - b^5 - 2*a^2*b^3 + a^5*tan(c/2 + (d*x)/2)
+ a*b^4*tan(c/2 + (d*x)/2) + 2*a^3*b^2*tan(c/2 + (d*x)/2))*(a^2 - b^2/2))/
(d*(a^2 + b^2)^(5/2)) - (log((a^2 + b^2)^(5/2) + a^4*b + b^5 + 2*a^2*b^3 -
 a^5*tan(c/2 + (d*x)/2) - a*b^4*tan(c/2 + (d*x)/2) - 2*a^3*b^2*tan(c/2 + (
d*x)/2))*(2*a^2 - b^2))/(2*d*(a^2 + b^2)^(5/2)) - ((4*a^2*b + b^3)/(a^4 +
b^4 + 2*a^2*b^2) - (tan(c/2 + (d*x)/2)^2*(a^2 - 2*b^2)*(4*a^2*b + b^3))/(a
^2*(a^4 + b^4 + 2*a^2*b^2)) + (b*tan(c/2 + (d*x)/2)*(11*a^2*b + 2*b^3))/(a
*(a^4 + b^4 + 2*a^2*b^2)) - (b*tan(c/2 + (d*x)/2)^3*(5*a^2*b + 2*b^3))/(a*
(a^4 + b^4 + 2*a^2*b^2)))/(d*(a^2*tan(c/2 + (d*x)/2)^4 - tan(c/2 + (d*x)/2
)^2*(2*a^2 - 4*b^2) + a^2 - 4*a*b*tan(c/2 + (d*x)/2)^3 + 4*a*b*tan(c/2 + (
d*x)/2)))
```

3.134 $\int \frac{\cos(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

3.134.1 Optimal result

Integrand size = 26, antiderivative size = 22

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = -\frac{1}{2bd(a+b\tan(c+dx))^2}$$

output
```
-1/2/b/d/(a+b*tan(d*x+c))^2
```

3.134.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 57 vs. 2(22) = 44.

Time = 0.49 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.59

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = \frac{-b\cos(2(c+dx))+a\sin(2(c+dx))}{2(a^2+b^2)\,d(a\cos(c+dx)+b\sin(c+dx))^2}$$

input
```
Integrate[Cos[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

output
```
(-(b*Cos[2*(c + d*x)]) + a*Sin[2*(c + d*x)])/(2*(a^2 + b^2)*d*(a*Cos[c + d
*x] + b*Sin[c + d*x])^2)
```

3.134.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.36, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3042, 3567, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}dx$$

$$\Big\downarrow \text{3567}$$

$$-\frac{\int \frac{\cot(c+dx)}{(b+a\cot(c+dx))^3}d\cot(c+dx)}{d}$$

$$\Big\downarrow \text{48}$$

$$-\frac{\cot^2(c+dx)}{2bd(a\cot(c+dx)+b)^2}$$

input
```
Int[Cos[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

output
```
-1/2*Cot[c + d*x]^2/(b*d*(b + a*Cot[c + d*x])^2)
```

3.134.3.1 Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3567
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[x^m*((b
+ a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a,
b, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[
n, 0] && GtQ[m, 1])
```

3.134.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$-\frac{1}{2bd(a+b\tan(dx+c))^2}$	21
default	$-\frac{1}{2bd(a+b\tan(dx+c))^2}$	21
risch	$\frac{2ia\,e^{2i(dx+c)}+2b\,e^{2i(dx+c)}+2ia}{(b\,e^{2i(dx+c)}+ia\,e^{2i(dx+c)}-b+ia)^2 d(ia+b)^2}$	77
parallelrisch	$-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{a^2 d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^2}$	78
norman	$\frac{\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{ad}+\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{a^2d}+\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{a^2d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^2}$	125

input
```
int(cos(d*x+c)/(cos(d*x+c)*a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/b/d/(a+b*tan(d*x+c))^2
```

3.134.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. 2(20) = 40.

Time = 0.24 (sec) , antiderivative size = 142, normalized size of antiderivative = 6.45

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = -\frac{4\,a^2b\cos(dx+c)^2-a^2b+b^3-2\,(a^3-ab^2)\cos(dx+c)\sin(dx+c)}{2\,((a^6+a^4b^2-a^2b^4-b^6)d\cos(dx+c)^2+2\,(a^5b+2\,a^3b^3+ab^5)d\cos(dx+c)\sin(dx+c)+(a^4b^2+2}$$

input
```
integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/2*(4*a^2*b*cos(d*x + c)^2 - a^2*b + b^3 - 2*(a^3 - a*b^2)*cos(d*x + c)*
sin(d*x + c))/((a^6 + a^4*b^2 - a^2*b^4 - b^6)*d*cos(d*x + c)^2 + 2*(a^5*b
 + 2*a^3*b^3 + a*b^5)*d*cos(d*x + c)*sin(d*x + c) + (a^4*b^2 + 2*a^2*b^4 +
 b^6)*d)
```

3.134.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

output
```
Timed out
```

3.134.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(20) = 40.

Time = 0.22 (sec) , antiderivative size = 171, normalized size of antiderivative = 7.77

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = \frac{2\left(\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\frac{b\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{\left(a^4+\frac{4a^3b\sin(dx+c)}{\cos(dx+c)+1}-\frac{4a^3b\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{2(a^4-2a^2b^2)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)d}$$

input
```
integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
2*(a*sin(d*x + c)/(cos(d*x + c) + 1) + b*sin(d*x + c)^2/(cos(d*x + c) + 1)
^2 - a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/((a^4 + 4*a^3*b*sin(d*x + c)/(
cos(d*x + c) + 1) - 4*a^3*b*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + a^4*sin(
d*x + c)^4/(cos(d*x + c) + 1)^4 - 2*(a^4 - 2*a^2*b^2)*sin(d*x + c)^2/(cos(
d*x + c) + 1)^2)*d)
```

3.134.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = -\frac{1}{2\,(b\tan(dx+c)+a)^2bd}$$

input
```
integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/2/((b*tan(d*x + c) + a)^2*b*d)
```

3.134.9 Mupad [B] (verification not implemented)

Time = 22.24 (sec) , antiderivative size = 85, normalized size of antiderivative = 3.86

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = -\frac{b\left(\frac{\cos(2\,c+2\,d\,x)}{2}-\frac{1}{2}\right)-a\,\sin(2\,c+2\,d\,x)}{a^2\,d\,\left(a^2+b^2+a^2\,\cos(2\,c+2\,d\,x)-b^2\,\cos(2\,c+2\,d\,x)+2\,a\,b\,\sin(2\,c+2\,d\,x)\right)}$$

input
```
int(cos(c + d*x)/(a*cos(c + d*x) + b*sin(c + d*x))^3,x)
```

output
```
-(b*(cos(2*c + 2*d*x)/2 - 1/2) - a*sin(2*c + 2*d*x))/(a^2*d*(a^2 + b^2 + a
^2*cos(2*c + 2*d*x) - b^2*cos(2*c + 2*d*x) + 2*a*b*sin(2*c + 2*d*x)))
```

3.135 $\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

3.135.1 Optimal result

Integrand size = 19, antiderivative size = 103

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = -\frac{\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{3/2}d} - \frac{b\cos(c+dx)-a\sin(c+dx)}{2\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^2}$$

output
```
-1/2*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)/
d+1/2*(-b*cos(d*x+c)+a*sin(d*x+c))/(a^2+b^2)/d/(a*cos(d*x+c)+b*sin(d*x+c))
^2
```

3.135.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.28

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = \frac{\left(a^2+b^2\right)(-b\cos(c+dx)+a\sin(c+dx))+2\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)(a\cos(c+dx)+b\sin(c}{2(a-ib)^2(a+ib)^2d(a\cos(c+dx)+b\sin(c+dx))^2}$$

input
```
Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-3),x]
```

output
```
((a^2 + b^2)*(-(b*Cos[c + d*x]) + a*Sin[c + d*x]) + 2*Sqrt[a^2 + b^2]*ArcT
anh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]]*(a*Cos[c + d*x] + b*Sin[c +
 d*x])^2)/(2*(a - I*b)^2*(a + I*b)^2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2
)
```

3.135.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 3555, 3042, 3553, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^3}dx$$

$$\Big\downarrow \text{3555}$$

$$\frac{\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{2(a^2+b^2)}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{2(a^2+b^2)}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^2}$$

$$\Big\downarrow \text{3553}$$

$$-\frac{\int \frac{1}{a^2+b^2-(b\cos(c+dx)-a\sin(c+dx))^2}d(b\cos(c+dx)-a\sin(c+dx))}{2d(a^2+b^2)}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^2}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2d(a^2+b^2)^{3/2}}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^2}$$

input
```
Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-3),x]
```

output
```
-1/2*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]]/((a^2 + b^
2)^(3/2)*d) - (b*Cos[c + d*x] - a*Sin[c + d*x])/(2*(a^2 + b^2)*d*(a*Cos[c
+ d*x] + b*Sin[c + d*x])^2)
```

3.135.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3555
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(b*Cos[c + d*x] - a*Sin[c + d*x])*((a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Simp[(n + 2)/((n + 1)*(a^
2 + b^2))   Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

3.135.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.85

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{\left(a^{2}+2b^{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{2\left(a^{2}+b^{2}\right)a}-\frac{b\left(a^{2}-2b^{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{2\left(a^{2}+b^{2}\right)a^{2}}-\frac{\left(a^{2}-2b^{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2\left(a^{2}+b^{2}\right)a}+\frac{b}{2a^{2}+2b^{2}}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^{2}}+\frac{\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right.}{\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}}{d}$
default	$\frac{-\frac{2\left(-\frac{\left(a^{2}+2b^{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{2\left(a^{2}+b^{2}\right)a}-\frac{b\left(a^{2}-2b^{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{2\left(a^{2}+b^{2}\right)a^{2}}-\frac{\left(a^{2}-2b^{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2\left(a^{2}+b^{2}\right)a}+\frac{b}{2a^{2}+2b^{2}}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^{2}}+\frac{\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right.}{\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}}{d}$
risch	$\frac{e^{i(dx+c)}\left(ia\,e^{2i(dx+c)}+b\,e^{2i(dx+c)}-ia+b\right)}{\left(b\,e^{2i(dx+c)}+ia\,e^{2i(dx+c)}-b+ia\right)^{2}(-ia+b)d(ia+b)}+\frac{\ln\left(e^{i(dx+c)}+\frac{ia^{3}+ia\,b^{2}-a^{2}b-b^{3}}{\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}\right)}{2(a^{2}+b^{2})^{\frac{3}{2}}d}-\frac{\ln\left(e^{i(dx+c)}-\frac{ia^{3}+ia\,b^{2}-}{\left(a^{2}+b^{2}\right.}\right.}{2(a^{2}+b^{2})^{\frac{3}{2}}d}$

input
```
int(1/(cos(d*x+c)*a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*(-1/2*(a^2+2*b^2)/(a^2+b^2)/a*tan(1/2*d*x+1/2*c)^3-1/2*b*(a^2-2*b^
2)/(a^2+b^2)/a^2*tan(1/2*d*x+1/2*c)^2-1/2*(a^2-2*b^2)/(a^2+b^2)/a*tan(1/2*
d*x+1/2*c)+1/2*b/(a^2+b^2))/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)
-a)^2+1/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)
^(1/2)))
```

3.135.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(95) = 190.

Time = 0.27 (sec) , antiderivative size = 294, normalized size of antiderivative = 2.85

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$$
$$=\frac{\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^{2}-b^{2}\right)\cos\left(dx+c\right)^{2}+b^{2}\right)\sqrt{a^{2}+b^{2}}\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^{2}-b^{2})\cos}{2\,ab\cos(dx+c)\sin}\right.}{4\left(\left(a^{6}+a^{4}b^{2}-a^{2}b^{4}-b^{6}\right)d\cos\left(dx+c\right)^{2}+2\left(a^{5}b+2\,a^{3}b^{3}+a\right.\right.}$$

input
```
integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/4*((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)*
sqrt(a^2 + b^2)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*
x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c
)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2))
- 2*(a^2*b + b^3)*cos(d*x + c) + 2*(a^3 + a*b^2)*sin(d*x + c))/((a^6 + a^4
*b^2 - a^2*b^4 - b^6)*d*cos(d*x + c)^2 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d*c
os(d*x + c)*sin(d*x + c) + (a^4*b^2 + 2*a^2*b^4 + b^6)*d)
```

3.135.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

output
```
Timed out
```

3.135.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(95) = 190$.

Time = 0.30 (sec) , antiderivative size = 326, normalized size of antiderivative = 3.17

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = -\frac{\frac{2\left(a^2b-\frac{(a^3-2\,ab^2)\sin(dx+c)}{\cos(dx+c)+1}-\frac{(a^2b-2\,b^3)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{(a^3+2\,ab^2)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^6+a^4b^2+\frac{4\,(a^5b+a^3b^3)\sin(dx+c)}{\cos(dx+c)+1}-\frac{2\,(a^6-a^4b^2-2\,a^2b^4)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{4\,(a^5b+a^3b^3)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{(a^6+a^4b^2)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\cdots}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\cdots}\right)}{(a^2+b^2)^{\frac{3}{2}}}}{2\,d}$$

input
```
integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/2*(2*(a^2*b - (a^3 - 2*a*b^2)*sin(d*x + c)/(cos(d*x + c) + 1) - (a^2*b
- 2*b^3)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - (a^3 + 2*a*b^2)*sin(d*x + c
)^3/(cos(d*x + c) + 1)^3)/(a^6 + a^4*b^2 + 4*(a^5*b + a^3*b^3)*sin(d*x + c
)/(cos(d*x + c) + 1) - 2*(a^6 - a^4*b^2 - 2*a^2*b^4)*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 - 4*(a^5*b + a^3*b^3)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 +
 (a^6 + a^4*b^2)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) + log((b - a*sin(d*x
 + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d*x +
 c) + 1) - sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2))/d
```

3.135.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(95) = 190$.

Time = 0.32 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.15

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = -\frac{\frac{\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^2+b^2)^{\frac{3}{2}}}-\frac{2\left(a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a^3\,\mathrm{ta}\cdots\right.}{(a^4+a^2b^2)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)^{\cdots}}}{2\,d}$$

input
```
integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/2*(log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*
tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(a^
3*tan(1/2*d*x + 1/2*c)^3 + 2*a*b^2*tan(1/2*d*x + 1/2*c)^3 + a^2*b*tan(1/2*
d*x + 1/2*c)^2 - 2*b^3*tan(1/2*d*x + 1/2*c)^2 + a^3*tan(1/2*d*x + 1/2*c) -
 2*a*b^2*tan(1/2*d*x + 1/2*c) - a^2*b)/((a^4 + a^2*b^2)*(a*tan(1/2*d*x + 1
/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c) - a)^2))/d
```

3.135.9 Mupad [B] (verification not implemented)

Time = 24.65 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.52

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = \frac{\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(a^2-2b^2)}{a(a^2+b^2)}-\frac{b}{a^2+b^2}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(a^2+2b^2)}{a(a^2+b^2)}+\frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(a^2-2b^2)}{a^2(a^2+b^2)}}{d\left(a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(2a^2-4b^2)+a^2-4ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+4ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)} + \frac{\operatorname{atanh}\left(\frac{\left(2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-\frac{2a^2b+2b^3}{a^2+b^2}\right)\left(\frac{a^2}{2}+\frac{b^2}{2}\right)}{(a^2+b^2)^{3/2}}\right)}{d(a^2+b^2)^{3/2}}$$

input
```
int(1/(a*cos(c + d*x) + b*sin(c + d*x))^3,x)
```

output
```
((tan(c/2 + (d*x)/2)*(a^2 - 2*b^2))/(a*(a^2 + b^2)) - b/(a^2 + b^2) + (tan
(c/2 + (d*x)/2)^3*(a^2 + 2*b^2))/(a*(a^2 + b^2)) + (b*tan(c/2 + (d*x)/2)^2
*(a^2 - 2*b^2))/(a^2*(a^2 + b^2)))/(d*(a^2*tan(c/2 + (d*x)/2)^4 - tan(c/2
+ (d*x)/2)^2*(2*a^2 - 4*b^2) + a^2 - 4*a*b*tan(c/2 + (d*x)/2)^3 + 4*a*b*ta
n(c/2 + (d*x)/2))) + atanh(((2*a*tan(c/2 + (d*x)/2) - (2*a^2*b + 2*b^3)/(a
^2 + b^2))*(a^2/2 + b^2/2))/(a^2 + b^2)^(3/2))/(d*(a^2 + b^2)^(3/2))
```

3.136 $\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

3.136.1 Optimal result

Integrand size = 26, antiderivative size = 86

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = -\frac{\frac{1}{b}+\frac{b}{a^2}}{2d(b+a\cot(c+dx))^2}+\frac{\frac{1}{a^2}-\frac{1}{b^2}}{d(b+a\cot(c+dx))}+\frac{\log(b+a\cot(c+dx))}{b^3d}+\frac{\log(\tan(c+dx))}{b^3d}$$

output
```
1/2*(-1/b-b/a^2)/d/(b+a*cot(d*x+c))^2+(1/a^2-1/b^2)/d/(b+a*cot(d*x+c))+ln(
b+a*cot(d*x+c))/b^3/d+ln(tan(d*x+c))/b^3/d
```

3.136.2 Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.66

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = \frac{\log(a+b\tan(c+dx))-\frac{a^2+b^2}{2(a+b\tan(c+dx))^2}+\frac{2a}{a+b\tan(c+dx)}}{b^3d}$$

input
```
Integrate[Sec[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

output
```
(Log[a + b*Tan[c + d*x]] - (a^2 + b^2)/(2*(a + b*Tan[c + d*x])^2) + (2*a)/
(a + b*Tan[c + d*x]))/(b^3*d)
```

3.136.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3042, 3567, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3}dx$$

$$\Big\downarrow \text{3567}$$

$$-\frac{\int \frac{(\cot^2(c+dx)+1)\tan(c+dx)}{(b+a\cot(c+dx))^3}d\cot(c+dx)}{d}$$

$$\Big\downarrow \text{522}$$

$$-\frac{\int \left(-\frac{a}{b^3(b+a\cot(c+dx))}+\frac{\tan(c+dx)}{b^3}+\frac{b^2-a^2}{b^2(b+a\cot(c+dx))^2a}+\frac{-a^2-b^2}{b(b+a\cot(c+dx))^3a}\right)d\cot(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{\frac{1}{a^2}-\frac{1}{b^2}}{a\cot(c+dx)+b}+\frac{\frac{b}{a^2}+\frac{1}{b}}{2(a\cot(c+dx)+b)^2}-\frac{\log(a\cot(c+dx)+b)}{b^3}+\frac{\log(\cot(c+dx))}{b^3}}{d}$$

input

```
Int[Sec[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

output

```
-(((b^(-1) + b/a^2)/(2*(b + a*Cot[c + d*x])^2) - (a^(-2) - b^(-2))/(b + a*
Cot[c + d*x]) + Log[Cot[c + d*x]]/b^3 - Log[b + a*Cot[c + d*x]]/b^3)/d)
```

3.136.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3567
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[x^m*((b
+ a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a,
b, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[
n, 0] && GtQ[m, 1])
```

3.136.4 Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.73

method	result
derivativedivides	$\frac{\frac{\ln(a+b\tan(dx+c))}{b^3}-\frac{a^2+b^2}{2b^3(a+b\tan(dx+c))^2}+\frac{2a}{b^3(a+b\tan(dx+c))}}{d}$
default	$\frac{\frac{\ln(a+b\tan(dx+c))}{b^3}-\frac{a^2+b^2}{2b^3(a+b\tan(dx+c))^2}+\frac{2a}{b^3(a+b\tan(dx+c))}}{d}$
risch	$\frac{-2a^2 e^{2i(dx+c)}+2b^2 e^{2i(dx+c)}+4iab\, e^{2i(dx+c)}-2a^2-2iba}{b^2(ia+b)\left(b\, e^{2i(dx+c)}+ia\, e^{2i(dx+c)}-b+ia\right)^2 d}+\frac{\ln\left(e^{2i(dx+c)}-\frac{ib+a}{ib-a}\right)}{b^3 d}-\frac{\ln\left(e^{2i(dx+c)}+1\right)}{b^3 d}$
norman	$\frac{-\frac{2(a^2-b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{b^2 da}+\frac{2(a^2-b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{b^2 da}-\frac{2(3a^2-b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{a^2 bd}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^2}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{b^3 d}$
parallelrisch	$\frac{((2a^2-2b^2)\cos(2dx+2c)+4ab\sin(2dx+2c)+2a^2+2b^2)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)+((-2a^2+2b^2)\cos(2dx}{2(}$

input
```
int(sec(d*x+c)/(cos(d*x+c)*a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/b^3*ln(a+b*tan(d*x+c))-1/2*(a^2+b^2)/b^3/(a+b*tan(d*x+c))^2+2*a/b^3
/(a+b*tan(d*x+c)))
```

3.136.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(84) = 168$.

Time = 0.28 (sec) , antiderivative size = 284, normalized size of antiderivative = 3.30

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = \frac{4\,a^2b^2\cos\left(dx+c\right)^2 - 3\,a^2b^2 - b^4 - 2\,(a^3b-ab^3)\cos\left(dx+c\right)\sin\left(dx+c\right) + \left(a^2b^2+b^4+(a^4-b^4)\cos\left(dx\right.\right.}{}$$

input
```
integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/2*(4*a^2*b^2*cos(d*x + c)^2 - 3*a^2*b^2 - b^4 - 2*(a^3*b - a*b^3)*cos(d*
x + c)*sin(d*x + c) + (a^2*b^2 + b^4 + (a^4 - b^4)*cos(d*x + c)^2 + 2*(a^3
*b + a*b^3)*cos(d*x + c)*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c)
 + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - (a^2*b^2 + b^4 + (a^4 - b^4)*cos(d*
x + c)^2 + 2*(a^3*b + a*b^3)*cos(d*x + c)*sin(d*x + c))*log(cos(d*x + c)^2
))/((a^4*b^3 - b^7)*d*cos(d*x + c)^2 + 2*(a^3*b^4 + a*b^6)*d*cos(d*x + c)*
sin(d*x + c) + (a^2*b^5 + b^7)*d)
```

3.136.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = \int \frac{\sec\left(c+dx\right)}{\left(a\cos\left(c+dx\right)+b\sin\left(c+dx\right)\right)^3}\,dx$$

input
```
integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)/(a*cos(c + d*x) + b*sin(c + d*x))**3, x)
```

3.136.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(84) = 168$.

Time = 0.24 (sec) , antiderivative size = 315, normalized size of antiderivative = 3.66

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = -\frac{\frac{2\left(\frac{(a^3-ab^2)\sin(dx+c)}{\cos(dx+c)+1}+\frac{(3a^2b-b^3)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{(a^3-ab^2)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^4b^2+\frac{4a^3b^3\sin(dx+c)}{\cos(dx+c)+1}-\frac{4a^3b^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{a^4b^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{2(a^4b^2-2a^2b^4)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}-\frac{\log\left(-a-\frac{2b\sin(dx+c)}{\cos(dx+c)+1}+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{b^3}+\frac{\log\left(\cdots\right.}{}}{d}$$

input
```
integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
-(2*((a^3 - a*b^2)*sin(d*x + c)/(cos(d*x + c) + 1) + (3*a^2*b - b^3)*sin(d
*x + c)^2/(cos(d*x + c) + 1)^2 - (a^3 - a*b^2)*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3)/(a^4*b^2 + 4*a^3*b^3*sin(d*x + c)/(cos(d*x + c) + 1) - 4*a^3*b^3
*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + a^4*b^2*sin(d*x + c)^4/(cos(d*x + c
) + 1)^4 - 2*(a^4*b^2 - 2*a^2*b^4)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2) -
log(-a - 2*b*sin(d*x + c)/(cos(d*x + c) + 1) + a*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2)/b^3 + log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/b^3 + log(sin(d
*x + c)/(cos(d*x + c) + 1) - 1)/b^3)/d
```

3.136.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.72

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = \frac{\frac{2\log(|b\tan(dx+c)+a|)}{b^3}-\frac{3b\tan(dx+c)^2+2a\tan(dx+c)+b}{(b\tan(dx+c)+a)^2b^2}}{2d}$$

input
```
integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
1/2*(2*log(abs(b*tan(d*x + c) + a))/b^3 - (3*b*tan(d*x + c)^2 + 2*a*tan(d*
x + c) + b)/((b*tan(d*x + c) + a)^2*b^2))/d
```

3.136.9 Mupad [B] (verification not implemented)

Time = 26.11 (sec) , antiderivative size = 396, normalized size of antiderivative = 4.60

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$$
$$= \frac{2\,\mathrm{atanh}\left(\frac{16\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{16\,a+\frac{32\,a^3}{b^2}-16\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-\frac{32\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{b^2}+\frac{32\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{b}}-\frac{16\,a}{16\,a+\frac{32\,a^3}{b^2}-16\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-\frac{32\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{b^2}+}\right.}{b^3\,d}$$
$$-\frac{\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,(3\,a^2-b^2)}{a^2\,b}-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,(a^2-b^2)}{a\,b^2}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(a^2-b^2)}{a\,b^2}}{d\,\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,(2\,a^2-4\,b^2)+a^2-4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}$$

input
```
int(1/(cos(c + d*x)*(a*cos(c + d*x) + b*sin(c + d*x))^3),x)
```

output
```
(2*atanh((16*a*tan(c/2 + (d*x)/2)^2)/(16*a + (32*a^3)/b^2 - 16*a*tan(c/2 +
 (d*x)/2)^2 - (32*a^3*tan(c/2 + (d*x)/2)^2)/b^2 + (32*a^2*tan(c/2 + (d*x)/
2))/b) - (16*a)/(16*a + (32*a^3)/b^2 - 16*a*tan(c/2 + (d*x)/2)^2 - (32*a^3
*tan(c/2 + (d*x)/2)^2)/b^2 + (32*a^2*tan(c/2 + (d*x)/2))/b) + (32*a^2*tan(
c/2 + (d*x)/2))/(16*a*b + (32*a^3)/b + 32*a^2*tan(c/2 + (d*x)/2) - (32*a^3
*tan(c/2 + (d*x)/2)^2)/b - 16*a*b*tan(c/2 + (d*x)/2)^2)))/(b^3*d) - ((2*ta
n(c/2 + (d*x)/2)^2*(3*a^2 - b^2))/(a^2*b) - (2*tan(c/2 + (d*x)/2)^3*(a^2 -
 b^2))/(a*b^2) + (2*tan(c/2 + (d*x)/2)*(a^2 - b^2))/(a*b^2))/(d*(a^2*tan(c
/2 + (d*x)/2)^4 - tan(c/2 + (d*x)/2)^2*(2*a^2 - 4*b^2) + a^2 - 4*a*b*tan(c
/2 + (d*x)/2)^3 + 4*a*b*tan(c/2 + (d*x)/2)))
```

3.137 $\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

3.137.1 Optimal result

Integrand size = 28, antiderivative size = 260

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = -\frac{3a\text{arctanh}(\sin(c+dx))}{b^4d} - \frac{2a^2\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^4\sqrt{a^2+b^2}d} - \frac{\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2b^2\sqrt{a^2+b^2}d} - \frac{\sqrt{a^2+b^2}\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^4d} + \frac{\sec(c+dx)}{b^3d} - \frac{b\cos(c+dx)-a\sin(c+dx)}{2b^2d(a\cos(c+dx)+b\sin(c+dx))^2} + \frac{2a}{b^3d(a\cos(c+dx)+b\sin(c+dx))}$$

output
```
-3*a*arctanh(sin(d*x+c))/b^4/d+sec(d*x+c)/b^3/d+1/2*(-b*cos(d*x+c)+a*sin(d
*x+c))/b^2/d/(a*cos(d*x+c)+b*sin(d*x+c))^2+2*a/b^3/d/(a*cos(d*x+c)+b*sin(d
*x+c))-2*a^2*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/b^4/d/(a
^2+b^2)^(1/2)-1/2*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/b^2
/d/(a^2+b^2)^(1/2)-arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))*(a
^2+b^2)^(1/2)/b^4/d
```

3.137.2 Mathematica [A] (verified)

Time = 2.94 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.52

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$$
$$= \frac{\sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))\left(\frac{b^2(a^2+b^2)\sin(c+dx)}{a}+\frac{(2a-b)b(2a+b)(a\cos(c+dx)+b\sin(c+dx))}{a}+2b(a\cos\right.}{}$$

input
```
Integrate[Sec[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

output
```
(Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])*((b^2*(a^2 + b^2)*Sin[c
+ d*x])/a + ((2*a - b)*b*(2*a + b)*(a*Cos[c + d*x] + b*Sin[c + d*x]))/a +
2*b*(a*Cos[c + d*x] + b*Sin[c + d*x])^2 + (6*(2*a^2 + b^2)*ArcTanh[(-b + a
*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/S
qrt[a^2 + b^2] + 6*a*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a*Cos[c + d
*x] + b*Sin[c + d*x])^2 - 6*a*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a*
Cos[c + d*x] + b*Sin[c + d*x])^2 + (2*b*Sin[(c + d*x)/2]*(a*Cos[c + d*x] +
 b*Sin[c + d*x])^2)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) - (2*b*Sin[(c +
d*x)/2]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(Cos[(c + d*x)/2] + Sin[(c +
d*x)/2])))/(2*b^4*d*(a + b*Tan[c + d*x])^3)
```

3.137.3 Rubi [A] (verified)

Time = 1.46 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.14, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3042, 3585, 3042, 3555, 3042, 3553, 219, 3573, 3042, 3553, 219, 3583, 3042, 3553, 219, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))^3}dx$$

$$\Big\downarrow \ 3585$$

$$\frac{(a^2+b^2)\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^3}dx}{b^2}+\frac{\int \frac{\sec^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx}{b^2}-\frac{2a\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{(a^2+b^2)\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^3}dx}{b^2}-\frac{2a\int \frac{1}{\cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}+\frac{\int \frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))}dx}{b^2}$$

$$\Big\downarrow \ 3555$$

$$\frac{(a^2+b^2)\left(\frac{\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{2(a^2+b^2)}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^2}\right)}{b^2}-\frac{2a\int \frac{1}{\cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}+\frac{\int \frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))}dx}{b^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{(a^2+b^2)\left(\frac{\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{2(a^2+b^2)}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^2}\right)}{b^2}-\frac{2a\int \frac{1}{\cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}+\frac{\int \frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))}dx}{b^2}$$

$$\Big\downarrow \ 3553$$

$$\frac{(a^2+b^2)\left(-\frac{\int \frac{1}{a^2+b^2-(b\cos(c+dx)-a\sin(c+dx))^2}d(b\cos(c+dx)-a\sin(c+dx))}{2d(a^2+b^2)}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^2}\right)}{b^2}-\frac{2a\int \frac{1}{\cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}+\frac{\int \frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))}dx}{b^2}$$

$$\Big\downarrow \ 219$$

$$-\frac{2a\int \frac{1}{\cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}+\frac{\int \frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))}dx}{b^2}+\frac{(a^2+b^2)\left(-\frac{\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2d(a^2+b^2)^{3/2}}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^2}\right)}{b^2}$$

$$\Big\downarrow \ 3573$$

3.137. $\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

$$\frac{\int \frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))}dx}{b^2}-\frac{2a\left(-\frac{a\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{b^2}+\frac{\int \sec(c+dx)dx}{b^2}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}\right)}{b^2}+\frac{(a^2+b^2)\left(-\frac{\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2d(a^2+b^2)^{3/2}}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^2}\right)}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))}dx}{b^2}-\frac{2a\left(-\frac{a\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{b^2}+\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}\right)}{b^2}+\frac{(a^2+b^2)\left(-\frac{\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2d(a^2+b^2)^{3/2}}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^2}\right)}{b^2}$$

$$\Big\downarrow \text{3553}$$

$$-\frac{2a\left(\frac{a\int \frac{1}{a^2+b^2-(b\cos(c+dx)-a\sin(c+dx))^2}d(b\cos(c+dx)-a\sin(c+dx))}{b^2d}+\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}\right)}{b^2}+\frac{\int \frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))}dx}{b^2}+\frac{(a^2+b^2)\left(-\frac{\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2d(a^2+b^2)^{3/2}}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^2}\right)}{b^2}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2a\left(\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}+\frac{a\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}\right)}{b^2}+\frac{\int \frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))}dx}{b^2}+\frac{(a^2+b^2)\left(-\frac{\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2d(a^2+b^2)^{3/2}}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^2}\right)}{b^2}$$

$$\Big\downarrow \text{3583}$$

3.137. $\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

$$-\frac{2a\left(\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}+\frac{a\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}\right)}{b^2}+\frac{\frac{(a^2+b^2)\int\frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{b^2}-\frac{a\int\sec(c+dx)dx}{b^2}+\frac{\sec(c+dx)}{bd}}{b^2}+\frac{(a^2+b^2)\left(-\frac{\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2d(a^2+b^2)^{3/2}}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^2}\right)}{b^2}$$

$\downarrow$ 3042

$$-\frac{2a\left(\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}+\frac{a\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}\right)}{b^2}+\frac{\frac{(a^2+b^2)\int\frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{b^2}-\frac{a\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}+\frac{\sec(c+dx)}{bd}}{b^2}+\frac{(a^2+b^2)\left(-\frac{\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2d(a^2+b^2)^{3/2}}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^2}\right)}{b^2}$$

$\downarrow$ 3553

$$-\frac{2a\left(\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}+\frac{a\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}\right)}{b^2}+\frac{-\frac{(a^2+b^2)\int\frac{1}{a^2+b^2-(b\cos(c+dx)-a\sin(c+dx))^2}d(b\cos(c+dx)-a\sin(c+dx))}{b^2d}-\frac{a\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}+\frac{\sec(c+dx)}{bd}}{b^2}+\frac{(a^2+b^2)\left(-\frac{\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2d(a^2+b^2)^{3/2}}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^2}\right)}{b^2}$$

$\downarrow$ 219

3.137. $\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

$$-\frac{2a\left(\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}+\frac{a\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}\right)}{b^2}+\frac{-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{\sqrt{a^2+b^2}\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d}+\frac{\sec(c+dx)}{bd}}{b^2}+\frac{(a^2+b^2)\left(-\frac{\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2d(a^2+b^2)^{3/2}}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^2}\right)}{b^2}$$

$$\Big\downarrow \; 4257$$

$$\frac{(a^2+b^2)\left(-\frac{\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2d(a^2+b^2)^{3/2}}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^2}\right)}{b^2}-\frac{2a\left(\frac{a\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}+\frac{\text{arctanh}(\sin(c+dx))}{b^2d}\right)}{b^2}+\frac{-\frac{\sqrt{a^2+b^2}\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d}-\frac{a\text{arctanh}(\sin(c+dx))}{b^2d}+\frac{\sec(c+dx)}{bd}}{b^2}$$

input
```
Int[Sec[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

output
```
(-((a*ArcTanh[Sin[c + d*x]])/(b^2*d)) - (Sqrt[a^2 + b^2]*ArcTanh[(b*Cos[c
+ d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(b^2*d) + Sec[c + d*x]/(b*d))/b
^2 + ((a^2 + b^2)*(-1/2*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2
 + b^2]]/((a^2 + b^2)^(3/2)*d) - (b*Cos[c + d*x] - a*Sin[c + d*x])/(2*(a^2
 + b^2)*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)))/b^2 - (2*a*(ArcTanh[Sin[c
 + d*x]]/(b^2*d) + (a*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 +
 b^2]])/(b^2*Sqrt[a^2 + b^2]*d) - 1/(b*d*(a*Cos[c + d*x] + b*Sin[c + d*x])
)))/b^2
```

3.137. $\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

3.137.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3555
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(b*Cos[c + d*x] - a*Sin[c + d*x])*((a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Simp[(n + 2)/((n + 1)*(a^
2 + b^2))   Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

rule 3573
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_)/co
s[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(a*Cos[c + d*x] + b*Sin[c + d*x])^
(n + 1)/(b*d*(n + 1)), x] + (Simp[1/b^2   Int[(a*Cos[c + d*x] + b*Sin[c + d
*x])^(n + 2)/Cos[c + d*x], x], x] - Simp[a/b^2   Int[(a*Cos[c + d*x] + b*Si
n[c + d*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
 && LtQ[n, -1]
```

rule 3583
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-Cos[c + d*x]^(m + 1)/(b*d*(m + 1)
), x] + (-Simp[a/b^2   Int[Cos[c + d*x]^(m + 1), x], x] + Simp[(a^2 + b^2)/
b^2   Int[Cos[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

3.137. $\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

rule 3585
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a^2 + b^2)/b^2   Int[Cos[c +
 d*x]^(m + 2)*(a*Cos[c + d*x] + b*Sin[c + d*x])^n, x], x] + (Simp[1/b^2   I
nt[Cos[c + d*x]^m*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] - Simp[
2*(a/b^2)   Int[Cos[c + d*x]^(m + 1)*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n +
 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] &
& LtQ[m, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.137.4 Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.03

method	result
derivativedivides	$\frac{\frac{1}{b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{3a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^4}-\frac{2\left(\frac{\frac{b^2(3a^2-2b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2a}+\frac{b(4a^4-9a^2b^2+2b^4)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2a^2}-\frac{b^2(13a^2-2b}{}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^2}\right.}{b^4}}{d}$
default	$\frac{\frac{1}{b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{3a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^4}-\frac{2\left(\frac{\frac{b^2(3a^2-2b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2a}+\frac{b(4a^4-9a^2b^2+2b^4)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2a^2}-\frac{b^2(13a^2-2b}{}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^2}\right.}{b^4}}{d}$
risch	$\frac{-9iab\,e^{5i(dx+c)}+6a^2e^{5i(dx+c)}-3b^2e^{5i(dx+c)}+12a^2e^{3i(dx+c)}+2b^2e^{3i(dx+c)}+9iab\,e^{i(dx+c)}+6a^2e^{i(dx+c)}-3b^2e^{i(dx+c)}}{\left(e^{2i(dx+c)}+1\right)\left(-ib\,e^{2i(dx+c)}+e^{2i(dx+c)}a+ib+a\right)^2b^3d}-$

input
```
int(sec(d*x+c)^2/(cos(d*x+c)*a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/b^3/(tan(1/2*d*x+1/2*c)+1)-3*a/b^4*ln(tan(1/2*d*x+1/2*c)+1)-2/b^4*(
(1/2*b^2*(3*a^2-2*b^2)/a*tan(1/2*d*x+1/2*c)^3+1/2*b*(4*a^4-9*a^2*b^2+2*b^4
)/a^2*tan(1/2*d*x+1/2*c)^2-1/2*b^2*(13*a^2-2*b^2)/a*tan(1/2*d*x+1/2*c)-2*a
^2*b+1/2*b^3)/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)^2-3/2*(2*a
^2+b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)
^(1/2)))-1/b^3/(tan(1/2*d*x+1/2*c)-1)+3*a/b^4*ln(tan(1/2*d*x+1/2*c)-1))
```

3.137.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 513 vs. 2(244) = 488.

Time = 0.32 (sec) , antiderivative size = 513, normalized size of antiderivative = 1.97

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$$
$$= \frac{4\,a^2b^3 + 4\,b^5 + 6\,(2\,a^4b + a^2b^3 - b^5)\cos(dx+c)^2 + 18\,(a^3b^2 + ab^4)\cos(dx+c)\sin(dx+c) + 3\,((2\,a^4 - a}{}$$

input
```
integrate(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas"
)
```

output
```
1/4*(4*a^2*b^3 + 4*b^5 + 6*(2*a^4*b + a^2*b^3 - b^5)*cos(d*x + c)^2 + 18*(
a^3*b^2 + a*b^4)*cos(d*x + c)*sin(d*x + c) + 3*((2*a^4 - a^2*b^2 - b^4)*co
s(d*x + c)^3 + 2*(2*a^3*b + a*b^3)*cos(d*x + c)^2*sin(d*x + c) + (2*a^2*b^
2 + b^4)*cos(d*x + c))*sqrt(a^2 + b^2)*log(-(2*a*b*cos(d*x + c)*sin(d*x +
c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d
*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*
cos(d*x + c)^2 + b^2)) - 6*((a^5 - a*b^4)*cos(d*x + c)^3 + 2*(a^4*b + a^2*
b^3)*cos(d*x + c)^2*sin(d*x + c) + (a^3*b^2 + a*b^4)*cos(d*x + c))*log(sin
(d*x + c) + 1) + 6*((a^5 - a*b^4)*cos(d*x + c)^3 + 2*(a^4*b + a^2*b^3)*cos
(d*x + c)^2*sin(d*x + c) + (a^3*b^2 + a*b^4)*cos(d*x + c))*log(-sin(d*x +
c) + 1))/((a^4*b^4 - b^8)*d*cos(d*x + c)^3 + 2*(a^3*b^5 + a*b^7)*d*cos(d*x
 + c)^2*sin(d*x + c) + (a^2*b^6 + b^8)*d*cos(d*x + c))
```

3.137.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = \int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)**2/(a*cos(c + d*x) + b*sin(c + d*x))**3, x)
```

3.137.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 518 vs. $2(244) = 488$.

Time = 0.34 (sec) , antiderivative size = 518, normalized size of antiderivative = 1.99

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$$
$$= \frac{\frac{2\left(6a^4-a^2b^2+\frac{(21a^3b-2ab^3)\sin(dx+c)}{\cos(dx+c)+1}-\frac{2(6a^4-9a^2b^2+b^4)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{4(6a^3b-ab^3)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{(6a^4-9a^2b^2+2b^4)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{(3a^3b-2ab^3)\,\mathrm{si}}{(\cos(dx+c)\cdot}\right.}{a^4b^3+\frac{4a^3b^4\sin(dx+c)}{\cos(dx+c)+1}-\frac{8a^3b^4\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{4a^3b^4\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{a^4b^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{(3a^4b^3-4a^2b^5)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{(3a^4b^3-4a^2b^5)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}}{2d}$$

input
```
integrate(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima"
)
```

output
```
1/2*(2*(6*a^4 - a^2*b^2 + (21*a^3*b - 2*a*b^3)*sin(d*x + c)/(cos(d*x + c)
+ 1) - 2*(6*a^4 - 9*a^2*b^2 + b^4)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 4
*(6*a^3*b - a*b^3)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + (6*a^4 - 9*a^2*b^
2 + 2*b^4)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + (3*a^3*b - 2*a*b^3)*sin(d
*x + c)^5/(cos(d*x + c) + 1)^5)/(a^4*b^3 + 4*a^3*b^4*sin(d*x + c)/(cos(d*x
 + c) + 1) - 8*a^3*b^4*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 4*a^3*b^4*sin
(d*x + c)^5/(cos(d*x + c) + 1)^5 - a^4*b^3*sin(d*x + c)^6/(cos(d*x + c) +
1)^6 - (3*a^4*b^3 - 4*a^2*b^5)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + (3*a^
4*b^3 - 4*a^2*b^5)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) - 6*a*log(sin(d*x
+ c)/(cos(d*x + c) + 1) + 1)/b^4 + 6*a*log(sin(d*x + c)/(cos(d*x + c) + 1)
 - 1)/b^4 - 3*(2*a^2 + b^2)*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + s
qrt(a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))
/(sqrt(a^2 + b^2)*b^4))/d
```

3.137.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.21

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx =$$
$$-\frac{\frac{6a\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{b^4}-\frac{6a\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{b^4}+\frac{3(2a^2+b^2)\log\left(\frac{\left|2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2b-2\sqrt{a^2+b^2}\right|}{\left|2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2b+2\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,b^4}+\frac{4}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\cdot\right.}}{}$$

input
```
integrate(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/2*(6*a*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^4 - 6*a*log(abs(tan(1/2*d*x
 + 1/2*c) - 1))/b^4 + 3*(2*a^2 + b^2)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2
*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 +
b^2)))/(sqrt(a^2 + b^2)*b^4) + 4/((tan(1/2*d*x + 1/2*c)^2 - 1)*b^3) + 2*(3
*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 2*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 4*a^4*tan
(1/2*d*x + 1/2*c)^2 - 9*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 + 2*b^4*tan(1/2*d*x
 + 1/2*c)^2 - 13*a^3*b*tan(1/2*d*x + 1/2*c) + 2*a*b^3*tan(1/2*d*x + 1/2*c)
 - 4*a^4 + a^2*b^2)/((a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c)
- a)^2*a^2*b^3))/d
```

3.137.9 Mupad [B] (verification not implemented)

Time = 24.90 (sec) , antiderivative size = 1311, normalized size of antiderivative = 5.04

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(c + d*x)^2*(a*cos(c + d*x) + b*sin(c + d*x))^3),x)
```

output
```
((6*a^2 - b^2)/b^3 - (2*tan(c/2 + (d*x)/2)^2*(6*a^4 + b^4 - 9*a^2*b^2))/(a
^2*b^3) + (tan(c/2 + (d*x)/2)*(21*a^2 - 2*b^2))/(a*b^2) + (tan(c/2 + (d*x)
/2)^4*(6*a^4 + 2*b^4 - 9*a^2*b^2))/(a^2*b^3) - (4*tan(c/2 + (d*x)/2)^3*(6*
a^2 - b^2))/(a*b^2) + (tan(c/2 + (d*x)/2)^5*(3*a^2 - 2*b^2))/(a*b^2))/(d*(
tan(c/2 + (d*x)/2)^4*(3*a^2 - 4*b^2) - tan(c/2 + (d*x)/2)^2*(3*a^2 - 4*b^2
) - a^2*tan(c/2 + (d*x)/2)^6 + a^2 - 8*a*b*tan(c/2 + (d*x)/2)^3 + 4*a*b*ta
n(c/2 + (d*x)/2)^5 + 4*a*b*tan(c/2 + (d*x)/2))) - (6*a*atanh(tan(c/2 + (d*
x)/2)))/(b^4*d) + (atan((((2*a^2 + b^2)*(a^2 + b^2)^(1/2)*((288*a^4)/b^5 +
 (8*tan(c/2 + (d*x)/2)*(9*a*b^7 + 108*a^3*b^5 + 72*a^5*b^3))/b^9 - (3*(2*a
^2 + b^2)*(a^2 + b^2)^(1/2)*((8*tan(c/2 + (d*x)/2)*(12*a*b^10 + 24*a^3*b^8
))/b^9 - 48*a^2 + (3*(2*a^2 + b^2)*(a^2 + b^2)^(1/2)*(32*a^2*b^3 + (8*tan(
c/2 + (d*x)/2)*(12*a*b^13 + 8*a^3*b^11))/b^9))/(2*(b^6 + a^2*b^4))))/(2*(b
^6 + a^2*b^4)))*3i)/(2*(b^6 + a^2*b^4)) + ((2*a^2 + b^2)*(a^2 + b^2)^(1/2)
*((288*a^4)/b^5 + (8*tan(c/2 + (d*x)/2)*(9*a*b^7 + 108*a^3*b^5 + 72*a^5*b^
3))/b^9 - (3*(2*a^2 + b^2)*(a^2 + b^2)^(1/2)*(48*a^2 - (8*tan(c/2 + (d*x)/
2)*(12*a*b^10 + 24*a^3*b^8))/b^9 + (3*(2*a^2 + b^2)*(a^2 + b^2)^(1/2)*(32*
a^2*b^3 + (8*tan(c/2 + (d*x)/2)*(12*a*b^13 + 8*a^3*b^11))/b^9))/(2*(b^6 +
a^2*b^4))))/(2*(b^6 + a^2*b^4)))*3i)/(2*(b^6 + a^2*b^4)))/((16*(54*a^4 + 2
7*a^2*b^2))/b^8 - (16*tan(c/2 + (d*x)/2)*(216*a^5 + 108*a^3*b^2))/b^9 - (3
*(2*a^2 + b^2)*(a^2 + b^2)^(1/2)*((288*a^4)/b^5 + (8*tan(c/2 + (d*x)/2)...
```

3.137. $\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

3.138 $\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

3.138.1 Optimal result

Integrand size = 28, antiderivative size = 161

$$\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = -\frac{(a^2+b^2)^2}{2a^2b^3d(b+a\cot(c+dx))^2} - \frac{(3a^2-b^2)(a^2+b^2)}{a^2b^4d(b+a\cot(c+dx))} + \frac{2(3a^2+b^2)\log(b+a\cot(c+dx))}{b^5d} + \frac{2(3a^2+b^2)\log(\tan(c+dx))}{b^5d} - \frac{3a\tan(c+dx)}{b^4d} + \frac{\tan^2(c+dx)}{2b^3d}$$

output
```
-1/2*(a^2+b^2)^2/a^2/b^3/d/(b+a*cot(d*x+c))^2-(3*a^2-b^2)*(a^2+b^2)/a^2/b^
4/d/(b+a*cot(d*x+c))+2*(3*a^2+b^2)*ln(b+a*cot(d*x+c))/b^5/d+2*(3*a^2+b^2)*
ln(tan(d*x+c))/b^5/d-3*a*tan(d*x+c)/b^4/d+1/2*tan(d*x+c)^2/b^3/d
```

3.138.2 Mathematica [A] (verified)

Time = 3.65 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.87

$$\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$$
$$= \frac{\frac{b^4\sec^4(c+dx)}{2(a+b\tan(c+dx))^2} - 2a\left(-2a\log(a+b\tan(c+dx))+b\tan(c+dx)-\frac{a^2+b^2}{a+b\tan(c+dx)}\right)+2(a^2+b^2)\left(\log(a+b\tan(c+dx)) \cdots\right.}{b^5d}$$

input
```
Integrate[Sec[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

output
```
((b^4*Sec[c + d*x]^4)/(2*(a + b*Tan[c + d*x])^2) - 2*a*(-2*a*Log[a + b*Tan
[c + d*x]] + b*Tan[c + d*x] - (a^2 + b^2)/(a + b*Tan[c + d*x])) + 2*(a^2 +
 b^2)*(Log[a + b*Tan[c + d*x]] + (3*a^2 - b^2 + 4*a*b*Tan[c + d*x])/(2*(a
+ b*Tan[c + d*x])^2)))/(b^5*d)
```

3.138.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3567, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\cos(c+dx)^3(a\cos(c+dx)+b\sin(c+dx))^3}dx$$

$$\Big\downarrow \; 3567$$

$$-\frac{\int \frac{(\cot^2(c+dx)+1)^2\tan^3(c+dx)}{(b+a\cot(c+dx))^3}d\cot(c+dx)}{d}$$

$$\Big\downarrow \; 522$$

$$-\frac{\int \left(\frac{\tan^3(c+dx)}{b^3}-\frac{3a\tan^2(c+dx)}{b^4}+\frac{2(3a^2+b^2)\tan(c+dx)}{b^5}-\frac{2a(3a^2+b^2)}{b^5(b+a\cot(c+dx))}+\frac{-3a^4-2b^2a^2+b^4}{ab^4(b+a\cot(c+dx))^2}-\frac{(a^2+b^2)^2}{ab^3(b+a\cot(c+dx))^3}\right)d\cot \cdots}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{2(3a^2+b^2)\log(\cot(c+dx))}{b^5}-\frac{2(3a^2+b^2)\log(a\cot(c+dx)+b)}{b^5}+\frac{(3a^2-b^2)(a^2+b^2)}{a^2b^4(a\cot(c+dx)+b)}+\frac{(a^2+b^2)^2}{2a^2b^3(a\cot(c+dx)+b)^2}+\frac{3a\tan(c+dx)}{b^4}-\frac{\tan^2(c+dx)}{2b^3}}{d}$$

input
```
Int[Sec[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

output
```
-(((a^2 + b^2)^2/(2*a^2*b^3*(b + a*Cot[c + d*x])^2) + ((3*a^2 - b^2)*(a^2
+ b^2))/(a^2*b^4*(b + a*Cot[c + d*x])) + (2*(3*a^2 + b^2)*Log[Cot[c + d*x]
])/b^5 - (2*(3*a^2 + b^2)*Log[b + a*Cot[c + d*x]])/b^5 + (3*a*Tan[c + d*x]
)/b^4 - Tan[c + d*x]^2/(2*b^3))/d)
```

3.138.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3567
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[x^m*((b
+ a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a,
b, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[
n, 0] && GtQ[m, 1])
```

3.138.4 Maple [A] (verified)

Time = 2.27 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.71

method	result
derivativedivides	$\frac{-\frac{-\frac{b\tan(dx+c)^2}{2}+3a\tan(dx+c)}{b^4}+\frac{(6a^2+2b^2)\ln(a+b\tan(dx+c))}{b^5}-\frac{a^4+2a^2b^2+b^4}{2b^5(a+b\tan(dx+c))^2}+\frac{4a(a^2+b^2)}{b^5(a+b\tan(dx+c))}}{d}$
default	$\frac{-\frac{-\frac{b\tan(dx+c)^2}{2}+3a\tan(dx+c)}{b^4}+\frac{(6a^2+2b^2)\ln(a+b\tan(dx+c))}{b^5}-\frac{a^4+2a^2b^2+b^4}{2b^5(a+b\tan(dx+c))^2}+\frac{4a(a^2+b^2)}{b^5(a+b\tan(dx+c))}}{d}$
risch	$\frac{-12iab^2-24a^2b+12ia^3+12a^2b\,e^{6i(dx+c)}+12ia^3e^{6i(dx+c)}+36ia^3e^{4i(dx+c)}+4b^3e^{2i(dx+c)}+4b^3e^{6i(dx+c)}+12iab^2e^{4i(dx+ \cdots}{(e^{2i(dx+c)}+1)^2(b\,e^{2i(dx+c)}+ia\,e^{2i(dx+c)}-b+ia)^2b \cdots}$
norman	$\frac{-\frac{2(18a^4+6a^2b^2-b^4)\tan(\frac{dx}{2}+\frac{c}{2})^2}{a^2db^3}-\frac{2(18a^4+6a^2b^2-b^4)\tan(\frac{dx}{2}+\frac{c}{2})^6}{a^2db^3}+\frac{2(18a^4-2a^2b^2-3b^4)\tan(\frac{dx}{2}+\frac{c}{2})^3}{adb^4}-\frac{2(18a^4-2a^2b^2- \cdots}{a \cdots}}{(\tan(\frac{dx}{2}+\frac{c}{2})^2-1)^2(\tan(\frac{dx}{2} \cdots}$
parallelrisch	$48\left(a^2+\frac{b^2}{3}\right)\left(\frac{(a^2-b^2)\cos(4dx+4c)}{4}+a^2\cos(2dx+2c)+ab\sin(2dx+2c)+\frac{ab\sin(4dx+4c)}{2}+\frac{3a^2}{4}+\frac{b^2}{4}\right)a^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a- \cdots$

input
```
int(sec(d*x+c)^3/(cos(d*x+c)*a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/b^4*(-1/2*b*tan(d*x+c)^2+3*a*tan(d*x+c))+(6*a^2+2*b^2)/b^5*ln(a+b*
tan(d*x+c))-1/2/b^5*(a^4+2*a^2*b^2+b^4)/(a+b*tan(d*x+c))^2+4*a/b^5*(a^2+b^
2)/(a+b*tan(d*x+c)))
```

3.138.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(157) = 314$.

Time = 0.29 (sec) , antiderivative size = 354, normalized size of antiderivative = 2.20

$$\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$$
$$= \frac{24\,a^2b^2\cos(dx+c)^4+b^4-2\,(9\,a^2b^2+b^4)\cos(dx+c)^2+2\,((3\,a^4-2\,a^2b^2-b^4)\cos(dx+c)^4+2\,(3\,a^3b+ \cdots}{\cdots}$$

input
```
integrate(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas"
)
```

output
```
1/2*(24*a^2*b^2*cos(d*x + c)^4 + b^4 - 2*(9*a^2*b^2 + b^4)*cos(d*x + c)^2
+ 2*((3*a^4 - 2*a^2*b^2 - b^4)*cos(d*x + c)^4 + 2*(3*a^3*b + a*b^3)*cos(d*
x + c)^3*sin(d*x + c) + (3*a^2*b^2 + b^4)*cos(d*x + c)^2)*log(2*a*b*cos(d*
x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 2*((3*a^4 - 2*a^
2*b^2 - b^4)*cos(d*x + c)^4 + 2*(3*a^3*b + a*b^3)*cos(d*x + c)^3*sin(d*x +
 c) + (3*a^2*b^2 + b^4)*cos(d*x + c)^2)*log(cos(d*x + c)^2) - 4*(a*b^3*cos
(d*x + c) + 3*(a^3*b - a*b^3)*cos(d*x + c)^3)*sin(d*x + c))/(2*a*b^6*d*cos
(d*x + c)^3*sin(d*x + c) + b^7*d*cos(d*x + c)^2 + (a^2*b^5 - b^7)*d*cos(d*
x + c)^4)
```

3.138.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = \int \frac{\sec^3\left(c+dx\right)}{\left(a\cos\left(c+dx\right)+b\sin\left(c+dx\right)\right)^3}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)**3/(a*cos(c + d*x) + b*sin(c + d*x))**3, x)
```

3.138.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 652 vs. $2(157) = 314$.

Time = 0.28 (sec) , antiderivative size = 652, normalized size of antiderivative = 4.05

$$\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = -\frac{2\left(\frac{\frac{(6a^5+2a^3b^2-ab^4)\sin(dx+c)}{\cos(dx+c)+1}+\frac{(18a^4b+6a^2b^3-b^5)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{(18a^5-2a^3b^2-3ab^4)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{2(18a^4b+8a^2b^3-b^5)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{(18a^5-2\cdots}{\cdots}}{a^4b^4+\frac{4a^3b^5\sin(dx+c)}{\cos(dx+c)+1}-\frac{12a^3b^5\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{12a^3b^5\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{4a^3b^5\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{a^4b^4\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{4(a^4b^4-a^2b^6)\sin\cdots}{(\cos(dx+c)+\cdots}}\right.}{\cdots}$$

input
```
integrate(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima"
)
```

output
```
-2*(((6*a^5 + 2*a^3*b^2 - a*b^4)*sin(d*x + c)/(cos(d*x + c) + 1) + (18*a^4
*b + 6*a^2*b^3 - b^5)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - (18*a^5 - 2*a^
3*b^2 - 3*a*b^4)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 2*(18*a^4*b + 8*a^2
*b^3 - b^5)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + (18*a^5 - 2*a^3*b^2 - 3*
a*b^4)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + (18*a^4*b + 6*a^2*b^3 - b^5)*
sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - (6*a^5 + 2*a^3*b^2 - a*b^4)*sin(d*x
+ c)^7/(cos(d*x + c) + 1)^7)/(a^4*b^4 + 4*a^3*b^5*sin(d*x + c)/(cos(d*x +
c) + 1) - 12*a^3*b^5*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 12*a^3*b^5*sin(
d*x + c)^5/(cos(d*x + c) + 1)^5 - 4*a^3*b^5*sin(d*x + c)^7/(cos(d*x + c) +
 1)^7 + a^4*b^4*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 4*(a^4*b^4 - a^2*b^6
)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 2*(3*a^4*b^4 - 4*a^2*b^6)*sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4 - 4*(a^4*b^4 - a^2*b^6)*sin(d*x + c)^6/(cos(d*
x + c) + 1)^6) - (3*a^2 + b^2)*log(-a - 2*b*sin(d*x + c)/(cos(d*x + c) + 1
) + a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/b^5 + (3*a^2 + b^2)*log(sin(d*x
 + c)/(cos(d*x + c) + 1) + 1)/b^5 + (3*a^2 + b^2)*log(sin(d*x + c)/(cos(d*
x + c) + 1) - 1)/b^5)/d
```

3.138.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.87

$$\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = \frac{\frac{4\,(3\,a^2+b^2)\log(|b\tan(dx+c)+a|)}{b^5}+\frac{b^3\tan(dx+c)^2-6\,ab^2\tan(dx+c)}{b^6}-\frac{18\,a^2b^2\tan(dx+c)^2+6\,b^4\tan(dx+c)^2+28\,a^3b\tan(dx+c)+4\,ab^3\tan(d}{(b\tan(dx+c)+a)^2b^5}}{2\,d}$$

input
```
integrate(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
1/2*(4*(3*a^2 + b^2)*log(abs(b*tan(d*x + c) + a))/b^5 + (b^3*tan(d*x + c)^
2 - 6*a*b^2*tan(d*x + c))/b^6 - (18*a^2*b^2*tan(d*x + c)^2 + 6*b^4*tan(d*x
 + c)^2 + 28*a^3*b*tan(d*x + c) + 4*a*b^3*tan(d*x + c) + 11*a^4 + b^4)/((b
*tan(d*x + c) + a)^2*b^5))/d
```

3.138.9 Mupad [B] (verification not implemented)

Time = 27.02 (sec) , antiderivative size = 1204, normalized size of antiderivative = 7.48

$$\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(c + d*x)^3*(a*cos(c + d*x) + b*sin(c + d*x))^3),x)
```

output
```
- ((2*tan(c/2 + (d*x)/2)*(6*a^4 - b^4 + 2*a^2*b^2))/(a*b^4) - (2*tan(c/2 +
 (d*x)/2)^7*(6*a^4 - b^4 + 2*a^2*b^2))/(a*b^4) + (2*tan(c/2 + (d*x)/2)^3*(
3*b^4 - 18*a^4 + 2*a^2*b^2))/(a*b^4) + (2*tan(c/2 + (d*x)/2)^2*(18*a^4 - b
^4 + 6*a^2*b^2))/(a^2*b^3) - (2*tan(c/2 + (d*x)/2)^5*(3*b^4 - 18*a^4 + 2*a
^2*b^2))/(a*b^4) - (4*tan(c/2 + (d*x)/2)^4*(18*a^4 - b^4 + 8*a^2*b^2))/(a^
2*b^3) + (2*tan(c/2 + (d*x)/2)^6*(18*a^4 - b^4 + 6*a^2*b^2))/(a^2*b^3))/(d
*(tan(c/2 + (d*x)/2)^4*(6*a^2 - 8*b^2) - tan(c/2 + (d*x)/2)^6*(4*a^2 - 4*b
^2) - tan(c/2 + (d*x)/2)^2*(4*a^2 - 4*b^2) + a^2*tan(c/2 + (d*x)/2)^8 + a^
2 - 12*a*b*tan(c/2 + (d*x)/2)^3 + 12*a*b*tan(c/2 + (d*x)/2)^5 - 4*a*b*tan(
c/2 + (d*x)/2)^7 + 4*a*b*tan(c/2 + (d*x)/2))) - (atan((((3*a^2 + b^2)*((2*
(3*a^2 + b^2)*((4*(a*b^10 + 4*a^3*b^8))/b^8 - (4*tan(c/2 + (d*x)/2)^2*(3*a
*b^10 + 4*a^3*b^8))/b^8 + 16*a^2*b*tan(c/2 + (d*x)/2)))/b^5 - (4*(4*a*b^7
+ 12*a^3*b^5))/b^8 + (4*tan(c/2 + (d*x)/2)^2*(4*a*b^7 + 12*a^3*b^5))/b^8 +
 (16*tan(c/2 + (d*x)/2)*(6*a^4 + 2*a^2*b^2))/b^4)*2i)/b^5 - ((3*a^2 + b^2)
*((4*(4*a*b^7 + 12*a^3*b^5))/b^8 + (2*(3*a^2 + b^2)*((4*(a*b^10 + 4*a^3*b^
8))/b^8 - (4*tan(c/2 + (d*x)/2)^2*(3*a*b^10 + 4*a^3*b^8))/b^8 + 16*a^2*b*t
an(c/2 + (d*x)/2)))/b^5 - (4*tan(c/2 + (d*x)/2)^2*(4*a*b^7 + 12*a^3*b^5))/
b^8 - (16*tan(c/2 + (d*x)/2)*(6*a^4 + 2*a^2*b^2))/b^4)*2i)/b^5)/((8*(4*a*b
^4 + 36*a^5 + 24*a^3*b^2))/b^8 + (2*(3*a^2 + b^2)*((2*(3*a^2 + b^2)*((4*(a
*b^10 + 4*a^3*b^8))/b^8 - (4*tan(c/2 + (d*x)/2)^2*(3*a*b^10 + 4*a^3*b^8...
```

3.139 $\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

3.139.1 Optimal result

Integrand size = 28, antiderivative size = 383

$$\begin{aligned}\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = &-\frac{4a^3\text{arctanh}(\sin(c+dx))}{b^6d}\\
&-\frac{3a\text{arctanh}(\sin(c+dx))}{2b^4d}\\
&-\frac{6a(a^2+b^2)\,\text{arctanh}(\sin(c+dx))}{b^6d}\\
&-\frac{8a^2\sqrt{a^2+b^2}\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^6d}\\
&-\frac{\sqrt{a^2+b^2}\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2b^4d}\\
&-\frac{2(a^2+b^2)^{3/2}\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^6d}\\
&+\frac{4a^2\sec(c+dx)}{b^5d}\\
&+\frac{2(a^2+b^2)\sec(c+dx)}{b^5d}+\frac{\sec^3(c+dx)}{3b^3d}\\
&-\frac{(a^2+b^2)\,(b\cos(c+dx)-a\sin(c+dx))}{2b^4d(a\cos(c+dx)+b\sin(c+dx))^2}\\
&+\frac{4a(a^2+b^2)}{b^5d(a\cos(c+dx)+b\sin(c+dx))}\\
&-\frac{3a\sec(c+dx)\tan(c+dx)}{2b^4d}\end{aligned}$$

output
```
-4*a^3*arctanh(sin(d*x+c))/b^6/d-3/2*a*arctanh(sin(d*x+c))/b^4/d-6*a*(a^2+
b^2)*arctanh(sin(d*x+c))/b^6/d-2*(a^2+b^2)^(3/2)*arctanh((b*cos(d*x+c)-a*s
in(d*x+c))/(a^2+b^2)^(1/2))/b^6/d+4*a^2*sec(d*x+c)/b^5/d+2*(a^2+b^2)*sec(d
*x+c)/b^5/d+1/3*sec(d*x+c)^3/b^3/d-1/2*(a^2+b^2)*(b*cos(d*x+c)-a*sin(d*x+c
))/b^4/d/(a*cos(d*x+c)+b*sin(d*x+c))^2+4*a*(a^2+b^2)/b^5/d/(a*cos(d*x+c)+b
*sin(d*x+c))-8*a^2*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))*(a
^2+b^2)^(1/2)/b^6/d-1/2*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2
))*(a^2+b^2)^(1/2)/b^4/d-3/2*a*sec(d*x+c)*tan(d*x+c)/b^4/d
```

3.139.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.66 (sec) , antiderivative size = 688, normalized size of antiderivative = 1.80

$$\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$$
$$= \frac{\sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))\left(\frac{6b^2(a^2+b^2)^2\sin(c+dx)}{a}+\frac{6(a-ib)(a+ib)b(8a^2-b^2)(a\cos(c+dx)+b\sin(c+dx))}{a}\right.}{}$$

input
```
Integrate[Sec[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

output
```
(Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])*((6*b^2*(a^2 + b^2)^2*Si
n[c + d*x])/a + (6*(a - I*b)*(a + I*b)*b*(8*a^2 - b^2)*(a*Cos[c + d*x] + b
*Sin[c + d*x]))/a + 2*b*(36*a^2 + 13*b^2)*(a*Cos[c + d*x] + b*Sin[c + d*x]
)^2 + 60*Sqrt[a^2 + b^2]*(4*a^2 + b^2)*ArcTanh[(-b + a*Tan[(c + d*x)/2])/S
qrt[a^2 + b^2]]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2 + 30*a*(4*a^2 + 3*b^2)
*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a*Cos[c + d*x] + b*Sin[c + d*x]
)^2 - 30*a*(4*a^2 + 3*b^2)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a*Cos
[c + d*x] + b*Sin[c + d*x])^2 + (b^2*(-9*a + b)*(a*Cos[c + d*x] + b*Sin[c
+ d*x])^2)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (2*b^3*Sin[(c + d*x)/
2]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/
2])^3 + (2*b*(36*a^2 + 13*b^2)*Sin[(c + d*x)/2]*(a*Cos[c + d*x] + b*Sin[c
+ d*x])^2)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) - (2*b^3*Sin[(c + d*x)/2]
*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
)^3 + (b^2*(9*a + b)*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(Cos[(c + d*x)/2
] + Sin[(c + d*x)/2])^2 - (2*b*(36*a^2 + 13*b^2)*Sin[(c + d*x)/2]*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^2)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))/(12*b
^6*d*(a + b*Tan[c + d*x])^3)
```

3.139.3 Rubi [A] (verified)

Time = 4.04 (sec) , antiderivative size = 697, normalized size of antiderivative = 1.82, number of steps used = 27, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.929, Rules used = {3042, 3585, 3042, 3583, 3042, 3583, 3042, 3553, 219, 3585, 3042, 3555, 3042, 3553, 219, 3573, 3042, 3553, 219, 3583, 3042, 3553, 219, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(c+dx)^4(a\cos(c+dx)+b\sin(c+dx))^3}dx$$

$$\Big\downarrow \text{3585}$$

$$\frac{(a^2+b^2)\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}dx}{b^2}+\frac{\int \frac{\sec^4(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx}{b^2}-\frac{2a\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a^2+b^2)\int \frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))^3}dx}{b^2}-\frac{2a\int \frac{1}{\cos(c+dx)^3(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}+\frac{\int \frac{1}{\cos(c+dx)^4(a\cos(c+dx)+b\sin(c+dx))}dx}{b^2}$$

$$\Big\downarrow \text{3583}$$

$$\frac{(a^2+b^2)\int \frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))^3}dx}{b^2}+\frac{\frac{(a^2+b^2)\int \frac{\sec^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx}{b^2}-\frac{a\int \sec^3(c+dx)dx}{b^2}+\frac{\sec^3(c+dx)}{3bd}}{b^2}-\frac{2a\int \frac{1}{\cos(c+dx)^3(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a^2+b^2)\int \frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))^3}dx}{b^2}+\frac{\frac{(a^2+b^2)\int \frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))}dx}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{b^2}+\frac{\sec^3(c+dx)}{3bd}}{b^2}-\frac{2a\int \frac{1}{\cos(c+dx)^3(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}$$

$$\Big\downarrow \text{3583}$$

$$\frac{\frac{\left(a^2+b^2\right)\int \frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))^3}dx}{b^2}+\frac{\frac{(a^2+b^2)\left(\frac{\left(a^2+b^2\right)\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{b^2}-\frac{a\int \sec(c+dx)dx}{b^2}+\frac{\sec(c+dx)}{bd}\right)}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{b^2}+\frac{\sec^3(c+dx)}{3bd}}{b^2}-\frac{2a\int \frac{1}{\cos(c+dx)^3(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}}{}$$

$\downarrow$ 3042

$$\frac{\left(a^2+b^2\right)\int \frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))^3}dx}{b^2}+\frac{\frac{(a^2+b^2)\left(\frac{\left(a^2+b^2\right)\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}+\frac{\sec(c+dx)}{bd}\right)}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{b^2}+\frac{\sec^3(c+dx)}{3bd}}{b^2}-\frac{2a\int \frac{1}{\cos(c+dx)^3(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}$$

$\downarrow$ 3553

$$\frac{\left(a^2+b^2\right)\int \frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))^3}dx}{b^2}+\frac{\frac{(a^2+b^2)\left(-\frac{\left(a^2+b^2\right)\int \frac{1}{a^2+b^2-(b\cos(c+dx)-a\sin(c+dx))^2}d(b\cos(c+dx)-a\sin(c+dx))}{b^2d}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}+\frac{\sec(c+dx)}{bd}\right)}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{b^2}+}{b^2}-\frac{2a\int \frac{1}{\cos(c+dx)^3(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}$$

$\downarrow$ 219

$$\frac{\frac{(a^2+b^2)\left(-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d}+\frac{\sec(c+dx)}{bd}\right)}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{b^2}+\frac{\sec^3(c+dx)}{3bd}}{b^2}+\frac{\left(a^2+b^2\right)\int \frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))^3}dx}{b^2}-\frac{2a\int \frac{1}{\cos(c+dx)^3(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}$$

$\downarrow$ 3585

3.139. $\displaystyle\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

$$\frac{\frac{(a^2+b^2)\left(-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{\sqrt{a^2+b^2}\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2 d}+\frac{\sec(c+dx)}{bd}\right)}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{b^2}+\frac{\sec^3(c+dx)}{3bd}}{b^2}+\frac{(a^2+b^2)\left(\frac{(a^2+b^2)\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^3}dx}{b^2}+\frac{\int \frac{\sec^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx}{b^2}-\frac{2a\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}\right)}{b^2}-\frac{2a\left(\frac{(a^2+b^2)\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}-\frac{2a\int\frac{\sec^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx}{b^2}+\frac{\int \sec^3(c+dx)dx}{b^2}\right)}{b^2}$$

$\big\downarrow$ 3042

$$\frac{\frac{(a^2+b^2)\left(-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{\sqrt{a^2+b^2}\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2 d}+\frac{\sec(c+dx)}{bd}\right)}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{b^2}+\frac{\sec^3(c+dx)}{3bd}}{b^2}+\frac{(a^2+b^2)\left(\frac{(a^2+b^2)\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^3}dx}{b^2}-\frac{2a\int \frac{1}{\cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}+\frac{\int \frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))}dx}{b^2}\right)}{b^2}$$
$$\frac{2a\left(\frac{(a^2+b^2)\int \frac{1}{\cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}-\frac{2a\int\frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))}dx}{b^2}+\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{b^2}\right)}{b^2}$$

$\big\downarrow$ 3555

$$\frac{\frac{(a^2+b^2)\left(-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{\sqrt{a^2+b^2}\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2 d}+\frac{\sec(c+dx)}{bd}\right)}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{b^2}+\frac{\sec^3(c+dx)}{3bd}}{b^2}+\frac{(a^2+b^2)\left(\frac{(a^2+b^2)\left(\frac{\int\frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{2\left(a^2+b^2\right)}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^2}\right)}{b^2}-\frac{2a\int \frac{1}{\cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}+\int\cdots\right)}{b^2}$$
$$\frac{2a\left(\frac{(a^2+b^2)\int \frac{1}{\cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}-\frac{2a\int\frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))}dx}{b^2}+\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{b^2}\right)}{b^2}$$

$\big\downarrow$ 3042

3.139. $\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

$$\frac{\frac{(a^2+b^2)\left(-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{\sqrt{a^2+b^2}\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d}+\frac{\sec(c+dx)}{bd}\right)}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{b^2}+\frac{\sec^3(c+dx)}{3bd}}{b^2}+$$

$$\frac{(a^2+b^2)\left(\frac{(a^2+b^2)\left(\frac{\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{2(a^2+b^2)}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^2}\right)}{b^2}-\frac{2a\int \frac{1}{\cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}+\frac{\int}{}\right.}{b^2}$$

$$\frac{2a\left(\frac{(a^2+b^2)\int \frac{1}{\cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}-\frac{2a\int \frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))}dx}{b^2}+\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{b^2}\right)}{b^2}$$

$$\Big\downarrow \; 3553$$

$$\frac{\frac{(a^2+b^2)\left(-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{\sqrt{a^2+b^2}\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d}+\frac{\sec(c+dx)}{bd}\right)}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{b^2}+\frac{\sec^3(c+dx)}{3bd}}{b^2}+$$

$$\frac{(a^2+b^2)\left(\frac{(a^2+b^2)\left(-\frac{\int \frac{1}{a^2+b^2-(b\cos(c+dx)-a\sin(c+dx))^2}d(b\cos(c+dx)-a\sin(c+dx))}{2d(a^2+b^2)}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^2}\right)}{b^2}-\frac{2a\int}{\cos(c+dx)}\right.}{b^2}$$

$$\frac{2a\left(\frac{(a^2+b^2)\int \frac{1}{\cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}-\frac{2a\int \frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))}dx}{b^2}+\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{b^2}\right)}{b^2}$$

$$\Big\downarrow \; 219$$

$$\frac{(a^2+b^2)\left(-\frac{2a\int \frac{1}{\cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}+\frac{\int \frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))}dx}{b^2}+\frac{(a^2+b^2)\left(-\frac{\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin}{\sqrt{a^2+b}}\right.}{2d(a^2+b^2)^{3/2}}\right.}{}\right.}{b^2}$$

$$\frac{\frac{(a^2+b^2)\left(-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{\sqrt{a^2+b^2}\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d}+\frac{\sec(c+dx)}{bd}\right)}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{b^2}+\frac{\sec^3(c+dx)}{3bd}}{b^2}-$$

$$\frac{2a\left(\frac{(a^2+b^2)\int \frac{1}{\cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}-\frac{2a\int \frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))}dx}{b^2}+\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{b^2}\right)}{b^2}$$

$$\Big\downarrow \; 3573$$

3.139. $\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

$$\frac{(a^2+b^2)\left(\frac{\int \frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))}dx}{b^2}-\frac{2a\left(-\frac{a\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{b^2}+\frac{\int \sec(c+dx)dx}{b^2}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}\right)}{b^2}+-\frac{(a\cdots}{}\right.}{b^2}$$

$$\frac{\frac{(a^2+b^2)\left(-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{\sqrt{a^2+b^2}\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d}+\frac{\sec(c+dx)}{bd}\right)}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{b^2}+\frac{\sec^3(c+dx)}{3bd}}{b^2}-$$

$$\frac{2a\left(\frac{(a^2+b^2)\left(-\frac{a\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{b^2}+\frac{\int \sec(c+dx)dx}{b^2}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}\right)}{b^2}-\frac{2a\int \frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))}dx}{b^2}+\frac{\int \mathrm{c}\cdots}{}\right.}{b^2}$$

$$\Big\downarrow \; \text{3042}$$

$$\frac{(a^2+b^2)\left(\frac{\int \frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))}dx}{b^2}-\frac{2a\left(-\frac{a\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{b^2}+\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}\right)}{b^2}+\cdots\right.}{b^2}$$

$$\frac{\frac{(a^2+b^2)\left(-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{\sqrt{a^2+b^2}\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d}+\frac{\sec(c+dx)}{bd}\right)}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{b^2}+\frac{\sec^3(c+dx)}{3bd}}{b^2}-$$

$$\frac{2a\left(\frac{(a^2+b^2)\left(-\frac{a\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{b^2}+\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}\right)}{b^2}-\frac{2a\int \frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))}dx}{b^2}+\cdots\right.}{b^2}$$

$$\Big\downarrow \; \text{3553}$$

3.139. $\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

$$\frac{\left(a^2+b^2\right)\left(-\frac{2a\left(\frac{a\int \frac{1}{a^2+b^2-(b\cos(c+dx)-a\sin(c+dx))^2}d(b\cos(c+dx)-a\sin(c+dx))}{b^2d}+\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}\right)}{b^2}+\frac{\int \cdots}{\cos(c+\cdots}\right.}{b^2}$$

$$\frac{\frac{(a^2+b^2)\left(-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{\sqrt{a^2+b^2}\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d}+\frac{\sec(c+dx)}{bd}\right)}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{b^2}+\frac{\sec^3(c+dx)}{3bd}}{b^2}-$$

$$\frac{2a\left(\frac{(a^2+b^2)\left(\frac{a\int \frac{1}{a^2+b^2-(b\cos(c+dx)-a\sin(c+dx))^2}d(b\cos(c+dx)-a\sin(c+dx))}{b^2d}+\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}\right)}{b^2}-\frac{2a\int \cdots}{\cos(c+dx)\cdots}\right.}{b^2}$$

$$\Big\downarrow \; 219$$

$$\frac{\left(a^2+b^2\right)\left(-\frac{2a\left(\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}+\frac{a\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}\right)}{b^2}+\frac{\int \frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+\cdots}}{b^2}\right.}{b^2}$$

$$\frac{2a\left(\frac{(a^2+b^2)\left(\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}+\frac{a\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}\right)}{b^2}-\frac{2a\int \frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx\cdots}}{b^2}\right.}{b^2}$$

$$\frac{\frac{(a^2+b^2)\left(-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{\sqrt{a^2+b^2}\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d}+\frac{\sec(c+dx)}{bd}\right)}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{b^2}+\frac{\sec^3(c+dx)}{3bd}}{b^2}$$

$$\Big\downarrow \; 3583$$

3.139. $\displaystyle\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

$$\frac{\frac{(a^2+b^2)\left(-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{\sqrt{a^2+b^2}\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d}+\frac{\sec(c+dx)}{bd}\right)}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{b^2}+\frac{\sec^3(c+dx)}{3bd}}{b^2}+$$

$$\frac{(a^2+b^2)\left(-\frac{2a\left(\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}+\frac{a\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}\right)}{b^2}+\frac{\frac{(a^2+b^2)\int \frac{1}{a\cos(c+dx)+b\sin(c+dx\cdots}}{b^2}}{b\cdots}\cdots\right.}{b^2}$$

$$\frac{2a\left(\frac{(a^2+b^2)\left(\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}+\frac{a\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}\right)}{b^2}-\frac{2a\left(\frac{(a^2+b^2)\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}\cdots}{b^2}\cdots\right.}{b^2}\cdots\right.}{b^2}$$

$\downarrow$ 3042

$$\frac{\frac{(a^2+b^2)\left(-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{\sqrt{a^2+b^2}\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d}+\frac{\sec(c+dx)}{bd}\right)}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{b^2}+\frac{\sec^3(c+dx)}{3bd}}{b^2}+$$

$$\frac{(a^2+b^2)\left(-\frac{2a\left(\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}+\frac{a\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}\right)}{b^2}+\frac{(a^2+b^2)\int \frac{1}{a\cos(c+dx)+b\sin(c+dx\cdots}}{b^2}\cdots\right.}{b^2}$$

$$\frac{2a\left(\frac{(a^2+b^2)\left(\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}+\frac{a\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}\right)}{b^2}-\frac{2a\left(\frac{(a^2+b^2)\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}}{b^2}\cdots\right.}{b^{2}\cdots}\cdots\right.}{b^2}$$

$\downarrow$ 3553

3.139. $\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

$$\frac{\frac{(a^2+b^2)\left(-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{\sqrt{a^2+b^2}\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d}+\frac{\sec(c+dx)}{bd}\right)}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{b^2}+\frac{\sec^3(c+dx)}{3bd}}{b^2}+$$

$$\frac{(a^2+b^2)\left(-\frac{2a\left(\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}+\frac{a\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}\right)}{b^2}+\frac{-\frac{\left(a^2+b^2\right)\int \frac{1}{a^2+b^2-(b\cos(c+dx)-\cdots}}{\cdots}}{\cdots}\right.}{\cdots}$$

$$\frac{2a\left(\frac{(a^2+b^2)\left(\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}+\frac{a\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}\right)}{b^2}-\frac{2a\left(-\frac{\left(a^2+b^2\right)\int \frac{1}{a^2+b^2-(b\cos(c+dx)-\cdots}}{\cdots}\right.}{\cdots}\right.}{b^2}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{(a^2+b^2)\left(-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{\sqrt{a^2+b^2}\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d}+\frac{\sec(c+dx)}{bd}\right)}{b^2}-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{b^2}+\frac{\sec^3(c+dx)}{3bd}}{b^2}+$$

$$\frac{(a^2+b^2)\left(-\frac{2a\left(\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}+\frac{a\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}\right)}{b^2}+\frac{-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{\sqrt{a^2+b^2}\,\text{ar}\cdots}{\cdots}}{\cdots}\right.}{b^2}$$

$$\frac{2a\left(\frac{(a^2+b^2)\left(\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}+\frac{a\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}\right)}{b^2}-\frac{2a\left(-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{\sqrt{a^2+b^2}\,\text{ar}\cdots}{\cdots}\right.}{\cdots}\right.}{b^2}$$

$$\Big\downarrow \; 4255$$

3.139. $\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

$$\left(a^2+b^2\right)\left(-\frac{2a\left(\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}+\frac{a\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}\right)}{b^2}+\frac{-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{\sqrt{a^2+b^2}\,\text{a}}{}}{}\right.$$

$$b^2$$

$$\frac{\frac{\left(a^2+b^2\right)\left(-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{\sqrt{a^2+b^2}\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d}+\frac{\sec(c+dx)}{bd}\right)}{b^2}-\frac{a\left(\frac{1}{2}\int \sec(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{b^2}+\sec^3}{b^2}$$

$$\frac{2a\left(\frac{\left(a^2+b^2\right)\left(\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}+\frac{a\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}\right)}{b^2}-\frac{2a\left(-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{\sqrt{a^2+b^2}\,\text{ar}}{}\right.}{}\right.}{b^2}$$

$\downarrow$ 3042

$$\left(a^2+b^2\right)\left(-\frac{2a\left(\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}+\frac{a\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}\right)}{b^2}+\frac{-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{\sqrt{a^2+b^2}\,\text{a}}{}}{}\right.$$

$$b^2$$

$$\frac{\frac{\left(a^2+b^2\right)\left(-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{\sqrt{a^2+b^2}\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d}+\frac{\sec(c+dx)}{bd}\right)}{b^2}-\frac{a\left(\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{b^2}+}{b^2}$$

$$\frac{2a\left(\frac{\left(a^2+b^2\right)\left(\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}+\frac{a\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}\right)}{b^2}-\frac{2a\left(-\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{\sqrt{a^2+b^2}\,\text{ar}}{}\right.}{}\right.}{b^2}$$

$\downarrow$ 4257

3.139. $\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

$$\frac{(a^2+b^2)\left(\frac{(a^2+b^2)\left(-\frac{\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2d(a^2+b^2)^{3/2}}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^2}\right)}{b^2}-\frac{2a\left(\frac{a\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+\ldots}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}\ldots\right.}{\ldots}\right.}{b\ldots}$$

$$\frac{\frac{(a^2+b^2)\left(-\frac{\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d}-\frac{a\operatorname{arctanh}(\sin(c+dx))}{b^2d}+\frac{\sec(c+dx)}{bd}\right)}{b^2}-\frac{a\left(\frac{\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{\tan(c+dx)\sec(c+dx)}{2d}\right)}{b^2}}{b^2}$$

$$\frac{2a\left(\frac{(a^2+b^2)\left(\frac{a\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}+\frac{\operatorname{arctanh}(\sin(c+dx))}{b^2d}\right)}{b^2}-\frac{2a\left(-\frac{\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{b\cos(c+\ldots}{\ldots}\right)}{b^2d}\ldots\right.}{\ldots}\right.}{b^2}$$

input
```
Int[Sec[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

output
```
((a^2 + b^2)*((-((a*ArcTanh[Sin[c + d*x]])/(b^2*d)) - (Sqrt[a^2 + b^2]*Arc
Tanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(b^2*d) + Sec[c +
 d*x]/(b*d))/b^2 + ((a^2 + b^2)*(-1/2*ArcTanh[(b*Cos[c + d*x] - a*Sin[c +
d*x])/Sqrt[a^2 + b^2]]/((a^2 + b^2)^(3/2)*d) - (b*Cos[c + d*x] - a*Sin[c +
 d*x])/(2*(a^2 + b^2)*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)))/b^2 - (2*a*
(ArcTanh[Sin[c + d*x]]/(b^2*d) + (a*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*
x])/Sqrt[a^2 + b^2]])/(b^2*Sqrt[a^2 + b^2]*d) - 1/(b*d*(a*Cos[c + d*x] + b
*Sin[c + d*x]))))/b^2))/b^2 - (2*a*((-2*a*(-((a*ArcTanh[Sin[c + d*x]])/(b^
2*d)) - (Sqrt[a^2 + b^2]*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^
2 + b^2]])/(b^2*d) + Sec[c + d*x]/(b*d)))/b^2 + ((a^2 + b^2)*(ArcTanh[Sin[
c + d*x]]/(b^2*d) + (a*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2
+ b^2]])/(b^2*Sqrt[a^2 + b^2]*d) - 1/(b*d*(a*Cos[c + d*x] + b*Sin[c + d*x]
))))/b^2 + (ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d
))/b^2))/b^2 + (Sec[c + d*x]^3/(3*b*d) + ((a^2 + b^2)*(-((a*ArcTanh[Sin[c
+ d*x]])/(b^2*d)) - (Sqrt[a^2 + b^2]*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d
*x])/Sqrt[a^2 + b^2]])/(b^2*d) + Sec[c + d*x]/(b*d)))/b^2 - (a*(ArcTanh[Si
n[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/b^2)/b^2
```

3.139. $\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

3.139.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3555
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(b*Cos[c + d*x] - a*Sin[c + d*x])*((a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Simp[(n + 2)/((n + 1)*(a^
2 + b^2))   Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

rule 3573
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_)/co
s[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(a*Cos[c + d*x] + b*Sin[c + d*x])^
(n + 1)/(b*d*(n + 1)), x] + (Simp[1/b^2   Int[(a*Cos[c + d*x] + b*Sin[c + d
*x])^(n + 2)/Cos[c + d*x], x], x] - Simp[a/b^2   Int[(a*Cos[c + d*x] + b*Si
n[c + d*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
 && LtQ[n, -1]
```

rule 3583
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-Cos[c + d*x]^(m + 1)/(b*d*(m + 1)
), x] + (-Simp[a/b^2   Int[Cos[c + d*x]^(m + 1), x], x] + Simp[(a^2 + b^2)/
b^2   Int[Cos[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

3.139. $\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

rule 3585
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a^2 + b^2)/b^2   Int[Cos[c +
 d*x]^(m + 2)*(a*Cos[c + d*x] + b*Sin[c + d*x])^n, x], x] + (Simp[1/b^2   I
nt[Cos[c + d*x]^m*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] - Simp[
2*(a/b^2)   Int[Cos[c + d*x]^(m + 1)*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n +
 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] &
& LtQ[m, -1]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.139.4 Maple [A] (verified)

Time = 3.25 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.16

method	result
derivativedivides	$-\frac{1}{3b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{3a+b}{2b^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{12a^2+3ab+5b^2}{2b^5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{5a\left(4a^2+3b^2\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2b^6}+\frac{1}{3b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}$
default	$-\frac{1}{3b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{3a+b}{2b^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{12a^2+3ab+5b^2}{2b^5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{5a\left(4a^2+3b^2\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2b^6}+\frac{1}{3b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}$
risch	$60ia\,b^3e^{i(dx+c)}-180ia^3b\,e^{7i(dx+c)}-60ia\,b^3e^{9i(dx+c)}+180ia^3b\,e^{3i(dx+c)}+240a^4e^{3i(dx+c)}+60a^4e^{i(dx+c)}-20b^4e^{3i(dx+c)}$

input
```
int(sec(d*x+c)^4/(cos(d*x+c)*a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/3/b^3/(tan(1/2*d*x+1/2*c)-1)^3-1/2*(3*a+b)/b^4/(tan(1/2*d*x+1/2*c)
-1)^2-1/2*(12*a^2+3*a*b+5*b^2)/b^5/(tan(1/2*d*x+1/2*c)-1)+5/2*a*(4*a^2+3*b
^2)/b^6*ln(tan(1/2*d*x+1/2*c)-1)+1/3/b^3/(tan(1/2*d*x+1/2*c)+1)^3-1/2*(-3*
a+b)/b^4/(tan(1/2*d*x+1/2*c)+1)^2-1/2*(-12*a^2+3*a*b-5*b^2)/b^5/(tan(1/2*d
*x+1/2*c)+1)-5/2*a*(4*a^2+3*b^2)/b^6*ln(tan(1/2*d*x+1/2*c)+1)-2/b^6*((1/2*
b^2*(7*a^4+5*a^2*b^2-2*b^4)/a*tan(1/2*d*x+1/2*c)^3+1/2*b*(8*a^6-9*a^4*b^2-
15*a^2*b^4+2*b^6)/a^2*tan(1/2*d*x+1/2*c)^2-1/2*b^2*(25*a^4+23*a^2*b^2-2*b^
4)/a*tan(1/2*d*x+1/2*c)-4*a^4*b-7/2*a^2*b^3+1/2*b^5)/(tan(1/2*d*x+1/2*c)^2
*a-2*b*tan(1/2*d*x+1/2*c)-a)^2-5/2*(4*a^4+5*a^2*b^2+b^4)/(a^2+b^2)^(1/2)*a
rctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))))
```

3.139.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 564, normalized size of antiderivative = 1.47

$$\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$$
$$= \frac{4\,b^5+30\,(4\,a^4b+a^2b^3-b^5)\cos{(dx+c)}^4+20\,(2\,a^2b^3+b^5)\cos{(dx+c)}^2+15\,\left((4\,a^4-3\,a^2b^2-b^4)\cos{(dx}\right.}{}$$

input
```
integrate(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas"
)
```

output
```
1/12*(4*b^5 + 30*(4*a^4*b + a^2*b^3 - b^5)*cos(d*x + c)^4 + 20*(2*a^2*b^3
+ b^5)*cos(d*x + c)^2 + 15*((4*a^4 - 3*a^2*b^2 - b^4)*cos(d*x + c)^5 + 2*(
4*a^3*b + a*b^3)*cos(d*x + c)^4*sin(d*x + c) + (4*a^2*b^2 + b^4)*cos(d*x +
 c)^3)*sqrt(a^2 + b^2)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)
*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(
d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 +
 b^2)) - 15*((4*a^5 - a^3*b^2 - 3*a*b^4)*cos(d*x + c)^5 + 2*(4*a^4*b + 3*a
^2*b^3)*cos(d*x + c)^4*sin(d*x + c) + (4*a^3*b^2 + 3*a*b^4)*cos(d*x + c)^3
)*log(sin(d*x + c) + 1) + 15*((4*a^5 - a^3*b^2 - 3*a*b^4)*cos(d*x + c)^5 +
 2*(4*a^4*b + 3*a^2*b^3)*cos(d*x + c)^4*sin(d*x + c) + (4*a^3*b^2 + 3*a*b^
4)*cos(d*x + c)^3)*log(-sin(d*x + c) + 1) - 10*(a*b^4*cos(d*x + c) - 6*(3*
a^3*b^2 + 2*a*b^4)*cos(d*x + c)^3)*sin(d*x + c))/(2*a*b^7*d*cos(d*x + c)^4
*sin(d*x + c) + b^8*d*cos(d*x + c)^3 + (a^2*b^6 - b^8)*d*cos(d*x + c)^5)
```

3.139.6 Sympy [F]

$$\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = \int \frac{\sec^4{(c+dx)}}{(a\cos{(c+dx)}+b\sin{(c+dx)})^3}\,dx$$

input
```
integrate(sec(d*x+c)**4/(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)**4/(a*cos(c + d*x) + b*sin(c + d*x))**3, x)
```

3.139.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 902 vs. $2(361) = 722$.

Time = 0.33 (sec) , antiderivative size = 902, normalized size of antiderivative = 2.36

$$\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima"
)
```

output
```
1/6*(2*(60*a^6 + 35*a^4*b^2 - 3*a^2*b^4 + (210*a^5*b + 125*a^3*b^3 - 6*a*b
^5)*sin(d*x + c)/(cos(d*x + c) + 1) - 2*(120*a^6 - 10*a^4*b^2 - 55*a^2*b^4
 + 3*b^6)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 2*(330*a^5*b + 205*a^3*b^3
 - 12*a*b^5)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2*(180*a^6 - 95*a^4*b^2
 - 120*a^2*b^4 + 9*b^6)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 12*(60*a^5*b
 + 35*a^3*b^3 - 3*a*b^5)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 6*(40*a^6 -
 30*a^4*b^2 - 35*a^2*b^4 + 3*b^6)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 6*
(50*a^5*b + 25*a^3*b^3 - 4*a*b^5)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 3*
(20*a^6 - 15*a^4*b^2 - 15*a^2*b^4 + 2*b^6)*sin(d*x + c)^8/(cos(d*x + c) +
1)^8 + 3*(10*a^5*b + 5*a^3*b^3 - 2*a*b^5)*sin(d*x + c)^9/(cos(d*x + c) + 1
)^9)/(a^4*b^5 + 4*a^3*b^6*sin(d*x + c)/(cos(d*x + c) + 1) - 16*a^3*b^6*sin
(d*x + c)^3/(cos(d*x + c) + 1)^3 + 24*a^3*b^6*sin(d*x + c)^5/(cos(d*x + c)
 + 1)^5 - 16*a^3*b^6*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 4*a^3*b^6*sin(d
*x + c)^9/(cos(d*x + c) + 1)^9 - a^4*b^5*sin(d*x + c)^10/(cos(d*x + c) + 1
)^10 - (5*a^4*b^5 - 4*a^2*b^7)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 2*(5*
a^4*b^5 - 6*a^2*b^7)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 2*(5*a^4*b^5 -
6*a^2*b^7)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + (5*a^4*b^5 - 4*a^2*b^7)*s
in(d*x + c)^8/(cos(d*x + c) + 1)^8) - 15*(4*a^3 + 3*a*b^2)*log(sin(d*x + c
)/(cos(d*x + c) + 1) + 1)/b^6 + 15*(4*a^3 + 3*a*b^2)*log(sin(d*x + c)/(cos
(d*x + c) + 1) - 1)/b^6 - 15*(4*a^4 + 5*a^2*b^2 + b^4)*log((b - a*sin(d...
```

3.139.8 Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 510, normalized size of antiderivative = 1.33

$$\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx =$$

$$-\frac{\frac{15\,(4\,a^3+3\,ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^6} - \frac{15\,(4\,a^3+3\,ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^6} + \frac{15\,(4\,a^4+5\,a^2b^2+b^4)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-\cdots\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+\cdots\right|}\right.}{\sqrt{a^2+b^2}\,b^6}}{\cdots}$$

input
```
integrate(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/6*(15*(4*a^3 + 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^6 - 15*(4*
a^3 + 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^6 + 15*(4*a^4 + 5*a^2*
b^2 + b^4)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs
(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^6
) + 2*(9*a*b*tan(1/2*d*x + 1/2*c)^5 + 36*a^2*tan(1/2*d*x + 1/2*c)^4 + 18*b
^2*tan(1/2*d*x + 1/2*c)^4 - 72*a^2*tan(1/2*d*x + 1/2*c)^2 - 24*b^2*tan(1/2
*d*x + 1/2*c)^2 - 9*a*b*tan(1/2*d*x + 1/2*c) + 36*a^2 + 14*b^2)/((tan(1/2*
d*x + 1/2*c)^2 - 1)^3*b^5) + 6*(7*a^5*b*tan(1/2*d*x + 1/2*c)^3 + 5*a^3*b^3
*tan(1/2*d*x + 1/2*c)^3 - 2*a*b^5*tan(1/2*d*x + 1/2*c)^3 + 8*a^6*tan(1/2*d
*x + 1/2*c)^2 - 9*a^4*b^2*tan(1/2*d*x + 1/2*c)^2 - 15*a^2*b^4*tan(1/2*d*x
+ 1/2*c)^2 + 2*b^6*tan(1/2*d*x + 1/2*c)^2 - 25*a^5*b*tan(1/2*d*x + 1/2*c)
- 23*a^3*b^3*tan(1/2*d*x + 1/2*c) + 2*a*b^5*tan(1/2*d*x + 1/2*c) - 8*a^6 -
 7*a^4*b^2 + a^2*b^4)/((a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c
) - a)^2*a^2*b^5))/d
```

3.139.9 Mupad [B] (verification not implemented)

Time = 26.43 (sec) , antiderivative size = 1203, normalized size of antiderivative = 3.14

$$\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(c + d*x)^4*(a*cos(c + d*x) + b*sin(c + d*x))^3),x)
```

output
```
((60*a^4 - 3*b^4 + 35*a^2*b^2)/(3*b^5) + (tan(c/2 + (d*x)/2)*(210*a^4 - 6*
b^4 + 125*a^2*b^2))/(3*a*b^4) + (tan(c/2 + (d*x)/2)^8*(20*a^6 + 2*b^6 - 15
*a^2*b^4 - 15*a^4*b^2))/(a^2*b^5) - (2*tan(c/2 + (d*x)/2)^6*(40*a^6 + 3*b^
6 - 35*a^2*b^4 - 30*a^4*b^2))/(a^2*b^5) - (2*tan(c/2 + (d*x)/2)^2*(120*a^6
 + 3*b^6 - 55*a^2*b^4 - 10*a^4*b^2))/(3*a^2*b^5) + (2*tan(c/2 + (d*x)/2)^4
*(180*a^6 + 9*b^6 - 120*a^2*b^4 - 95*a^4*b^2))/(3*a^2*b^5) + (tan(c/2 + (d
*x)/2)^9*(10*a^4 - 2*b^4 + 5*a^2*b^2))/(a*b^4) - (2*tan(c/2 + (d*x)/2)^7*(
50*a^4 - 4*b^4 + 25*a^2*b^2))/(a*b^4) + (4*tan(c/2 + (d*x)/2)^5*(60*a^4 -
3*b^4 + 35*a^2*b^2))/(a*b^4) - (2*tan(c/2 + (d*x)/2)^3*(330*a^4 - 12*b^4 +
 205*a^2*b^2))/(3*a*b^4))/(d*(tan(c/2 + (d*x)/2)^8*(5*a^2 - 4*b^2) - tan(c
/2 + (d*x)/2)^2*(5*a^2 - 4*b^2) + tan(c/2 + (d*x)/2)^4*(10*a^2 - 12*b^2) -
 tan(c/2 + (d*x)/2)^6*(10*a^2 - 12*b^2) - a^2*tan(c/2 + (d*x)/2)^10 + a^2
- 16*a*b*tan(c/2 + (d*x)/2)^3 + 24*a*b*tan(c/2 + (d*x)/2)^5 - 16*a*b*tan(c
/2 + (d*x)/2)^7 + 4*a*b*tan(c/2 + (d*x)/2)^9 + 4*a*b*tan(c/2 + (d*x)/2)))
- (atanh((3000*a^2*tan(c/2 + (d*x)/2))/(3000*a^2 + (7000*a^4)/b^2 + (4000*
a^6)/b^4) + (7000*a^4*tan(c/2 + (d*x)/2))/(7000*a^4 + 3000*a^2*b^2 + (4000
*a^6)/b^2) + (4000*a^6*tan(c/2 + (d*x)/2))/(4000*a^6 + 3000*a^2*b^4 + 7000
*a^4*b^2))*(15*a*b^2 + 20*a^3))/(b^6*d) + (5*atanh((1000*a^2*(a^2 + b^2)^(
1/2))/(1000*a^2*b + (5000*a^4)/b + (4000*a^6)/b^3 + 10000*a^3*tan(c/2 + (d
*x)/2) + 2000*a*b^2*tan(c/2 + (d*x)/2) + (8000*a^5*tan(c/2 + (d*x)/2))/...
```

3.139. $\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

3.140 $\int \frac{\sec^5(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

3.140.1 Optimal result

Integrand size = 28, antiderivative size = 232

$$\int \frac{\sec^5(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = -\frac{(a^2+b^2)^3}{2a^2b^5d(b+a\cot(c+dx))^2} - \frac{(5a^2-b^2)(a^2+b^2)^2}{a^2b^6d(b+a\cot(c+dx))} + \frac{3(a^2+b^2)(5a^2+b^2)\log(b+a\cot(c+dx))}{b^7d} + \frac{3(a^2+b^2)(5a^2+b^2)\log(\tan(c+dx))}{b^7d} - \frac{a(10a^2+9b^2)\tan(c+dx)}{b^6d} + \frac{3(2a^2+b^2)\tan^2(c+dx)}{2b^5d} - \frac{a\tan^3(c+dx)}{b^4d} + \frac{\tan^4(c+dx)}{4b^3d}$$

output
```
-1/2*(a^2+b^2)^3/a^2/b^5/d/(b+a*cot(d*x+c))^2-(5*a^2-b^2)*(a^2+b^2)^2/a^2/
b^6/d/(b+a*cot(d*x+c))+3*(a^2+b^2)*(5*a^2+b^2)*ln(b+a*cot(d*x+c))/b^7/d+3*
(a^2+b^2)*(5*a^2+b^2)*ln(tan(d*x+c))/b^7/d-a*(10*a^2+9*b^2)*tan(d*x+c)/b^6
/d+3/2*(2*a^2+b^2)*tan(d*x+c)^2/b^5/d-a*tan(d*x+c)^3/b^4/d+1/4*tan(d*x+c)^
4/b^3/d
```

3.140.2 Mathematica [A] (verified)

Time = 1.43 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.17

$$\int \frac{\sec^5(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$$
$$= \frac{2(a^2+b^2)\left(19a^4+16a^2b^2-3b^4+6a^2(5a^2+b^2)\log(a+b\tan(c+dx))\right)+b^6\sec^6(c+dx)+4ab(4a^4+17a^2b^2\cdots}{\cdots}$$

input
```
Integrate[Sec[c + d*x]^5/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

output
```
(2*(a^2 + b^2)*(19*a^4 + 16*a^2*b^2 - 3*b^4 + 6*a^2*(5*a^2 + b^2)*Log[a +
b*Tan[c + d*x]]) + b^6*Sec[c + d*x]^6 + 4*a*b*(4*a^4 + 17*a^2*b^2 + 11*b^4
 + 6*(5*a^4 + 6*a^2*b^2 + b^4)*Log[a + b*Tan[c + d*x]])*Tan[c + d*x] + 4*b
^2*(-13*a^4 - 10*a^2*b^2 + 3*(5*a^4 + 6*a^2*b^2 + b^4)*Log[a + b*Tan[c + d
*x]])*Tan[c + d*x]^2 - 20*a*b^3*(a^2 + b^2)*Tan[c + d*x]^3 + 4*a^2*b^4*Tan
[c + d*x]^4 + b^4*Sec[c + d*x]^4*(a^2 + 3*b^2 - 2*a*b*Tan[c + d*x]))/(4*b^
7*d*(a + b*Tan[c + d*x])^2)
```

3.140.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3567, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\cos(c+dx)^5(a\cos(c+dx)+b\sin(c+dx))^3}dx$$
$$\Big\downarrow \text{3567}$$
$$-\frac{\int \frac{\left(\cot^2(c+dx)+1\right)^3\tan^5(c+dx)}{(b+a\cot(c+dx))^3}d\cot(c+dx)}{d}$$
$$\Big\downarrow \text{522}$$

$$-\frac{\int \left(\frac{\tan^5(c+dx)}{b^3} - \frac{3a\tan^4(c+dx)}{b^4} + \frac{3(2a^2+b^2)\tan^3(c+dx)}{b^5} + \frac{(-10a^3-9b^2a)\tan^2(c+dx)}{b^6} + \frac{3(5a^4+6b^2a^2+b^4)\tan(c+dx)}{b^7} - \frac{3a(5a^4+6b}{b^7(b+a\cot}\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{3(a^2+b^2)(5a^2+b^2)\log(\cot(c+dx))}{b^7} - \frac{3(a^2+b^2)(5a^2+b^2)\log(a\cot(c+dx)+b)}{b^7} + \frac{a(10a^2+9b^2)\tan(c+dx)}{b^6} + \frac{(5a^2-b^2)(a^2+b^2)^2}{a^2b^6(a\cot(c+dx)+b)} - \frac{3(2a^2}{}}{d}$$

input
```
Int[Sec[c + d*x]^5/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

output
```
-(((a^2 + b^2)^3/(2*a^2*b^5*(b + a*Cot[c + d*x])^2) + ((5*a^2 - b^2)*(a^2
+ b^2)^2)/(a^2*b^6*(b + a*Cot[c + d*x])) + (3*(a^2 + b^2)*(5*a^2 + b^2)*Lo
g[Cot[c + d*x]])/b^7 - (3*(a^2 + b^2)*(5*a^2 + b^2)*Log[b + a*Cot[c + d*x]
])/b^7 + (a*(10*a^2 + 9*b^2)*Tan[c + d*x])/b^6 - (3*(2*a^2 + b^2)*Tan[c +
d*x]^2)/(2*b^5) + (a*Tan[c + d*x]^3)/b^4 - Tan[c + d*x]^4/(4*b^3))/d)
```

3.140.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3567
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[x^m*((b
+ a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a,
b, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[
n, 0] && GtQ[m, 1])
```

3.140. $\int \frac{\sec^5(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

3.140.4 Maple [A] (verified)

Time = 3.79 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.84

method	result
derivativedivides	$-\frac{-\frac{-\frac{\tan(dx+c)^4 b^3}{4}+a\tan(dx+c)^3 b^2-3a^2 b\tan(dx+c)^2-\frac{3b^3\tan(dx+c)^2}{2}+10\tan(dx+c)a^3+9\tan(dx+c)a\,b^2}{b^6}+\frac{\left(15a^4+18a^2b^2+3b^4\right)\ldots}{b^7}\ldots}{d}$
default	$-\frac{-\frac{-\frac{\tan(dx+c)^4 b^3}{4}+a\tan(dx+c)^3 b^2-3a^2 b\tan(dx+c)^2-\frac{3b^3\tan(dx+c)^2}{2}+10\tan(dx+c)a^3+9\tan(dx+c)a\,b^2}{b^6}+\frac{\left(15a^4+18a^2b^2+3b^4\right)\ldots}{b^7}\ldots}{d}$
norman	$\frac{\left(360a^6+452a^4b^2+96a^2b^4-8b^6\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{a^2 d\,b^5}+\frac{\left(360a^6+452a^4b^2+96a^2b^4-8b^6\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{a^2 d\,b^5}-\frac{2\left(45a^6+54a^4b^2+9a^2b^4-b^6\right)\tan\ldots}{a^2 d\,b^5}\ldots$
risch	$-60a^4b-52a^2b^3+12b^5\mathrm{e}^{4i(dx+c)}+320ia^3b^2\mathrm{e}^{6i(dx+c)}-58ia\,b^4\mathrm{e}^{2i(dx+c)}+36ia^3b^2\mathrm{e}^{10i(dx+c)}+6ia\,b^4\mathrm{e}^{10i(dx+c)}+180ia^3b^2\mathrm{e}\ldots$
parallelrisch	$\frac{900\left(a^2+\frac{b^2}{5}\right)\left(\left(a^2+\frac{b^2}{15}\right)\cos(2dx+2c)+\frac{2\left(a^2-\frac{b^2}{3}\right)\cos(4dx+4c)}{5}+\frac{\left(a^2-b^2\right)\cos(6dx+6c)}{15}+\frac{8ab\sin(4dx+4c)}{15}+\frac{2ab\sin(6dx+6c)}{15}\ldots\right.}{\ldots}$

input
```
int(sec(d*x+c)^5/(cos(d*x+c)*a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/b^6*(-1/4*tan(d*x+c)^4*b^3+a*tan(d*x+c)^3*b^2-3*a^2*b*tan(d*x+c)^2
-3/2*b^3*tan(d*x+c)^2+10*tan(d*x+c)*a^3+9*tan(d*x+c)*a*b^2)+(15*a^4+18*a^2
*b^2+3*b^4)/b^7*ln(a+b*tan(d*x+c))-1/2/b^7*(a^6+3*a^4*b^2+3*a^2*b^4+b^6)/(
a+b*tan(d*x+c))^2+6*a/b^7*(a^4+2*a^2*b^2+b^4)/(a+b*tan(d*x+c)))
```

3.140.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 476 vs. 2(226) = 452.

Time = 0.30 (sec) , antiderivative size = 476, normalized size of antiderivative = 2.05

$$\int \frac{\sec^5(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = \frac{8\left(15\,a^4b^2+13\,a^2b^4\right)\cos\left(dx+c\right)^6+b^6-2\left(45\,a^4b^2+44\,a^2b^4+3\,b^6\right)\cos\left(dx+c\right)^4+\left(5\,a^2b^4+3\,b^6\right)\cos\left(dx\ldots\right.}{\ldots}$$

input
```
integrate(sec(d*x+c)^5/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas"
)
```

output
```
1/4*(8*(15*a^4*b^2 + 13*a^2*b^4)*cos(d*x + c)^6 + b^6 - 2*(45*a^4*b^2 + 44
*a^2*b^4 + 3*b^6)*cos(d*x + c)^4 + (5*a^2*b^4 + 3*b^6)*cos(d*x + c)^2 + 6*
((5*a^6 + a^4*b^2 - 5*a^2*b^4 - b^6)*cos(d*x + c)^6 + 2*(5*a^5*b + 6*a^3*b
^3 + a*b^5)*cos(d*x + c)^5*sin(d*x + c) + (5*a^4*b^2 + 6*a^2*b^4 + b^6)*co
s(d*x + c)^4)*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x +
c)^2 + b^2) - 6*((5*a^6 + a^4*b^2 - 5*a^2*b^4 - b^6)*cos(d*x + c)^6 + 2*(5
*a^5*b + 6*a^3*b^3 + a*b^5)*cos(d*x + c)^5*sin(d*x + c) + (5*a^4*b^2 + 6*a
^2*b^4 + b^6)*cos(d*x + c)^4)*log(cos(d*x + c)^2) - 2*(a*b^5*cos(d*x + c)
+ 2*(15*a^5*b - 2*a^3*b^3 - 13*a*b^5)*cos(d*x + c)^5 + 10*(a^3*b^3 + a*b^5
)*cos(d*x + c)^3)*sin(d*x + c))/(2*a*b^8*d*cos(d*x + c)^5*sin(d*x + c) + b
^9*d*cos(d*x + c)^4 + (a^2*b^7 - b^9)*d*cos(d*x + c)^6)
```

3.140.6 Sympy [F]

$$\int \frac{\sec^5(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = \int \frac{\sec^5\left(c+dx\right)}{\left(a\cos\left(c+dx\right)+b\sin\left(c+dx\right)\right)^3}\,dx$$

input
```
integrate(sec(d*x+c)**5/(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)**5/(a*cos(c + d*x) + b*sin(c + d*x))**3, x)
```

3.140.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1053 vs. 2(226) = 452.

Time = 0.27 (sec) , antiderivative size = 1053, normalized size of antiderivative = 4.54

$$\int \frac{\sec^5(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^5/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima"
)
```

output
```
-(2*((15*a^7 + 18*a^5*b^2 + 3*a^3*b^4 - a*b^6)*sin(d*x + c)/(cos(d*x + c)
+ 1) + (45*a^6*b + 54*a^4*b^3 + 9*a^2*b^5 - b^7)*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 - (75*a^7 + 70*a^5*b^2 - 9*a^3*b^4 - 5*a*b^6)*sin(d*x + c)^3/(c
os(d*x + c) + 1)^3 - 2*(90*a^6*b + 113*a^4*b^3 + 24*a^2*b^5 - 2*b^7)*sin(d
*x + c)^4/(cos(d*x + c) + 1)^4 + 2*(75*a^7 + 60*a^5*b^2 - 17*a^3*b^4 - 5*a
*b^6)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 2*(135*a^6*b + 172*a^4*b^3 + 3
5*a^2*b^5 - 3*b^7)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 2*(75*a^7 + 60*a^
5*b^2 - 17*a^3*b^4 - 5*a*b^6)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 2*(90*
a^6*b + 113*a^4*b^3 + 24*a^2*b^5 - 2*b^7)*sin(d*x + c)^8/(cos(d*x + c) + 1
)^8 + (75*a^7 + 70*a^5*b^2 - 9*a^3*b^4 - 5*a*b^6)*sin(d*x + c)^9/(cos(d*x
+ c) + 1)^9 + (45*a^6*b + 54*a^4*b^3 + 9*a^2*b^5 - b^7)*sin(d*x + c)^10/(c
os(d*x + c) + 1)^10 - (15*a^7 + 18*a^5*b^2 + 3*a^3*b^4 - a*b^6)*sin(d*x +
c)^11/(cos(d*x + c) + 1)^11)/(a^4*b^6 + 4*a^3*b^7*sin(d*x + c)/(cos(d*x +
c) + 1) - 20*a^3*b^7*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 40*a^3*b^7*sin(
d*x + c)^5/(cos(d*x + c) + 1)^5 - 40*a^3*b^7*sin(d*x + c)^7/(cos(d*x + c)
+ 1)^7 + 20*a^3*b^7*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 4*a^3*b^7*sin(d*
x + c)^11/(cos(d*x + c) + 1)^11 + a^4*b^6*sin(d*x + c)^12/(cos(d*x + c) +
1)^12 - 2*(3*a^4*b^6 - 2*a^2*b^8)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + (1
5*a^4*b^6 - 16*a^2*b^8)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 4*(5*a^4*b^6
 - 6*a^2*b^8)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + (15*a^4*b^6 - 16*a^...
```

3.140.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.05

$$\int \frac{\sec^5(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = \frac{\frac{12\,(5\,a^4+6\,a^2b^2+b^4)\log(|b\tan(dx+c)+a|)}{b^7} - \frac{2\left(45\,a^4b^2\tan(dx+c)^2+54\,a^2b^4\tan(dx+c)^2+9\,b^6\tan(dx+c)^2+78\,a^5b\tan(dx+c)+84\,a^3b^3\tan(dx\cdots\right.}{(b\tan(dx+c)+a)^2b^7}}{\cdots}$$

input
```
integrate(sec(d*x+c)^5/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
1/4*(12*(5*a^4 + 6*a^2*b^2 + b^4)*log(abs(b*tan(d*x + c) + a))/b^7 - 2*(45
*a^4*b^2*tan(d*x + c)^2 + 54*a^2*b^4*tan(d*x + c)^2 + 9*b^6*tan(d*x + c)^2
 + 78*a^5*b*tan(d*x + c) + 84*a^3*b^3*tan(d*x + c) + 6*a*b^5*tan(d*x + c)
+ 34*a^6 + 33*a^4*b^2 + b^6)/((b*tan(d*x + c) + a)^2*b^7) + (b^9*tan(d*x +
 c)^4 - 4*a*b^8*tan(d*x + c)^3 + 12*a^2*b^7*tan(d*x + c)^2 + 6*b^9*tan(d*x
 + c)^2 - 40*a^3*b^6*tan(d*x + c) - 36*a*b^8*tan(d*x + c))/b^12)/d
```

3.140.9 Mupad [B] (verification not implemented)

Time = 30.62 (sec) , antiderivative size = 1712, normalized size of antiderivative = 7.38

$$\int \frac{\sec^5(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(c + d*x)^5*(a*cos(c + d*x) + b*sin(c + d*x))^3),x)
```

output
```
- ((2*tan(c/2 + (d*x)/2)*(15*a^6 - b^6 + 3*a^2*b^4 + 18*a^4*b^2))/(a*b^6)
- (2*tan(c/2 + (d*x)/2)^11*(15*a^6 - b^6 + 3*a^2*b^4 + 18*a^4*b^2))/(a*b^6
) + (2*tan(c/2 + (d*x)/2)^2*(45*a^6 - b^6 + 9*a^2*b^4 + 54*a^4*b^2))/(a^2*
b^5) + (2*tan(c/2 + (d*x)/2)^10*(45*a^6 - b^6 + 9*a^2*b^4 + 54*a^4*b^2))/(
a^2*b^5) - (2*tan(c/2 + (d*x)/2)^3*(75*a^6 - 5*b^6 - 9*a^2*b^4 + 70*a^4*b^
2))/(a*b^6) + (4*tan(c/2 + (d*x)/2)^5*(75*a^6 - 5*b^6 - 17*a^2*b^4 + 60*a^
4*b^2))/(a*b^6) - (4*tan(c/2 + (d*x)/2)^7*(75*a^6 - 5*b^6 - 17*a^2*b^4 + 6
0*a^4*b^2))/(a*b^6) + (2*tan(c/2 + (d*x)/2)^9*(75*a^6 - 5*b^6 - 9*a^2*b^4
+ 70*a^4*b^2))/(a*b^6) - (4*tan(c/2 + (d*x)/2)^4*(90*a^6 - 2*b^6 + 24*a^2*
b^4 + 113*a^4*b^2))/(a^2*b^5) - (4*tan(c/2 + (d*x)/2)^8*(90*a^6 - 2*b^6 +
24*a^2*b^4 + 113*a^4*b^2))/(a^2*b^5) + (4*tan(c/2 + (d*x)/2)^6*(135*a^6 -
3*b^6 + 35*a^2*b^4 + 172*a^4*b^2))/(a^2*b^5))/(d*(tan(c/2 + (d*x)/2)^4*(15
*a^2 - 16*b^2) - tan(c/2 + (d*x)/2)^10*(6*a^2 - 4*b^2) - tan(c/2 + (d*x)/2
)^2*(6*a^2 - 4*b^2) + tan(c/2 + (d*x)/2)^8*(15*a^2 - 16*b^2) - tan(c/2 + (
d*x)/2)^6*(20*a^2 - 24*b^2) + a^2*tan(c/2 + (d*x)/2)^12 + a^2 - 20*a*b*tan
(c/2 + (d*x)/2)^3 + 40*a*b*tan(c/2 + (d*x)/2)^5 - 40*a*b*tan(c/2 + (d*x)/2
)^7 + 20*a*b*tan(c/2 + (d*x)/2)^9 - 4*a*b*tan(c/2 + (d*x)/2)^11 + 4*a*b*ta
n(c/2 + (d*x)/2))) - (atan((((5*a^2 + b^2)*(a^2 + b^2)*((16*tan(c/2 + (d*x
)/2)*(15*a^6 + 3*a^2*b^4 + 18*a^4*b^2))/b^6 - (4*(6*a*b^11 + 36*a^3*b^9 +
30*a^5*b^7))/b^12 + (4*tan(c/2 + (d*x)/2)^2*(6*a*b^11 + 36*a^3*b^9 + 30...
```

3.141 $\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

3.141.1 Optimal result

Integrand size = 28, antiderivative size = 165

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = \frac{(a^4-6a^2b^2+b^4)\,x}{(a^2+b^2)^4} + \frac{4ab(a^2-b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^4\,d} - \frac{b}{3\,(a^2+b^2)\,d(a+b\tan(c+dx))^3} - \frac{ab}{(a^2+b^2)^2\,d(a+b\tan(c+dx))^2} - \frac{b(3a^2-b^2)}{(a^2+b^2)^3\,d(a+b\tan(c+dx))}$$

output

```
(a^4-6*a^2*b^2+b^4)*x/(a^2+b^2)^4+4*a*b*(a^2-b^2)*ln(a*cos(d*x+c)+b*sin(d*
x+c))/(a^2+b^2)^4/d-1/3*b/(a^2+b^2)/d/(a+b*tan(d*x+c))^3-a*b/(a^2+b^2)^2/d
/(a+b*tan(d*x+c))^2-b*(3*a^2-b^2)/(a^2+b^2)^3/d/(a+b*tan(d*x+c))
```

3.141.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.10 (sec) , antiderivative size = 419, normalized size of antiderivative = 2.54

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$$
$$= \frac{(a^2-2ab-b^2)(a^2+2ab-b^2)(c+dx)}{(a-ib)^4(a+ib)^4d}$$
$$+ \frac{4(ia^{10}b+a^9b^2+2ia^8b^3+2a^7b^4-2ia^4b^7-2a^3b^8-ia^2b^9-ab^{10})(c+dx)}{(a-ib)^8(a+ib)^7d}$$
$$- \frac{4i(a^3b-ab^3)\arctan(\tan(c+dx))}{(a^2+b^2)^4d} + \frac{2(a^3b-ab^3)\log\left((a\cos(c+dx)+b\sin(c+dx))^2\right)}{(a^2+b^2)^4d}$$
$$+ \frac{b^4\sin(c+dx)}{3a(a-ib)^2(a+ib)^2d(a\cos(c+dx)+b\sin(c+dx))^3}$$
$$- \frac{b^3(6a^2+b^2)}{3a(a-ib)^3(a+ib)^3d(a\cos(c+dx)+b\sin(c+dx))^2}$$
$$+ \frac{2(9a^2b^2\sin(c+dx)-2b^4\sin(c+dx))}{3a(a-ib)^3(a+ib)^3d(a\cos(c+dx)+b\sin(c+dx))}$$

input
```
Integrate[Cos[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

output
```
((a^2 - 2*a*b - b^2)*(a^2 + 2*a*b - b^2)*(c + d*x))/((a - I*b)^4*(a + I*b)
^4*d) + (4*(I*a^10*b + a^9*b^2 + (2*I)*a^8*b^3 + 2*a^7*b^4 - (2*I)*a^4*b^7
 - 2*a^3*b^8 - I*a^2*b^9 - a*b^10)*(c + d*x))/((a - I*b)^8*(a + I*b)^7*d)
- ((4*I)*(a^3*b - a*b^3)*ArcTan[Tan[c + d*x]])/((a^2 + b^2)^4*d) + (2*(a^3
*b - a*b^3)*Log[(a*Cos[c + d*x] + b*Sin[c + d*x])^2])/((a^2 + b^2)^4*d) +
(b^4*Sin[c + d*x])/(3*a*(a - I*b)^2*(a + I*b)^2*d*(a*Cos[c + d*x] + b*Sin[
c + d*x])^3) - (b^3*(6*a^2 + b^2))/(3*a*(a - I*b)^3*(a + I*b)^3*d*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^2) + (2*(9*a^2*b^2*Sin[c + d*x] - 2*b^4*Sin[c +
d*x]))/(3*a*(a - I*b)^3*(a + I*b)^3*d*(a*Cos[c + d*x] + b*Sin[c + d*x]))
```

3.141.3 Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.20, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3565, 3042, 3964, 3042, 4012, 3042, 4012, 3042, 4014, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(c+dx)^4}{(a\cos(c+dx)+b\sin(c+dx))^4}dx$$

$$\Big\downarrow \text{3565}$$

$$\int \frac{1}{(a+b\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+b\tan(c+dx))^4}dx$$

$$\Big\downarrow \text{3964}$$

$$\frac{\int \frac{a-b\tan(c+dx)}{(a+b\tan(c+dx))^3}dx}{a^2+b^2}-\frac{b}{3d(a^2+b^2)(a+b\tan(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{a-b\tan(c+dx)}{(a+b\tan(c+dx))^3}dx}{a^2+b^2}-\frac{b}{3d(a^2+b^2)(a+b\tan(c+dx))^3}$$

$$\Big\downarrow \text{4012}$$

$$\frac{\frac{\int \frac{a^2-2b\tan(c+dx)a-b^2}{(a+b\tan(c+dx))^2}dx}{a^2+b^2}-\frac{ab}{d(a^2+b^2)(a+b\tan(c+dx))^2}}{a^2+b^2}-\frac{b}{3d(a^2+b^2)(a+b\tan(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{a^2-2b\tan(c+dx)a-b^2}{(a+b\tan(c+dx))^2}dx}{a^2+b^2}-\frac{ab}{d(a^2+b^2)(a+b\tan(c+dx))^2}}{a^2+b^2}-\frac{b}{3d(a^2+b^2)(a+b\tan(c+dx))^3}$$

$$\Big\downarrow \text{4012}$$

$$\frac{\frac{\frac{\int \frac{a(a^2-3b^2)-b(3a^2-b^2)\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}-\frac{b(3a^2-b^2)}{d(a^2+b^2)(a+b\tan(c+dx))}}{a^2+b^2}-\frac{ab}{d(a^2+b^2)(a+b\tan(c+dx))^2}}{a^2+b^2}-\frac{b}{3d\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{\frac{\int \frac{a(a^2-3b^2)-b(3a^2-b^2)\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}-\frac{b(3a^2-b^2)}{d(a^2+b^2)(a+b\tan(c+dx))}}{a^2+b^2}-\frac{ab}{d(a^2+b^2)(a+b\tan(c+dx))^2}}{a^2+b^2}-\frac{b}{3d\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)^3}$$

$$\Big\downarrow \text{ 4014}$$

$$\frac{\frac{\frac{\frac{4ab(a^2-b^2)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{x(a^4-6a^2b^2+b^4)}{a^2+b^2}}{a^2+b^2}-\frac{b(3a^2-b^2)}{d(a^2+b^2)(a+b\tan(c+dx))}}{a^2+b^2}-\frac{ab}{d(a^2+b^2)(a+b\tan(c+dx))^2}}{a^2+b^2}-\frac{b}{3d\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{\frac{\frac{4ab(a^2-b^2)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}dx}{a^2+b^2}+\frac{x(a^4-6a^2b^2+b^4)}{a^2+b^2}}{a^2+b^2}-\frac{b(3a^2-b^2)}{d(a^2+b^2)(a+b\tan(c+dx))}}{a^2+b^2}-\frac{ab}{d(a^2+b^2)(a+b\tan(c+dx))^2}}{a^2+b^2}-\frac{b}{3d\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)^3}$$

$$\Big\downarrow \text{ 4013}$$

$$\frac{\frac{\frac{\frac{4ab(a^2-b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{d(a^2+b^2)}+\frac{x(a^4-6a^2b^2+b^4)}{a^2+b^2}}{a^2+b^2}-\frac{b(3a^2-b^2)}{d(a^2+b^2)(a+b\tan(c+dx))}}{a^2+b^2}-\frac{ab}{d(a^2+b^2)(a+b\tan(c+dx))^2}}{a^2+b^2}-\frac{b}{3d\left(a^2+b^2\right)\left(a+b\tan(c+dx)\right)^3}$$

input
```
Int[Cos[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

3.141. $\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

output
```
-1/3*b/((a^2 + b^2)*d*(a + b*Tan[c + d*x])^3) + (-((a*b)/((a^2 + b^2)*d*(a
 + b*Tan[c + d*x])^2)) + ((((a^4 - 6*a^2*b^2 + b^4)*x)/(a^2 + b^2) + (4*a*
b*(a^2 - b^2)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)*d))/(a^2
+ b^2) - (b*(3*a^2 - b^2))/((a^2 + b^2)*d*(a + b*Tan[c + d*x])))/(a^2 + b^
2))/(a^2 + b^2)
```

3.141.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3565
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(a + b*Tan[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

rule 3964
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)
 Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

3.141. $\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

3.141.4 Maple [A] (verified)

Time = 1.71 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.11

method	result
derivativedivides	$\frac{\frac{\frac{\left(-4a^3b+4a\,b^3\right)\ln\left(1+\tan(dx+c)^2\right)}{2}+\left(a^4-6a^2b^2+b^4\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^4}-\frac{b}{3\left(a^2+b^2\right)(a+b\tan(dx+c))^3}-\frac{b\left(3a^2-b^2\right)}{\left(a^2+b^2\right)^3(a+b\tan(dx+c}}{d}$
default	$\frac{\frac{\frac{\left(-4a^3b+4a\,b^3\right)\ln\left(1+\tan(dx+c)^2\right)}{2}+\left(a^4-6a^2b^2+b^4\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^4}-\frac{b}{3\left(a^2+b^2\right)(a+b\tan(dx+c))^3}-\frac{b\left(3a^2-b^2\right)}{\left(a^2+b^2\right)^3(a+b\tan(dx+c}}{d}$
risch	$-\frac{x}{4ia^3b-4ia\,b^3-a^4+6a^2b^2-b^4}-\frac{8ia^3bx}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8}+\frac{8ia\,b^3x}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8}-\frac{8i}{d(a^8+4a^6b^2+6}$
parallelrisch	$\frac{36b\left(\left(\frac{1}{3}a^3-a\,b^2\right)\cos(3dx+3c)+\left(a^2b-\frac{1}{3}b^3\right)\sin(3dx+3c)+\left(a^2+b^2\right)(\cos(dx+c)a+b\sin(dx+c))\right)a^4(a+b)(a-b)\ln\left(\tan\left(\frac{dx}{2}\right.\right.}{}$
norman	Expression too large to display

input
```
int(cos(d*x+c)^4/(cos(d*x+c)*a+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/(a^2+b^2)^4*(1/2*(-4*a^3*b+4*a*b^3)*ln(1+tan(d*x+c)^2)+(a^4-6*a^2*b
^2+b^4)*arctan(tan(d*x+c)))-1/3*b/(a^2+b^2)/(a+b*tan(d*x+c))^3-b*(3*a^2-b^
2)/(a^2+b^2)^3/(a+b*tan(d*x+c))-a*b/(a^2+b^2)^2/(a+b*tan(d*x+c))^2+4*a*b*(
a^2-b^2)/(a^2+b^2)^4*ln(a+b*tan(d*x+c)))
```

3.141.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 575 vs. $2(163) = 326$.

Time = 0.28 (sec) , antiderivative size = 575, normalized size of antiderivative = 3.48

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx =$$
$$-\frac{\left(54\,a^4b^3-30\,a^2b^5+4\,b^7-3\,(a^7-9\,a^5b^2+19\,a^3b^4-3\,ab^6)dx\right)\cos\left(dx+c\right)^3-3\left(10\,a^4b^3-11\,a^2b^5+b^7\right.}{}$$

input
```
integrate(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas"
)
```

output
```
-1/3*((54*a^4*b^3 - 30*a^2*b^5 + 4*b^7 - 3*(a^7 - 9*a^5*b^2 + 19*a^3*b^4 -
 3*a*b^6)*d*x)*cos(d*x + c)^3 - 3*(10*a^4*b^3 - 11*a^2*b^5 + b^7 + 3*(a^5*
b^2 - 6*a^3*b^4 + a*b^6)*d*x)*cos(d*x + c) - 6*((a^6*b - 4*a^4*b^3 + 3*a^2
*b^5)*cos(d*x + c)^3 + 3*(a^4*b^3 - a^2*b^5)*cos(d*x + c) + (a^3*b^4 - a*b
^6 + (3*a^5*b^2 - 4*a^3*b^4 + a*b^6)*cos(d*x + c)^2)*sin(d*x + c))*log(2*a
*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - (13*a^3
*b^4 - 9*a*b^6 + 3*(a^4*b^3 - 6*a^2*b^5 + b^7)*d*x + (18*a^5*b^2 - 58*a^3*
b^4 + 12*a*b^6 + 3*(3*a^6*b - 19*a^4*b^3 + 9*a^2*b^5 - b^7)*d*x)*cos(d*x +
 c)^2)*sin(d*x + c))/((a^11 + a^9*b^2 - 6*a^7*b^4 - 14*a^5*b^6 - 11*a^3*b^
8 - 3*a*b^10)*d*cos(d*x + c)^3 + 3*(a^9*b^2 + 4*a^7*b^4 + 6*a^5*b^6 + 4*a^
3*b^8 + a*b^10)*d*cos(d*x + c) + ((3*a^10*b + 11*a^8*b^3 + 14*a^6*b^5 + 6*
a^4*b^7 - a^2*b^9 - b^11)*d*cos(d*x + c)^2 + (a^8*b^3 + 4*a^6*b^5 + 6*a^4*
b^7 + 4*a^2*b^9 + b^11)*d)*sin(d*x + c))
```

3.141.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**4/(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

output
```
Timed out
```

3.141.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(163) = 326$.

Time = 0.33 (sec) , antiderivative size = 385, normalized size of antiderivative = 2.33

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$$
$$= \frac{\frac{3\,(a^4-6\,a^2b^2+b^4)\,(dx+c)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8} + \frac{12\,(a^3b-ab^3)\,\log(b\tan(dx+c)+a)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8} - \frac{6\,(a^3b-ab^3)\,\log\left(\tan(dx+c)^2+1\right)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8} - \frac{}{a^9+3\,a^7b^2+3\,a^5b^4+a^3b^6+(a^6b^3}}{3\,d}$$

input
```
integrate(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima"
)
```

output
```
1/3*(3*(a^4 - 6*a^2*b^2 + b^4)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*
a^2*b^6 + b^8) + 12*(a^3*b - a*b^3)*log(b*tan(d*x + c) + a)/(a^8 + 4*a^6*b
^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - 6*(a^3*b - a*b^3)*log(tan(d*x + c)^2 +
 1)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - (13*a^4*b + 2*a^2*b^
3 + b^5 + 3*(3*a^2*b^3 - b^5)*tan(d*x + c)^2 + 3*(7*a^3*b^2 - a*b^4)*tan(d
*x + c))/(a^9 + 3*a^7*b^2 + 3*a^5*b^4 + a^3*b^6 + (a^6*b^3 + 3*a^4*b^5 + 3
*a^2*b^7 + b^9)*tan(d*x + c)^3 + 3*(a^7*b^2 + 3*a^5*b^4 + 3*a^3*b^6 + a*b^
8)*tan(d*x + c)^2 + 3*(a^8*b + 3*a^6*b^3 + 3*a^4*b^5 + a^2*b^7)*tan(d*x +
c)))/d
```

3.141.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(163) = 326$.

Time = 0.38 (sec) , antiderivative size = 370, normalized size of antiderivative = 2.24

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$$
$$= \frac{\frac{3\,(a^4-6\,a^2b^2+b^4)(dx+c)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8} - \frac{6\,(a^3b-ab^3)\log\left(\tan(dx+c)^2+1\right)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8} + \frac{12\,(a^3b^2-ab^4)\log(|b\tan(dx+c)+a|)}{a^8b+4\,a^6b^3+6\,a^4b^5+4\,a^2b^7+b^9} - \frac{22\,a^3b^4\tan(dx+c)^3-22\,ab^6\tan}{}}{}$$

input
```
integrate(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

output
```
1/3*(3*(a^4 - 6*a^2*b^2 + b^4)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*
a^2*b^6 + b^8) - 6*(a^3*b - a*b^3)*log(tan(d*x + c)^2 + 1)/(a^8 + 4*a^6*b^
2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) + 12*(a^3*b^2 - a*b^4)*log(abs(b*tan(d*x
+ c) + a))/(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9) - (22*a^3*b^4
*tan(d*x + c)^3 - 22*a*b^6*tan(d*x + c)^3 + 75*a^4*b^3*tan(d*x + c)^2 - 60
*a^2*b^5*tan(d*x + c)^2 - 3*b^7*tan(d*x + c)^2 + 87*a^5*b^2*tan(d*x + c) -
 48*a^3*b^4*tan(d*x + c) - 3*a*b^6*tan(d*x + c) + 35*a^6*b - 7*a^4*b^3 + 3
*a^2*b^5 + b^7)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*(b*tan(d*
x + c) + a)^3))/d
```

3.141.9 Mupad [B] (verification not implemented)

Time = 37.65 (sec) , antiderivative size = 8586, normalized size of antiderivative = 52.04

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
int(cos(c + d*x)^4/(a*cos(c + d*x) + b*sin(c + d*x))^4,x)
```

output
```
((4*tan(c/2 + (d*x)/2)^2*(b^7 + 3*a^2*b^5 + 10*a^4*b^3))/(a^2*(a^6 + b^6 +
 3*a^2*b^4 + 3*a^4*b^2)) - (4*tan(c/2 + (d*x)/2)^4*(b^7 + 3*a^2*b^5 + 10*a
^4*b^3))/(a^2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (2*b*tan(c/2 + (d*x)/
2)^5*(6*a^4*b + b^5 + 3*a^2*b^3))/(a*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))
+ (4*b*tan(c/2 + (d*x)/2)^3*(2*b^7 - 18*a^6*b + a^2*b^5 + 17*a^4*b^3))/(3*
a^3*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (2*b*tan(c/2 + (d*x)/2)*(6*a^4*
b + b^5 + 3*a^2*b^3))/(a*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))/(d*(tan(c/2
 + (d*x)/2)^2*(12*a*b^2 - 3*a^3) - a^3*tan(c/2 + (d*x)/2)^6 - tan(c/2 + (d
*x)/2)^4*(12*a*b^2 - 3*a^3) - tan(c/2 + (d*x)/2)^3*(12*a^2*b - 8*b^3) + a^
3 + 6*a^2*b*tan(c/2 + (d*x)/2) + 6*a^2*b*tan(c/2 + (d*x)/2)^5)) - (log(a +
 2*b*tan(c/2 + (d*x)/2) - a*tan(c/2 + (d*x)/2)^2)*(4*a*b^3 - 4*a^3*b))/(d*
(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)) + (log((((-(a^4 + b^4 - 6
*a^2*b^2)^2/(a^2 + b^2)^8)^(1/2) - (4*a*b*(a^2 - b^2))/(a^2 + b^2)^4)*(((-
(a^4 + b^4 - 6*a^2*b^2)^2/(a^2 + b^2)^8)^(1/2) - (4*a*b*(a^2 - b^2))/(a^2
+ b^2)^4)*((32*a*(a^6 - b^6 + 11*a^2*b^4 - 11*a^4*b^2))/(a^2 + b^2)^3 + 96
*a*b*((-(a^4 + b^4 - 6*a^2*b^2)^2/(a^2 + b^2)^8)^(1/2) - (4*a*b*(a^2 - b^2
))/(a^2 + b^2)^4)*(a + b*tan(c/2 + (d*x)/2))*(a^2 + b^2) - (64*a^2*b*tan(c
/2 + (d*x)/2)*(b^4 - 5*a^4 + 8*a^2*b^2))/(a^2 + b^2)^3) - (32*a^2*b*(7*a^4
 + 7*b^4 - 18*a^2*b^2))/(a^2 + b^2)^5 + (32*a*tan(c/2 + (d*x)/2)*(a^8 + 2*
b^8 - 57*a^2*b^6 + 105*a^4*b^4 - 27*a^6*b^2))/(a^2 + b^2)^6) + (128*a^3...
```

3.142 $\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

3.142.1 Optimal result

Integrand size = 28, antiderivative size = 157

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = \frac{a(2a^2-3b^2)\,\text{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}\,d} + \frac{-3(3a^4b-a^2b^3+b^5)\cos(2(c+dx))+\frac{1}{2}b(-9a^2+b^2)\left(2(a^2+b^2)+3ab\sin(2(c+dx))\right)}{6\,(a^2+b^2)^3\,d(a\cos(c+dx)+b\sin(c+dx))^3}$$

output
```
a*(2*a^2-3*b^2)*arctanh((-b+a*tan(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/(a^2+b^
2)^(7/2)/d+1/6*(-3*(3*a^4*b-a^2*b^3+b^5)*cos(2*d*x+2*c)+1/2*b*(-9*a^2+b^2)
*(2*a^2+2*b^2+3*a*b*sin(2*d*x+2*c)))/(a^2+b^2)^3/d/(a*cos(d*x+c)+b*sin(d*x
+c))^3
```

3.142.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.06 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.05

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = \frac{\frac{6a(2a^2-3b^2)\text{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}} + \frac{-3(3a^4b-a^2b^3+b^5)\cos(2(c+dx))+\frac{1}{2}b(-9a^2+b^2)\left(2(a^2+b^2)+3ab\sin(2(c+dx))\right)}{(a-ib)^3(a+ib)^3(a\cos(c+dx)+b\sin(c+dx))^3}}{6d}$$

input
```
Integrate[Cos[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

output
```
((6*a*(2*a^2 - 3*b^2)*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/
(a^2 + b^2)^(7/2) + (-3*(3*a^4*b - a^2*b^3 + b^5)*Cos[2*(c + d*x)] + (b*(-
9*a^2 + b^2)*(2*(a^2 + b^2) + 3*a*b*Sin[2*(c + d*x)]))/2)/((a - I*b)^3*(a
+ I*b)^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^3))/(6*d)
```

3.142.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 391 vs. $2(157) = 314$.

Time = 1.12 (sec) , antiderivative size = 391, normalized size of antiderivative = 2.49, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 4902, 2191, 27, 2191, 27, 2191, 27, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(c+dx)^3}{(a\cos(c+dx)+b\sin(c+dx))^4}dx$$

$$\Big\downarrow \text{4902}$$

$$\frac{2\int \frac{\left(1-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^3}{\left(-a\tan^2\left(\frac{1}{2}(c+dx)\right)+2b\tan\left(\frac{1}{2}(c+dx)\right)+a\right)^4}d\tan\left(\frac{1}{2}(c+dx)\right)}{d}$$

$$\Big\downarrow \text{2191}$$

$$\frac{2\left(-\frac{\int -\frac{4\left(3\left(\frac{b^2}{a}+a\right)\tan^4\left(\frac{1}{2}(c+dx)\right)+6b\left(\frac{b^2}{a^2}+1\right)\tan^3\left(\frac{1}{2}(c+dx)\right)-6\left(-\frac{2b^4}{a^3}-\frac{b^2}{a}+a\right)\tan^2\left(\frac{1}{2}(c+dx)\right)-6b\left(-\frac{4b^4}{a^4}-\frac{3b^2}{a^2}+1\right)\tan\left(\frac{1}{2}(c+dx)\right)+\frac{3a^6+3b^2a^4-12b}{a^5}}{\left(-a\tan^2\left(\frac{1}{2}(c+dx)\right)+2b\tan\left(\frac{1}{2}(c+dx)\right)+a\right)^3}}{12(a^2+b^2)}\right.}{d}$$

$$\Big\downarrow \text{27}$$

$$2\left(\frac{\int \frac{-\frac{32b^6}{a^5}-\frac{12b^4}{a^3}+\frac{3b^2}{a}+6\left(\frac{b^2}{a^2}+1\right)\tan^3\left(\frac{1}{2}(c+dx)\right)b-6\left(-\frac{4b^4}{a^4}-\frac{3b^2}{a^2}+1\right)\tan\left(\frac{1}{2}(c+dx)\right)b+3\left(\frac{b^2}{a}+a\right)\tan^4\left(\frac{1}{2}(c+dx)\right)-6\left(-\frac{2b^4}{a^3}-\frac{b^2}{a}+a\right)\tan^2\left(\frac{1}{2}(c+dx)\right)+3a}{\left(-a\tan^2\left(\frac{1}{2}(c+dx)\right)+2b\tan\left(\frac{1}{2}(c+dx)\right)+a\right)^3}}{3(a^2+b^2)}\right.$$
$$\overline{\qquad\qquad d\qquad\qquad}$$

$$\Big\downarrow \; 2191$$

$$2\left(\frac{\frac{b^2\left(a\left(9a^4+30a^2b^2+16b^4\right)\tan\left(\frac{1}{2}(c+dx)\right)+b\left(15a^4+18a^2b^2+8b^4\right)\right)}{a^5\left(a^2+b^2\right)\left(-a\tan^2\left(\frac{1}{2}(c+dx)\right)+a+2b\tan\left(\frac{1}{2}(c+dx)\right)\right)^2}-\frac{\int -\frac{24\left(-\frac{\tan^2\left(\frac{1}{2}(c+dx)\right)\left(a^2+b^2\right)^2}{a^2}-\frac{4b\tan\left(\frac{1}{2}(c+dx)\right)\left(a^2+b^2\right)^2}{a^3}+\frac{a^6-b^2a^4+7b^4a^2+}{a^4}\right.}{\left(-a\tan^2\left(\frac{1}{2}(c+dx)\right)+2b\tan\left(\frac{1}{2}(c+dx)\right)+a\right)^2}}{8\left(a^2+b^2\right)}}{3(a^2+b^2)}\right.$$
$$\overline{\qquad\qquad d\qquad\qquad}$$

$$\Big\downarrow \; 27$$

$$2\left(\frac{\frac{3\int \frac{\frac{4b^6}{a^4}+\frac{7b^4}{a^2}-b^2-\frac{4\left(a^2+b^2\right)^2\tan\left(\frac{1}{2}(c+dx)\right)b}{a^3}+a^2-\frac{\left(a^2+b^2\right)^2\tan^2\left(\frac{1}{2}(c+dx)\right)}{a^2}}{\left(-a\tan^2\left(\frac{1}{2}(c+dx)\right)+2b\tan\left(\frac{1}{2}(c+dx)\right)+a\right)^2}d\tan\left(\frac{1}{2}(c+dx)\right)}{a^2+b^2}+\frac{b^2\left(a\left(9a^4+30a^2b^2+16b^4\right)\tan\left(\frac{1}{2}(c+dx)\right)+b\left(15a^4+18a^2\right.\right.}{a^5\left(a^2+b^2\right)\left(-a\tan^2\left(\frac{1}{2}(c+dx)\right)+a+2b\tan\left(\frac{1}{2}(c+d\right.\right.}}{3(a^2+b^2)}\right.$$
$$\overline{\qquad\qquad d\qquad\qquad}$$

$$\Big\downarrow \; 2191$$

$$2\left(\frac{\frac{3\left(-\frac{\int -\frac{2a\left(2a^2-3b^2\right)}{-a\tan^2\left(\frac{1}{2}(c+dx)\right)+2b\tan\left(\frac{1}{2}(c+dx)\right)+a}d\tan\left(\frac{1}{2}(c+dx)\right)}{4\left(a^2+b^2\right)}-\frac{b\left(b\left(9a^4+6a^2b^2+2b^4\right)\tan\left(\frac{1}{2}(c+dx)\right)+a^3\left(\frac{4b^6}{a^4}+\frac{12b^4}{a^2}+6a^2+9b^2\right)\right)}{2a^3\left(a^2+b^2\right)\left(-a\tan^2\left(\frac{1}{2}(c+dx)\right)+a+2b\tan\left(\frac{1}{2}(c+dx)\right)\right)}\right)}{a^2+b^2}+\frac{b^2\left(a\left(9a^4+30\right.\right.}{a^5\left(a^2+b\right.}}{3(a^2+b^2)}\right.$$
$$\overline{\qquad\qquad d\qquad\qquad}$$

$$\Big\downarrow \; 27$$

3.142. $\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

$$2\left(\frac{\frac{3\left(\frac{a\left(2a^2-3b^2\right)\int \frac{1}{-a\tan^2\left(\frac{1}{2}(c+dx)\right)+2b\tan\left(\frac{1}{2}(c+dx)\right)+a}d\tan\left(\frac{1}{2}(c+dx)\right)}{2\left(a^2+b^2\right)}-\frac{b\left(b\left(9a^4+6a^2b^2+2b^4\right)\tan\left(\frac{1}{2}(c+dx)\right)+a^3\left(\frac{4b^6}{a^4}+\frac{12b^4}{a^2}+6a^2+9b^2\right)\right)}{2a^3\left(a^2+b^2\right)\left(-a\tan^2\left(\frac{1}{2}(c+dx)\right)+a+2b\tan\left(\frac{1}{2}(c+dx)\right)\right)}\right)}{a^2+b^2}+\frac{b^2(a}{}}{3(a^2+b^2)}\right.$$
$$\overline{\qquad\qquad d\qquad\qquad}$$

$\big\downarrow$ 1083

$$2\left(\frac{\frac{3\left(-\frac{a\left(2a^2-3b^2\right)\int \frac{1}{4\left(a^2+b^2\right)-\left(2b-2a\tan\left(\frac{1}{2}(c+dx)\right)\right)^2}d\left(2b-2a\tan\left(\frac{1}{2}(c+dx)\right)\right)}{a^2+b^2}-\frac{b\left(b\left(9a^4+6a^2b^2+2b^4\right)\tan\left(\frac{1}{2}(c+dx)\right)+a^3\left(\frac{4b^6}{a^4}+\frac{12b^4}{a^2}+6a^2+9b^2\right)\right)}{2a^3\left(a^2+b^2\right)\left(-a\tan^2\left(\frac{1}{2}(c+dx)\right)+a+2b\tan\left(\frac{1}{2}(c+dx)\right)\right)}\right)}{a^2+b^2}}{3(a^2+b^2)}\right.$$
$$\overline{\qquad\qquad d\qquad\qquad}$$

$\big\downarrow$ 219

$$2\left(\frac{\frac{b^2\left(a\left(9a^4+30a^2b^2+16b^4\right)\tan\left(\frac{1}{2}(c+dx)\right)+b\left(15a^4+18a^2b^2+8b^4\right)\right)}{a^5\left(a^2+b^2\right)\left(-a\tan^2\left(\frac{1}{2}(c+dx)\right)+a+2b\tan\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{3\left(-\frac{a\left(2a^2-3b^2\right)\text{arctanh}\left(\frac{2b-2a\tan\left(\frac{1}{2}(c+dx)\right)}{2\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{3/2}}-\frac{b\left(b\left(9a^4+6a^2b^2+2b^4\right)\tan\left(\frac{1}{2}(\right.\right.}{2a^3\left(a^2+b^2\right)\left(-a\tan^2\left(\frac{1}{2}\right.\right.}\right.}{a^2+b^2}}{3(a^2+b^2)}\right.$$
$$\overline{\qquad\qquad d\qquad\qquad}$$

input
```
Int[Cos[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

3.142. $\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

output
```
(2*((-4*b^3*(a*(a^2 + 2*b^2) + b*(3*a^2 + 4*b^2)*Tan[(c + d*x)/2]))/(3*a^5
*(a^2 + b^2)*(a + 2*b*Tan[(c + d*x)/2] - a*Tan[(c + d*x)/2]^2)^3) + ((b^2*
(b*(15*a^4 + 18*a^2*b^2 + 8*b^4) + a*(9*a^4 + 30*a^2*b^2 + 16*b^4)*Tan[(c
+ d*x)/2]))/(a^5*(a^2 + b^2)*(a + 2*b*Tan[(c + d*x)/2] - a*Tan[(c + d*x)/2
]^2)^2) + (3*(-1/2*(a*(2*a^2 - 3*b^2)*ArcTanh[(2*b - 2*a*Tan[(c + d*x)/2])
/(2*Sqrt[a^2 + b^2])])/(a^2 + b^2)^(3/2) - (b*(a^3*(6*a^2 + 9*b^2 + (12*b^
4)/a^2 + (4*b^6)/a^4) + b*(9*a^4 + 6*a^2*b^2 + 2*b^4)*Tan[(c + d*x)/2]))/(
2*a^3*(a^2 + b^2)*(a + 2*b*Tan[(c + d*x)/2] - a*Tan[(c + d*x)/2]^2))))/(a^
2 + b^2))/(3*(a^2 + b^2))))/d
```

3.142.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 2191
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c))   Int
[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*
(2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^
2 - 4*a*c, 0] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.142. $\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

rule 4902
```
Int[u_, x_Symbol] :> With[{w = Block[{$ShowSteps = False, $StepCounter = Nu
ll}, Int[SubstFor[1/(1 + FreeFactors[Tan[FunctionOfTrig[u, x]/2], x]^2*x^2)
, Tan[FunctionOfTrig[u, x]/2]/FreeFactors[Tan[FunctionOfTrig[u, x]/2], x],
u, x], x]]}, Module[{v = FunctionOfTrig[u, x], d}, Simp[d = FreeFactors[Tan
[v/2], x]; 2*(d/Coefficient[v, x, 1])   Subst[Int[SubstFor[1/(1 + d^2*x^2),
 Tan[v/2]/d, u, x], x], x, Tan[v/2]/d], x]] /; CalculusFreeQ[w, x]] /; Inve
rseFunctionFreeQ[u, x] &&  !FalseQ[FunctionOfTrig[u, x]]
```

3.142.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 493 vs. $2(151) = 302$.

Time = 1.46 (sec) , antiderivative size = 494, normalized size of antiderivative = 3.15

method	result
derivativedivides	$-\frac{2\left(-\frac{b^{2}\left(9a^{4}+6a^{2}b^{2}+2b^{4}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{2a\left(a^{6}+3a^{4}b^{2}+3a^{2}b^{4}+b^{6}\right)}-\frac{b\left(6a^{6}-27a^{4}b^{2}-12a^{2}b^{4}-4b^{6}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{2a^{2}\left(a^{6}+3a^{4}b^{2}+3a^{2}b^{4}+b^{6}\right)}+\frac{b^{2}\left(54a^{6}-21a^{4}b^{2}-4a^{2}b^{4}-4b^{6}\right)\tan\left(\frac{dx}{2}\right.}{3a^{3}\left(a^{6}+3a^{4}b^{2}+3a^{2}b^{4}+b^{6}\right)}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-2b\tan\right.}$
default	$-\frac{2\left(-\frac{b^{2}\left(9a^{4}+6a^{2}b^{2}+2b^{4}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{2a\left(a^{6}+3a^{4}b^{2}+3a^{2}b^{4}+b^{6}\right)}-\frac{b\left(6a^{6}-27a^{4}b^{2}-12a^{2}b^{4}-4b^{6}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{2a^{2}\left(a^{6}+3a^{4}b^{2}+3a^{2}b^{4}+b^{6}\right)}+\frac{b^{2}\left(54a^{6}-21a^{4}b^{2}-4a^{2}b^{4}-4b^{6}\right)\tan\left(\frac{dx}{2}\right.}{3a^{3}\left(a^{6}+3a^{4}b^{2}+3a^{2}b^{4}+b^{6}\right)}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-2b\tan\right.}$
risch	$\frac{ib\,e^{i(dx+c)}\left(-27ia^{3}b\,e^{4i(dx+c)}+3ia\,b^{3}e^{4i(dx+c)}+18a^{4}e^{4i(dx+c)}-6a^{2}b^{2}e^{4i(dx+c)}+6b^{4}e^{4i(dx+c)}+36a^{4}e^{2i(dx+c)}+32a^{2}b^{2}e\right.}{3\left(b\,e^{2i(dx+c)}+ia\,e^{2i(dx+c)}-b+ia\right)^{3}(-ia+b)^{3}d(ia+b)^{3}}$

input
```
int(cos(d*x+c)^3/(cos(d*x+c)*a+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*(-1/2*b^2*(9*a^4+6*a^2*b^2+2*b^4)/a/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*
tan(1/2*d*x+1/2*c)^5-1/2*b*(6*a^6-27*a^4*b^2-12*a^2*b^4-4*b^6)/a^2/(a^6+3*
a^4*b^2+3*a^2*b^4+b^6)*tan(1/2*d*x+1/2*c)^4+1/3/a^3*b^2*(54*a^6-21*a^4*b^2
-4*a^2*b^4-4*b^6)/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*tan(1/2*d*x+1/2*c)^3+1/a^2
*b*(6*a^6-20*a^4*b^2-3*a^2*b^4-2*b^6)/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*tan(1/
2*d*x+1/2*c)^2-1/2/a*b^2*(27*a^4+4*a^2*b^2+2*b^4)/(a^6+3*a^4*b^2+3*a^2*b^4
+b^6)*tan(1/2*d*x+1/2*c)-1/6*b*(18*a^4+5*a^2*b^2+2*b^4)/(a^6+3*a^4*b^2+3*a
^2*b^4+b^6))/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)-a)^3+a*(2*a^2-
3*b^2)/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(
1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

3.142.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 524 vs. $2(152) = 304$.

Time = 0.28 (sec) , antiderivative size = 524, normalized size of antiderivative = 3.34

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = -\frac{22\,a^4b^3+14\,a^2b^5-8\,b^7+12\,(3\,a^6b+2\,a^4b^3+b^7)\cos{(dx+c)}^2+6\,(9\,a^5b^2+8\,a^3b^4-ab^6)\cos{(dx+c)}\,\mathrm{s}}{12\,((a^{11}+a^9b^2-6\,a^7b^4-14\,a^5b^6-11\,a^3b^8-3\,ab^{10})}$$

input
```
integrate(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas"
)
```

output
```
-1/12*(22*a^4*b^3 + 14*a^2*b^5 - 8*b^7 + 12*(3*a^6*b + 2*a^4*b^3 + b^7)*co
s(d*x + c)^2 + 6*(9*a^5*b^2 + 8*a^3*b^4 - a*b^6)*cos(d*x + c)*sin(d*x + c)
 + 3*((2*a^6 - 9*a^4*b^2 + 9*a^2*b^4)*cos(d*x + c)^3 + 3*(2*a^4*b^2 - 3*a^
2*b^4)*cos(d*x + c) + (2*a^3*b^3 - 3*a*b^5 + (6*a^5*b - 11*a^3*b^3 + 3*a*b
^5)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 + b^2)*log((2*a*b*cos(d*x + c)*
sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2
)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a
^2 - b^2)*cos(d*x + c)^2 + b^2)))/((a^11 + a^9*b^2 - 6*a^7*b^4 - 14*a^5*b^
6 - 11*a^3*b^8 - 3*a*b^10)*d*cos(d*x + c)^3 + 3*(a^9*b^2 + 4*a^7*b^4 + 6*a
^5*b^6 + 4*a^3*b^8 + a*b^10)*d*cos(d*x + c) + ((3*a^10*b + 11*a^8*b^3 + 14
*a^6*b^5 + 6*a^4*b^7 - a^2*b^9 - b^11)*d*cos(d*x + c)^2 + (a^8*b^3 + 4*a^6
*b^5 + 6*a^4*b^7 + 4*a^2*b^9 + b^11)*d)*sin(d*x + c))
```

3.142.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3/(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

output
```
Timed out
```

3.142.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 724 vs. $2(152) = 304$.

Time = 0.34 (sec) , antiderivative size = 724, normalized size of antiderivative = 4.61

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx =$$

$$-\frac{\frac{3\,(2\,a^2-3\,b^2)a\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\sqrt{a^2+b^2}}+\frac{2\left(18\,a^7b+5\,a^5b^3+2\,a^3b^5+\frac{3\left(27\,a^6b^2+4\,a^4b^4+2\,a^2b^6\right)\sin(}{\cos(dx+c)+1}\right.}{a^{12}+3\,a^{10}b^2+3\,a^8b^4+a^6b^6+\frac{6\left(a^{11}b+3\,a^9b^3+3\,a^7b^5+a^5b^7\right)\sin(dx+c)}{\cos(dx+c)+1}-\frac{3\left(a^{12}-a^{10}b^2-9\,a^8b^4\right.}{(\cos(}}}{}$$

input
```
integrate(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima"
)
```

output
```
-1/6*(3*(2*a^2 - 3*b^2)*a*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqr
t(a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(
(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) + 2*(18*a^7*b + 5*a^5
*b^3 + 2*a^3*b^5 + 3*(27*a^6*b^2 + 4*a^4*b^4 + 2*a^2*b^6)*sin(d*x + c)/(co
s(d*x + c) + 1) - 6*(6*a^7*b - 20*a^5*b^3 - 3*a^3*b^5 - 2*a*b^7)*sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 - 2*(54*a^6*b^2 - 21*a^4*b^4 - 4*a^2*b^6 - 4*b^
8)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*(6*a^7*b - 27*a^5*b^3 - 12*a^3*
b^5 - 4*a*b^7)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 3*(9*a^6*b^2 + 6*a^4*
b^4 + 2*a^2*b^6)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^12 + 3*a^10*b^2 +
 3*a^8*b^4 + a^6*b^6 + 6*(a^11*b + 3*a^9*b^3 + 3*a^7*b^5 + a^5*b^7)*sin(d*
x + c)/(cos(d*x + c) + 1) - 3*(a^12 - a^10*b^2 - 9*a^8*b^4 - 11*a^6*b^6 -
4*a^4*b^8)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 4*(3*a^11*b + 7*a^9*b^3 +
 3*a^7*b^5 - 3*a^5*b^7 - 2*a^3*b^9)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 +
3*(a^12 - a^10*b^2 - 9*a^8*b^4 - 11*a^6*b^6 - 4*a^4*b^8)*sin(d*x + c)^4/(c
os(d*x + c) + 1)^4 + 6*(a^11*b + 3*a^9*b^3 + 3*a^7*b^5 + a^5*b^7)*sin(d*x
+ c)^5/(cos(d*x + c) + 1)^5 - (a^12 + 3*a^10*b^2 + 3*a^8*b^4 + a^6*b^6)*si
n(d*x + c)^6/(cos(d*x + c) + 1)^6))/d
```

3.142.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 524 vs. $2(152) = 304$.

Time = 0.41 (sec) , antiderivative size = 524, normalized size of antiderivative = 3.34

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = -\frac{\frac{3\,(2\,a^3-3\,ab^2)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\sqrt{a^2+b^2}} - \frac{2\left(27\,a^6b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+18\,a^4b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,a^2b^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+18\cdots\right.}{\cdots}}{\cdots}$$

input
```
integrate(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/6*(3*(2*a^3 - 3*a*b^2)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(
a^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^6
 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) - 2*(27*a^6*b^2*tan(1/2*d
*x + 1/2*c)^5 + 18*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 + 6*a^2*b^6*tan(1/2*d*x
+ 1/2*c)^5 + 18*a^7*b*tan(1/2*d*x + 1/2*c)^4 - 81*a^5*b^3*tan(1/2*d*x + 1/
2*c)^4 - 36*a^3*b^5*tan(1/2*d*x + 1/2*c)^4 - 12*a*b^7*tan(1/2*d*x + 1/2*c)
^4 - 108*a^6*b^2*tan(1/2*d*x + 1/2*c)^3 + 42*a^4*b^4*tan(1/2*d*x + 1/2*c)^
3 + 8*a^2*b^6*tan(1/2*d*x + 1/2*c)^3 + 8*b^8*tan(1/2*d*x + 1/2*c)^3 - 36*a
^7*b*tan(1/2*d*x + 1/2*c)^2 + 120*a^5*b^3*tan(1/2*d*x + 1/2*c)^2 + 18*a^3*
b^5*tan(1/2*d*x + 1/2*c)^2 + 12*a*b^7*tan(1/2*d*x + 1/2*c)^2 + 81*a^6*b^2*
tan(1/2*d*x + 1/2*c) + 12*a^4*b^4*tan(1/2*d*x + 1/2*c) + 6*a^2*b^6*tan(1/2
*d*x + 1/2*c) + 18*a^7*b + 5*a^5*b^3 + 2*a^3*b^5)/((a^9 + 3*a^7*b^2 + 3*a^
5*b^4 + a^3*b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c) - a)
^3))/d
```

3.142.9 Mupad [B] (verification not implemented)

Time = 26.65 (sec) , antiderivative size = 764, normalized size of antiderivative = 4.87

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$$
$$= \frac{\ln\left((a^2+b^2)^{7/2}+a^6\,b+b^7+3\,a^2\,b^5+3\,a^4\,b^3-a^7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-a\,b^6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-3\,a^3\,b^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right.}{d\,(a^2+b^2)^{7/2}}$$
$$-\frac{\frac{18\,a^4\,b+5\,a^2\,b^3+2\,b^5}{3\,(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6)}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2(-6\,a^6\,b+20\,a^4\,b^3+3\,a^2\,b^5+2\,b^7)}{a^2\,(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4(-6\,a^6\,b+27\,a^4\,b^3+12\,a^2\,b^5+4\,b^7)}{a^2\,(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6)}+\frac{b\tan}{}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2(12\,a\,b^2-3\,a^3)-a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4(12\,a\,b^2-3\,a^3)-\right.}$$
$$+\frac{a\ln\left((a^2+b^2)^{7/2}-a^6\,b-b^7-3\,a^2\,b^5-3\,a^4\,b^3+a^7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\,b^6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+3\,a^3\,b^4\tan\left(\frac{c}{2}+\right.\right.}{2\,d\,(a^2+b^2)^{7/2}}$$

input
```
int(cos(c + d*x)^3/(a*cos(c + d*x) + b*sin(c + d*x))^4,x)
```

output
```
(log((a^2 + b^2)^(7/2) + a^6*b + b^7 + 3*a^2*b^5 + 3*a^4*b^3 - a^7*tan(c/2
 + (d*x)/2) - a*b^6*tan(c/2 + (d*x)/2) - 3*a^3*b^4*tan(c/2 + (d*x)/2) - 3*
a^5*b^2*tan(c/2 + (d*x)/2))*((3*a*b^2)/2 - a^3))/(d*(a^2 + b^2)^(7/2)) - (
(18*a^4*b + 2*b^5 + 5*a^2*b^3)/(3*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (
2*tan(c/2 + (d*x)/2)^2*(2*b^7 - 6*a^6*b + 3*a^2*b^5 + 20*a^4*b^3))/(a^2*(a
^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (tan(c/2 + (d*x)/2)^4*(4*b^7 - 6*a^6*
b + 12*a^2*b^5 + 27*a^4*b^3))/(a^2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) +
(b*tan(c/2 + (d*x)/2)*(27*a^4*b + 2*b^5 + 4*a^2*b^3))/(a*(a^6 + b^6 + 3*a^
2*b^4 + 3*a^4*b^2)) + (b*tan(c/2 + (d*x)/2)^5*(9*a^4*b + 2*b^5 + 6*a^2*b^3
))/(a*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (2*b*tan(c/2 + (d*x)/2)^3*(3*
a^2 - 2*b^2)*(18*a^4*b + 2*b^5 + 5*a^2*b^3))/(3*a^3*(a^6 + b^6 + 3*a^2*b^4
 + 3*a^4*b^2)))/(d*(tan(c/2 + (d*x)/2)^2*(12*a*b^2 - 3*a^3) - a^3*tan(c/2
+ (d*x)/2)^6 - tan(c/2 + (d*x)/2)^4*(12*a*b^2 - 3*a^3) - tan(c/2 + (d*x)/2
)^3*(12*a^2*b - 8*b^3) + a^3 + 6*a^2*b*tan(c/2 + (d*x)/2) + 6*a^2*b*tan(c/
2 + (d*x)/2)^5)) + (a*log((a^2 + b^2)^(7/2) - a^6*b - b^7 - 3*a^2*b^5 - 3*
a^4*b^3 + a^7*tan(c/2 + (d*x)/2) + a*b^6*tan(c/2 + (d*x)/2) + 3*a^3*b^4*ta
n(c/2 + (d*x)/2) + 3*a^5*b^2*tan(c/2 + (d*x)/2))*(2*a^2 - 3*b^2))/(2*d*(a^
2 + b^2)^(7/2))
```

3.143 $\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4} \, dx$

3.143.1 Optimal result

Integrand size = 28, antiderivative size = 30

$$\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4} \, dx = -\frac{\cot^3(c+dx)}{3bd(b+a\cot(c+dx))^3}$$

output
```
-1/3*cot(d*x+c)^3/b/d/(b+a*cot(d*x+c))^3
```

3.143.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 124 vs. 2(30) = 60.

Time = 1.52 (sec) , antiderivative size = 124, normalized size of antiderivative = 4.13

$$\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4} \, dx = \frac{-6ab(a^2+b^2)\cos(c+dx)+(-6a^3b+2ab^3)\cos(3(c+dx))+2(a^2-b^2)\left(3a^2+b^2+(3a^2-b^2)\cos(2(c+\right.}{12a\left(a^2+b^2\right)^2 d(a\cos(c+dx)+b\sin(c+dx))^3}$$

input
```
Integrate[Cos[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

output
```
(-6*a*b*(a^2 + b^2)*Cos[c + d*x] + (-6*a^3*b + 2*a*b^3)*Cos[3*(c + d*x)] +
 2*(a^2 - b^2)*(3*a^2 + b^2 + (3*a^2 - b^2)*Cos[2*(c + d*x)])*Sin[c + d*x]
)/(12*a*(a^2 + b^2)^2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3)
```

3.143.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3042, 3567, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\cos(c+dx)^2}{(a\cos(c+dx)+b\sin(c+dx))^4}dx$$

$$\Big\downarrow \; 3567$$

$$-\frac{\int \frac{\cot^2(c+dx)}{(b+a\cot(c+dx))^4}d\cot(c+dx)}{d}$$

$$\Big\downarrow \; 48$$

$$-\frac{\cot^3(c+dx)}{3bd(a\cot(c+dx)+b)^3}$$

input
```
Int[Cos[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

output
```
-1/3*Cot[c + d*x]^3/(b*d*(b + a*Cot[c + d*x])^3)
```

3.143.3.1 Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3567
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[x^m*((b
+ a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a,
b, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[
n, 0] && GtQ[m, 1])
```

3.143.4 Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$-\frac{1}{3db(a+b\tan(dx+c))^3}$	21
default	$-\frac{1}{3db(a+b\tan(dx+c))^3}$	21
parallelrisch	$-\frac{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4a^2-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3ab+\left(-2a^2+\frac{4b^2}{3}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)ab+a^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^3}$	119
risch	$\frac{2i\left(6iab\,e^{4i(dx+c)}-3a^2e^{4i(dx+c)}+3b^2e^{4i(dx+c)}+6iab\,e^{2i(dx+c)}-6a^2e^{2i(dx+c)}-3a^2+b^2\right)}{3\left(b\,e^{2i(dx+c)}+ia\,e^{2i(dx+c)}-b+ia\right)^3d(ia+b)^3}$	128
norman	$\frac{\frac{1}{3bd}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{3bd}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{3db}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{3db}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{3db}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{3db}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^3}$	152

input
```
int(cos(d*x+c)^2/(cos(d*x+c)*a+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3/d/b/(a+b*tan(d*x+c))^3
```

3.143.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(28) = 56$.

Time = 0.26 (sec) , antiderivative size = 255, normalized size of antiderivative = 8.50

$$\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx =$$
$$-\frac{(9\,a^4b-6\,a^2b^3+b^5)\cos{(dx+c)}^3-3\,(a^4b-3\,a^2b^3)\cos{(dx+c)}-(a^3b^2-}{3\,((a^9-6\,a^5b^4-8\,a^3b^6-3\,ab^8)d\cos{(dx+c)}^3+3\,(a^7b^2+3\,a^5b^4+3\,a^3b^6+ab^8)d\cos{(dx+c)}+((3\,a^8}$$

input
```
integrate(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas"
)
```

output
```
-1/3*((9*a^4*b - 6*a^2*b^3 + b^5)*cos(d*x + c)^3 - 3*(a^4*b - 3*a^2*b^3)*c
os(d*x + c) - (a^3*b^2 - 3*a*b^4 + (3*a^5 - 10*a^3*b^2 + 3*a*b^4)*cos(d*x
+ c)^2)*sin(d*x + c))/((a^9 - 6*a^5*b^4 - 8*a^3*b^6 - 3*a*b^8)*d*cos(d*x +
 c)^3 + 3*(a^7*b^2 + 3*a^5*b^4 + 3*a^3*b^6 + a*b^8)*d*cos(d*x + c) + ((3*a
^8*b + 8*a^6*b^3 + 6*a^4*b^5 - b^9)*d*cos(d*x + c)^2 + (a^6*b^3 + 3*a^4*b^
5 + 3*a^2*b^7 + b^9)*d)*sin(d*x + c))
```

3.143.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**2/(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

output
```
Timed out
```

3.143.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.77

$$\begin{aligned}&\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx\\&=-\frac{1}{3\left(b^4\tan\left(dx+c\right)^3+3\,ab^3\tan\left(dx+c\right)^2+3\,a^2b^2\tan\left(dx+c\right)+a^3b\right)d}\end{aligned}$$

input
```
integrate(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima"
)
```

output
```
-1/3/((b^4*tan(d*x + c)^3 + 3*a*b^3*tan(d*x + c)^2 + 3*a^2*b^2*tan(d*x + c
) + a^3*b)*d)
```

3.143.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

$$\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx=-\frac{1}{3\,(b\tan\left(dx+c\right)+a)^3bd}$$

input
```
integrate(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/3/((b*tan(d*x + c) + a)^3*b*d)
```

3.143.9 Mupad [B] (verification not implemented)

Time = 24.26 (sec) , antiderivative size = 224, normalized size of antiderivative = 7.47

$$\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$$
$$=\frac{\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{a}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a}+\frac{4\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{a^2}-\frac{4\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{a^2}-\frac{4}{}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(12\,a\,b^2-3\,a^3\right)-a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(12\,a\,b^2-3\,a^3\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(12\right.\right.}$$

input
```
int(cos(c + d*x)^2/(a*cos(c + d*x) + b*sin(c + d*x))^4,x)
```

output
```
((2*tan(c/2 + (d*x)/2)^5)/a + (2*tan(c/2 + (d*x)/2))/a + (4*b*tan(c/2 + (d
*x)/2)^2)/a^2 - (4*b*tan(c/2 + (d*x)/2)^4)/a^2 - (4*tan(c/2 + (d*x)/2)^3*(
3*a^2 - 2*b^2))/(3*a^3))/(d*(tan(c/2 + (d*x)/2)^2*(12*a*b^2 - 3*a^3) - a^3
*tan(c/2 + (d*x)/2)^6 - tan(c/2 + (d*x)/2)^4*(12*a*b^2 - 3*a^3) - tan(c/2
+ (d*x)/2)^3*(12*a^2*b - 8*b^3) + a^3 + 6*a^2*b*tan(c/2 + (d*x)/2) + 6*a^2
*b*tan(c/2 + (d*x)/2)^5))
```

3.144 $\int \frac{\cos(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

3.144.1 Optimal result

Integrand size = 26, antiderivative size = 141

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = -\frac{a\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{5/2}d} - \frac{b}{3(a^2+b^2)d(a\cos(c+dx)+b\sin(c+dx))^3} - \frac{a(b\cos(c+dx)-a\sin(c+dx))}{2(a^2+b^2)^2d(a\cos(c+dx)+b\sin(c+dx))^2}$$

output
```
-1/2*a*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2
)/d-1/3*b/(a^2+b^2)/d/(a*cos(d*x+c)+b*sin(d*x+c))^3-1/2*a*(b*cos(d*x+c)-a*
sin(d*x+c))/(a^2+b^2)^2/d/(a*cos(d*x+c)+b*sin(d*x+c))^2
```

3.144.2 Mathematica [A] (verified)

Time = 1.32 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.91

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = \frac{\frac{6a\text{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} + \frac{-4b(a^2+b^2)-6a^2b\cos(2(c+dx))+3(a^3-ab^2)\sin(2(c+dx))}{2(a^2+b^2)^2(a\cos(c+dx)+b\sin(c+dx))^3}}{6d}$$

input
```
Integrate[Cos[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

output
```
((6*a*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2
) + (-4*b*(a^2 + b^2) - 6*a^2*b*Cos[2*(c + d*x)] + 3*(a^3 - a*b^2)*Sin[2*(
c + d*x)])/(2*(a^2 + b^2)^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3))/(6*d)
```

3.144.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 3637, 27, 3042, 3555, 3042, 3553, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}dx$$

$$\Big\downarrow \text{3637}$$

$$\frac{\int \frac{3a}{(a\cos(c+dx)+b\sin(c+dx))^3}dx}{3(a^2+b^2)}-\frac{b}{3d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^3}dx}{a^2+b^2}-\frac{b}{3d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^3}dx}{a^2+b^2}-\frac{b}{3d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^3}$$

$$\Big\downarrow \text{3555}$$

$$\frac{a\left(\frac{\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{2(a^2+b^2)}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^2}\right)}{a^2+b^2}-\frac{b}{3d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\left(\frac{\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{2(a^2+b^2)}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^2}\right)}{a^2+b^2}-\frac{b}{3d\,(a^2+b^2)\,(a\cos(c+dx)+b\sin(c+dx))^3}$$

$\Big\downarrow$ 3553

$$\frac{a\left(-\frac{\int \frac{1}{a^2+b^2-(b\cos(c+dx)-a\sin(c+dx))^2}d(b\cos(c+dx)-a\sin(c+dx))}{2d(a^2+b^2)}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^2}\right)}{a^2+b^2}-\frac{b}{3d\,(a^2+b^2)\,(a\cos(c+dx)+b\sin(c+dx))^3}$$

$\Big\downarrow$ 219

$$\frac{a\left(-\frac{\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2d(a^2+b^2)^{3/2}}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^2}\right)}{a^2+b^2}-\frac{b}{3d\,(a^2+b^2)\,(a\cos(c+dx)+b\sin(c+dx))^3}$$

input
```
Int[Cos[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

output
```
-1/3*b/((a^2 + b^2)*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) + (a*(-1/2*ArcT
anh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]]/((a^2 + b^2)^(3/2)*
d) - (b*Cos[c + d*x] - a*Sin[c + d*x])/(2*(a^2 + b^2)*d*(a*Cos[c + d*x] +
b*Sin[c + d*x])^2)))/(a^2 + b^2)
```

3.144.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3555
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(b*Cos[c + d*x] - a*Sin[c + d*x])*((a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Simp[(n + 2)/((n + 1)*(a^
2 + b^2))   Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

rule 3637
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))*((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(c*B + c
*A*Cos[d + e*x] + (a*B - b*A)*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d
 + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Simp[1/((n + 1)*(a^2
- b^2 - c^2))   Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n +
 1)*(a*A - b*B) + (n + 2)*(a*B - b*A)*Cos[d + e*x] - (n + 2)*c*A*Sin[d + e*
x], x], x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && LtQ[n, -1] && NeQ[a^2
- b^2 - c^2, 0] && NeQ[n, -2]
```

3.144.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.21 (sec) , antiderivative size = 335, normalized size of antiderivative = 2.38

method	result
risch	$-\frac{i\left(-8ib^3 e^{3i(dx+c)}-8ia^2b\,e^{3i(dx+c)}+3a\,b^2e^{i(dx+c)}-6ia^2b\,e^{i(dx+c)}+3a^3e^{5i(dx+c)}-6ia^2b\,e^{5i(dx+c)}-3a\,b^2e^{5i(dx+c)}-3a\right.}{3\left(-ib\,e^{2i(dx+c)}+e^{2i(dx+c)}a+ib+a\right)^3(ib+a)^2d(-ib+a)^2}$
derivativedivides	$-\frac{\frac{2\left(-\frac{\left(a^4+4a^2b^2+2b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{2a\left(a^4+2a^2b^2+b^4\right)}-\frac{b\left(a^4-8a^2b^2-4b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{2a^2\left(a^4+2a^2b^2+b^4\right)}+\frac{b^2\left(15a^4-4a^2b^2-4b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3a^3\left(a^4+2a^2b^2+b^4\right)}+\frac{b\left(2a^4-5a^2b^2-\right.}{a^2\left(a^4-\right.}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^3}}{d}$
default	$-\frac{\frac{2\left(-\frac{\left(a^4+4a^2b^2+2b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{2a\left(a^4+2a^2b^2+b^4\right)}-\frac{b\left(a^4-8a^2b^2-4b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{2a^2\left(a^4+2a^2b^2+b^4\right)}+\frac{b^2\left(15a^4-4a^2b^2-4b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3a^3\left(a^4+2a^2b^2+b^4\right)}+\frac{b\left(2a^4-5a^2b^2-\right.}{a^2\left(a^4-\right.}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^3}}{d}$

input
```
int(cos(d*x+c)/(cos(d*x+c)*a+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*I*(-8*I*b^3*exp(3*I*(d*x+c))-8*I*a^2*b*exp(3*I*(d*x+c))+3*a*b^2*exp(I
*(d*x+c))-6*I*a^2*b*exp(I*(d*x+c))+3*a^3*exp(5*I*(d*x+c))-6*I*a^2*b*exp(5*
I*(d*x+c))-3*a*b^2*exp(5*I*(d*x+c))-3*a^3*exp(I*(d*x+c)))/(-I*b*exp(2*I*(d
*x+c))+exp(2*I*(d*x+c))*a+I*b+a)^3/(I*b+a)^2/d/(-I*b+a)^2+1/2/(a^2+b^2)^(5
/2)*a/d*ln(exp(I*(d*x+c))+(I*a^5+2*I*a^3*b^2+I*a*b^4-a^4*b-2*a^2*b^3-b^5)/
(a^2+b^2)^(5/2))-1/2/(a^2+b^2)^(5/2)*a/d*ln(exp(I*(d*x+c))-(I*a^5+2*I*a^3*
b^2+I*a*b^4-a^4*b-2*a^2*b^3-b^5)/(a^2+b^2)^(5/2))
```

3.144.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 420 vs. $2(131) = 262$.

Time = 0.27 (sec) , antiderivative size = 420, normalized size of antiderivative = 2.98

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = \frac{2\,a^4b-2\,a^2b^3-4\,b^5-12\,(a^4b+a^2b^3)\cos(dx+c)^2+6\,(a^5-ab^4)\cos(dx+c)\sin(dx+c)+3\,(3\,a^2b^2\cos}{12\,((a^9-6\,a^5b^4-8\,a^3b^6-3\,ab^8)d\cos(dx+c)^3+3\,(a^7b^2+}$$

input
```
integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/12*(2*a^4*b - 2*a^2*b^3 - 4*b^5 - 12*(a^4*b + a^2*b^3)*cos(d*x + c)^2 +
6*(a^5 - a*b^4)*cos(d*x + c)*sin(d*x + c) + 3*(3*a^2*b^2*cos(d*x + c) + (a
^4 - 3*a^2*b^2)*cos(d*x + c)^3 + (a*b^3 + (3*a^3*b - a*b^3)*cos(d*x + c)^2
)*sin(d*x + c))*sqrt(a^2 + b^2)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a
^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c)
 - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x
 + c)^2 + b^2)))/((a^9 - 6*a^5*b^4 - 8*a^3*b^6 - 3*a*b^8)*d*cos(d*x + c)^3
 + 3*(a^7*b^2 + 3*a^5*b^4 + 3*a^3*b^6 + a*b^8)*d*cos(d*x + c) + ((3*a^8*b
+ 8*a^6*b^3 + 6*a^4*b^5 - b^9)*d*cos(d*x + c)^2 + (a^6*b^3 + 3*a^4*b^5 + 3
*a^2*b^7 + b^9)*d)*sin(d*x + c))
```

3.144.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

output
```
Timed out
```

3.144.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 606 vs. $2(131) = 262$.

Time = 0.34 (sec) , antiderivative size = 606, normalized size of antiderivative = 4.30

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = -\frac{\frac{3a\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{(a^4+2a^2b^2+b^4)\sqrt{a^2+b^2}}+\frac{2\left(5a^5b+2a^3b^3-\frac{3(a^6-6a^4b^2-2a^2b^4)\sin(dx+c)}{\cos(dx+c)+1}-\frac{6(2a^5b-5a^3b^3-2ab^5)\sin\cdots}{(\cos(dx+c)+1)^2}\cdots\right.}{a^{10}+2a^8b^2+a^6b^4+\frac{6(a^9b+2a^7b^3+a^5b^5)\sin(dx+c)}{\cos(dx+c)+1}-\frac{3(a^{10}-2a^8b^2-7a^6b^4-4a^4b^6)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{4(3a^9b-\cdots}{\cdots}\cdots}}{6d\cdots}$$

input
```
integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/6*(3*a*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b
 - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2
 + b^4)*sqrt(a^2 + b^2)) + 2*(5*a^5*b + 2*a^3*b^3 - 3*(a^6 - 6*a^4*b^2 - 2
*a^2*b^4)*sin(d*x + c)/(cos(d*x + c) + 1) - 6*(2*a^5*b - 5*a^3*b^3 - 2*a*b
^5)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 2*(15*a^4*b^2 - 4*a^2*b^4 - 4*b^
6)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*(a^5*b - 8*a^3*b^3 - 4*a*b^5)*s
in(d*x + c)^4/(cos(d*x + c) + 1)^4 + 3*(a^6 + 4*a^4*b^2 + 2*a^2*b^4)*sin(d
*x + c)^5/(cos(d*x + c) + 1)^5)/(a^10 + 2*a^8*b^2 + a^6*b^4 + 6*(a^9*b + 2
*a^7*b^3 + a^5*b^5)*sin(d*x + c)/(cos(d*x + c) + 1) - 3*(a^10 - 2*a^8*b^2
- 7*a^6*b^4 - 4*a^4*b^6)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 4*(3*a^9*b
+ 4*a^7*b^3 - a^5*b^5 - 2*a^3*b^7)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3
*(a^10 - 2*a^8*b^2 - 7*a^6*b^4 - 4*a^4*b^6)*sin(d*x + c)^4/(cos(d*x + c) +
 1)^4 + 6*(a^9*b + 2*a^7*b^3 + a^5*b^5)*sin(d*x + c)^5/(cos(d*x + c) + 1)^
5 - (a^10 + 2*a^8*b^2 + a^6*b^4)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6))/d
```

3.144.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 426 vs. $2(131) = 262$.

Time = 0.39 (sec) , antiderivative size = 426, normalized size of antiderivative = 3.02

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = -\frac{\frac{3\,a\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}} - \frac{2\left(3\,a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+12\,a^4b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,a^2b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,a^5b\tan\left(\frac{1}{2}\,dx+\right.}{}}{}$$

input
```
integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/6*(3*a*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(
2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b
^4)*sqrt(a^2 + b^2)) - 2*(3*a^6*tan(1/2*d*x + 1/2*c)^5 + 12*a^4*b^2*tan(1/
2*d*x + 1/2*c)^5 + 6*a^2*b^4*tan(1/2*d*x + 1/2*c)^5 + 3*a^5*b*tan(1/2*d*x
+ 1/2*c)^4 - 24*a^3*b^3*tan(1/2*d*x + 1/2*c)^4 - 12*a*b^5*tan(1/2*d*x + 1/
2*c)^4 - 30*a^4*b^2*tan(1/2*d*x + 1/2*c)^3 + 8*a^2*b^4*tan(1/2*d*x + 1/2*c
)^3 + 8*b^6*tan(1/2*d*x + 1/2*c)^3 - 12*a^5*b*tan(1/2*d*x + 1/2*c)^2 + 30*
a^3*b^3*tan(1/2*d*x + 1/2*c)^2 + 12*a*b^5*tan(1/2*d*x + 1/2*c)^2 - 3*a^6*t
an(1/2*d*x + 1/2*c) + 18*a^4*b^2*tan(1/2*d*x + 1/2*c) + 6*a^2*b^4*tan(1/2*
d*x + 1/2*c) + 5*a^5*b + 2*a^3*b^3)/((a^7 + 2*a^5*b^2 + a^3*b^4)*(a*tan(1/
2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c) - a)^3))/d
```

3.144.9 Mupad [B] (verification not implemented)

Time = 26.97 (sec) , antiderivative size = 505, normalized size of antiderivative = 3.58

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = -\frac{\frac{\frac{5\,a^2\,b}{3}+\frac{2\,b^3}{3}}{a^4+2\,a^2\,b^2+b^4} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(-a^4\,b+8\,a^2\,b^3+4\,b^5\right)}{a^2\left(a^4+2\,a^2\,b^2+b^4\right)} + \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(-4\,a^4\,b+10\,a^2\,b^3+4\,b^5\right)}{a^2\left(a^4+2\,a^2\,b^2+b^4\right)} + \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(-a^4+6\,a^2\,b^2+2\,b^4\right)}{a\left(a^4+2\,a^2\,b^2+b^4\right)}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(12\,a\,b^2-3\,a^3\right)-a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(12\,a\,b^2-3\,a^3\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\right.} - \frac{a\,\mathrm{atanh}\left(\frac{a^4\,b+2\,a^2\,b^3+b^5}{\left(a^2+b^2\right)^{5/2}}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(a^4+2\,a^2\,b^2+b^4\right)}{\left(a^2+b^2\right)^{5/2}}\right)}{d\left(a^2+b^2\right)^{5/2}}$$

input
```
int(cos(c + d*x)/(a*cos(c + d*x) + b*sin(c + d*x))^4,x)
```

output
```
- (((5*a^2*b)/3 + (2*b^3)/3)/(a^4 + b^4 + 2*a^2*b^2) - (tan(c/2 + (d*x)/2)
^4*(4*b^5 - a^4*b + 8*a^2*b^3))/(a^2*(a^4 + b^4 + 2*a^2*b^2)) + (tan(c/2 +
 (d*x)/2)^2*(4*b^5 - 4*a^4*b + 10*a^2*b^3))/(a^2*(a^4 + b^4 + 2*a^2*b^2))
+ (tan(c/2 + (d*x)/2)*(2*b^4 - a^4 + 6*a^2*b^2))/(a*(a^4 + b^4 + 2*a^2*b^2
)) + (tan(c/2 + (d*x)/2)^5*(a^4 + 2*b^4 + 4*a^2*b^2))/(a*(a^4 + b^4 + 2*a^
2*b^2)) - (2*b*tan(c/2 + (d*x)/2)^3*(5*a^2*b + 2*b^3)*(3*a^2 - 2*b^2))/(3*
a^3*(a^4 + b^4 + 2*a^2*b^2)))/(d*(tan(c/2 + (d*x)/2)^2*(12*a*b^2 - 3*a^3)
- a^3*tan(c/2 + (d*x)/2)^6 - tan(c/2 + (d*x)/2)^4*(12*a*b^2 - 3*a^3) - tan
(c/2 + (d*x)/2)^3*(12*a^2*b - 8*b^3) + a^3 + 6*a^2*b*tan(c/2 + (d*x)/2) +
6*a^2*b*tan(c/2 + (d*x)/2)^5)) - (a*atanh((a^4*b + b^5 + 2*a^2*b^3)/(a^2 +
 b^2)^(5/2) - (a*tan(c/2 + (d*x)/2)*(a^4 + b^4 + 2*a^2*b^2))/(a^2 + b^2)^(
5/2)))/(d*(a^2 + b^2)^(5/2))
```

3.144. $\int \frac{\cos(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

3.145 $\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

3.145.1 Optimal result

Integrand size = 19, antiderivative size = 98

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = -\frac{b\cos(c+dx)-a\sin(c+dx)}{3(a^2+b^2)\,d(a\cos(c+dx)+b\sin(c+dx))^3} + \frac{2\sin(c+dx)}{3a(a^2+b^2)\,d(a\cos(c+dx)+b\sin(c+dx))}$$

output
```
1/3*(-b*cos(d*x+c)+a*sin(d*x+c))/(a^2+b^2)/d/(a*cos(d*x+c)+b*sin(d*x+c))^3
+2/3*sin(d*x+c)/a/(a^2+b^2)/d/(a*cos(d*x+c)+b*sin(d*x+c))
```

3.145.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.87

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = \frac{-ab\cos(3(c+dx))+(2a^2+b^2+(a^2-b^2)\cos(2(c+dx)))\sin(c+dx)}{3a(a^2+b^2)\,d(a\cos(c+dx)+b\sin(c+dx))^3}$$

input
```
Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-4),x]
```

output
```
(-(a*b*Cos[3*(c + d*x)]) + (2*a^2 + b^2 + (a^2 - b^2)*Cos[2*(c + d*x)])*Si
n[c + d*x])/(3*a*(a^2 + b^2)*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3)
```

3.145.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 3555, 3042, 3554}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^4}dx$$

$$\Big\downarrow \text{3555}$$

$$\frac{2\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^2}dx}{3(a^2+b^2)}-\frac{b\cos(c+dx)-a\sin(c+dx)}{3d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^2}dx}{3(a^2+b^2)}-\frac{b\cos(c+dx)-a\sin(c+dx)}{3d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^3}$$

$$\Big\downarrow \text{3554}$$

$$\frac{2\sin(c+dx)}{3ad(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))}-\frac{b\cos(c+dx)-a\sin(c+dx)}{3d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^3}$$

input
```
Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-4),x]
```

output
```
-1/3*(b*Cos[c + d*x] - a*Sin[c + d*x])/((a^2 + b^2)*d*(a*Cos[c + d*x] + b*
Sin[c + d*x])^3) + (2*Sin[c + d*x])/(3*a*(a^2 + b^2)*d*(a*Cos[c + d*x] + b
*Sin[c + d*x]))
```

3.145.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3554
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3555
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(b*Cos[c + d*x] - a*Sin[c + d*x])*((a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Simp[(n + 2)/((n + 1)*(a^
2 + b^2))   Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

3.145.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.65

method	result	size
derivativedivides	$\frac{-\frac{a^2+b^2}{3b^3(a+b\tan(dx+c))^3}+\frac{a}{b^3(a+b\tan(dx+c))^2}-\frac{1}{b^3(a+b\tan(dx+c))}}{d}$	64
default	$\frac{-\frac{a^2+b^2}{3b^3(a+b\tan(dx+c))^3}+\frac{a}{b^3(a+b\tan(dx+c))^2}-\frac{1}{b^3(a+b\tan(dx+c))}}{d}$	64
risch	$\frac{4i\left(3ia\,e^{2i(dx+c)}+3b\,e^{2i(dx+c)}+ia-b\right)}{3\left(b\,e^{2i(dx+c)}+ia\,e^{2i(dx+c)}-b+ia\right)^3 d(ia+b)^2}$	82
norman	$\frac{\frac{1}{3bd}-\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3da}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{3db}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{db}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{db}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^3}$	117
parallelrisch	$-\frac{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4a^2-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3ab+\frac{2\left(-a^2+2b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{3}+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)ab+a^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^3}$	120

input
```
int(1/(cos(d*x+c)*a+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/3*(a^2+b^2)/b^3/(a+b*tan(d*x+c))^3+a/b^3/(a+b*tan(d*x+c))^2-1/b^3/
(a+b*tan(d*x+c)))
```

3.145.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(94) = 188$.

Time = 0.24 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.21

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = -\frac{2\,(3\,a^2b-b^3)\cos\left(dx+c\right)^3-3\,(a^2b-b^3)\cos\left(dx+c\right)-\left(a^3+3\,ab^2+2\,(a\right.}{3\,\left((a^7-a^5b^2-5\,a^3b^4-3\,ab^6)d\cos\left(dx+c\right)^3+3\,(a^5b^2+2\,a^3b^4+ab^6)d\cos\left(dx+c\right)+\left((3\,a^6b+5\,a^4b^3\right.\right.}$$

input
```
integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

output
```
-1/3*(2*(3*a^2*b - b^3)*cos(d*x + c)^3 - 3*(a^2*b - b^3)*cos(d*x + c) - (a
^3 + 3*a*b^2 + 2*(a^3 - 3*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/((a^7 - a^5
*b^2 - 5*a^3*b^4 - 3*a*b^6)*d*cos(d*x + c)^3 + 3*(a^5*b^2 + 2*a^3*b^4 + a*
b^6)*d*cos(d*x + c) + ((3*a^6*b + 5*a^4*b^3 + a^2*b^5 - b^7)*d*cos(d*x + c
)^2 + (a^4*b^3 + 2*a^2*b^5 + b^7)*d)*sin(d*x + c))
```

3.145.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

output
```
Timed out
```

3.145.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.87

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = -\frac{3\,b^2\tan\left(dx+c\right)^2+3\,ab\tan\left(dx+c\right)+a^2+b^2}{3\,\left(b^6\tan\left(dx+c\right)^3+3\,ab^5\tan\left(dx+c\right)^2+3\,a^2b^4\tan\left(dx+c\right)+a^3b^3\right)d}$$

input
```
integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/3*(3*b^2*tan(d*x + c)^2 + 3*a*b*tan(d*x + c) + a^2 + b^2)/((b^6*tan(d*x
 + c)^3 + 3*a*b^5*tan(d*x + c)^2 + 3*a^2*b^4*tan(d*x + c) + a^3*b^3)*d)
```

3.145.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.51

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = -\frac{3\,b^2\tan\left(dx+c\right)^2+3\,ab\tan\left(dx+c\right)+a^2+b^2}{3\left(b\tan\left(dx+c\right)+a\right)^3b^3d}$$

input
```
integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/3*(3*b^2*tan(d*x + c)^2 + 3*a*b*tan(d*x + c) + a^2 + b^2)/((b*tan(d*x +
 c) + a)^3*b^3*d)
```

3.145.9 Mupad [B] (verification not implemented)

Time = 23.67 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.27

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$$

$$= \frac{\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{a}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a}-\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(a^2-2\,b^2\right)}{3\,a^3}+\frac{4\,b\tan\left(\frac{c}{2}+\frac{d}{2}\right.}{a^2}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(12\,a\,b^2-3\,a^3\right)-a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(12\,a\,b^2-3\,a^3\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(12\right.\right.}$$

input
```
int(1/(a*cos(c + d*x) + b*sin(c + d*x))^4,x)
```

output
```
((2*tan(c/2 + (d*x)/2)^5)/a + (2*tan(c/2 + (d*x)/2))/a - (4*tan(c/2 + (d*x
)/2)^3*(a^2 - 2*b^2))/(3*a^3) + (4*b*tan(c/2 + (d*x)/2)^2)/a^2 - (4*b*tan(
c/2 + (d*x)/2)^4)/a^2)/(d*(tan(c/2 + (d*x)/2)^2*(12*a*b^2 - 3*a^3) - a^3*t
an(c/2 + (d*x)/2)^6 - tan(c/2 + (d*x)/2)^4*(12*a*b^2 - 3*a^3) - tan(c/2 +
(d*x)/2)^3*(12*a^2*b - 8*b^3) + a^3 + 6*a^2*b*tan(c/2 + (d*x)/2) + 6*a^2*b
*tan(c/2 + (d*x)/2)^5))
```

3.146 $\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

3.146.1 Optimal result

Integrand size = 26, antiderivative size = 231

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{b^4d} + \frac{a\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2b^2(a^2+b^2)^{3/2}d} + \frac{a\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^4\sqrt{a^2+b^2}d} - \frac{1}{3bd(a\cos(c+dx)+b\sin(c+dx))^3} + \frac{a(b\cos(c+dx)-a\sin(c+dx))}{2b^2(a^2+b^2)d(a\cos(c+dx)+b\sin(c+dx))^2} - \frac{1}{b^3d(a\cos(c+dx)+b\sin(c+dx))}$$

output
```
arctanh(sin(d*x+c))/b^4/d+1/2*a*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b
^2)^(1/2))/b^2/(a^2+b^2)^(3/2)/d-1/3/b/d/(a*cos(d*x+c)+b*sin(d*x+c))^3+1/2
*a*(b*cos(d*x+c)-a*sin(d*x+c))/b^2/(a^2+b^2)/d/(a*cos(d*x+c)+b*sin(d*x+c))
^2-1/b^3/d/(a*cos(d*x+c)+b*sin(d*x+c))+a*arctanh((b*cos(d*x+c)-a*sin(d*x+c
))/(a^2+b^2)^(1/2))/b^4/d/(a^2+b^2)^(1/2)
```

3.146.2 Mathematica [A] (verified)

Time = 4.26 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.26

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = -\frac{\sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))\left(2b^3\sec(c+dx)+3b^2(a\cos(c+dx)+b\sin(c+dx))\tan(c\right.}{}$$

input
```
Integrate[Sec[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

output
```
-1/6*(Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])*(2*b^3*Sec[c + d*x]
 + 3*b^2*(a*Cos[c + d*x] + b*Sin[c + d*x])*Tan[c + d*x] + (3*b*(2*a^2 + b^
2)*Cos[c + d*x]*(a + b*Tan[c + d*x])^2)/(a^2 + b^2) + (6*a*(2*a^2 + 3*b^2)
*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]]*Cos[c + d*x]^2*(a + b*
Tan[c + d*x])^3)/(a^2 + b^2)^(3/2) + 6*Cos[c + d*x]^2*Log[Cos[(c + d*x)/2]
 - Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^3 - 6*Cos[c + d*x]^2*Log[Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^3))/(b^4*d*(a + b*Tan[c
 + d*x])^4)
```

3.146.3 Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.02, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3042, 3573, 3042, 3555, 3042, 3553, 219, 3573, 3042, 3553, 219, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4}dx$$

$$\Big\downarrow \text{ 3573}$$

$$-\frac{a\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^3}dx}{b^2}+\frac{\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}-\frac{1}{3bd(a\cos(c+dx)+b\sin(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^3}dx}{b^2}+\frac{\int \frac{1}{\cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}-\frac{1}{3bd(a\cos(c+dx)+b\sin(c+dx))^3}$$

$$\Big\downarrow \text{3555}$$

$$-\frac{a\left(\frac{\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{2(a^2+b^2)}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^2}\right)}{b^2}+\frac{\int \frac{1}{\cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}-\frac{1}{3bd(a\cos(c+dx)+b\sin(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\left(\frac{\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{2(a^2+b^2)}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^2}\right)}{b^2}+\frac{\int \frac{1}{\cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}-\frac{1}{3bd(a\cos(c+dx)+b\sin(c+dx))^3}$$

$$\Big\downarrow \text{3553}$$

$$-\frac{a\left(-\frac{\int \frac{1}{a^2+b^2-(b\cos(c+dx)-a\sin(c+dx))^2}d(b\cos(c+dx)-a\sin(c+dx))}{2d(a^2+b^2)}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^2}\right)}{b^2}+\frac{\int \frac{1}{\cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}-\frac{1}{3bd(a\cos(c+dx)+b\sin(c+dx))^3}$$

$$\Big\downarrow \text{219}$$

$$\frac{\int \frac{1}{\cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}-\frac{a\left(-\frac{\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2d(a^2+b^2)^{3/2}}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^2}\right)}{b^2}-\frac{1}{3bd(a\cos(c+dx)+b\sin(c+dx))^3}$$

$$\Big\downarrow \text{3573}$$

3.146. $\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

$$\frac{-\frac{a\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{b^2}+\frac{\int \sec(c+dx)dx}{b^2}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}}{b^2}-\frac{a\left(-\frac{\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2d(a^2+b^2)^{3/2}}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^2}\right)}{b^2}-\frac{1}{3bd(a\cos(c+dx)+b\sin(c+dx))^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{-\frac{a\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{b^2}+\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}}{b^2}-\frac{a\left(-\frac{\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2d(a^2+b^2)^{3/2}}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^2}\right)}{b^2}-\frac{1}{3bd(a\cos(c+dx)+b\sin(c+dx))^3}$$

$$\Big\downarrow \text{ 3553}$$

$$\frac{\frac{a\int \frac{1}{a^2+b^2-(b\cos(c+dx)-a\sin(c+dx))^2}d(b\cos(c+dx)-a\sin(c+dx))}{b^2d}+\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}}{b^2}-\frac{a\left(-\frac{\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2d(a^2+b^2)^{3/2}}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^2}\right)}{b^2}-\frac{1}{3bd(a\cos(c+dx)+b\sin(c+dx))^3}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}+\frac{a\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}}{b^2}-\frac{a\left(-\frac{\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2d(a^2+b^2)^{3/2}}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^2}\right)}{b^2}-\frac{1}{3bd(a\cos(c+dx)+b\sin(c+dx))^3}$$

$$\Big\downarrow \text{ 4257}$$

3.146. $\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

$$-\frac{a\left(-\frac{\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2d(a^2+b^2)^{3/2}}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^2}\right)}{b^2}+\frac{\frac{a\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}+\frac{\text{arctanh}(\sin(c+dx))}{b^2d}}{b^2}-\frac{1}{3bd(a\cos(c+dx)+b\sin(c+dx))^3}$$

input
```
Int[Sec[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

output
```
-1/3*1/(b*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (a*(-1/2*ArcTanh[(b*Cos
[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]]/((a^2 + b^2)^(3/2)*d) - (b*Co
s[c + d*x] - a*Sin[c + d*x])/(2*(a^2 + b^2)*d*(a*Cos[c + d*x] + b*Sin[c +
d*x])^2)))/b^2 + (ArcTanh[Sin[c + d*x]]/(b^2*d) + (a*ArcTanh[(b*Cos[c + d*
x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(b^2*Sqrt[a^2 + b^2]*d) - 1/(b*d*(a
*Cos[c + d*x] + b*Sin[c + d*x])))/b^2
```

3.146.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3555
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(b*Cos[c + d*x] - a*Sin[c + d*x])*((a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Simp[(n + 2)/((n + 1)*(a^
2 + b^2))   Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

rule 3573
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_)/co
s[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(a*Cos[c + d*x] + b*Sin[c + d*x])^
(n + 1)/(b*d*(n + 1)), x] + (Simp[1/b^2   Int[(a*Cos[c + d*x] + b*Sin[c + d
*x])^(n + 2)/Cos[c + d*x], x], x] - Simp[a/b^2   Int[(a*Cos[c + d*x] + b*Si
n[c + d*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
 && LtQ[n, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.146.4 Maple [A] (verified)

Time = 2.58 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.78

method	result
derivativedivides	$\frac{2\left(\frac{b^2\left(a^4+2a^2b^2+2b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{2\left(a^2+b^2\right)a}+\frac{b\left(2a^6-3a^4b^2-4a^2b^4-4b^6\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{2\left(a^2+b^2\right)a^2}-\frac{b^2\left(18a^6+3a^4b^2-4a^2b^4-4b^6\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3a^3\left(a^2+b^2\right)}-\cdots}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\cdots\right.\right.}$
default	$\frac{2\left(\frac{b^2\left(a^4+2a^2b^2+2b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{2\left(a^2+b^2\right)a}+\frac{b\left(2a^6-3a^4b^2-4a^2b^4-4b^6\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{2\left(a^2+b^2\right)a^2}-\frac{b^2\left(18a^6+3a^4b^2-4a^2b^4-4b^6\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3a^3\left(a^2+b^2\right)}-\cdots}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\cdots\right.\right.}$
risch	$-\frac{-6b^4e^{i(dx+c)}-15ia^3b\,e^{5i(dx+c)}+6a^4e^{5i(dx+c)}+12a^4e^{3i(dx+c)}+20b^4e^{3i(dx+c)}-6b^4e^{5i(dx+c)}+32a^2b^2e^{3i(dx+c)}-6a^2b\cdots}{3(ib+a)b^3\left(-ib\,e^{2i(dx+c)}+e^{2i(dx+c)}a+ib-\cdots\right.}$

input
```
int(sec(d*x+c)/(cos(d*x+c)*a+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/b^4*((1/2*b^2*(a^4+2*a^2*b^2+2*b^4)/(a^2+b^2)/a*tan(1/2*d*x+1/2*c)^
5+1/2*b*(2*a^6-3*a^4*b^2-4*a^2*b^4-4*b^6)/(a^2+b^2)/a^2*tan(1/2*d*x+1/2*c)
^4-1/3/a^3*b^2*(18*a^6+3*a^4*b^2-4*a^2*b^4-4*b^6)/(a^2+b^2)*tan(1/2*d*x+1/
2*c)^3-1/a^2*b*(2*a^6-8*a^4*b^2-7*a^2*b^4-2*b^6)/(a^2+b^2)*tan(1/2*d*x+1/2
*c)^2+1/2/a*b^2*(11*a^4+8*a^2*b^2+2*b^4)/(a^2+b^2)*tan(1/2*d*x+1/2*c)+1/6*
b*(6*a^4+5*a^2*b^2+2*b^4)/(a^2+b^2))/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d
*x+1/2*c)-a)^3-1/2*a*(2*a^2+3*b^2)/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tan(1/
2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))-1/b^4*ln(tan(1/2*d*x+1/2*c)-1)+1/b^4*l
n(tan(1/2*d*x+1/2*c)+1))
```

3.146.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 745 vs. $2(217) = 434$.

Time = 0.39 (sec) , antiderivative size = 745, normalized size of antiderivative = 3.23

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx =$$
$$-\frac{22\,a^4b^3+38\,a^2b^5+16\,b^7+12\,(a^6b-2\,a^2b^5-b^7)\cos{(dx+c)}^2+6\,(5\,a^5b^2+8\,a^3b^4+3\,ab^6)\cos{(dx+c)}}{}$$

input
```
integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

output
```
-1/12*(22*a^4*b^3 + 38*a^2*b^5 + 16*b^7 + 12*(a^6*b - 2*a^2*b^5 - b^7)*cos
(d*x + c)^2 + 6*(5*a^5*b^2 + 8*a^3*b^4 + 3*a*b^6)*cos(d*x + c)*sin(d*x + c
) - 3*((2*a^6 - 3*a^4*b^2 - 9*a^2*b^4)*cos(d*x + c)^3 + 3*(2*a^4*b^2 + 3*a
^2*b^4)*cos(d*x + c) + (2*a^3*b^3 + 3*a*b^5 + (6*a^5*b + 7*a^3*b^3 - 3*a*b
^5)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 + b^2)*log((2*a*b*cos(d*x + c)*
sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2
)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a
^2 - b^2)*cos(d*x + c)^2 + b^2)) - 6*((a^7 - a^5*b^2 - 5*a^3*b^4 - 3*a*b^6
)*cos(d*x + c)^3 + 3*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*cos(d*x + c) + (a^4*b^3
 + 2*a^2*b^5 + b^7 + (3*a^6*b + 5*a^4*b^3 + a^2*b^5 - b^7)*cos(d*x + c)^2)
*sin(d*x + c))*log(sin(d*x + c) + 1) + 6*((a^7 - a^5*b^2 - 5*a^3*b^4 - 3*a
*b^6)*cos(d*x + c)^3 + 3*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*cos(d*x + c) + (a^4
*b^3 + 2*a^2*b^5 + b^7 + (3*a^6*b + 5*a^4*b^3 + a^2*b^5 - b^7)*cos(d*x + c
)^2)*sin(d*x + c))*log(-sin(d*x + c) + 1))/((a^7*b^4 - a^5*b^6 - 5*a^3*b^8
 - 3*a*b^10)*d*cos(d*x + c)^3 + 3*(a^5*b^6 + 2*a^3*b^8 + a*b^10)*d*cos(d*x
 + c) + ((3*a^6*b^5 + 5*a^4*b^7 + a^2*b^9 - b^11)*d*cos(d*x + c)^2 + (a^4*
b^7 + 2*a^2*b^9 + b^11)*d)*sin(d*x + c))
```

3.146.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = \int \frac{\sec{(c+dx)}}{(a\cos{(c+dx)}+b\sin{(c+dx)})^4}\,dx$$

input
```
integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

output
```
Integral(sec(c + d*x)/(a*cos(c + d*x) + b*sin(c + d*x))**4, x)
```

3.146.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 661 vs. $2(217) = 434$.

Time = 0.35 (sec) , antiderivative size = 661, normalized size of antiderivative = 2.86

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$$

$$= \frac{\frac{3\,(2\,a^2+3\,b^2)a\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{(a^2b^4+b^6)\sqrt{a^2+b^2}} - \frac{2\left(6\,a^7+5\,a^5b^2+2\,a^3b^4+\frac{3\,\left(11\,a^6b+8\,a^4b^3+2\,a^2b^5\right)\sin(dx+c)}{\cos(dx+c)+1}-\frac{6\,\left(2\,a^7-8\,a^5b^2-7\,a^3b^4-2\,ab^6\right)\sin(}{(\cos(dx+c)+1)^2}\right.}{a^8b^3+a^6b^5+\frac{6\,\left(a^7b^4+a^5b^6\right)\sin(dx+c)}{\cos(dx+c)+1}-\frac{3\,\left(a^8b^3-3\,a^6b^5-4\,a^4b^7\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{4\,\left(3\,a^7b\right.}{}}}{}$$

input
```
integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/6*(3*(2*a^2 + 3*b^2)*a*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt
(a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/((
a^2*b^4 + b^6)*sqrt(a^2 + b^2)) - 2*(6*a^7 + 5*a^5*b^2 + 2*a^3*b^4 + 3*(11
*a^6*b + 8*a^4*b^3 + 2*a^2*b^5)*sin(d*x + c)/(cos(d*x + c) + 1) - 6*(2*a^7
 - 8*a^5*b^2 - 7*a^3*b^4 - 2*a*b^6)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 -
2*(18*a^6*b + 3*a^4*b^3 - 4*a^2*b^5 - 4*b^7)*sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3 + 3*(2*a^7 - 3*a^5*b^2 - 4*a^3*b^4 - 4*a*b^6)*sin(d*x + c)^4/(cos(d
*x + c) + 1)^4 + 3*(a^6*b + 2*a^4*b^3 + 2*a^2*b^5)*sin(d*x + c)^5/(cos(d*x
 + c) + 1)^5)/(a^8*b^3 + a^6*b^5 + 6*(a^7*b^4 + a^5*b^6)*sin(d*x + c)/(cos
(d*x + c) + 1) - 3*(a^8*b^3 - 3*a^6*b^5 - 4*a^4*b^7)*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 - 4*(3*a^7*b^4 + a^5*b^6 - 2*a^3*b^8)*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 + 3*(a^8*b^3 - 3*a^6*b^5 - 4*a^4*b^7)*sin(d*x + c)^4/(cos(d
*x + c) + 1)^4 + 6*(a^7*b^4 + a^5*b^6)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5
 - (a^8*b^3 + a^6*b^5)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + 6*log(sin(d*
x + c)/(cos(d*x + c) + 1) + 1)/b^4 - 6*log(sin(d*x + c)/(cos(d*x + c) + 1)
 - 1)/b^4)/d
```

3.146.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 527 vs. $2(217) = 434$.

Time = 0.39 (sec) , antiderivative size = 527, normalized size of antiderivative = 2.28

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$$

$$= \frac{\frac{3\,(2\,a^3+3\,ab^2)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^2b^4+b^6)\sqrt{a^2+b^2}} + \frac{2\left(3\,a^6b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,a^4b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,a^2b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,a^7\tan(\right.}{}}{}$$

input
```
integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

output
```
1/6*(3*(2*a^3 + 3*a*b^2)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a
^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^2*
b^4 + b^6)*sqrt(a^2 + b^2)) + 2*(3*a^6*b*tan(1/2*d*x + 1/2*c)^5 + 6*a^4*b^
3*tan(1/2*d*x + 1/2*c)^5 + 6*a^2*b^5*tan(1/2*d*x + 1/2*c)^5 + 6*a^7*tan(1/
2*d*x + 1/2*c)^4 - 9*a^5*b^2*tan(1/2*d*x + 1/2*c)^4 - 12*a^3*b^4*tan(1/2*d
*x + 1/2*c)^4 - 12*a*b^6*tan(1/2*d*x + 1/2*c)^4 - 36*a^6*b*tan(1/2*d*x + 1
/2*c)^3 - 6*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 + 8*a^2*b^5*tan(1/2*d*x + 1/2*c
)^3 + 8*b^7*tan(1/2*d*x + 1/2*c)^3 - 12*a^7*tan(1/2*d*x + 1/2*c)^2 + 48*a^
5*b^2*tan(1/2*d*x + 1/2*c)^2 + 42*a^3*b^4*tan(1/2*d*x + 1/2*c)^2 + 12*a*b^
6*tan(1/2*d*x + 1/2*c)^2 + 33*a^6*b*tan(1/2*d*x + 1/2*c) + 24*a^4*b^3*tan(
1/2*d*x + 1/2*c) + 6*a^2*b^5*tan(1/2*d*x + 1/2*c) + 6*a^7 + 5*a^5*b^2 + 2*
a^3*b^4)/((a^5*b^3 + a^3*b^5)*(a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x
+ 1/2*c) - a)^3) + 6*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^4 - 6*log(abs(ta
n(1/2*d*x + 1/2*c) - 1))/b^4)/d
```

3.146.9 Mupad [B] (verification not implemented)

Time = 27.55 (sec) , antiderivative size = 2848, normalized size of antiderivative = 12.33

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(c + d*x)*(a*cos(c + d*x) + b*sin(c + d*x))^4),x)
```

output
```
(2*atanh((64*a*b^5*tan(c/2 + (d*x)/2))/((176*a^3*b^15)/(b^12 + 2*a^2*b^10
+ a^4*b^8) + (160*a^5*b^13)/(b^12 + 2*a^2*b^10 + a^4*b^8) + (48*a^7*b^11)/
(b^12 + 2*a^2*b^10 + a^4*b^8) + (64*a*b^17)/(b^12 + 2*a^2*b^10 + a^4*b^8))
 + (48*a^3*b^3*tan(c/2 + (d*x)/2))/((176*a^3*b^15)/(b^12 + 2*a^2*b^10 + a^
4*b^8) + (160*a^5*b^13)/(b^12 + 2*a^2*b^10 + a^4*b^8) + (48*a^7*b^11)/(b^1
2 + 2*a^2*b^10 + a^4*b^8) + (64*a*b^17)/(b^12 + 2*a^2*b^10 + a^4*b^8))))/(
b^4*d) - ((6*a^4 + 2*b^4 + 5*a^2*b^2)/(3*b^3*(a^2 + b^2)) + (tan(c/2 + (d*
x)/2)*(11*a^4 + 2*b^4 + 8*a^2*b^2))/(a*b^2*(a^2 + b^2)) + (tan(c/2 + (d*x)
/2)^5*(a^4 + 2*b^4 + 2*a^2*b^2))/(a*b^2*(a^2 + b^2)) - (tan(c/2 + (d*x)/2)
^4*(4*b^6 - 2*a^6 + 4*a^2*b^4 + 3*a^4*b^2))/(a^2*b^3*(a^2 + b^2)) + (2*tan
(c/2 + (d*x)/2)^2*(2*b^6 - 2*a^6 + 7*a^2*b^4 + 8*a^4*b^2))/(a^2*b^3*(a^2 +
 b^2)) - (2*tan(c/2 + (d*x)/2)^3*(3*a^2 - 2*b^2)*(6*a^4 + 2*b^4 + 5*a^2*b^
2))/(3*a^3*b^2*(a^2 + b^2)))/(d*(tan(c/2 + (d*x)/2)^2*(12*a*b^2 - 3*a^3) -
 a^3*tan(c/2 + (d*x)/2)^6 - tan(c/2 + (d*x)/2)^4*(12*a*b^2 - 3*a^3) - tan(
c/2 + (d*x)/2)^3*(12*a^2*b - 8*b^3) + a^3 + 6*a^2*b*tan(c/2 + (d*x)/2) + 6
*a^2*b*tan(c/2 + (d*x)/2)^5)) - (a*atan(((a*((a^2 + b^2)^3)^(1/2)*(2*a^2 +
 3*b^2)*((8*(4*a^2*b^7 + 8*a^4*b^5 + 4*a^6*b^3))/(b^12 + 2*a^2*b^10 + a^4*
b^8) + (8*tan(c/2 + (d*x)/2)*(8*a*b^9 + 29*a^3*b^7 + 28*a^5*b^5 + 8*a^7*b^
3))/(b^13 + 2*a^2*b^11 + a^4*b^9) - (a*((a^2 + b^2)^3)^(1/2)*(2*a^2 + 3*b^
2)*((8*tan(c/2 + (d*x)/2)*(12*a^2*b^12 + 20*a^4*b^10 + 8*a^6*b^8))/(b^1...
```

3.146. $\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

3.147 $\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

3.147.1 Optimal result

Integrand size = 28, antiderivative size = 138

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = \frac{(a^2+b^2)^2}{3a^3b^2d(b+a\cot(c+dx))^3} + \frac{\frac{a}{b^3}-\frac{b}{a^3}}{d(b+a\cot(c+dx))^2} + \frac{\frac{1}{a^3}+\frac{3a}{b^4}}{d(b+a\cot(c+dx))} - \frac{4a\log(b+a\cot(c+dx))}{b^5d} - \frac{4a\log(\tan(c+dx))}{b^5d} + \frac{\tan(c+dx)}{b^4d}$$

output
```
1/3*(a^2+b^2)^2/a^3/b^2/d/(b+a*cot(d*x+c))^3+(a/b^3-b/a^3)/d/(b+a*cot(d*x+
c))^2+(1/a^3+3*a/b^4)/d/(b+a*cot(d*x+c))-4*a*ln(b+a*cot(d*x+c))/b^5/d-4*a*
ln(tan(d*x+c))/b^5/d+tan(d*x+c)/b^4/d
```

3.147.2 Mathematica [A] (verified)

Time = 2.38 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.96

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = \frac{3b^4\sec^4(c+dx) - 4(a^2+b^2)\left(a^2+b^2+3ab\tan(c+dx)+3b^2\tan^2(c+dx)\right) + 6a(a+b\tan(c+dx))\,(a^2 -}{3b^5d(a+b\tan(c+dx))^3}$$

input
```
Integrate[Sec[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

output
```
(3*b^4*Sec[c + d*x]^4 - 4*(a^2 + b^2)*(a^2 + b^2 + 3*a*b*Tan[c + d*x] + 3*
b^2*Tan[c + d*x]^2) + 6*a*(a + b*Tan[c + d*x])*(a^2 + b^2 - 4*a*(a + b*Tan
[c + d*x]) - 2*Log[a + b*Tan[c + d*x]]*(a + b*Tan[c + d*x])^2))/(3*b^5*d*(
a + b*Tan[c + d*x])^3)
```

3.147.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3567, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))^4}dx$$

$$\Big\downarrow \text{3567}$$

$$-\frac{\int \frac{\left(\cot^2(c+dx)+1\right)^2\tan^2(c+dx)}{(b+a\cot(c+dx))^4}d\cot(c+dx)}{d}$$

$$\Big\downarrow \text{522}$$

$$-\frac{\int \left(\frac{4a^2}{b^5(b+a\cot(c+dx))}-\frac{4\tan(c+dx)a}{b^5}+\frac{\tan^2(c+dx)}{b^4}+\frac{3a^4+b^4}{b^4(b+a\cot(c+dx))^2a^2}+\frac{2\left(a^4-b^4\right)}{b^3(b+a\cot(c+dx))^3a^2}+\frac{\left(a^2+b^2\right)^2}{b^2(b+a\cot(c+dx))^4a^2}\right)d}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{\frac{1}{a^3}+\frac{3a}{b^4}}{a\cot(c+dx)+b}-\frac{\frac{a}{b^3}-\frac{b}{a^3}}{(a\cot(c+dx)+b)^2}-\frac{\left(a^2+b^2\right)^2}{3a^3b^2(a\cot(c+dx)+b)^3}-\frac{4a\log(\cot(c+dx))}{b^5}+\frac{4a\log(a\cot(c+dx)+b)}{b^5}-\frac{\tan(c+dx)}{b^4}}{d}$$

input
```
Int[Sec[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

output
```
-((-1/3*(a^2 + b^2)^2/(a^3*b^2*(b + a*Cot[c + d*x])^3) - (a/b^3 - b/a^3)/(
b + a*Cot[c + d*x])^2 - (a^(-3) + (3*a)/b^4)/(b + a*Cot[c + d*x]) - (4*a*L
og[Cot[c + d*x]])/b^5 + (4*a*Log[b + a*Cot[c + d*x]])/b^5 - Tan[c + d*x]/b
^4)/d)
```

3.147.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3567
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[x^m*((b
+ a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a,
b, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[
n, 0] && GtQ[m, 1])
```

3.147.4 Maple [A] (verified)

Time = 2.99 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{\frac{\tan(dx+c)}{b^4}-\frac{a^4+2a^2b^2+b^4}{3b^5(a+b\tan(dx+c))^3}+\frac{2a(a^2+b^2)}{b^5(a+b\tan(dx+c))^2}-\frac{6a^2+2b^2}{b^5(a+b\tan(dx+c))}-\frac{4a\ln(a+b\tan(dx+c))}{b^5}}{d}$
default	$\frac{\frac{\tan(dx+c)}{b^4}-\frac{a^4+2a^2b^2+b^4}{3b^5(a+b\tan(dx+c))^3}+\frac{2a(a^2+b^2)}{b^5(a+b\tan(dx+c))^2}-\frac{6a^2+2b^2}{b^5(a+b\tan(dx+c))}-\frac{4a\ln(a+b\tan(dx+c))}{b^5}}{d}$
risch	$\frac{8i(iab^3e^{2i(dx+c)}+3ia^3be^{2i(dx+c)}+3a^4e^{6i(dx+c)}-9a^2b^2e^{6i(dx+c)}-9ia^3be^{6i(dx+c)}+6ia^3b+9a^4e^{4i(dx+c)}+3a^2b^2e^{4i(dx+}}{3(e^{2i(dx+c)}+1)(be^{2i(dx+c}}$
norman	$\frac{-\frac{8a^4+2b^4}{6b^5d}-\frac{(8a^4+2b^4)\tan(\frac{dx}{2}+\frac{c}{2})^8}{6b^5d}+\frac{(144a^3+16ab^2)\tan(\frac{dx}{2}+\frac{c}{2})^3}{3da^2b^2}-\frac{(144a^3+16ab^2)\tan(\frac{dx}{2}+\frac{c}{2})^5}{3da^2b^2}-\frac{(8a^5+48a^3b^2-14ab^4)}{dab^5}}{(\tan(\frac{dx}{2}+\frac{c}{2})^2-1)(\tan(\frac{dx}{2}+\frac{c}{2})^2a-2b\tan(}$
parallelrisch	$\frac{4((-a^5+3a^3b^2)\cos(4dx+4c)+2(-3a^4b-a^2b^3)\sin(2dx+2c)+(-3a^4b+a^2b^3)\sin(4dx+4c)-4a^5\cos(2dx+2c)-3a^5-3a^3}{}$

input
```
int(sec(d*x+c)^2/(cos(d*x+c)*a+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(tan(d*x+c)/b^4-1/3/b^5*(a^4+2*a^2*b^2+b^4)/(a+b*tan(d*x+c))^3+2*a/b^5
*(a^2+b^2)/(a+b*tan(d*x+c))^2-(6*a^2+2*b^2)/b^5/(a+b*tan(d*x+c))-4*a/b^5*l
n(a+b*tan(d*x+c)))
```

3.147.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 537 vs. $2(136) = 272$.

Time = 0.31 (sec) , antiderivative size = 537, normalized size of antiderivative = 3.89

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = \frac{3\,a^2b^4+3\,b^6-4\,(9\,a^4b^2+3\,a^2b^4-2\,b^6)\cos(dx+c)^4+6\,(5\,a^4b^2+a^2b^4-2\,b^6)\cos(dx+c)^2-6\,((a^6-2\,a}{}$$

input
```
integrate(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas"
)
```

output
```
1/3*(3*a^2*b^4 + 3*b^6 - 4*(9*a^4*b^2 + 3*a^2*b^4 - 2*b^6)*cos(d*x + c)^4
+ 6*(5*a^4*b^2 + a^2*b^4 - 2*b^6)*cos(d*x + c)^2 - 6*((a^6 - 2*a^4*b^2 - 3
*a^2*b^4)*cos(d*x + c)^4 + 3*(a^4*b^2 + a^2*b^4)*cos(d*x + c)^2 + ((3*a^5*
b + 2*a^3*b^3 - a*b^5)*cos(d*x + c)^3 + (a^3*b^3 + a*b^5)*cos(d*x + c))*si
n(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)
^2 + b^2) + 6*((a^6 - 2*a^4*b^2 - 3*a^2*b^4)*cos(d*x + c)^4 + 3*(a^4*b^2 +
 a^2*b^4)*cos(d*x + c)^2 + ((3*a^5*b + 2*a^3*b^3 - a*b^5)*cos(d*x + c)^3 +
 (a^3*b^3 + a*b^5)*cos(d*x + c))*sin(d*x + c))*log(cos(d*x + c)^2) + 2*(2*
(3*a^5*b - 7*a^3*b^3 - 6*a*b^5)*cos(d*x + c)^3 + (11*a^3*b^3 + 9*a*b^5)*co
s(d*x + c))*sin(d*x + c))/((a^5*b^5 - 2*a^3*b^7 - 3*a*b^9)*d*cos(d*x + c)^
4 + 3*(a^3*b^7 + a*b^9)*d*cos(d*x + c)^2 + ((3*a^4*b^6 + 2*a^2*b^8 - b^10)
*d*cos(d*x + c)^3 + (a^2*b^8 + b^10)*d*cos(d*x + c))*sin(d*x + c))
```

3.147.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = \int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

output
```
Integral(sec(c + d*x)**2/(a*cos(c + d*x) + b*sin(c + d*x))**4, x)
```

3.147.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.04

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = -\frac{\frac{13\,a^4+2\,a^2b^2+b^4+6\,(3\,a^2b^2+b^4)\tan(dx+c)^2+6\,(5\,a^3b+ab^3)\tan(dx+c)}{b^8\tan(dx+c)^3+3\,ab^7\tan(dx+c)^2+3\,a^2b^6\tan(dx+c)+a^3b^5}+\frac{12\,a\log(b\tan(dx+c)+a)}{b^5}-\frac{3\,\tan(dx+c)}{b^4}}{3\,d}$$

input
```
integrate(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima"
)
```

output
```
-1/3*((13*a^4 + 2*a^2*b^2 + b^4 + 6*(3*a^2*b^2 + b^4)*tan(d*x + c)^2 + 6*(
5*a^3*b + a*b^3)*tan(d*x + c))/(b^8*tan(d*x + c)^3 + 3*a*b^7*tan(d*x + c)^
2 + 3*a^2*b^6*tan(d*x + c) + a^3*b^5) + 12*a*log(b*tan(d*x + c) + a)/b^5 -
 3*tan(d*x + c)/b^4)/d
```

3.147.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx =$$
$$-\frac{\frac{12\,a\log(|b\tan(dx+c)+a|)}{b^5}-\frac{3\,\tan(dx+c)}{b^4}-\frac{22\,ab^3\tan(dx+c)^3+48\,a^2b^2\tan(dx+c)^2-6\,b^4\tan(dx+c)^2+36\,a^3b\tan(dx+c)-6\,ab^3\tan(dx+c}{(b\tan(dx+c)+a)^3b^5}}{3\,d}$$

input
```
integrate(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/3*(12*a*log(abs(b*tan(d*x + c) + a))/b^5 - 3*tan(d*x + c)/b^4 - (22*a*b
^3*tan(d*x + c)^3 + 48*a^2*b^2*tan(d*x + c)^2 - 6*b^4*tan(d*x + c)^2 + 36*
a^3*b*tan(d*x + c) - 6*a*b^3*tan(d*x + c) + 9*a^4 - 2*a^2*b^2 - b^4)/((b*t
an(d*x + c) + a)^3*b^5))/d
```

3.147.9 Mupad [B] (verification not implemented)

Time = 27.24 (sec) , antiderivative size = 666, normalized size of antiderivative = 4.83

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$$
$$=\frac{\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(10\,a^4+b^4)}{a^2\,b^3}-\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7(4\,a^4+b^4)}{a\,b^4}-\frac{8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4(10\,a^4-2\,a^2\,b^2+b^4)}{a^2\,b^3}+\frac{4\tan\left(\frac{c}{2}+\frac{d}{2}\right.}{}}{d\left(a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(12\,a\,b^2-4\,a^3)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6(12\,a\,b^2-4\,a^3)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4(24\right.}$$
$$-\frac{8\,a\,\mathrm{atanh}\left(\frac{256\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{256\,a^3-256\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+\frac{512\,a^5}{b^2}-\frac{512\,a^5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{b^2}+\frac{512\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{b}}-\frac{256\,a^3}{256\,a^3-256\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+\frac{512\,a^5}{b^2}-512}\right.}{b^5\,d}$$

input
```
int(1/(cos(c + d*x)^2*(a*cos(c + d*x) + b*sin(c + d*x))^4),x)
```

output
```
((4*tan(c/2 + (d*x)/2)^2*(10*a^4 + b^4))/(a^2*b^3) - (2*tan(c/2 + (d*x)/2)
^7*(4*a^4 + b^4))/(a*b^4) - (8*tan(c/2 + (d*x)/2)^4*(10*a^4 + b^4 - 2*a^2*
b^2))/(a^2*b^3) + (4*tan(c/2 + (d*x)/2)^6*(10*a^4 + b^4))/(a^2*b^3) - (2*t
an(c/2 + (d*x)/2)^3*(36*a^6 - 4*b^6 + a^2*b^4 - 88*a^4*b^2))/(3*a^3*b^4) +
 (2*tan(c/2 + (d*x)/2)^5*(36*a^6 - 4*b^6 + a^2*b^4 - 88*a^4*b^2))/(3*a^3*b
^4) + (2*tan(c/2 + (d*x)/2)*(4*a^4 + b^4))/(a*b^4))/(d*(a^3*tan(c/2 + (d*x
)/2)^8 + tan(c/2 + (d*x)/2)^2*(12*a*b^2 - 4*a^3) + tan(c/2 + (d*x)/2)^6*(1
2*a*b^2 - 4*a^3) - tan(c/2 + (d*x)/2)^4*(24*a*b^2 - 6*a^3) - tan(c/2 + (d*
x)/2)^3*(18*a^2*b - 8*b^3) + tan(c/2 + (d*x)/2)^5*(18*a^2*b - 8*b^3) + a^3
 + 6*a^2*b*tan(c/2 + (d*x)/2) - 6*a^2*b*tan(c/2 + (d*x)/2)^7)) - (8*a*atan
h((256*a^3*tan(c/2 + (d*x)/2)^2)/(256*a^3 - 256*a^3*tan(c/2 + (d*x)/2)^2 +
 (512*a^5)/b^2 - (512*a^5*tan(c/2 + (d*x)/2)^2)/b^2 + (512*a^4*tan(c/2 + (
d*x)/2))/b) - (256*a^3)/(256*a^3 - 256*a^3*tan(c/2 + (d*x)/2)^2 + (512*a^5
)/b^2 - (512*a^5*tan(c/2 + (d*x)/2)^2)/b^2 + (512*a^4*tan(c/2 + (d*x)/2))/
b) + (512*a^4*tan(c/2 + (d*x)/2))/(256*a^3*b + (512*a^5)/b + 512*a^4*tan(c
/2 + (d*x)/2) - (512*a^5*tan(c/2 + (d*x)/2)^2)/b - 256*a^3*b*tan(c/2 + (d*
x)/2)^2)))/(b^5*d)
```

3.147. $\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

3.148 $\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

3.148.1 Optimal result

Integrand size = 28, antiderivative size = 400

$$\begin{aligned}\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx &= \frac{8a^2\operatorname{arctanh}(\sin(c+dx))}{b^6d}+\frac{\operatorname{arctanh}(\sin(c+dx))}{2b^4d}\\
&+\frac{2(a^2+b^2)\operatorname{arctanh}(\sin(c+dx))}{b^6d}\\
&+\frac{4a^3\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^6\sqrt{a^2+b^2}\,d}\\
&+\frac{3a\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2b^4\sqrt{a^2+b^2}\,d}\\
&+\frac{6a\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^6d}\\
&-\frac{4a\sec(c+dx)}{b^5d}\\
&-\frac{a^2+b^2}{3b^3d(a\cos(c+dx)+b\sin(c+dx))^3}\\
&+\frac{3a(b\cos(c+dx)-a\sin(c+dx))}{2b^4d(a\cos(c+dx)+b\sin(c+dx))^2}\\
&-\frac{4a^2}{b^5d(a\cos(c+dx)+b\sin(c+dx))}\\
&-\frac{2(a^2+b^2)}{b^5d(a\cos(c+dx)+b\sin(c+dx))}\\
&+\frac{\sec(c+dx)\tan(c+dx)}{2b^4d}\end{aligned}$$

output
```
8*a^2*arctanh(sin(d*x+c))/b^6/d+1/2*arctanh(sin(d*x+c))/b^4/d+2*(a^2+b^2)*
arctanh(sin(d*x+c))/b^6/d-4*a*sec(d*x+c)/b^5/d+1/3*(-a^2-b^2)/b^3/d/(a*cos
(d*x+c)+b*sin(d*x+c))^3+3/2*a*(b*cos(d*x+c)-a*sin(d*x+c))/b^4/d/(a*cos(d*x
+c)+b*sin(d*x+c))^2-4*a^2/b^5/d/(a*cos(d*x+c)+b*sin(d*x+c))-2*(a^2+b^2)/b^
5/d/(a*cos(d*x+c)+b*sin(d*x+c))+4*a^3*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/
(a^2+b^2)^(1/2))/b^6/d/(a^2+b^2)^(1/2)+3/2*a*arctanh((b*cos(d*x+c)-a*sin(d
*x+c))/(a^2+b^2)^(1/2))/b^4/d/(a^2+b^2)^(1/2)+6*a*arctanh((b*cos(d*x+c)-a*
sin(d*x+c))/(a^2+b^2)^(1/2))*(a^2+b^2)^(1/2)/b^6/d+1/2*sec(d*x+c)*tan(d*x+
c)/b^4/d
```

3.148.2 Mathematica [A] (verified)

Time = 4.48 (sec) , antiderivative size = 538, normalized size of antiderivative = 1.34

$$\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx =$$

$$-\frac{\sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))\left(4b^3(a^2+b^2)+18b^2(a^2+b^2)\sin(c+dx)(a\cos(c+dx)+b\right.}{}$$

input
```
Integrate[Sec[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

output
```
-1/12*(Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])*(4*b^3*(a^2 + b^2)
 + 18*b^2*(a^2 + b^2)*Sin[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x]) + 6*b
*(12*a^2 + b^2)*(a*Cos[c + d*x] + b*Sin[c + d*x])^2 + 48*a*b*(a*Cos[c + d*
x] + b*Sin[c + d*x])^3 + (60*a*(4*a^2 + 3*b^2)*ArcTanh[(-b + a*Tan[(c + d*
x)/2])/Sqrt[a^2 + b^2]]*(a*Cos[c + d*x] + b*Sin[c + d*x])^3)/Sqrt[a^2 + b^
2] + 30*(4*a^2 + b^2)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a*Cos[c +
d*x] + b*Sin[c + d*x])^3 - 30*(4*a^2 + b^2)*Log[Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2]]*(a*Cos[c + d*x] + b*Sin[c + d*x])^3 - (3*b^2*(a*Cos[c + d*x] +
b*Sin[c + d*x])^3)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (48*a*b*Sin[(
c + d*x)/2]*(a*Cos[c + d*x] + b*Sin[c + d*x])^3)/(Cos[(c + d*x)/2] - Sin[(
c + d*x)/2]) + (3*b^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3)/(Cos[(c + d*x)/
2] + Sin[(c + d*x)/2])^2 - (48*a*b*Sin[(c + d*x)/2]*(a*Cos[c + d*x] + b*Si
n[c + d*x])^3)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))/(b^6*d*(a + b*Tan[c
 + d*x])^4)
```

3.148.3 Rubi [A] (verified)

Time = 4.27 (sec) , antiderivative size = 782, normalized size of antiderivative = 1.96, number of steps used = 31, number of rules used = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.071, Rules used = {3042, 3585, 3042, 3573, 3042, 3555, 3042, 3553, 219, 3573, 3042, 3553, 219, 3585, 3042, 3555, 3042, 3553, 219, 3573, 3042, 3553, 219, 3583, 3042, 3553, 219, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\cos(c+dx)^3(a\cos(c+dx)+b\sin(c+dx))^4}dx$$

$$\Big\downarrow \ 3585$$

$$\frac{(a^2+b^2)\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}dx}{b^2}+\frac{\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}-\frac{2a\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}dx}{b^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{(a^2+b^2)\int \frac{1}{\cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4}dx}{b^2}-\frac{2a\int \frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))^3}dx}{b^2}+\frac{\int \frac{1}{\cos(c+dx)^3(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}$$

$$\Big\downarrow \ 3573$$

$$\frac{(a^2+b^2)\left(-\frac{a\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^3}dx}{b^2}+\frac{\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}-\frac{1}{3bd(a\cos(c+dx)+b\sin(c+dx))^3}\right)}{b^2}-\frac{2a\int \frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))^3}dx}{b^2}+\frac{\int \frac{1}{\cos(c+dx)^3(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{(a^2+b^2)\left(-\frac{a\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^3}dx}{b^2}+\frac{\int \frac{1}{\cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}-\frac{1}{3bd(a\cos(c+dx)+b\sin(c+dx))^3}\right)}{b^2}-\frac{2a\int \frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))^3}dx}{b^2}+\frac{\int \frac{1}{\cos(c+dx)^3(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}$$

$$\Big\downarrow \ 3555$$

$$\frac{(a^2+b^2)\left(-\frac{a\left(\frac{\int\frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{2\left(a^2+b^2\right)}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^2}\right)}{b^2}+\frac{\int\frac{1}{\cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}-\frac{}{3bd(a\cos(}\right.}{b^2}$$

$$\frac{2a\int\frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))^3}dx}{b^2}+\frac{\int\frac{1}{\cos(c+dx)^3(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{(a^2+b^2)\left(-\frac{a\left(\frac{\int\frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{2\left(a^2+b^2\right)}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^2}\right)}{b^2}+\frac{\int\frac{1}{\cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}-\frac{}{3bd(a\cos(}\right.}{b^2}$$

$$\frac{2a\int\frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))^3}dx}{b^2}+\frac{\int\frac{1}{\cos(c+dx)^3(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}$$

$$\Big\downarrow \; 3553$$

$$\frac{(a^2+b^2)\left(-\frac{a\left(-\frac{\int\frac{1}{a^2+b^2-(b\cos(c+dx)-a\sin(c+dx))^2}d(b\cos(c+dx)-a\sin(c+dx))}{2d\left(a^2+b^2\right)}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^2}\right)}{b^2}+\frac{\int\frac{}{\cos(c+dx)(a\cos(c+}}{b}\right.}{b^2}$$

$$\frac{2a\int\frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))^3}dx}{b^2}+\frac{\int\frac{1}{\cos(c+dx)^3(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}$$

$$\Big\downarrow \; 219$$

$$\frac{(a^2+b^2)\left(\frac{\int\frac{1}{\cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}-\frac{a\left(-\frac{\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2d\left(a^2+b^2\right)^{3/2}}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^2}\right)}{b^2}-\frac{}{3}\right.}{b^2}$$

$$\frac{2a\int\frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))^3}dx}{b^2}+\frac{\int\frac{1}{\cos(c+dx)^3(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}$$

$$\Big\downarrow \; 3573$$

3.148. $\int\frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

$$\frac{(a^2+b^2)\left(\frac{-\frac{a\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{b^2}+\frac{\int \sec(c+dx)dx}{b^2}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}}{b^2}-\frac{a\left(-\frac{\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2d\left(a^2+b^2\right)^{3/2}}-\frac{b\,c}{2d\left(a^2+b^2\right.}\cdots\right.}{b^2}\cdots\right.}{b^2}$$

$$\frac{2a\int \frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))^3}dx}{b^2}+\frac{\int \frac{1}{\cos(c+dx)^3(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{(a^2+b^2)\left(\frac{-\frac{a\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{b^2}+\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}}{b^2}-\frac{a\left(-\frac{\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2d\left(a^2+b^2\right)^{3/2}}-\frac{\cdots}{2d\left(a\right.}\cdots\right.}{b^2}\cdots\right.}{b^2}$$

$$\frac{2a\int \frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))^3}dx}{b^2}+\frac{\int \frac{1}{\cos(c+dx)^3(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}$$

$$\Big\downarrow \; 3553$$

$$\frac{(a^2+b^2)\left(\frac{\frac{a\int \frac{1}{a^2+b^2-(b\cos(c+dx)-a\sin(c+dx))^2}d(b\cos(c+dx)-a\sin(c+dx))}{b^2d}+\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}}{b^2}-\frac{a\left(-\frac{\text{arctanh}\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{b^2}$$

$$\frac{2a\int \frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))^3}dx}{b^2}+\frac{\int \frac{1}{\cos(c+dx)^3(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}$$

$$\Big\downarrow \; 219$$

$$\frac{(a^2+b^2)\left(\frac{\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}+\frac{a\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}}{b^2}-\frac{a\left(-\frac{\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2d\left(a^2+b^2\right)^{3/2}}\cdots\right.}{b^2}\cdots\right.}{b^2}$$

$$\frac{2a\int \frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))^3}dx}{b^2}+\frac{\int \frac{1}{\cos(c+dx)^3(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}$$

$$\Big\downarrow \; 3585$$

3.148. $\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

$$\frac{(a^2+b^2)\left(\frac{\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}+\frac{a\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}}{b^2}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}-\frac{a\left(-\frac{\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2d\left(a^2+b^2\right)^{3/2}}\right.}{b^{2}}\right.}{b^2}$$

$$\frac{2a\left(\frac{(a^2+b^2)\int\frac{1}{(a\cos(c+dx)+b\sin(c+dx))^3}dx}{b^2}+\frac{\int\frac{\sec^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx}{b^2}-\frac{2a\int\frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}\right)}{b^2}+$$

$$\frac{\frac{(a^2+b^2)\int\frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}-\frac{2a\int\frac{\sec^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}dx}{b^2}+\frac{\int\sec^3(c+dx)dx}{b^2}}{b^2}$$

$\downarrow$ 3042

$$\frac{(a^2+b^2)\left(\frac{\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}+\frac{a\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}}{b^2}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}-\frac{a\left(-\frac{\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2d\left(a^2+b^2\right)^{3/2}}\right.}{b^{2}}\right.}{b^2}$$

$$\frac{2a\left(\frac{(a^2+b^2)\int\frac{1}{(a\cos(c+dx)+b\sin(c+dx))^3}dx}{b^2}-\frac{2a\int\frac{1}{\cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}+\frac{\int\frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))}dx}{b^2}\right)}{b^2}+$$

$$\frac{\frac{(a^2+b^2)\int\frac{1}{\cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}-\frac{2a\int\frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))}dx}{b^2}+\frac{\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{b^2}}{b^2}$$

$\downarrow$ 3555

$$\frac{(a^2+b^2)\left(\frac{\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}+\frac{a\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}}{b^2}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}-\frac{a\left(-\frac{\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2d\left(a^2+b^2\right)^{3/2}}\right.}{b^{2}}\right.}{b^2}$$

$$\frac{2a\left(\frac{(a^2+b^2)\left(\frac{\int\frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{2\left(a^2+b^2\right)}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^2}\right)}{b^2}-\frac{2a\int\frac{1}{\cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}+\frac{\int\frac{}{\cos(c+dx)^2}}{}\right.}{b^2}$$

$$\frac{\frac{(a^2+b^2)\int\frac{1}{\cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}-\frac{2a\int\frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))}dx}{b^2}+\frac{\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{b^2}}{b^2}$$

$\downarrow$ 3042

3.148. $\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

$$\frac{(a^2+b^2)\left(\frac{\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}+\frac{a\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}}{b^2}-\frac{a\left(-\frac{\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2d\left(a^2+b^2\right)^{3/2}}\cdots\right.}{b^2}\right.}{b^2}$$

$$\frac{2a\left(\frac{(a^2+b^2)\left(\frac{\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{2\left(a^2+b^2\right)}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^2}\right)}{b^2}-\frac{2a\int \frac{1}{\cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}+\frac{\int \frac{}{\cos(c+dx)^2}\cdots}{}\right.}{b^2}$$

$$\frac{\frac{(a^2+b^2)\int \frac{1}{\cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}-\frac{2a\int \frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))}dx}{b^2}+\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{b^2}}{b^2}$$

$\downarrow$ 3553

$$\frac{(a^2+b^2)\left(\frac{\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}+\frac{a\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}}{b^2}-\frac{a\left(-\frac{\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2d\left(a^2+b^2\right)^{3/2}}\cdots\right.}{b^2}\right.}{b^2}$$

$$\frac{2a\left(\frac{(a^2+b^2)\left(-\frac{\int \frac{1}{a^2+b^2-(b\cos(c+dx)-a\sin(c+dx))^2}d(b\cos(c+dx)-a\sin(c+dx))}{2d\left(a^2+b^2\right)}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^2}\right)}{b^2}-\frac{2a\int \frac{1}{\cos(c+dx)(a\cos(c+d\cdots}}{b^2}\right.}{b^2}$$

$$\frac{\frac{(a^2+b^2)\int \frac{1}{\cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}-\frac{2a\int \frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))}dx}{b^2}+\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{b^2}}{b^2}$$

$\downarrow$ 219

$$-\frac{2a\left(-\frac{2a\int \frac{1}{\cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}+\frac{\int \frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))}dx}{b^2}+\frac{(a^2+b^2)\left(-\frac{\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+d\cdots}{\sqrt{a^2+b^2}}\right)}{2d\left(a^2+b^2\right)^{3/2}}\cdots\right.}{b^2}\right.}{b^2}$$

$$\frac{(a^2+b^2)\left(\frac{\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}+\frac{a\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}}{b^2}-\frac{a\left(-\frac{\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2d\left(a^2+b^2\right)^{3/2}}\cdots\right.}{b^2}\right.}{b^2}$$

$$\frac{\frac{(a^2+b^2)\int \frac{1}{\cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2}dx}{b^2}-\frac{2a\int \frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))}dx}{b^2}+\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{b^2}}{b^2}$$

$\downarrow$ 3573

3.148. $\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

$$\frac{(a^2+b^2)\left(\frac{\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}+\frac{a\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}}{b^2}-\frac{a\left(-\frac{\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2d\left(a^2+b^2\right)^{3/2}}\right.}{b^2}\right.}{b^2}$$

$$\frac{2a\left(\frac{\int \frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))}dx}{b^2}-\frac{2a\left(-\frac{a\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{b^2}+\frac{\int \sec(c+dx)dx}{b^2}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}\right)}{b^2}+\frac{(a^2+b^2)\left(-\right.}{}\right.}{b^2}$$

$$\frac{\frac{(a^2+b^2)\left(-\frac{a\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{b^2}+\frac{\int \sec(c+dx)dx}{b^2}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}\right)}{b^2}-\frac{2a\int \frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))}dx}{b^2}+\frac{\int \csc(c+}{}}{b^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{(a^2+b^2)\left(\frac{\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}+\frac{a\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}}{b^2}-\frac{a\left(-\frac{\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2d\left(a^2+b^2\right)^{3/2}}\right.}{b^2}\right.}{b^2}$$

$$\frac{2a\left(\frac{\int \frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))}dx}{b^2}-\frac{2a\left(-\frac{a\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{b^2}+\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}\right)}{b^2}+\frac{(a^2+b^2}{}\right.}{b^2}$$

$$\frac{\frac{(a^2+b^2)\left(-\frac{a\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{b^2}+\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}\right)}{b^2}-\frac{2a\int \frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))}dx}{b^2}+\frac{\int \csc}{}}{b^2}$$

$$\Big\downarrow \; 3553$$

3.148. $\displaystyle\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

$$\frac{(a^2+b^2)\left(\frac{\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}+\frac{a\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2 d\sqrt{a^2+b^2}}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}}{b^2}-\frac{a\left(-\frac{\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2d\left(a^2+b^2\right)^{3/2}}\right.}{b^{\cdots}}\right.}{b^2}$$

$$\frac{2a\left(-\frac{2a\left(\frac{a\int\frac{1}{a^2+b^2-(b\cos(c+dx)-a\sin(c+dx))^2}d(b\cos(c+dx)-a\sin(c+dx))}{b^2 d}+\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}\right)}{b^2}+\frac{\int\frac{\cdots}{\cos(c+dx)^2(a\cos\cdots}}{\cdots}\right.}{b^2}$$

$$\frac{\frac{(a^2+b^2)\left(\frac{a\int\frac{1}{a^2+b^2-(b\cos(c+dx)-a\sin(c+dx))^2}d(b\cos(c+dx)-a\sin(c+dx))}{b^2 d}+\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}\right)}{b^2}-\frac{2a\int\frac{\cdots}{\cos(c+dx)^2(a\cos\cdots}}{\cdots}}{b^2}$$

$$\Big\downarrow \; 219$$

$$\frac{(a^2+b^2)\left(\frac{\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}+\frac{a\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2 d\sqrt{a^2+b^2}}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}}{b^2}-\frac{a\left(-\frac{\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2d\left(a^2+b^2\right)^{3/2}}\right.}{b^{\cdots}}\right.}{b^2}$$

$$\frac{2a\left(-\frac{2a\left(\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}+\frac{a\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2 d\sqrt{a^2+b^2}}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}\right)}{b^2}+\frac{\int\frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))}dx}{b^2}+\cdots\right.}{b^2}$$

$$\frac{\frac{(a^2+b^2)\left(\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}+\frac{a\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2 d\sqrt{a^2+b^2}}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}\right)}{b^2}-\frac{2a\int\frac{1}{\cos(c+dx)^2(a\cos(c+dx)+b\sin(c+dx))}dx}{b^2}\cdots}{b^2}$$

$$\Big\downarrow \; 3583$$

3.148. $\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

$$\frac{\left(a^2+b^2\right)\left(\frac{\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}+\frac{a\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}}{b^2}-\frac{a\left(-\frac{\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2d\left(a^2+b^2\right)^{3/2}}\right.}{b^{}}\right.}{b^2}$$

$$\frac{2a\left(-\frac{2a\left(\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}+\frac{a\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}\right)}{b^2}+\frac{\frac{\left(a^2+b^2\right)\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{b^2}-\frac{a\int}{}}{b^2}\right.}{b^2}$$

$$\frac{\frac{\left(a^2+b^2\right)\left(\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}+\frac{a\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}\right)}{b^2}-\frac{2a\left(\frac{\left(a^2+b^2\right)\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{b^2}-\frac{a}{}\right.}{b^2}}{b^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\left(a^2+b^2\right)\left(\frac{\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}+\frac{a\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}}{b^2}-\frac{a\left(-\frac{\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2d\left(a^2+b^2\right)^{3/2}}\right.}{b^{}}\right.}{b^2}$$

$$\frac{2a\left(-\frac{2a\left(\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}+\frac{a\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}\right)}{b^2}+\frac{\frac{\left(a^2+b^2\right)\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{b^2}-\frac{a\int}{}}{b^2}\right.}{b^2}$$

$$\frac{\frac{\left(a^2+b^2\right)\left(\frac{\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}+\frac{a\,\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}\right)}{b^2}-\frac{2a\left(\frac{\left(a^2+b^2\right)\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{b^2}-\frac{a}{}\right.}{b^2}}{b^2}$$

$$\Big\downarrow \; 3553$$

3.148. $\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

$$\frac{(a^2+b^2)\left(-\frac{a\left(-\frac{\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{3/2}d}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2(a^2+b^2)d(a\cos(c+dx)+b\sin(c+dx))^2}\right)}{b^2}+\frac{\frac{a\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}d}+\frac{\int c}{}}{b}\right.}{b^2}$$

$$2a\left(\frac{(a^2+b^2)\left(-\frac{\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{3/2}d}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2(a^2+b^2)d(a\cos(c+dx)+b\sin(c+dx))^2}\right)}{b^2}-\frac{2a\left(\frac{a\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}d}+\frac{\int c}{}\right.}{b^2}\right.$$

$$\frac{\frac{(a^2+b^2)\left(\frac{a\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}d}+\frac{\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}\right)}{b^2}+\frac{\int\csc\left(c+dx+\frac{\pi}{2}\right)^3dx}{b^2}-\frac{2a\left(\frac{\sec(c+dx)}{bd}-\frac{a\int}{}\right.}{}}{b^2}$$

$$\Big\downarrow \; 219$$

$$\frac{(a^2+b^2)\left(\frac{\frac{\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}+\frac{a\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}}{b^2}-\frac{a\left(-\frac{\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2d(a^2+b^2)^{3/2}}\right.}{b}\right.}{b^2}$$

$$2a\left(-\frac{2a\left(\frac{\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}+\frac{a\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}\right)}{b^2}+\frac{-\frac{a\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{\sqrt{a^2+b^2}\operatorname{arctanh}}{}}{b^2}\right.$$

$$\frac{\frac{(a^2+b^2)\left(\frac{\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}+\frac{a\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}\right)}{b^2}-\frac{2a\left(-\frac{a\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{\sqrt{a^2+b^2}\operatorname{arctan}}{}\right.}{b^2}}{b^2}$$

$$\Big\downarrow \; 4255$$

3.148. $\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

$$\frac{(a^2+b^2)\left(-\frac{a\left(-\frac{\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{3/2}d}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^2}\right)}{b^2}+\frac{\frac{a\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}d}+\frac{\int c}{}}{b}\right.}{b^2}$$

$$\frac{2a\left(\frac{(a^2+b^2)\left(-\frac{\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{3/2}d}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^2}\right)}{b^2}-\frac{2a\left(\frac{a\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}d}+\frac{\int c}{}\right.}{b^2}\right.}{b^2}$$

$$\frac{\frac{(a^2+b^2)\left(\frac{a\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}d}+\frac{\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}\right)}{b^2}-\frac{2a\left(-\frac{\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c}{\sqrt{a^2+b^2}}\right.}{b^2d}\right.}{b^2}}{b^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{(a^2+b^2)\left(-\frac{a\left(-\frac{\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{3/2}d}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^2}\right)}{b^2}+\frac{\frac{a\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}d}+\frac{\int c}{}}{b}\right.}{b^2}$$

$$\frac{2a\left(\frac{(a^2+b^2)\left(-\frac{\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{3/2}d}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^2}\right)}{b^2}-\frac{2a\left(\frac{a\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}d}+\frac{\int c}{}\right.}{b^2}\right.}{b^2}$$

$$\frac{\frac{\frac{\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}+\frac{(a^2+b^2)\left(\frac{a\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}d}+\frac{\int\csc\left(c+dx+\frac{\pi}{2}\right)dx}{b^2}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}\right)}{b^2}}{b^2}$$

$$\Big\downarrow \; 4257$$

3.148. $\displaystyle\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

$$\frac{\frac{(a^2+b^2)\left(-\frac{a\left(-\frac{\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2d(a^2+b^2)^{3/2}}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^2}\right)}{b^2}+\frac{\frac{a\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{}{bd(}}{}\right.}{b^2}}{\frac{2a\left(\frac{(a^2+b^2)\left(-\frac{\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2d(a^2+b^2)^{3/2}}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^2}\right)}{b^2}-\frac{2a\left(\frac{a\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{}{bd(}\right.}{b}\right.}{b^2}}$$

$$\frac{\frac{(a^2+b^2)\left(\frac{a\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}+\frac{\operatorname{arctanh}(\sin(c+dx))}{b^2d}\right)}{b^2}-\frac{2a\left(-\frac{\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\,s}{\sqrt{a^2+b}}\right.}{b^2d}\right.}{}}{b^2}$$

input
```
Int[Sec[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

output
```
((a^2 + b^2)*(-1/3*1/(b*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (a*(-1/2*
ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]]/((a^2 + b^2)^(3
/2)*d) - (b*Cos[c + d*x] - a*Sin[c + d*x])/(2*(a^2 + b^2)*d*(a*Cos[c + d*x
] + b*Sin[c + d*x])^2)))/b^2 + (ArcTanh[Sin[c + d*x]]/(b^2*d) + (a*ArcTanh
[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(b^2*Sqrt[a^2 + b^2]*
d) - 1/(b*d*(a*Cos[c + d*x] + b*Sin[c + d*x])))/b^2))/b^2 - (2*a*((-((a*Ar
cTanh[Sin[c + d*x]])/(b^2*d)) - (Sqrt[a^2 + b^2]*ArcTanh[(b*Cos[c + d*x] -
 a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(b^2*d) + Sec[c + d*x]/(b*d))/b^2 + ((a
^2 + b^2)*(-1/2*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]]
/((a^2 + b^2)^(3/2)*d) - (b*Cos[c + d*x] - a*Sin[c + d*x])/(2*(a^2 + b^2)*
d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)))/b^2 - (2*a*(ArcTanh[Sin[c + d*x]]
/(b^2*d) + (a*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/
(b^2*Sqrt[a^2 + b^2]*d) - 1/(b*d*(a*Cos[c + d*x] + b*Sin[c + d*x]))))/b^2)
)/b^2 + ((-2*a*(-((a*ArcTanh[Sin[c + d*x]])/(b^2*d)) - (Sqrt[a^2 + b^2]*Ar
cTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(b^2*d) + Sec[c
+ d*x]/(b*d)))/b^2 + ((a^2 + b^2)*(ArcTanh[Sin[c + d*x]]/(b^2*d) + (a*ArcT
anh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(b^2*Sqrt[a^2 + b^
2]*d) - 1/(b*d*(a*Cos[c + d*x] + b*Sin[c + d*x]))))/b^2 + (ArcTanh[Sin[c +
 d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d))/b^2)/b^2
```

3.148. $\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

3.148.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3555
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(b*Cos[c + d*x] - a*Sin[c + d*x])*((a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Simp[(n + 2)/((n + 1)*(a^
2 + b^2))   Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

rule 3573
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_)/co
s[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(a*Cos[c + d*x] + b*Sin[c + d*x])^
(n + 1)/(b*d*(n + 1)), x] + (Simp[1/b^2   Int[(a*Cos[c + d*x] + b*Sin[c + d
*x])^(n + 2)/Cos[c + d*x], x], x] - Simp[a/b^2   Int[(a*Cos[c + d*x] + b*Si
n[c + d*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
 && LtQ[n, -1]
```

rule 3583
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-Cos[c + d*x]^(m + 1)/(b*d*(m + 1)
), x] + (-Simp[a/b^2   Int[Cos[c + d*x]^(m + 1), x], x] + Simp[(a^2 + b^2)/
b^2   Int[Cos[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

3.148. $\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

rule 3585
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a^2 + b^2)/b^2   Int[Cos[c +
 d*x]^(m + 2)*(a*Cos[c + d*x] + b*Sin[c + d*x])^n, x], x] + (Simp[1/b^2   I
nt[Cos[c + d*x]^m*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] - Simp[
2*(a/b^2)   Int[Cos[c + d*x]^(m + 1)*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n +
 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] &
& LtQ[m, -1]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.148.4 Maple [A] (verified)

Time = 5.32 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.13

method	result
derivativedivides	$\frac{\frac{2\left(\frac{b^2(9a^4+2b^4)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{2a}+\frac{b(12a^6-39a^4b^2-4b^6)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{2a^2}-\frac{b^2(108a^6-57a^4b^2-4a^2b^4-4b^6)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3a^3}-\frac{b(12a^6-}{}\right.}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}}{}$
default	$\frac{\frac{2\left(\frac{b^2(9a^4+2b^4)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{2a}+\frac{b(12a^6-39a^4b^2-4b^6)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{2a^2}-\frac{b^2(108a^6-57a^4b^2-4a^2b^4-4b^6)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3a^3}-\frac{b(12a^6-}{}\right.}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}}{}$
risch	$-\frac{20a^2b^2e^{3i(dx+c)}+250a^2b^2e^{5i(dx+c)}-105a^2b^2e^{i(dx+c)}+60iab^3e^{3i(dx+c)}+150ia^3be^{i(dx+c)}+300ia^3be^{3i(dx+c)}-300ia}{}$

input
```
int(sec(d*x+c)^3/(cos(d*x+c)*a+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/b^6*((1/2*b^2*(9*a^4+2*b^4)/a*tan(1/2*d*x+1/2*c)^5+1/2*b*(12*a^6-39
*a^4*b^2-4*b^6)/a^2*tan(1/2*d*x+1/2*c)^4-1/3/a^3*b^2*(108*a^6-57*a^4*b^2-4
*a^2*b^4-4*b^6)*tan(1/2*d*x+1/2*c)^3-1/a^2*b*(12*a^6-50*a^4*b^2-9*a^2*b^4-
2*b^6)*tan(1/2*d*x+1/2*c)^2+1/2/a*b^2*(63*a^4+10*a^2*b^2+2*b^4)*tan(1/2*d*
x+1/2*c)+1/6*b*(36*a^4+5*a^2*b^2+2*b^4))/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1
/2*d*x+1/2*c)-a)^3-5/2*a*(4*a^2+3*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*ta
n(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))+1/2/b^4/(tan(1/2*d*x+1/2*c)-1)^2-1
/2*(-8*a-b)/b^5/(tan(1/2*d*x+1/2*c)-1)+1/2/b^6*(-20*a^2-5*b^2)*ln(tan(1/2*
d*x+1/2*c)-1)-1/2/b^4/(tan(1/2*d*x+1/2*c)+1)^2-1/2*(8*a-b)/b^5/(tan(1/2*d*
x+1/2*c)+1)+1/2/b^6*(20*a^2+5*b^2)*ln(tan(1/2*d*x+1/2*c)+1))
```

3.148.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 820 vs. $2(378) = 756$.

Time = 0.37 (sec) , antiderivative size = 820, normalized size of antiderivative = 2.05

$$\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$$
$$= \frac{6\,a^2b^5+6\,b^7-30\left(4\,a^6b-3\,a^4b^3-8\,a^2b^5-b^7\right)\cos\left(dx+c\right)^4-20\left(11\,a^4b^3+13\,a^2b^5+2\,b^7\right)\cos\left(dx+c\right)^2}{}$$

input
```
integrate(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas"
)
```

output
```
1/12*(6*a^2*b^5 + 6*b^7 - 30*(4*a^6*b - 3*a^4*b^3 - 8*a^2*b^5 - b^7)*cos(d
*x + c)^4 - 20*(11*a^4*b^3 + 13*a^2*b^5 + 2*b^7)*cos(d*x + c)^2 + 15*((4*a
^6 - 9*a^4*b^2 - 9*a^2*b^4)*cos(d*x + c)^5 + 3*(4*a^4*b^2 + 3*a^2*b^4)*cos
(d*x + c)^3 + ((12*a^5*b + 5*a^3*b^3 - 3*a*b^5)*cos(d*x + c)^4 + (4*a^3*b^
3 + 3*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 + b^2)*log((2*a*b*cos(
d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(
a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x
+ c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) + 15*((4*a^7 - 7*a^5*b^2 - 14*a^
3*b^4 - 3*a*b^6)*cos(d*x + c)^5 + 3*(4*a^5*b^2 + 5*a^3*b^4 + a*b^6)*cos(d*
x + c)^3 + ((12*a^6*b + 11*a^4*b^3 - 2*a^2*b^5 - b^7)*cos(d*x + c)^4 + (4*
a^4*b^3 + 5*a^2*b^5 + b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(sin(d*x + c)
+ 1) - 15*((4*a^7 - 7*a^5*b^2 - 14*a^3*b^4 - 3*a*b^6)*cos(d*x + c)^5 + 3*(
4*a^5*b^2 + 5*a^3*b^4 + a*b^6)*cos(d*x + c)^3 + ((12*a^6*b + 11*a^4*b^3 -
2*a^2*b^5 - b^7)*cos(d*x + c)^4 + (4*a^4*b^3 + 5*a^2*b^5 + b^7)*cos(d*x +
c)^2)*sin(d*x + c))*log(-sin(d*x + c) + 1) - 30*(10*(a^5*b^2 + a^3*b^4)*co
s(d*x + c)^3 + (a^3*b^4 + a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^5*b^6 - 2
*a^3*b^8 - 3*a*b^10)*d*cos(d*x + c)^5 + 3*(a^3*b^8 + a*b^10)*d*cos(d*x + c
)^3 + ((3*a^4*b^7 + 2*a^2*b^9 - b^11)*d*cos(d*x + c)^4 + (a^2*b^9 + b^11)*
d*cos(d*x + c)^2)*sin(d*x + c))
```

3.148.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = \int \frac{\sec^3{(c+dx)}}{(a\cos{(c+dx)}+b\sin{(c+dx)})^4}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

output
```
Integral(sec(c + d*x)**3/(a*cos(c + d*x) + b*sin(c + d*x))**4, x)
```

3.148.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 936 vs. $2(378) = 756$.

Time $= 0.38$ (sec) , antiderivative size $= 936$, normalized size of antiderivative $= 2.34$

$$\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima"
)
```

output
```
-1/6*(2*(60*a^7 + 5*a^5*b^2 + 2*a^3*b^4 + 6*(55*a^6*b + 5*a^4*b^3 + a^2*b^
5)*sin(d*x + c)/(cos(d*x + c) + 1) - 2*(120*a^7 - 280*a^5*b^2 - 25*a^3*b^4
 - 6*a*b^6)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 2*(510*a^6*b - 105*a^4*b
^3 + 2*a^2*b^5 - 4*b^7)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2*(180*a^7 -
 635*a^5*b^2 - 65*a^3*b^4 - 18*a*b^6)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4
+ 2*(540*a^6*b - 195*a^4*b^3 - 2*a^2*b^5 - 8*b^7)*sin(d*x + c)^5/(cos(d*x
+ c) + 1)^5 - 6*(40*a^7 - 140*a^5*b^2 - 5*a^3*b^4 - 6*a*b^6)*sin(d*x + c)^
6/(cos(d*x + c) + 1)^6 - 2*(210*a^6*b - 75*a^4*b^3 + 2*a^2*b^5 - 4*b^7)*si
n(d*x + c)^7/(cos(d*x + c) + 1)^7 + 3*(20*a^7 - 45*a^5*b^2 - 4*a*b^6)*sin(
d*x + c)^8/(cos(d*x + c) + 1)^8 + 6*(5*a^6*b + a^2*b^5)*sin(d*x + c)^9/(co
s(d*x + c) + 1)^9)/(a^6*b^5 + 6*a^5*b^6*sin(d*x + c)/(cos(d*x + c) + 1) +
6*a^5*b^6*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - a^6*b^5*sin(d*x + c)^10/(c
os(d*x + c) + 1)^10 - (5*a^6*b^5 - 12*a^4*b^7)*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 - 8*(3*a^5*b^6 - a^3*b^8)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2
*(5*a^6*b^5 - 18*a^4*b^7)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*(9*a^5*b
^6 - 4*a^3*b^8)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 2*(5*a^6*b^5 - 18*a^
4*b^7)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 8*(3*a^5*b^6 - a^3*b^8)*sin(d
*x + c)^7/(cos(d*x + c) + 1)^7 + (5*a^6*b^5 - 12*a^4*b^7)*sin(d*x + c)^8/(
cos(d*x + c) + 1)^8) - 15*(4*a^2 + 3*b^2)*a*log((b - a*sin(d*x + c)/(cos(d
*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) ...
```

3.148.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 548, normalized size of antiderivative = 1.37

$$\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$$

$$= \frac{\frac{15\,(4\,a^2+b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^6} - \frac{15\,(4\,a^2+b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^6} + \frac{15\,(4\,a^3+3\,ab^2)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,b^6} + \cdots}{\cdots}$$

input
```
integrate(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

output
```
1/6*(15*(4*a^2 + b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^6 - 15*(4*a^2 +
 b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^6 + 15*(4*a^3 + 3*a*b^2)*log(ab
s(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*d*x
+ 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^6) + 6*(b*tan(1/2*
d*x + 1/2*c)^3 + 8*a*tan(1/2*d*x + 1/2*c)^2 + b*tan(1/2*d*x + 1/2*c) - 8*a
)/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*b^5) + 2*(27*a^6*b*tan(1/2*d*x + 1/2*c)^
5 + 6*a^2*b^5*tan(1/2*d*x + 1/2*c)^5 + 36*a^7*tan(1/2*d*x + 1/2*c)^4 - 117
*a^5*b^2*tan(1/2*d*x + 1/2*c)^4 - 12*a*b^6*tan(1/2*d*x + 1/2*c)^4 - 216*a^
6*b*tan(1/2*d*x + 1/2*c)^3 + 114*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 + 8*a^2*b^
5*tan(1/2*d*x + 1/2*c)^3 + 8*b^7*tan(1/2*d*x + 1/2*c)^3 - 72*a^7*tan(1/2*d
*x + 1/2*c)^2 + 300*a^5*b^2*tan(1/2*d*x + 1/2*c)^2 + 54*a^3*b^4*tan(1/2*d*
x + 1/2*c)^2 + 12*a*b^6*tan(1/2*d*x + 1/2*c)^2 + 189*a^6*b*tan(1/2*d*x + 1
/2*c) + 30*a^4*b^3*tan(1/2*d*x + 1/2*c) + 6*a^2*b^5*tan(1/2*d*x + 1/2*c) +
 36*a^7 + 5*a^5*b^2 + 2*a^3*b^4)/((a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*
d*x + 1/2*c) - a)^3*a^3*b^5))/d
```

3.148.9 Mupad [B] (verification not implemented)

Time = 28.16 (sec) , antiderivative size = 1961, normalized size of antiderivative = 4.90

$$\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(c + d*x)^3*(a*cos(c + d*x) + b*sin(c + d*x))^4),x)
```

output
```
(atanh((4000*a^3*tan(c/2 + (d*x)/2))/(1000*a*b^2 + 4000*a^3) + (1000*a*tan
(c/2 + (d*x)/2))/(1000*a + (4000*a^3)/b^2))*(20*a^2 + 5*b^2))/(b^6*d) - ((
60*a^4 + 2*b^4 + 5*a^2*b^2)/(3*b^5) + (2*tan(c/2 + (d*x)/2)^9*(5*a^4 + b^4
))/(a*b^4) + (2*tan(c/2 + (d*x)/2)^6*(6*b^6 - 40*a^6 + 5*a^2*b^4 + 140*a^4
*b^2))/(a^2*b^5) - (2*tan(c/2 + (d*x)/2)^7*(210*a^6 - 4*b^6 + 2*a^2*b^4 -
75*a^4*b^2))/(3*a^3*b^4) + (2*tan(c/2 + (d*x)/2)^2*(6*b^6 - 120*a^6 + 25*a
^2*b^4 + 280*a^4*b^2))/(3*a^2*b^5) - (2*tan(c/2 + (d*x)/2)^3*(510*a^6 - 4*
b^6 + 2*a^2*b^4 - 105*a^4*b^2))/(3*a^3*b^4) - (2*tan(c/2 + (d*x)/2)^4*(18*
b^6 - 180*a^6 + 65*a^2*b^4 + 635*a^4*b^2))/(3*a^2*b^5) - (tan(c/2 + (d*x)/
2)^8*(4*b^6 - 20*a^6 + 45*a^4*b^2))/(a^2*b^5) + (2*tan(c/2 + (d*x)/2)*(55*
a^4 + b^4 + 5*a^2*b^2))/(a*b^4) + (2*tan(c/2 + (d*x)/2)^5*(9*a^2 - 4*b^2)*
(60*a^4 + 2*b^4 + 5*a^2*b^2))/(3*a^3*b^4))/(d*(tan(c/2 + (d*x)/2)^2*(12*a*
b^2 - 5*a^3) - a^3*tan(c/2 + (d*x)/2)^10 - tan(c/2 + (d*x)/2)^8*(12*a*b^2
- 5*a^3) - tan(c/2 + (d*x)/2)^4*(36*a*b^2 - 10*a^3) + tan(c/2 + (d*x)/2)^6
*(36*a*b^2 - 10*a^3) - tan(c/2 + (d*x)/2)^3*(24*a^2*b - 8*b^3) - tan(c/2 +
 (d*x)/2)^7*(24*a^2*b - 8*b^3) + tan(c/2 + (d*x)/2)^5*(36*a^2*b - 16*b^3)
+ a^3 + 6*a^2*b*tan(c/2 + (d*x)/2) + 6*a^2*b*tan(c/2 + (d*x)/2)^9)) - (a*a
tan(((a*(4*a^2 + 3*b^2)*(a^2 + b^2)^(1/2)*((8*(25*a^2*b^9 + 200*a^4*b^7 +
400*a^6*b^5))/b^14 + (8*tan(c/2 + (d*x)/2)*(50*a*b^11 + 650*a^3*b^9 + 1600
*a^5*b^7 + 800*a^7*b^5))/b^15 - (5*a*(4*a^2 + 3*b^2)*(a^2 + b^2)^(1/2)*...
```

3.148. $\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

3.149 $\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

3.149.1 Optimal result

Integrand size = 28, antiderivative size = 232

$$\begin{aligned}\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx &= \frac{(a^2+b^2)^3}{3a^3b^4d(b+a\cot(c+dx))^3} \\ &+ \frac{2a^6+3a^4b^2-b^6}{a^3b^5d(b+a\cot(c+dx))^2} \\ &+ \frac{10a^6+9a^4b^2+b^6}{a^3b^6d(b+a\cot(c+dx))} \\ &- \frac{4a(5a^2+3b^2)\log(b+a\cot(c+dx))}{b^7d} \\ &- \frac{4a(5a^2+3b^2)\log(\tan(c+dx))}{b^7d} \\ &+ \frac{(10a^2+3b^2)\tan(c+dx)}{b^6d} \\ &- \frac{2a\tan^2(c+dx)}{b^5d} + \frac{\tan^3(c+dx)}{3b^4d}\end{aligned}$$

output
```
1/3*(a^2+b^2)^3/a^3/b^4/d/(b+a*cot(d*x+c))^3+(2*a^6+3*a^4*b^2-b^6)/a^3/b^5
/d/(b+a*cot(d*x+c))^2+(10*a^6+9*a^4*b^2+b^6)/a^3/b^6/d/(b+a*cot(d*x+c))-4*
a*(5*a^2+3*b^2)*ln(b+a*cot(d*x+c))/b^7/d-4*a*(5*a^2+3*b^2)*ln(tan(d*x+c))/
b^7/d+(10*a^2+3*b^2)*tan(d*x+c)/b^6/d-2*a*tan(d*x+c)^2/b^5/d+1/3*tan(d*x+c
)^3/b^4/d
```

3.149.2 Mathematica [A] (verified)

Time = 2.27 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.27

$$\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$$
$$= \frac{b^6\sec^6(c+dx)+3b^4\sec^4(c+dx)\left(a^2+2b^2-ab\tan(c+dx)\right)-2\left(37a^6+36a^4b^2+3a^2b^4+4b^6+6a^4(5a^2\right.}{}$$

input
```
Integrate[Sec[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

output
```
(b^6*Sec[c + d*x]^6 + 3*b^4*Sec[c + d*x]^4*(a^2 + 2*b^2 - a*b*Tan[c + d*x]
) - 2*(37*a^6 + 36*a^4*b^2 + 3*a^2*b^4 + 4*b^6 + 6*a^4*(5*a^2 + 3*b^2)*Log
[a + b*Tan[c + d*x]] + 3*a*b*(27*a^4 + 30*a^2*b^2 + b^4 + 6*a^2*(5*a^2 + 3
*b^2)*Log[a + b*Tan[c + d*x]])*Tan[c + d*x] + 6*b^2*(6*a^4 + 11*a^2*b^2 +
2*b^4 + 3*a^2*(5*a^2 + 3*b^2)*Log[a + b*Tan[c + d*x]])*Tan[c + d*x]^2 + 6*
a*b^3*(-3*a^2 + (5*a^2 + 3*b^2)*Log[a + b*Tan[c + d*x]])*Tan[c + d*x]^3 -
6*a^2*b^4*Tan[c + d*x]^4))/(3*b^7*d*(a + b*Tan[c + d*x])^3)
```

3.149.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3567, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\cos(c+dx)^4(a\cos(c+dx)+b\sin(c+dx))^4}dx$$

$$\Big\downarrow \text{ 3567}$$

$$-\frac{\int \frac{(\cot^2(c+dx)+1)^3\tan^4(c+dx)}{(b+a\cot(c+dx))^4}d\cot(c+dx)}{d}$$

$$\Big\downarrow \text{ 522}$$

$$-\frac{\int \left(\frac{\tan^4(c+dx)}{b^4} - \frac{4a\tan^3(c+dx)}{b^5} + \frac{(10a^2+3b^2)\tan^2(c+dx)}{b^6} - \frac{4(5a^3+3b^2a)\tan(c+dx)}{b^7} + \frac{4(5a^4+3b^2a^2)}{b^7(b+a\cot(c+dx))} + \frac{10a^6+9b^2a^4+b^6}{a^2b^6(b+a\cot(c+dx))^2}\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{4a(5a^2+3b^2)\log(\cot(c+dx))}{b^7} + \frac{4a(5a^2+3b^2)\log(a\cot(c+dx)+b)}{b^7} - \frac{(10a^2+3b^2)\tan(c+dx)}{b^6} - \frac{(a^2+b^2)^3}{3a^3b^4(a\cot(c+dx)+b)^3} - \frac{10a^6+9a^4b^2}{a^3b^6(a\cot(c+}}{d}$$

input
```
Int[Sec[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

output
```
-((-1/3*(a^2 + b^2)^3/(a^3*b^4*(b + a*Cot[c + d*x])^3) - (2*a^6 + 3*a^4*b^
2 - b^6)/(a^3*b^5*(b + a*Cot[c + d*x])^2) - (10*a^6 + 9*a^4*b^2 + b^6)/(a^
3*b^6*(b + a*Cot[c + d*x])) - (4*a*(5*a^2 + 3*b^2)*Log[Cot[c + d*x]])/b^7
+ (4*a*(5*a^2 + 3*b^2)*Log[b + a*Cot[c + d*x]])/b^7 - ((10*a^2 + 3*b^2)*Ta
n[c + d*x])/b^6 + (2*a*Tan[c + d*x]^2)/b^5 - Tan[c + d*x]^3/(3*b^4))/d)
```

3.149.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3567
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[x^m*((b
+ a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a,
b, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[
n, 0] && GtQ[m, 1])
```

3.149. $\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

3.149.4 Maple [A] (verified)

Time = 5.42 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{\frac{\frac{b^2\tan(dx+c)^3}{3}-2a\tan(dx+c)^2 b+10\tan(dx+c)a^2+3\tan(dx+c)b^2}{b^6}-\frac{4a\left(5a^2+3b^2\right)\ln(a+b\tan(dx+c))}{b^7}-\frac{a^6+3a^4b^2+3a^2b^4+b^6}{3b^7(a+b\tan(dx+c))^3}-\frac{15a^4}{b^7(a}}{d}$
default	$\frac{\frac{\frac{b^2\tan(dx+c)^3}{3}-2a\tan(dx+c)^2 b+10\tan(dx+c)a^2+3\tan(dx+c)b^2}{b^6}-\frac{4a\left(5a^2+3b^2\right)\ln(a+b\tan(dx+c))}{b^7}-\frac{a^6+3a^4b^2+3a^2b^4+b^6}{3b^7(a+b\tan(dx+c))^3}-\frac{15a^4}{b^7(a}}{d}$
risch	$\frac{8i(-4b^5+45a^4b-3a^2b^3+12b^5e^{4i(dx+c)}-130ia^3b^2e^{6i(dx+c)}+15iab^4e^{2i(dx+c)}+6ia^3b^2e^{10i(dx+c)}+9iab^4e^{10i(dx+c)}-45i}{}$
norman	$\frac{4\left(100a^8-160a^6b^2-140a^4b^4-a^2b^6-2b^8\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{a^3db^6}-\frac{4\left(100a^8-160a^6b^2-140a^4b^4-a^2b^6-2b^8\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{a^3db^6}-\frac{2\left(300a^8-260a\right.}{}$
parallelrisch	$\frac{-900a^2\left(a^2+\frac{3b^2}{5}\right)\left(\left(a^3+\frac{1}{5}ab^2\right)\cos(2dx+2c)+\frac{2\left(a^3-ab^2\right)\cos(4dx+4c)}{5}+\frac{\left(\frac{1}{3}a^3-ab^2\right)\cos(6dx+6c)}{5}+\left(a^2b+\frac{1}{5}b^3\right)\sin(2dx+2\right.\right.}{}$

input
```
int(sec(d*x+c)^4/(cos(d*x+c)*a+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/b^6*(1/3*b^2*tan(d*x+c)^3-2*a*tan(d*x+c)^2*b+10*tan(d*x+c)*a^2+3*ta
n(d*x+c)*b^2)-4*a/b^7*(5*a^2+3*b^2)*ln(a+b*tan(d*x+c))-1/3/b^7*(a^6+3*a^4*
b^2+3*a^2*b^4+b^6)/(a+b*tan(d*x+c))^3-(15*a^4+18*a^2*b^2+3*b^4)/b^7/(a+b*t
an(d*x+c))+3*a/b^7*(a^4+2*a^2*b^2+b^4)/(a+b*tan(d*x+c))^2)
```

3.149.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 553 vs. 2(228) = 456.

Time = 0.29 (sec) , antiderivative size = 553, normalized size of antiderivative = 2.38

$$\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx =$$
$$-\frac{4\left(45a^4b^2-3a^2b^4-4b^6\right)\cos(dx+c)^6-b^6-6\left(25a^4b^2-5a^2b^4-4b^6\right)\cos(dx+c)^4-3\left(5a^2b^4+2b^6\right.}{}$$

input
```
integrate(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas"
)
```

output
```
-1/3*(4*(45*a^4*b^2 - 3*a^2*b^4 - 4*b^6)*cos(d*x + c)^6 - b^6 - 6*(25*a^4*
b^2 - 5*a^2*b^4 - 4*b^6)*cos(d*x + c)^4 - 3*(5*a^2*b^4 + 2*b^6)*cos(d*x +
c)^2 + 6*((5*a^6 - 12*a^4*b^2 - 9*a^2*b^4)*cos(d*x + c)^6 + 3*(5*a^4*b^2 +
 3*a^2*b^4)*cos(d*x + c)^4 + ((15*a^5*b + 4*a^3*b^3 - 3*a*b^5)*cos(d*x + c
)^5 + (5*a^3*b^3 + 3*a*b^5)*cos(d*x + c)^3)*sin(d*x + c))*log(2*a*b*cos(d*
x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 6*((5*a^6 - 12*a
^4*b^2 - 9*a^2*b^4)*cos(d*x + c)^6 + 3*(5*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c
)^4 + ((15*a^5*b + 4*a^3*b^3 - 3*a*b^5)*cos(d*x + c)^5 + (5*a^3*b^3 + 3*a*
b^5)*cos(d*x + c)^3)*sin(d*x + c))*log(cos(d*x + c)^2) + (3*a*b^5*cos(d*x
+ c) - 4*(15*a^5*b - 41*a^3*b^3 - 12*a*b^5)*cos(d*x + c)^5 - 2*(55*a^3*b^3
 + 21*a*b^5)*cos(d*x + c)^3)*sin(d*x + c))/(3*a*b^9*d*cos(d*x + c)^4 + (a^
3*b^7 - 3*a*b^9)*d*cos(d*x + c)^6 + (b^10*d*cos(d*x + c)^3 + (3*a^2*b^8 -
b^10)*d*cos(d*x + c)^5)*sin(d*x + c))
```

3.149.6 Sympy [F]

$$\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = \int \frac{\sec^4\left(c+dx\right)}{\left(a\cos\left(c+dx\right)+b\sin\left(c+dx\right)\right)^4}\,dx$$

input
```
integrate(sec(d*x+c)**4/(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

output
```
Integral(sec(c + d*x)**4/(a*cos(c + d*x) + b*sin(c + d*x))**4, x)
```

3.149.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.94

$$\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = -\frac{\frac{37\,a^6+39\,a^4b^2+3\,a^2b^4+b^6+9\,\left(5\,a^4b^2+6\,a^2b^4+b^6\right)\tan(dx+c)^2+9\,\left(9\,a^5b+10\,a^3b^3+ab^5\right)\tan(dx+c)}{b^{10}\tan(dx+c)^3+3\,ab^9\tan(dx+c)^2+3\,a^2b^8\tan(dx+c)+a^3b^7}-\frac{b^2\tan(dx+c)^3-6\,ab\tan(dx+c)^2+3\,(10\cdots}{b^6}}{3\,d}$$

input
```
integrate(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima"
)
```

output
```
-1/3*((37*a^6 + 39*a^4*b^2 + 3*a^2*b^4 + b^6 + 9*(5*a^4*b^2 + 6*a^2*b^4 +
b^6)*tan(d*x + c)^2 + 9*(9*a^5*b + 10*a^3*b^3 + a*b^5)*tan(d*x + c))/(b^10
*tan(d*x + c)^3 + 3*a*b^9*tan(d*x + c)^2 + 3*a^2*b^8*tan(d*x + c) + a^3*b^
7) - (b^2*tan(d*x + c)^3 - 6*a*b*tan(d*x + c)^2 + 3*(10*a^2 + 3*b^2)*tan(d
*x + c))/b^6 + 12*(5*a^3 + 3*a*b^2)*log(b*tan(d*x + c) + a)/b^7)/d
```

3.149.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.07

$$\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx =$$
$$-\frac{\frac{12\,(5\,a^3+3\,ab^2)\log(|b\tan(dx+c)+a|)}{b^7} - \frac{110\,a^3b^3\tan(dx+c)^3+66\,ab^5\tan(dx+c)^3+285\,a^4b^2\tan(dx+c)^2+144\,a^2b^4\tan(dx+c)^2-9\,b^6\tan(dx\cdots}{(b\tan(dx+c)\cdots}}{\cdots}$$

input
```
integrate(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/3*(12*(5*a^3 + 3*a*b^2)*log(abs(b*tan(d*x + c) + a))/b^7 - (110*a^3*b^3
*tan(d*x + c)^3 + 66*a*b^5*tan(d*x + c)^3 + 285*a^4*b^2*tan(d*x + c)^2 + 1
44*a^2*b^4*tan(d*x + c)^2 - 9*b^6*tan(d*x + c)^2 + 249*a^5*b*tan(d*x + c)
+ 108*a^3*b^3*tan(d*x + c) - 9*a*b^5*tan(d*x + c) + 73*a^6 + 27*a^4*b^2 -
3*a^2*b^4 - b^6)/((b*tan(d*x + c) + a)^3*b^7) - (b^8*tan(d*x + c)^3 - 6*a*
b^7*tan(d*x + c)^2 + 30*a^2*b^6*tan(d*x + c) + 9*b^8*tan(d*x + c))/b^12)/d
```

3.149.9 Mupad [B] (verification not implemented)

Time = 32.43 (sec) , antiderivative size = 1599, normalized size of antiderivative = 6.89

$$\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(c + d*x)^4*(a*cos(c + d*x) + b*sin(c + d*x))^4),x)
```

output
```
((4*tan(c/2 + (d*x)/2)^2*(50*a^6 + b^6 + 30*a^4*b^2))/(a^2*b^5) - (16*tan(
c/2 + (d*x)/2)^8*(50*a^6 + b^6 - 3*a^2*b^4 + 25*a^4*b^2))/(a^2*b^5) - (2*t
an(c/2 + (d*x)/2)^11*(20*a^6 + b^6 + 12*a^4*b^2))/(a*b^6) - (16*tan(c/2 +
(d*x)/2)^4*(50*a^6 + b^6 - 3*a^2*b^4 + 25*a^4*b^2))/(a^2*b^5) + (4*tan(c/2
 + (d*x)/2)^10*(50*a^6 + b^6 + 30*a^4*b^2))/(a^2*b^5) - (4*tan(c/2 + (d*x)
/2)^5*(2*b^8 - 100*a^8 + a^2*b^6 + 140*a^4*b^4 + 160*a^6*b^2))/(a^3*b^6) +
 (4*tan(c/2 + (d*x)/2)^7*(2*b^8 - 100*a^8 + a^2*b^6 + 140*a^4*b^4 + 160*a^
6*b^2))/(a^3*b^6) + (2*tan(c/2 + (d*x)/2)^3*(4*b^8 - 300*a^8 - 3*a^2*b^6 +
 264*a^4*b^4 + 260*a^6*b^2))/(3*a^3*b^6) - (2*tan(c/2 + (d*x)/2)^9*(4*b^8
- 300*a^8 - 3*a^2*b^6 + 264*a^4*b^4 + 260*a^6*b^2))/(3*a^3*b^6) + (8*tan(c
/2 + (d*x)/2)^6*(450*a^6 + 9*b^6 - 28*a^2*b^4 + 210*a^4*b^2))/(3*a^2*b^5)
+ (2*tan(c/2 + (d*x)/2)*(20*a^6 + b^6 + 12*a^4*b^2))/(a*b^6))/(d*(a^3*tan(
c/2 + (d*x)/2)^12 + tan(c/2 + (d*x)/2)^2*(12*a*b^2 - 6*a^3) + tan(c/2 + (d
*x)/2)^10*(12*a*b^2 - 6*a^3) - tan(c/2 + (d*x)/2)^4*(48*a*b^2 - 15*a^3) -
tan(c/2 + (d*x)/2)^8*(48*a*b^2 - 15*a^3) + tan(c/2 + (d*x)/2)^6*(72*a*b^2
- 20*a^3) - tan(c/2 + (d*x)/2)^3*(30*a^2*b - 8*b^3) + tan(c/2 + (d*x)/2)^9
*(30*a^2*b - 8*b^3) + tan(c/2 + (d*x)/2)^5*(60*a^2*b - 24*b^3) - tan(c/2 +
 (d*x)/2)^7*(60*a^2*b - 24*b^3) + a^3 + 6*a^2*b*tan(c/2 + (d*x)/2) - 6*a^2
*b*tan(c/2 + (d*x)/2)^11)) + (a*atan(((a*(5*a^2 + 3*b^2)*((16*tan(c/2 + (d
*x)/2)*(20*a^5 + 12*a^3*b^2))/b^6 - (4*(24*a^2*b^9 + 40*a^4*b^7))/b^12 ...
```

3.149. $\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

3.150 $\int \frac{\cos^5(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

3.150.1 Optimal result

Integrand size = 31, antiderivative size = 99

$$\int \frac{\cos^5(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \frac{5x}{16a}+\frac{i\cos^6(c+dx)}{6ad}+\frac{5\cos(c+dx)\sin(c+dx)}{16ad}+\frac{5\cos^3(c+dx)\sin(c+dx)}{24ad}+\frac{\cos^5(c+dx)\sin(c+dx)}{6ad}$$

output
```
5/16*x/a+1/6*I*cos(d*x+c)^6/a/d+5/16*cos(d*x+c)*sin(d*x+c)/a/d+5/24*cos(d*
x+c)^3*sin(d*x+c)/a/d+1/6*cos(d*x+c)^5*sin(d*x+c)/a/d
```

3.150.2 Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.83

$$\int \frac{\cos^5(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \frac{60c+60dx+15i\cos(2(c+dx))+6i\cos(4(c+dx))+i\cos(6(c+dx))+45\sin(2(c+dx))+9\sin(4(c+}{192ad}$$

input
```
Integrate[Cos[c + d*x]^5/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

output
```
(60*c + 60*d*x + (15*I)*Cos[2*(c + d*x)] + (6*I)*Cos[4*(c + d*x)] + I*Cos[
6*(c + d*x)] + 45*Sin[2*(c + d*x)] + 9*Sin[4*(c + d*x)] + Sin[6*(c + d*x)]
)/(192*a*d)
```

3.150.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3042, 3571, 3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^5(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(c+dx)^5}{a\cos(c+dx)+ia\sin(c+dx)}dx$$

$$\Big\downarrow \text{3571}$$

$$-\frac{i\int \cos^5(c+dx)(ia\cos(c+dx)+a\sin(c+dx))dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{i\int \cos(c+dx)^5(ia\cos(c+dx)+a\sin(c+dx))dx}{a^2}$$

$$\Big\downarrow \text{3569}$$

$$-\frac{i\int \left(ia\cos^6(c+dx)+a\sin(c+dx)\cos^5(c+dx)\right)dx}{a^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(-\frac{a\cos^6(c+dx)}{6d}+\frac{ia\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{5ia\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{5ia\sin(c+dx)\cos(c+dx)}{16d}+\frac{5iax}{16}\right)}{a^2}$$

input
```
Int[Cos[c + d*x]^5/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

output
```
((-I)*(((5*I)/16)*a*x - (a*Cos[c + d*x]^6)/(6*d) + (((5*I)/16)*a*Cos[c + d
*x]*Sin[c + d*x])/d + (((5*I)/24)*a*Cos[c + d*x]^3*Sin[c + d*x])/d + ((I/6
)*a*Cos[c + d*x]^5*Sin[c + d*x])/d))/a^2
```

3.150.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

rule 3571
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^n*b^n   Int[Cos[c + d*x]^m
/(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &
& EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

3.150.4 Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.97

method	result
risch	$\frac{5x}{16a}+\frac{i\,\mathrm{e}^{-6i(dx+c)}}{192ad}+\frac{i\cos(4dx+4c)}{32ad}+\frac{3\sin(4dx+4c)}{64ad}+\frac{5i\cos(2dx+2c)}{64ad}+\frac{15\sin(2dx+2c)}{64ad}$
derivativedivides	$\frac{-\frac{5i\ln(\tan(dx+c)-i)}{32}-\frac{3i}{32(\tan(dx+c)-i)^2}-\frac{1}{24(\tan(dx+c)-i)^3}+\frac{3}{16(\tan(dx+c)-i)}+\frac{i}{32(\tan(dx+c)+i)^2}+\frac{5i\ln(\tan(dx+c)+i)}{32}+\cdots}{da}$
default	$\frac{-\frac{5i\ln(\tan(dx+c)-i)}{32}-\frac{3i}{32(\tan(dx+c)-i)^2}-\frac{1}{24(\tan(dx+c)-i)^3}+\frac{3}{16(\tan(dx+c)-i)}+\frac{i}{32(\tan(dx+c)+i)^2}+\frac{5i\ln(\tan(dx+c)+i)}{32}+\cdots}{da}$
parallelrisch	$\frac{120idx\sin(dx+c)+120dx\cos(dx+c)+16i\cos(dx+c)-i\cos(5dx+5c)-15i\cos(3dx+3c)+104\sin(dx+c)+5\sin(5dx+5c)+\cdots}{384ad(i\sin(dx+c)+\cos(dx+c))}$

input
```
int(cos(d*x+c)^5/(cos(d*x+c)*a+I*a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
5/16*x/a+1/192*I/a/d*exp(-6*I*(d*x+c))+1/32*I/a/d*cos(4*d*x+4*c)+3/64/a/d*
sin(4*d*x+4*c)+5/64*I/a/d*cos(2*d*x+2*c)+15/64/a/d*sin(2*d*x+2*c)
```

3.150.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.77

$$\int \frac{\cos^5(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \frac{\left(120\,dxe^{(6i\,dx+6i\,c)} - 3i\,e^{(10i\,dx+10i\,c)} - 30i\,e^{(8i\,dx+8i\,c)} + 60i\,e^{(4i\,dx+4i\,c)} + 15i\,e^{(2i\,dx+2i\,c)} + 2i\right)e^{(-6i\,dx-6i\,c)}}{384\,ad}$$

input
```
integrate(cos(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="fricas"
)
```

output
```
1/384*(120*d*x*e^(6*I*d*x + 6*I*c) - 3*I*e^(10*I*d*x + 10*I*c) - 30*I*e^(8
*I*d*x + 8*I*c) + 60*I*e^(4*I*d*x + 4*I*c) + 15*I*e^(2*I*d*x + 2*I*c) + 2*
I)*e^(-6*I*d*x - 6*I*c)/(a*d)
```

3.150.6 Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.21

$$\int \frac{\cos^5(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \begin{cases} \frac{(-50331648ia^4d^4e^{16ic}e^{4idx}-503316480ia^4d^4e^{14ic}e^{2idx}+1006632960ia^4d^4e^{10ic}e^{-2idx}+251658240ia^4d^4e^{8ic}e^{-4idx}+33554432ia^4d^4e^{6ic}e^{-6id}}{6442450944a^5d^5} \\ x\left(\frac{(e^{10ic}+5e^{8ic}+10e^{6ic}+10e^{4ic}+5e^{2ic}+1)e^{-6ic}}{32a}-\frac{5}{16a}\right) \end{cases} + \frac{5x}{16a}$$

input
```
integrate(cos(d*x+c)**5/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

output
```
Piecewise(((-50331648*I*a**4*d**4*exp(16*I*c)*exp(4*I*d*x) - 503316480*I*a
**4*d**4*exp(14*I*c)*exp(2*I*d*x) + 1006632960*I*a**4*d**4*exp(10*I*c)*exp
(-2*I*d*x) + 251658240*I*a**4*d**4*exp(8*I*c)*exp(-4*I*d*x) + 33554432*I*a
**4*d**4*exp(6*I*c)*exp(-6*I*d*x))*exp(-12*I*c)/(6442450944*a**5*d**5), Ne
(a**5*d**5*exp(12*I*c), 0)), (x*((exp(10*I*c) + 5*exp(8*I*c) + 10*exp(6*I*
c) + 10*exp(4*I*c) + 5*exp(2*I*c) + 1)*exp(-6*I*c)/(32*a) - 5/(16*a)), Tru
e)) + 5*x/(16*a)
```

3.150.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^5(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="maxima"
)
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.150.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.17

$$\int \frac{\cos^5(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = -\frac{-\frac{30i\log(\tan(dx+c)+i)}{a}+\frac{30i\log(\tan(dx+c)-i)}{a}+\frac{3\left(-15i\tan(dx+c)^2+38\tan(dx+c)+25i\right)}{a(-i\tan(dx+c)+1)^2}-\frac{55i\tan(dx+c)^3+201\tan(dx+c)^2-25}{a(\tan(dx+c)-i)^3}}{192\,d}$$

input
```
integrate(cos(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="giac")
```

output
```
-1/192*(-30*I*log(tan(d*x + c) + I)/a + 30*I*log(tan(d*x + c) - I)/a + 3*(
-15*I*tan(d*x + c)^2 + 38*tan(d*x + c) + 25*I)/(a*(-I*tan(d*x + c) + 1)^2)
 - (55*I*tan(d*x + c)^3 + 201*tan(d*x + c)^2 - 255*I*tan(d*x + c) - 117)/(
a*(tan(d*x + c) - I)^3))/d
```

3.150.9 Mupad [B] (verification not implemented)

Time = 29.37 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.66

$$\int \frac{\cos^5(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \frac{5x}{16a} + \frac{\frac{11\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9}{8}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8 3i}{4}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{3}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 1i}{12}+\frac{13\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{4}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 1i}{12}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3}}{a\,d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1i\right)^4\left(1+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)1i\right)^6}$$

input
```
int(cos(c + d*x)^5/(a*cos(c + d*x) + a*sin(c + d*x)*1i),x)
```

output
```
(5*x)/(16*a) + ((11*tan(c/2 + (d*x)/2))/8 + (tan(c/2 + (d*x)/2)^2*3i)/4 -
tan(c/2 + (d*x)/2)^3/3 + (tan(c/2 + (d*x)/2)^4*1i)/12 + (13*tan(c/2 + (d*x
)/2)^5)/4 - (tan(c/2 + (d*x)/2)^6*1i)/12 - tan(c/2 + (d*x)/2)^7/3 - (tan(c
/2 + (d*x)/2)^8*3i)/4 + (11*tan(c/2 + (d*x)/2)^9)/8)/(a*d*(tan(c/2 + (d*x)
/2) + 1i)^4*(tan(c/2 + (d*x)/2)*1i + 1)^6)
```

3.150. $\int \frac{\cos^5(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

3.151 $\int \frac{\cos^4(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

3.151.1 Optimal result

Integrand size = 31, antiderivative size = 70

$$\int \frac{\cos^4(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \frac{i\cos^5(c+dx)}{5ad}+\frac{\sin(c+dx)}{ad}-\frac{2\sin^3(c+dx)}{3ad}+\frac{\sin^5(c+dx)}{5ad}$$

output
```
1/5*I*cos(d*x+c)^5/a/d+sin(d*x+c)/a/d-2/3*sin(d*x+c)^3/a/d+1/5*sin(d*x+c)^
5/a/d
```

3.151.2 Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.59

$$\int \frac{\cos^4(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \frac{i\cos(c+dx)}{8ad}+\frac{i\cos(3(c+dx))}{16ad}+\frac{i\cos(5(c+dx))}{80ad}+\frac{5\sin(c+dx)}{8ad}+\frac{5\sin(3(c+dx))}{48ad}+\frac{\sin(5(c+dx))}{80ad}$$

input
```
Integrate[Cos[c + d*x]^4/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

output
```
((I/8)*Cos[c + d*x])/(a*d) + ((I/16)*Cos[3*(c + d*x)])/(a*d) + ((I/80)*Cos
[5*(c + d*x)])/(a*d) + (5*Sin[c + d*x])/(8*a*d) + (5*Sin[3*(c + d*x)])/(48
*a*d) + Sin[5*(c + d*x)]/(80*a*d)
```

3.151.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3042, 3571, 3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(c+dx)^4}{a\cos(c+dx)+ia\sin(c+dx)}dx$$

$$\Big\downarrow \text{ 3571}$$

$$-\frac{i\int \cos^4(c+dx)(ia\cos(c+dx)+a\sin(c+dx))dx}{a^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{i\int \cos(c+dx)^4(ia\cos(c+dx)+a\sin(c+dx))dx}{a^2}$$

$$\Big\downarrow \text{ 3569}$$

$$-\frac{i\int \left(ia\cos^5(c+dx)+a\sin(c+dx)\cos^4(c+dx)\right)dx}{a^2}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{i\left(\frac{ia\sin^5(c+dx)}{5d}-\frac{2ia\sin^3(c+dx)}{3d}+\frac{ia\sin(c+dx)}{d}-\frac{a\cos^5(c+dx)}{5d}\right)}{a^2}$$

input
```
Int[Cos[c + d*x]^4/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

output
```
((-I)*(-1/5*(a*Cos[c + d*x]^5)/d + (I*a*Sin[c + d*x])/d - (((2*I)/3)*a*Sin
[c + d*x]^3)/d + ((I/5)*a*Sin[c + d*x]^5)/d))/a^2
```

3.151.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

rule 3571
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^n*b^n   Int[Cos[c + d*x]^m
/(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &
& EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

3.151.4 Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.20

method	result
risch	$\frac{i e^{-5i(dx+c)}}{80ad}+\frac{i\cos(dx+c)}{8ad}+\frac{5\sin(dx+c)}{8ad}+\frac{i\cos(3dx+3c)}{16ad}+\frac{5\sin(3dx+3c)}{48ad}$
derivativedivides	$\frac{-\frac{i}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^2}-\frac{1}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^3}+\frac{5}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)}-\frac{i}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^4}+\frac{3i}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^2}+\frac{2}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right.}}{ad}$
default	$\frac{-\frac{i}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^2}-\frac{1}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^3}+\frac{5}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)}-\frac{i}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^4}+\frac{3i}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^2}+\frac{2}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right.}}{ad}$
parallelrisch	$\frac{\frac{2i}{5}+\frac{6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{5}-\frac{26\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{15}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{3}-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7+2i\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+\frac{10i\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{3}+\frac{14i\tan\left(\frac{dx}{2}+\right.}{5}}{a\left(2i\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3 d}$

input
```
int(cos(d*x+c)^4/(cos(d*x+c)*a+I*a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/80*I/a/d*exp(-5*I*(d*x+c))+1/8*I/a/d*cos(d*x+c)+5/8*sin(d*x+c)/a/d+1/16*
I/a/d*cos(3*d*x+3*c)+5/48/a/d*sin(3*d*x+3*c)
```

3.151.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.90

$$\int \frac{\cos^4(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \frac{\left(-5i\,e^{(8i\,dx+8i\,c)}-60i\,e^{(6i\,dx+6i\,c)}+90i\,e^{(4i\,dx+4i\,c)}+20i\,e^{(2i\,dx+2i\,c)}+3i\right)e^{(-5i\,dx-5i\,c)}}{240\,ad}$$

input
```
integrate(cos(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="fricas"
)
```

output
```
1/240*(-5*I*e^(8*I*d*x + 8*I*c) - 60*I*e^(6*I*d*x + 6*I*c) + 90*I*e^(4*I*d
*x + 4*I*c) + 20*I*e^(2*I*d*x + 2*I*c) + 3*I)*e^(-5*I*d*x - 5*I*c)/(a*d)
```

3.151.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 196 vs. 2(53) = 106.

Time = 0.25 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.80

$$\int \frac{\cos^4(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \begin{cases} \frac{\left(-30720ia^4d^4e^{12ic}e^{3idx}-368640ia^4d^4e^{10ic}e^{idx}+552960ia^4d^4e^{8ic}e^{-idx}+122880ia^4d^4e^{6ic}e^{-3idx}+18432ia^4d^4e^{4ic}e^{-5idx}\right)e^{-9ic}}{1474560a^5d^5} & \text{for } a^5d^5 \\ \frac{x\left(e^{8ic}+4e^{6ic}+6e^{4ic}+4e^{2ic}+1\right)e^{-5ic}}{16a} & \text{otherwi} \end{cases}$$

input
```
integrate(cos(d*x+c)**4/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

output
```
Piecewise(((-30720*I*a**4*d**4*exp(12*I*c)*exp(3*I*d*x) - 368640*I*a**4*d*
*4*exp(10*I*c)*exp(I*d*x) + 552960*I*a**4*d**4*exp(8*I*c)*exp(-I*d*x) + 12
2880*I*a**4*d**4*exp(6*I*c)*exp(-3*I*d*x) + 18432*I*a**4*d**4*exp(4*I*c)*e
xp(-5*I*d*x))*exp(-9*I*c)/(1474560*a**5*d**5), Ne(a**5*d**5*exp(9*I*c), 0)
), (x*(exp(8*I*c) + 4*exp(6*I*c) + 6*exp(4*I*c) + 4*exp(2*I*c) + 1)*exp(-5
*I*c)/(16*a), True))
```

3.151.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^4(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="maxima"
)
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.151.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.70

$$\int \frac{\cos^4(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \frac{\frac{5\left(15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+24i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-13\right)}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+i\right)^3}+\frac{165\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-480i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-650\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+400i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)^5}}{120\,d}$$

input
```
integrate(cos(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="giac")
```

output
```
1/120*(5*(15*tan(1/2*d*x + 1/2*c)^2 + 24*I*tan(1/2*d*x + 1/2*c) - 13)/(a*(
tan(1/2*d*x + 1/2*c) + I)^3) + (165*tan(1/2*d*x + 1/2*c)^4 - 480*I*tan(1/2
*d*x + 1/2*c)^3 - 650*tan(1/2*d*x + 1/2*c)^2 + 400*I*tan(1/2*d*x + 1/2*c)
+ 113)/(a*(tan(1/2*d*x + 1/2*c) - I)^5))/d
```

3.151.9 Mupad [B] (verification not implemented)

Time = 24.95 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.91

$$\int \frac{\cos^4(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = -\frac{\left(-15\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 15\mathrm{i}-5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 25\mathrm{i}-13\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+\tan}{15\,a\,d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\mathrm{i}\right)^3\left(1+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)1\mathrm{i}\right)^5}$$

input
```
int(cos(c + d*x)^4/(a*cos(c + d*x) + a*sin(c + d*x)*1i),x)
```

output
```
-((9*tan(c/2 + (d*x)/2) + tan(c/2 + (d*x)/2)^2*21i - 13*tan(c/2 + (d*x)/2)
^3 + tan(c/2 + (d*x)/2)^4*25i - 5*tan(c/2 + (d*x)/2)^5 + tan(c/2 + (d*x)/2
)^6*15i - 15*tan(c/2 + (d*x)/2)^7 + 3i)*2i)/(15*a*d*(tan(c/2 + (d*x)/2) +
1i)^3*(tan(c/2 + (d*x)/2)*1i + 1)^5)
```

3.151. $\int \frac{\cos^4(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

3.152 $\int \frac{\cos^3(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

3.152.1 Optimal result

Integrand size = 31, antiderivative size = 75

$$\int \frac{\cos^3(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \frac{3x}{8a}+\frac{i\cos^4(c+dx)}{4ad}+\frac{3\cos(c+dx)\sin(c+dx)}{8ad} + \frac{\cos^3(c+dx)\sin(c+dx)}{4ad}$$

output
```
3/8*x/a+1/4*I*cos(d*x+c)^4/a/d+3/8*cos(d*x+c)*sin(d*x+c)/a/d+1/4*cos(d*x+c
)^3*sin(d*x+c)/a/d
```

3.152.2 Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.80

$$\int \frac{\cos^3(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \frac{12c+12dx+4i\cos(2(c+dx))+i\cos(4(c+dx))+8\sin(2(c+dx))+\sin(4(c+dx))}{32ad}$$

input
```
Integrate[Cos[c + d*x]^3/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

output
```
(12*c + 12*d*x + (4*I)*Cos[2*(c + d*x)] + I*Cos[4*(c + d*x)] + 8*Sin[2*(c
+ d*x)] + Sin[4*(c + d*x)])/(32*a*d)
```

3.152.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3042, 3571, 3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(c+dx)^3}{a\cos(c+dx)+ia\sin(c+dx)}dx$$

$$\Big\downarrow \text{3571}$$

$$-\frac{i\int \cos^3(c+dx)(ia\cos(c+dx)+a\sin(c+dx))dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{i\int \cos(c+dx)^3(ia\cos(c+dx)+a\sin(c+dx))dx}{a^2}$$

$$\Big\downarrow \text{3569}$$

$$-\frac{i\int \left(ia\cos^4(c+dx)+a\sin(c+dx)\cos^3(c+dx)\right)dx}{a^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(-\frac{a\cos^4(c+dx)}{4d}+\frac{ia\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3ia\sin(c+dx)\cos(c+dx)}{8d}+\frac{3iax}{8}\right)}{a^2}$$

input
```
Int[Cos[c + d*x]^3/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

output
```
((-I)*(((3*I)/8)*a*x - (a*Cos[c + d*x]^4)/(4*d) + (((3*I)/8)*a*Cos[c + d*x
]*Sin[c + d*x])/d + ((I/4)*a*Cos[c + d*x]^3*Sin[c + d*x])/d))/a^2
```

3.152.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

rule 3571
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^n*b^n   Int[Cos[c + d*x]^m
/(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &
& EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

3.152.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.80

method	result	size
parallelrisch	$\frac{4i\cos(2dx+2c)+i\cos(4dx+4c)+12dx-21i+8\sin(2dx+2c)+\sin(4dx+4c)}{32ad}$	60
risch	$\frac{3x}{8a}+\frac{i\mathrm{e}^{-4i(dx+c)}}{32ad}+\frac{i\cos(2dx+2c)}{8ad}+\frac{\sin(2dx+2c)}{4ad}$	61
derivativedivides	$\frac{-\frac{3i\ln(\tan(dx+c)-i)}{16}-\frac{i}{8(\tan(dx+c)-i)^2}+\frac{1}{4\tan(dx+c)-4i}+\frac{3i\ln(\tan(dx+c)+i)}{16}+\frac{1}{8\tan(dx+c)+8i}}{da}$	75
default	$\frac{-\frac{3i\ln(\tan(dx+c)-i)}{16}-\frac{i}{8(\tan(dx+c)-i)^2}+\frac{1}{4\tan(dx+c)-4i}+\frac{3i\ln(\tan(dx+c)+i)}{16}+\frac{1}{8\tan(dx+c)+8i}}{da}$	75

input
```
int(cos(d*x+c)^3/(cos(d*x+c)*a+I*a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/32*(4*I*cos(2*d*x+2*c)+I*cos(4*d*x+4*c)+12*d*x-21*I+8*sin(2*d*x+2*c)+sin
(4*d*x+4*c))/a/d
```

3.152.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.72

$$\int \frac{\cos^3(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \frac{\left(12\,dxe^{(4i\,dx+4i\,c)}-2i\,e^{(6i\,dx+6i\,c)}+6i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-4i\,dx-4i\,c)}}{32\,ad}$$

input
```
integrate(cos(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="fricas"
)
```

output
```
1/32*(12*d*x*e^(4*I*d*x + 4*I*c) - 2*I*e^(6*I*d*x + 6*I*c) + 6*I*e^(2*I*d*
x + 2*I*c) + I)*e^(-4*I*d*x - 4*I*c)/(a*d)
```

3.152.6 Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.01

$$\int \frac{\cos^3(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \begin{cases} \frac{(-512ia^2d^2e^{8ic}e^{2idx}+1536ia^2d^2e^{4ic}e^{-2idx}+256ia^2d^2e^{2ic}e^{-4idx})e^{-6ic}}{8192a^3d^3} & \text{for } a^3d^3e^{6ic}\neq 0 \\ x\left(\frac{(e^{6ic}+3e^{4ic}+3e^{2ic}+1)e^{-4ic}}{8a}-\frac{3}{8a}\right) & \text{otherwise} \end{cases} + \frac{3x}{8a}$$

input
```
integrate(cos(d*x+c)**3/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

output
```
Piecewise(((-512*I*a**2*d**2*exp(8*I*c)*exp(2*I*d*x) + 1536*I*a**2*d**2*ex
p(4*I*c)*exp(-2*I*d*x) + 256*I*a**2*d**2*exp(2*I*c)*exp(-4*I*d*x))*exp(-6*
I*c)/(8192*a**3*d**3), Ne(a**3*d**3*exp(6*I*c), 0)), (x*((exp(6*I*c) + 3*e
xp(4*I*c) + 3*exp(2*I*c) + 1)*exp(-4*I*c)/(8*a) - 3/(8*a)), True)) + 3*x/(
8*a)
```

3.152.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^3(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="maxima"
)
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.152.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.27

$$\int \frac{\cos^3(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = -\frac{-\frac{6i\log(\tan(dx+c)+i)}{a}+\frac{6i\log(\tan(dx+c)-i)}{a}+\frac{2(3\tan(dx+c)+5i)}{a(-i\tan(dx+c)+1)}+\frac{-9i\tan(dx+c)^2-26\tan(dx+c)+21i}{a(\tan(dx+c)-i)^2}}{32\,d}$$

input
```
integrate(cos(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="giac")
```

output
```
-1/32*(-6*I*log(tan(d*x + c) + I)/a + 6*I*log(tan(d*x + c) - I)/a + 2*(3*t
an(d*x + c) + 5*I)/(a*(-I*tan(d*x + c) + 1)) + (-9*I*tan(d*x + c)^2 - 26*t
an(d*x + c) + 21*I)/(a*(tan(d*x + c) - I)^2))/d
```

3.152.9 Mupad [B] (verification not implemented)

Time = 26.04 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.48

$$\int \frac{\cos^3(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \frac{3\,x}{8\,a}-\frac{\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,1{}\mathrm{i}}{2}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,1{}\mathrm{i}}{2}+\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{a\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1{}\mathrm{i}\right)^2\,\left(1+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}\right)^4}$$

input
```
int(cos(c + d*x)^3/(a*cos(c + d*x) + a*sin(c + d*x)*1i),x)
```

output
```
(3*x)/(8*a) - ((5*tan(c/2 + (d*x)/2))/4 + (tan(c/2 + (d*x)/2)^2*1i)/2 - ta
n(c/2 + (d*x)/2)^3/2 - (tan(c/2 + (d*x)/2)^4*1i)/2 + (5*tan(c/2 + (d*x)/2)
^5)/4)/(a*d*(tan(c/2 + (d*x)/2) + 1i)^2*(tan(c/2 + (d*x)/2)*1i + 1)^4)
```

3.152. $\int \frac{\cos^3(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

3.153 $\int \frac{\cos^2(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

3.153.1 Optimal result

Integrand size = 31, antiderivative size = 52

$$\int \frac{\cos^2(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx=\frac{i\cos^3(c+dx)}{3ad}+\frac{\sin(c+dx)}{ad}-\frac{\sin^3(c+dx)}{3ad}$$

output
```
1/3*I*cos(d*x+c)^3/a/d+sin(d*x+c)/a/d-1/3*sin(d*x+c)^3/a/d
```

3.153.2 Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.40

$$\int \frac{\cos^2(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx=\frac{i\cos(c+dx)}{4ad}+\frac{i\cos(3(c+dx))}{12ad}+\frac{3\sin(c+dx)}{4ad}+\frac{\sin(3(c+dx))}{12ad}$$

input
```
Integrate[Cos[c + d*x]^2/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

output
```
((I/4)*Cos[c + d*x])/(a*d) + ((I/12)*Cos[3*(c + d*x)])/(a*d) + (3*Sin[c +
d*x])/(4*a*d) + Sin[3*(c + d*x)]/(12*a*d)
```

3.153.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3042, 3571, 3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(c+dx)^2}{a\cos(c+dx)+ia\sin(c+dx)}dx$$

$$\Big\downarrow \text{3571}$$

$$-\frac{i\int \cos^2(c+dx)(ia\cos(c+dx)+a\sin(c+dx))dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{i\int \cos(c+dx)^2(ia\cos(c+dx)+a\sin(c+dx))dx}{a^2}$$

$$\Big\downarrow \text{3569}$$

$$-\frac{i\int \left(ia\cos^3(c+dx)+a\sin(c+dx)\cos^2(c+dx)\right)dx}{a^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(-\frac{ia\sin^3(c+dx)}{3d}+\frac{ia\sin(c+dx)}{d}-\frac{a\cos^3(c+dx)}{3d}\right)}{a^2}$$

input
```
Int[Cos[c + d*x]^2/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

output
```
((-I)*(-1/3*(a*Cos[c + d*x]^3)/d + (I*a*Sin[c + d*x])/d - ((I/3)*a*Sin[c +
 d*x]^3)/d))/a^2
```

3.153.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

rule 3571
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^n*b^n   Int[Cos[c + d*x]^m
/(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &
& EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

3.153.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.94

method	result	size
risch	$\frac{i e^{-3i(dx+c)}}{12ad}+\frac{i\cos(dx+c)}{4ad}+\frac{3\sin(dx+c)}{4ad}$	49
derivativedivides	$\frac{-\frac{2}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^{3}}+\frac{i}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^{2}}+\frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)}+\frac{2}{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+4i}}{ad}$	75
default	$\frac{-\frac{2}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^{3}}+\frac{i}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^{2}}+\frac{3}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)}+\frac{2}{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+4i}}{ad}$	75
parallelrisch	$\frac{2i+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+6i\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{3ad\left(2i\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+2i\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$	93

input
```
int(cos(d*x+c)^2/(cos(d*x+c)*a+I*a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/12*I/a/d*exp(-3*I*(d*x+c))+1/4*I/a/d*cos(d*x+c)+3/4*sin(d*x+c)/a/d
```

3.153.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.79

$$\int \frac{\cos^2(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \frac{\left(-3i\,e^{(4i\,dx+4i\,c)}+6i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-3i\,dx-3i\,c)}}{12\,ad}$$

input
```
integrate(cos(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="fricas"
)
```

output
```
1/12*(-3*I*e^(4*I*d*x + 4*I*c) + 6*I*e^(2*I*d*x + 2*I*c) + I)*e^(-3*I*d*x
- 3*I*c)/(a*d)
```

3.153.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(37) = 74.

Time = 0.17 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.42

$$\int \frac{\cos^2(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \begin{cases} \frac{(-24ia^2d^2e^{5ic}e^{idx}+48ia^2d^2e^{3ic}e^{-idx}+8ia^2d^2e^{ic}e^{-3idx})e^{-4ic}}{96a^3d^3} & \text{for } a^3d^3e^{4ic}\neq 0 \\ \frac{x(e^{4ic}+2e^{2ic}+1)e^{-3ic}}{4a} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**2/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

output
```
Piecewise(((-24*I*a**2*d**2*exp(5*I*c)*exp(I*d*x) + 48*I*a**2*d**2*exp(3*I
*c)*exp(-I*d*x) + 8*I*a**2*d**2*exp(I*c)*exp(-3*I*d*x))*exp(-4*I*c)/(96*a*
*3*d**3), Ne(a**3*d**3*exp(4*I*c), 0)), (x*(exp(4*I*c) + 2*exp(2*I*c) + 1)
*exp(-3*I*c)/(4*a), True))
```

3.153.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^2(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="maxima"
)
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.153.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.29

$$\int \frac{\cos^2(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \frac{\frac{3}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)}+\frac{9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-12i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-7}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^3}}{6\,d}$$

input
```
integrate(cos(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="giac")
```

output
```
1/6*(3/(a*(tan(1/2*d*x + 1/2*c) + I)) + (9*tan(1/2*d*x + 1/2*c)^2 - 12*I*t
an(1/2*d*x + 1/2*c) - 7)/(a*(tan(1/2*d*x + 1/2*c) - I)^3))/d
```

3.153.9 Mupad [B] (verification not implemented)

Time = 23.33 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.50

$$\int \frac{\cos^2(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \frac{\left(-3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 3\mathrm{i}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\mathrm{i}\right)\,2\mathrm{i}}{3\,a\,d\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\mathrm{i}\right)\,{\left(1+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,1\mathrm{i}\right)}^3}$$

input
```
int(cos(c + d*x)^2/(a*cos(c + d*x) + a*sin(c + d*x)*1i),x)
```

output
```
((tan(c/2 + (d*x)/2) + tan(c/2 + (d*x)/2)^2*3i - 3*tan(c/2 + (d*x)/2)^3 +
1i)*2i)/(3*a*d*(tan(c/2 + (d*x)/2) + 1i)*(tan(c/2 + (d*x)/2)*1i + 1)^3)
```

3.154 $\int \frac{\cos(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

3.154.1 Optimal result

Integrand size = 29, antiderivative size = 46

$$\int \frac{\cos(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \frac{x}{2a}+\frac{i\cos(c+dx)}{2d(a\cos(c+dx)+ia\sin(c+dx))}$$

output
```
1/2*x/a+1/2*I*cos(d*x+c)/d/(a*cos(d*x+c)+I*a*sin(d*x+c))
```

3.154.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{\cos(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \frac{2(c+dx)+i\cos(2(c+dx))+\sin(2(c+dx))}{4ad}$$

input
```
Integrate[Cos[c + d*x]/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

output
```
(2*(c + d*x) + I*Cos[2*(c + d*x)] + Sin[2*(c + d*x)])/(4*a*d)
```

3.154.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {3042, 3561, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}dx$$

$$\Big\downarrow \text{3561}$$

$$\frac{\int 1dx}{2a}+\frac{i\cos(c+dx)}{2d(a\cos(c+dx)+ia\sin(c+dx))}$$

$$\Big\downarrow \text{24}$$

$$\frac{x}{2a}+\frac{i\cos(c+dx)}{2d(a\cos(c+dx)+ia\sin(c+dx))}$$

input
```
Int[Cos[c + d*x]/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

output
```
x/(2*a) + ((I/2)*Cos[c + d*x])/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x]))
```

3.154.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3561
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*((a*Cos[c + d*x] + b*Si
n[c + d*x])^n/(2*a*d*n*Cos[c + d*x]^n)), x] + Simp[1/(2*a)   Int[(a*Cos[c +
 d*x] + b*Sin[c + d*x])^(n + 1)/Cos[c + d*x]^(n + 1), x], x] /; FreeQ[{a, b
, c, d}, x] && EqQ[m + n, 0] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

3.154.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.57

method	result	size
risch	$\frac{x}{2a}+\frac{i\mathrm{e}^{-2i(dx+c)}}{4ad}$	26
derivativedivides	$\frac{\frac{i\ln(\tan(dx+c)+i)}{4}-\frac{i\ln(\tan(dx+c)-i)}{4}+\frac{1}{2\tan(dx+c)-2i}}{da}$	48
default	$\frac{\frac{i\ln(\tan(dx+c)+i)}{4}-\frac{i\ln(\tan(dx+c)-i)}{4}+\frac{1}{2\tan(dx+c)-2i}}{da}$	48

input
```
int(cos(d*x+c)/(cos(d*x+c)*a+I*a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x/a+1/4*I/a/d*exp(-2*I*(d*x+c))
```

3.154.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.70

$$\int \frac{\cos(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx=\frac{\left(2\,dxe^{(2i\,dx+2i\,c)}+i\right)e^{(-2i\,dx-2i\,c)}}{4\,ad}$$

input
```
integrate(cos(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*(2*d*x*e^(2*I*d*x + 2*I*c) + I)*e^(-2*I*d*x - 2*I*c)/(a*d)
```

3.154.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.30

$$\int \frac{\cos(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx=\begin{cases}\frac{ie^{-2ic}e^{-2idx}}{4ad} & \text{for } ade^{2ic}\neq 0\\ x\left(\frac{(e^{2ic}+1)e^{-2ic}}{2a}-\frac{1}{2a}\right) & \text{otherwise}\end{cases}+\frac{x}{2a}$$

input
```
integrate(cos(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

output
```
Piecewise((I*exp(-2*I*c)*exp(-2*I*d*x)/(4*a*d), Ne(a*d*exp(2*I*c), 0)), (x
*((exp(2*I*c) + 1)*exp(-2*I*c)/(2*a) - 1/(2*a)), True)) + x/(2*a)
```

3.154.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.154.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.26

$$\int \frac{\cos(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx=-\frac{-\frac{i\log(\tan(dx+c)+i)}{a}+\frac{i\log(\tan(dx+c)-i)}{a}+\frac{-i\tan(dx+c)-3}{a(\tan(dx+c)-i)}}{4d}$$

input
```
integrate(cos(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="giac")
```

output
```
-1/4*(-I*log(tan(d*x + c) + I)/a + I*log(tan(d*x + c) - I)/a + (-I*tan(d*x
 + c) - 3)/(a*(tan(d*x + c) - I)))/d
```

3.154.9 Mupad [B] (verification not implemented)

Time = 22.66 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{\cos(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \frac{x}{2\,a}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,d\,\left(1+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}\right)^2}$$

input
```
int(cos(c + d*x)/(a*cos(c + d*x) + a*sin(c + d*x)*1i),x)
```

output
```
x/(2*a) + tan(c/2 + (d*x)/2)/(a*d*(tan(c/2 + (d*x)/2)*1i + 1)^2)
```

3.155 $\int \frac{1}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

3.155.1 Optimal result

Integrand size = 22, antiderivative size = 29

$$\int \frac{1}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \frac{i}{d(a\cos(c+dx)+ia\sin(c+dx))}$$

output
```
I/d/(a*cos(d*x+c)+I*a*sin(d*x+c))
```

3.155.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{1}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \frac{i}{d(a\cos(c+dx)+ia\sin(c+dx))}$$

input
```
Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-1),x]
```

output
```
I/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x]))
```

3.155.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3042, 3550}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{a\cos(c+dx)+ia\sin(c+dx)}dx$$

$$\Big\downarrow \ 3550$$

$$\frac{i}{d(a\cos(c+dx)+ia\sin(c+dx))}$$

input
```
Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-1),x]
```

output
```
I/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x]))
```

3.155.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3550
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

3.155.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

method	result	size
risch	$\frac{i\mathrm{e}^{-i(dx+c)}}{ad}$	19
derivativedivides	$\frac{2}{da\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)}$	23
default	$\frac{2}{da\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)}$	23
norman	$\frac{-\frac{2i\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{ad}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}$	55

input
```
int(1/(cos(d*x+c)*a+I*a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
I/a/d*exp(-I*(d*x+c))
```

3.155.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.59

$$\int \frac{1}{a\cos(c+dx)+ia\sin(c+dx)}\,dx=\frac{i\,e^{(-i\,dx-i\,c)}}{ad}$$

input
```
integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
I*e^(-I*d*x - I*c)/(a*d)
```

3.155.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{a\cos(c+dx)+ia\sin(c+dx)}\,dx=\begin{cases}\frac{ie^{-ic}e^{-idx}}{ad} & \text{for } ade^{ic}\neq 0\\ \frac{xe^{-ic}}{a} & \text{otherwise}\end{cases}$$

input
```
integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

output
```
Piecewise((I*exp(-I*c)*exp(-I*d*x)/(a*d), Ne(a*d*exp(I*c), 0)), (x*exp(-I*
c)/a, True))
```

3.155.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{1}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \frac{2}{\left(-i\,a+\frac{a\sin(dx+c)}{\cos(dx+c)+1}\right)d}$$

input
```
integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
2/((-I*a + a*sin(d*x + c)/(cos(d*x + c) + 1))*d)
```

3.155.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{1}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \frac{2}{ad\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)}$$

input
```
integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="giac")
```

output
```
2/(a*d*(tan(1/2*d*x + 1/2*c) - I))
```

3.155.9 Mupad [B] (verification not implemented)

Time = 22.59 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{1}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \frac{2\mathrm{i}}{a\,d\,\left(1+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}\right)}$$

input
```
int(1/(a*cos(c + d*x) + a*sin(c + d*x)*1i),x)
```

output
```
2i/(a*d*(tan(c/2 + (d*x)/2)*1i + 1))
```

3.156 $\int \frac{\sec(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

3.156.1 Optimal result

Integrand size = 29, antiderivative size = 23

$$\int \frac{\sec(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \frac{x}{a}+\frac{i\log(\cos(c+dx))}{ad}$$

output
```
x/a+I*ln(cos(d*x+c))/a/d
```

3.156.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sec(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = -\frac{i\log(i-\tan(c+dx))}{ad}$$

input
```
Integrate[Sec[c + d*x]/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

output
```
((-I)*Log[I - Tan[c + d*x]])/(a*d)
```

3.156.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {3042, 3571, 3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(c+dx)(a\cos(c+dx)+ia\sin(c+dx))}dx$$

$$\Big\downarrow \text{3571}$$

$$-\frac{i\int \sec(c+dx)(ia\cos(c+dx)+a\sin(c+dx))dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{i\int \frac{ia\cos(c+dx)+a\sin(c+dx)}{\cos(c+dx)}dx}{a^2}$$

$$\Big\downarrow \text{3569}$$

$$-\frac{i\int (\tan(c+dx)a+ia)dx}{a^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(-\frac{a\log(\cos(c+dx))}{d}+iax\right)}{a^2}$$

input
```
Int[Sec[c + d*x]/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

output
```
((-I)*(I*a*x - (a*Log[Cos[c + d*x]])/d))/a^2
```

3.156.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

rule 3571
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^n*b^n   Int[Cos[c + d*x]^m
/(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &
& EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

3.156.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$-\frac{i \ln(i \tan(dx+c)+1)}{da}$	22
default	$-\frac{i \ln(i \tan(dx+c)+1)}{da}$	22
risch	$\frac{2x}{a}+\frac{2c}{ad}+\frac{i \ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{ad}$	38
norman	$\frac{x}{a}+\frac{i \ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{ad}+\frac{i \ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{ad}-\frac{i \ln\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}{ad}$	72

input
```
int(sec(d*x+c)/(cos(d*x+c)*a+I*a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-I/d/a*ln(I*tan(d*x+c)+1)
```

3.156.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \frac{\sec(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \frac{2\,dx+i\,\log\left(e^{(2i\,dx+2i\,c)}+1\right)}{ad}$$

input
```
integrate(sec(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
(2*d*x + I*log(e^(2*I*d*x + 2*I*c) + 1))/(a*d)
```

3.156.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \frac{\int \frac{\sec(c+dx)}{i\sin(c+dx)+\cos(c+dx)}\,dx}{a}$$

input
```
integrate(sec(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)/(I*sin(c + d*x) + cos(c + d*x)), x)/a
```

3.156.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(21) = 42.

Time = 0.25 (sec) , antiderivative size = 101, normalized size of antiderivative = 4.39

$$\int \frac{\sec(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = -\frac{-\frac{i\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{i\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}+\frac{i\,\log\left(-\frac{2i\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)}{a}}{d}$$

input
```
integrate(sec(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-(-I*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - I*log(sin(d*x + c)/(cos(
d*x + c) + 1) - 1)/a + I*log(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*
x + c)^2/(cos(d*x + c) + 1)^2 - 1)/a)/d
```

3.156.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(21) = 42$.

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.48

$$\int \frac{\sec(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = -\frac{-\frac{i\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a}+\frac{2i\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)}{a}-\frac{i\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a}}{d}$$

input
```
integrate(sec(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="giac")
```

output
```
-(-I*log(tan(1/2*d*x + 1/2*c) + 1)/a + 2*I*log(tan(1/2*d*x + 1/2*c) - I)/a
 - I*log(tan(1/2*d*x + 1/2*c) - 1)/a)/d
```

3.156.9 Mupad [B] (verification not implemented)

Time = 22.63 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.78

$$\int \frac{\sec(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = -\frac{\left(2\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-\mathrm{i}\right)-\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)\right)\,1\mathrm{i}}{a\,d}$$

input
```
int(1/(cos(c + d*x)*(a*cos(c + d*x) + a*sin(c + d*x)*1i)),x)
```

output
```
-((2*log(tan(c/2 + (d*x)/2) - 1i) - log(tan(c/2 + (d*x)/2)^2 - 1))*1i)/(a*
d)
```

3.157 $\int \frac{\sec^2(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

3.157.1 Optimal result

Integrand size = 31, antiderivative size = 31

$$\int \frac{\sec^2(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{ad}-\frac{i\sec(c+dx)}{ad}$$

output
```
arctanh(sin(d*x+c))/a/d-I*sec(d*x+c)/a/d
```

3.157.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13

$$\int \frac{\sec^2(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = -\frac{i\left(2i\operatorname{arctanh}\left(\sin(c)+\cos(c)\tan\left(\frac{dx}{2}\right)\right)+\sec(c+dx)\right)}{ad}$$

input
```
Integrate[Sec[c + d*x]^2/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

output
```
((-I)*((2*I)*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]] + Sec[c + d*x]))/(a*d)
```

3.157.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3042, 3571, 3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\cos(c+dx)^2(a\cos(c+dx)+ia\sin(c+dx))}dx$$

$$\Big\downarrow \text{ 3571}$$

$$-\frac{i\int \sec^2(c+dx)(ia\cos(c+dx)+a\sin(c+dx))dx}{a^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{i\int \frac{ia\cos(c+dx)+a\sin(c+dx)}{\cos(c+dx)^2}dx}{a^2}$$

$$\Big\downarrow \text{ 3569}$$

$$-\frac{i\int (ia\sec(c+dx)+a\tan(c+dx)\sec(c+dx))dx}{a^2}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{i\left(\frac{a\sec(c+dx)}{d}+\frac{ia\text{arctanh}(\sin(c+dx))}{d}\right)}{a^2}$$

input
```
Int[Sec[c + d*x]^2/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

output
```
((-I)*((I*a*ArcTanh[Sin[c + d*x]])/d + (a*Sec[c + d*x])/d))/a^2
```

3.157.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

rule 3571
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^n*b^n   Int[Cos[c + d*x]^m
/(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &
& EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

3.157.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(30) = 60$.

Time = 0.59 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.10

method	result	size
norman	$\frac{2i}{ad\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{ad}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{ad}$	65
derivativedivides	$\frac{\frac{2i}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2}-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{i}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{ad}$	70
default	$\frac{\frac{2i}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2}-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{i}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{ad}$	70
risch	$-\frac{2i\,\mathrm{e}^{i(dx+c)}}{da\left(\mathrm{e}^{2i(dx+c)}+1\right)}+\frac{\ln\left(i+\mathrm{e}^{i(dx+c)}\right)}{ad}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{ad}$	74

input
```
int(sec(d*x+c)^2/(cos(d*x+c)*a+I*a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2*I/a/d/(tan(1/2*d*x+1/2*c)^2-1)+1/a/d*ln(tan(1/2*d*x+1/2*c)+1)-1/a/d*ln(t
an(1/2*d*x+1/2*c)-1)
```

3.157.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(29) = 58$.

Time = 0.24 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.58

$$\int \frac{\sec^2(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \frac{\left(e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-\left(e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}-i\right)-2i\,e^{(i\,dx+i\,c)}}{ade^{(2i\,dx+2i\,c)}+ad}$$

input
```
integrate(sec(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="fricas"
)
```

output
```
((e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x + I*c) + I) - (e^(2*I*d*x + 2*I*c)
 + 1)*log(e^(I*d*x + I*c) - I) - 2*I*e^(I*d*x + I*c))/(a*d*e^(2*I*d*x + 2*
I*c) + a*d)
```

3.157.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \frac{\int \frac{\sec^2(c+dx)}{i\sin(c+dx)+\cos(c+dx)}\,dx}{a}$$

input
```
integrate(sec(d*x+c)**2/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**2/(I*sin(c + d*x) + cos(c + d*x)), x)/a
```

3.157.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(29) = 58$.

Time = 0.22 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.68

$$\int \frac{\sec^2(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \frac{\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a} - \frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a} - \frac{2}{-i\,a+\frac{i\,a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}}{d}$$

input
```
integrate(sec(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="maxima"
)
```

output
```
(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x +
c) + 1) - 1)/a - 2/(-I*a + I*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2))/d
```

3.157.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.87

$$\int \frac{\sec^2(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \frac{\frac{\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a} - \frac{\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a} + \frac{2i}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)a}}{d}$$

input
```
integrate(sec(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="giac")
```

output
```
(log(tan(1/2*d*x + 1/2*c) + 1)/a - log(tan(1/2*d*x + 1/2*c) - 1)/a + 2*I/(
(tan(1/2*d*x + 1/2*c)^2 - 1)*a))/d
```

3.157.9 Mupad [B] (verification not implemented)

Time = 23.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.39

$$\int \frac{\sec^2(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d}+\frac{2\mathrm{i}}{a\,d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

input
```
int(1/(cos(c + d*x)^2*(a*cos(c + d*x) + a*sin(c + d*x)*1i)),x)
```

output
```
(2*atanh(tan(c/2 + (d*x)/2)))/(a*d) + 2i/(a*d*(tan(c/2 + (d*x)/2)^2 - 1))
```

3.158 $\int \frac{\sec^3(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

3.158.1 Optimal result

Integrand size = 31, antiderivative size = 34

$$\int \frac{\sec^3(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx=-\frac{i\sec^2(c+dx)}{2ad}+\frac{\tan(c+dx)}{ad}$$

output
```
-1/2*I*sec(d*x+c)^2/a/d+tan(d*x+c)/a/d
```

3.158.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int \frac{\sec^3(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx=-\frac{i\tan(c+dx)(2i+\tan(c+dx))}{2ad}$$

input
```
Integrate[Sec[c + d*x]^3/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

output
```
((-1/2*I)*Tan[c + d*x]*(2*I + Tan[c + d*x]))/(a*d)
```

3.158.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3042, 3571, 3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(c+dx)^3(a\cos(c+dx)+ia\sin(c+dx))}dx$$

$$\Big\downarrow \text{3571}$$

$$-\frac{i\int \sec^3(c+dx)(ia\cos(c+dx)+a\sin(c+dx))dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{i\int \frac{ia\cos(c+dx)+a\sin(c+dx)}{\cos(c+dx)^3}dx}{a^2}$$

$$\Big\downarrow \text{3569}$$

$$-\frac{i\int \left(ia\sec^2(c+dx)+a\tan(c+dx)\sec^2(c+dx)\right)dx}{a^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{a\sec^2(c+dx)}{2d}+\frac{ia\tan(c+dx)}{d}\right)}{a^2}$$

input
```
Int[Sec[c + d*x]^3/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

output
```
((-I)*((a*Sec[c + d*x]^2)/(2*d) + (I*a*Tan[c + d*x])/d))/a^2
```

3.158.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

rule 3571
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^n*b^n   Int[Cos[c + d*x]^m
/(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &
& EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

3.158.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

method	result	size
risch	$\frac{2i}{da\left(\mathrm{e}^{2i(dx+c)}+1\right)^{2}}$	23
derivativedivides	$-\frac{i\left(\frac{\tan(dx+c)^{2}}{2}+i\tan(dx+c)\right)}{da}$	30
default	$-\frac{i\left(\frac{\tan(dx+c)^{2}}{2}+i\tan(dx+c)\right)}{da}$	30
norman	$\frac{\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{ad}-\frac{2i\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{ad}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)^{2}}$	74

input
```
int(sec(d*x+c)^3/(cos(d*x+c)*a+I*a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2*I/d/a/(exp(2*I*(d*x+c))+1)^2
```

3.158.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int \frac{\sec^3(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \frac{2i}{ade^{(4i\,dx+4i\,c)}+2\,ade^{(2i\,dx+2i\,c)}+ad}$$

input
```
integrate(sec(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="fricas"
)
```

output
```
2*I/(a*d*e^(4*I*d*x + 4*I*c) + 2*a*d*e^(2*I*d*x + 2*I*c) + a*d)
```

3.158.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \frac{\int \frac{\sec^3(c+dx)}{i\sin(c+dx)+\cos(c+dx)}\,dx}{a}$$

input
```
integrate(sec(d*x+c)**3/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**3/(I*sin(c + d*x) + cos(c + d*x)), x)/a
```

3.158.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 108 vs. 2(30) = 60.

Time = 0.22 (sec) , antiderivative size = 108, normalized size of antiderivative = 3.18

$$\int \frac{\sec^3(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \frac{2\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{i\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{\left(a-\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)d}$$

input
```
integrate(sec(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="maxima"
)
```

output
```
2*(sin(d*x + c)/(cos(d*x + c) + 1) - I*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
 - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/((a - 2*a*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 + a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)*d)
```

3.158.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int \frac{\sec^3(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = -\frac{i\,\tan\left(dx+c\right)^2-2\,\tan\left(dx+c\right)}{2\,ad}$$

input
```
integrate(sec(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="giac")
```

output
```
-1/2*(I*tan(d*x + c)^2 - 2*tan(d*x + c))/(a*d)
```

3.158.9 Mupad [B] (verification not implemented)

Time = 24.35 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

$$\int \frac{\sec^3(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = -\frac{\tan(c+d\,x)\,\left(-2+\tan(c+d\,x)\,1\mathrm{i}\right)}{2\,a\,d}$$

input
```
int(1/(cos(c + d*x)^3*(a*cos(c + d*x) + a*sin(c + d*x)*1i)),x)
```

output
```
-(tan(c + d*x)*(tan(c + d*x)*1i - 2))/(2*a*d)
```

3.159 $\int \frac{\sec^4(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

3.159.1 Optimal result

Integrand size = 31, antiderivative size = 60

$$\int \frac{\sec^4(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{2ad} - \frac{i\sec^3(c+dx)}{3ad} + \frac{\sec(c+dx)\tan(c+dx)}{2ad}$$

output
```
1/2*arctanh(sin(d*x+c))/a/d-1/3*I*sec(d*x+c)^3/a/d+1/2*sec(d*x+c)*tan(d*x+
c)/a/d
```

3.159.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.90

$$\int \frac{\sec^4(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = -\frac{i\left(12i\operatorname{arctanh}\left(\sin(c)+\cos(c)\tan\left(\frac{dx}{2}\right)\right)+\sec^3(c+dx)(4+3i\sin(2(c+dx)))\right)}{12ad}$$

input
```
Integrate[Sec[c + d*x]^4/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

output
```
((-1/12*I)*((12*I)*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]] + Sec[c + d*x]^3*
(4 + (3*I)*Sin[2*(c + d*x)])))/(a*d)
```

3.159.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3042, 3571, 3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\cos(c+dx)^4(a\cos(c+dx)+ia\sin(c+dx))}dx$$

$$\Big\downarrow \text{ 3571}$$

$$-\frac{i\int \sec^4(c+dx)(ia\cos(c+dx)+a\sin(c+dx))dx}{a^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{i\int \frac{ia\cos(c+dx)+a\sin(c+dx)}{\cos(c+dx)^4}dx}{a^2}$$

$$\Big\downarrow \text{ 3569}$$

$$-\frac{i\int \left(ia\sec^3(c+dx)+a\tan(c+dx)\sec^3(c+dx)\right)dx}{a^2}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{i\left(\frac{ia\text{arctanh}(\sin(c+dx))}{2d}+\frac{a\sec^3(c+dx)}{3d}+\frac{ia\tan(c+dx)\sec(c+dx)}{2d}\right)}{a^2}$$

input
```
Int[Sec[c + d*x]^4/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

output
```
((-I)*(((I/2)*a*ArcTanh[Sin[c + d*x]])/d + (a*Sec[c + d*x]^3)/(3*d) + ((I/
2)*a*Sec[c + d*x]*Tan[c + d*x])/d))/a^2
```

3.159.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

rule 3571
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^n*b^n   Int[Cos[c + d*x]^m
/(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &
& EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

3.159.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.67

method	result
risch	$-\frac{i\left(3\,\mathrm{e}^{5i(dx+c)}+8\,\mathrm{e}^{3i(dx+c)}-3\,\mathrm{e}^{i(dx+c)}\right)}{3da\left(\mathrm{e}^{2i(dx+c)}+1\right)^{3}}+\frac{\ln\left(i+\mathrm{e}^{i(dx+c)}\right)}{2ad}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2ad}$
norman	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{2i}{3ad}+\frac{2i\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{ad}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)^{3}}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2ad}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2ad}$
derivativedivides	$\frac{\frac{i}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}+\frac{2\left(\frac{1}{4}+\frac{i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{2\left(\frac{1}{4}+\frac{i}{4}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}-\frac{i}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{2\left(\frac{1}{4}-\frac{i}{4}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\ldots}{ad}$
default	$\frac{\frac{i}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}+\frac{2\left(\frac{1}{4}+\frac{i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{2\left(\frac{1}{4}+\frac{i}{4}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}-\frac{i}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{2\left(\frac{1}{4}-\frac{i}{4}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\ldots}{ad}$

input
```
int(sec(d*x+c)^4/(cos(d*x+c)*a+I*a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/3*I/d/a/(exp(2*I*(d*x+c))+1)^3*(3*exp(5*I*(d*x+c))+8*exp(3*I*(d*x+c))-3
*exp(I*(d*x+c)))+1/2/a/d*ln(I+exp(I*(d*x+c)))-1/2/a/d*ln(exp(I*(d*x+c))-I)
```

3.159.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(52) = 104$.

Time = 0.25 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.90

$$\int \frac{\sec^4(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$$
$$= \frac{3\left(e^{(6i\,dx+6i\,c)}+3\,e^{(4i\,dx+4i\,c)}+3\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-3\left(e^{(6i\,dx+6i\,c)}+3\,e^{(4i\,dx+4i\,c)}+3\,e^{(2i\,dx+}\right.}{6\left(ade^{(6i\,dx+6i\,c)}+3\,ade^{(4i\,dx+4i\,c)}+3\,ade^{(2i\,dx}\right.}$$

input
```
integrate(sec(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="fricas"
)
```

output
```
1/6*(3*(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x + 4*I*c) + 3*e^(2*I*d*x + 2*I*c
) + 1)*log(e^(I*d*x + I*c) + I) - 3*(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x +
4*I*c) + 3*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x + I*c) - I) - 6*I*e^(5*I*
d*x + 5*I*c) - 16*I*e^(3*I*d*x + 3*I*c) + 6*I*e^(I*d*x + I*c))/(a*d*e^(6*I
*d*x + 6*I*c) + 3*a*d*e^(4*I*d*x + 4*I*c) + 3*a*d*e^(2*I*d*x + 2*I*c) + a*
d)
```

3.159.6 Sympy [F]

$$\int \frac{\sec^4(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \frac{\int \frac{\sec^4(c+dx)}{i\sin(c+dx)+\cos(c+dx)}\,dx}{a}$$

input
```
integrate(sec(d*x+c)**4/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**4/(I*sin(c + d*x) + cos(c + d*x)), x)/a
```

3.159.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(52) = 104$.

Time = 0.20 (sec) , antiderivative size = 186, normalized size of antiderivative = 3.10

$$\int \frac{\sec^4(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \frac{\frac{4\left(\frac{3i\sin(dx+c)}{\cos(dx+c)+1}+\frac{6\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{3i\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+2\right)}{6i\,a-\frac{18i\,a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{18i\,a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{6i\,a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}}{2d}$$

input
```
integrate(sec(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="maxima"
)
```

output
```
1/2*(4*(3*I*sin(d*x + c)/(cos(d*x + c) + 1) + 6*sin(d*x + c)^4/(cos(d*x +
c) + 1)^4 - 3*I*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 2)/(6*I*a - 18*I*a*s
in(d*x + c)^2/(cos(d*x + c) + 1)^2 + 18*I*a*sin(d*x + c)^4/(cos(d*x + c) +
 1)^4 - 6*I*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + log(sin(d*x + c)/(cos
(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a)/d
```

3.159.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.65

$$\int \frac{\sec^4(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \frac{\frac{3\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a}-\frac{3\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{a}+\frac{2\left(3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+6i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2i\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^3a}}{6d}$$

input
```
integrate(sec(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="giac")
```

output
```
1/6*(3*log(tan(1/2*d*x + 1/2*c) + 1)/a - 3*log(tan(1/2*d*x + 1/2*c) - 1)/a
 + 2*(3*tan(1/2*d*x + 1/2*c)^5 + 6*I*tan(1/2*d*x + 1/2*c)^4 - 3*tan(1/2*d*
x + 1/2*c) + 2*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*a))/d
```

3.159.9 Mupad [B] (verification not implemented)

Time = 24.86 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.93

$$\int \frac{\sec^4(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$$
$$=\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a\,d}+\frac{\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{a}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\,2i}{a}+\frac{2i}{3\,a}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)}$$

input
```
int(1/(cos(c + d*x)^4*(a*cos(c + d*x) + a*sin(c + d*x)*1i)),x)
```

output
```
atanh(tan(c/2 + (d*x)/2))/(a*d) + ((tan(c/2 + (d*x)/2)^4*2i)/a + tan(c/2 +
 (d*x)/2)^5/a + 2i/(3*a) - tan(c/2 + (d*x)/2)/a)/(d*(3*tan(c/2 + (d*x)/2)^
2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

3.160 $\int \frac{\sec^5(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

3.160.1 Optimal result

Integrand size = 31, antiderivative size = 52

$$\int \frac{\sec^5(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = -\frac{i\sec^4(c+dx)}{4ad}+\frac{\tan(c+dx)}{ad}+\frac{\tan^3(c+dx)}{3ad}$$

output
```
-1/4*I*sec(d*x+c)^4/a/d+tan(d*x+c)/a/d+1/3*tan(d*x+c)^3/a/d
```

3.160.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \frac{\sec^5(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = -\frac{i\tan(c+dx)\left(12i+6\tan(c+dx)+4i\tan^2(c+dx)+3\tan^3(c+dx)\right)}{12ad}$$

input
```
Integrate[Sec[c + d*x]^5/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

output
```
((-1/12*I)*Tan[c + d*x]*(12*I + 6*Tan[c + d*x] + (4*I)*Tan[c + d*x]^2 + 3*
Tan[c + d*x]^3))/(a*d)
```

3.160.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3042, 3571, 3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(c+dx)^5(a\cos(c+dx)+ia\sin(c+dx))}dx$$

$$\Big\downarrow \text{3571}$$

$$-\frac{i\int \sec^5(c+dx)(ia\cos(c+dx)+a\sin(c+dx))dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{i\int \frac{ia\cos(c+dx)+a\sin(c+dx)}{\cos(c+dx)^5}dx}{a^2}$$

$$\Big\downarrow \text{3569}$$

$$-\frac{i\int \left(ia\sec^4(c+dx)+a\tan(c+dx)\sec^4(c+dx)\right)dx}{a^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{ia\tan^3(c+dx)}{3d}+\frac{ia\tan(c+dx)}{d}+\frac{a\sec^4(c+dx)}{4d}\right)}{a^2}$$

input
```
Int[Sec[c + d*x]^5/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

output
```
((-I)*((a*Sec[c + d*x]^4)/(4*d) + (I*a*Tan[c + d*x])/d + ((I/3)*a*Tan[c +
d*x]^3)/d))/a^2
```

3.160.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

rule 3571
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^n*b^n   Int[Cos[c + d*x]^m
/(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &
& EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

3.160.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.69

method	result	size
risch	$\frac{4i\left(4\,{\mathrm e}^{2i(dx+c)}+1\right)}{3da\left({\mathrm e}^{2i(dx+c)}+1\right)^{4}}$	36
derivativedivides	$\frac{i\left(-i\tan(dx+c)-\frac{\tan(dx+c)^{4}}{4}-\frac{i\tan(dx+c)^{3}}{3}-\frac{\tan(dx+c)^{2}}{2}\right)}{da}$	51
default	$\frac{i\left(-i\tan(dx+c)-\frac{\tan(dx+c)^{4}}{4}-\frac{i\tan(dx+c)^{3}}{3}-\frac{\tan(dx+c)^{2}}{2}\right)}{da}$	51
norman	$\frac{\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3ad}+\frac{10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{3ad}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{ad}-\frac{2i\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{ad}-\frac{2i\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}}{ad}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)^{4}}$	132

input
```
int(sec(d*x+c)^5/(cos(d*x+c)*a+I*a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
4/3*I*(4*exp(2*I*(d*x+c))+1)/d/a/(exp(2*I*(d*x+c))+1)^4
```

3.160.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.38

$$\int \frac{\sec^5(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = -\frac{4\left(-4i\,e^{(2i\,dx+2i\,c)}-i\right)}{3\left(ade^{(8i\,dx+8i\,c)}+4\,ade^{(6i\,dx+6i\,c)}+6\,ade^{(4i\,dx+4i\,c)}+4\,ade^{(2i\,dx+2i\,c)}+ad\right)}$$

input
```
integrate(sec(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="fricas"
)
```

output
```
-4/3*(-4*I*e^(2*I*d*x + 2*I*c) - I)/(a*d*e^(8*I*d*x + 8*I*c) + 4*a*d*e^(6*
I*d*x + 6*I*c) + 6*a*d*e^(4*I*d*x + 4*I*c) + 4*a*d*e^(2*I*d*x + 2*I*c) + a
*d)
```

3.160.6 Sympy [F]

$$\int \frac{\sec^5(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \frac{\int \frac{\sec^5(c+dx)}{i\sin(c+dx)+\cos(c+dx)}\,dx}{a}$$

input
```
integrate(sec(d*x+c)**5/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**5/(I*sin(c + d*x) + cos(c + d*x)), x)/a
```

3.160.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(46) = 92$.

Time = 0.23 (sec) , antiderivative size = 211, normalized size of antiderivative = 4.06

$$\int \frac{\sec^5(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \frac{2\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{3i\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{5\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{5\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{3i\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{3\left(a-\frac{4a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right)d}$$

input
```
integrate(sec(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="maxima"
)
```

output
```
2/3*(3*sin(d*x + c)/(cos(d*x + c) + 1) - 3*I*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 - 5*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*sin(d*x + c)^5/(cos(d*x
 + c) + 1)^5 - 3*I*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 3*sin(d*x + c)^7/
(cos(d*x + c) + 1)^7)/((a - 4*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a*
sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 4*a*sin(d*x + c)^6/(cos(d*x + c) + 1
)^6 + a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*d)
```

3.160.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.90

$$\int \frac{\sec^5(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = -\frac{3i\,\tan\left(dx+c\right)^4-4\,\tan\left(dx+c\right)^3+6i\,\tan\left(dx+c\right)^2-12\,\tan\left(dx+c\right)}{12\,ad}$$

input
```
integrate(sec(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="giac")
```

output
```
-1/12*(3*I*tan(d*x + c)^4 - 4*tan(d*x + c)^3 + 6*I*tan(d*x + c)^2 - 12*tan
(d*x + c))/(a*d)
```

3.160.9 Mupad [B] (verification not implemented)

Time = 23.56 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.90

$$\int \frac{\sec^5(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = -\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,3\mathrm{i}-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,3}{3\,a\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)^4}$$

input
```
int(1/(cos(c + d*x)^5*(a*cos(c + d*x) + a*sin(c + d*x)*1i)),x)
```

output
```
-(2*tan(c/2 + (d*x)/2)*(tan(c/2 + (d*x)/2)*3i + 5*tan(c/2 + (d*x)/2)^2 - 5
*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^5*3i + 3*tan(c/2 + (d*x)/2)^6 -
 3))/(3*a*d*(tan(c/2 + (d*x)/2)^2 - 1)^4)
```

3.160. $\int \frac{\sec^5(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

3.161 $\int \frac{\sec^6(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

3.161.1 Optimal result

Integrand size = 31, antiderivative size = 84

$$\int \frac{\sec^6(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \frac{3\text{arctanh}(\sin(c+dx))}{8ad} - \frac{i\sec^5(c+dx)}{5ad} + \frac{3\sec(c+dx)\tan(c+dx)}{8ad} + \frac{\sec^3(c+dx)\tan(c+dx)}{4ad}$$

output
```
3/8*arctanh(sin(d*x+c))/a/d-1/5*I*sec(d*x+c)^5/a/d+3/8*sec(d*x+c)*tan(d*x+
c)/a/d+1/4*sec(d*x+c)^3*tan(d*x+c)/a/d
```

3.161.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.79

$$\int \frac{\sec^6(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = -\frac{i\left(240i\text{arctanh}\left(\sin(c)+\cos(c)\tan\left(\frac{dx}{2}\right)\right)+\sec^5(c+dx)(64+70i\sin(2(c+dx))+15i\sin(4(c+dx)))\right)}{320ad}$$

input
```
Integrate[Sec[c + d*x]^6/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

output
```
((-1/320*I)*((240*I)*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]] + Sec[c + d*x]^
5*(64 + (70*I)*Sin[2*(c + d*x)] + (15*I)*Sin[4*(c + d*x)])))/(a*d)
```

3.161.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3042, 3571, 3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^6(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(c+dx)^6(a\cos(c+dx)+ia\sin(c+dx))}dx$$

$$\Big\downarrow \text{3571}$$

$$-\frac{i\int \sec^6(c+dx)(ia\cos(c+dx)+a\sin(c+dx))dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{i\int \frac{ia\cos(c+dx)+a\sin(c+dx)}{\cos(c+dx)^6}dx}{a^2}$$

$$\Big\downarrow \text{3569}$$

$$-\frac{i\int \left(ia\sec^5(c+dx)+a\tan(c+dx)\sec^5(c+dx)\right)dx}{a^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{3ia\,\text{arctanh}(\sin(c+dx))}{8d}+\frac{a\sec^5(c+dx)}{5d}+\frac{ia\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3ia\tan(c+dx)\sec(c+dx)}{8d}\right)}{a^2}$$

input
```
Int[Sec[c + d*x]^6/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

output
```
((-I)*((((3*I)/8)*a*ArcTanh[Sin[c + d*x]])/d + (a*Sec[c + d*x]^5)/(5*d) +
(((3*I)/8)*a*Sec[c + d*x]*Tan[c + d*x])/d + ((I/4)*a*Sec[c + d*x]^3*Tan[c
+ d*x])/d))/a^2
```

3.161.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

rule 3571
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^n*b^n   Int[Cos[c + d*x]^m
/(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &
& EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

3.161.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.45

method	result
risch	$-\frac{i\left(15\,\mathrm{e}^{9i(dx+c)}+70\,\mathrm{e}^{7i(dx+c)}+128\,\mathrm{e}^{5i(dx+c)}-70\,\mathrm{e}^{3i(dx+c)}-15\,\mathrm{e}^{i(dx+c)}\right)}{20da\left(\mathrm{e}^{2i(dx+c)}+1\right)^{5}}-\frac{3\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{8ad}+\frac{3\ln\left(i+\mathrm{e}^{i(dx+c)}\right)}{8ad}$
norman	$\frac{-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{2ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{2ad}+\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{4ad}+\frac{2i}{5ad}+\frac{4i\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{ad}+\frac{2i\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{8}}{ad}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)^{5}}-\frac{3\ln\left(\tan\left(\frac{dx}{2}\cdots\right.\right.}{8ad\cdots}$
derivativedivides	$\frac{-\frac{i}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}+\frac{2\left(\frac{5}{16}-\frac{3i}{16}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\frac{2\left(\frac{1}{4}-\frac{3i}{8}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{2\left(-\frac{1}{8}+\frac{i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}+\frac{2\left(-\frac{7}{16}+\frac{5i}{16}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{8}\cdots}{a\cdots}$
default	$\frac{-\frac{i}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}+\frac{2\left(\frac{5}{16}-\frac{3i}{16}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\frac{2\left(\frac{1}{4}-\frac{3i}{8}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{2\left(-\frac{1}{8}+\frac{i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}+\frac{2\left(-\frac{7}{16}+\frac{5i}{16}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{8}\cdots}{a\cdots}$

input
```
int(sec(d*x+c)^6/(cos(d*x+c)*a+I*a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/20*I/d/a/(exp(2*I*(d*x+c))+1)^5*(15*exp(9*I*(d*x+c))+70*exp(7*I*(d*x+c)
)+128*exp(5*I*(d*x+c))-70*exp(3*I*(d*x+c))-15*exp(I*(d*x+c)))-3/8/a/d*ln(e
xp(I*(d*x+c))-I)+3/8/a/d*ln(I+exp(I*(d*x+c)))
```

3.161.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 266 vs. $2(74) = 148$.

Time = 0.25 (sec) , antiderivative size = 266, normalized size of antiderivative = 3.17

$$\int \frac{\sec^6(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \frac{15\left(e^{(10i\,dx+10i\,c)}+5\,e^{(8i\,dx+8i\,c)}+10\,e^{(6i\,dx+6i\,c)}+10\,e^{(4i\,dx+4i\,c)}+5\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-15\,(}{40\,(ade^{(10i}}$$

input
```
integrate(sec(d*x+c)^6/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="fricas"
)
```

output
```
1/40*(15*(e^(10*I*d*x + 10*I*c) + 5*e^(8*I*d*x + 8*I*c) + 10*e^(6*I*d*x +
6*I*c) + 10*e^(4*I*d*x + 4*I*c) + 5*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x
+ I*c) + I) - 15*(e^(10*I*d*x + 10*I*c) + 5*e^(8*I*d*x + 8*I*c) + 10*e^(6*
I*d*x + 6*I*c) + 10*e^(4*I*d*x + 4*I*c) + 5*e^(2*I*d*x + 2*I*c) + 1)*log(e
^(I*d*x + I*c) - I) - 30*I*e^(9*I*d*x + 9*I*c) - 140*I*e^(7*I*d*x + 7*I*c)
 - 256*I*e^(5*I*d*x + 5*I*c) + 140*I*e^(3*I*d*x + 3*I*c) + 30*I*e^(I*d*x +
 I*c))/(a*d*e^(10*I*d*x + 10*I*c) + 5*a*d*e^(8*I*d*x + 8*I*c) + 10*a*d*e^(
6*I*d*x + 6*I*c) + 10*a*d*e^(4*I*d*x + 4*I*c) + 5*a*d*e^(2*I*d*x + 2*I*c)
+ a*d)
```

3.161.6 Sympy [F]

$$\int \frac{\sec^6(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \frac{\int \frac{\sec^6(c+dx)}{i\sin(c+dx)+\cos(c+dx)}\,dx}{a}$$

input
```
integrate(sec(d*x+c)**6/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**6/(I*sin(c + d*x) + cos(c + d*x)), x)/a
```

3.161.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(74) = 148$.

Time = 0.21 (sec) , antiderivative size = 288, normalized size of antiderivative = 3.43

$$\int \frac{\sec^6(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = -\frac{3\left(\frac{16\left(\frac{25i\sin(dx+c)}{\cos(dx+c)+1}-\frac{10i\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{80\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{10i\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{40\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{25i\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+8\right)}{-120ia+\frac{600ia\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{1200ia\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{1200ia\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{600ia\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{120ia\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}+\right.}{8d}$$

input
```
integrate(sec(d*x+c)^6/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="maxima"
)
```

output
```
-3/8*(16*(25*I*sin(d*x + c)/(cos(d*x + c) + 1) - 10*I*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3 + 80*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*I*sin(d*x +
c)^7/(cos(d*x + c) + 1)^7 + 40*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 25*I*
sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 8)/(-120*I*a + 600*I*a*sin(d*x + c)^
2/(cos(d*x + c) + 1)^2 - 1200*I*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 12
00*I*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 600*I*a*sin(d*x + c)^8/(cos(d
*x + c) + 1)^8 + 120*I*a*sin(d*x + c)^10/(cos(d*x + c) + 1)^10) - log(sin(
d*x + c)/(cos(d*x + c) + 1) + 1)/a + log(sin(d*x + c)/(cos(d*x + c) + 1) -
 1)/a)/d
```

3.161.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.64

$$\int \frac{\sec^6(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \frac{\frac{15\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a}-\frac{15\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{a}+\frac{2\left(25\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9+40i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8-10\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+80i\tan\left(\frac{1}{2}dx+\right.\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^5a}}{40d}$$

input
```
integrate(sec(d*x+c)^6/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="giac")
```

output
```
1/40*(15*log(tan(1/2*d*x + 1/2*c) + 1)/a - 15*log(tan(1/2*d*x + 1/2*c) - 1
)/a + 2*(25*tan(1/2*d*x + 1/2*c)^9 + 40*I*tan(1/2*d*x + 1/2*c)^8 - 10*tan(
1/2*d*x + 1/2*c)^7 + 80*I*tan(1/2*d*x + 1/2*c)^4 + 10*tan(1/2*d*x + 1/2*c)
^3 - 25*tan(1/2*d*x + 1/2*c) + 8*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^5*a))/d
```

3.161.9 Mupad [B] (verification not implemented)

Time = 26.61 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.30

$$\int \frac{\sec^6(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \frac{3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,a\,d} + \frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2\,a}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{2\,a}+\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4\,a}-\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,a}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,4\mathrm{i}}{a}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\,2\mathrm{i}}{a}+\frac{2\mathrm{i}}{5\,a}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

input
```
int(1/(cos(c + d*x)^6*(a*cos(c + d*x) + a*sin(c + d*x)*1i)),x)
```

output
```
(3*atanh(tan(c/2 + (d*x)/2)))/(4*a*d) + (tan(c/2 + (d*x)/2)^3/(2*a) + (tan
(c/2 + (d*x)/2)^4*4i)/a - tan(c/2 + (d*x)/2)^7/(2*a) + (tan(c/2 + (d*x)/2)
^8*2i)/a + (5*tan(c/2 + (d*x)/2)^9)/(4*a) + 2i/(5*a) - (5*tan(c/2 + (d*x)/
2))/(4*a))/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c
/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

3.162 $\int \frac{\sec^7(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

3.162.1 Optimal result

Integrand size = 31, antiderivative size = 70

$$\int \frac{\sec^7(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = -\frac{i\sec^6(c+dx)}{6ad}+\frac{\tan(c+dx)}{ad}+\frac{2\tan^3(c+dx)}{3ad}+\frac{\tan^5(c+dx)}{5ad}$$

output
```
-1/6*I*sec(d*x+c)^6/a/d+tan(d*x+c)/a/d+2/3*tan(d*x+c)^3/a/d+1/5*tan(d*x+c)
^5/a/d
```

3.162.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.06

$$\int \frac{\sec^7(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = -\frac{i\tan(c+dx)\left(30i+15\tan(c+dx)+20i\tan^2(c+dx)+15\tan^3(c+dx)+6i\tan^4(c+dx)+5\tan^5(c-\right.}{30ad}$$

input
```
Integrate[Sec[c + d*x]^7/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

output
```
((-1/30*I)*Tan[c + d*x]*(30*I + 15*Tan[c + d*x] + (20*I)*Tan[c + d*x]^2 +
15*Tan[c + d*x]^3 + (6*I)*Tan[c + d*x]^4 + 5*Tan[c + d*x]^5))/(a*d)
```

3.162.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3042, 3571, 3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^7(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(c+dx)^7(a\cos(c+dx)+ia\sin(c+dx))}dx$$

$$\Big\downarrow \text{3571}$$

$$-\frac{i\int \sec^7(c+dx)(ia\cos(c+dx)+a\sin(c+dx))dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{i\int \frac{ia\cos(c+dx)+a\sin(c+dx)}{\cos(c+dx)^7}dx}{a^2}$$

$$\Big\downarrow \text{3569}$$

$$-\frac{i\int \left(ia\sec^6(c+dx)+a\tan(c+dx)\sec^6(c+dx)\right)dx}{a^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i\left(\frac{ia\tan^5(c+dx)}{5d}+\frac{2ia\tan^3(c+dx)}{3d}+\frac{ia\tan(c+dx)}{d}+\frac{a\sec^6(c+dx)}{6d}\right)}{a^2}$$

input
```
Int[Sec[c + d*x]^7/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

output
```
((-I)*((a*Sec[c + d*x]^6)/(6*d) + (I*a*Tan[c + d*x])/d + (((2*I)/3)*a*Tan[
c + d*x]^3)/d + ((I/5)*a*Tan[c + d*x]^5)/d))/a^2
```

3.162.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

rule 3571
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^n*b^n   Int[Cos[c + d*x]^m
/(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &
& EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

3.162.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.67

method	result
risch	$\frac{16 i \left(15 \,{\mathrm e}^{4 i \left(d x +c \right)}+6 \,{\mathrm e}^{2 i \left(d x +c \right)}+1\right)}{15 d a \left({\mathrm e}^{2 i \left(d x +c \right)}+1\right)^{6}}$
derivativedivides	$-\frac{i \left(\frac{\tan \left(d x +c \right)^{6}}{6}+\frac{\tan \left(d x +c \right)^{4}}{2}+\frac{i \tan \left(d x +c \right)^{5}}{5}+\frac{\tan \left(d x +c \right)^{2}}{2}+\frac{2 i \tan \left(d x +c \right)^{3}}{3}+i \tan \left(d x +c \right)\right)}{d a}$
default	$-\frac{i \left(\frac{\tan \left(d x +c \right)^{6}}{6}+\frac{\tan \left(d x +c \right)^{4}}{2}+\frac{i \tan \left(d x +c \right)^{5}}{5}+\frac{\tan \left(d x +c \right)^{2}}{2}+\frac{2 i \tan \left(d x +c \right)^{3}}{3}+i \tan \left(d x +c \right)\right)}{d a}$
norman	$\frac{\frac{2 \tan \left(\frac{d x}{2}+\frac{c}{2}\right)}{a d}-\frac{14 \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{3}}{3 a d}+\frac{52 \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{5}}{5 a d}-\frac{52 \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{7}}{5 a d}+\frac{14 \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{9}}{3 a d}-\frac{2 \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{11}}{a d}-\frac{2 i \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}}{a d}-}{\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}-1\right)^{6}}$

input
```
int(sec(d*x+c)^7/(cos(d*x+c)*a+I*a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
16/15*I*(15*exp(4*I*(d*x+c))+6*exp(2*I*(d*x+c))+1)/d/a/(exp(2*I*(d*x+c))+1
)^6
```

3.162.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.56

$$\int \frac{\sec^7(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = -\frac{16\left(-15i\,e^{(4i\,dx+4i\,c)}-6i\,e^{(2i\,dx+2i\,c)}-i\right)}{15\left(ade^{(12i\,dx+12i\,c)}+6\,ade^{(10i\,dx+10i\,c)}+15\,ade^{(8i\,dx+8i\,c)}+20\,ade^{(6i\,dx+6i\,c)}+15\,ade^{(4i\,dx+4i\,c)}+6\,ade^{(2i\,dx}\right.}$$

input
```
integrate(sec(d*x+c)^7/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="fricas"
)
```

output
```
-16/15*(-15*I*e^(4*I*d*x + 4*I*c) - 6*I*e^(2*I*d*x + 2*I*c) - I)/(a*d*e^(1
2*I*d*x + 12*I*c) + 6*a*d*e^(10*I*d*x + 10*I*c) + 15*a*d*e^(8*I*d*x + 8*I*
c) + 20*a*d*e^(6*I*d*x + 6*I*c) + 15*a*d*e^(4*I*d*x + 4*I*c) + 6*a*d*e^(2*
I*d*x + 2*I*c) + a*d)
```

3.162.6 Sympy [F]

$$\int \frac{\sec^7(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \frac{\int \frac{\sec^7(c+dx)}{i\sin(c+dx)+\cos(c+dx)}\,dx}{a}$$

input
```
integrate(sec(d*x+c)**7/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**7/(I*sin(c + d*x) + cos(c + d*x)), x)/a
```

3.162.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 313 vs. 2(62) = 124.

Time = 0.22 (sec) , antiderivative size = 313, normalized size of antiderivative = 4.47

$$\int \frac{\sec^7(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \frac{2\left(\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{15i\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{35\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{78\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{50i\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{78\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{35\sin(dx+c)^9}{(\cos(dx+c)+1)^9}\right.}{15\left(a-\frac{6a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{20a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{15a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{6a\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\right.}$$

input
```
integrate(sec(d*x+c)^7/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="maxima"
)
```

output
```
2/15*(15*sin(d*x + c)/(cos(d*x + c) + 1) - 15*I*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 - 35*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 78*sin(d*x + c)^5/(co
s(d*x + c) + 1)^5 - 50*I*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 78*sin(d*x
+ c)^7/(cos(d*x + c) + 1)^7 + 35*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 15*
I*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - 15*sin(d*x + c)^11/(cos(d*x + c)
 + 1)^11)/((a - 6*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 15*a*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4 - 20*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 15*a
*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 6*a*sin(d*x + c)^10/(cos(d*x + c) +
 1)^10 + a*sin(d*x + c)^12/(cos(d*x + c) + 1)^12)*d)
```

3.162.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.96

$$\int \frac{\sec^7(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = -\frac{5i\tan(dx+c)^6-6\tan(dx+c)^5+15i\tan(dx+c)^4-20\tan(dx+c)^3+15i\tan(dx+c)^2-30\tan}{30\,ad}$$

input
```
integrate(sec(d*x+c)^7/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="giac")
```

output
```
-1/30*(5*I*tan(d*x + c)^6 - 6*tan(d*x + c)^5 + 15*I*tan(d*x + c)^4 - 20*ta
n(d*x + c)^3 + 15*I*tan(d*x + c)^2 - 30*tan(d*x + c))/(a*d)
```

3.162.9 Mupad [B] (verification not implemented)

Time = 24.33 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.99

$$\int \frac{\sec^7(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = -\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(15\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}\,15\mathrm{i}-35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}+78\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}+\tan\left(\frac{c}{2}+\right.\right.}{15\,a\,d\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}-1\right)^{6}}$$

input
```
int(1/(cos(c + d*x)^7*(a*cos(c + d*x) + a*sin(c + d*x)*1i)),x)
```

output
```
-(2*tan(c/2 + (d*x)/2)*(tan(c/2 + (d*x)/2)*15i + 35*tan(c/2 + (d*x)/2)^2 -
 78*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^5*50i + 78*tan(c/2 + (d*x)/2
)^6 - 35*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^9*15i + 15*tan(c/2 + (d
*x)/2)^10 - 15))/(15*a*d*(tan(c/2 + (d*x)/2)^2 - 1)^6)
```

3.162. $\int \frac{\sec^7(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

3.163 $\int \frac{\cos^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$

3.163.1 Optimal result

Integrand size = 31, antiderivative size = 85

$$\int \frac{\cos^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \frac{2i\cos^7(c+dx)}{7a^2d}+\frac{\sin(c+dx)}{a^2d}-\frac{4\sin^3(c+dx)}{3a^2d}+\frac{\sin^5(c+dx)}{a^2d}-\frac{2\sin^7(c+dx)}{7a^2d}$$

output
```
2/7*I*cos(d*x+c)^7/a^2/d+sin(d*x+c)/a^2/d-4/3*sin(d*x+c)^3/a^2/d+sin(d*x+c
)^5/a^2/d-2/7*sin(d*x+c)^7/a^2/d
```

3.163.2 Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.75

$$\int \frac{\cos^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \frac{5i\cos(c+dx)}{32a^2d}+\frac{3i\cos(3(c+dx))}{32a^2d}+\frac{i\cos(5(c+dx))}{32a^2d}+\frac{i\cos(7(c+dx))}{224a^2d}+\frac{15\sin(c+dx)}{32a^2d}+\frac{11\sin(3(c+dx))}{96a^2d}+\frac{\sin(5(c+dx))}{32a^2d}+\frac{\sin(7(c+dx))}{224a^2d}$$

input
```
Integrate[Cos[c + d*x]^5/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

output
```
(((5*I)/32)*Cos[c + d*x])/(a^2*d) + (((3*I)/32)*Cos[3*(c + d*x)])/(a^2*d)
+ ((I/32)*Cos[5*(c + d*x)])/(a^2*d) + ((I/224)*Cos[7*(c + d*x)])/(a^2*d) +
 (15*Sin[c + d*x])/(32*a^2*d) + (11*Sin[3*(c + d*x)])/(96*a^2*d) + Sin[5*(
c + d*x)]/(32*a^2*d) + Sin[7*(c + d*x)]/(224*a^2*d)
```

3.163.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3042, 3571, 3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(c+dx)^5}{(a\cos(c+dx)+ia\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{3571}$$

$$-\frac{\int \cos^5(c+dx)(ia\cos(c+dx)+a\sin(c+dx))^2dx}{a^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \cos(c+dx)^5(ia\cos(c+dx)+a\sin(c+dx))^2dx}{a^4}$$

$$\Big\downarrow \text{3569}$$

$$-\frac{\int \left(-a^2\cos^7(c+dx)+2ia^2\sin(c+dx)\cos^6(c+dx)+a^2\sin^2(c+dx)\cos^5(c+dx)\right)dx}{a^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{2a^2\sin^7(c+dx)}{7d}-\frac{a^2\sin^5(c+dx)}{d}+\frac{4a^2\sin^3(c+dx)}{3d}-\frac{a^2\sin(c+dx)}{d}-\frac{2ia^2\cos^7(c+dx)}{7d}}{a^4}$$

input
```
Int[Cos[c + d*x]^5/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

output
```
-(((((-2*I)/7)*a^2*Cos[c + d*x]^7)/d - (a^2*Sin[c + d*x])/d + (4*a^2*Sin[c
 + d*x]^3)/(3*d) - (a^2*Sin[c + d*x]^5)/d + (2*a^2*Sin[c + d*x]^7)/(7*d))/
a^4)
```

3.163.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

rule 3571
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^n*b^n   Int[Cos[c + d*x]^m
/(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &
& EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

3.163.4 Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.20

method	result
risch	$\frac{i\mathrm{e}^{-5i(dx+c)}}{32a^2d}+\frac{i\mathrm{e}^{-7i(dx+c)}}{224a^2d}+\frac{5i\cos(dx+c)}{32a^2d}+\frac{15\sin(dx+c)}{32a^2d}+\frac{3i\cos(3dx+3c)}{32a^2d}+\frac{11\sin(3dx+3c)}{96a^2d}$
derivativedivides	$\frac{\frac{2i}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^6}-\frac{5i}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^4}+\frac{23i}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^2}-\frac{4}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^7}+\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^5}-\frac{55}{12\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)}\cdots}{a^2d}$
default	$\frac{\frac{2i}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^6}-\frac{5i}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^4}+\frac{23i}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^2}-\frac{4}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^7}+\frac{4}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^5}-\frac{55}{12\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)}\cdots}{a^2d}$

input
```
int(cos(d*x+c)^5/(cos(d*x+c)*a+I*a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/32*I/a^2/d*exp(-5*I*(d*x+c))+1/224*I/a^2/d*exp(-7*I*(d*x+c))+5/32*I/a^2/
d*cos(d*x+c)+15/32*sin(d*x+c)/a^2/d+3/32*I/a^2/d*cos(3*d*x+3*c)+11/96/a^2/
d*sin(3*d*x+3*c)
```

3.163.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.87

$$\int \frac{\cos^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \frac{\left(-7i\,e^{(10i\,dx+10i\,c)}-105i\,e^{(8i\,dx+8i\,c)}+210i\,e^{(6i\,dx+6i\,c)}+70i\,e^{(4i\,dx+4i\,c)}+21i\,e^{(2i\,dx+2i\,c)}+3i\right)e^{(-7i\,dx-7i\,c)}}{672\,a^2d}$$

input
```
integrate(cos(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="frica
s")
```

output
```
1/672*(-7*I*e^(10*I*d*x + 10*I*c) - 105*I*e^(8*I*d*x + 8*I*c) + 210*I*e^(6
*I*d*x + 6*I*c) + 70*I*e^(4*I*d*x + 4*I*c) + 21*I*e^(2*I*d*x + 2*I*c) + 3*
I)*e^(-7*I*d*x - 7*I*c)/(a^2*d)
```

3.163.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 231 vs. 2(76) = 152.

Time = 0.29 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.72

$$\int \frac{\cos^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \begin{cases} \frac{(-176160768ia^{10}d^5e^{19ic}e^{3idx}-2642411520ia^{10}d^5e^{17ic}e^{idx}+5284823040ia^{10}d^5e^{15ic}e^{-idx}+1761607680ia^{10}d^5e^{13ic}e^{-3idx}+528482304ia^{10}d^5}{16911433728a^{12}d^6} \\ \frac{x\left(e^{10ic}+5e^{8ic}+10e^{6ic}+10e^{4ic}+5e^{2ic}+1\right)e^{-7ic}}{32a^2} \end{cases}$$

input
```
integrate(cos(d*x+c)**5/(a*cos(d*x+c)+I*a*sin(d*x+c))**2,x)
```

output
```
Piecewise(((-176160768*I*a**10*d**5*exp(19*I*c)*exp(3*I*d*x) - 2642411520*
I*a**10*d**5*exp(17*I*c)*exp(I*d*x) + 5284823040*I*a**10*d**5*exp(15*I*c)*
exp(-I*d*x) + 1761607680*I*a**10*d**5*exp(13*I*c)*exp(-3*I*d*x) + 52848230
4*I*a**10*d**5*exp(11*I*c)*exp(-5*I*d*x) + 75497472*I*a**10*d**5*exp(9*I*c
)*exp(-7*I*d*x))*exp(-16*I*c)/(16911433728*a**12*d**6), Ne(a**12*d**6*exp(
16*I*c), 0)), (x*(exp(10*I*c) + 5*exp(8*I*c) + 10*exp(6*I*c) + 10*exp(4*I*
c) + 5*exp(2*I*c) + 1)*exp(-7*I*c)/(32*a**2), True))
```

3.163.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="maxim
a")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.163.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.71

$$\int \frac{\cos^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \frac{\frac{7\left(9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+15i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-8\right)}{a^2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+i\right)^3}+\frac{273\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-1155i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-2450\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+2870i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3}{a^2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)^7}}{168\,d}$$

input
```
integrate(cos(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="giac"
)
```

output
```
1/168*(7*(9*tan(1/2*d*x + 1/2*c)^2 + 15*I*tan(1/2*d*x + 1/2*c) - 8)/(a^2*(
tan(1/2*d*x + 1/2*c) + I)^3) + (273*tan(1/2*d*x + 1/2*c)^6 - 1155*I*tan(1/
2*d*x + 1/2*c)^5 - 2450*tan(1/2*d*x + 1/2*c)^4 + 2870*I*tan(1/2*d*x + 1/2*
c)^3 + 2037*tan(1/2*d*x + 1/2*c)^2 - 791*I*tan(1/2*d*x + 1/2*c) - 152)/(a^
2*(tan(1/2*d*x + 1/2*c) - I)^7))/d
```

3.163.9 Mupad [B] (verification not implemented)

Time = 27.17 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.89

$$\int \frac{\cos^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$$
$$= \frac{\left(-21\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8 42\mathrm{i}+28\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 56\mathrm{i}+42\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\tan\left(\right.\right.}{21\,a^2\,d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\mathrm{i}\right)^3\left(1+\tan\left(\frac{c}{2}\right.\right.}$$

input
```
int(cos(c + d*x)^5/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^2,x)
```

output
```
((3*tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)/2)^2*24i + 76*tan(c/2 + (d*x)/2)^
3 + tan(c/2 + (d*x)/2)^4*28i + 42*tan(c/2 + (d*x)/2)^5 + tan(c/2 + (d*x)/2
)^6*56i + 28*tan(c/2 + (d*x)/2)^7 + tan(c/2 + (d*x)/2)^8*42i - 21*tan(c/2
+ (d*x)/2)^9 - 6i)*2i)/(21*a^2*d*(tan(c/2 + (d*x)/2) + 1i)^3*(tan(c/2 + (d
*x)/2)*1i + 1)^7)
```

3.164 $\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$

3.164.1 Optimal result

Integrand size = 31, antiderivative size = 101

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \frac{x}{4a^2}-\frac{1}{16a^2d(i-\cot(c+dx))} - \frac{1}{12a^2d(i+\cot(c+dx))^3} - \frac{3i}{8a^2d(i+\cot(c+dx))^2}+\frac{11}{16a^2d(i+\cot(c+dx))}$$

output
```
1/4*x/a^2-1/16/a^2/d/(I-cot(d*x+c))-1/12/a^2/d/(I+cot(d*x+c))^3-3/8*I/a^2/
d/(I+cot(d*x+c))^2+11/16/a^2/d/(I+cot(d*x+c))
```

3.164.2 Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.81

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \frac{24c+24dx+15i\cos(2(c+dx))+6i\cos(4(c+dx))+i\cos(6(c+dx))+21\sin(2(c+dx))+6\sin(4(c+ \cdots}{96a^2d}$$

input
```
Integrate[Cos[c + d*x]^4/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

output
```
(24*c + 24*d*x + (15*I)*Cos[2*(c + d*x)] + (6*I)*Cos[4*(c + d*x)] + I*Cos[
6*(c + d*x)] + 21*Sin[2*(c + d*x)] + 6*Sin[4*(c + d*x)] + Sin[6*(c + d*x)]
)/(96*a^2*d)
```

3.164.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.87, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3042, 3567, 27, 516, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(c+dx)^4}{(a\cos(c+dx)+ia\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{3567}$$

$$-\frac{\int \frac{\cot^4(c+dx)}{a^2(\cot(c+dx)+i)^2(\cot^2(c+dx)+1)^2}d\cot(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\cot^4(c+dx)}{(\cot(c+dx)+i)^2(\cot^2(c+dx)+1)^2}d\cot(c+dx)}{a^2d}$$

$$\Big\downarrow \text{516}$$

$$-\frac{\int \frac{\cot^4(c+dx)}{(\cot(c+dx)-i)^2(\cot(c+dx)+i)^4}d\cot(c+dx)}{a^2d}$$

$$\Big\downarrow \text{99}$$

$$-\frac{\int \left(\frac{1}{16(\cot(c+dx)-i)^2}+\frac{11}{16(\cot(c+dx)+i)^2}-\frac{3i}{4(\cot(c+dx)+i)^3}-\frac{1}{4(\cot(c+dx)+i)^4}+\frac{1}{4(\cot^2(c+dx)+1)}\right)d\cot(c+dx)}{a^2d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{1}{4}\arctan(\cot(c+dx))+\frac{1}{16(-\cot(c+dx)+i)}-\frac{11}{16(\cot(c+dx)+i)}+\frac{3i}{8(\cot(c+dx)+i)^2}+\frac{1}{12(\cot(c+dx)+i)^3}}{a^2d}$$

input
```
Int[Cos[c + d*x]^4/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

output
```
-((ArcTan[Cot[c + d*x]]/4 + 1/(16*(I - Cot[c + d*x])) + 1/(12*(I + Cot[c +
 d*x])^3) + ((3*I)/8)/(I + Cot[c + d*x])^2 - 11/(16*(I + Cot[c + d*x])))/(
a^2*d))
```

3.164.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 516
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[(e*x)^m*(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; Free
Q[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] ||
(GtQ[a, 0] && GtQ[c, 0] &&  !IntegerQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3567
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[x^m*((b
+ a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a,
b, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[
n, 0] && GtQ[m, 1])
```

3.164.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.78

method	result	size
risch	$\frac{x}{4a^2}+\frac{i\mathrm{e}^{-4i(dx+c)}}{16a^2d}+\frac{i\mathrm{e}^{-6i(dx+c)}}{96a^2d}+\frac{5i\cos(2dx+2c)}{32a^2d}+\frac{7\sin(2dx+2c)}{32a^2d}$	79
derivativedivides	$\frac{-\frac{i\ln(\tan(dx+c)-i)}{8}-\frac{i}{8(\tan(dx+c)-i)^2}-\frac{1}{12(\tan(dx+c)-i)^3}+\frac{3}{16(\tan(dx+c)-i)}+\frac{i\ln(\tan(dx+c)+i)}{8}+\frac{1}{16\tan(dx+c)+16i}}{d\,a^2}$	88
default	$\frac{-\frac{i\ln(\tan(dx+c)-i)}{8}-\frac{i}{8(\tan(dx+c)-i)^2}-\frac{1}{12(\tan(dx+c)-i)^3}+\frac{3}{16(\tan(dx+c)-i)}+\frac{i\ln(\tan(dx+c)+i)}{8}+\frac{1}{16\tan(dx+c)+16i}}{d\,a^2}$	88

input
```
int(cos(d*x+c)^4/(cos(d*x+c)*a+I*a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*x/a^2+1/16*I/a^2/d*exp(-4*I*(d*x+c))+1/96*I/a^2/d*exp(-6*I*(d*x+c))+5/
32*I/a^2/d*cos(2*d*x+2*c)+7/32/a^2/d*sin(2*d*x+2*c)
```

3.164.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.64

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \frac{\left(24\,dxe^{(6i\,dx+6i\,c)}-3i\,e^{(8i\,dx+8i\,c)}+18i\,e^{(4i\,dx+4i\,c)}+6i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-6i\,dx-6i\,c)}}{96\,a^2d}$$

input
```
integrate(cos(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="frica
s")
```

output
```
1/96*(24*d*x*e^(6*I*d*x + 6*I*c) - 3*I*e^(8*I*d*x + 8*I*c) + 18*I*e^(4*I*d
*x + 4*I*c) + 6*I*e^(2*I*d*x + 2*I*c) + I)*e^(-6*I*d*x - 6*I*c)/(a^2*d)
```

3.164.6 Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.87

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \begin{cases} \frac{(-24576ia^6d^3e^{14ic}e^{2idx}+147456ia^6d^3e^{10ic}e^{-2idx}+49152ia^6d^3e^{8ic}e^{-4idx}+8192ia^6d^3e^{6ic}e^{-6idx})e^{-12ic}}{786432a^8d^4} & \text{for } a^8d^4e^{12ic}\neq 0 \\ x\left(\frac{(e^{8ic}+4e^{6ic}+6e^{4ic}+4e^{2ic}+1)e^{-6ic}}{16a^2}-\frac{1}{4a^2}\right) & \text{otherwise} \end{cases} + \frac{x}{4a^2}$$

input
```
integrate(cos(d*x+c)**4/(a*cos(d*x+c)+I*a*sin(d*x+c))**2,x)
```

output
```
Piecewise(((-24576*I*a**6*d**3*exp(14*I*c)*exp(2*I*d*x) + 147456*I*a**6*d*
*3*exp(10*I*c)*exp(-2*I*d*x) + 49152*I*a**6*d**3*exp(8*I*c)*exp(-4*I*d*x)
+ 8192*I*a**6*d**3*exp(6*I*c)*exp(-6*I*d*x))*exp(-12*I*c)/(786432*a**8*d**
4), Ne(a**8*d**4*exp(12*I*c), 0)), (x*((exp(8*I*c) + 4*exp(6*I*c) + 6*exp(
4*I*c) + 4*exp(2*I*c) + 1)*exp(-6*I*c)/(16*a**2) - 1/(4*a**2)), True)) + x
/(4*a**2)
```

3.164.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="maxim
a")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.164.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.02

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = -\frac{-\frac{6i\log(\tan(dx+c)+i)}{a^2}+\frac{6i\log(\tan(dx+c)-i)}{a^2}+\frac{3(2i\tan(dx+c)-3)}{a^2(\tan(dx+c)+i)}+\frac{-11i\tan(dx+c)^3-42\tan(dx+c)^2+57i\tan(dx+c)+30}{a^2(\tan(dx+c)-i)^3}}{48\,d}$$

input
```
integrate(cos(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="giac"
)
```

output
```
-1/48*(-6*I*log(tan(d*x + c) + I)/a^2 + 6*I*log(tan(d*x + c) - I)/a^2 + 3*
(2*I*tan(d*x + c) - 3)/(a^2*(tan(d*x + c) + I)) + (-11*I*tan(d*x + c)^3 -
42*tan(d*x + c)^2 + 57*I*tan(d*x + c) + 30)/(a^2*(tan(d*x + c) - I)^3))/d
```

3.164.9 Mupad [B] (verification not implemented)

Time = 27.41 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.37

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \frac{x}{4\,a^2} - \frac{-\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{2}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 2\mathrm{i}+\frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{6}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 4\mathrm{i}}{3}-\frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{6}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 2\mathrm{i}+\frac{3\tan}{}}{a^2\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)^2\left(1+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}\right)^6}$$

input
```
int(cos(c + d*x)^4/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^2,x)
```

output
```
x/(4*a^2) - ((3*tan(c/2 + (d*x)/2))/2 + tan(c/2 + (d*x)/2)^2*2i - (7*tan(c
/2 + (d*x)/2)^3)/6 + (tan(c/2 + (d*x)/2)^4*4i)/3 + (7*tan(c/2 + (d*x)/2)^5
)/6 + tan(c/2 + (d*x)/2)^6*2i - (3*tan(c/2 + (d*x)/2)^7)/2)/(a^2*d*(tan(c/
2 + (d*x)/2) + 1i)^2*(tan(c/2 + (d*x)/2)*1i + 1)^6)
```

3.165 $\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$

3.165.1 Optimal result

Integrand size = 31, antiderivative size = 68

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \frac{2i\cos^5(c+dx)}{5a^2d}+\frac{\sin(c+dx)}{a^2d}-\frac{\sin^3(c+dx)}{a^2d}+\frac{2\sin^5(c+dx)}{5a^2d}$$

output
```
2/5*I*cos(d*x+c)^5/a^2/d+sin(d*x+c)/a^2/d-sin(d*x+c)^3/a^2/d+2/5*sin(d*x+c
)^5/a^2/d
```

3.165.2 Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.63

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \frac{i\cos(c+dx)}{4a^2d}+\frac{i\cos(3(c+dx))}{8a^2d}+\frac{i\cos(5(c+dx))}{40a^2d}+\frac{\sin(c+dx)}{2a^2d}+\frac{\sin(3(c+dx))}{8a^2d}+\frac{\sin(5(c+dx))}{40a^2d}$$

input
```
Integrate[Cos[c + d*x]^3/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

output
```
((I/4)*Cos[c + d*x])/(a^2*d) + ((I/8)*Cos[3*(c + d*x)])/(a^2*d) + ((I/40)*
Cos[5*(c + d*x)])/(a^2*d) + Sin[c + d*x]/(2*a^2*d) + Sin[3*(c + d*x)]/(8*a
^2*d) + Sin[5*(c + d*x)]/(40*a^2*d)
```

3.165.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3042, 3571, 3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(c+dx)^3}{(a\cos(c+dx)+ia\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{ 3571}$$

$$-\frac{\int \cos^3(c+dx)(ia\cos(c+dx)+a\sin(c+dx))^2dx}{a^4}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\int \cos(c+dx)^3(ia\cos(c+dx)+a\sin(c+dx))^2dx}{a^4}$$

$$\Big\downarrow \text{ 3569}$$

$$-\frac{\int \left(-a^2\cos^5(c+dx)+2ia^2\sin(c+dx)\cos^4(c+dx)+a^2\sin^2(c+dx)\cos^3(c+dx)\right)dx}{a^4}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{-\frac{2a^2\sin^5(c+dx)}{5d}+\frac{a^2\sin^3(c+dx)}{d}-\frac{a^2\sin(c+dx)}{d}-\frac{2ia^2\cos^5(c+dx)}{5d}}{a^4}$$

input
```
Int[Cos[c + d*x]^3/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

output
```
-(((((-2*I)/5)*a^2*Cos[c + d*x]^5)/d - (a^2*Sin[c + d*x])/d + (a^2*Sin[c +
 d*x]^3)/d - (2*a^2*Sin[c + d*x]^5)/(5*d))/a^4)
```

3.165.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

rule 3571
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^n*b^n   Int[Cos[c + d*x]^m
/(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &
& EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

3.165.4 Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.99

method	result
risch	$\frac{i e^{-3 i (d x+c)}}{8 a^{2} d}+\frac{i e^{-5 i (d x+c)}}{40 a^{2} d}+\frac{i \cos (d x+c)}{4 a^{2} d}+\frac{\sin (d x+c)}{2 a^{2} d}$
derivativedivides	$\frac{\frac{2}{8 \tan \left(\frac{d x}{2}+\frac{c}{2}\right)+8 i}-\frac{2 i}{\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-i\right)^{4}}+\frac{5 i}{2\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-i\right)^{2}}+\frac{4}{5\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-i\right)^{5}}-\frac{3}{\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-i\right)^{3}}+\frac{7}{4\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-i\right)}}{a^{2} d}$
default	$\frac{\frac{2}{8 \tan \left(\frac{d x}{2}+\frac{c}{2}\right)+8 i}-\frac{2 i}{\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-i\right)^{4}}+\frac{5 i}{2\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-i\right)^{2}}+\frac{4}{5\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-i\right)^{5}}-\frac{3}{\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-i\right)^{3}}+\frac{7}{4\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-i\right)}}{a^{2} d}$
parallelrisch	$\frac{4 i \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{4}-2 \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{5}+4 \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{3}-\frac{4 i}{5}+\frac{6 \tan \left(\frac{d x}{2}+\frac{c}{2}\right)}{5}}{a^{2} d\left(4 i \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{5}-\tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{6}+5 \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{4}-4 i \tan \left(\frac{d x}{2}+\frac{c}{2}\right)+5 \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}-1\right)}$

input
```
int(cos(d*x+c)^3/(cos(d*x+c)*a+I*a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*I/a^2/d*exp(-3*I*(d*x+c))+1/40*I/a^2/d*exp(-5*I*(d*x+c))+1/4*I/a^2/d*c
os(d*x+c)+1/2*sin(d*x+c)/a^2/d
```

3.165.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.76

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \frac{\left(-5i\,e^{(6i\,dx+6i\,c)}+15i\,e^{(4i\,dx+4i\,c)}+5i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-5i\,dx-5i\,c)}}{40\,a^2d}$$

input
```
integrate(cos(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="frica
s")
```

output
```
1/40*(-5*I*e^(6*I*d*x + 6*I*c) + 15*I*e^(4*I*d*x + 4*I*c) + 5*I*e^(2*I*d*x
 + 2*I*c) + I)*e^(-5*I*d*x - 5*I*c)/(a^2*d)
```

3.165.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(60) = 120.

Time = 0.20 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.40

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \begin{cases} \frac{(-2560ia^6d^3e^{10ic}e^{idx}+7680ia^6d^3e^{8ic}e^{-idx}+2560ia^6d^3e^{6ic}e^{-3idx}+512ia^6d^3e^{4ic}e^{-5idx})e^{-9ic}}{20480a^8d^4} & \text{for } a^8d^4e^{9ic}\neq 0 \\ \frac{x(e^{6ic}+3e^{4ic}+3e^{2ic}+1)e^{-5ic}}{8a^2} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**3/(a*cos(d*x+c)+I*a*sin(d*x+c))**2,x)
```

output
```
Piecewise(((-2560*I*a**6*d**3*exp(10*I*c)*exp(I*d*x) + 7680*I*a**6*d**3*ex
p(8*I*c)*exp(-I*d*x) + 2560*I*a**6*d**3*exp(6*I*c)*exp(-3*I*d*x) + 512*I*a
**6*d**3*exp(4*I*c)*exp(-5*I*d*x))*exp(-9*I*c)/(20480*a**8*d**4), Ne(a**8*
d**4*exp(9*I*c), 0)), (x*(exp(6*I*c) + 3*exp(4*I*c) + 3*exp(2*I*c) + 1)*ex
p(-5*I*c)/(8*a**2), True))
```

3.165.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="maxim
a")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.165.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.37

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \frac{\frac{5}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)} + \frac{35\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-90i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-120\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+70i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+21}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^5}}{20\,d}$$

input
```
integrate(cos(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="giac"
)
```

output
```
1/20*(5/(a^2*(tan(1/2*d*x + 1/2*c) + I)) + (35*tan(1/2*d*x + 1/2*c)^4 - 90
*I*tan(1/2*d*x + 1/2*c)^3 - 120*tan(1/2*d*x + 1/2*c)^2 + 70*I*tan(1/2*d*x
+ 1/2*c) + 21)/(a^2*(tan(1/2*d*x + 1/2*c) - I)^5))/d
```

3.165.9 Mupad [B] (verification not implemented)

Time = 24.29 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.32

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = -\frac{2\left(-5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 10\mathrm{i}+10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-2\mathrm{i}\right)}{5\,a^2\,d\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-\mathrm{i}\right)^5\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\mathrm{i}\right)}$$

input
```
int(cos(c + d*x)^3/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^2,x)
```

output
```
-(2*(3*tan(c/2 + (d*x)/2) + 10*tan(c/2 + (d*x)/2)^3 + tan(c/2 + (d*x)/2)^4
*10i - 5*tan(c/2 + (d*x)/2)^5 - 2i))/(5*a^2*d*(tan(c/2 + (d*x)/2) - 1i)^5*
(tan(c/2 + (d*x)/2) + 1i))
```

3.165. $\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$

3.166 $\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$

3.166.1 Optimal result

Integrand size = 31, antiderivative size = 89

$$\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \frac{x}{4a^2}+\frac{i\cos^2(c+dx)}{4d(a\cos(c+dx)+ia\sin(c+dx))^2}+\frac{i\cos(c+dx)}{4d\left(a^2\cos(c+dx)+ia^2\sin(c+dx)\right)}$$

output
```
1/4*x/a^2+1/4*I*cos(d*x+c)^2/d/(a*cos(d*x+c)+I*a*sin(d*x+c))^2+1/4*I*cos(d
*x+c)/d/(a^2*cos(d*x+c)+I*a^2*sin(d*x+c))
```

3.166.2 Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.67

$$\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \frac{4c+4dx+4i\cos(2(c+dx))+i\cos(4(c+dx))+4\sin(2(c+dx))+\sin(4(c+dx))}{16a^2d}$$

input
```
Integrate[Cos[c + d*x]^2/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

output
```
(4*c + 4*d*x + (4*I)*Cos[2*(c + d*x)] + I*Cos[4*(c + d*x)] + 4*Sin[2*(c +
d*x)] + Sin[4*(c + d*x)])/(16*a^2*d)
```

3.166.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3042, 3561, 3042, 3561, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(c+dx)^2}{(a\cos(c+dx)+ia\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{ 3561}$$

$$\frac{\int \frac{\cos(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}dx}{2a}+\frac{i\cos^2(c+dx)}{4d(a\cos(c+dx)+ia\sin(c+dx))^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{\cos(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}dx}{2a}+\frac{i\cos^2(c+dx)}{4d(a\cos(c+dx)+ia\sin(c+dx))^2}$$

$$\Big\downarrow \text{ 3561}$$

$$\frac{\frac{\int 1dx}{2a}+\frac{i\cos(c+dx)}{2d(a\cos(c+dx)+ia\sin(c+dx))}}{2a}+\frac{i\cos^2(c+dx)}{4d(a\cos(c+dx)+ia\sin(c+dx))^2}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{i\cos^2(c+dx)}{4d(a\cos(c+dx)+ia\sin(c+dx))^2}+\frac{\frac{x}{2a}+\frac{i\cos(c+dx)}{2d(a\cos(c+dx)+ia\sin(c+dx))}}{2a}$$

input
```
Int[Cos[c + d*x]^2/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

output
```
((I/4)*Cos[c + d*x]^2)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2) + (x/(2*a
) + ((I/2)*Cos[c + d*x])/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])))/(2*a)
```

3.166.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3561
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*((a*Cos[c + d*x] + b*Si
n[c + d*x])^n/(2*a*d*n*Cos[c + d*x]^n)), x] + Simp[1/(2*a)   Int[(a*Cos[c +
 d*x] + b*Sin[c + d*x])^(n + 1)/Cos[c + d*x]^(n + 1), x], x] /; FreeQ[{a, b
, c, d}, x] && EqQ[m + n, 0] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

3.166.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.49

method	result	size
risch	$\frac{x}{4a^2}+\frac{i\,\mathrm{e}^{-2i(dx+c)}}{4a^2d}+\frac{i\,\mathrm{e}^{-4i(dx+c)}}{16a^2d}$	44
derivativedivides	$\frac{\frac{i\ln(\tan(dx+c)+i)}{8}-\frac{i\ln(\tan(dx+c)-i)}{8}-\frac{i}{4(\tan(dx+c)-i)^2}+\frac{1}{4\tan(dx+c)-4i}}{d\,a^2}$	62
default	$\frac{\frac{i\ln(\tan(dx+c)+i)}{8}-\frac{i\ln(\tan(dx+c)-i)}{8}-\frac{i}{4(\tan(dx+c)-i)^2}+\frac{1}{4\tan(dx+c)-4i}}{d\,a^2}$	62

input
```
int(cos(d*x+c)^2/(cos(d*x+c)*a+I*a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*x/a^2+1/4*I/a^2/d*exp(-2*I*(d*x+c))+1/16*I/a^2/d*exp(-4*I*(d*x+c))
```

3.166.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.48

$$\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx=\frac{\left(4\,dxe^{(4i\,dx+4i\,c)}+4i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-4i\,dx-4i\,c)}}{16\,a^2d}$$

input
```
integrate(cos(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="frica
s")
```

output
```
1/16*(4*d*x*e^(4*I*d*x + 4*I*c) + 4*I*e^(2*I*d*x + 2*I*c) + I)*e^(-4*I*d*x
 - 4*I*c)/(a^2*d)
```

3.166.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.31

$$\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \begin{cases} \frac{\left(16ia^2de^{4ic}e^{-2idx}+4ia^2de^{2ic}e^{-4idx}\right)e^{-6ic}}{64a^4d^2} & \text{for } a^4d^2e^{6ic}\neq 0 \\ x\left(\frac{\left(e^{4ic}+2e^{2ic}+1\right)e^{-4ic}}{4a^2}-\frac{1}{4a^2}\right) & \text{otherwise} \end{cases} + \frac{x}{4a^2}$$

input
```
integrate(cos(d*x+c)**2/(a*cos(d*x+c)+I*a*sin(d*x+c))**2,x)
```

output
```
Piecewise(((16*I*a**2*d*exp(4*I*c)*exp(-2*I*d*x) + 4*I*a**2*d*exp(2*I*c)*e
xp(-4*I*d*x))*exp(-6*I*c)/(64*a**4*d**2), Ne(a**4*d**2*exp(6*I*c), 0)), (x
*((exp(4*I*c) + 2*exp(2*I*c) + 1)*exp(-4*I*c)/(4*a**2) - 1/(4*a**2)), True
)) + x/(4*a**2)
```

3.166.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="maxim
a")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.166.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

$$\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = -\frac{-\frac{2i\log(\tan(dx+c)+i)}{a^2}+\frac{2i\log(\tan(dx+c)-i)}{a^2}+\frac{-3i\tan(dx+c)^2-10\tan(dx+c)+11i}{a^2(\tan(dx+c)-i)^2}}{16\,d}$$

input
```
integrate(cos(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="giac"
)
```

output
```
-1/16*(-2*I*log(tan(d*x + c) + I)/a^2 + 2*I*log(tan(d*x + c) - I)/a^2 + (-
3*I*tan(d*x + c)^2 - 10*tan(d*x + c) + 11*I)/(a^2*(tan(d*x + c) - I)^2))/d
```

3.166.9 Mupad [B] (verification not implemented)

Time = 24.89 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.78

$$\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \frac{x}{4\,a^2}+\frac{-\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,2\mathrm{i}+\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}}{a^2\,d\,\left(1+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}\right)^4}$$

input
```
int(cos(c + d*x)^2/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^2,x)
```

output
```
x/(4*a^2) + ((3*tan(c/2 + (d*x)/2))/2 + tan(c/2 + (d*x)/2)^2*2i - (3*tan(c
/2 + (d*x)/2)^3)/2)/(a^2*d*(tan(c/2 + (d*x)/2)*1i + 1)^4)
```

3.167 $\int \frac{\cos(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$

3.167.1 Optimal result

Integrand size = 29, antiderivative size = 52

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \frac{2i\cos^3(c+dx)}{3a^2d}+\frac{\sin(c+dx)}{a^2d}-\frac{2\sin^3(c+dx)}{3a^2d}$$

output
```
2/3*I*cos(d*x+c)^3/a^2/d+sin(d*x+c)/a^2/d-2/3*sin(d*x+c)^3/a^2/d
```

3.167.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.40

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \frac{i\cos(c+dx)}{2a^2d}+\frac{i\cos(3(c+dx))}{6a^2d}+\frac{\sin(c+dx)}{2a^2d}+\frac{\sin(3(c+dx))}{6a^2d}$$

input
```
Integrate[Cos[c + d*x]/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

output
```
((I/2)*Cos[c + d*x])/(a^2*d) + ((I/6)*Cos[3*(c + d*x)])/(a^2*d) + Sin[c +
d*x]/(2*a^2*d) + Sin[3*(c + d*x)]/(6*a^2*d)
```

3.167.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {3042, 3571, 3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{3571}$$

$$-\frac{\int \cos(c+dx)(ia\cos(c+dx)+a\sin(c+dx))^2dx}{a^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \cos(c+dx)(ia\cos(c+dx)+a\sin(c+dx))^2dx}{a^4}$$

$$\Big\downarrow \text{3569}$$

$$-\frac{\int \left(-a^2\cos^3(c+dx)+2ia^2\sin(c+dx)\cos^2(c+dx)+a^2\sin^2(c+dx)\cos(c+dx)\right)dx}{a^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{2a^2\sin^3(c+dx)}{3d}-\frac{a^2\sin(c+dx)}{d}-\frac{2ia^2\cos^3(c+dx)}{3d}}{a^4}$$

input
```
Int[Cos[c + d*x]/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

output
```
-(((((-2*I)/3)*a^2*Cos[c + d*x]^3)/d - (a^2*Sin[c + d*x])/d + (2*a^2*Sin[c
 + d*x]^3)/(3*d))/a^4)
```

3.167.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

rule 3571
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^n*b^n   Int[Cos[c + d*x]^m
/(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &
& EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

3.167.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.73

method	result	size
risch	$\frac{i e^{-i(dx+c)}}{2a^2d}+\frac{i e^{-3i(dx+c)}}{6a^2d}$	38
derivativedivides	$\frac{\frac{2i}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^2}-\frac{4}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^3}+\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i}}{a^2d}$	57
default	$\frac{\frac{2i}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^2}-\frac{4}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^3}+\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i}}{a^2d}$	57
norman	$\frac{\frac{4i\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{ad}+\frac{4i}{3ad}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3ad}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{ad}}{a\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^3}$	105

input
```
int(cos(d*x+c)/(cos(d*x+c)*a+I*a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*I/a^2/d*exp(-I*(d*x+c))+1/6*I/a^2/d*exp(-3*I*(d*x+c))
```

3.167.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.58

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \frac{\left(3i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-3i\,dx-3i\,c)}}{6\,a^2d}$$

input
```
integrate(cos(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="fricas"
)
```

output
```
1/6*(3*I*e^(2*I*d*x + 2*I*c) + I)*e^(-3*I*d*x - 3*I*c)/(a^2*d)
```

3.167.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.77

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \begin{cases} \frac{(6ia^2de^{3ic}e^{-idx}+2ia^2de^{ic}e^{-3idx})e^{-4ic}}{12a^4d^2} & \text{for } a^4d^2e^{4ic}\neq 0 \\ \frac{x(e^{2ic}+1)e^{-3ic}}{2a^2} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))**2,x)
```

output
```
Piecewise(((6*I*a**2*d*exp(3*I*c)*exp(-I*d*x) + 2*I*a**2*d*exp(I*c)*exp(-3
*I*d*x))*exp(-4*I*c)/(12*a**4*d**2), Ne(a**4*d**2*exp(4*I*c), 0)), (x*(exp
(2*I*c) + 1)*exp(-3*I*c)/(2*a**2), True))
```

3.167.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.87

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \frac{i\,\cos(3\,dx+3\,c)+3i\,\cos(dx+c)+\sin(3\,dx+3\,c)+3\,\sin(dx+c)}{6\,a^2d}$$

input
```
integrate(cos(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="maxima"
)
```

output
```
1/6*(I*cos(3*d*x + 3*c) + 3*I*cos(d*x + c) + sin(3*d*x + 3*c) + 3*sin(d*x
+ c))/(a^2*d)
```

3.167.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.90

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \frac{2\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\right)}{3\,a^2d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^3}$$

input
```
integrate(cos(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
2/3*(3*tan(1/2*d*x + 1/2*c)^2 - 3*I*tan(1/2*d*x + 1/2*c) - 2)/(a^2*d*(tan(
1/2*d*x + 1/2*c) - I)^3)
```

3.167.9 Mupad [B] (verification not implemented)

Time = 22.73 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.52

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = -\frac{2\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,3\mathrm{i}+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2\mathrm{i}\right)}{3\,a^2\,d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,1\mathrm{i}-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,3\mathrm{i}+1\right)}$$

input
```
int(cos(c + d*x)/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^2,x)
```

output
```
-(2*(3*tan(c/2 + (d*x)/2) + tan(c/2 + (d*x)/2)^2*3i - 2i))/(3*a^2*d*(tan(c
/2 + (d*x)/2)*3i - 3*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)^3*1i + 1))
```

3.168 $\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$

3.168.1 Optimal result

Integrand size = 22, antiderivative size = 31

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \frac{i}{2d(a\cos(c+dx)+ia\sin(c+dx))^2}$$

output
```
1/2*I/d/(a*cos(d*x+c)+I*a*sin(d*x+c))^2
```

3.168.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \frac{i}{2d(a\cos(c+dx)+ia\sin(c+dx))^2}$$

input
```
Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-2),x]
```

output
```
(I/2)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2)
```

3.168.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3042, 3550}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{3550}$$

$$\frac{i}{2d(a\cos(c+dx)+ia\sin(c+dx))^2}$$

input
```
Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-2),x]
```

output
```
(I/2)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2)
```

3.168.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3550
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

3.168.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

method	result	size
risch	$\frac{i e^{-2i(dx+c)}}{2a^2 d}$	19
derivativedivides	$\frac{i}{d\,a^2(i\tan(dx+c)+1)}$	23
default	$\frac{i}{d\,a^2(i\tan(dx+c)+1)}$	23
norman	$\frac{\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{ad}-\frac{4i\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{ad}}{a\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}$	77

input
```
int(1/(cos(d*x+c)*a+I*a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*I/a^2/d*exp(-2*I*(d*x+c))
```

3.168.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.55

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \frac{i\,e^{(-2i\,dx-2i\,c)}}{2\,a^2d}$$

input
```
integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/2*I*e^(-2*I*d*x - 2*I*c)/(a^2*d)
```

3.168.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.42

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \begin{cases} \frac{ie^{-2ic}e^{-2idx}}{2a^2d} & \text{for } a^2de^{2ic} \neq 0 \\ \frac{xe^{-2ic}}{a^2} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))**2,x)
```

output
```
Piecewise((I*exp(-2*I*c)*exp(-2*I*d*x)/(2*a**2*d), Ne(a**2*d*exp(2*I*c), 0
)), (x*exp(-2*I*c)/a**2, True))
```

3.168.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \frac{1}{(a^2\tan(dx+c)-i\,a^2)d}$$

input
```
integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/((a^2*tan(d*x + c) - I*a^2)*d)
```

3.168.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = -\frac{2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^2}$$

input
```
integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
-2*tan(1/2*d*x + 1/2*c)/(a^2*d*(tan(1/2*d*x + 1/2*c) - I)^2)
```

3.168.9 Mupad [B] (verification not implemented)

Time = 22.44 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = -\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^2\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\mathrm{i}\right)^2}$$

input
```
int(1/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^2,x)
```

output
```
-(2*tan(c/2 + (d*x)/2))/(a^2*d*(tan(c/2 + (d*x)/2) - 1i)^2)
```

3.169 $\int \frac{\sec(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$

3.169.1 Optimal result

Integrand size = 29, antiderivative size = 46

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = -\frac{\operatorname{arctanh}(\sin(c+dx))}{a^2d} + \frac{2i\cos(c+dx)}{a^2d} + \frac{2\sin(c+dx)}{a^2d}$$

output
```
-arctanh(sin(d*x+c))/a^2/d+2*I*cos(d*x+c)/a^2/d+2*sin(d*x+c)/a^2/d
```

3.169.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 184 vs. 2(46) = 92.

Time = 0.36 (sec) , antiderivative size = 184, normalized size of antiderivative = 4.00

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = -\frac{\sec^2(c+dx)\left(\cos\left(\frac{1}{2}(c+dx)\right)\left(2i+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\right.\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

output
```
-((Sec[c + d*x]^2*(Cos[(c + d*x)/2]*(2*I + Log[Cos[(c + d*x)/2] - Sin[(c +
 d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + (2 + I*Log[Cos[(c
+ d*x)/2] - Sin[(c + d*x)/2]] - I*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]
)*Sin[(c + d*x)/2])*(Cos[(3*(c + d*x))/2] + I*Sin[(3*(c + d*x))/2]))/(a^2*
d*(-I + Tan[c + d*x])^2))
```

3.169.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {3042, 3571, 3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(c+dx)(a\cos(c+dx)+ia\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{3571}$$

$$-\frac{\int \sec(c+dx)(ia\cos(c+dx)+a\sin(c+dx))^2dx}{a^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{(ia\cos(c+dx)+a\sin(c+dx))^2}{\cos(c+dx)}dx}{a^4}$$

$$\Big\downarrow \text{3569}$$

$$-\frac{\int \left(-\cos(c+dx)a^2+2i\sin(c+dx)a^2+\sin(c+dx)\tan(c+dx)a^2\right)dx}{a^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{a^2\text{arctanh}(\sin(c+dx))}{d}-\frac{2a^2\sin(c+dx)}{d}-\frac{2ia^2\cos(c+dx)}{d}}{a^4}$$

input
```
Int[Sec[c + d*x]/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

output
```
-(((a^2*ArcTanh[Sin[c + d*x]])/d - ((2*I)*a^2*Cos[c + d*x])/d - (2*a^2*Sin
[c + d*x])/d)/a^4)
```

3.169.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

rule 3571
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^n*b^n   Int[Cos[c + d*x]^m
/(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &
& EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

3.169.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.17

method	result	size
derivativedivides	$\frac{-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\frac{4}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i}}{a^2 d}$	54
default	$\frac{-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\frac{4}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i}}{a^2 d}$	54
risch	$\frac{2i\,e^{-i(dx+c)}}{a^2 d}+\frac{\ln\left(e^{i(dx+c)}-i\right)}{a^2 d}-\frac{\ln\left(i+e^{i(dx+c)}\right)}{a^2 d}$	61
norman	$\frac{\frac{4i}{ad}+\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)a}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^2 d}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^2 d}$	87

input
```
int(sec(d*x+c)/(cos(d*x+c)*a+I*a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
2/d/a^2*(-1/2*ln(tan(1/2*d*x+1/2*c)+1)+1/2*ln(tan(1/2*d*x+1/2*c)-1)+2/(tan
(1/2*d*x+1/2*c)-I))
```

3.169.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.39

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = -\frac{\left(e^{(i\,dx+i\,c)}\log\left(e^{(i\,dx+i\,c)}+i\right)-e^{(i\,dx+i\,c)}\log\left(e^{(i\,dx+i\,c)}-i\right)-2i\right)e^{(-i\,dx-i\,c)}}{a^2d}$$

input
```
integrate(sec(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="fricas"
)
```

output
```
-(e^(I*d*x + I*c)*log(e^(I*d*x + I*c) + I) - e^(I*d*x + I*c)*log(e^(I*d*x
+ I*c) - I) - 2*I)*e^(-I*d*x - I*c)/(a^2*d)
```

3.169.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \frac{\int \frac{\sec(c+dx)}{-\sin^2(c+dx)+2i\sin(c+dx)\cos(c+dx)+\cos^2(c+dx)}\,dx}{a^2}$$

input
```
integrate(sec(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)/(-sin(c + d*x)**2 + 2*I*sin(c + d*x)*cos(c + d*x) +
cos(c + d*x)**2), x)/a**2
```

3.169.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(44) = 88.

Time = 0.31 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.54

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = -\frac{-2i\,\arctan\left(\cos\left(dx+c\right),\sin\left(dx+c\right)+1\right)-2i\,\arctan\left(\cos\left(dx+c\right),-\sin\left(dx+c\right)+1\right)-4i\,\cos\left(dx-\right.}{}$$

input
```
integrate(sec(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="maxima"
)
```

output
```
-1/2*(-2*I*arctan2(cos(d*x + c), sin(d*x + c) + 1) - 2*I*arctan2(cos(d*x +
 c), -sin(d*x + c) + 1) - 4*I*cos(d*x + c) + log(cos(d*x + c)^2 + sin(d*x
+ c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin
(d*x + c) + 1) - 4*sin(d*x + c))/(a^2*d)
```

3.169.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.24

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = -\frac{\frac{\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^2}-\frac{\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^2}-\frac{4}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)}}{d}$$

input
```
integrate(sec(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
-(log(tan(1/2*d*x + 1/2*c) + 1)/a^2 - log(tan(1/2*d*x + 1/2*c) - 1)/a^2 -
4/(a^2*(tan(1/2*d*x + 1/2*c) - I)))/d
```

3.169.9 Mupad [B] (verification not implemented)

Time = 22.95 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$$
$$=-\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}+\frac{4\mathrm{i}}{a^2\,d\,\left(1+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}\right)}$$

input
```
int(1/(cos(c + d*x)*(a*cos(c + d*x) + a*sin(c + d*x)*1i)^2),x)
```

output
```
4i/(a^2*d*(tan(c/2 + (d*x)/2)*1i + 1)) - (2*atanh(tan(c/2 + (d*x)/2)))/(a^
2*d)
```

3.170 $\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$

3.170.1 Optimal result

Integrand size = 31, antiderivative size = 55

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \frac{2x}{a^2}+\frac{2i\log(\sin(c+dx))}{a^2d}-\frac{2i\log(\tan(c+dx))}{a^2d}-\frac{\tan(c+dx)}{a^2d}$$

output
```
2*x/a^2+2*I*ln(sin(d*x+c))/a^2/d-2*I*ln(tan(d*x+c))/a^2/d-tan(d*x+c)/a^2/d
```

3.170.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.65

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = -\frac{\frac{2i\log(i-\tan(c+dx))}{d}+\frac{\tan(c+dx)}{d}}{a^2}$$

input
```
Integrate[Sec[c + d*x]^2/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

output
```
-((((2*I)*Log[I - Tan[c + d*x]])/d + Tan[c + d*x]/d)/a^2)
```

3.170.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.75, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3042, 3567, 27, 516, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\cos(c+dx)^2(a\cos(c+dx)+ia\sin(c+dx))^2}dx$$
$$\Big\downarrow \text{3567}$$
$$-\frac{\int \frac{(\cot^2(c+dx)+1)\tan^2(c+dx)}{a^2(\cot(c+dx)+i)^2}d\cot(c+dx)}{d}$$
$$\Big\downarrow \text{27}$$
$$-\frac{\int \frac{(\cot^2(c+dx)+1)\tan^2(c+dx)}{(\cot(c+dx)+i)^2}d\cot(c+dx)}{a^2d}$$
$$\Big\downarrow \text{516}$$
$$-\frac{\int \frac{(\cot(c+dx)-i)\tan^2(c+dx)}{\cot(c+dx)+i}d\cot(c+dx)}{a^2d}$$
$$\Big\downarrow \text{86}$$
$$-\frac{\int \left(-\tan^2(c+dx)-2i\tan(c+dx)+\frac{2i}{\cot(c+dx)+i}\right)d\cot(c+dx)}{a^2d}$$
$$\Big\downarrow \text{2009}$$
$$-\frac{\tan(c+dx)-2i\log(\cot(c+dx))+2i\log(\cot(c+dx)+i)}{a^2d}$$

input
```
Int[Sec[c + d*x]^2/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

output
```
-(((-2*I)*Log[Cot[c + d*x]] + (2*I)*Log[I + Cot[c + d*x]] + Tan[c + d*x])/
(a^2*d))
```

3.170.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 516
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[(e*x)^m*(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; Free
Q[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] ||
(GtQ[a, 0] && GtQ[c, 0] &&  !IntegerQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3567
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[x^m*((b
+ a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a,
b, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[
n, 0] && GtQ[m, 1])
```

3.170. $\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$

3.170.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.55

method	result
derivativedivides	$\frac{-\tan(dx+c)-2i\ln(\tan(dx+c)-i)}{d\,a^2}$
default	$\frac{-\tan(dx+c)-2i\ln(\tan(dx+c)-i)}{d\,a^2}$
risch	$\frac{4x}{a^2}+\frac{4c}{a^2d}-\frac{2i}{a^2d(\mathrm{e}^{2i(dx+c)}+1)}+\frac{2i\ln(\mathrm{e}^{2i(dx+c)}+1)}{a^2d}$
norman	$\frac{-\frac{2x}{a}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{2x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{a}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{2i\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^2d}+\frac{2i\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^2d}-\frac{2i\ln\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{a^2d}$

input
```
int(sec(d*x+c)^2/(cos(d*x+c)*a+I*a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^2*(-tan(d*x+c)-2*I*ln(tan(d*x+c)-I))
```

3.170.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.27

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \frac{2\left(2\,dxe^{(2i\,dx+2i\,c)}+2\,dx-\left(-i\,e^{(2i\,dx+2i\,c)}-i\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)-i\right)}{a^2de^{(2i\,dx+2i\,c)}+a^2d}$$

input
```
integrate(sec(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="frica
s")
```

output
```
2*(2*d*x*e^(2*I*d*x + 2*I*c) + 2*d*x - (-I*e^(2*I*d*x + 2*I*c) - I)*log(e^
(2*I*d*x + 2*I*c) + 1) - I)/(a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

3.170.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \frac{\int \frac{\sec^2(c+dx)}{-\sin^2(c+dx)+2i\sin(c+dx)\cos(c+dx)+\cos^2(c+dx)}\,dx}{a^2}$$

input
```
integrate(sec(d*x+c)**2/(a*cos(d*x+c)+I*a*sin(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)**2/(-sin(c + d*x)**2 + 2*I*sin(c + d*x)*cos(c + d*x)
 + cos(c + d*x)**2), x)/a**2
```

3.170.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.55

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \frac{-\frac{2i\log(\tan(dx+c)-i)}{a^2}-\frac{\tan(dx+c)}{a^2}}{d}$$

input
```
integrate(sec(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="maxim
a")
```

output
```
(-2*I*log(tan(d*x + c) - I)/a^2 - tan(d*x + c)/a^2)/d
```

3.170.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.82

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$$
$$= \frac{2\left(\frac{i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^2}-\frac{2i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)}{a^2}+\frac{i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{a^2}+\frac{-i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+i}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a^2}\right)}{d}$$

input
```
integrate(sec(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="giac"
)
```

output
```
2*(I*log(tan(1/2*d*x + 1/2*c) + 1)/a^2 - 2*I*log(tan(1/2*d*x + 1/2*c) - I)
/a^2 + I*log(tan(1/2*d*x + 1/2*c) - 1)/a^2 + (-I*tan(1/2*d*x + 1/2*c)^2 +
tan(1/2*d*x + 1/2*c) + I)/((tan(1/2*d*x + 1/2*c)^2 - 1)*a^2))/d
```

3.170.9 Mupad [B] (verification not implemented)

Time = 22.70 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.51

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-a^2\right)} - \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-\mathrm{i}\right)\,4\mathrm{i}}{a^2\,d} + \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)\,2\mathrm{i}}{a^2\,d}$$

input
```
int(1/(cos(c + d*x)^2*(a*cos(c + d*x) + a*sin(c + d*x)*1i)^2),x)
```

output
```
(2*tan(c/2 + (d*x)/2))/(d*(a^2*tan(c/2 + (d*x)/2)^2 - a^2)) - (log(tan(c/2
 + (d*x)/2) - 1i)*4i)/(a^2*d) + (log(tan(c/2 + (d*x)/2)^2 - 1)*2i)/(a^2*d)
```

3.171 $\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$

3.171.1 Optimal result

Integrand size = 31, antiderivative size = 56

$$\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \frac{3\text{arctanh}(\sin(c+dx))}{2a^2d} - \frac{2i\sec(c+dx)}{a^2d} - \frac{\sec(c+dx)\tan(c+dx)}{2a^2d}$$

output
```
3/2*arctanh(sin(d*x+c))/a^2/d-2*I*sec(d*x+c)/a^2/d-1/2*sec(d*x+c)*tan(d*x+
c)/a^2/d
```

3.171.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 146 vs. 2(56) = 112.

Time = 0.55 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.61

$$\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = -\frac{\sec^2(c+dx)\left(8i\cos(c+dx)+3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+3\cos(2(c+dx))\left(\log\left(\cos\left(\frac{1}{2}\right.\right.\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]^3/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

output
```
-1/4*(Sec[c + d*x]^2*((8*I)*Cos[c + d*x] + 3*Log[Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2]] + 3*Cos[2*(c + d*x)]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
 - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 3*Log[Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]] + 2*Sin[c + d*x]))/(a^2*d)
```

3.171.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3042, 3571, 3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(c+dx)^3(a\cos(c+dx)+ia\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{3571}$$

$$-\frac{\int \sec^3(c+dx)(ia\cos(c+dx)+a\sin(c+dx))^2dx}{a^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{(ia\cos(c+dx)+a\sin(c+dx))^2}{\cos(c+dx)^3}dx}{a^4}$$

$$\Big\downarrow \text{3569}$$

$$-\frac{\int \left(\sec(c+dx)\tan^2(c+dx)a^2-\sec(c+dx)a^2+2i\sec(c+dx)\tan(c+dx)a^2\right)dx}{a^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{3a^2\text{arctanh}(\sin(c+dx))}{2d}+\frac{2ia^2\sec(c+dx)}{d}+\frac{a^2\tan(c+dx)\sec(c+dx)}{2d}}{a^4}$$

input
```
Int[Sec[c + d*x]^3/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

output
```
-(((-3*a^2*ArcTanh[Sin[c + d*x]])/(2*d) + ((2*I)*a^2*Sec[c + d*x])/d + (a^
2*Sec[c + d*x]*Tan[c + d*x])/(2*d))/a^4)
```

3.171.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

rule 3571
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^n*b^n   Int[Cos[c + d*x]^m
/(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &
& EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

3.171.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.59

method	result
risch	$-\frac{i\left(3\,\mathrm{e}^{3i(dx+c)}+5\,\mathrm{e}^{i(dx+c)}\right)}{d\,a^{2}\left(\mathrm{e}^{2i(dx+c)}+1\right)^{2}}+\frac{3\ln\left(i+\mathrm{e}^{i(dx+c)}\right)}{2a^{2}d}-\frac{3\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2a^{2}d}$
derivativedivides	$\frac{\frac{2\left(-\frac{1}{4}-i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2}+\frac{2\left(-\frac{1}{4}+i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}}{a^{2}d}$
default	$\frac{\frac{2\left(-\frac{1}{4}-i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2}+\frac{2\left(-\frac{1}{4}+i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}}{a^{2}d}$
norman	$\frac{-\frac{4i}{ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{ad}+\frac{4i\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{ad}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}a}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2a^{2}d}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2a^{2}d}$

input
```
int(sec(d*x+c)^3/(cos(d*x+c)*a+I*a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-I/d/a^2/(exp(2*I*(d*x+c))+1)^2*(3*exp(3*I*(d*x+c))+5*exp(I*(d*x+c)))+3/2/
a^2/d*ln(I+exp(I*(d*x+c)))-3/2/a^2/d*ln(exp(I*(d*x+c))-I)
```

3.171.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(50) = 100$.

Time = 0.27 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.39

$$\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \frac{3\left(e^{(4i\,dx+4i\,c)}+2\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-3\left(e^{(4i\,dx+4i\,c)}+2\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}-i\right)-}{2\left(a^2de^{(4i\,dx+4i\,c)}+2\,a^2de^{(2i\,dx+2i\,c)}+a^2d\right)}$$

input
```
integrate(sec(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="frica
s")
```

output
```
1/2*(3*(e^(4*I*d*x + 4*I*c) + 2*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x + I*
c) + I) - 3*(e^(4*I*d*x + 4*I*c) + 2*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x
 + I*c) - I) - 6*I*e^(3*I*d*x + 3*I*c) - 10*I*e^(I*d*x + I*c))/(a^2*d*e^(4
*I*d*x + 4*I*c) + 2*a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

3.171.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \frac{\int \frac{\sec^3(c+dx)}{-\sin^2(c+dx)+2i\sin(c+dx)\cos(c+dx)+\cos^2(c+dx)}\,dx}{a^2}$$

input
```
integrate(sec(d*x+c)**3/(a*cos(d*x+c)+I*a*sin(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)**3/(-sin(c + d*x)**2 + 2*I*sin(c + d*x)*cos(c + d*x)
 + cos(c + d*x)**2), x)/a**2
```

3.171.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(50) = 100$.

Time = 0.23 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.98

$$\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = -\frac{\frac{2\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{4i\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+4i\right)}{a^2-\frac{2a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}}{2d}$$

input
```
integrate(sec(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="maxim
a")
```

output
```
-1/2*(2*(sin(d*x + c)/(cos(d*x + c) + 1) - 4*I*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 4*I)/(a^2 - 2*a^2*sin(d*x
 + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) -
3*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 3*log(sin(d*x + c)/(cos(d
*x + c) + 1) - 1)/a^2)/d
```

3.171.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.70

$$\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \frac{\frac{3\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^2}-\frac{3\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{a^2}-\frac{2\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-4i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+4i\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a^2}}{2d}$$

input
```
integrate(sec(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="giac"
)
```

output
```
1/2*(3*log(tan(1/2*d*x + 1/2*c) + 1)/a^2 - 3*log(tan(1/2*d*x + 1/2*c) - 1)
/a^2 - 2*(tan(1/2*d*x + 1/2*c)^3 - 4*I*tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d*
x + 1/2*c) + 4*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^2))/d
```

3.171.9 Mupad [B] (verification not implemented)

Time = 23.16 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.86

$$\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \frac{3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d} - \frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{a^2}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^2}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,4\mathrm{i}}{a^2}+\frac{4\mathrm{i}}{a^2}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

input
```
int(1/(cos(c + d*x)^3*(a*cos(c + d*x) + a*sin(c + d*x)*1i)^2),x)
```

output
```
(3*atanh(tan(c/2 + (d*x)/2)))/(a^2*d) - (tan(c/2 + (d*x)/2)^3/a^2 - (tan(c
/2 + (d*x)/2)^2*4i)/a^2 + 4i/a^2 + tan(c/2 + (d*x)/2)/a^2)/(d*(tan(c/2 + (
d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^2 + 1))
```

3.172 $\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$

3.172.1 Optimal result

Integrand size = 31, antiderivative size = 34

$$\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = -\frac{i(i-\cot(c+dx))^3\tan^3(c+dx)}{3a^2d}$$

output
```
-1/3*I*(I-cot(d*x+c))^3*tan(d*x+c)^3/a^2/d
```

3.172.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = -\frac{\tan(c+dx)\left(-3+3i\tan(c+dx)+\tan^2(c+dx)\right)}{3a^2d}$$

input
```
Integrate[Sec[c + d*x]^4/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

output
```
-1/3*(Tan[c + d*x]*(-3 + (3*I)*Tan[c + d*x] + Tan[c + d*x]^2))/(a^2*d)
```

3.172.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3042, 3567, 27, 516, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(c+dx)^4(a\cos(c+dx)+ia\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{3567}$$

$$-\frac{\int \frac{(\cot^2(c+dx)+1)^2\tan^4(c+dx)}{a^2(\cot(c+dx)+i)^2}d\cot(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{(\cot^2(c+dx)+1)^2\tan^4(c+dx)}{(\cot(c+dx)+i)^2}d\cot(c+dx)}{a^2d}$$

$$\Big\downarrow \text{516}$$

$$-\frac{\int (\cot(c+dx)-i)^2\tan^4(c+dx)d\cot(c+dx)}{a^2d}$$

$$\Big\downarrow \text{48}$$

$$-\frac{i\tan^3(c+dx)(-\cot(c+dx)+i)^3}{3a^2d}$$

input
```
Int[Sec[c + d*x]^4/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

output
```
((-1/3*I)*(I - Cot[c + d*x])^3*Tan[c + d*x]^3)/(a^2*d)
```

3.172.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 516
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[(e*x)^m*(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; Free
Q[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] ||
(GtQ[a, 0] && GtQ[c, 0] &&  !IntegerQ[n]))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3567
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[x^m*((b
+ a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a,
b, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[
n, 0] && GtQ[m, 1])
```

3.172.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.59

method	result	size
derivativedivides	$-\frac{(\tan(dx+c)+i)^3}{3d\,a^2}$	20
default	$-\frac{(\tan(dx+c)+i)^3}{3d\,a^2}$	20
risch	$\frac{8i}{3a^2d\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}$	23
norman	$\frac{\frac{4i\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{ad}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{20\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3ad}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{ad}-\frac{4i\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{ad}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3 a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}$	127

input
```
int(sec(d*x+c)^4/(cos(d*x+c)*a+I*a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3/d/a^2*(tan(d*x+c)+I)^3
```

3.172.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(26) = 52$.

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.59

$$\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \frac{8i}{3\left(a^2de^{(6i\,dx+6i\,c)}+3\,a^2de^{(4i\,dx+4i\,c)}+3\,a^2de^{(2i\,dx+2i\,c)}+a^2d\right)}$$

input
```
integrate(sec(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="frica
s")
```

output
```
8/3*I/(a^2*d*e^(6*I*d*x + 6*I*c) + 3*a^2*d*e^(4*I*d*x + 4*I*c) + 3*a^2*d*e
^(2*I*d*x + 2*I*c) + a^2*d)
```

3.172.6 Sympy [F]

$$\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \frac{\int \frac{\sec^4(c+dx)}{-\sin^2(c+dx)+2i\sin(c+dx)\cos(c+dx)+\cos^2(c+dx)}\,dx}{a^2}$$

input
```
integrate(sec(d*x+c)**4/(a*cos(d*x+c)+I*a*sin(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)**4/(-sin(c + d*x)**2 + 2*I*sin(c + d*x)*cos(c + d*x)
 + cos(c + d*x)**2), x)/a**2
```

3.172.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = -\frac{\tan{(dx+c)}^3+3i\,\tan{(dx+c)}^2-3\,\tan{(dx+c)}}{3\,a^2d}$$

input
```
integrate(sec(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="maxim
a")
```

output
```
-1/3*(tan(d*x + c)^3 + 3*I*tan(d*x + c)^2 - 3*tan(d*x + c))/(a^2*d)
```

3.172.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = -\frac{\tan{(dx+c)}^3+3i\,\tan{(dx+c)}^2-3\,\tan{(dx+c)}}{3\,a^2d}$$

input
```
integrate(sec(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="giac"
)
```

output
```
-1/3*(tan(d*x + c)^3 + 3*I*tan(d*x + c)^2 - 3*tan(d*x + c))/(a^2*d)
```

3.172.9 Mupad [B] (verification not implemented)

Time = 23.71 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.44

$$\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = -\frac{\sin{(c+d\,x)}\,\left(-4\cos{(c+d\,x)}^2+3\mathrm{i}\,\sin{(c+d\,x)}\,\cos{(c+d\,x)}+1\right)}{3\,a^2\,d\cos{(c+d\,x)}^3}$$

input
```
int(1/(cos(c + d*x)^4*(a*cos(c + d*x) + a*sin(c + d*x)*1i)^2),x)
```

output
```
-(sin(c + d*x)*(cos(c + d*x)*sin(c + d*x)*3i - 4*cos(c + d*x)^2 + 1))/(3*a
^2*d*cos(c + d*x)^3)
```

3.173 $\int \frac{\sec^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$

3.173.1 Optimal result

Integrand size = 31, antiderivative size = 84

$$\int \frac{\sec^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \frac{5\text{arctanh}(\sin(c+dx))}{8a^2d} - \frac{2i\sec^3(c+dx)}{3a^2d} + \frac{5\sec(c+dx)\tan(c+dx)}{8a^2d} - \frac{\sec^3(c+dx)\tan(c+dx)}{4a^2d}$$

output
```
5/8*arctanh(sin(d*x+c))/a^2/d-2/3*I*sec(d*x+c)^3/a^2/d+5/8*sec(d*x+c)*tan(
d*x+c)/a^2/d-1/4*sec(d*x+c)^3*tan(d*x+c)/a^2/d
```

3.173.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 215 vs. 2(84) = 168.

Time = 1.23 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.56

$$\int \frac{\sec^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = -\frac{\sec^4(c+dx)\left(128i\cos(c+dx)+45\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+60\cos(2(c+dx))\left(\log\left(\mathrm{c}\right.\right.\right.}{}$$

input
```
Integrate[Sec[c + d*x]^5/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

output
```
-1/192*(Sec[c + d*x]^4*((128*I)*Cos[c + d*x] + 45*Log[Cos[(c + d*x)/2] - S
in[(c + d*x)/2]] + 60*Cos[2*(c + d*x)]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*
x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 15*Cos[4*(c + d*x)]*(
Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2]]) - 45*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 18*Sin[c + d*x]
 - 30*Sin[3*(c + d*x)]))/(a^2*d)
```

3.173.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3042, 3571, 3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(c+dx)^5(a\cos(c+dx)+ia\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{3571}$$

$$-\frac{\int \sec^5(c+dx)(ia\cos(c+dx)+a\sin(c+dx))^2dx}{a^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{(ia\cos(c+dx)+a\sin(c+dx))^2}{\cos(c+dx)^5}dx}{a^4}$$

$$\Big\downarrow \text{3569}$$

$$-\frac{\int \left(-a^2\sec^3(c+dx)+a^2\tan^2(c+dx)\sec^3(c+dx)+2ia^2\tan(c+dx)\sec^3(c+dx)\right)dx}{a^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{5a^2\text{arctanh}(\sin(c+dx))}{8d}+\frac{2ia^2\sec^3(c+dx)}{3d}+\frac{a^2\tan(c+dx)\sec^3(c+dx)}{4d}-\frac{5a^2\tan(c+dx)\sec(c+dx)}{8d}}{a^4}$$

input
```
Int[Sec[c + d*x]^5/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

output
```
-(((-5*a^2*ArcTanh[Sin[c + d*x]])/(8*d) + (((2*I)/3)*a^2*Sec[c + d*x]^3)/d
 - (5*a^2*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a^2*Sec[c + d*x]^3*Tan[c + d
*x])/(4*d))/a^4)
```

3.173.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

rule 3571
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^n*b^n   Int[Cos[c + d*x]^m
/(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &
& EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

3.173.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.32

method	result
risch	$-\frac{i\left(15\,\mathrm{e}^{7i(dx+c)}+55\,\mathrm{e}^{5i(dx+c)}+73\,\mathrm{e}^{3i(dx+c)}-15\,\mathrm{e}^{i(dx+c)}\right)}{12d\,a^2\left(\mathrm{e}^{2i(dx+c)}+1\right)^4}+\frac{5\ln\left(i+\mathrm{e}^{i(dx+c)}\right)}{8a^2d}-\frac{5\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{8a^2d}$
derivativedivides	$\frac{\frac{2\left(\frac{3}{16}-\frac{i}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\frac{2\left(-\frac{1}{16}+\frac{i}{2}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{2\left(-\frac{1}{4}-\frac{i}{3}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}+\frac{5\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{8}+\frac{2\left(\frac{3}{16}+\frac{i}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{}{(\mathrm{ta}}}{a^2d}$
default	$\frac{\frac{2\left(\frac{3}{16}-\frac{i}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\frac{2\left(-\frac{1}{16}+\frac{i}{2}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{2\left(-\frac{1}{4}-\frac{i}{3}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}+\frac{5\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{8}+\frac{2\left(\frac{3}{16}+\frac{i}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{}{(\mathrm{ta}}}{a^2d}$
norman	$\frac{\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}-\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{4ad}-\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{4ad}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{4ad}-\frac{4i}{3ad}+\frac{4i\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{ad}-\frac{4i\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{ad}+\frac{4i\tan\left(\frac{dx}{2}+\frac{c}{2}\right.}{3ad}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}$

input
```
int(sec(d*x+c)^5/(cos(d*x+c)*a+I*a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/12*I/d/a^2/(exp(2*I*(d*x+c))+1)^4*(15*exp(7*I*(d*x+c))+55*exp(5*I*(d*x+
c))+73*exp(3*I*(d*x+c))-15*exp(I*(d*x+c)))+5/8/a^2/d*ln(I+exp(I*(d*x+c)))-
5/8/a^2/d*ln(exp(I*(d*x+c))-I)
```

3.173.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 230 vs. 2(74) = 148.

Time = 0.24 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.74

$$\int \frac{\sec^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$$
$$=\frac{15\left(e^{(8i\,dx+8i\,c)}+4\,e^{(6i\,dx+6i\,c)}+6\,e^{(4i\,dx+4i\,c)}+4\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-15\left(e^{(8i\,dx+8i\,c)}+4\,e^{(6i\,c}\right.}{24\left(a^2de^{(8i\,dx+8i\,c)}+4\,a^2de^{(6i\,d}\right.}$$

input
```
integrate(sec(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="frica
s")
```

output
```
1/24*(15*(e^(8*I*d*x + 8*I*c) + 4*e^(6*I*d*x + 6*I*c) + 6*e^(4*I*d*x + 4*I
*c) + 4*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x + I*c) + I) - 15*(e^(8*I*d*x
 + 8*I*c) + 4*e^(6*I*d*x + 6*I*c) + 6*e^(4*I*d*x + 4*I*c) + 4*e^(2*I*d*x +
 2*I*c) + 1)*log(e^(I*d*x + I*c) - I) - 30*I*e^(7*I*d*x + 7*I*c) - 110*I*e
^(5*I*d*x + 5*I*c) - 146*I*e^(3*I*d*x + 3*I*c) + 30*I*e^(I*d*x + I*c))/(a^
2*d*e^(8*I*d*x + 8*I*c) + 4*a^2*d*e^(6*I*d*x + 6*I*c) + 6*a^2*d*e^(4*I*d*x
 + 4*I*c) + 4*a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

3.173.6 Sympy [F]

$$\int \frac{\sec^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \frac{\int \frac{\sec^5(c+dx)}{-\sin^2(c+dx)+2i\sin(c+dx)\cos(c+dx)+\cos^2(c+dx)}\,dx}{a^2}$$

input
```
integrate(sec(d*x+c)**5/(a*cos(d*x+c)+I*a*sin(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)**5/(-sin(c + d*x)**2 + 2*I*sin(c + d*x)*cos(c + d*x)
 + cos(c + d*x)**2), x)/a**2
```

3.173.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(74) = 148$.

Time = 0.23 (sec) , antiderivative size = 295, normalized size of antiderivative = 3.51

$$\int \frac{\sec^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$$
$$= \frac{\frac{2\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{16i\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{33\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{48i\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{33\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{48i\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{9\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-16i\right)}{a^2-\frac{4a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}+\frac{15\log\left(\frac{\sin(dx+}{\cos(dx+c}\right.}{a^2}}{24d}$$

input
```
integrate(sec(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="maxim
a")
```

output
```
1/24*(2*(9*sin(d*x + c)/(cos(d*x + c) + 1) + 16*I*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 - 33*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 48*I*sin(d*x + c)^4
/(cos(d*x + c) + 1)^4 - 33*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 48*I*sin(
d*x + c)^6/(cos(d*x + c) + 1)^6 + 9*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 -
16*I)/(a^2 - 4*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a^2*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4 - 4*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a^2
*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) + 15*log(sin(d*x + c)/(cos(d*x + c)
+ 1) + 1)/a^2 - 15*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2)/d
```

3.173.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(74) = 148$.

Time = 0.33 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.80

$$\int \frac{\sec^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$$
$$= \frac{\frac{15\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^2} - \frac{15\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{a^2} + \frac{2\left(9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+48i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-33\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-48i\tan\left(\frac{1}{2}dx+\frac{1}{2}\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right.}}{24\,d}$$

input
```
integrate(sec(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="giac"
)
```

output
```
1/24*(15*log(tan(1/2*d*x + 1/2*c) + 1)/a^2 - 15*log(tan(1/2*d*x + 1/2*c) -
 1)/a^2 + 2*(9*tan(1/2*d*x + 1/2*c)^7 + 48*I*tan(1/2*d*x + 1/2*c)^6 - 33*t
an(1/2*d*x + 1/2*c)^5 - 48*I*tan(1/2*d*x + 1/2*c)^4 - 33*tan(1/2*d*x + 1/2
*c)^3 + 16*I*tan(1/2*d*x + 1/2*c)^2 + 9*tan(1/2*d*x + 1/2*c) - 16*I)/((tan
(1/2*d*x + 1/2*c)^2 - 1)^4*a^2))/d
```

3.173.9 Mupad [B] (verification not implemented)

Time = 26.62 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.62

$$\int \frac{\sec^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \frac{5\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{4\,a^2\,d}$$
$$+ \frac{\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{4}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 4\mathrm{i}-\frac{11\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{4}-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 4\mathrm{i}-\frac{11\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{4}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 4\mathrm{i}}{3}+\frac{3\tan}{}}{a^2\,d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)^4}$$

input
```
int(1/(cos(c + d*x)^5*(a*cos(c + d*x) + a*sin(c + d*x)*1i)^2),x)
```

output
```
(5*atanh(tan(c/2 + (d*x)/2)))/(4*a^2*d) + ((3*tan(c/2 + (d*x)/2))/4 + (tan
(c/2 + (d*x)/2)^2*4i)/3 - (11*tan(c/2 + (d*x)/2)^3)/4 - tan(c/2 + (d*x)/2)
^4*4i - (11*tan(c/2 + (d*x)/2)^5)/4 + tan(c/2 + (d*x)/2)^6*4i + (3*tan(c/2
 + (d*x)/2)^7)/4 - 4i/3)/(a^2*d*(tan(c/2 + (d*x)/2)^2 - 1)^4)
```

3.174 $\int \frac{\sec^6(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$

3.174.1 Optimal result

Integrand size = 31, antiderivative size = 70

$$\int \frac{\sec^6(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \frac{\tan(c+dx)}{a^2d} - \frac{i\tan^2(c+dx)}{a^2d} - \frac{i\tan^4(c+dx)}{2a^2d} - \frac{\tan^5(c+dx)}{5a^2d}$$

output
```
tan(d*x+c)/a^2/d-I*tan(d*x+c)^2/a^2/d-1/2*I*tan(d*x+c)^4/a^2/d-1/5*tan(d*x
+c)^5/a^2/d
```

3.174.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.71

$$\int \frac{\sec^6(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = -\frac{\tan(c+dx)\left(-10+10i\tan(c+dx)+5i\tan^3(c+dx)+2\tan^4(c+dx)\right)}{10a^2d}$$

input
```
Integrate[Sec[c + d*x]^6/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

output
```
-1/10*(Tan[c + d*x]*(-10 + (10*I)*Tan[c + d*x] + (5*I)*Tan[c + d*x]^3 + 2*
Tan[c + d*x]^4))/(a^2*d)
```

3.174.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.79, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3042, 3567, 27, 516, 84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^6(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}dx$$

$$\Big\downarrow \textbf{3042}$$

$$\int \frac{1}{\cos(c+dx)^6(a\cos(c+dx)+ia\sin(c+dx))^2}dx$$

$$\Big\downarrow \textbf{3567}$$

$$-\frac{\int \frac{(\cot^2(c+dx)+1)^3\tan^6(c+dx)}{a^2(\cot(c+dx)+i)^2}d\cot(c+dx)}{d}$$

$$\Big\downarrow \textbf{27}$$

$$-\frac{\int \frac{(\cot^2(c+dx)+1)^3\tan^6(c+dx)}{(\cot(c+dx)+i)^2}d\cot(c+dx)}{a^2d}$$

$$\Big\downarrow \textbf{516}$$

$$-\frac{\int (\cot(c+dx)-i)^3(\cot(c+dx)+i)\tan^6(c+dx)d\cot(c+dx)}{a^2d}$$

$$\Big\downarrow \textbf{84}$$

$$-\frac{\int \left(-\tan^6(c+dx)-2i\tan^5(c+dx)-2i\tan^3(c+dx)+\tan^2(c+dx)\right)d\cot(c+dx)}{a^2d}$$

$$\Big\downarrow \textbf{2009}$$

$$-\frac{\frac{1}{5}\tan^5(c+dx)+\frac{1}{2}i\tan^4(c+dx)+i\tan^2(c+dx)-\tan(c+dx)}{a^2d}$$

input
```
Int[Sec[c + d*x]^6/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

output
```
-((-Tan[c + d*x] + I*Tan[c + d*x]^2 + (I/2)*Tan[c + d*x]^4 + Tan[c + d*x]^
5/5)/(a^2*d))
```

3.174.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 516
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[(e*x)^m*(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; Free
Q[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] ||
(GtQ[a, 0] && GtQ[c, 0] &&  !IntegerQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3567
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[x^m*((b
+ a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a,
b, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[
n, 0] && GtQ[m, 1])
```

3.174. $\int \frac{\sec^6(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$

3.174.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.51

method	result
risch	$\frac{8i\left(5\,\mathrm{e}^{2i(dx+c)}+1\right)}{5d\,a^{2}\left(\mathrm{e}^{2i(dx+c)}+1\right)^{5}}$
derivativedivides	$\frac{\tan(dx+c)-\frac{\tan(dx+c)^{5}}{5}-\frac{i\tan(dx+c)^{4}}{2}-i\tan(dx+c)^{2}}{d\,a^{2}}$
default	$\frac{\tan(dx+c)-\frac{\tan(dx+c)^{5}}{5}-\frac{i\tan(dx+c)^{4}}{2}-i\tan(dx+c)^{2}}{d\,a^{2}}$
norman	$\frac{\frac{4i\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{ad}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{ad}-\frac{28\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{5ad}+\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{ad}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}{ad}-\frac{4i\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{ad}+4i\,\mathrm{t}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{5}a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}$

input
```
int(sec(d*x+c)^6/(cos(d*x+c)*a+I*a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
8/5*I*(5*exp(2*I*(d*x+c))+1)/d/a^2/(exp(2*I*(d*x+c))+1)^5
```

3.174.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.39

$$\int \frac{\sec^6(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = -\frac{8\left(-5i\,e^{(2i\,dx+2i\,c)}-i\right)}{5\left(a^2de^{(10i\,dx+10i\,c)}+5\,a^2de^{(8i\,dx+8i\,c)}+10\,a^2de^{(6i\,dx+6i\,c)}+10\,a^2de^{(4i\,dx+4i\,c)}+5\,a^2de^{(2i\,dx+2i\,c)}+a^2d\right)}$$

input
```
integrate(sec(d*x+c)^6/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="frica
s")
```

output
```
-8/5*(-5*I*e^(2*I*d*x + 2*I*c) - I)/(a^2*d*e^(10*I*d*x + 10*I*c) + 5*a^2*d
*e^(8*I*d*x + 8*I*c) + 10*a^2*d*e^(6*I*d*x + 6*I*c) + 10*a^2*d*e^(4*I*d*x
+ 4*I*c) + 5*a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

3.174.6 Sympy [F]

$$\int \frac{\sec^6(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \frac{\int \frac{\sec^6(c+dx)}{-\sin^2(c+dx)+2i\sin(c+dx)\cos(c+dx)+\cos^2(c+dx)}\,dx}{a^2}$$

input
```
integrate(sec(d*x+c)**6/(a*cos(d*x+c)+I*a*sin(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)**6/(-sin(c + d*x)**2 + 2*I*sin(c + d*x)*cos(c + d*x)
 + cos(c + d*x)**2), x)/a**2
```

3.174.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.67

$$\int \frac{\sec^6(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = -\frac{2\tan(dx+c)^5+5i\tan(dx+c)^4+10i\tan(dx+c)^2-10\tan(dx+c)}{10\,a^2d}$$

input
```
integrate(sec(d*x+c)^6/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="maxim
a")
```

output
```
-1/10*(2*tan(d*x + c)^5 + 5*I*tan(d*x + c)^4 + 10*I*tan(d*x + c)^2 - 10*ta
n(d*x + c))/(a^2*d)
```

3.174.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.67

$$\int \frac{\sec^6(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = -\frac{2\tan(dx+c)^5+5i\tan(dx+c)^4+10i\tan(dx+c)^2-10\tan(dx+c)}{10\,a^2d}$$

input
```
integrate(sec(d*x+c)^6/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="giac"
)
```

output
```
-1/10*(2*tan(d*x + c)^5 + 5*I*tan(d*x + c)^4 + 10*I*tan(d*x + c)^2 - 10*ta
n(d*x + c))/(a^2*d)
```

3.174.9 Mupad [B] (verification not implemented)

Time = 23.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.09

$$\int \frac{\sec^6(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = -\frac{\sin(c+dx)\left(-4\cos(c+dx)^4+\frac{5\mathrm{i}\sin(c+dx)\cos(c+dx)^3}{2}-2\cos(c+dx)^2+\frac{5\mathrm{i}\sin(c+dx)\cos(c+dx)}{2}+1\right)}{5a^2d\cos(c+dx)^5}$$

input
```
int(1/(cos(c + d*x)^6*(a*cos(c + d*x) + a*sin(c + d*x)*1i)^2),x)
```

output
```
-(sin(c + d*x)*((cos(c + d*x)*sin(c + d*x)*5i)/2 + (cos(c + d*x)^3*sin(c +
 d*x)*5i)/2 - 2*cos(c + d*x)^2 - 4*cos(c + d*x)^4 + 1))/(5*a^2*d*cos(c + d
*x)^5)
```

3.175 $\int \frac{\cos^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$

3.175.1 Optimal result

Integrand size = 31, antiderivative size = 125

$$\int \frac{\cos^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{5x}{32a^3} - \frac{1}{32a^3d(i-\cot(c+dx))} + \frac{i}{16a^3d(i+\cot(c+dx))^4} - \frac{1}{3a^3d(i+\cot(c+dx))^3} - \frac{23i}{32a^3d(i+\cot(c+dx))^2} + \frac{13}{16a^3d(i+\cot(c+dx))}$$

output
```
5/32*x/a^3-1/32/a^3/d/(I-cot(d*x+c))+1/16*I/a^3/d/(I+cot(d*x+c))^4-1/3/a^3
/d/(I+cot(d*x+c))^3-23/32*I/a^3/d/(I+cot(d*x+c))^2+13/16/a^3/d/(I+cot(d*x+
c))
```

3.175.2 Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.85

$$\int \frac{\cos^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{120c+120dx+108i\cos(2(c+dx))+60i\cos(4(c+dx))+20i\cos(6(c+dx))+3i\cos(8(c+dx))+132\,\mathrm{s}}{768a^3d}$$

input
```
Integrate[Cos[c + d*x]^5/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

output
```
(120*c + 120*d*x + (108*I)*Cos[2*(c + d*x)] + (60*I)*Cos[4*(c + d*x)] + (2
0*I)*Cos[6*(c + d*x)] + (3*I)*Cos[8*(c + d*x)] + 132*Sin[2*(c + d*x)] + 60
*Sin[4*(c + d*x)] + 20*Sin[6*(c + d*x)] + 3*Sin[8*(c + d*x)])/(768*a^3*d)
```

3.175.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.85, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3042, 3567, 27, 516, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(c+dx)^5}{(a\cos(c+dx)+ia\sin(c+dx))^3}dx$$

$$\Big\downarrow \text{3567}$$

$$-\frac{\int \frac{\cot^5(c+dx)}{a^3(\cot(c+dx)+i)^3(\cot^2(c+dx)+1)^2}d\cot(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\cot^5(c+dx)}{(\cot(c+dx)+i)^3(\cot^2(c+dx)+1)^2}d\cot(c+dx)}{a^3d}$$

$$\Big\downarrow \text{516}$$

$$-\frac{\int \frac{\cot^5(c+dx)}{(\cot(c+dx)-i)^2(\cot(c+dx)+i)^5}d\cot(c+dx)}{a^3d}$$

$$\Big\downarrow \text{99}$$

$$-\frac{\int \left(\frac{1}{32(\cot(c+dx)-i)^2}+\frac{13}{16(\cot(c+dx)+i)^2}-\frac{23i}{16(\cot(c+dx)+i)^3}-\frac{1}{(\cot(c+dx)+i)^4}+\frac{i}{4(\cot(c+dx)+i)^5}+\frac{5}{32(\cot^2(c+dx)+1)}\right)d\cot(c+dx)}{a^3d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{5}{32}\arctan(\cot(c+dx))+\frac{1}{32(-\cot(c+dx)+i)}-\frac{13}{16(\cot(c+dx)+i)}+\frac{23i}{32(\cot(c+dx)+i)^2}+\frac{1}{3(\cot(c+dx)+i)^3}-\frac{i}{16(\cot(c+dx)+i)^4}}{a^3d}$$

input
```
Int[Cos[c + d*x]^5/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

output
```
-(((5*ArcTan[Cot[c + d*x]])/32 + 1/(32*(I - Cot[c + d*x])) - (I/16)/(I + C
ot[c + d*x])^4 + 1/(3*(I + Cot[c + d*x])^3) + ((23*I)/32)/(I + Cot[c + d*x
])^2 - 13/(16*(I + Cot[c + d*x])))/(a^3*d))
```

3.175.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 516
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[(e*x)^m*(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; Free
Q[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] ||
(GtQ[a, 0] && GtQ[c, 0] &&  !IntegerQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3567
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[x^m*((b
+ a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a,
b, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[
n, 0] && GtQ[m, 1])
```

3.175.4 Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.78

method	result
risch	$\frac{5x}{32a^3}+\frac{5i\mathrm{e}^{-4i(dx+c)}}{64d\,a^3}+\frac{5i\mathrm{e}^{-6i(dx+c)}}{192d\,a^3}+\frac{i\mathrm{e}^{-8i(dx+c)}}{256d\,a^3}+\frac{9i\cos(2dx+2c)}{64d\,a^3}+\frac{11\sin(2dx+2c)}{64d\,a^3}$
derivativedivides	$\frac{\frac{5i\ln(\tan(dx+c)+i)}{64}+\frac{1}{32\tan(dx+c)+32i}-\frac{5i\ln(\tan(dx+c)-i)}{64}+\frac{i}{16(\tan(dx+c)-i)^4}-\frac{3i}{32(\tan(dx+c)-i)^2}-\frac{1}{12(\tan(dx+c)-i)^3}+\frac{}{8\tan}}{d\,a^3}$
default	$\frac{\frac{5i\ln(\tan(dx+c)+i)}{64}+\frac{1}{32\tan(dx+c)+32i}-\frac{5i\ln(\tan(dx+c)-i)}{64}+\frac{i}{16(\tan(dx+c)-i)^4}-\frac{3i}{32(\tan(dx+c)-i)^2}-\frac{1}{12(\tan(dx+c)-i)^3}+\frac{}{8\tan}}{d\,a^3}$

input
```
int(cos(d*x+c)^5/(cos(d*x+c)*a+I*a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
5/32*x/a^3+5/64*I/d/a^3*exp(-4*I*(d*x+c))+5/192*I/d/a^3*exp(-6*I*(d*x+c))+
1/256*I/d/a^3*exp(-8*I*(d*x+c))+9/64*I/d/a^3*cos(2*d*x+2*c)+11/64/d/a^3*si
n(2*d*x+2*c)
```

3.175.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.61

$$\int \frac{\cos^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$$
$$=\frac{\left(120\,dxe^{(8i\,dx+8i\,c)}-12i\,e^{(10i\,dx+10i\,c)}+120i\,e^{(6i\,dx+6i\,c)}+60i\,e^{(4i\,dx+4i\,c)}+20i\,e^{(2i\,dx+2i\,c)}+3i\right)e^{(-8i\,dx-8i\,c)}}{768\,a^3d}$$

input
```
integrate(cos(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="frica
s")
```

output
```
1/768*(120*d*x*e^(8*I*d*x + 8*I*c) - 12*I*e^(10*I*d*x + 10*I*c) + 120*I*e^
(6*I*d*x + 6*I*c) + 60*I*e^(4*I*d*x + 4*I*c) + 20*I*e^(2*I*d*x + 2*I*c) +
3*I)*e^(-8*I*d*x - 8*I*c)/(a^3*d)
```

3.175.6 Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.79

$$\int \frac{\cos^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \begin{cases} \frac{(-100663296ia^{12}d^4e^{22ic}e^{2idx}+1006632960ia^{12}d^4e^{18ic}e^{-2idx}+503316480ia^{12}d^4e^{16ic}e^{-4idx}+167772160ia^{12}d^4e^{14ic}e^{-6idx}+25165824ia^{12}d^4 \cdots}{6442450944a^{15}d^5} \\ x\left(\frac{(e^{10ic}+5e^{8ic}+10e^{6ic}+10e^{4ic}+5e^{2ic}+1)e^{-8ic}}{32a^3}-\frac{5}{32a^3}\right) \end{cases} + \frac{5x}{32a^3}$$

input
```
integrate(cos(d*x+c)**5/(a*cos(d*x+c)+I*a*sin(d*x+c))**3,x)
```

output
```
Piecewise(((-100663296*I*a**12*d**4*exp(22*I*c)*exp(2*I*d*x) + 1006632960*
I*a**12*d**4*exp(18*I*c)*exp(-2*I*d*x) + 503316480*I*a**12*d**4*exp(16*I*c
)*exp(-4*I*d*x) + 167772160*I*a**12*d**4*exp(14*I*c)*exp(-6*I*d*x) + 25165
824*I*a**12*d**4*exp(12*I*c)*exp(-8*I*d*x))*exp(-20*I*c)/(6442450944*a**15
*d**5), Ne(a**15*d**5*exp(20*I*c), 0)), (x*((exp(10*I*c) + 5*exp(8*I*c) +
10*exp(6*I*c) + 10*exp(4*I*c) + 5*exp(2*I*c) + 1)*exp(-8*I*c)/(32*a**3) -
5/(32*a**3)), True)) + 5*x/(32*a**3)
```

3.175.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="maxim
a")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.175.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.92

$$\int \frac{\cos^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx =$$
$$-\frac{-\frac{60i\log(\tan(dx+c)+i)}{a^3}+\frac{60i\log(\tan(dx+c)-i)}{a^3}-\frac{12(5\tan(dx+c)+7i)}{a^3(i\tan(dx+c)-1)}+\frac{-125i\tan(dx+c)^4-596\tan(dx+c)^3+1110i\tan(dx+c)^2+9}{a^3(\tan(dx+c)-i)^4}}{768\,d}$$

input
```
integrate(cos(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="giac"
)
```

output
```
-1/768*(-60*I*log(tan(d*x + c) + I)/a^3 + 60*I*log(tan(d*x + c) - I)/a^3 -
 12*(5*tan(d*x + c) + 7*I)/(a^3*(I*tan(d*x + c) - 1)) + (-125*I*tan(d*x +
c)^4 - 596*tan(d*x + c)^3 + 1110*I*tan(d*x + c)^2 + 996*tan(d*x + c) - 405
*I)/(a^3*(tan(d*x + c) - I)^4))/d
```

3.175.9 Mupad [B] (verification not implemented)

Time = 27.76 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.31

$$\int \frac{\cos^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{5\,x}{32\,a^3}$$
$$+\frac{-\frac{27\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9}{16}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8 33\mathrm{i}}{8}+\frac{31\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{6}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 9\mathrm{i}}{8}+\frac{89\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{24}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 9\mathrm{i}}{8}+\frac{31\tan\left(\frac{c}{2}+\right.}{6}}{a^3\,d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\mathrm{i}\right)^2\left(1+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,1\mathrm{i}\right)^8}$$

input
```
int(cos(c + d*x)^5/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^3,x)
```

output
```
(5*x)/(32*a^3) + ((31*tan(c/2 + (d*x)/2)^3)/6 - (tan(c/2 + (d*x)/2)^2*33i)
/8 - (27*tan(c/2 + (d*x)/2))/16 - (tan(c/2 + (d*x)/2)^4*9i)/8 + (89*tan(c/
2 + (d*x)/2)^5)/24 + (tan(c/2 + (d*x)/2)^6*9i)/8 + (31*tan(c/2 + (d*x)/2)^
7)/6 + (tan(c/2 + (d*x)/2)^8*33i)/8 - (27*tan(c/2 + (d*x)/2)^9)/16)/(a^3*d
*(tan(c/2 + (d*x)/2) + 1i)^2*(tan(c/2 + (d*x)/2)*1i + 1)^8)
```

3.176 $\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$

3.176.1 Optimal result

Integrand size = 31, antiderivative size = 106

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = -\frac{i\cos^5(c+dx)}{5a^3d}+\frac{4i\cos^7(c+dx)}{7a^3d}+\frac{\sin(c+dx)}{a^3d}-\frac{2\sin^3(c+dx)}{a^3d}+\frac{9\sin^5(c+dx)}{5a^3d}-\frac{4\sin^7(c+dx)}{7a^3d}$$

output
```
-1/5*I*cos(d*x+c)^5/a^3/d+4/7*I*cos(d*x+c)^7/a^3/d+sin(d*x+c)/a^3/d-2*sin(
d*x+c)^3/a^3/d+9/5*sin(d*x+c)^5/a^3/d-4/7*sin(d*x+c)^7/a^3/d
```

3.176.2 Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.41

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{3i\cos(c+dx)}{16a^3d}+\frac{i\cos(3(c+dx))}{8a^3d}+\frac{i\cos(5(c+dx))}{20a^3d}+\frac{i\cos(7(c+dx))}{112a^3d}+\frac{5\sin(c+dx)}{16a^3d}+\frac{\sin(3(c+dx))}{8a^3d}+\frac{\sin(5(c+dx))}{20a^3d}+\frac{\sin(7(c+dx))}{112a^3d}$$

input
```
Integrate[Cos[c + d*x]^4/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

output
```
(((3*I)/16)*Cos[c + d*x])/(a^3*d) + ((I/8)*Cos[3*(c + d*x)])/(a^3*d) + ((I
/20)*Cos[5*(c + d*x)])/(a^3*d) + ((I/112)*Cos[7*(c + d*x)])/(a^3*d) + (5*S
in[c + d*x])/(16*a^3*d) + Sin[3*(c + d*x)]/(8*a^3*d) + Sin[5*(c + d*x)]/(2
0*a^3*d) + Sin[7*(c + d*x)]/(112*a^3*d)
```

3.176.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3042, 3571, 3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(c+dx)^4}{(a\cos(c+dx)+ia\sin(c+dx))^3}dx$$

$$\Big\downarrow \text{3571}$$

$$\frac{i\int \cos^4(c+dx)(ia\cos(c+dx)+a\sin(c+dx))^3dx}{a^6}$$

$$\Big\downarrow \text{3042}$$

$$\frac{i\int \cos(c+dx)^4(ia\cos(c+dx)+a\sin(c+dx))^3dx}{a^6}$$

$$\Big\downarrow \text{3569}$$

$$\frac{i\int \left(-ia^3\cos^7(c+dx)-3a^3\sin(c+dx)\cos^6(c+dx)+3ia^3\sin^2(c+dx)\cos^5(c+dx)+a^3\sin^3(c+dx)\cos^4(c+d$$
$$a^6$$

$$\Big\downarrow \text{2009}$$

$$\frac{i\left(\frac{4ia^3\sin^7(c+dx)}{7d}-\frac{9ia^3\sin^5(c+dx)}{5d}+\frac{2ia^3\sin^3(c+dx)}{d}-\frac{ia^3\sin(c+dx)}{d}+\frac{4a^3\cos^7(c+dx)}{7d}-\frac{a^3\cos^5(c+dx)}{5d}\right)}{a^6}$$

input
```
Int[Cos[c + d*x]^4/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

output
```
(I*(-1/5*(a^3*Cos[c + d*x]^5)/d + (4*a^3*Cos[c + d*x]^7)/(7*d) - (I*a^3*Si
n[c + d*x])/d + ((2*I)*a^3*Sin[c + d*x]^3)/d - (((9*I)/5)*a^3*Sin[c + d*x]
^5)/d + (((4*I)/7)*a^3*Sin[c + d*x]^7)/d))/a^6
```

3.176.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

rule 3571
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^n*b^n   Int[Cos[c + d*x]^m
/(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &
& EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

3.176.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.80

method	result
risch	$\frac{i\mathrm{e}^{-3 i (d x +c)}}{8 d \,a^{3}}+\frac{i\mathrm{e}^{-5 i (d x +c)}}{20 d \,a^{3}}+\frac{i\mathrm{e}^{-7 i (d x +c)}}{112 d \,a^{3}}+\frac{3 i \cos \left(d x +c \right)}{16 d \,a^{3}}+\frac{5 \sin \left(d x +c \right)}{16 a^{3} d}$
derivativedivides	$\frac{\frac{2}{16 \tan \left(\frac{d x}{2}+\frac{c}{2}\right)+16 i}+\frac{4 i}{\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-i \right)^{6}}-\frac{9 i}{\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-i \right)^{4}}+\frac{17 i}{4 \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-i \right)^{2}}-\frac{8}{7 \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-i \right)^{7}}+\frac{38}{5 \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-i \right)}}{d \,a^{3}}$
default	$\frac{\frac{2}{16 \tan \left(\frac{d x}{2}+\frac{c}{2}\right)+16 i}+\frac{4 i}{\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-i \right)^{6}}-\frac{9 i}{\left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-i \right)^{4}}+\frac{17 i}{4 \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-i \right)^{2}}-\frac{8}{7 \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-i \right)^{7}}+\frac{38}{5 \left(\tan \left(\frac{d x}{2}+\frac{c}{2}\right)-i \right)}}{d \,a^{3}}$

input
```
int(cos(d*x+c)^4/(cos(d*x+c)*a+I*a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*I/d/a^3*exp(-3*I*(d*x+c))+1/20*I/d/a^3*exp(-5*I*(d*x+c))+1/112*I/d/a^3
*exp(-7*I*(d*x+c))+3/16*I/d/a^3*cos(d*x+c)+5/16*sin(d*x+c)/a^3/d
```

3.176.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.59

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{\left(-35i\,e^{(8i\,dx+8i\,c)}+140i\,e^{(6i\,dx+6i\,c)}+70i\,e^{(4i\,dx+4i\,c)}+28i\,e^{(2i\,dx+2i\,c)}+5i\right)e^{(-7i\,dx-7i\,c)}}{560\,a^3d}$$

input
```
integrate(cos(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="frica
s")
```

output
```
1/560*(-35*I*e^(8*I*d*x + 8*I*c) + 140*I*e^(6*I*d*x + 6*I*c) + 70*I*e^(4*I
*d*x + 4*I*c) + 28*I*e^(2*I*d*x + 2*I*c) + 5*I)*e^(-7*I*d*x - 7*I*c)/(a^3*
d)
```

3.176.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(95) = 190$.

Time = 0.26 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.86

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \begin{cases} \frac{(-71680ia^{12}d^4e^{17ic}e^{idx}+286720ia^{12}d^4e^{15ic}e^{-idx}+143360ia^{12}d^4e^{13ic}e^{-3idx}+57344ia^{12}d^4e^{11ic}e^{-5idx}+10240ia^{12}d^4e^{9ic}e^{-7idx})e^{-16ic}}{1146880a^{15}d^5} & \text{fo} \\ \frac{x(e^{8ic}+4e^{6ic}+6e^{4ic}+4e^{2ic}+1)e^{-7ic}}{16a^3} & \text{o} \end{cases}$$

input
```
integrate(cos(d*x+c)**4/(a*cos(d*x+c)+I*a*sin(d*x+c))**3,x)
```

output
```
Piecewise(((-71680*I*a**12*d**4*exp(17*I*c)*exp(I*d*x) + 286720*I*a**12*d*
*4*exp(15*I*c)*exp(-I*d*x) + 143360*I*a**12*d**4*exp(13*I*c)*exp(-3*I*d*x)
 + 57344*I*a**12*d**4*exp(11*I*c)*exp(-5*I*d*x) + 10240*I*a**12*d**4*exp(9
*I*c)*exp(-7*I*d*x))*exp(-16*I*c)/(1146880*a**15*d**5), Ne(a**15*d**5*exp(
16*I*c), 0)), (x*(exp(8*I*c) + 4*exp(6*I*c) + 6*exp(4*I*c) + 4*exp(2*I*c)
+ 1)*exp(-7*I*c)/(16*a**3), True))
```

3.176.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="maxim
a")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.176.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.12

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{\frac{35}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)} + \frac{525\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-1960i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-4025\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+4480i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3143\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^7}}{280\,d}$$

input
```
integrate(cos(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="giac"
)
```

output
```
1/280*(35/(a^3*(tan(1/2*d*x + 1/2*c) + I)) + (525*tan(1/2*d*x + 1/2*c)^6 -
 1960*I*tan(1/2*d*x + 1/2*c)^5 - 4025*tan(1/2*d*x + 1/2*c)^4 + 4480*I*tan(
1/2*d*x + 1/2*c)^3 + 3143*tan(1/2*d*x + 1/2*c)^2 - 1176*I*tan(1/2*d*x + 1/
2*c) - 243)/(a^3*(tan(1/2*d*x + 1/2*c) - I)^7))/d
```

3.176.9 Mupad [B] (verification not implemented)

Time = 25.11 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.26

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = -\frac{\left(35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 105\mathrm{i}-175\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 105\mathrm{i}-7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+\mathrm{ta}\right.}{35\,a^3\,d\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\mathrm{i}\right)\,\left(1+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,1\mathrm{i}\right)^7}$$

input
```
int(cos(c + d*x)^4/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^3,x)
```

output
```
-((43*tan(c/2 + (d*x)/2) + tan(c/2 + (d*x)/2)^2*77i - 7*tan(c/2 + (d*x)/2)
^3 + tan(c/2 + (d*x)/2)^4*105i - 175*tan(c/2 + (d*x)/2)^5 - tan(c/2 + (d*x
)/2)^6*105i + 35*tan(c/2 + (d*x)/2)^7 - 13i)*2i)/(35*a^3*d*(tan(c/2 + (d*x
)/2) + 1i)*(tan(c/2 + (d*x)/2)*1i + 1)^7)
```

3.177 $\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$

3.177.1 Optimal result

Integrand size = 31, antiderivative size = 131

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{x}{8a^3} + \frac{i\cos^3(c+dx)}{6d(a\cos(c+dx)+ia\sin(c+dx))^3} + \frac{i\cos^2(c+dx)}{8ad(a\cos(c+dx)+ia\sin(c+dx))^2} + \frac{i\cos(c+dx)}{8d\left(a^3\cos(c+dx)+ia^3\sin(c+dx)\right)}$$

output
```
1/8*x/a^3+1/6*I*cos(d*x+c)^3/d/(a*cos(d*x+c)+I*a*sin(d*x+c))^3+1/8*I*cos(d
*x+c)^2/a/d/(a*cos(d*x+c)+I*a*sin(d*x+c))^2+1/8*I*cos(d*x+c)/d/(a^3*cos(d*
x+c)+I*a^3*sin(d*x+c))
```

3.177.2 Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.64

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{12c+12dx+18i\cos(2(c+dx))+9i\cos(4(c+dx))+2i\cos(6(c+dx))+18\sin(2(c+dx))+9\sin(4(c+}{96a^3d}$$

input
```
Integrate[Cos[c + d*x]^3/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

output
```
(12*c + 12*d*x + (18*I)*Cos[2*(c + d*x)] + (9*I)*Cos[4*(c + d*x)] + (2*I)*
Cos[6*(c + d*x)] + 18*Sin[2*(c + d*x)] + 9*Sin[4*(c + d*x)] + 2*Sin[6*(c +
 d*x)])/(96*a^3*d)
```

3.177.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {3042, 3561, 3042, 3561, 3042, 3561, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(c+dx)^3}{(a\cos(c+dx)+ia\sin(c+dx))^3}dx$$

$$\Big\downarrow \text{3561}$$

$$\frac{\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}dx}{2a}+\frac{i\cos^3(c+dx)}{6d(a\cos(c+dx)+ia\sin(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\cos(c+dx)^2}{(a\cos(c+dx)+ia\sin(c+dx))^2}dx}{2a}+\frac{i\cos^3(c+dx)}{6d(a\cos(c+dx)+ia\sin(c+dx))^3}$$

$$\Big\downarrow \text{3561}$$

$$\frac{\frac{\int \frac{\cos(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}dx}{2a}+\frac{i\cos^2(c+dx)}{4d(a\cos(c+dx)+ia\sin(c+dx))^2}}{2a}+\frac{i\cos^3(c+dx)}{6d(a\cos(c+dx)+ia\sin(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{\cos(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}dx}{2a}+\frac{i\cos^2(c+dx)}{4d(a\cos(c+dx)+ia\sin(c+dx))^2}}{2a}+\frac{i\cos^3(c+dx)}{6d(a\cos(c+dx)+ia\sin(c+dx))^3}$$

$$\Big\downarrow \text{3561}$$

$$\frac{\frac{\frac{\int 1dx}{2a}+\frac{i\cos(c+dx)}{2d(a\cos(c+dx)+ia\sin(c+dx))}}{2a}+\frac{i\cos^2(c+dx)}{4d(a\cos(c+dx)+ia\sin(c+dx))^2}}{2a}+\frac{i\cos^3(c+dx)}{6d(a\cos(c+dx)+ia\sin(c+dx))^3}$$

$$\downarrow 24$$

$$\frac{i\cos^3(c+dx)}{6d(a\cos(c+dx)+ia\sin(c+dx))^3}+\frac{\frac{i\cos^2(c+dx)}{4d(a\cos(c+dx)+ia\sin(c+dx))^2}+\frac{\frac{x}{2a}+\frac{i\cos(c+dx)}{2d(a\cos(c+dx)+ia\sin(c+dx))}}{2a}}{2a}$$

input
```
Int[Cos[c + d*x]^3/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

output
```
((I/6)*Cos[c + d*x]^3)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3) + (((I/4)
*Cos[c + d*x]^2)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2) + (x/(2*a) + ((
I/2)*Cos[c + d*x])/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])))/(2*a))/(2*a)
```

3.177.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3561
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*((a*Cos[c + d*x] + b*Si
n[c + d*x])^n/(2*a*d*n*Cos[c + d*x]^n)), x] + Simp[1/(2*a)   Int[(a*Cos[c +
 d*x] + b*Sin[c + d*x])^(n + 1)/Cos[c + d*x]^(n + 1), x], x] /; FreeQ[{a, b
, c, d}, x] && EqQ[m + n, 0] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

3.177.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.47

method	result	size
risch	$\frac{x}{8a^3}+\frac{3i e^{-2i(dx+c)}}{16d\,a^3}+\frac{3i e^{-4i(dx+c)}}{32d\,a^3}+\frac{i e^{-6i(dx+c)}}{48d\,a^3}$	62
derivativedivides	$\frac{\frac{i\ln(\tan(dx+c)+i)}{16}-\frac{i\ln(\tan(dx+c)-i)}{16}-\frac{i}{8(\tan(dx+c)-i)^2}-\frac{1}{6(\tan(dx+c)-i)^3}+\frac{1}{8\tan(dx+c)-8i}}{d\,a^3}$	75
default	$\frac{\frac{i\ln(\tan(dx+c)+i)}{16}-\frac{i\ln(\tan(dx+c)-i)}{16}-\frac{i}{8(\tan(dx+c)-i)^2}-\frac{1}{6(\tan(dx+c)-i)^3}+\frac{1}{8\tan(dx+c)-8i}}{d\,a^3}$	75

input
```
int(cos(d*x+c)^3/(cos(d*x+c)*a+I*a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*x/a^3+3/16*I/d/a^3*exp(-2*I*(d*x+c))+3/32*I/d/a^3*exp(-4*I*(d*x+c))+1/
48*I/d/a^3*exp(-6*I*(d*x+c))
```

3.177.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.41

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{\left(12\,dxe^{(6i\,dx+6i\,c)}+18i\,e^{(4i\,dx+4i\,c)}+9i\,e^{(2i\,dx+2i\,c)}+2i\right)e^{(-6i\,dx-6i\,c)}}{96\,a^3d}$$

input
```
integrate(cos(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="frica
s")
```

output
```
1/96*(12*d*x*e^(6*I*d*x + 6*I*c) + 18*I*e^(4*I*d*x + 4*I*c) + 9*I*e^(2*I*d
*x + 2*I*c) + 2*I)*e^(-6*I*d*x - 6*I*c)/(a^3*d)
```

3.177.6 Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.18

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \begin{cases} \frac{(4608ia^6d^2e^{10ic}e^{-2idx}+2304ia^6d^2e^{8ic}e^{-4idx}+512ia^6d^2e^{6ic}e^{-6idx})e^{-12ic}}{24576a^9d^3} & \text{for } a^9d^3e^{12ic}\neq 0 \\ x\left(\frac{(e^{6ic}+3e^{4ic}+3e^{2ic}+1)e^{-6ic}}{8a^3}-\frac{1}{8a^3}\right) & \text{otherwise} \end{cases} + \frac{x}{8a^3}$$

input
```
integrate(cos(d*x+c)**3/(a*cos(d*x+c)+I*a*sin(d*x+c))**3,x)
```

output
```
Piecewise(((4608*I*a**6*d**2*exp(10*I*c)*exp(-2*I*d*x) + 2304*I*a**6*d**2*
exp(8*I*c)*exp(-4*I*d*x) + 512*I*a**6*d**2*exp(6*I*c)*exp(-6*I*d*x))*exp(-
12*I*c)/(24576*a**9*d**3), Ne(a**9*d**3*exp(12*I*c), 0)), (x*((exp(6*I*c)
+ 3*exp(4*I*c) + 3*exp(2*I*c) + 1)*exp(-6*I*c)/(8*a**3) - 1/(8*a**3)), Tru
e)) + x/(8*a**3)
```

3.177.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="maxim
a")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.177.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.60

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$$
$$= -\frac{-\frac{6i\log(\tan(dx+c)+i)}{a^3}+\frac{6i\log(\tan(dx+c)-i)}{a^3}+\frac{-11i\tan(dx+c)^3-45\tan(dx+c)^2+69i\tan(dx+c)+51}{a^3(\tan(dx+c)-i)^3}}{96\,d}$$

input
```
integrate(cos(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="giac"
)
```

output
```
-1/96*(-6*I*log(tan(d*x + c) + I)/a^3 + 6*I*log(tan(d*x + c) - I)/a^3 + (-
11*I*tan(d*x + c)^3 - 45*tan(d*x + c)^2 + 69*I*tan(d*x + c) + 51)/(a^3*(ta
n(d*x + c) - I)^3))/d
```

3.177.9 Mupad [B] (verification not implemented)

Time = 25.84 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.73

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$$
$$= \frac{x}{8\,a^3}+\frac{\frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,9{}\mathrm{i}}{2}-\frac{41\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{6}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,9{}\mathrm{i}}{2}+\frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{a^3\,d\,\left(1+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}\right)^6}$$

input
```
int(cos(c + d*x)^3/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^3,x)
```

output
```
x/(8*a^3) + ((7*tan(c/2 + (d*x)/2))/4 + (tan(c/2 + (d*x)/2)^2*9i)/2 - (41*
tan(c/2 + (d*x)/2)^3)/6 - (tan(c/2 + (d*x)/2)^4*9i)/2 + (7*tan(c/2 + (d*x)
/2)^5)/4)/(a^3*d*(tan(c/2 + (d*x)/2)*1i + 1)^6)
```

3.177. $\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$

3.178 $\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$

3.178.1 Optimal result

Integrand size = 31, antiderivative size = 90

$$\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = -\frac{i\cos^3(c+dx)}{3a^3d}+\frac{4i\cos^5(c+dx)}{5a^3d}+\frac{\sin(c+dx)}{a^3d}-\frac{5\sin^3(c+dx)}{3a^3d}+\frac{4\sin^5(c+dx)}{5a^3d}$$

output
```
-1/3*I*cos(d*x+c)^3/a^3/d+4/5*I*cos(d*x+c)^5/a^3/d+sin(d*x+c)/a^3/d-5/3*si
n(d*x+c)^3/a^3/d+4/5*sin(d*x+c)^5/a^3/d
```

3.178.2 Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.23

$$\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{i\cos(c+dx)}{4a^3d}+\frac{i\cos(3(c+dx))}{6a^3d}+\frac{i\cos(5(c+dx))}{20a^3d}+\frac{\sin(c+dx)}{4a^3d}+\frac{\sin(3(c+dx))}{6a^3d}+\frac{\sin(5(c+dx))}{20a^3d}$$

input
```
Integrate[Cos[c + d*x]^2/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

output
```
((I/4)*Cos[c + d*x])/(a^3*d) + ((I/6)*Cos[3*(c + d*x)])/(a^3*d) + ((I/20)*
Cos[5*(c + d*x)])/(a^3*d) + Sin[c + d*x]/(4*a^3*d) + Sin[3*(c + d*x)]/(6*a
^3*d) + Sin[5*(c + d*x)]/(20*a^3*d)
```

3.178.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3042, 3571, 3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(c+dx)^2}{(a\cos(c+dx)+ia\sin(c+dx))^3}dx$$

$$\Big\downarrow \text{3571}$$

$$\frac{i\int \cos^2(c+dx)(ia\cos(c+dx)+a\sin(c+dx))^3dx}{a^6}$$

$$\Big\downarrow \text{3042}$$

$$\frac{i\int \cos(c+dx)^2(ia\cos(c+dx)+a\sin(c+dx))^3dx}{a^6}$$

$$\Big\downarrow \text{3569}$$

$$\frac{i\int \left(-ia^3\cos^5(c+dx)-3a^3\sin(c+dx)\cos^4(c+dx)+3ia^3\sin^2(c+dx)\cos^3(c+dx)+a^3\sin^3(c+dx)\cos^2(c+d$$

$$\Big\downarrow \text{2009}$$

$$\frac{i\left(-\frac{4ia^3\sin^5(c+dx)}{5d}+\frac{5ia^3\sin^3(c+dx)}{3d}-\frac{ia^3\sin(c+dx)}{d}+\frac{4a^3\cos^5(c+dx)}{5d}-\frac{a^3\cos^3(c+dx)}{3d}\right)}{a^6}$$

input
```
Int[Cos[c + d*x]^2/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

output
```
(I*(-1/3*(a^3*Cos[c + d*x]^3)/d + (4*a^3*Cos[c + d*x]^5)/(5*d) - (I*a^3*Si
n[c + d*x])/d + (((5*I)/3)*a^3*Sin[c + d*x]^3)/d - (((4*I)/5)*a^3*Sin[c +
d*x]^5)/d))/a^6
```

3.178.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

rule 3571
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^n*b^n   Int[Cos[c + d*x]^m
/(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &
& EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

3.178.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.62

method	result
risch	$\frac{i e^{-i(dx+c)}}{4d\,a^3}+\frac{i e^{-3i(dx+c)}}{6d\,a^3}+\frac{i e^{-5i(dx+c)}}{20d\,a^3}$
derivativedivides	$\frac{\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i}-\frac{4i}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^4}+\frac{4i}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^2}+\frac{8}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^5}-\frac{16}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^3}}{d\,a^3}$
default	$\frac{\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i}-\frac{4i}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^4}+\frac{4i}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^2}+\frac{8}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^5}-\frac{16}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^3}}{d\,a^3}$
norman	$\frac{\frac{6i\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{ad}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3ad}+\frac{164\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{15ad}-\frac{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{3ad}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{ad}+\frac{14i}{15ad}-\frac{4i\tan\left(\frac{dx}{2}+\right.}{3ad}}{a^2\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^5}$

input
```
int(cos(d*x+c)^2/(cos(d*x+c)*a+I*a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*I/d/a^3*exp(-I*(d*x+c))+1/6*I/d/a^3*exp(-3*I*(d*x+c))+1/20*I/d/a^3*exp
(-5*I*(d*x+c))
```

3.178.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.46

$$\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{\left(15i\,e^{(4i\,dx+4i\,c)}+10i\,e^{(2i\,dx+2i\,c)}+3i\right)e^{(-5i\,dx-5i\,c)}}{60\,a^3d}$$

input
```
integrate(cos(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="frica
s")
```

output
```
1/60*(15*I*e^(4*I*d*x + 4*I*c) + 10*I*e^(2*I*d*x + 2*I*c) + 3*I)*e^(-5*I*d
*x - 5*I*c)/(a^3*d)
```

3.178.6 Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.46

$$\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \begin{cases} \frac{(120ia^6d^2e^{8ic}e^{-idx}+80ia^6d^2e^{6ic}e^{-3idx}+24ia^6d^2e^{4ic}e^{-5idx})e^{-9ic}}{480a^9d^3} & \text{for } a^9d^3e^{9ic}\neq 0 \\ \frac{x(e^{4ic}+2e^{2ic}+1)e^{-5ic}}{4a^3} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)**2/(a*cos(d*x+c)+I*a*sin(d*x+c))**3,x)
```

output
```
Piecewise(((120*I*a**6*d**2*exp(8*I*c)*exp(-I*d*x) + 80*I*a**6*d**2*exp(6*
I*c)*exp(-3*I*d*x) + 24*I*a**6*d**2*exp(4*I*c)*exp(-5*I*d*x))*exp(-9*I*c)/
(480*a**9*d**3), Ne(a**9*d**3*exp(9*I*c), 0)), (x*(exp(4*I*c) + 2*exp(2*I*
c) + 1)*exp(-5*I*c)/(4*a**3), True))
```

3.178.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.77

$$\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{3i\,\cos(5\,dx+5\,c)+10i\,\cos(3\,dx+3\,c)+15i\,\cos(dx+c)+3\,\sin(5\,dx+5\,c)+10\,\sin(3\,dx+3\,c)+15}{60\,a^3d}$$

input
```
integrate(cos(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="maxim
a")
```

output
```
1/60*(3*I*cos(5*d*x + 5*c) + 10*I*cos(3*d*x + 3*c) + 15*I*cos(d*x + c) + 3
*sin(5*d*x + 5*c) + 10*sin(3*d*x + 3*c) + 15*sin(d*x + c))/(a^3*d)
```

3.178.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.81

$$\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{2\left(15\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-30i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-40\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+20i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+7\right)}{15\,a^3d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^5}$$

input
```
integrate(cos(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="giac"
)
```

output
```
2/15*(15*tan(1/2*d*x + 1/2*c)^4 - 30*I*tan(1/2*d*x + 1/2*c)^3 - 40*tan(1/2
*d*x + 1/2*c)^2 + 20*I*tan(1/2*d*x + 1/2*c) + 7)/(a^3*d*(tan(1/2*d*x + 1/2
*c) - I)^5)
```

3.178.9 Mupad [B] (verification not implemented)

Time = 23.00 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.48

$$\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{2\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 15\mathrm{i}+30\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 40\mathrm{i}-20\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+7\mathrm{i}\right)}{15\,a^3\,d\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5 1\mathrm{i}+5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3 10\mathrm{i}-10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{dx}{2}\right) 5\mathrm{i}+1\right.}$$

input
```
int(cos(c + d*x)^2/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^3,x)
```

output
```
(2*(30*tan(c/2 + (d*x)/2)^3 - tan(c/2 + (d*x)/2)^2*40i - 20*tan(c/2 + (d*x
)/2) + tan(c/2 + (d*x)/2)^4*15i + 7i))/(15*a^3*d*(tan(c/2 + (d*x)/2)*5i -
10*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)^3*10i + 5*tan(c/2 + (d*x)/2)^
4 + tan(c/2 + (d*x)/2)^5*1i + 1))
```

3.178. $\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$

3.179 $\int \frac{\cos(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$

3.179.1 Optimal result

Integrand size = 29, antiderivative size = 32

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{i\cot^2(c+dx)}{2a^3d(i+\cot(c+dx))^2}$$

output
```
1/2*I*cot(d*x+c)^2/a^3/d/(I+cot(d*x+c))^2
```

3.179.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 77 vs. 2(32) = 64.

Time = 0.44 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.41

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{i\cos(2(c+dx))}{4a^3d}+\frac{i\cos(4(c+dx))}{8a^3d} + \frac{\sin(2(c+dx))}{4a^3d}+\frac{\sin(4(c+dx))}{8a^3d}$$

input
```
Integrate[Cos[c + d*x]/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

output
```
((I/4)*Cos[2*(c + d*x)])/(a^3*d) + ((I/8)*Cos[4*(c + d*x)])/(a^3*d) + Sin[
2*(c + d*x)]/(4*a^3*d) + Sin[4*(c + d*x)]/(8*a^3*d)
```

3.179.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {3042, 3567, 27, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}dx$$
$$\Big\downarrow \text{3567}$$
$$-\frac{\int \frac{\cot(c+dx)}{a^3(\cot(c+dx)+i)^3}d\cot(c+dx)}{d}$$
$$\Big\downarrow \text{27}$$
$$-\frac{\int \frac{\cot(c+dx)}{(\cot(c+dx)+i)^3}d\cot(c+dx)}{a^3d}$$
$$\Big\downarrow \text{48}$$
$$\frac{i\cot^2(c+dx)}{2a^3d(\cot(c+dx)+i)^2}$$

input
```
Int[Cos[c + d*x]/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

output
```
((I/2)*Cot[c + d*x]^2)/(a^3*d*(I + Cot[c + d*x])^2)
```

3.179.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3567
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[x^m*((b
+ a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a,
b, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[
n, 0] && GtQ[m, 1])
```

3.179.4 Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.72

method	result	s
derivativedivides	$\frac{i}{2d\,a^{3}\left(i\tan\left(dx+c\right)+1\right)^{2}}$	2
default	$\frac{i}{2d\,a^{3}\left(i\tan\left(dx+c\right)+1\right)^{2}}$	2
risch	$\frac{i\,\mathrm{e}^{-2i(dx+c)}}{4d\,a^{3}}+\frac{i\,\mathrm{e}^{-4i(dx+c)}}{8d\,a^{3}}$	3
norman	$\frac{\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{ad}+\frac{6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{ad}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{ad}-\frac{6i\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{ad}-\frac{6i\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}}{ad}+\frac{4i\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{ad}}{a^{2}\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{4}}$	1

input
```
int(cos(d*x+c)/(cos(d*x+c)*a+I*a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*I/d/a^3/(I*tan(d*x+c)+1)^2
```

3.179.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx=\frac{\left(2i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-4i\,dx-4i\,c)}}{8\,a^3d}$$

input
```
integrate(cos(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="fricas"
)
```

output
```
1/8*(2*I*e^(2*I*d*x + 2*I*c) + I)*e^(-4*I*d*x - 4*I*c)/(a^3*d)
```

3.179.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(26) = 52.

Time = 0.12 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.97

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \begin{cases} \frac{(8ia^3de^{4ic}e^{-2idx}+4ia^3de^{2ic}e^{-4idx})e^{-6ic}}{32a^6d^2} & \text{for } a^6d^2e^{6ic}\neq 0 \\ \frac{x(e^{2ic}+1)e^{-4ic}}{2a^3} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))**3,x)
```

output
```
Piecewise(((8*I*a**3*d*exp(4*I*c)*exp(-2*I*d*x) + 4*I*a**3*d*exp(2*I*c)*ex
p(-4*I*d*x))*exp(-6*I*c)/(32*a**6*d**2), Ne(a**6*d**2*exp(6*I*c), 0)), (x*
(exp(2*I*c) + 1)*exp(-4*I*c)/(2*a**3), True))
```

3.179.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.59

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{i\cos(4\,dx+4\,c)+2i\cos(2\,dx+2\,c)+\sin(4\,dx+4\,c)+2\sin(2\,dx+2\,c)}{8\,a^3d}$$

input
```
integrate(cos(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="maxima"
)
```

output
```
1/8*(I*cos(4*d*x + 4*c) + 2*I*cos(2*d*x + 2*c) + sin(4*d*x + 4*c) + 2*sin(
2*d*x + 2*c))/(a^3*d)
```

3.179.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(26) = 52$.

Time = 0.33 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.78

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = -\frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a^3d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^4}$$

input
```
integrate(cos(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
-2*(tan(1/2*d*x + 1/2*c)^3 - I*tan(1/2*d*x + 1/2*c)^2 - tan(1/2*d*x + 1/2*
c))/(a^3*d*(tan(1/2*d*x + 1/2*c) - I)^4)
```

3.179.9 Mupad [B] (verification not implemented)

Time = 22.95 (sec) , antiderivative size = 100, normalized size of antiderivative = 3.12

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = -\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,1\mathrm{i}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\mathrm{i}\right)}{a^3\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,1\mathrm{i}+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,6\mathrm{i}-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)}$$

input
```
int(cos(c + d*x)/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^3,x)
```

output
```
-(2*tan(c/2 + (d*x)/2)*(tan(c/2 + (d*x)/2) + tan(c/2 + (d*x)/2)^2*1i - 1i)
)/(a^3*d*(4*tan(c/2 + (d*x)/2)^3 - tan(c/2 + (d*x)/2)^2*6i - 4*tan(c/2 + (
d*x)/2) + tan(c/2 + (d*x)/2)^4*1i + 1i))
```

3.180 $\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$

3.180.1 Optimal result

Integrand size = 22, antiderivative size = 31

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx=\frac{i}{3d(a\cos(c+dx)+ia\sin(c+dx))^3}$$

output
```
1/3*I/d/(a*cos(d*x+c)+I*a*sin(d*x+c))^3
```

3.180.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx=\frac{i}{3d(a\cos(c+dx)+ia\sin(c+dx))^3}$$

input
```
Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-3),x]
```

output
```
(I/3)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3)
```

3.180.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3042, 3550}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^3}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^3}dx$$
$$\Big\downarrow \text{ 3550}$$
$$\frac{i}{3d(a\cos(c+dx)+ia\sin(c+dx))^3}$$

input
```
Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-3),x]
```

output
```
(I/3)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3)
```

3.180.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3550
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

3.180.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

method	result	size
risch	$\frac{i\mathrm{e}^{-3i(dx+c)}}{3d\,a^{3}}$	19
derivativedivides	$\frac{\frac{4i}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^{2}}+\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i}-\frac{8}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^{3}}}{d\,a^{3}}$	57
default	$\frac{\frac{4i}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^{2}}+\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i}-\frac{8}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^{3}}}{d\,a^{3}}$	57
norman	$\frac{-\frac{4i\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{ad}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{20\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3ad}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{ad}+\frac{2i}{3ad}+\frac{6i\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{ad}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{3}a^{2}}$	125

input
```
int(1/(cos(d*x+c)*a+I*a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*I/d/a^3*exp(-3*I*(d*x+c))
```

3.180.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.55

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{i\,e^{(-3i\,dx-3i\,c)}}{3\,a^3d}$$

input
```
integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/3*I*e^(-3*I*d*x - 3*I*c)/(a^3*d)
```

3.180.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.42

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \begin{cases} \frac{ie^{-3ic}e^{-3idx}}{3a^3d} & \text{for } a^3de^{3ic}\neq 0 \\ \frac{xe^{-3ic}}{a^3} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))**3,x)
```

output
```
Piecewise((I*exp(-3*I*c)*exp(-3*I*d*x)/(3*a**3*d), Ne(a**3*d*exp(3*I*c), 0
)), (x*exp(-3*I*c)/a**3, True))
```

3.180.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{i\,\cos\left(3\,dx+3\,c\right)+\sin\left(3\,dx+3\,c\right)}{3\,a^3d}$$

input
```
integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/3*(I*cos(3*d*x + 3*c) + sin(3*d*x + 3*c))/(a^3*d)
```

3.180.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{2\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}{3\,a^3d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^3}$$

input
```
integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
2/3*(3*tan(1/2*d*x + 1/2*c)^2 - 1)/(a^3*d*(tan(1/2*d*x + 1/2*c) - I)^3)
```

3.180.9 Mupad [B] (verification not implemented)

Time = 24.44 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.19

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = -\frac{2\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,3\mathrm{i}-\mathrm{i}\right)}{3\,a^3\,d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,1\mathrm{i}-3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,3\mathrm{i}+1\right)}$$

input
```
int(1/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^3,x)
```

output
```
-(2*(tan(c/2 + (d*x)/2)^2*3i - 1i))/(3*a^3*d*(tan(c/2 + (d*x)/2)*3i - 3*ta
n(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)^3*1i + 1))
```

3.180. $\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$

3.181 $\int \frac{\sec(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$

3.181.1 Optimal result

Integrand size = 29, antiderivative size = 61

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = -\frac{x}{a^3}+\frac{2}{a^3d(i+\cot(c+dx))} - \frac{i\log(\sin(c+dx))}{a^3d}+\frac{i\log(\tan(c+dx))}{a^3d}$$

output
```
-x/a^3+2/a^3/d/(I+cot(d*x+c))-I*ln(sin(d*x+c))/a^3/d+I*ln(tan(d*x+c))/a^3/
d
```

3.181.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.66

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{i\left(\log(i-\tan(c+dx))-\frac{2i}{-i+\tan(c+dx)}\right)}{a^3d}$$

input
```
Integrate[Sec[c + d*x]/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

output
```
(I*(Log[I - Tan[c + d*x]] - (2*I)/(-I + Tan[c + d*x])))/(a^3*d)
```

3.181.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.80, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {3042, 3567, 27, 516, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(c+dx)(a\cos(c+dx)+ia\sin(c+dx))^3}dx$$

$$\Big\downarrow \text{3567}$$

$$-\frac{\int \frac{(\cot^2(c+dx)+1)\tan(c+dx)}{a^3(\cot(c+dx)+i)^3}d\cot(c+dx)}{d}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{(\cot^2(c+dx)+1)\tan(c+dx)}{(\cot(c+dx)+i)^3}d\cot(c+dx)}{a^3d}$$

$$\Big\downarrow \text{516}$$

$$-\frac{\int \frac{(\cot(c+dx)-i)\tan(c+dx)}{(\cot(c+dx)+i)^2}d\cot(c+dx)}{a^3d}$$

$$\Big\downarrow \text{86}$$

$$-\frac{\int \left(i\tan(c+dx)-\frac{i}{\cot(c+dx)+i}+\frac{2}{(\cot(c+dx)+i)^2}\right)d\cot(c+dx)}{a^3d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{2}{\cot(c+dx)+i}+i\log(\cot(c+dx))-i\log(\cot(c+dx)+i)}{a^3d}$$

input
```
Int[Sec[c + d*x]/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

output
```
-((-2/(I + Cot[c + d*x]) + I*Log[Cot[c + d*x]] - I*Log[I + Cot[c + d*x]])/
(a^3*d))
```

3.181.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 516
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[(e*x)^m*(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; Free
Q[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] ||
(GtQ[a, 0] && GtQ[c, 0] &&  !IntegerQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3567
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[x^m*((b
+ a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a,
b, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[
n, 0] && GtQ[m, 1])
```

3.181.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.57

method	result
derivativedivides	$\frac{\frac{2}{\tan(dx+c)-i}+i\ln(\tan(dx+c)-i)}{d\,a^3}$
default	$\frac{\frac{2}{\tan(dx+c)-i}+i\ln(\tan(dx+c)-i)}{d\,a^3}$
risch	$\frac{i e^{-2i(dx+c)}}{d\,a^3}-\frac{2x}{a^3}-\frac{2c}{d\,a^3}-\frac{i\ln(e^{2i(dx+c)}+1)}{d\,a^3}$
norman	$\frac{-\frac{x}{a}+\frac{4\tan(\frac{dx}{2}+\frac{c}{2})}{ad}-\frac{4\tan(\frac{dx}{2}+\frac{c}{2})^3}{ad}-\frac{2x\tan(\frac{dx}{2}+\frac{c}{2})^2}{a}-\frac{x\tan(\frac{dx}{2}+\frac{c}{2})^4}{a}-\frac{8i\tan(\frac{dx}{2}+\frac{c}{2})^2}{ad}}{\left(1+\tan(\frac{dx}{2}+\frac{c}{2})^2\right)^2 a^2}+\frac{i\ln\left(1+\tan(\frac{dx}{2}+\frac{c}{2})^2\right)}{d\,a^3}-$

input
```
int(sec(d*x+c)/(cos(d*x+c)*a+I*a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^3*(2/(tan(d*x+c)-I)+I*ln(tan(d*x+c)-I))
```

3.181.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.90

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = -\frac{\left(2\,dxe^{(2i\,dx+2i\,c)}+i\,e^{(2i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-i\right)e^{(-2i\,dx-2i\,c)}}{a^3d}$$

input
```
integrate(sec(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="fricas"
)
```

output
```
-(2*d*x*e^(2*I*d*x + 2*I*c) + I*e^(2*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c
) + 1) - I)*e^(-2*I*d*x - 2*I*c)/(a^3*d)
```

3.181.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{\int \frac{\sec(c+dx)}{-i\sin^3(c+dx)-3\sin^2(c+dx)\cos(c+dx)+3i\sin(c+dx)\cos^2(c+dx)+\cos^3(c+dx)}\,dx}{a^3}$$

input
```
integrate(sec(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)/(-I*sin(c + d*x)**3 - 3*sin(c + d*x)**2*cos(c + d*x)
 + 3*I*sin(c + d*x)*cos(c + d*x)**2 + cos(c + d*x)**3), x)/a**3
```

3.181.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.62

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = -\frac{4\,dx+4\,c-2\,\arctan\left(\sin\left(2\,dx+2\,c\right),\cos\left(2\,dx+2\,c\right)+1\right)-2i\,\cos\left(2\,dx+2\,c\right)+i\,\log\left(\cos\left(2\,dx+2\,c\right.\right.}{2\,a^3d}$$

input
```
integrate(sec(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="maxima"
)
```

output
```
-1/2*(4*d*x + 4*c - 2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1) - 2*
I*cos(2*d*x + 2*c) + I*log(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1) - 2*sin(2*d*x + 2*c))/(a^3*d)
```

3.181.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.64

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = -\frac{\frac{i\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^3}-\frac{2i\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)}{a^3}+\frac{i\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^3}+\frac{3i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+10\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3i}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^2}}{d}$$

input
```
integrate(sec(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
-(I*log(tan(1/2*d*x + 1/2*c) + 1)/a^3 - 2*I*log(tan(1/2*d*x + 1/2*c) - I)/
a^3 + I*log(tan(1/2*d*x + 1/2*c) - 1)/a^3 + (3*I*tan(1/2*d*x + 1/2*c)^2 +
10*tan(1/2*d*x + 1/2*c) - 3*I)/(a^3*(tan(1/2*d*x + 1/2*c) - I)^2))/d
```

3.181.9 Mupad [B] (verification not implemented)

Time = 23.36 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.66

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = -\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,4\mathrm{i}}{d\,\left(a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\,1\mathrm{i}+2\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-a^3\,1\mathrm{i}\right)} + \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-\mathrm{i}\right)\,2\mathrm{i}}{a^3\,d} - \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)\,1\mathrm{i}}{a^3\,d}$$

input
```
int(1/(cos(c + d*x)*(a*cos(c + d*x) + a*sin(c + d*x)*1i)^3),x)
```

output
```
(log(tan(c/2 + (d*x)/2) - 1i)*2i)/(a^3*d) - (tan(c/2 + (d*x)/2)*4i)/(d*(a^
3*tan(c/2 + (d*x)/2)^2*1i - a^3*1i + 2*a^3*tan(c/2 + (d*x)/2))) - (log(tan
(c/2 + (d*x)/2)^2 - 1)*1i)/(a^3*d)
```

3.182 $\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$

3.182.1 Optimal result

Integrand size = 31, antiderivative size = 62

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = -\frac{3\text{arctanh}(\sin(c+dx))}{a^3d}+\frac{4i\cos(c+dx)}{a^3d}+\frac{i\sec(c+dx)}{a^3d}+\frac{4\sin(c+dx)}{a^3d}$$

output
```
-3*arctanh(sin(d*x+c))/a^3/d+4*I*cos(d*x+c)/a^3/d+I*sec(d*x+c)/a^3/d+4*sin
(d*x+c)/a^3/d
```

3.182.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.76

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = -\frac{i\sec^3(c+dx)(\cos(dx)+i\sin(dx))^3\left(6\text{arctanh}\left(\sin(c)+\cos(c)\tan\left(\frac{dx}{2}\right)\right)(\cos(3c)+i\sin(3c))+(\cos(2c\right.}{a^3d(-i+\tan(c+dx))^3}$$

input
```
Integrate[Sec[c + d*x]^2/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

output
```
((-I)*Sec[c + d*x]^3*(Cos[d*x] + I*Sin[d*x])^3*(6*ArcTanh[Sin[c] + Cos[c]*
Tan[(d*x)/2]]*(Cos[3*c] + I*Sin[3*c]) + (Cos[2*c - d*x] + I*Sin[2*c - d*x]
)*(-5*I + Tan[c + d*x])))/(a^3*d*(-I + Tan[c + d*x])^3)
```

3.182.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3042, 3571, 3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(c+dx)^2(a\cos(c+dx)+ia\sin(c+dx))^3}dx$$

$$\Big\downarrow \text{3571}$$

$$\frac{i\int \sec^2(c+dx)(ia\cos(c+dx)+a\sin(c+dx))^3dx}{a^6}$$

$$\Big\downarrow \text{3042}$$

$$\frac{i\int \frac{(ia\cos(c+dx)+a\sin(c+dx))^3}{\cos(c+dx)^2}dx}{a^6}$$

$$\Big\downarrow \text{3569}$$

$$\frac{i\int \left(\sin(c+dx)\tan^2(c+dx)a^3-i\cos(c+dx)a^3-3\sin(c+dx)a^3+3i\sin(c+dx)\tan(c+dx)a^3\right)dx}{a^6}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i\left(\frac{3ia^3\text{arctanh}(\sin(c+dx))}{d}-\frac{4ia^3\sin(c+dx)}{d}+\frac{4a^3\cos(c+dx)}{d}+\frac{a^3\sec(c+dx)}{d}\right)}{a^6}$$

input
```
Int[Sec[c + d*x]^2/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

output
```
(I*(((3*I)*a^3*ArcTanh[Sin[c + d*x]])/d + (4*a^3*Cos[c + d*x])/d + (a^3*Se
c[c + d*x])/d - ((4*I)*a^3*Sin[c + d*x])/d))/a^6
```

3.182.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

rule 3571
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^n*b^n   Int[Cos[c + d*x]^m
/(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &
& EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

3.182.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.39

method	result	size
derivativedivides	$\frac{\frac{8}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i}+\frac{2i}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2}-3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{i}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d\,a^{3}}$	86
default	$\frac{\frac{8}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i}+\frac{2i}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2}-3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{i}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d\,a^{3}}$	86
risch	$\frac{4i\,\mathrm{e}^{-i(dx+c)}}{d\,a^{3}}+\frac{2i\,\mathrm{e}^{i(dx+c)}}{d\,a^{3}\left(\mathrm{e}^{2i(dx+c)}+1\right)}+\frac{3\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{d\,a^{3}}-\frac{3\ln\left(i+\mathrm{e}^{i(dx+c)}\right)}{d\,a^{3}}$	93
norman	$\frac{-\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{ad}-\frac{10i}{ad}+\frac{6i\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{ad}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)a^{2}}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d\,a^{3}}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d\,a^{3}}$	142

input
```
int(sec(d*x+c)^2/(cos(d*x+c)*a+I*a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
2/d/a^3*(4/(tan(1/2*d*x+1/2*c)-I)+1/2*I/(tan(1/2*d*x+1/2*c)+1)-3/2*ln(tan(
1/2*d*x+1/2*c)+1)-1/2*I/(tan(1/2*d*x+1/2*c)-1)+3/2*ln(tan(1/2*d*x+1/2*c)-1
))
```

3.182.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.81

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = -\frac{3\left(e^{(3i\,dx+3i\,c)}+e^{(i\,dx+i\,c)}\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-3\left(e^{(3i\,dx+3i\,c)}+e^{(i\,dx+i\,c)}\right)\log\left(e^{(i\,dx+i\,c)}-i\right)-6i\,e^{(2i\,dx+2i\,c}}{a^3de^{(3i\,dx+3i\,c)}+a^3de^{(i\,dx+i\,c)}}$$

input
```
integrate(sec(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="frica
s")
```

output
```
-(3*(e^(3*I*d*x + 3*I*c) + e^(I*d*x + I*c))*log(e^(I*d*x + I*c) + I) - 3*(
e^(3*I*d*x + 3*I*c) + e^(I*d*x + I*c))*log(e^(I*d*x + I*c) - I) - 6*I*e^(2
*I*d*x + 2*I*c) - 4*I)/(a^3*d*e^(3*I*d*x + 3*I*c) + a^3*d*e^(I*d*x + I*c))
```

3.182.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{\int \frac{\sec^2(c+dx)}{-i\sin^3(c+dx)-3\sin^2(c+dx)\cos(c+dx)+3i\sin(c+dx)\cos^2(c+dx)+\cos^3(c+dx)}\,dx}{a^3}$$

input
```
integrate(sec(d*x+c)**2/(a*cos(d*x+c)+I*a*sin(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)**2/(-I*sin(c + d*x)**3 - 3*sin(c + d*x)**2*cos(c + d
*x) + 3*I*sin(c + d*x)*cos(c + d*x)**2 + cos(c + d*x)**3), x)/a**3
```

3.182.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 319 vs. 2(58) = 116.

Time = 0.34 (sec) , antiderivative size = 319, normalized size of antiderivative = 5.15

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{6\left(\cos(3\,dx+3\,c)+\cos(dx+c)+i\,\sin(3\,dx+3\,c)+i\,\sin(dx+c)\right)\arctan\left(\cos(dx+c),\sin(dx+c)+\right.}{}$$

input
```
integrate(sec(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="maxim
a")
```

output
```
(6*(cos(3*d*x + 3*c) + cos(d*x + c) + I*sin(3*d*x + 3*c) + I*sin(d*x + c))
*arctan2(cos(d*x + c), sin(d*x + c) + 1) + 6*(cos(3*d*x + 3*c) + cos(d*x +
 c) + I*sin(3*d*x + 3*c) + I*sin(d*x + c))*arctan2(cos(d*x + c), -sin(d*x
+ c) + 1) + 3*(I*cos(3*d*x + 3*c) + I*cos(d*x + c) - sin(3*d*x + 3*c) - si
n(d*x + c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) + 3*
(-I*cos(3*d*x + 3*c) - I*cos(d*x + c) + sin(3*d*x + 3*c) + sin(d*x + c))*l
og(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1) + 12*cos(2*d*x +
2*c) + 12*I*sin(2*d*x + 2*c) + 8)/((-2*I*a^3*cos(3*d*x + 3*c) - 2*I*a^3*co
s(d*x + c) + 2*a^3*sin(3*d*x + 3*c) + 2*a^3*sin(d*x + c))*d)
```

3.182.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.77

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = -\frac{\frac{3\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^3}-\frac{3\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{a^3}-\frac{2\left(4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-5\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+i\right)a^3}}{d}$$

input
```
integrate(sec(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="giac"
)
```

output
```
-(3*log(tan(1/2*d*x + 1/2*c) + 1)/a^3 - 3*log(tan(1/2*d*x + 1/2*c) - 1)/a^
3 - 2*(4*tan(1/2*d*x + 1/2*c)^2 - I*tan(1/2*d*x + 1/2*c) - 5)/((tan(1/2*d*
x + 1/2*c)^3 - I*tan(1/2*d*x + 1/2*c)^2 - tan(1/2*d*x + 1/2*c) + I)*a^3))/
d
```

3.182.9 Mupad [B] (verification not implemented)

Time = 23.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.69

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$$
$$= -\frac{6\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^3\,d}$$
$$-\frac{\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^3}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 8\mathrm{i}}{a^3}-\frac{10\mathrm{i}}{a^3}}{d\left(-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3 1\mathrm{i}-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,1\mathrm{i}+1\right)}$$

input
```
int(1/(cos(c + d*x)^2*(a*cos(c + d*x) + a*sin(c + d*x)*1i)^3),x)
```

output
```
- (6*atanh(tan(c/2 + (d*x)/2)))/(a^3*d) - ((tan(c/2 + (d*x)/2)^2*8i)/a^3 -
 10i/a^3 + (2*tan(c/2 + (d*x)/2))/a^3)/(d*(tan(c/2 + (d*x)/2)*1i - tan(c/2
 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)^3*1i + 1))
```

3.183 $\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$

3.183.1 Optimal result

Integrand size = 31, antiderivative size = 75

$$\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{4x}{a^3}+\frac{4i\log(\sin(c+dx))}{a^3d}-\frac{4i\log(\tan(c+dx))}{a^3d}-\frac{3\tan(c+dx)}{a^3d}+\frac{i\tan^2(c+dx)}{2a^3d}$$

output
```
4*x/a^3+4*I*ln(sin(d*x+c))/a^3/d-4*I*ln(tan(d*x+c))/a^3/d-3*tan(d*x+c)/a^3
/d+1/2*I*tan(d*x+c)^2/a^3/d
```

3.183.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.64

$$\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{i\left(-4\log(i-\tan(c+dx))+3i\tan(c+dx)+\frac{1}{2}\tan^2(c+dx)\right)}{a^3d}$$

input
```
Integrate[Sec[c + d*x]^3/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

output
```
(I*(-4*Log[I - Tan[c + d*x]] + (3*I)*Tan[c + d*x] + Tan[c + d*x]^2/2))/(a^
3*d)
```

3.183.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.76, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3042, 3567, 27, 516, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\cos(c+dx)^3(a\cos(c+dx)+ia\sin(c+dx))^3}dx$$
$$\Big\downarrow \text{3567}$$
$$-\frac{\int \frac{(\cot^2(c+dx)+1)^2\tan^3(c+dx)}{a^3(\cot(c+dx)+i)^3}d\cot(c+dx)}{d}$$
$$\Big\downarrow \text{27}$$
$$-\frac{\int \frac{(\cot^2(c+dx)+1)^2\tan^3(c+dx)}{(\cot(c+dx)+i)^3}d\cot(c+dx)}{a^3d}$$
$$\Big\downarrow \text{516}$$
$$-\frac{\int \frac{(\cot(c+dx)-i)^2\tan^3(c+dx)}{\cot(c+dx)+i}d\cot(c+dx)}{a^3d}$$
$$\Big\downarrow \text{99}$$
$$-\frac{\int \left(i\tan^3(c+dx)-3\tan^2(c+dx)-4i\tan(c+dx)+\frac{4i}{\cot(c+dx)+i}\right)d\cot(c+dx)}{a^3d}$$
$$\Big\downarrow \text{2009}$$
$$-\frac{-\frac{1}{2}i\tan^2(c+dx)+3\tan(c+dx)-4i\log(\cot(c+dx))+4i\log(\cot(c+dx)+i)}{a^3d}$$

input
```
Int[Sec[c + d*x]^3/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

output
```
-(((-4*I)*Log[Cot[c + d*x]] + (4*I)*Log[I + Cot[c + d*x]] + 3*Tan[c + d*x]
 - (I/2)*Tan[c + d*x]^2)/(a^3*d))
```

3.183. $\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}dx$

3.183.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 516
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[(e*x)^m*(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; Free
Q[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] ||
(GtQ[a, 0] && GtQ[c, 0] &&  !IntegerQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3567
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[x^m*((b
+ a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a,
b, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[
n, 0] && GtQ[m, 1])
```

3.183. $\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$

3.183.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.55

method	result
derivativedivides	$\frac{-3\tan(dx+c)+\frac{i\tan(dx+c)^2}{2}-4i\ln(\tan(dx+c)-i)}{d\,a^3}$
default	$\frac{-3\tan(dx+c)+\frac{i\tan(dx+c)^2}{2}-4i\ln(\tan(dx+c)-i)}{d\,a^3}$
risch	$\frac{8x}{a^3}+\frac{8c}{d\,a^3}-\frac{2i(2\,\mathrm{e}^{2i(dx+c)}+3)}{a^3d(\mathrm{e}^{2i(dx+c)}+1)^2}+\frac{4i\ln(\mathrm{e}^{2i(dx+c)}+1)}{d\,a^3}$
norman	$\frac{\frac{4x}{a}-\frac{6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{ad}-\frac{8x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{a}+\frac{4x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{a}+\frac{2i\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{ad}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{4i\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d\,a^3}+$

input
```
int(sec(d*x+c)^3/(cos(d*x+c)*a+I*a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^3*(-3*tan(d*x+c)+1/2*I*tan(d*x+c)^2-4*I*ln(tan(d*x+c)-I))
```

3.183.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.51

$$\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$$
$$=\frac{2\left(4\,dxe^{(4i\,dx+4i\,c)}+4\,dx+2\,(4\,dx-i)e^{(2i\,dx+2i\,c)}-2\left(-i\,e^{(4i\,dx+4i\,c)}-2i\,e^{(2i\,dx+2i\,c)}-i\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right.}{a^3de^{(4i\,dx+4i\,c)}+2\,a^3de^{(2i\,dx+2i\,c)}+a^3d}$$

input
```
integrate(sec(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="frica
s")
```

output
```
2*(4*d*x*e^(4*I*d*x + 4*I*c) + 4*d*x + 2*(4*d*x - I)*e^(2*I*d*x + 2*I*c) -
 2*(-I*e^(4*I*d*x + 4*I*c) - 2*I*e^(2*I*d*x + 2*I*c) - I)*log(e^(2*I*d*x +
 2*I*c) + 1) - 3*I)/(a^3*d*e^(4*I*d*x + 4*I*c) + 2*a^3*d*e^(2*I*d*x + 2*I*
c) + a^3*d)
```

3.183.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{\int \frac{\sec^3(c+dx)}{-i\sin^3(c+dx)-3\sin^2(c+dx)\cos(c+dx)+3i\sin(c+dx)\cos^2(c+dx)+\cos^3(c+dx)}\,dx}{a^3}$$

input
```
integrate(sec(d*x+c)**3/(a*cos(d*x+c)+I*a*sin(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)**3/(-I*sin(c + d*x)**3 - 3*sin(c + d*x)**2*cos(c + d
*x) + 3*I*sin(c + d*x)*cos(c + d*x)**2 + cos(c + d*x)**3), x)/a**3
```

3.183.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(67) = 134$.

Time = 0.32 (sec) , antiderivative size = 301, normalized size of antiderivative = 4.01

$$\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = -\frac{2\,(4i\,dx+2\,(-i\,\cos(4\,dx+4\,c)-2i\,\cos(2\,dx+2\,c)+\sin(4\,dx+4\,c)+2\,\sin(2\,dx+2\,c)-i)\arctan(}{}$$

input
```
integrate(sec(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="maxim
a")
```

output
```
-2*(4*I*d*x + 2*(-I*cos(4*d*x + 4*c) - 2*I*cos(2*d*x + 2*c) + sin(4*d*x +
4*c) + 2*sin(2*d*x + 2*c) - I)*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
+ 1) + 4*(I*d*x + I*c)*cos(4*d*x + 4*c) + 2*(4*I*d*x + 4*I*c + 1)*cos(2*d*
x + 2*c) - (cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + I*sin(4*d*x + 4*c) + 2
*I*sin(2*d*x + 2*c) + 1)*log(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*c
os(2*d*x + 2*c) + 1) - 4*(d*x + c)*sin(4*d*x + 4*c) - 2*(4*d*x + 4*c - I)*
sin(2*d*x + 2*c) + 4*I*c + 3)/((-I*a^3*cos(4*d*x + 4*c) - 2*I*a^3*cos(2*d*
x + 2*c) + a^3*sin(4*d*x + 4*c) + 2*a^3*sin(2*d*x + 2*c) - I*a^3)*d)
```

3.183.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.71

$$\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$$
$$= \frac{2\left(\frac{2i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^3} - \frac{4i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)}{a^3} + \frac{2i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{a^3} + \frac{-3i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+7i\tan}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\right.}\right.}{d}$$

input
```
integrate(sec(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="giac"
)
```

output
```
2*(2*I*log(tan(1/2*d*x + 1/2*c) + 1)/a^3 - 4*I*log(tan(1/2*d*x + 1/2*c) -
I)/a^3 + 2*I*log(tan(1/2*d*x + 1/2*c) - 1)/a^3 + (-3*I*tan(1/2*d*x + 1/2*c
)^4 + 3*tan(1/2*d*x + 1/2*c)^3 + 7*I*tan(1/2*d*x + 1/2*c)^2 - 3*tan(1/2*d*
x + 1/2*c) - 3*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^3))/d
```

3.183.9 Mupad [B] (verification not implemented)

Time = 23.22 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.39

$$\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$$
$$= -\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\mathrm{i}\right)\,8\mathrm{i} - \ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)\,4\mathrm{i}}{a^3\,d}$$
$$+ \frac{6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,2\mathrm{i}-6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^3\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)^2}$$

input
```
int(1/(cos(c + d*x)^3*(a*cos(c + d*x) + a*sin(c + d*x)*1i)^3),x)
```

output
```
(tan(c/2 + (d*x)/2)^2*2i - 6*tan(c/2 + (d*x)/2) + 6*tan(c/2 + (d*x)/2)^3)/
(a^3*d*(tan(c/2 + (d*x)/2)^2 - 1)^2) - (log(tan(c/2 + (d*x)/2) - 1i)*8i -
log(tan(c/2 + (d*x)/2)^2 - 1)*4i)/(a^3*d)
```

3.184 $\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$

3.184.1 Optimal result

Integrand size = 31, antiderivative size = 76

$$\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{5\text{arctanh}(\sin(c+dx))}{2a^3d} - \frac{4i\sec(c+dx)}{a^3d} + \frac{i\sec^3(c+dx)}{3a^3d} - \frac{3\sec(c+dx)\tan(c+dx)}{2a^3d}$$

output
```
5/2*arctanh(sin(d*x+c))/a^3/d-4*I*sec(d*x+c)/a^3/d+1/3*I*sec(d*x+c)^3/a^3/
d-3/2*sec(d*x+c)*tan(d*x+c)/a^3/d
```

3.184.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.84

$$\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{i\left(-60i\text{arctanh}\left(\sin(c)+\cos(c)\tan\left(\frac{dx}{2}\right)\right)+\sec^3(c+dx)(-20-24\cos(2(c+dx))+9i\sin(2(c+dx)))\right)}{12a^3d}$$

input
```
Integrate[Sec[c + d*x]^4/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

output
```
((I/12)*((-60*I)*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]] + Sec[c + d*x]^3*(-
20 - 24*Cos[2*(c + d*x)] + (9*I)*Sin[2*(c + d*x)])))/(a^3*d)
```

3.184.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3042, 3571, 3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(c+dx)^4(a\cos(c+dx)+ia\sin(c+dx))^3}dx$$

$$\Big\downarrow \text{3571}$$

$$\frac{i\int \sec^4(c+dx)(ia\cos(c+dx)+a\sin(c+dx))^3dx}{a^6}$$

$$\Big\downarrow \text{3042}$$

$$\frac{i\int \frac{(ia\cos(c+dx)+a\sin(c+dx))^3}{\cos(c+dx)^4}dx}{a^6}$$

$$\Big\downarrow \text{3569}$$

$$\frac{i\int \left(\sec(c+dx)\tan^3(c+dx)a^3+3i\sec(c+dx)\tan^2(c+dx)a^3-i\sec(c+dx)a^3-3\sec(c+dx)\tan(c+dx)a^3\right)d}{a^6}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i\left(-\frac{5ia^3\text{arctanh}(\sin(c+dx))}{2d}+\frac{a^3\sec^3(c+dx)}{3d}-\frac{4a^3\sec(c+dx)}{d}+\frac{3ia^3\tan(c+dx)\sec(c+dx)}{2d}\right)}{a^6}$$

input
```
Int[Sec[c + d*x]^4/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

output
```
(I*((((-5*I)/2)*a^3*ArcTanh[Sin[c + d*x]])/d - (4*a^3*Sec[c + d*x])/d + (a
^3*Sec[c + d*x]^3)/(3*d) + (((3*I)/2)*a^3*Sec[c + d*x]*Tan[c + d*x])/d))/a
^6
```

3.184.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

rule 3571
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^n*b^n   Int[Cos[c + d*x]^m
/(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &
& EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

3.184.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.32

method	result
risch	$-\frac{i\left(15\,e^{5i(dx+c)}+40\,e^{3i(dx+c)}+33\,e^{i(dx+c)}\right)}{3d\,a^{3}\left(e^{2i(dx+c)}+1\right)^{3}}-\frac{5\ln\left(e^{i(dx+c)}-i\right)}{2d\,a^{3}}+\frac{5\ln\left(i+e^{i(dx+c)}\right)}{2d\,a^{3}}$
derivativedivides	$\frac{-\frac{i}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}+\frac{2\left(-\frac{3}{4}-\frac{i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{2\left(-\frac{3}{4}+\frac{7i}{4}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}-\frac{5\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}+\frac{i}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{2\left(\frac{3}{4}-\frac{i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}}{d\,a^{3}}$
default	$\frac{-\frac{i}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}+\frac{2\left(-\frac{3}{4}-\frac{i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{2\left(-\frac{3}{4}+\frac{7i}{4}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}-\frac{5\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}+\frac{i}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{2\left(\frac{3}{4}-\frac{i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}}{d\,a^{3}}$
norman	$\frac{-\frac{16i\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{ad}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{ad}+\frac{22i}{3ad}+\frac{6i\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{ad}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}a^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}-\frac{5\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d\,a^{3}}+\frac{5\ln\left(\tan\left(\frac{dx}{2}+\right.\right.}{2d\,a^{3}}$

input
```
int(sec(d*x+c)^4/(cos(d*x+c)*a+I*a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3*I/d/a^3/(exp(2*I*(d*x+c))+1)^3*(15*exp(5*I*(d*x+c))+40*exp(3*I*(d*x+c
))+33*exp(I*(d*x+c)))-5/2/d/a^3*ln(exp(I*(d*x+c))-I)+5/2/d/a^3*ln(I+exp(I*
(d*x+c)))
```

3.184.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(66) = 132.

Time = 0.25 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.39

$$\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{15\left(e^{(6i\,dx+6i\,c)}+3\,e^{(4i\,dx+4i\,c)}+3\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-15\left(e^{(6i\,dx+6i\,c)}+3\,e^{(4i\,dx+4i\,c)}+3\,e^{(2i\,c}\right.}{6\left(a^3de^{(6i\,dx+6i\,c)}+3\,a^3de^{(4i\,dx+4i\,c)}+3\,a^3de^{(\right.}}$$

input
```
integrate(sec(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="frica
s")
```

output
```
1/6*(15*(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x + 4*I*c) + 3*e^(2*I*d*x + 2*I*
c) + 1)*log(e^(I*d*x + I*c) + I) - 15*(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x
+ 4*I*c) + 3*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x + I*c) - I) - 30*I*e^(5
*I*d*x + 5*I*c) - 80*I*e^(3*I*d*x + 3*I*c) - 66*I*e^(I*d*x + I*c))/(a^3*d*
e^(6*I*d*x + 6*I*c) + 3*a^3*d*e^(4*I*d*x + 4*I*c) + 3*a^3*d*e^(2*I*d*x + 2
*I*c) + a^3*d)
```

3.184.6 Sympy [F]

$$\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{\int \frac{\sec^4(c+dx)}{-i\sin^3(c+dx)-3\sin^2(c+dx)\cos(c+dx)+3i\sin(c+dx)\cos^2(c+dx)+\cos^3(c+dx)}\,dx}{a^3}$$

input
```
integrate(sec(d*x+c)**4/(a*cos(d*x+c)+I*a*sin(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)**4/(-I*sin(c + d*x)**3 - 3*sin(c + d*x)**2*cos(c + d
*x) + 3*I*sin(c + d*x)*cos(c + d*x)**2 + cos(c + d*x)**3), x)/a**3
```

3.184.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(66) = 132$.

Time = 0.23 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.83

$$\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$$
$$= \frac{\frac{4\left(-\frac{9i\sin(dx+c)}{\cos(dx+c)+1}-\frac{48\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{18\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{9i\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+22\right)}{6i\,a^3-\frac{18i\,a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{18i\,a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{6i\,a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{5\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}-\frac{5\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}}{2\,d}$$

input
```
integrate(sec(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="maxim
a")
```

output
```
1/2*(4*(-9*I*sin(d*x + c)/(cos(d*x + c) + 1) - 48*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2 + 18*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 9*I*sin(d*x + c)^5/
(cos(d*x + c) + 1)^5 + 22)/(6*I*a^3 - 18*I*a^3*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 18*I*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 6*I*a^3*sin(d*x
+ c)^6/(cos(d*x + c) + 1)^6) + 5*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/
a^3 - 5*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^3)/d
```

3.184.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.47

$$\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$$
$$= \frac{\frac{15\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^3}-\frac{15\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{a^3}-\frac{2\left(9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-18i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+48i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-9\tan\left(\frac{1}{2}dx+\frac{1}{2}\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^3a^3}}{6\,d}$$

input
```
integrate(sec(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="giac"
)
```

output
```
1/6*(15*log(tan(1/2*d*x + 1/2*c) + 1)/a^3 - 15*log(tan(1/2*d*x + 1/2*c) -
1)/a^3 - 2*(9*tan(1/2*d*x + 1/2*c)^5 - 18*I*tan(1/2*d*x + 1/2*c)^4 + 48*I*
tan(1/2*d*x + 1/2*c)^2 - 9*tan(1/2*d*x + 1/2*c) - 22*I)/((tan(1/2*d*x + 1/
2*c)^2 - 1)^3*a^3))/d
```

3.184.9 Mupad [B] (verification not implemented)

Time = 24.93 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.78

$$\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$$
$$= \frac{5\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d} + \frac{\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^3} - \frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{a^3} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,16\mathrm{i}}{a^3} + \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,6\mathrm{i}}{a^3} + \frac{22\mathrm{i}}{3\,a^3}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 - 3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 + 3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 - 1\right)}$$

input
```
int(1/(cos(c + d*x)^4*(a*cos(c + d*x) + a*sin(c + d*x)*1i)^3),x)
```

output
```
(5*atanh(tan(c/2 + (d*x)/2)))/(a^3*d) + ((tan(c/2 + (d*x)/2)^4*6i)/a^3 - (
tan(c/2 + (d*x)/2)^2*16i)/a^3 - (3*tan(c/2 + (d*x)/2)^5)/a^3 + 22i/(3*a^3)
 + (3*tan(c/2 + (d*x)/2))/a^3)/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2 + (d
*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

3.185 $\int \frac{\sec^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$

3.185.1 Optimal result

Integrand size = 31, antiderivative size = 34

$$\int \frac{\sec^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{i(i-\cot(c+dx))^4\tan^4(c+dx)}{4a^3d}$$

output
```
1/4*I*(I-cot(d*x+c))^4*tan(d*x+c)^4/a^3/d
```

3.185.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.47

$$\int \frac{\sec^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{i\tan(c+dx)\left(-4i-6\tan(c+dx)+4i\tan^2(c+dx)+\tan^3(c+dx)\right)}{4a^3d}$$

input
```
Integrate[Sec[c + d*x]^5/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

output
```
((I/4)*Tan[c + d*x]*(-4*I - 6*Tan[c + d*x] + (4*I)*Tan[c + d*x]^2 + Tan[c
+ d*x]^3))/(a^3*d)
```

3.185.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3042, 3567, 27, 516, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\cos(c+dx)^5(a\cos(c+dx)+ia\sin(c+dx))^3}dx$$
$$\Big\downarrow \text{3567}$$
$$-\frac{\int \frac{(\cot^2(c+dx)+1)^3\tan^5(c+dx)}{a^3(\cot(c+dx)+i)^3}d\cot(c+dx)}{d}$$
$$\Big\downarrow \text{27}$$
$$-\frac{\int \frac{(\cot^2(c+dx)+1)^3\tan^5(c+dx)}{(\cot(c+dx)+i)^3}d\cot(c+dx)}{a^3d}$$
$$\Big\downarrow \text{516}$$
$$-\frac{\int (\cot(c+dx)-i)^3\tan^5(c+dx)d\cot(c+dx)}{a^3d}$$
$$\Big\downarrow \text{48}$$
$$\frac{i\tan^4(c+dx)(-\cot(c+dx)+i)^4}{4a^3d}$$

input
```
Int[Sec[c + d*x]^5/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

output
```
((I/4)*(I - Cot[c + d*x])^4*Tan[c + d*x]^4)/(a^3*d)
```

3.185.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 516
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[(e*x)^m*(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; Free
Q[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] ||
(GtQ[a, 0] && GtQ[c, 0] &&  !IntegerQ[n]))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3567
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[x^m*((b
+ a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a,
b, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[
n, 0] && GtQ[m, 1])
```

3.185.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.62

method	result
derivativedivides	$\frac{i(\tan(dx+c)+i)^4}{4d\,a^3}$
default	$\frac{i(\tan(dx+c)+i)^4}{4d\,a^3}$
risch	$\frac{4i}{d\,a^3\left(e^{2i(dx+c)}+1\right)^4}$
norman	$\frac{\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{14\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{ad}+\frac{14\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{ad}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{ad}+\frac{16i\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{ad}-\frac{6i\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{ad}-\frac{6i\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{ad}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4 a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}$

input
```
int(sec(d*x+c)^5/(cos(d*x+c)*a+I*a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*I/d/a^3*(tan(d*x+c)+I)^4
```

3.185.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(26) = 52$.

Time = 0.23 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.03

$$\int \frac{\sec^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{4i}{a^3de^{(8i\,dx+8i\,c)}+4\,a^3de^{(6i\,dx+6i\,c)}+6\,a^3de^{(4i\,dx+4i\,c)}+4\,a^3de^{(2i\,dx+2i\,c)}+a^3d}$$

input
```
integrate(sec(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="frica
s")
```

output
```
4*I/(a^3*d*e^(8*I*d*x + 8*I*c) + 4*a^3*d*e^(6*I*d*x + 6*I*c) + 6*a^3*d*e^(
4*I*d*x + 4*I*c) + 4*a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)
```

3.185.6 Sympy [F]

$$\int \frac{\sec^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{\int \frac{\sec^5(c+dx)}{-i\sin^3(c+dx)-3\sin^2(c+dx)\cos(c+dx)+3i\sin(c+dx)\cos^2(c+dx)+\cos^3(c+dx)}\,dx}{a^3}$$

input
```
integrate(sec(d*x+c)**5/(a*cos(d*x+c)+I*a*sin(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)**5/(-I*sin(c + d*x)**3 - 3*sin(c + d*x)**2*cos(c + d
*x) + 3*I*sin(c + d*x)*cos(c + d*x)**2 + cos(c + d*x)**3), x)/a**3
```

3.185.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 240 vs. 2(26) = 52.

Time = 0.22 (sec) , antiderivative size = 240, normalized size of antiderivative = 7.06

$$\int \frac{\sec^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{2\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{3i\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{7\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{8i\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{7\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{3i\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{\left(a^3-\frac{4a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right)d}$$

input
```
integrate(sec(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="maxim
a")
```

output
```
2*(sin(d*x + c)/(cos(d*x + c) + 1) - 3*I*sin(d*x + c)^2/(cos(d*x + c) + 1)
^2 - 7*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 8*I*sin(d*x + c)^4/(cos(d*x +
 c) + 1)^4 + 7*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 3*I*sin(d*x + c)^6/(c
os(d*x + c) + 1)^6 - sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/((a^3 - 4*a^3*si
n(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1
)^4 - 4*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a^3*sin(d*x + c)^8/(cos(
d*x + c) + 1)^8)*d)
```

3.185.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.38

$$\int \frac{\sec^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = -\frac{-i\tan(dx+c)^4+4\tan(dx+c)^3+6i\tan(dx+c)^2-4\tan(dx+c)}{4a^3d}$$

input
```
integrate(sec(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="giac"
)
```

output
```
-1/4*(-I*tan(d*x + c)^4 + 4*tan(d*x + c)^3 + 6*I*tan(d*x + c)^2 - 4*tan(d*
x + c))/(a^3*d)
```

3.185.9 Mupad [B] (verification not implemented)

Time = 22.62 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.62

$$\int \frac{\sec^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{\sin\left(c+d\,x\right)^2\,1\mathrm{i}-\frac{\sin(2\,c+2\,d\,x)^2\,7\mathrm{i}}{4}+\sin\left(4\,c+4\,d\,x\right)}{4\,a^3\,d\,\left(\sin\left(c+d\,x\right)^2-1\right)^2}$$

input
```
int(1/(cos(c + d*x)^5*(a*cos(c + d*x) + a*sin(c + d*x)*1i)^3),x)
```

output
```
(sin(4*c + 4*d*x) - (sin(2*c + 2*d*x)^2*7i)/4 + sin(c + d*x)^2*1i)/(4*a^3*
d*(sin(c + d*x)^2 - 1)^2)
```

3.186 $\int \frac{\sec^6(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$

3.186.1 Optimal result

Integrand size = 31, antiderivative size = 104

$$\int \frac{\sec^6(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{7\text{arctanh}(\sin(c+dx))}{8a^3d} - \frac{4i\sec^3(c+dx)}{3a^3d} + \frac{i\sec^5(c+dx)}{5a^3d} + \frac{7\sec(c+dx)\tan(c+dx)}{8a^3d} - \frac{3\sec^3(c+dx)\tan(c+dx)}{4a^3d}$$

output
```
7/8*arctanh(sin(d*x+c))/a^3/d-4/3*I*sec(d*x+c)^3/a^3/d+1/5*I*sec(d*x+c)^5/
a^3/d+7/8*sec(d*x+c)*tan(d*x+c)/a^3/d-3/4*sec(d*x+c)^3*tan(d*x+c)/a^3/d
```

3.186.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.11

$$\int \frac{\sec^6(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{i\sec^8(c+dx)(-i\cos(3(c+dx))+\sin(3(c+dx)))\left(448+1680i\text{arctanh}\left(\sin(c)+\cos(c)\tan\left(\frac{dx}{2}\right)\right)\cos^5(c+\right.}{960a^3d(-i+\tan(c+dx))^3}$$

input
```
Integrate[Sec[c + d*x]^6/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

output
```
((I/960)*Sec[c + d*x]^8*((-I)*Cos[3*(c + d*x)] + Sin[3*(c + d*x)])*(448 +
(1680*I)*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]]*Cos[c + d*x]^5 + 640*Cos[2*
(c + d*x)] - (150*I)*Sin[2*(c + d*x)] + (105*I)*Sin[4*(c + d*x)]))/(a^3*d*
(-I + Tan[c + d*x])^3)
```

3.186.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3042, 3571, 3042, 3569, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^6(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(c+dx)^6(a\cos(c+dx)+ia\sin(c+dx))^3}dx$$

$$\Big\downarrow \text{3571}$$

$$\frac{i\int \sec^6(c+dx)(ia\cos(c+dx)+a\sin(c+dx))^3dx}{a^6}$$

$$\Big\downarrow \text{3042}$$

$$\frac{i\int \frac{(ia\cos(c+dx)+a\sin(c+dx))^3}{\cos(c+dx)^6}dx}{a^6}$$

$$\Big\downarrow \text{3569}$$

$$\frac{i\int \left(-ia^3\sec^3(c+dx)+a^3\tan^3(c+dx)\sec^3(c+dx)+3ia^3\tan^2(c+dx)\sec^3(c+dx)-3a^3\tan(c+dx)\sec^3(c+\right.}{a^6}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i\left(-\frac{7ia^3\text{arctanh}(\sin(c+dx))}{8d}+\frac{a^3\sec^5(c+dx)}{5d}-\frac{4a^3\sec^3(c+dx)}{3d}+\frac{3ia^3\tan(c+dx)\sec^3(c+dx)}{4d}-\frac{7ia^3\tan(c+dx)\sec(c+dx)}{8d}\right)}{a^6}$$

input
```
Int[Sec[c + d*x]^6/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

output
```
(I*((((-7*I)/8)*a^3*ArcTanh[Sin[c + d*x]])/d - (4*a^3*Sec[c + d*x]^3)/(3*d
) + (a^3*Sec[c + d*x]^5)/(5*d) - (((7*I)/8)*a^3*Sec[c + d*x]*Tan[c + d*x])
/d + (((3*I)/4)*a^3*Sec[c + d*x]^3*Tan[c + d*x])/d))/a^6
```

3.186.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3569
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

rule 3571
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^n*b^n   Int[Cos[c + d*x]^m
/(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &
& EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

3.186.4 Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.17

method	result
risch	$-\frac{i\left(105\,\mathrm{e}^{9i(dx+c)}+490\,\mathrm{e}^{7i(dx+c)}+896\,\mathrm{e}^{5i(dx+c)}+790\,\mathrm{e}^{3i(dx+c)}-105\,\mathrm{e}^{i(dx+c)}\right)}{60d\,a^3\left(\mathrm{e}^{2i(dx+c)}+1\right)^5}+\frac{7\ln\left(i+\mathrm{e}^{i(dx+c)}\right)}{8d\,a^3}-\frac{7\ln\left(\mathrm{e}^{i(dx+c)}-i\right.}{8d\,a^3}$
derivativedivides	$\frac{-\frac{i}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5}+\frac{2\left(\frac{1}{16}+\frac{13i}{16}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{2\left(-\frac{3}{8}-\frac{i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}+\frac{2\left(-\frac{5}{16}+\frac{11i}{16}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{2\left(-\frac{3}{4}+\frac{7i}{24}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{7\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8}}{d}$
default	$\frac{-\frac{i}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5}+\frac{2\left(\frac{1}{16}+\frac{13i}{16}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{2\left(-\frac{3}{8}-\frac{i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}+\frac{2\left(-\frac{5}{16}+\frac{11i}{16}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{2\left(-\frac{3}{4}+\frac{7i}{24}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{7\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8}}{d}$
norman	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}+\frac{13\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2ad}-\frac{13\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{2ad}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{4ad}+\frac{34i}{15ad}-\frac{16i\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{ad}+\frac{6i\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{ad}-\frac{16i\tan\left(\frac{dx}{2}+\right.}{3ad}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}$

input
```
int(sec(d*x+c)^6/(cos(d*x+c)*a+I*a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/60*I/d/a^3/(exp(2*I*(d*x+c))+1)^5*(105*exp(9*I*(d*x+c))+490*exp(7*I*(d*
x+c))+896*exp(5*I*(d*x+c))+790*exp(3*I*(d*x+c))-105*exp(I*(d*x+c)))+7/8/d/
a^3*ln(I+exp(I*(d*x+c)))-7/8/d/a^3*ln(exp(I*(d*x+c))-I)
```

3.186.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(90) = 180$.

Time = 0.25 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.67

$$\int \frac{\sec^6(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$$
$$= \frac{105\left(e^{(10i\,dx+10i\,c)}+5\,e^{(8i\,dx+8i\,c)}+10\,e^{(6i\,dx+6i\,c)}+10\,e^{(4i\,dx+4i\,c)}+5\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-10}{120\,(a^3de^{(}}$$

input
```
integrate(sec(d*x+c)^6/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="frica
s")
```

output
```
1/120*(105*(e^(10*I*d*x + 10*I*c) + 5*e^(8*I*d*x + 8*I*c) + 10*e^(6*I*d*x
+ 6*I*c) + 10*e^(4*I*d*x + 4*I*c) + 5*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*
x + I*c) + I) - 105*(e^(10*I*d*x + 10*I*c) + 5*e^(8*I*d*x + 8*I*c) + 10*e^
(6*I*d*x + 6*I*c) + 10*e^(4*I*d*x + 4*I*c) + 5*e^(2*I*d*x + 2*I*c) + 1)*lo
g(e^(I*d*x + I*c) - I) - 210*I*e^(9*I*d*x + 9*I*c) - 980*I*e^(7*I*d*x + 7*
I*c) - 1792*I*e^(5*I*d*x + 5*I*c) - 1580*I*e^(3*I*d*x + 3*I*c) + 210*I*e^(
I*d*x + I*c))/(a^3*d*e^(10*I*d*x + 10*I*c) + 5*a^3*d*e^(8*I*d*x + 8*I*c) +
 10*a^3*d*e^(6*I*d*x + 6*I*c) + 10*a^3*d*e^(4*I*d*x + 4*I*c) + 5*a^3*d*e^(
2*I*d*x + 2*I*c) + a^3*d)
```

3.186.6 Sympy [F]

$$\int \frac{\sec^6(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{\int \frac{\sec^6(c+dx)}{-i\sin^3(c+dx)-3\sin^2(c+dx)\cos(c+dx)+3i\sin(c+dx)\cos^2(c+dx)+\cos^3(c+dx)}\,dx}{a^3}$$

input
```
integrate(sec(d*x+c)**6/(a*cos(d*x+c)+I*a*sin(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)**6/(-I*sin(c + d*x)**3 - 3*sin(c + d*x)**2*cos(c + d
*x) + 3*I*sin(c + d*x)*cos(c + d*x)**2 + cos(c + d*x)**3), x)/a**3
```

3.186.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 341 vs. 2(90) = 180.

Time = 0.24 (sec) , antiderivative size = 341, normalized size of antiderivative = 3.28

$$\int \frac{\sec^6(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{\frac{16\left(-\frac{15i\sin(dx+c)}{\cos(dx+c)+1}+\frac{320\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{390i\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{400\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{960\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{390i\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{360\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{15i\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-136\right)}{-120ia^3+\frac{600ia^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{1200ia^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{1200ia^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{600ia^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{120ia^3\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}}{8d}$$

input
```
integrate(sec(d*x+c)^6/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="maxim
a")
```

output
```
1/8*(16*(-15*I*sin(d*x + c)/(cos(d*x + c) + 1) + 320*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 + 390*I*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 400*sin(d*x +
 c)^4/(cos(d*x + c) + 1)^4 + 960*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 390
*I*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 360*sin(d*x + c)^8/(cos(d*x + c)
+ 1)^8 + 15*I*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 136)/(-120*I*a^3 + 600
*I*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1200*I*a^3*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4 + 1200*I*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 600*I
*a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 120*I*a^3*sin(d*x + c)^10/(cos(
d*x + c) + 1)^10) + 7*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 - 7*log
(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^3)/d
```

3.186.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.58

$$\int \frac{\sec^6(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$$
$$= \frac{\frac{105\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^3} - \frac{105\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{a^3} + \frac{2\left(15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9+360i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8-390\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-960i\tan\left(\right.\right.}{}}{120\,d}$$

input
```
integrate(sec(d*x+c)^6/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="giac"
)
```

output
```
1/120*(105*log(tan(1/2*d*x + 1/2*c) + 1)/a^3 - 105*log(tan(1/2*d*x + 1/2*c
) - 1)/a^3 + 2*(15*tan(1/2*d*x + 1/2*c)^9 + 360*I*tan(1/2*d*x + 1/2*c)^8 -
 390*tan(1/2*d*x + 1/2*c)^7 - 960*I*tan(1/2*d*x + 1/2*c)^6 + 400*I*tan(1/2
*d*x + 1/2*c)^4 + 390*tan(1/2*d*x + 1/2*c)^3 - 320*I*tan(1/2*d*x + 1/2*c)^
2 - 15*tan(1/2*d*x + 1/2*c) + 136*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^5*a^3))
/d
```

3.186.9 Mupad [B] (verification not implemented)

Time = 25.99 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.44

$$\int \frac{\sec^6(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{7\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,a^3\,d}$$
$$+ \frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4} + \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8 6\mathrm{i} - \frac{13\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{2} - \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 16\mathrm{i} + \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 20\mathrm{i}}{3} + \frac{13\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2} - \frac{\tan\left(\right.}{}}{a^3\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)^5}$$

input
```
int(1/(cos(c + d*x)^6*(a*cos(c + d*x) + a*sin(c + d*x)*1i)^3),x)
```

output
```
(7*atanh(tan(c/2 + (d*x)/2)))/(4*a^3*d) + ((13*tan(c/2 + (d*x)/2)^3)/2 - (
tan(c/2 + (d*x)/2)^2*16i)/3 - tan(c/2 + (d*x)/2)/4 + (tan(c/2 + (d*x)/2)^4
*20i)/3 - tan(c/2 + (d*x)/2)^6*16i - (13*tan(c/2 + (d*x)/2)^7)/2 + tan(c/2
 + (d*x)/2)^8*6i + tan(c/2 + (d*x)/2)^9/4 + 34i/15)/(a^3*d*(tan(c/2 + (d*x
)/2)^2 - 1)^5)
```

3.187 $\int \cos^{-n}(c+dx)(a\cos(c+dx)+ia\sin(c+dx))^n\,dx$

3.187.1 Optimal result

Integrand size = 33, antiderivative size = 66

$$\int \cos^{-n}(c+dx)(a\cos(c+dx)+ia\sin(c+dx))^n\,dx = -\frac{i\cos^{-n}(c+dx)\,\text{Hypergeometric2F1}\left(1,n,1+n,\frac{1}{2}(1+i\tan(c+dx))\right)(a\cos(c+dx)+ia\sin(c+dx))^n}{2dn}$$

output
```
-1/2*I*hypergeom([1, n],[1+n],1/2+1/2*I*tan(d*x+c))*(a*cos(d*x+c)+I*a*sin(
d*x+c))^n/d/n/(cos(d*x+c)^n)
```

3.187.2 Mathematica [A] (verified)

Time = 4.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.36

$$\int \cos^{-n}(c+dx)(a\cos(c+dx)+ia\sin(c+dx))^n\,dx = \frac{\cos^{-n}(c+dx)(a(\cos(c+dx)+i\sin(c+dx)))^n\left(-2i(1+n)+n\,\text{Hypergeometric2F1}\left(1,1+n,2+n,\frac{1}{2}(1\right.\right.}{4dn(1+n)}$$

input
```
Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^n/Cos[c + d*x]^n,x]
```

output
```
((a*(Cos[c + d*x] + I*Sin[c + d*x]))^n*((-2*I)*(1 + n) + n*Hypergeometric2
F1[1, 1 + n, 2 + n, (1 + I*Tan[c + d*x])/2]*(-I + Tan[c + d*x])))/(4*d*n*(
1 + n)*Cos[c + d*x]^n)
```

3.187.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {3042, 3563}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{-n}(c+dx)(a\cos(c+dx)+ia\sin(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(c+dx)^{-n}(a\cos(c+dx)+ia\sin(c+dx))^n dx$$

$$\Big\downarrow \text{3563}$$

$$-\frac{i\cos^{-n}(c+dx)\,\text{Hypergeometric2F1}\left(1,n,n+1,\frac{1}{2}(i\tan(c+dx)+1)\right)(a\cos(c+dx)+ia\sin(c+dx))^n}{2dn}$$

input
```
Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^n/Cos[c + d*x]^n,x]
```

output
```
((-1/2*I)*Hypergeometric2F1[1, n, 1 + n, (1 + I*Tan[c + d*x])/2]*(a*Cos[c
+ d*x] + I*a*Sin[c + d*x])^n)/(d*n*Cos[c + d*x]^n)
```

3.187.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3563
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*((a*Cos[c + d*x] + b*Si
n[c + d*x])^n/(2*a*d*n*Cos[c + d*x]^n))*Hypergeometric2F1[1, n, n + 1, (a +
 b*Tan[c + d*x])/(2*a)], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[m + n, 0] &
& EqQ[a^2 + b^2, 0] &&  !IntegerQ[n]
```

3.187.4 Maple [F]

$$\int (\cos(dx+c)\,a+ia\sin(dx+c))^n\cos(dx+c)^{-n}\,dx$$

input
```
int((cos(d*x+c)*a+I*a*sin(d*x+c))^n/(cos(d*x+c)^n),x)
```

output
```
int((cos(d*x+c)*a+I*a*sin(d*x+c))^n/(cos(d*x+c)^n),x)
```

3.187.5 Fricas [F]

$$\int \cos^{-n}(c+dx)(a\cos(c+dx)+ia\sin(c+dx))^n\,dx=\int \frac{(a\cos(dx+c)+i\,a\sin(dx+c))^n}{\cos(dx+c)^n}\,dx$$

input
```
integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^n/(cos(d*x+c)^n),x, algorithm="fri
cas")
```

output
```
integral(e^(I*d*n*x + I*c*n + n*log(a))/(1/2*(e^(2*I*d*x + 2*I*c) + 1)*e^(
-I*d*x - I*c))^n, x)
```

3.187.6 Sympy [F]

$$\int \cos^{-n}(c+dx)(a\cos(c+dx)+ia\sin(c+dx))^n\,dx$$
$$=\int (a(i\sin(c+dx)+\cos(c+dx)))^n\cos^{-n}(c+dx)\,dx$$

input
```
integrate((a*cos(d*x+c)+I*a*sin(d*x+c))**n/(cos(d*x+c)**n),x)
```

output
```
Integral((a*(I*sin(c + d*x) + cos(c + d*x)))**n/cos(c + d*x)**n, x)
```

3.187.7 Maxima [F]

$$\int \cos^{-n}(c+dx)(a\cos(c+dx)+ia\sin(c+dx))^n\,dx = \int \frac{(a\cos(dx+c)+i\,a\sin(dx+c))^n}{\cos(dx+c)^n}\,dx$$

input
```
integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^n/(cos(d*x+c)^n),x, algorithm="max
ima")
```

output
```
integrate((a*cos(d*x + c) + I*a*sin(d*x + c))^n*cos(d*x + c)^(-n), x)
```

3.187.8 Giac [F]

$$\int \cos^{-n}(c+dx)(a\cos(c+dx)+ia\sin(c+dx))^n\,dx = \int \frac{(a\cos(dx+c)+i\,a\sin(dx+c))^n}{\cos(dx+c)^n}\,dx$$

input
```
integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^n/(cos(d*x+c)^n),x, algorithm="gia
c")
```

output
```
integrate((a*cos(d*x + c) + I*a*sin(d*x + c))^n/cos(d*x + c)^n, x)
```

3.187.9 Mupad [F(-1)]

Timed out.

$$\int \cos^{-n}(c+dx)(a\cos(c+dx)+ia\sin(c+dx))^n\,dx = \int \frac{(a\cos(c+d\,x)+a\sin(c+d\,x)\;1\mathrm{i})^n}{\cos(c+d\,x)^n}\,dx$$

input
```
int((a*cos(c + d*x) + a*sin(c + d*x)*1i)^n/cos(c + d*x)^n,x)
```

output
```
int((a*cos(c + d*x) + a*sin(c + d*x)*1i)^n/cos(c + d*x)^n, x)
```

3.188 $\int \frac{1}{\sec(x)+\tan(x)} dx$

3.188.1 Optimal result

Integrand size = 7, antiderivative size = 5

$$\int \frac{1}{\sec(x)+\tan(x)} dx = \log(1+\sin(x))$$

output
```
ln(1+sin(x))
```

3.188.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 16 vs. 2(5) = 10.

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 3.20

$$\int \frac{1}{\sec(x)+\tan(x)} dx = 2\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[(Sec[x] + Tan[x])^(-1),x]
```

output
```
2*Log[Cos[x/2] + Sin[x/2]]
```

3.188.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 3638, 3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\tan(x)+\sec(x)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\tan(x)+\sec(x)}dx$$
$$\Big\downarrow \text{3638}$$
$$\int \frac{\cos(x)}{\sin(x)+1}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos(x)}{\sin(x)+1}dx$$
$$\Big\downarrow \text{3146}$$
$$\int \frac{1}{\sin(x)+1}d\sin(x)$$
$$\Big\downarrow \text{16}$$
$$\log(\sin(x)+1)$$

input
```
Int[(Sec[x] + Tan[x])^(-1),x]
```

output
```
Log[1 + Sin[x]]
```

3.188.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3638
```
Int[((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)])
^(-1), x_Symbol] :> Int[Cos[d + e*x]/(b + a*Cos[d + e*x] + c*Sin[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

3.188.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

method	result	size
default	$\ln(1+\sin(x))$	6
risch	$-ix+2\ln(i+e^{ix})$	17

input
```
int(1/(sec(x)+tan(x)),x,method=_RETURNVERBOSE)
```

output
```
ln(1+sin(x))
```

3.188.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sec(x)+\tan(x)}\,dx = \log\left(\sin\left(x\right)+1\right)$$

input
```
integrate(1/(sec(x)+tan(x)),x, algorithm="fricas")
```

output
```
log(sin(x) + 1)
```

3.188.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(5) = 10.

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 3.40

$$\int \frac{1}{\sec(x)+\tan(x)}\,dx = \log\left(\tan\left(x\right)+\sec\left(x\right)\right)-\frac{\log\left(\tan^{2}\left(x\right)+1\right)}{2}$$

input
```
integrate(1/(sec(x)+tan(x)),x)
```

output
```
log(tan(x) + sec(x)) - log(tan(x)**2 + 1)/2
```

3.188.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(5) = 10.

Time = 0.22 (sec) , antiderivative size = 31, normalized size of antiderivative = 6.20

$$\int \frac{1}{\sec(x)+\tan(x)}\,dx = 2\,\log\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}+1\right)-\log\left(\frac{\sin\left(x\right)^{2}}{\left(\cos\left(x\right)+1\right)^{2}}+1\right)$$

input
```
integrate(1/(sec(x)+tan(x)),x, algorithm="maxima")
```

output
```
2*log(sin(x)/(cos(x) + 1) + 1) - log(sin(x)^2/(cos(x) + 1)^2 + 1)
```

3.188.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(5) = 10$.

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 4.40

$$\int \frac{1}{\sec(x)+\tan(x)}\,dx = -\log\left(\tan\left(\frac{1}{2}x\right)^2+1\right)+2\log\left(\left|\tan\left(\frac{1}{2}x\right)+1\right|\right)$$

input
```
integrate(1/(sec(x)+tan(x)),x, algorithm="giac")
```

output
```
-log(tan(1/2*x)^2 + 1) + 2*log(abs(tan(1/2*x) + 1))
```

3.188.9 Mupad [B] (verification not implemented)

Time = 24.59 (sec) , antiderivative size = 21, normalized size of antiderivative = 4.20

$$\int \frac{1}{\sec(x)+\tan(x)}\,dx = 2\ln\left(\tan\left(\frac{x}{2}\right)+1\right)-\ln\left(\tan\left(\frac{x}{2}\right)^2+1\right)$$

input
```
int(1/(tan(x) + 1/cos(x)),x)
```

output
```
2*log(tan(x/2) + 1) - log(tan(x/2)^2 + 1)
```

3.189 $\int \frac{\sin(x)}{\sec(x)+\tan(x)}\,dx$

3.189.1 Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\sin(x)}{\sec(x)+\tan(x)}\,dx = -\log(1+\sin(x))+\sin(x)$$

output
```
-ln(1+sin(x))+sin(x)
```

3.189.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.90

$$\int \frac{\sin(x)}{\sec(x)+\tan(x)}\,dx = -2\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)+\sin(x)$$

input
```
Integrate[Sin[x]/(Sec[x] + Tan[x]),x]
```

output
```
-2*Log[Cos[x/2] + Sin[x/2]] + Sin[x]
```

3.189.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 4891, 3042, 3312, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)}{\tan(x)+\sec(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)}{\tan(x)+\sec(x)}dx$$

$$\Big\downarrow \text{4891}$$

$$\int \frac{\sin(x)\cos(x)}{\sin(x)+1}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)\cos(x)}{\sin(x)+1}dx$$

$$\Big\downarrow \text{3312}$$

$$\int \frac{\sin(x)}{\sin(x)+1}d\sin(x)$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{1}{-\sin(x)-1}+1\right)d\sin(x)$$

$$\Big\downarrow \text{2009}$$

$$\sin(x)-\log(\sin(x)+1)$$

input
```
Int[Sin[x]/(Sec[x] + Tan[x]),x]
```

output
```
-Log[1 + Sin[x]] + Sin[x]
```

3.189.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3312
```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b*f)   Su
bst[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x]
```

rule 4891
```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.189.4 Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$-\ln\left(1+\sin\left(x\right)\right)+\sin\left(x\right)$	11
default	$-\ln\left(1+\sin\left(x\right)\right)+\sin\left(x\right)$	11
risch	$ix-\frac{i\mathrm{e}^{ix}}{2}+\frac{i\mathrm{e}^{-ix}}{2}-2\ln\left(i+\mathrm{e}^{ix}\right)$	33

input
```
int(sin(x)/(sec(x)+tan(x)),x,method=_RETURNVERBOSE)
```

output
```
-ln(1+sin(x))+sin(x)
```

3.189.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{\sec(x)+\tan(x)}\,dx = -\log\left(\sin\left(x\right)+1\right)+\sin\left(x\right)$$

input
```
integrate(sin(x)/(sec(x)+tan(x)),x, algorithm="fricas")
```

output
```
-log(sin(x) + 1) + sin(x)
```

3.189.6 Sympy [F]

$$\int \frac{\sin(x)}{\sec(x)+\tan(x)}\,dx = \int \frac{\sin\left(x\right)}{\tan\left(x\right)+\sec\left(x\right)}\,dx$$

input
```
integrate(sin(x)/(sec(x)+tan(x)),x)
```

output
```
Integral(sin(x)/(tan(x) + sec(x)), x)
```

3.189.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(10) = 20.

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 5.40

$$\int \frac{\sin(x)}{\sec(x)+\tan(x)}\,dx = \frac{2\,\sin\left(x\right)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)\left(\cos\left(x\right)+1\right)} - 2\,\log\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}+1\right)+\log\left(\frac{\sin\left(x\right)^2}{\left(\cos\left(x\right)+1\right)^2}+1\right)$$

input
```
integrate(sin(x)/(sec(x)+tan(x)),x, algorithm="maxima")
```

output
```
2*sin(x)/((sin(x)^2/(cos(x) + 1)^2 + 1)*(cos(x) + 1)) - 2*log(sin(x)/(cos(
x) + 1) + 1) + log(sin(x)^2/(cos(x) + 1)^2 + 1)
```

3.189.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{\sec(x)+\tan(x)}\,dx = -\log\left(\sin\left(x\right)+1\right)+\sin\left(x\right)$$

input
```
integrate(sin(x)/(sec(x)+tan(x)),x, algorithm="giac")
```

output
```
-log(sin(x) + 1) + sin(x)
```

3.189.9 Mupad [B] (verification not implemented)

Time = 22.56 (sec) , antiderivative size = 21, normalized size of antiderivative = 2.10

$$\int \frac{\sin(x)}{\sec(x)+\tan(x)}\,dx = \ln\left(\tan\left(\frac{x}{2}\right)^2+1\right)-2\,\ln\left(\tan\left(\frac{x}{2}\right)+1\right)+\sin\left(x\right)$$

input
```
int(sin(x)/(tan(x) + 1/cos(x)),x)
```

output
```
log(tan(x/2)^2 + 1) - 2*log(tan(x/2) + 1) + sin(x)
```

3.190 $\int \frac{\cos(x)}{\sec(x)+\tan(x)}\,dx$

3.190.1 Optimal result

Integrand size = 10, antiderivative size = 4

$$\int \frac{\cos(x)}{\sec(x)+\tan(x)}\,dx = x+\cos(x)$$

output
```
x+cos(x)
```

3.190.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 63 vs. $2(4) = 8$.

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 15.75

$$\int \frac{\cos(x)}{\sec(x)+\tan(x)}\,dx = -\frac{\cos^3(x)\left(2\arcsin\left(\frac{\sqrt{1-\sin(x)}}{\sqrt{2}}\right)\sqrt{1-\sin(x)}+(-1+\sin(x))\sqrt{1+\sin(x)}\right)}{(-1+\sin(x))^2(1+\sin(x))^{3/2}}$$

input
```
Integrate[Cos[x]/(Sec[x] + Tan[x]),x]
```

output
```
-((Cos[x]^3*(2*ArcSin[Sqrt[1 - Sin[x]]/Sqrt[2]]*Sqrt[1 - Sin[x]] + (-1 + S
in[x])*Sqrt[1 + Sin[x]]))/((-1 + Sin[x])^2*(1 + Sin[x])^(3/2)))
```

3.190.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4891, 3042, 3161, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(x)}{\tan(x)+\sec(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(x)}{\tan(x)+\sec(x)}dx$$

$$\Big\downarrow \text{4891}$$

$$\int \frac{\cos^2(x)}{\sin(x)+1}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(x)^2}{\sin(x)+1}dx$$

$$\Big\downarrow \text{3161}$$

$$\int 1dx+\cos(x)$$

$$\Big\downarrow \text{24}$$

$$x+\cos(x)$$

input
```
Int[Cos[x]/(Sec[x] + Tan[x]),x]
```

output
```
x + Cos[x]
```

3.190.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3161
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Si
mp[g^2/a   Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x
] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

rule 4891
```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.190.4 Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

method	result	size
risch	$x + \cos(x)$	5
default	$\frac{2}{1+\tan\left(\frac{x}{2}\right)^2} + 2\arctan\left(\tan\left(\frac{x}{2}\right)\right)$	21

input
```
int(cos(x)/(sec(x)+tan(x)),x,method=_RETURNVERBOSE)
```

output
```
x+cos(x)
```

3.190.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{\sec(x) + \tan(x)}\, dx = x + \cos{(x)}$$

input
```
integrate(cos(x)/(sec(x)+tan(x)),x, algorithm="fricas")
```

output
```
x + cos(x)
```

3.190.6 Sympy [F]

$$\int \frac{\cos(x)}{\sec(x) + \tan(x)}\, dx = \int \frac{\cos{(x)}}{\tan{(x)} + \sec{(x)}}\, dx$$

input
```
integrate(cos(x)/(sec(x)+tan(x)),x)
```

output
```
Integral(cos(x)/(tan(x) + sec(x)), x)
```

3.190.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. 2(4) = 8.

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 7.50

$$\int \frac{\cos(x)}{\sec(x) + \tan(x)}\, dx = \frac{2}{\frac{\sin(x)^2}{(\cos(x)+1)^2} + 1} + 2\arctan\left(\frac{\sin{(x)}}{\cos{(x)} + 1}\right)$$

input
```
integrate(cos(x)/(sec(x)+tan(x)),x, algorithm="maxima")
```

output
```
2/(sin(x)^2/(cos(x) + 1)^2 + 1) + 2*arctan(sin(x)/(cos(x) + 1))
```

3.190.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14 vs. 2(4) = 8.

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 3.50

$$\int \frac{\cos(x)}{\sec(x)+\tan(x)}\,dx = x + \frac{2}{\tan\left(\frac{1}{2}\,x\right)^2+1}$$

input
```
integrate(cos(x)/(sec(x)+tan(x)),x, algorithm="giac")
```

output
```
x + 2/(tan(1/2*x)^2 + 1)
```

3.190.9 Mupad [B] (verification not implemented)

Time = 22.65 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{\sec(x)+\tan(x)}\,dx = x + \cos\left(x\right)$$

input
```
int(cos(x)/(tan(x) + 1/cos(x)),x)
```

output
```
x + cos(x)
```

3.191 $\int \frac{\tan(x)}{\sec(x)+\tan(x)}\,dx$

3.191.1 Optimal result

Integrand size = 10, antiderivative size = 11

$$\int \frac{\tan(x)}{\sec(x)+\tan(x)}\,dx = x+\frac{\cos(x)}{1+\sin(x)}$$

output
```
x+cos(x)/(1+sin(x))
```

3.191.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 25 vs. 2(11) = 22.

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.27

$$\int \frac{\tan(x)}{\sec(x)+\tan(x)}\,dx = x-\frac{2\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}$$

input
```
Integrate[Tan[x]/(Sec[x] + Tan[x]),x]
```

output
```
x - (2*Sin[x/2])/(Cos[x/2] + Sin[x/2])
```

3.191.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 4891, 3042, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(x)}{\tan(x)+\sec(x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\tan(x)}{\tan(x)+\sec(x)}dx$$
$$\Big\downarrow \text{4891}$$
$$\int \frac{\sin(x)}{\sin(x)+1}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin(x)}{\sin(x)+1}dx$$
$$\Big\downarrow \text{3214}$$
$$x-\int \frac{1}{\sin(x)+1}dx$$
$$\Big\downarrow \text{3042}$$
$$x-\int \frac{1}{\sin(x)+1}dx$$
$$\Big\downarrow \text{3127}$$
$$x+\frac{\cos(x)}{\sin(x)+1}$$

input
```
Int[Tan[x]/(Sec[x] + Tan[x]),x]
```

output
```
x + Cos[x]/(1 + Sin[x])
```

3.191.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 4891
```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.191.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

method	result	size
risch	$x+\frac{2}{i+{\mathrm e}^{i x}}$	15
default	$\frac{2}{\tan\left(\frac{x}{2}\right)+1}+2 \arctan\left(\tan\left(\frac{x}{2}\right)\right)$	19

input
```
int(tan(x)/(sec(x)+tan(x)),x,method=_RETURNVERBOSE)
```

output
```
x+2/(I+exp(I*x))
```

3.191.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. 2(11) = 22.

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.18

$$\int \frac{\tan(x)}{\sec(x)+\tan(x)}\,dx = \frac{(x+1)\cos(x)+(x-1)\sin(x)+x+1}{\cos(x)+\sin(x)+1}$$

input
```
integrate(tan(x)/(sec(x)+tan(x)),x, algorithm="fricas")
```

output
```
((x + 1)*cos(x) + (x - 1)*sin(x) + x + 1)/(cos(x) + sin(x) + 1)
```

3.191.6 Sympy [F]

$$\int \frac{\tan(x)}{\sec(x)+\tan(x)}\,dx = \int \frac{\tan(x)}{\tan(x)+\sec(x)}\,dx$$

input
```
integrate(tan(x)/(sec(x)+tan(x)),x)
```

output
```
Integral(tan(x)/(tan(x) + sec(x)), x)
```

3.191.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. 2(11) = 22.

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.55

$$\int \frac{\tan(x)}{\sec(x)+\tan(x)}\,dx = \frac{2}{\frac{\sin(x)}{\cos(x)+1}+1} + 2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

input
```
integrate(tan(x)/(sec(x)+tan(x)),x, algorithm="maxima")
```

output
```
2/(sin(x)/(cos(x) + 1) + 1) + 2*arctan(sin(x)/(cos(x) + 1))
```

3.191.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{\tan(x)}{\sec(x) + \tan(x)}\, dx = x + \frac{2}{\tan\left(\frac{1}{2}\,x\right) + 1}$$

input
```
integrate(tan(x)/(sec(x)+tan(x)),x, algorithm="giac")
```

output
```
x + 2/(tan(1/2*x) + 1)
```

3.191.9 Mupad [B] (verification not implemented)

Time = 22.61 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{\tan(x)}{\sec(x) + \tan(x)}\, dx = x + \frac{2}{\tan\left(\frac{x}{2}\right) + 1}$$

input
```
int(tan(x)/(tan(x) + 1/cos(x)),x)
```

output
```
x + 2/(tan(x/2) + 1)
```

3.192 $\int \frac{\cot(x)}{\sec(x)+\tan(x)}\,dx$

3.192.1 Optimal result

Integrand size = 10, antiderivative size = 9

$$\int \frac{\cot(x)}{\sec(x)+\tan(x)}\,dx=-x-\operatorname{arctanh}(\cos(x))$$

output
```
-x-arctanh(cos(x))
```

3.192.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 20 vs. 2(9) = 18.

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.22

$$\int \frac{\cot(x)}{\sec(x)+\tan(x)}\,dx=-x-\log\left(\cos\left(\frac{x}{2}\right)\right)+\log\left(\sin\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[Cot[x]/(Sec[x] + Tan[x]),x]
```

output
```
-x - Log[Cos[x/2]] + Log[Sin[x/2]]
```

3.192.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {3042, 4891, 3042, 3318, 24, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(x)}{\tan(x)+\sec(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cot(x)}{\tan(x)+\sec(x)}dx$$

$$\Big\downarrow \text{4891}$$

$$\int \frac{\cos(x)\cot(x)}{\sin(x)+1}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(x)^2}{\sin(x)(\sin(x)+1)}dx$$

$$\Big\downarrow \text{3318}$$

$$\int \csc(x)dx-\int 1dx$$

$$\Big\downarrow \text{24}$$

$$\int \csc(x)dx-x$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(x)dx-x$$

$$\Big\downarrow \text{4257}$$

$$-\text{arctanh}(\cos(x))-x$$

input
```
Int[Cot[x]/(Sec[x] + Tan[x]),x]
```

output
```
-x - ArcTanh[Cos[x]]
```

3.192. $\int \frac{\cot(x)}{\sec(x)+\tan(x)}\,dx$

3.192.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3318
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g^2/a   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Simp[g^2/(b*d)   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b,
d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4891
```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.192.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.56

method	result	size
default	$\ln\left(\tan\left(\frac{x}{2}\right)\right)-2\arctan\left(\tan\left(\frac{x}{2}\right)\right)$	14
risch	$-x+\ln\left(\mathrm{e}^{ix}-1\right)-\ln\left(\mathrm{e}^{ix}+1\right)$	23

input
```
int(cot(x)/(sec(x)+tan(x)),x,method=_RETURNVERBOSE)
```

output
```
ln(tan(1/2*x))-2*arctan(tan(1/2*x))
```

3.192.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(9) = 18$.

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.44

$$\int \frac{\cot(x)}{\sec(x)+\tan(x)}\,dx = -x - \frac{1}{2}\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right) + \frac{1}{2}\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)$$

input
```
integrate(cot(x)/(sec(x)+tan(x)),x, algorithm="fricas")
```

output
```
-x - 1/2*log(1/2*cos(x) + 1/2) + 1/2*log(-1/2*cos(x) + 1/2)
```

3.192.6 Sympy [F]

$$\int \frac{\cot(x)}{\sec(x)+\tan(x)}\,dx = \int \frac{\cot(x)}{\tan(x)+\sec(x)}\,dx$$

input
```
integrate(cot(x)/(sec(x)+tan(x)),x)
```

output
```
Integral(cot(x)/(tan(x) + sec(x)), x)
```

3.192.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(9) = 18$.

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.56

$$\int \frac{\cot(x)}{\sec(x)+\tan(x)}\,dx = -2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right) + \log\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

input
```
integrate(cot(x)/(sec(x)+tan(x)),x, algorithm="maxima")
```

output
```
-2*arctan(sin(x)/(cos(x) + 1)) + log(sin(x)/(cos(x) + 1))
```

3.192.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

$$\int \frac{\cot(x)}{\sec(x)+\tan(x)}\,dx = -x+\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)$$

input
```
integrate(cot(x)/(sec(x)+tan(x)),x, algorithm="giac")
```

output
```
-x + log(abs(tan(1/2*x)))
```

3.192.9 Mupad [B] (verification not implemented)

Time = 22.43 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.56

$$\int \frac{\cot(x)}{\sec(x)+\tan(x)}\,dx = 2\,\mathrm{atan}\left(\frac{8}{4\tan\left(\frac{x}{2}\right)+4}-1\right)+\ln\left(\tan\left(\frac{x}{2}\right)\right)$$

input
```
int(cot(x)/(tan(x) + 1/cos(x)),x)
```

output
```
2*atan(8/(4*tan(x/2) + 4) - 1) + log(tan(x/2))
```

3.193 $\int \frac{\sec(x)}{\sec(x)+\tan(x)}\,dx$

3.193.1 Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\sec(x)}{\sec(x)+\tan(x)}\,dx = -\frac{\cos(x)}{1+\sin(x)}$$

output
```
-cos(x)/(1+sin(x))
```

3.193.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 23 vs. 2(10) = 20.

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.30

$$\int \frac{\sec(x)}{\sec(x)+\tan(x)}\,dx = \frac{2\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}$$

input
```
Integrate[Sec[x]/(Sec[x] + Tan[x]),x]
```

output
```
(2*Sin[x/2])/(Cos[x/2] + Sin[x/2])
```

3.193.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3644, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(x)}{\tan(x)+\sec(x)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sec(x)}{\tan(x)+\sec(x)}dx$$

$$\Big\downarrow \text{ 3644}$$

$$\int \frac{1}{\sin(x)+1}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin(x)+1}dx$$

$$\Big\downarrow \text{ 3127}$$

$$-\frac{\cos(x)}{\sin(x)+1}$$

input
```
Int[Sec[x]/(Sec[x] + Tan[x]),x]
```

output
```
-(Cos[x]/(1 + Sin[x]))
```

3.193.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3644
```
Int[sec[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] +
(c_.)*tan[(d_.) + (e_.)*(x_)])^(m_), x_Symbol] :> Int[1/(b + a*Cos[d + e*x]
 + c*Sin[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && I
ntegerQ[n]
```

3.193.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
default	$-\frac{2}{\tan\left(\frac{x}{2}\right)+1}$	11
risch	$-\frac{2}{i+e^{ix}}$	13

input
```
int(sec(x)/(sec(x)+tan(x)),x,method=_RETURNVERBOSE)
```

output
```
-2/(tan(1/2*x)+1)
```

3.193.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

$$\int \frac{\sec(x)}{\sec(x)+\tan(x)}\,dx = -\frac{\cos(x)-\sin(x)+1}{\cos(x)+\sin(x)+1}$$

input
```
integrate(sec(x)/(sec(x)+tan(x)),x, algorithm="fricas")
```

output
```
-(cos(x) - sin(x) + 1)/(cos(x) + sin(x) + 1)
```

3.193.6 Sympy [F]

$$\int \frac{\sec(x)}{\sec(x)+\tan(x)}\,dx = \int \frac{\sec\left(x\right)}{\tan\left(x\right)+\sec\left(x\right)}\,dx$$

input
```
integrate(sec(x)/(sec(x)+tan(x)),x)
```

output
```
Integral(sec(x)/(tan(x) + sec(x)), x)
```

3.193.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.50

$$\int \frac{\sec(x)}{\sec(x)+\tan(x)}\,dx = -\frac{2}{\frac{\sin(x)}{\cos(x)+1}+1}$$

input
```
integrate(sec(x)/(sec(x)+tan(x)),x, algorithm="maxima")
```

output
```
-2/(sin(x)/(cos(x) + 1) + 1)
```

3.193.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\sec(x)}{\sec(x)+\tan(x)}\,dx = -\frac{2}{\tan\left(\frac{1}{2}\,x\right)+1}$$

input
```
integrate(sec(x)/(sec(x)+tan(x)),x, algorithm="giac")
```

output
```
-2/(tan(1/2*x) + 1)
```

3.193.9 Mupad [B] (verification not implemented)

Time = 22.47 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\sec(x)}{\sec(x)+\tan(x)}\,dx = -\frac{2}{\tan\left(\frac{x}{2}\right)+1}$$

input
```
int(1/(cos(x)*(tan(x) + 1/cos(x))),x)
```

output
```
-2/(tan(x/2) + 1)
```

3.194 $\int \frac{\csc(x)}{\sec(x)+\tan(x)}\,dx$

3.194.1 Optimal result

Integrand size = 10, antiderivative size = 11

$$\int \frac{\csc(x)}{\sec(x)+\tan(x)}\,dx = \log(\sin(x)) - \log(1+\sin(x))$$

output
```
ln(sin(x))-ln(1+sin(x))
```

3.194.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.82

$$\int \frac{\csc(x)}{\sec(x)+\tan(x)}\,dx = -2\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)+\log(\sin(x))$$

input
```
Integrate[Csc[x]/(Sec[x] + Tan[x]),x]
```

output
```
-2*Log[Cos[x/2] + Sin[x/2]] + Log[Sin[x]]
```

3.194.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {3042, 4891, 3042, 3186, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(x)}{\tan(x)+\sec(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc(x)}{\tan(x)+\sec(x)}dx$$

$$\Big\downarrow \text{ 4891}$$

$$\int \frac{\cot(x)}{\sin(x)+1}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{(\sin(x)+1)\tan(x)}dx$$

$$\Big\downarrow \text{ 3186}$$

$$\int \frac{\csc(x)}{\sin(x)+1}d\sin(x)$$

$$\Big\downarrow \text{ 47}$$

$$\int \csc(x)d\sin(x)-\int \frac{1}{\sin(x)+1}d\sin(x)$$

$$\Big\downarrow \text{ 14}$$

$$\log(\sin(x))-\int \frac{1}{\sin(x)+1}d\sin(x)$$

$$\Big\downarrow \text{ 16}$$

$$\log(\sin(x))-\log(\sin(x)+1)$$

input
```
Int[Csc[x]/(Sec[x] + Tan[x]),x]
```

output
```
Log[Sin[x]] - Log[1 + Sin[x]]
```

3.194. $\int \frac{\csc(x)}{\sec(x)+\tan(x)}\,dx$

3.194.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

rule 4891
```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.194.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$-\ln\left(\csc\left(x\right)+1\right)$	8
default	$-\ln\left(\csc\left(x\right)+1\right)$	8
risch	$-2\ln\left(i+{\mathrm e}^{i x}\right)+\ln\left({\mathrm e}^{2 i x}-1\right)$	21

input
```
int(csc(x)/(sec(x)+tan(x)),x,method=_RETURNVERBOSE)
```

output
```
-ln(csc(x)+1)
```

3.194.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{\csc(x)}{\sec(x)+\tan(x)}\,dx = \log\left(\frac{1}{2}\sin\left(x\right)\right) - \log\left(\sin\left(x\right)+1\right)$$

input
```
integrate(csc(x)/(sec(x)+tan(x)),x, algorithm="fricas")
```

output
```
log(1/2*sin(x)) - log(sin(x) + 1)
```

3.194.6 Sympy [F]

$$\int \frac{\csc(x)}{\sec(x)+\tan(x)}\,dx = \int \frac{\csc\left(x\right)}{\tan\left(x\right)+\sec\left(x\right)}\,dx$$

input
```
integrate(csc(x)/(sec(x)+tan(x)),x)
```

output
```
Integral(csc(x)/(tan(x) + sec(x)), x)
```

3.194.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(11) = 22.

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.27

$$\int \frac{\csc(x)}{\sec(x)+\tan(x)}\,dx = -2\log\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}+1\right) + \log\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}\right)$$

input
```
integrate(csc(x)/(sec(x)+tan(x)),x, algorithm="maxima")
```

output
```
-2*log(sin(x)/(cos(x) + 1) + 1) + log(sin(x)/(cos(x) + 1))
```

3.194.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{\csc(x)}{\sec(x)+\tan(x)}\,dx = -\log(\sin(x)+1)+\log(|\sin(x)|)$$

input
```
integrate(csc(x)/(sec(x)+tan(x)),x, algorithm="giac")
```

output
```
-log(sin(x) + 1) + log(abs(sin(x)))
```

3.194.9 Mupad [B] (verification not implemented)

Time = 22.41 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \frac{\csc(x)}{\sec(x)+\tan(x)}\,dx = \ln\left(\tan\left(\frac{x}{2}\right)\right)-2\ln\left(\tan\left(\frac{x}{2}\right)+1\right)$$

input
```
int(1/(sin(x)*(tan(x) + 1/cos(x))),x)
```

output
```
log(tan(x/2)) - 2*log(tan(x/2) + 1)
```

3.195 $\int \frac{1}{\sec(x)-\tan(x)}\,dx$

3.195.1 Optimal result

Integrand size = 9, antiderivative size = 9

$$\int \frac{1}{\sec(x)-\tan(x)}\,dx = -\log(1-\sin(x))$$

output
```
-ln(1-sin(x))
```

3.195.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sec(x)-\tan(x)}\,dx = -2\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[(Sec[x] - Tan[x])^(-1),x]
```

output
```
-2*Log[Cos[x/2] - Sin[x/2]]
```

3.195.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {3042, 3638, 3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec(x)-\tan(x)}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{\sec(x)-\tan(x)}dx$$
$$\Big\downarrow \text{ 3638}$$
$$\int \frac{\cos(x)}{1-\sin(x)}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\cos(x)}{1-\sin(x)}dx$$
$$\Big\downarrow \text{ 3146}$$
$$-\int \frac{1}{1-\sin(x)}d(-\sin(x))$$
$$\Big\downarrow \text{ 16}$$
$$-\log(1-\sin(x))$$

input
```
Int[(Sec[x] - Tan[x])^(-1),x]
```

output
```
-Log[1 - Sin[x]]
```

3.195.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3638
```
Int[((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)])
^(-1), x_Symbol] :> Int[Cos[d + e*x]/(b + a*Cos[d + e*x] + c*Sin[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

3.195.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

method	result	size
default	$-\ln\left(\sin\left(x\right)-1\right)$	8
risch	$ix-2\ln\left(\mathrm{e}^{ix}-i\right)$	17

input
```
int(1/(sec(x)-tan(x)),x,method=_RETURNVERBOSE)
```

output
```
-ln(sin(x)-1)
```

3.195.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sec(x)-\tan(x)}\,dx = -\log\left(-\sin\left(x\right)+1\right)$$

input
```
integrate(1/(sec(x)-tan(x)),x, algorithm="fricas")
```

output
```
-log(-sin(x) + 1)
```

3.195.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(7) = 14.

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.89

$$\int \frac{1}{\sec(x)-\tan(x)}\,dx = -\log\left(\tan\left(x\right)-\sec\left(x\right)\right)+\frac{\log\left(\tan^{2}\left(x\right)+1\right)}{2}$$

input
```
integrate(1/(sec(x)-tan(x)),x)
```

output
```
-log(tan(x) - sec(x)) + log(tan(x)**2 + 1)/2
```

3.195.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(9) = 18.

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 3.22

$$\int \frac{1}{\sec(x)-\tan(x)}\,dx = -2\log\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}-1\right)+\log\left(\frac{\sin\left(x\right)^{2}}{\left(\cos\left(x\right)+1\right)^{2}}+1\right)$$

input
```
integrate(1/(sec(x)-tan(x)),x, algorithm="maxima")
```

output
```
-2*log(sin(x)/(cos(x) + 1) - 1) + log(sin(x)^2/(cos(x) + 1)^2 + 1)
```

3.195.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. 2(9) = 18.

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.22

$$\int \frac{1}{\sec(x) - \tan(x)}\, dx = \log\left(\tan\left(\frac{1}{2}\,x\right)^2 + 1\right) - 2\,\log\left(\left|\tan\left(\frac{1}{2}\,x\right) - 1\right|\right)$$

input
```
integrate(1/(sec(x)-tan(x)),x, algorithm="giac")
```

output
```
log(tan(1/2*x)^2 + 1) - 2*log(abs(tan(1/2*x) - 1))
```

3.195.9 Mupad [B] (verification not implemented)

Time = 22.96 (sec) , antiderivative size = 19, normalized size of antiderivative = 2.11

$$\int \frac{1}{\sec(x) - \tan(x)}\, dx = \ln\left(\tan\left(\frac{x}{2}\right)^2 + 1\right) - 2\,\ln\left(\tan\left(\frac{x}{2}\right) - 1\right)$$

input
```
int(-1/(tan(x) - 1/cos(x)),x)
```

output
```
log(tan(x/2)^2 + 1) - 2*log(tan(x/2) - 1)
```

3.196 $\int \frac{\sin(x)}{\sec(x)-\tan(x)}\,dx$

3.196.1 Optimal result

Integrand size = 12, antiderivative size = 14

$$\int \frac{\sin(x)}{\sec(x)-\tan(x)}\,dx = -\log(1-\sin(x))-\sin(x)$$

output
```
-ln(1-sin(x))-sin(x)
```

3.196.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.64

$$\int \frac{\sin(x)}{\sec(x)-\tan(x)}\,dx = -2\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\sin(x)$$

input
```
Integrate[Sin[x]/(Sec[x] - Tan[x]),x]
```

output
```
-2*Log[Cos[x/2] - Sin[x/2]] - Sin[x]
```

3.196.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {3042, 4891, 3042, 3312, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)}{\sec(x)-\tan(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)}{\sec(x)-\tan(x)}dx$$

$$\Big\downarrow \text{4891}$$

$$\int \frac{\sin(x)\cos(x)}{1-\sin(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)\cos(x)}{1-\sin(x)}dx$$

$$\Big\downarrow \text{3312}$$

$$-\int \frac{\sin(x)}{1-\sin(x)}d(-\sin(x))$$

$$\Big\downarrow \text{25}$$

$$\int -\frac{\sin(x)}{1-\sin(x)}d(-\sin(x))$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{1}{\sin(x)-1}+1\right)d(-\sin(x))$$

$$\Big\downarrow \text{2009}$$

$$-\sin(x)-\log(1-\sin(x))$$

input
```
Int[Sin[x]/(Sec[x] - Tan[x]),x]
```

output
```
-Log[1 - Sin[x]] - Sin[x]
```

3.196.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3312
```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b*f)   Su
bst[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x]
```

rule 4891
```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.196.4 Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$-\sin(x)-\ln(\sin(x)-1)$	13
default	$-\sin(x)-\ln(\sin(x)-1)$	13
risch	$ix+\frac{i e^{ix}}{2}-\frac{i e^{-ix}}{2}-2\ln(e^{ix}-i)$	33

input
```
int(sin(x)/(sec(x)-tan(x)),x,method=_RETURNVERBOSE)
```

output
```
-sin(x)-ln(sin(x)-1)
```

3.196.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{\sec(x)-\tan(x)}\,dx = -\log\left(-\sin\left(x\right)+1\right)-\sin\left(x\right)$$

input
```
integrate(sin(x)/(sec(x)-tan(x)),x, algorithm="fricas")
```

output
```
-log(-sin(x) + 1) - sin(x)
```

3.196.6 Sympy [F]

$$\int \frac{\sin(x)}{\sec(x)-\tan(x)}\,dx = \int \frac{\sin\left(x\right)}{-\tan\left(x\right)+\sec\left(x\right)}\,dx$$

input
```
integrate(sin(x)/(sec(x)-tan(x)),x)
```

output
```
Integral(sin(x)/(-tan(x) + sec(x)), x)
```

3.196.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(14) = 28.

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 3.86

$$\int \frac{\sin(x)}{\sec(x)-\tan(x)}\,dx = -\frac{2\,\sin\left(x\right)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)\left(\cos\left(x\right)+1\right)} - 2\,\log\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}-1\right)+\log\left(\frac{\sin\left(x\right)^2}{\left(\cos\left(x\right)+1\right)^2}+1\right)$$

input
```
integrate(sin(x)/(sec(x)-tan(x)),x, algorithm="maxima")
```

output
```
-2*sin(x)/((sin(x)^2/(cos(x) + 1)^2 + 1)*(cos(x) + 1)) - 2*log(sin(x)/(cos
(x) + 1) - 1) + log(sin(x)^2/(cos(x) + 1)^2 + 1)
```

3.196.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{\sec(x) - \tan(x)}\, dx = -\log\left(-\sin\left(x\right) + 1\right) - \sin\left(x\right)$$

input
```
integrate(sin(x)/(sec(x)-tan(x)),x, algorithm="giac")
```

output
```
-log(-sin(x) + 1) - sin(x)
```

3.196.9 Mupad [B] (verification not implemented)

Time = 23.41 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.64

$$\int \frac{\sin(x)}{\sec(x) - \tan(x)}\, dx = \ln\left(\tan\left(\frac{x}{2}\right)^2 + 1\right) - 2\,\ln\left(\tan\left(\frac{x}{2}\right) - 1\right) - \sin\left(x\right)$$

input
```
int(-sin(x)/(tan(x) - 1/cos(x)),x)
```

output
```
log(tan(x/2)^2 + 1) - 2*log(tan(x/2) - 1) - sin(x)
```

3.197 $\int \frac{\cos(x)}{\sec(x)-\tan(x)}\,dx$

3.197.1 Optimal result

Integrand size = 12, antiderivative size = 6

$$\int \frac{\cos(x)}{\sec(x)-\tan(x)}\,dx = x-\cos(x)$$

output
```
x-cos(x)
```

3.197.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 34 vs. 2(6) = 12.

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 5.67

$$\int \frac{\cos(x)}{\sec(x)-\tan(x)}\,dx = -\cos(x)-2\arcsin\left(\frac{\sqrt{1-\sin(x)}}{\sqrt{2}}\right)\sqrt{\cos^2(x)}\sec(x)$$

input
```
Integrate[Cos[x]/(Sec[x] - Tan[x]),x]
```

output
```
-Cos[x] - 2*ArcSin[Sqrt[1 - Sin[x]]/Sqrt[2]]*Sqrt[Cos[x]^2]*Sec[x]
```

3.197.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3042, 4891, 3042, 3161, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(x)}{\sec(x)-\tan(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(x)}{\sec(x)-\tan(x)}dx$$

$$\Big\downarrow \text{4891}$$

$$\int \frac{\cos^2(x)}{1-\sin(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(x)^2}{1-\sin(x)}dx$$

$$\Big\downarrow \text{3161}$$

$$\int 1dx-\cos(x)$$

$$\Big\downarrow \text{24}$$

$$x-\cos(x)$$

input
```
Int[Cos[x]/(Sec[x] - Tan[x]),x]
```

output
```
x - Cos[x]
```

3.197.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3161
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Si
mp[g^2/a   Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x
] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

rule 4891
```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.197.4 Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

method	result	size
risch	$x-\cos(x)$	7
default	$-\frac{2}{1+\tan\left(\frac{x}{2}\right)^{2}}+2\arctan\left(\tan\left(\frac{x}{2}\right)\right)$	21

input
```
int(cos(x)/(sec(x)-tan(x)),x,method=_RETURNVERBOSE)
```

output
```
x-cos(x)
```

3.197.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{\sec(x) - \tan(x)}\, dx = x - \cos(x)$$

input
```
integrate(cos(x)/(sec(x)-tan(x)),x, algorithm="fricas")
```

output
```
x - cos(x)
```

3.197.6 Sympy [F]

$$\int \frac{\cos(x)}{\sec(x) - \tan(x)}\, dx = \int \frac{\cos(x)}{-\tan(x) + \sec(x)}\, dx$$

input
```
integrate(cos(x)/(sec(x)-tan(x)),x)
```

output
```
Integral(cos(x)/(-tan(x) + sec(x)), x)
```

3.197.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. 2(6) = 12.

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 5.00

$$\int \frac{\cos(x)}{\sec(x) - \tan(x)}\, dx = -\frac{2}{\frac{\sin(x)^2}{(\cos(x)+1)^2} + 1} + 2\arctan\left(\frac{\sin(x)}{\cos(x) + 1}\right)$$

input
```
integrate(cos(x)/(sec(x)-tan(x)),x, algorithm="maxima")
```

output
```
-2/(sin(x)^2/(cos(x) + 1)^2 + 1) + 2*arctan(sin(x)/(cos(x) + 1))
```

3.197.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14 vs. 2(6) = 12.

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 2.33

$$\int \frac{\cos(x)}{\sec(x) - \tan(x)}\, dx = x - \frac{2}{\tan\left(\frac{1}{2}\,x\right)^2 + 1}$$

input
```
integrate(cos(x)/(sec(x)-tan(x)),x, algorithm="giac")
```

output
```
x - 2/(tan(1/2*x)^2 + 1)
```

3.197.9 Mupad [B] (verification not implemented)

Time = 22.34 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{\sec(x) - \tan(x)}\, dx = x - \cos(x)$$

input
```
int(-cos(x)/(tan(x) - 1/cos(x)),x)
```

output
```
x - cos(x)
```

3.198 $\int \frac{\tan(x)}{\sec(x)-\tan(x)}\,dx$

3.198.1 Optimal result

Integrand size = 12, antiderivative size = 15

$$\int \frac{\tan(x)}{\sec(x)-\tan(x)}\,dx = -x+\frac{\cos(x)}{1-\sin(x)}$$

output
```
-x+cos(x)/(1-sin(x))
```

3.198.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.93

$$\int \frac{\tan(x)}{\sec(x)-\tan(x)}\,dx = -x+\frac{2\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)}$$

input
```
Integrate[Tan[x]/(Sec[x] - Tan[x]),x]
```

output
```
-x + (2*Sin[x/2])/(Cos[x/2] - Sin[x/2])
```

3.198.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4891, 3042, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(x)}{\sec(x)-\tan(x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\tan(x)}{\sec(x)-\tan(x)}dx$$
$$\Big\downarrow \text{4891}$$
$$\int \frac{\sin(x)}{1-\sin(x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin(x)}{1-\sin(x)}dx$$
$$\Big\downarrow \text{3214}$$
$$\int \frac{1}{1-\sin(x)}dx-x$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{1-\sin(x)}dx-x$$
$$\Big\downarrow \text{3127}$$
$$\frac{\cos(x)}{1-\sin(x)}-x$$

input
```
Int[Tan[x]/(Sec[x] - Tan[x]),x]
```

output
```
-x + Cos[x]/(1 - Sin[x])
```

3.198.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 4891
```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.198.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

method	result	size
risch	$-x+\frac{2}{{\mathrm e}^{i x}-i}$	17
default	$-2 \arctan\left(\tan\left(\frac{x}{2}\right)\right)-\frac{2}{\tan\left(\frac{x}{2}\right)-1}$	19

input
```
int(tan(x)/(sec(x)-tan(x)),x,method=_RETURNVERBOSE)
```

output
```
-x+2/(exp(I*x)-I)
```

3.198.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.87

$$\int \frac{\tan(x)}{\sec(x) - \tan(x)}\, dx = -\frac{(x-1)\cos(x) - (x+1)\sin(x) + x - 1}{\cos(x) - \sin(x) + 1}$$

input
```
integrate(tan(x)/(sec(x)-tan(x)),x, algorithm="fricas")
```

output
```
-((x - 1)*cos(x) - (x + 1)*sin(x) + x - 1)/(cos(x) - sin(x) + 1)
```

3.198.6 Sympy [F]

$$\int \frac{\tan(x)}{\sec(x) - \tan(x)}\, dx = \int \frac{\tan(x)}{-\tan(x) + \sec(x)}\, dx$$

input
```
integrate(tan(x)/(sec(x)-tan(x)),x)
```

output
```
Integral(tan(x)/(-tan(x) + sec(x)), x)
```

3.198.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.87

$$\int \frac{\tan(x)}{\sec(x) - \tan(x)}\, dx = -\frac{2}{\frac{\sin(x)}{\cos(x)+1} - 1} - 2\arctan\left(\frac{\sin(x)}{\cos(x) + 1}\right)$$

input
```
integrate(tan(x)/(sec(x)-tan(x)),x, algorithm="maxima")
```

output
```
-2/(sin(x)/(cos(x) + 1) - 1) - 2*arctan(sin(x)/(cos(x) + 1))
```

3.198.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{\tan(x)}{\sec(x)-\tan(x)}\,dx = -x - \frac{2}{\tan\left(\frac{1}{2}\,x\right)-1}$$

input
```
integrate(tan(x)/(sec(x)-tan(x)),x, algorithm="giac")
```

output
```
-x - 2/(tan(1/2*x) - 1)
```

3.198.9 Mupad [B] (verification not implemented)

Time = 22.80 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{\tan(x)}{\sec(x)-\tan(x)}\,dx = -x - \frac{2}{\tan\left(\frac{x}{2}\right)-1}$$

input
```
int(-tan(x)/(tan(x) - 1/cos(x)),x)
```

output
```
- x - 2/(tan(x/2) - 1)
```

3.199 $\int \frac{\cot(x)}{\sec(x)-\tan(x)}\,dx$

3.199.1 Optimal result

Integrand size = 12, antiderivative size = 7

$$\int \frac{\cot(x)}{\sec(x)-\tan(x)}\,dx = x-\operatorname{arctanh}(\cos(x))$$

output
```
x-arctanh(cos(x))
```

3.199.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 18 vs. 2(7) = 14.

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.57

$$\int \frac{\cot(x)}{\sec(x)-\tan(x)}\,dx = x-\log\left(\cos\left(\frac{x}{2}\right)\right)+\log\left(\sin\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[Cot[x]/(Sec[x] - Tan[x]),x]
```

output
```
x - Log[Cos[x/2]] + Log[Sin[x/2]]
```

3.199.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {3042, 4891, 3042, 3318, 24, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(x)}{\sec(x)-\tan(x)}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\cot(x)}{\sec(x)-\tan(x)}dx$$

$$\Big\downarrow \ 4891$$

$$\int \frac{\cos(x)\cot(x)}{1-\sin(x)}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\cos(x)^2}{(1-\sin(x))\sin(x)}dx$$

$$\Big\downarrow \ 3318$$

$$\int 1dx+\int \csc(x)dx$$

$$\Big\downarrow \ 24$$

$$\int \csc(x)dx+x$$

$$\Big\downarrow \ 3042$$

$$\int \csc(x)dx+x$$

$$\Big\downarrow \ 4257$$

$$x-\text{arctanh}(\cos(x))$$

input
```
Int[Cot[x]/(Sec[x] - Tan[x]),x]
```

output
```
x - ArcTanh[Cos[x]]
```

3.199.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3318
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g^2/a   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Simp[g^2/(b*d)   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b,
d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4891
```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.199.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 14, normalized size of antiderivative = 2.00

method	result	size
default	$2 \arctan\left(\tan\left(\frac{x}{2}\right)\right)+\ln\left(\tan\left(\frac{x}{2}\right)\right)$	14
risch	$x+\ln\left({\mathrm e}^{i x}-1\right)-\ln\left({\mathrm e}^{i x}+1\right)$	21

input
```
int(cot(x)/(sec(x)-tan(x)),x,method=_RETURNVERBOSE)
```

output
```
2*arctan(tan(1/2*x))+ln(tan(1/2*x))
```

3.199.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. 2(7) = 14.

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.86

$$\int \frac{\cot(x)}{\sec(x)-\tan(x)}\,dx = x-\frac{1}{2}\log\left(\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)+\frac{1}{2}\log\left(-\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)$$

input
```
integrate(cot(x)/(sec(x)-tan(x)),x, algorithm="fricas")
```

output
```
x - 1/2*log(1/2*cos(x) + 1/2) + 1/2*log(-1/2*cos(x) + 1/2)
```

3.199.6 Sympy [F]

$$\int \frac{\cot(x)}{\sec(x)-\tan(x)}\,dx = \int \frac{\cot\left(x\right)}{-\tan\left(x\right)+\sec\left(x\right)}\,dx$$

input
```
integrate(cot(x)/(sec(x)-tan(x)),x)
```

output
```
Integral(cot(x)/(-tan(x) + sec(x)), x)
```

3.199.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. 2(7) = 14.

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 3.29

$$\int \frac{\cot(x)}{\sec(x)-\tan(x)}\,dx = 2\arctan\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}\right)+\log\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}\right)$$

input
```
integrate(cot(x)/(sec(x)-tan(x)),x, algorithm="maxima")
```

output
```
2*arctan(sin(x)/(cos(x) + 1)) + log(sin(x)/(cos(x) + 1))
```

3.199.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

$$\int \frac{\cot(x)}{\sec(x)-\tan(x)}\,dx = x+\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)$$

input
```
integrate(cot(x)/(sec(x)-tan(x)),x, algorithm="giac")
```

output
```
x + log(abs(tan(1/2*x)))
```

3.199.9 Mupad [B] (verification not implemented)

Time = 22.69 (sec) , antiderivative size = 23, normalized size of antiderivative = 3.29

$$\int \frac{\cot(x)}{\sec(x)-\tan(x)}\,dx = \ln\left(\tan\left(\frac{x}{2}\right)\right)-2\,\mathrm{atan}\left(\frac{8}{4\tan\left(\frac{x}{2}\right)-4}+1\right)$$

input
```
int(-cot(x)/(tan(x) - 1/cos(x)),x)
```

output
```
log(tan(x/2)) - 2*atan(8/(4*tan(x/2) - 4) + 1)
```

3.200 $\int \frac{\sec(x)}{\sec(x)-\tan(x)}\,dx$

3.200.1 Optimal result

Integrand size = 12, antiderivative size = 11

$$\int \frac{\sec(x)}{\sec(x)-\tan(x)}\,dx = \frac{\cos(x)}{1-\sin(x)}$$

output
```
cos(x)/(1-sin(x))
```

3.200.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 25 vs. 2(11) = 22.

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.27

$$\int \frac{\sec(x)}{\sec(x)-\tan(x)}\,dx = \frac{2\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)}$$

input
```
Integrate[Sec[x]/(Sec[x] - Tan[x]),x]
```

output
```
(2*Sin[x/2])/(Cos[x/2] - Sin[x/2])
```

3.200.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3644, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(x)}{\sec(x)-\tan(x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sec(x)}{\sec(x)-\tan(x)}dx$$
$$\Big\downarrow \text{3644}$$
$$\int \frac{1}{1-\sin(x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{1-\sin(x)}dx$$
$$\Big\downarrow \text{3127}$$
$$\frac{\cos(x)}{1-\sin(x)}$$

input
```
Int[Sec[x]/(Sec[x] - Tan[x]),x]
```

output
```
Cos[x]/(1 - Sin[x])
```

3.200.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3644
```
Int[sec[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] +
(c_.)*tan[(d_.) + (e_.)*(x_)])^(m_), x_Symbol] :> Int[1/(b + a*Cos[d + e*x]
 + c*Sin[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && I
ntegerQ[n]
```

3.200.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

method	result	size
default	$-\frac{2}{\tan\left(\frac{x}{2}\right)-1}$	11
risch	$\frac{2}{e^{ix}-i}$	13

input
```
int(sec(x)/(sec(x)-tan(x)),x,method=_RETURNVERBOSE)
```

output
```
-2/(tan(1/2*x)-1)
```

3.200.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \frac{\sec(x)}{\sec(x)-\tan(x)}\,dx=\frac{\cos\left(x\right)+\sin\left(x\right)+1}{\cos\left(x\right)-\sin\left(x\right)+1}$$

input
```
integrate(sec(x)/(sec(x)-tan(x)),x, algorithm="fricas")
```

output
```
(cos(x) + sin(x) + 1)/(cos(x) - sin(x) + 1)
```

3.200.6 Sympy [F]

$$\int \frac{\sec(x)}{\sec(x) - \tan(x)}\,dx = \int \frac{\sec{(x)}}{-\tan{(x)} + \sec{(x)}}\,dx$$

input
```
integrate(sec(x)/(sec(x)-tan(x)),x)
```

output
```
Integral(sec(x)/(-tan(x) + sec(x)), x)
```

3.200.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \frac{\sec(x)}{\sec(x) - \tan(x)}\,dx = -\frac{2}{\frac{\sin(x)}{\cos(x)+1} - 1}$$

input
```
integrate(sec(x)/(sec(x)-tan(x)),x, algorithm="maxima")
```

output
```
-2/(sin(x)/(cos(x) + 1) - 1)
```

3.200.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{\sec(x)}{\sec(x) - \tan(x)}\,dx = -\frac{2}{\tan\left(\frac{1}{2}\,x\right) - 1}$$

input
```
integrate(sec(x)/(sec(x)-tan(x)),x, algorithm="giac")
```

output
```
-2/(tan(1/2*x) - 1)
```

3.200.9 Mupad [B] (verification not implemented)

Time = 22.35 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{\sec(x)}{\sec(x)-\tan(x)}\,dx = -\frac{2}{\tan\left(\frac{x}{2}\right)-1}$$

input
```
int(-1/(cos(x)*(tan(x) - 1/cos(x))),x)
```

output
```
-2/(tan(x/2) - 1)
```

3.201 $\int \frac{\csc(x)}{\sec(x)-\tan(x)}\,dx$

3.201.1 Optimal result

Integrand size = 12, antiderivative size = 13

$$\int \frac{\csc(x)}{\sec(x)-\tan(x)}\,dx = -\log(1-\sin(x))+\log(\sin(x))$$

output
```
-ln(1-sin(x))+ln(sin(x))
```

3.201.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int \frac{\csc(x)}{\sec(x)-\tan(x)}\,dx = -2\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\log(\sin(x))$$

input
```
Integrate[Csc[x]/(Sec[x] - Tan[x]),x]
```

output
```
-2*Log[Cos[x/2] - Sin[x/2]] + Log[Sin[x]]
```

3.201.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {3042, 4891, 3042, 3186, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(x)}{\sec(x)-\tan(x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\csc(x)}{\sec(x)-\tan(x)}dx$$
$$\Big\downarrow \text{4891}$$
$$\int \frac{\cot(x)}{1-\sin(x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(1-\sin(x))\tan(x)}dx$$
$$\Big\downarrow \text{3186}$$
$$\int -\frac{\csc(x)}{1-\sin(x)}d(-\sin(x))$$
$$\Big\downarrow \text{47}$$
$$\int -\csc(x)d(-\sin(x))-\int \frac{1}{1-\sin(x)}d(-\sin(x))$$
$$\Big\downarrow \text{14}$$
$$\log(-\sin(x))-\int \frac{1}{1-\sin(x)}d(-\sin(x))$$
$$\Big\downarrow \text{16}$$
$$\log(-\sin(x))-\log(1-\sin(x))$$

input
```
Int[Csc[x]/(Sec[x] - Tan[x]),x]
```

output
```
-Log[1 - Sin[x]] + Log[-Sin[x]]
```

3.201.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

rule 4891
```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.201.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.62

method	result	size
derivativedivides	$-\ln\left(\csc\left(x\right)-1\right)$	8
default	$-\ln\left(\csc\left(x\right)-1\right)$	8
risch	$-2\ln\left({\mathrm e}^{i x}-i\right)+\ln\left({\mathrm e}^{2 i x}-1\right)$	21

input
```
int(csc(x)/(sec(x)-tan(x)),x,method=_RETURNVERBOSE)
```

output
```
-ln(csc(x)-1)
```

3.201.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{\csc(x)}{\sec(x)-\tan(x)}\,dx = \log\left(\frac{1}{2}\sin(x)\right) - \log(-\sin(x)+1)$$

input
```
integrate(csc(x)/(sec(x)-tan(x)),x, algorithm="fricas")
```

output
```
log(1/2*sin(x)) - log(-sin(x) + 1)
```

3.201.6 Sympy [F]

$$\int \frac{\csc(x)}{\sec(x)-\tan(x)}\,dx = \int \frac{\csc(x)}{-\tan(x)+\sec(x)}\,dx$$

input
```
integrate(csc(x)/(sec(x)-tan(x)),x)
```

output
```
Integral(csc(x)/(-tan(x) + sec(x)), x)
```

3.201.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.92

$$\int \frac{\csc(x)}{\sec(x)-\tan(x)}\,dx = -2\log\left(\frac{\sin(x)}{\cos(x)+1}-1\right) + \log\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

input
```
integrate(csc(x)/(sec(x)-tan(x)),x, algorithm="maxima")
```

output
```
-2*log(sin(x)/(cos(x) + 1) - 1) + log(sin(x)/(cos(x) + 1))
```

3.201.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{\csc(x)}{\sec(x)-\tan(x)}\,dx = -\log\left(-\sin\left(x\right)+1\right)+\log\left(\left|\sin\left(x\right)\right|\right)$$

input
```
integrate(csc(x)/(sec(x)-tan(x)),x, algorithm="giac")
```

output
```
-log(-sin(x) + 1) + log(abs(sin(x)))
```

3.201.9 Mupad [B] (verification not implemented)

Time = 22.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{\csc(x)}{\sec(x)-\tan(x)}\,dx = \ln\left(\tan\left(\frac{x}{2}\right)\right)-2\,\ln\left(\tan\left(\frac{x}{2}\right)-1\right)$$

input
```
int(-1/(sin(x)*(tan(x) - 1/cos(x))),x)
```

output
```
log(tan(x/2)) - 2*log(tan(x/2) - 1)
```

3.202 $\int \csc(c+dx)(\cot(c+dx)+\csc(c+dx))\,dx$

3.202.1 Optimal result

Integrand size = 20, antiderivative size = 23

$$\int \csc(c+dx)(\cot(c+dx)+\csc(c+dx))\,dx = -\frac{\cot(c+dx)}{d}-\frac{\csc(c+dx)}{d}$$

output
```
-cot(d*x+c)/d-csc(d*x+c)/d
```

3.202.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \csc(c+dx)(\cot(c+dx)+\csc(c+dx))\,dx = -\frac{\cot\left(\frac{1}{2}(c+dx)\right)}{d}$$

input
```
Integrate[Csc[c + d*x]*(Cot[c + d*x] + Csc[c + d*x]),x]
```

output
```
-(Cot[(c + d*x)/2]/d)
```

3.202.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {3042, 4897, 3042, 3148, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(c+dx)(\cot(c+dx)+\csc(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(c+dx)(\cot(c+dx)+\csc(c+dx))dx$$

$$\Big\downarrow \text{4897}$$

$$\int (\cos(c+dx)+1)\csc^2(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1-\sin\left(c+dx-\frac{\pi}{2}\right)}{\cos\left(c+dx-\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3148}$$

$$\int \csc^2(c+dx)dx-\frac{\csc(c+dx)}{d}$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(c+dx)^2dx-\frac{\csc(c+dx)}{d}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\int 1d\cot(c+dx)}{d}-\frac{\csc(c+dx)}{d}$$

$$\Big\downarrow \text{24}$$

$$-\frac{\cot(c+dx)}{d}-\frac{\csc(c+dx)}{d}$$

input
```
Int[Csc[c + d*x]*(Cot[c + d*x] + Csc[c + d*x]),x]
```

output
```
-(Cot[c + d*x]/d) - Csc[c + d*x]/d
```

3.202.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.202.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
risch	$-\frac{2i}{d\left(e^{i(dx+c)}-1\right)}$	20
derivativedivides	$\frac{-\cot(dx+c)-\frac{1}{\sin(dx+c)}}{d}$	24
default	$\frac{-\cot(dx+c)-\frac{1}{\sin(dx+c)}}{d}$	24
parts	$-\frac{\cot(dx+c)}{d}-\frac{\csc(dx+c)}{d}$	24

input
```
int(csc(d*x+c)*(csc(d*x+c)+cot(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2*I/d/(exp(I*(d*x+c))-1)
```

3.202.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \csc(c+dx)(\cot(c+dx)+\csc(c+dx))\,dx = -\frac{\cos(dx+c)+1}{d\sin(dx+c)}$$

input
```
integrate(csc(d*x+c)*(cot(d*x+c)+csc(d*x+c)),x, algorithm="fricas")
```

output
```
-(cos(d*x + c) + 1)/(d*sin(d*x + c))
```

3.202.6 Sympy [A] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \csc(c+dx)(\cot(c+dx)+\csc(c+dx))\,dx = \begin{cases} \frac{-\cot(c+dx)-\csc(c+dx)}{d} & \text{for } d\neq 0 \\ x(\cot(c)+\csc(c))\csc(c) & \text{otherwise} \end{cases}$$

input
```
integrate(csc(d*x+c)*(cot(d*x+c)+csc(d*x+c)),x)
```

output
```
Piecewise(((-cot(c + d*x) - csc(c + d*x))/d, Ne(d, 0)), (x*(cot(c) + csc(c
))*csc(c), True))
```

3.202.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \csc(c+dx)(\cot(c+dx)+\csc(c+dx))\,dx = -\frac{\frac{1}{\sin(dx+c)}+\frac{1}{\tan(dx+c)}}{d}$$

input
```
integrate(csc(d*x+c)*(cot(d*x+c)+csc(d*x+c)),x, algorithm="maxima")
```

output
```
-(1/sin(d*x + c) + 1/tan(d*x + c))/d
```

3.202.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.70

$$\int \csc(c+dx)(\cot(c+dx)+\csc(c+dx))\,dx = -\frac{1}{d\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}$$

input
```
integrate(csc(d*x+c)*(cot(d*x+c)+csc(d*x+c)),x, algorithm="giac")
```

output
```
-1/(d*tan(1/2*d*x + 1/2*c))
```

3.202.9 Mupad [B] (verification not implemented)

Time = 22.54 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.61

$$\int \csc(c+dx)(\cot(c+dx)+\csc(c+dx))\,dx = -\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d}$$

input
```
int((cot(c + d*x) + 1/sin(c + d*x))/sin(c + d*x),x)
```

output
```
-cot(c/2 + (d*x)/2)/d
```

3.203 $\int \frac{\sin(x)}{\cot(x)+\csc(x)}\,dx$

3.203.1 Optimal result

Integrand size = 10, antiderivative size = 6

$$\int \frac{\sin(x)}{\cot(x)+\csc(x)}\,dx = x-\sin(x)$$

output
```
x-sin(x)
```

3.203.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 14 vs. 2(6) = 12.

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 2.33

$$\int \frac{\sin(x)}{\cot(x)+\csc(x)}\,dx = 2\left(\frac{x}{2}-\frac{\sin(x)}{2}\right)$$

input
```
Integrate[Sin[x]/(Cot[x] + Csc[x]),x]
```

output
```
2*(x/2 - Sin[x]/2)
```

3.203.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4892, 3042, 3161, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)}{\cot(x)+\csc(x)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(x)}{\cot(x)+\csc(x)}dx$$

$$\Big\downarrow \text{ 4892}$$

$$\int \frac{\sin^2(x)}{\cos(x)+1}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)^2}{1-\sin\left(x-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 3161}$$

$$\int 1dx-\sin(x)$$

$$\Big\downarrow \text{ 24}$$

$$x-\sin(x)$$

input
```
Int[Sin[x]/(Cot[x] + Csc[x]),x]
```

output
```
x - Sin[x]
```

3.203.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3161
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Si
mp[g^2/a   Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x
] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

rule 4892
```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.203.4 Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

method	result	size
risch	$x-\sin\left(x\right)$	7
default	$-\frac{2\tan\left(\frac{x}{2}\right)}{1+\tan\left(\frac{x}{2}\right)^{2}}+2\arctan\left(\tan\left(\frac{x}{2}\right)\right)$	25

input
```
int(sin(x)/(cot(x)+csc(x)),x,method=_RETURNVERBOSE)
```

output
```
x-sin(x)
```

3.203.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{\cot(x)+\csc(x)}\,dx = x - \sin{(x)}$$

input
```
integrate(sin(x)/(cot(x)+csc(x)),x, algorithm="fricas")
```

output
```
x - sin(x)
```

3.203.6 Sympy [F]

$$\int \frac{\sin(x)}{\cot(x)+\csc(x)}\,dx = \int \frac{\sin{(x)}}{\cot{(x)}+\csc{(x)}}\,dx$$

input
```
integrate(sin(x)/(cot(x)+csc(x)),x)
```

output
```
Integral(sin(x)/(cot(x) + csc(x)), x)
```

3.203.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(6) = 12.

Time = 0.34 (sec) , antiderivative size = 38, normalized size of antiderivative = 6.33

$$\int \frac{\sin(x)}{\cot(x)+\csc(x)}\,dx = -\frac{2\,\sin{(x)}}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)(\cos{(x)}+1)} + 2\,\arctan\left(\frac{\sin{(x)}}{\cos{(x)}+1}\right)$$

input
```
integrate(sin(x)/(cot(x)+csc(x)),x, algorithm="maxima")
```

output
```
-2*sin(x)/((sin(x)^2/(cos(x) + 1)^2 + 1)*(cos(x) + 1)) + 2*arctan(sin(x)/(
cos(x) + 1))
```

3.203.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. 2(6) = 12.

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 3.00

$$\int \frac{\sin(x)}{\cot(x)+\csc(x)}\,dx = x - \frac{2\tan\left(\frac{1}{2}\,x\right)}{\tan\left(\frac{1}{2}\,x\right)^2+1}$$

input
```
integrate(sin(x)/(cot(x)+csc(x)),x, algorithm="giac")
```

output
```
x - 2*tan(1/2*x)/(tan(1/2*x)^2 + 1)
```

3.203.9 Mupad [B] (verification not implemented)

Time = 23.78 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{\cot(x)+\csc(x)}\,dx = x - \sin(x)$$

input
```
int(sin(x)/(cot(x) + 1/sin(x)),x)
```

output
```
x - sin(x)
```

3.204 $\int \frac{\cos(x)}{\cot(x)+\csc(x)}\,dx$

3.204.1 Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\cos(x)}{\cot(x)+\csc(x)}\,dx = -\cos(x)+\log(1+\cos(x))$$

output
```
-cos(x)+ln(1+cos(x))
```

3.204.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.00

$$\int \frac{\cos(x)}{\cot(x)+\csc(x)}\,dx = -2\cos^2\left(\frac{x}{2}\right)+2\log\left(\cos\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[Cos[x]/(Cot[x] + Csc[x]),x]
```

output
```
-2*Cos[x/2]^2 + 2*Log[Cos[x/2]]
```

3.204.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {3042, 4892, 3042, 25, 3312, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(x)}{\cot(x)+\csc(x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos(x)}{\cot(x)+\csc(x)}dx$$
$$\Big\downarrow \text{4892}$$
$$\int \frac{\sin(x)\cos(x)}{\cos(x)+1}dx$$
$$\Big\downarrow \text{3042}$$
$$\int -\frac{\sin\left(x+\frac{\pi}{2}\right)\cos\left(x+\frac{\pi}{2}\right)}{\sin\left(x+\frac{\pi}{2}\right)+1}dx$$
$$\Big\downarrow \text{25}$$
$$-\int \frac{\cos\left(x+\frac{\pi}{2}\right)\sin\left(x+\frac{\pi}{2}\right)}{\sin\left(x+\frac{\pi}{2}\right)+1}dx$$
$$\Big\downarrow \text{3312}$$
$$-\int \frac{\cos(x)}{\cos(x)+1}d\cos(x)$$
$$\Big\downarrow \text{49}$$
$$-\int \left(1+\frac{1}{-\cos(x)-1}\right)d\cos(x)$$
$$\Big\downarrow \text{2009}$$
$$\log(\cos(x)+1)-\cos(x)$$

input
```
Int[Cos[x]/(Cot[x] + Csc[x]),x]
```

output
```
-Cos[x] + Log[1 + Cos[x]]
```

3.204.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3312
```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b*f)   Su
bst[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x]
```

rule 4892
```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.204.4 Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$-\cos(x)+\ln(\cos(x)+1)$	11
default	$-\cos(x)+\ln(\cos(x)+1)$	11
risch	$-ix-\frac{e^{ix}}{2}-\frac{e^{-ix}}{2}+2\ln(e^{ix}+1)$	30

input
```
int(cos(x)/(cot(x)+csc(x)),x,method=_RETURNVERBOSE)
```

output
```
-cos(x)+ln(cos(x)+1)
```

3.204.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\cos(x)}{\cot(x)+\csc(x)}\,dx = -\cos(x) + \log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right)$$

input
```
integrate(cos(x)/(cot(x)+csc(x)),x, algorithm="fricas")
```

output
```
-cos(x) + log(1/2*cos(x) + 1/2)
```

3.204.6 Sympy [F]

$$\int \frac{\cos(x)}{\cot(x)+\csc(x)}\,dx = \int \frac{\cos(x)}{\cot(x)+\csc(x)}\,dx$$

input
```
integrate(cos(x)/(cot(x)+csc(x)),x)
```

output
```
Integral(cos(x)/(cot(x) + csc(x)), x)
```

3.204.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(10) = 20.

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 3.40

$$\int \frac{\cos(x)}{\cot(x)+\csc(x)}\,dx = -\frac{2}{\frac{\sin(x)^2}{(\cos(x)+1)^2}+1} - \log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)$$

input
```
integrate(cos(x)/(cot(x)+csc(x)),x, algorithm="maxima")
```

output
```
-2/(sin(x)^2/(cos(x) + 1)^2 + 1) - log(sin(x)^2/(cos(x) + 1)^2 + 1)
```

3.204.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{\cot(x)+\csc(x)}\,dx = -\cos{(x)} + \log{(\cos{(x)}+1)}$$

input
```
integrate(cos(x)/(cot(x)+csc(x)),x, algorithm="giac")
```

output
```
-cos(x) + log(cos(x) + 1)
```

3.204.9 Mupad [B] (verification not implemented)

Time = 23.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.40

$$\int \frac{\cos(x)}{\cot(x)+\csc(x)}\,dx = -\ln\left(\tan\left(\frac{x}{2}\right)^2+1\right) - \frac{2}{\tan\left(\frac{x}{2}\right)^2+1}$$

input
```
int(cos(x)/(cot(x) + 1/sin(x)),x)
```

output
```
- log(tan(x/2)^2 + 1) - 2/(tan(x/2)^2 + 1)
```

3.205 $\int \frac{\tan(x)}{\cot(x)+\csc(x)}\,dx$

3.205.1 Optimal result

Integrand size = 10, antiderivative size = 7

$$\int \frac{\tan(x)}{\cot(x)+\csc(x)}\,dx = -x+\operatorname{arctanh}(\sin(x))$$

output
```
-x+arctanh(sin(x))
```

3.205.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 36 vs. 2(7) = 14.

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 5.14

$$\int \frac{\tan(x)}{\cot(x)+\csc(x)}\,dx = -x-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[Tan[x]/(Cot[x] + Csc[x]),x]
```

output
```
-x - Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]]
```

3.205.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {3042, 4892, 3042, 3318, 24, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(x)}{\cot(x)+\csc(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(x)}{\cot(x)+\csc(x)}dx$$

$$\Big\downarrow \text{4892}$$

$$\int \frac{\sin(x)\tan(x)}{\cos(x)+1}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos\left(x+\frac{\pi}{2}\right)^2}{\sin\left(x+\frac{\pi}{2}\right)\left(\sin\left(x+\frac{\pi}{2}\right)+1\right)}dx$$

$$\Big\downarrow \text{3318}$$

$$\int \sec(x)dx-\int 1dx$$

$$\Big\downarrow \text{24}$$

$$\int \sec(x)dx-x$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(x+\frac{\pi}{2}\right)dx-x$$

$$\Big\downarrow \text{4257}$$

$$\operatorname{arctanh}(\sin(x))-x$$

input
```
Int[Tan[x]/(Cot[x] + Csc[x]),x]
```

output
```
-x + ArcTanh[Sin[x]]
```

3.205.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3318
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g^2/a   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Simp[g^2/(b*d)   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b,
d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4892
```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.205.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(7) = 14$.

Time = 0.32 (sec) , antiderivative size = 25, normalized size of antiderivative = 3.57

method	result	size
default	$\ln\left(\tan\left(\frac{x}{2}\right)+1\right)-\ln\left(\tan\left(\frac{x}{2}\right)-1\right)-2\arctan\left(\tan\left(\frac{x}{2}\right)\right)$	25
risch	$-x-\ln\left(\mathrm{e}^{ix}-i\right)+\ln\left(i+\mathrm{e}^{ix}\right)$	25

input
```
int(tan(x)/(cot(x)+csc(x)),x,method=_RETURNVERBOSE)
```

output
```
ln(tan(1/2*x)+1)-ln(tan(1/2*x)-1)-2*arctan(tan(1/2*x))
```

3.205.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. 2(7) = 14.

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.86

$$\int \frac{\tan(x)}{\cot(x)+\csc(x)}\,dx = -x+\frac{1}{2}\log\left(\sin\left(x\right)+1\right)-\frac{1}{2}\log\left(-\sin\left(x\right)+1\right)$$

input
```
integrate(tan(x)/(cot(x)+csc(x)),x, algorithm="fricas")
```

output
```
-x + 1/2*log(sin(x) + 1) - 1/2*log(-sin(x) + 1)
```

3.205.6 Sympy [F]

$$\int \frac{\tan(x)}{\cot(x)+\csc(x)}\,dx = \int \frac{\tan\left(x\right)}{\cot\left(x\right)+\csc\left(x\right)}\,dx$$

input
```
integrate(tan(x)/(cot(x)+csc(x)),x)
```

output
```
Integral(tan(x)/(cot(x) + csc(x)), x)
```

3.205.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(7) = 14.

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 5.57

$$\int \frac{\tan(x)}{\cot(x)+\csc(x)}\,dx = -2\arctan\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}\right) + \log\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}+1\right)-\log\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}-1\right)$$

input
```
integrate(tan(x)/(cot(x)+csc(x)),x, algorithm="maxima")
```

output
```
-2*arctan(sin(x)/(cos(x) + 1)) + log(sin(x)/(cos(x) + 1) + 1) - log(sin(x)
/(cos(x) + 1) - 1)
```

3.205.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(7) = 14$.

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 3.14

$$\int \frac{\tan(x)}{\cot(x)+\csc(x)}\,dx = -x+\log\left(\left|\tan\left(\frac{1}{2}\,x\right)+1\right|\right)-\log\left(\left|\tan\left(\frac{1}{2}\,x\right)-1\right|\right)$$

input
```
integrate(tan(x)/(cot(x)+csc(x)),x, algorithm="giac")
```

output
```
-x + log(abs(tan(1/2*x) + 1)) - log(abs(tan(1/2*x) - 1))
```

3.205.9 Mupad [B] (verification not implemented)

Time = 23.40 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.57

$$\int \frac{\tan(x)}{\cot(x)+\csc(x)}\,dx = 2\,\mathrm{atanh}\left(\tan\left(\frac{x}{2}\right)\right)-x$$

input
```
int(tan(x)/(cot(x) + 1/sin(x)),x)
```

output
```
2*atanh(tan(x/2)) - x
```

3.206 $\int \frac{\cot(x)}{\cot(x)+\csc(x)}\,dx$

3.206.1 Optimal result

Integrand size = 10, antiderivative size = 12

$$\int \frac{\cot(x)}{\cot(x)+\csc(x)}\,dx = x-\frac{\sin(x)}{1+\cos(x)}$$

output
```
x-sin(x)/(1+cos(x))
```

3.206.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\cot(x)}{\cot(x)+\csc(x)}\,dx = x-\tan\left(\frac{x}{2}\right)$$

input
```
Integrate[Cot[x]/(Cot[x] + Csc[x]),x]
```

output
```
x - Tan[x/2]
```

3.206.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 4892, 3042, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(x)}{\cot(x)+\csc(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cot(x)}{\cot(x)+\csc(x)}dx$$

$$\Big\downarrow \text{4892}$$

$$\int \frac{\cos(x)}{\cos(x)+1}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(x+\frac{\pi}{2}\right)}{\sin\left(x+\frac{\pi}{2}\right)+1}dx$$

$$\Big\downarrow \text{3214}$$

$$x-\int \frac{1}{\cos(x)+1}dx$$

$$\Big\downarrow \text{3042}$$

$$x-\int \frac{1}{\sin\left(x+\frac{\pi}{2}\right)+1}dx$$

$$\Big\downarrow \text{3127}$$

$$x-\frac{\sin(x)}{\cos(x)+1}$$

input
```
Int[Cot[x]/(Cot[x] + Csc[x]),x]
```

output
```
x - Sin[x]/(1 + Cos[x])
```

3.206.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 4892
```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.206.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

method	result	size
default	$-\tan\left(\frac{x}{2}\right)+2\arctan\left(\tan\left(\frac{x}{2}\right)\right)$	15
risch	$x-\frac{2i}{{\mathrm e}^{i x}+1}$	15

input
```
int(cot(x)/(cot(x)+csc(x)),x,method=_RETURNVERBOSE)
```

output
```
-tan(1/2*x)+2*arctan(tan(1/2*x))
```

3.206.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \frac{\cot(x)}{\cot(x)+\csc(x)}\,dx = \frac{x\cos(x)+x-\sin(x)}{\cos(x)+1}$$

input
```
integrate(cot(x)/(cot(x)+csc(x)),x, algorithm="fricas")
```

output
```
(x*cos(x) + x - sin(x))/(cos(x) + 1)
```

3.206.6 Sympy [F]

$$\int \frac{\cot(x)}{\cot(x)+\csc(x)}\,dx = \int \frac{\cot(x)}{\cot(x)+\csc(x)}\,dx$$

input
```
integrate(cot(x)/(cot(x)+csc(x)),x)
```

output
```
Integral(cot(x)/(cot(x) + csc(x)), x)
```

3.206.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.92

$$\int \frac{\cot(x)}{\cot(x)+\csc(x)}\,dx = -\frac{\sin(x)}{\cos(x)+1} + 2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

input
```
integrate(cot(x)/(cot(x)+csc(x)),x, algorithm="maxima")
```

output
```
-sin(x)/(cos(x) + 1) + 2*arctan(sin(x)/(cos(x) + 1))
```

3.206.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{\cot(x)}{\cot(x)+\csc(x)}\,dx = x-\tan\left(\frac{1}{2}\,x\right)$$

input
```
integrate(cot(x)/(cot(x)+csc(x)),x, algorithm="giac")
```

output
```
x - tan(1/2*x)
```

3.206.9 Mupad [B] (verification not implemented)

Time = 23.24 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{\cot(x)}{\cot(x)+\csc(x)}\,dx = x-\tan\left(\frac{x}{2}\right)$$

input
```
int(cot(x)/(cot(x) + 1/sin(x)),x)
```

output
```
x - tan(x/2)
```

3.207 $\int \frac{\sec(x)}{\cot(x)+\csc(x)}\,dx$

3.207.1 Optimal result

Integrand size = 10, antiderivative size = 11

$$\int \frac{\sec(x)}{\cot(x)+\csc(x)}\,dx = -\log(\cos(x))+\log(1+\cos(x))$$

output
```
-ln(cos(x))+ln(1+cos(x))
```

3.207.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 25 vs. 2(11) = 22.

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.27

$$\int \frac{\sec(x)}{\cot(x)+\csc(x)}\,dx = 2\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(1-2\cos^2\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[Sec[x]/(Cot[x] + Csc[x]),x]
```

output
```
2*Log[Cos[x/2]] - Log[1 - 2*Cos[x/2]^2]
```

3.207.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {3042, 4892, 3042, 25, 3186, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(x)}{\cot(x)+\csc(x)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sec(x)}{\cot(x)+\csc(x)}dx$$
$$\Big\downarrow \text{4892}$$
$$\int \frac{\tan(x)}{\cos(x)+1}dx$$
$$\Big\downarrow \text{3042}$$
$$\int -\frac{1}{\left(1-\sin\left(x-\frac{\pi}{2}\right)\right)\tan\left(x-\frac{\pi}{2}\right)}dx$$
$$\Big\downarrow \text{25}$$
$$-\int \frac{1}{\left(1-\sin\left(x-\frac{\pi}{2}\right)\right)\tan\left(x-\frac{\pi}{2}\right)}dx$$
$$\Big\downarrow \text{3186}$$
$$-\int \frac{\sec(x)}{\cos(x)+1}d\cos(x)$$
$$\Big\downarrow \text{47}$$
$$\int \frac{1}{\cos(x)+1}d\cos(x)-\int \sec(x)d\cos(x)$$
$$\Big\downarrow \text{14}$$
$$\int \frac{1}{\cos(x)+1}d\cos(x)-\log(\cos(x))$$
$$\Big\downarrow \text{16}$$
$$\log(\cos(x)+1)-\log(\cos(x))$$

input
```
Int[Sec[x]/(Cot[x] + Csc[x]),x]
```

output
```
-Log[Cos[x]] + Log[1 + Cos[x]]
```

3.207.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

rule 4892
```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.207. $\int \frac{\sec(x)}{\cot(x)+\csc(x)} dx$

3.207.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.55

method	result	size
derivativedivides	$\ln(1+\sec(x))$	6
default	$\ln(1+\sec(x))$	6
risch	$2\ln(e^{ix}+1)-\ln(e^{2ix}+1)$	22

input
```
int(sec(x)/(cot(x)+csc(x)),x,method=_RETURNVERBOSE)
```

output
```
ln(1+sec(x))
```

3.207.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \frac{\sec(x)}{\cot(x)+\csc(x)}\,dx = -\log(-\cos(x)) + \log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)$$

input
```
integrate(sec(x)/(cot(x)+csc(x)),x, algorithm="fricas")
```

output
```
-log(-cos(x)) + log(1/2*cos(x) + 1/2)
```

3.207.6 Sympy [F]

$$\int \frac{\sec(x)}{\cot(x)+\csc(x)}\,dx = \int \frac{\sec(x)}{\cot(x)+\csc(x)}\,dx$$

input
```
integrate(sec(x)/(cot(x)+csc(x)),x)
```

output
```
Integral(sec(x)/(cot(x) + csc(x)), x)
```

3.207.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(11) = 22$.

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.64

$$\int \frac{\sec(x)}{\cot(x)+\csc(x)}\,dx = -\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right) - \log\left(\frac{\sin(x)}{\cos(x)+1}-1\right)$$

input
```
integrate(sec(x)/(cot(x)+csc(x)),x, algorithm="maxima")
```

output
```
-log(sin(x)/(cos(x) + 1) + 1) - log(sin(x)/(cos(x) + 1) - 1)
```

3.207.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{\sec(x)}{\cot(x)+\csc(x)}\,dx = \log(\cos(x)+1) - \log(|\cos(x)|)$$

input
```
integrate(sec(x)/(cot(x)+csc(x)),x, algorithm="giac")
```

output
```
log(cos(x) + 1) - log(abs(cos(x)))
```

3.207.9 Mupad [B] (verification not implemented)

Time = 22.99 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\sec(x)}{\cot(x)+\csc(x)}\,dx = -\ln\left(\tan\left(\frac{x}{2}\right)^2-1\right)$$

input
```
int(1/(cos(x)*(cot(x) + 1/sin(x))),x)
```

output
```
-log(tan(x/2)^2 - 1)
```

3.208 $\int \frac{\csc(x)}{\cot(x)+\csc(x)} \, dx$

3.208.1 Optimal result

Integrand size = 10, antiderivative size = 9

$$\int \frac{\csc(x)}{\cot(x)+\csc(x)} \, dx = \frac{\sin(x)}{1+\cos(x)}$$

output
```
sin(x)/(1+cos(x))
```

3.208.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int \frac{\csc(x)}{\cot(x)+\csc(x)} \, dx = \tan\left(\frac{x}{2}\right)$$

input
```
Integrate[Csc[x]/(Cot[x] + Csc[x]),x]
```

output
```
Tan[x/2]
```

3.208.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3645, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(x)}{\cot(x)+\csc(x)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\csc(x)}{\cot(x)+\csc(x)}dx$$

$$\Big\downarrow \ 3645$$

$$\int \frac{1}{\cos(x)+1}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\sin\left(x+\frac{\pi}{2}\right)+1}dx$$

$$\Big\downarrow \ 3127$$

$$\frac{\sin(x)}{\cos(x)+1}$$

input
```
Int[Csc[x]/(Cot[x] + Csc[x]),x]
```

output
```
Sin[x]/(1 + Cos[x])
```

3.208.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3645
```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Int[1/(b + a*Sin[d + e*x]
 + c*Cos[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && I
ntegerQ[n]
```

3.208.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.56

method	result	size
default	$\tan\left(\frac{x}{2}\right)$	5
risch	$\frac{2i}{e^{ix}+1}$	13

input
```
int(csc(x)/(cot(x)+csc(x)),x,method=_RETURNVERBOSE)
```

output
```
tan(1/2*x)
```

3.208.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{\csc(x)}{\cot(x)+\csc(x)}\,dx = \frac{\sin\left(x\right)}{\cos\left(x\right)+1}$$

input
```
integrate(csc(x)/(cot(x)+csc(x)),x, algorithm="fricas")
```

output
```
sin(x)/(cos(x) + 1)
```

3.208.6 Sympy [F]

$$\int \frac{\csc(x)}{\cot(x)+\csc(x)}\,dx = \int \frac{\csc{(x)}}{\cot{(x)}+\csc{(x)}}\,dx$$

input
```
integrate(csc(x)/(cot(x)+csc(x)),x)
```

output
```
Integral(csc(x)/(cot(x) + csc(x)), x)
```

3.208.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{\csc(x)}{\cot(x)+\csc(x)}\,dx = \frac{\sin{(x)}}{\cos{(x)}+1}$$

input
```
integrate(csc(x)/(cot(x)+csc(x)),x, algorithm="maxima")
```

output
```
sin(x)/(cos(x) + 1)
```

3.208.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.44

$$\int \frac{\csc(x)}{\cot(x)+\csc(x)}\,dx = \tan\left(\frac{1}{2}\,x\right)$$

input
```
integrate(csc(x)/(cot(x)+csc(x)),x, algorithm="giac")
```

output
```
tan(1/2*x)
```

3.208.9 Mupad [B] (verification not implemented)

Time = 23.16 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.44

$$\int \frac{\csc(x)}{\cot(x)+\csc(x)}\,dx = \tan\left(\frac{x}{2}\right)$$

input
```
int(1/(sin(x)*(cot(x) + 1/sin(x))),x)
```

output
```
tan(x/2)
```

3.209 $\int \frac{\sin(x)}{-\cot(x)+\csc(x)}\,dx$

3.209.1 Optimal result

Integrand size = 12, antiderivative size = 4

$$\int \frac{\sin(x)}{-\cot(x)+\csc(x)}\,dx = x+\sin(x)$$

output
```
x+sin(x)
```

3.209.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 14 vs. 2(4) = 8.

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 3.50

$$\int \frac{\sin(x)}{-\cot(x)+\csc(x)}\,dx = 2\left(\frac{x}{2}+\frac{\sin(x)}{2}\right)$$

input
```
Integrate[Sin[x]/(-Cot[x] + Csc[x]),x]
```

output
```
2*(x/2 + Sin[x]/2)
```

3.209.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3042, 4892, 3042, 3161, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)}{\csc(x)-\cot(x)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sin(x)}{\csc(x)-\cot(x)}dx$$

$$\Big\downarrow \ 4892$$

$$\int \frac{\sin^2(x)}{1-\cos(x)}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)^2}{\sin\left(x-\frac{\pi}{2}\right)+1}dx$$

$$\Big\downarrow \ 3161$$

$$\int 1dx+\sin(x)$$

$$\Big\downarrow \ 24$$

$$x+\sin(x)$$

input
```
Int[Sin[x]/(-Cot[x] + Csc[x]),x]
```

output
```
x + Sin[x]
```

3.209.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3161
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Si
mp[g^2/a   Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x
] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

rule 4892
```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.209.4 Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

method	result	size
risch	$x+\sin\left(x\right)$	5
default	$\frac{2\tan\left(\frac{x}{2}\right)}{1+\tan\left(\frac{x}{2}\right)^{2}}+2\arctan\left(\tan\left(\frac{x}{2}\right)\right)$	25

input
```
int(sin(x)/(csc(x)-cot(x)),x,method=_RETURNVERBOSE)
```

output
```
x+sin(x)
```

3.209.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{-\cot(x)+\csc(x)}\,dx = x+\sin(x)$$

input
```
integrate(sin(x)/(-cot(x)+csc(x)),x, algorithm="fricas")
```

output
```
x + sin(x)
```

3.209.6 Sympy [F]

$$\int \frac{\sin(x)}{-\cot(x)+\csc(x)}\,dx = -\int \frac{\sin(x)}{\cot(x)-\csc(x)}\,dx$$

input
```
integrate(sin(x)/(-cot(x)+csc(x)),x)
```

output
```
-Integral(sin(x)/(cot(x) - csc(x)), x)
```

3.209.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(4) = 8.

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 9.50

$$\int \frac{\sin(x)}{-\cot(x)+\csc(x)}\,dx = \frac{2\sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)(\cos(x)+1)} + 2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

input
```
integrate(sin(x)/(-cot(x)+csc(x)),x, algorithm="maxima")
```

output
```
2*sin(x)/((sin(x)^2/(cos(x) + 1)^2 + 1)*(cos(x) + 1)) + 2*arctan(sin(x)/(c
os(x) + 1))
```

3.209.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. $2(4) = 8$.

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 4.50

$$\int \frac{\sin(x)}{-\cot(x) + \csc(x)} dx = x + \frac{2 \tan\left(\frac{1}{2} x\right)}{\tan\left(\frac{1}{2} x\right)^2 + 1}$$

input
```
integrate(sin(x)/(-cot(x)+csc(x)),x, algorithm="giac")
```

output
```
x + 2*tan(1/2*x)/(tan(1/2*x)^2 + 1)
```

3.209.9 Mupad [B] (verification not implemented)

Time = 23.28 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{-\cot(x) + \csc(x)} dx = x + \sin(x)$$

input
```
int(-sin(x)/(cot(x) - 1/sin(x)),x)
```

output
```
x + sin(x)
```

3.210 $\int \frac{\cos(x)}{-\cot(x)+\csc(x)}\,dx$

3.210.1 Optimal result

Integrand size = 12, antiderivative size = 10

$$\int \frac{\cos(x)}{-\cot(x)+\csc(x)}\,dx = \cos(x)+\log(1-\cos(x))$$

output
```
cos(x)+ln(1-cos(x))
```

3.210.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.00

$$\int \frac{\cos(x)}{-\cot(x)+\csc(x)}\,dx = 2\log\left(\sin\left(\frac{x}{2}\right)\right)-2\sin^2\left(\frac{x}{2}\right)$$

input
```
Integrate[Cos[x]/(-Cot[x] + Csc[x]),x]
```

output
```
2*Log[Sin[x/2]] - 2*Sin[x/2]^2
```

3.210.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 4892, 3042, 25, 3312, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(x)}{\csc(x)-\cot(x)}dx$$

$$\Big\downarrow \textbf{3042}$$

$$\int \frac{\cos(x)}{\csc(x)-\cot(x)}dx$$

$$\Big\downarrow \textbf{4892}$$

$$\int \frac{\sin(x)\cos(x)}{1-\cos(x)}dx$$

$$\Big\downarrow \textbf{3042}$$

$$\int -\frac{\sin\left(x+\frac{\pi}{2}\right)\cos\left(x+\frac{\pi}{2}\right)}{1-\sin\left(x+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \textbf{25}$$

$$-\int \frac{\cos\left(x+\frac{\pi}{2}\right)\sin\left(x+\frac{\pi}{2}\right)}{1-\sin\left(x+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \textbf{3312}$$

$$\int \frac{\cos(x)}{1-\cos(x)}d(-\cos(x))$$

$$\Big\downarrow \textbf{25}$$

$$-\int -\frac{\cos(x)}{1-\cos(x)}d(-\cos(x))$$

$$\Big\downarrow \textbf{49}$$

$$-\int \left(1+\frac{1}{\cos(x)-1}\right)d(-\cos(x))$$

$$\Big\downarrow \textbf{2009}$$

$$\cos(x)+\log(1-\cos(x))$$

input
```
Int[Cos[x]/(-Cot[x] + Csc[x]),x]
```

output
```
Cos[x] + Log[1 - Cos[x]]
```

3.210.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3312
```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b*f)   Su
bst[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x]
```

rule 4892
```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.210.4 Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$\cos(x) + \ln(\cos(x) - 1)$	9
default	$\cos(x) + \ln(\cos(x) - 1)$	9
risch	$-ix + \frac{e^{ix}}{2} + \frac{e^{-ix}}{2} + 2\ln(e^{ix} - 1)$	30

input
```
int(cos(x)/(csc(x)-cot(x)),x,method=_RETURNVERBOSE)
```

output
```
cos(x)+ln(cos(x)-1)
```

3.210.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{-\cot(x) + \csc(x)}\, dx = \cos(x) + \log\left(-\frac{1}{2}\cos(x) + \frac{1}{2}\right)$$

input
```
integrate(cos(x)/(-cot(x)+csc(x)),x, algorithm="fricas")
```

output
```
cos(x) + log(-1/2*cos(x) + 1/2)
```

3.210.6 Sympy [F]

$$\int \frac{\cos(x)}{-\cot(x) + \csc(x)}\, dx = -\int \frac{\cos(x)}{\cot(x) - \csc(x)}\, dx$$

input
```
integrate(cos(x)/(-cot(x)+csc(x)),x)
```

output
```
-Integral(cos(x)/(cot(x) - csc(x)), x)
```

3.210.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(10) = 20.

Time = 0.31 (sec) , antiderivative size = 46, normalized size of antiderivative = 4.60

$$\int \frac{\cos(x)}{-\cot(x)+\csc(x)}\,dx = \frac{2}{\frac{\sin(x)^2}{(\cos(x)+1)^2}+1} + 2\log\left(\frac{\sin(x)}{\cos(x)+1}\right) - \log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)$$

input
```
integrate(cos(x)/(-cot(x)+csc(x)),x, algorithm="maxima")
```

output
```
2/(sin(x)^2/(cos(x) + 1)^2 + 1) + 2*log(sin(x)/(cos(x) + 1)) - log(sin(x)^
2/(cos(x) + 1)^2 + 1)
```

3.210.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{-\cot(x)+\csc(x)}\,dx = \cos(x) + \log(-\cos(x)+1)$$

input
```
integrate(cos(x)/(-cot(x)+csc(x)),x, algorithm="giac")
```

output
```
cos(x) + log(-cos(x) + 1)
```

3.210.9 Mupad [B] (verification not implemented)

Time = 22.59 (sec) , antiderivative size = 31, normalized size of antiderivative = 3.10

$$\int \frac{\cos(x)}{-\cot(x)+\csc(x)}\,dx = 2\ln\left(\tan\left(\frac{x}{2}\right)\right) - \ln\left(\tan\left(\frac{x}{2}\right)^2+1\right) + \frac{2}{\tan\left(\frac{x}{2}\right)^2+1}$$

input
```
int(-cos(x)/(cot(x) - 1/sin(x)),x)
```

output
```
2*log(tan(x/2)) - log(tan(x/2)^2 + 1) + 2/(tan(x/2)^2 + 1)
```

3.211 $\int \frac{\tan(x)}{-\cot(x)+\csc(x)}\,dx$

3.211.1 Optimal result

Integrand size = 12, antiderivative size = 5

$$\int \frac{\tan(x)}{-\cot(x)+\csc(x)}\,dx = x+\operatorname{arctanh}(\sin(x))$$

output
```
x+arctanh(sin(x))
```

3.211.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 46 vs. 2(5) = 10.

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 9.20

$$\int \frac{\tan(x)}{-\cot(x)+\csc(x)}\,dx = 2\left(\frac{x}{2}-\frac{1}{2}\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\frac{1}{2}\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\right)$$

input
```
Integrate[Tan[x]/(-Cot[x] + Csc[x]),x]
```

output
```
2*(x/2 - Log[Cos[x/2] - Sin[x/2]]/2 + Log[Cos[x/2] + Sin[x/2]]/2)
```

3.211.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {3042, 4892, 3042, 3318, 24, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(x)}{\csc(x)-\cot(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(x)}{\csc(x)-\cot(x)}dx$$

$$\Big\downarrow \text{4892}$$

$$\int \frac{\sin(x)\tan(x)}{1-\cos(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos\left(x+\frac{\pi}{2}\right)^2}{\left(1-\sin\left(x+\frac{\pi}{2}\right)\right)\sin\left(x+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3318}$$

$$\int 1dx+\int \sec(x)dx$$

$$\Big\downarrow \text{24}$$

$$\int \sec(x)dx+x$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(x+\frac{\pi}{2}\right)dx+x$$

$$\Big\downarrow \text{4257}$$

$$\text{arctanh}(\sin(x))+x$$

input
```
Int[Tan[x]/(-Cot[x] + Csc[x]),x]
```

output
```
x + ArcTanh[Sin[x]]
```

3.211.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3318
```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g^2/a   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Simp[g^2/(b*d)   Int
[(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b,
d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4892
```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.211.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 23, normalized size of antiderivative = 4.60

method	result	size
risch	$x+\ln \left(i+{\mathrm e}^{i x}\right)-\ln \left({\mathrm e}^{i x}-i\right)$	23
default	$-\ln \left(\tan \left(\frac{x}{2}\right)-1\right)+2 \arctan \left(\tan \left(\frac{x}{2}\right)\right)+\ln \left(\tan \left(\frac{x}{2}\right)+1\right)$	25

input
```
int(tan(x)/(csc(x)-cot(x)),x,method=_RETURNVERBOSE)
```

output
```
x+ln(I+exp(I*x))-ln(exp(I*x)-I)
```

3.211.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. 2(5) = 10.

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 3.60

$$\int \frac{\tan(x)}{-\cot(x)+\csc(x)}\,dx = x + \frac{1}{2}\log\left(\sin\left(x\right)+1\right) - \frac{1}{2}\log\left(-\sin\left(x\right)+1\right)$$

input
```
integrate(tan(x)/(-cot(x)+csc(x)),x, algorithm="fricas")
```

output
```
x + 1/2*log(sin(x) + 1) - 1/2*log(-sin(x) + 1)
```

3.211.6 Sympy [F]

$$\int \frac{\tan(x)}{-\cot(x)+\csc(x)}\,dx = -\int \frac{\tan\left(x\right)}{\cot\left(x\right)-\csc\left(x\right)}\,dx$$

input
```
integrate(tan(x)/(-cot(x)+csc(x)),x)
```

output
```
-Integral(tan(x)/(cot(x) - csc(x)), x)
```

3.211.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(5) = 10.

Time = 0.31 (sec) , antiderivative size = 39, normalized size of antiderivative = 7.80

$$\int \frac{\tan(x)}{-\cot(x)+\csc(x)}\,dx = 2\arctan\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}\right) + \log\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}+1\right) - \log\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}-1\right)$$

input
```
integrate(tan(x)/(-cot(x)+csc(x)),x, algorithm="maxima")
```

output
```
2*arctan(sin(x)/(cos(x) + 1)) + log(sin(x)/(cos(x) + 1) + 1) - log(sin(x)/
(cos(x) + 1) - 1)
```

3.211.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(5) = 10$.

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 4.00

$$\int \frac{\tan(x)}{-\cot(x)+\csc(x)}\,dx = x+\log\left(\left|\tan\left(\frac{1}{2}\,x\right)+1\right|\right)-\log\left(\left|\tan\left(\frac{1}{2}\,x\right)-1\right|\right)$$

input
```
integrate(tan(x)/(-cot(x)+csc(x)),x, algorithm="giac")
```

output
```
x + log(abs(tan(1/2*x) + 1)) - log(abs(tan(1/2*x) - 1))
```

3.211.9 Mupad [B] (verification not implemented)

Time = 23.36 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.80

$$\int \frac{\tan(x)}{-\cot(x)+\csc(x)}\,dx = x+2\,\mathrm{atanh}\left(\tan\left(\frac{x}{2}\right)\right)$$

input
```
int(-tan(x)/(cot(x) - 1/sin(x)),x)
```

output
```
x + 2*atanh(tan(x/2))
```

3.212 $\int \frac{\cot(x)}{-\cot(x)+\csc(x)}\,dx$

3.212.1 Optimal result

Integrand size = 12, antiderivative size = 16

$$\int \frac{\cot(x)}{-\cot(x)+\csc(x)}\,dx = -x - \frac{\sin(x)}{1-\cos(x)}$$

output
```
-x-sin(x)/(1-cos(x))
```

3.212.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\cot(x)}{-\cot(x)+\csc(x)}\,dx = \frac{1}{2}\left(-2x-2\cot\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[Cot[x]/(-Cot[x] + Csc[x]),x]
```

output
```
(-2*x - 2*Cot[x/2])/2
```

3.212.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4892, 3042, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(x)}{\csc(x)-\cot(x)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\cot(x)}{\csc(x)-\cot(x)}dx$$

$$\Big\downarrow \ 4892$$

$$\int \frac{\cos(x)}{1-\cos(x)}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sin\left(x+\frac{\pi}{2}\right)}{1-\sin\left(x+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \ 3214$$

$$\int \frac{1}{1-\cos(x)}dx-x$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{1-\sin\left(x+\frac{\pi}{2}\right)}dx-x$$

$$\Big\downarrow \ 3127$$

$$-x-\frac{\sin(x)}{1-\cos(x)}$$

input
```
Int[Cot[x]/(-Cot[x] + Csc[x]),x]
```

output
```
-x - Sin[x]/(1 - Cos[x])
```

3.212.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 4892
```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.212.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

method	result	size
default	$-\frac{1}{\tan\left(\frac{x}{2}\right)}-2\arctan\left(\tan\left(\frac{x}{2}\right)\right)$	17
risch	$-x-\frac{2i}{{\rm e}^{ix}-1}$	17

input
```
int(cot(x)/(csc(x)-cot(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/tan(1/2*x)-2*arctan(tan(1/2*x))
```

3.212.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\cot(x)}{-\cot(x)+\csc(x)}\,dx = -\frac{x\sin\left(x\right)+\cos\left(x\right)+1}{\sin\left(x\right)}$$

input
```
integrate(cot(x)/(-cot(x)+csc(x)),x, algorithm="fricas")
```

output
```
-(x*sin(x) + cos(x) + 1)/sin(x)
```

3.212.6 Sympy [F]

$$\int \frac{\cot(x)}{-\cot(x)+\csc(x)}\,dx = -\int \frac{\cot\left(x\right)}{\cot\left(x\right)-\csc\left(x\right)}\,dx$$

input
```
integrate(cot(x)/(-cot(x)+csc(x)),x)
```

output
```
-Integral(cot(x)/(cot(x) - csc(x)), x)
```

3.212.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.44

$$\int \frac{\cot(x)}{-\cot(x)+\csc(x)}\,dx = -\frac{\cos\left(x\right)+1}{\sin\left(x\right)} - 2\arctan\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}\right)$$

input
```
integrate(cot(x)/(-cot(x)+csc(x)),x, algorithm="maxima")
```

output
```
-(cos(x) + 1)/sin(x) - 2*arctan(sin(x)/(cos(x) + 1))
```

3.212.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{\cot(x)}{-\cot(x)+\csc(x)}\,dx = -x-\frac{1}{\tan\left(\frac{1}{2}\,x\right)}$$

input
```
integrate(cot(x)/(-cot(x)+csc(x)),x, algorithm="giac")
```

output
```
-x - 1/tan(1/2*x)
```

3.212.9 Mupad [B] (verification not implemented)

Time = 22.77 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{\cot(x)}{-\cot(x)+\csc(x)}\,dx = -x-\cot\left(\frac{x}{2}\right)$$

input
```
int(-cot(x)/(cot(x) - 1/sin(x)),x)
```

output
```
- x - cot(x/2)
```

3.213 $\int \frac{\sec(x)}{-\cot(x)+\csc(x)}\,dx$

3.213.1 Optimal result

Integrand size = 12, antiderivative size = 13

$$\int \frac{\sec(x)}{-\cot(x)+\csc(x)}\,dx = \log(1-\cos(x)) - \log(\cos(x))$$

output
```
ln(1-cos(x))-ln(cos(x))
```

3.213.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.92

$$\int \frac{\sec(x)}{-\cot(x)+\csc(x)}\,dx = 2\log\left(\sin\left(\frac{x}{2}\right)\right) - \log\left(1-2\sin^2\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[Sec[x]/(-Cot[x] + Csc[x]),x]
```

output
```
2*Log[Sin[x/2]] - Log[1 - 2*Sin[x/2]^2]
```

3.213.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 4892, 3042, 25, 3186, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(x)}{\csc(x)-\cot(x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sec(x)}{\csc(x)-\cot(x)}dx$$
$$\Big\downarrow \text{4892}$$
$$\int \frac{\tan(x)}{1-\cos(x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int -\frac{1}{\left(\sin\left(x-\frac{\pi}{2}\right)+1\right)\tan\left(x-\frac{\pi}{2}\right)}dx$$
$$\Big\downarrow \text{25}$$
$$-\int \frac{1}{\left(\sin\left(x-\frac{\pi}{2}\right)+1\right)\tan\left(x-\frac{\pi}{2}\right)}dx$$
$$\Big\downarrow \text{3186}$$
$$-\int -\frac{\sec(x)}{1-\cos(x)}d(-\cos(x))$$
$$\Big\downarrow \text{47}$$
$$\int \frac{1}{1-\cos(x)}d(-\cos(x))-\int -\sec(x)d(-\cos(x))$$
$$\Big\downarrow \text{14}$$
$$\int \frac{1}{1-\cos(x)}d(-\cos(x))-\log(-\cos(x))$$
$$\Big\downarrow \text{16}$$
$$\log(1-\cos(x))-\log(-\cos(x))$$

input
```
Int[Sec[x]/(-Cot[x] + Csc[x]),x]
```

output
```
Log[1 - Cos[x]] - Log[-Cos[x]]
```

3.213.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

rule 4892
```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.213. $\int \frac{\sec(x)}{-\cot(x)+\csc(x)}\,dx$

3.213.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.46

method	result	size
derivativedivides	$\ln\left(\sec\left(x\right)-1\right)$	6
default	$\ln\left(\sec\left(x\right)-1\right)$	6
risch	$2\ln\left(\mathrm{e}^{ix}-1\right)-\ln\left(\mathrm{e}^{2ix}+1\right)$	22

input
```
int(sec(x)/(csc(x)-cot(x)),x,method=_RETURNVERBOSE)
```

output
```
ln(sec(x)-1)
```

3.213.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{\sec(x)}{-\cot(x)+\csc(x)}\,dx = -\log\left(-\cos\left(x\right)\right)+\log\left(-\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)$$

input
```
integrate(sec(x)/(-cot(x)+csc(x)),x, algorithm="fricas")
```

output
```
-log(-cos(x)) + log(-1/2*cos(x) + 1/2)
```

3.213.6 Sympy [F]

$$\int \frac{\sec(x)}{-\cot(x)+\csc(x)}\,dx = -\int \frac{\sec\left(x\right)}{\cot\left(x\right)-\csc\left(x\right)}\,dx$$

input
```
integrate(sec(x)/(-cot(x)+csc(x)),x)
```

output
```
-Integral(sec(x)/(cot(x) - csc(x)), x)
```

3.213.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(13) = 26$.

Time = 0.23 (sec) , antiderivative size = 41, normalized size of antiderivative = 3.15

$$\int \frac{\sec(x)}{-\cot(x)+\csc(x)}\,dx = -\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right) - \log\left(\frac{\sin(x)}{\cos(x)+1}-1\right) + 2\log\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

input
```
integrate(sec(x)/(-cot(x)+csc(x)),x, algorithm="maxima")
```

output
```
-log(sin(x)/(cos(x) + 1) + 1) - log(sin(x)/(cos(x) + 1) - 1) + 2*log(sin(x
)/(cos(x) + 1))
```

3.213.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{\sec(x)}{-\cot(x)+\csc(x)}\,dx = \log(-\cos(x)+1) - \log(|\cos(x)|)$$

input
```
integrate(sec(x)/(-cot(x)+csc(x)),x, algorithm="giac")
```

output
```
log(-cos(x) + 1) - log(abs(cos(x)))
```

3.213.9 Mupad [B] (verification not implemented)

Time = 22.65 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int \frac{\sec(x)}{-\cot(x)+\csc(x)}\,dx = 2\ln\left(\tan\left(\frac{x}{2}\right)\right) - \ln\left(\tan\left(\frac{x}{2}\right)^2-1\right)$$

input
```
int(-1/(cos(x)*(cot(x) - 1/sin(x))),x)
```

output
```
2*log(tan(x/2)) - log(tan(x/2)^2 - 1)
```

3.214 $\int \frac{\csc(x)}{-\cot(x)+\csc(x)}\,dx$

3.214.1 Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\csc(x)}{-\cot(x)+\csc(x)}\,dx = -\frac{\sin(x)}{1-\cos(x)}$$

output
```
-sin(x)/(1-cos(x))
```

3.214.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{\csc(x)}{-\cot(x)+\csc(x)}\,dx = -\cot\left(\frac{x}{2}\right)$$

input
```
Integrate[Csc[x]/(-Cot[x] + Csc[x]),x]
```

output
```
-Cot[x/2]
```

3.214.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3645, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(x)}{\csc(x)-\cot(x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\csc(x)}{\csc(x)-\cot(x)}dx$$
$$\Big\downarrow \text{3645}$$
$$\int \frac{1}{1-\cos(x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{1-\sin\left(x+\frac{\pi}{2}\right)}dx$$
$$\Big\downarrow \text{3127}$$
$$-\frac{\sin(x)}{1-\cos(x)}$$

input
```
Int[Csc[x]/(-Cot[x] + Csc[x]),x]
```

output
```
-(Sin[x]/(1 - Cos[x]))
```

3.214.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3645
```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Int[1/(b + a*Sin[d + e*x]
 + c*Cos[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && I
ntegerQ[n]
```

3.214.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

method	result	size
default	$-\frac{1}{\tan\left(\frac{x}{2}\right)}$	9
risch	$-\frac{2i}{e^{ix}-1}$	13

input
```
int(csc(x)/(csc(x)-cot(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/tan(1/2*x)
```

3.214.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\csc(x)}{-\cot(x)+\csc(x)}\,dx = -\frac{\cos\left(x\right)+1}{\sin\left(x\right)}$$

input
```
integrate(csc(x)/(-cot(x)+csc(x)),x, algorithm="fricas")
```

output
```
-(cos(x) + 1)/sin(x)
```

3.214.6 Sympy [F]

$$\int \frac{\csc(x)}{-\cot(x)+\csc(x)}\,dx = -\int \frac{\csc(x)}{\cot(x)-\csc(x)}\,dx$$

input
```
integrate(csc(x)/(-cot(x)+csc(x)),x)
```

output
```
-Integral(csc(x)/(cot(x) - csc(x)), x)
```

3.214.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\csc(x)}{-\cot(x)+\csc(x)}\,dx = -\frac{\cos(x)+1}{\sin(x)}$$

input
```
integrate(csc(x)/(-cot(x)+csc(x)),x, algorithm="maxima")
```

output
```
-(cos(x) + 1)/sin(x)
```

3.214.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{\csc(x)}{-\cot(x)+\csc(x)}\,dx = -\frac{1}{\tan\left(\frac{1}{2}x\right)}$$

input
```
integrate(csc(x)/(-cot(x)+csc(x)),x, algorithm="giac")
```

output
```
-1/tan(1/2*x)
```

3.214.9 Mupad [B] (verification not implemented)

Time = 22.68 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.50

$$\int \frac{\csc(x)}{-\cot(x)+\csc(x)}\,dx = -\cot\left(\frac{x}{2}\right)$$

input
```
int(-1/(sin(x)*(cot(x) - 1/sin(x))),x)
```

output
```
-cot(x/2)
```

3.215 $\int \frac{1}{\csc(c+dx)+\sin(c+dx)}\,dx$

3.215.1 Optimal result

Integrand size = 15, antiderivative size = 23

$$\int \frac{1}{\csc(c+dx)+\sin(c+dx)}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\cos(c+dx)}{\sqrt{2}}\right)}{\sqrt{2}d}$$

output
```
-1/2*arctanh(1/2*cos(d*x+c)*2^(1/2))/d*2^(1/2)
```

3.215.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.65

$$\int \frac{1}{\csc(c+dx)+\sin(c+dx)}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\cos(c)-(-i+\sin(c))\tan\left(\frac{dx}{2}\right)}{\sqrt{2}}\right)+\operatorname{arctanh}\left(\frac{\cos(c)-(i+\sin(c))\tan\left(\frac{dx}{2}\right)}{\sqrt{2}}\right)}{\sqrt{2}d}$$

input
```
Integrate[(Csc[c + d*x] + Sin[c + d*x])^(-1),x]
```

output
```
-((ArcTanh[(Cos[c] - (-I + Sin[c])*Tan[(d*x)/2])/Sqrt[2]] + ArcTanh[(Cos[c
] - (I + Sin[c])*Tan[(d*x)/2])/Sqrt[2]])/(Sqrt[2]*d))
```

3.215.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4897, 3042, 3665, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sin(c+dx)+\csc(c+dx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sin(c+dx)+\csc(c+dx)}dx$$
$$\Big\downarrow \text{4897}$$
$$\int \frac{\sin(c+dx)}{\sin^2(c+dx)+1}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin(c+dx)}{\sin(c+dx)^2+1}dx$$
$$\Big\downarrow \text{3665}$$
$$-\frac{\int \frac{1}{2-\cos^2(c+dx)}d\cos(c+dx)}{d}$$
$$\Big\downarrow \text{219}$$
$$-\frac{\text{arctanh}\left(\frac{\cos(c+dx)}{\sqrt{2}}\right)}{\sqrt{2}d}$$

input
```
Int[(Csc[c + d*x] + Sin[c + d*x])^(-1),x]
```

output
```
-(ArcTanh[Cos[c + d*x]/Sqrt[2]]/(Sqrt[2]*d))
```

3.215.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3665
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Simp[-ff/f
Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
 f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.215.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$-\frac{\operatorname{arctanh}\left(\frac{\cos(dx+c)\sqrt{2}}{2}\right)\sqrt{2}}{2d}$	21
default	$-\frac{\operatorname{arctanh}\left(\frac{\cos(dx+c)\sqrt{2}}{2}\right)\sqrt{2}}{2d}$	21
risch	$\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2i(dx+c)}-2\sqrt{2}\,\mathrm{e}^{i(dx+c)}+1\right)}{4d}-\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2i(dx+c)}+2\sqrt{2}\,\mathrm{e}^{i(dx+c)}+1\right)}{4d}$	70

input
```
int(1/(csc(d*x+c)+sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*arctanh(1/2*cos(d*x+c)*2^(1/2))/d*2^(1/2)
```

3.215.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(20) = 40$.

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.91

$$\int \frac{1}{\csc(c+dx)+\sin(c+dx)}\,dx = \frac{\sqrt{2}\log\left(-\frac{\cos(dx+c)^2-2\sqrt{2}\cos(dx+c)+2}{\cos(dx+c)^2-2}\right)}{4\,d}$$

input
```
integrate(1/(csc(d*x+c)+sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*log(-(cos(d*x + c)^2 - 2*sqrt(2)*cos(d*x + c) + 2)/(cos(d*x +
c)^2 - 2))/d
```

3.215.6 Sympy [F]

$$\int \frac{1}{\csc(c+dx)+\sin(c+dx)}\,dx = \int \frac{1}{\sin(c+dx)+\csc(c+dx)}\,dx$$

input
```
integrate(1/(csc(d*x+c)+sin(d*x+c)),x)
```

output
```
Integral(1/(sin(c + d*x) + csc(c + d*x)), x)
```

3.215.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(20) = 40$.

Time = 0.31 (sec) , antiderivative size = 176, normalized size of antiderivative = 7.65

$$\int \frac{1}{\csc(c+dx)+\sin(c+dx)}\,dx$$
$$= \frac{\sqrt{2}\log\left(-\frac{2\left(\sqrt{2}+1\right)\cos(dx+c)-\cos(dx+c)^2-\sin(dx+c)^2-2\sqrt{2}-3}{2\left(\sqrt{2}-1\right)\cos(dx+c)+\cos(dx+c)^2+\sin(dx+c)^2-2\sqrt{2}+3}\right)+\sqrt{2}\log\left(-\frac{2\left(\sqrt{2}-1\right)\cos(dx+c)-\cos(dx+c)^2-\sin(dx+c)^2+2}{2\left(\sqrt{2}+1\right)\cos(dx+c)+\cos(dx+c)^2+\sin(dx+c)^2+2}\right.}{8\,d}$$

input
```
integrate(1/(csc(d*x+c)+sin(d*x+c)),x, algorithm="maxima")
```

output
```
1/8*(sqrt(2)*log(-(2*(sqrt(2) + 1)*cos(d*x + c) - cos(d*x + c)^2 - sin(d*x
 + c)^2 - 2*sqrt(2) - 3)/(2*(sqrt(2) - 1)*cos(d*x + c) + cos(d*x + c)^2 +
sin(d*x + c)^2 - 2*sqrt(2) + 3)) + sqrt(2)*log(-(2*(sqrt(2) - 1)*cos(d*x +
 c) - cos(d*x + c)^2 - sin(d*x + c)^2 + 2*sqrt(2) - 3)/(2*(sqrt(2) + 1)*co
s(d*x + c) + cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sqrt(2) + 3)))/d
```

3.215.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(20) = 40$.

Time = 0.30 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.96

$$\int \frac{1}{\csc(c+dx)+\sin(c+dx)}\,dx = \frac{\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}-\frac{2(\cos(dx+c)-1)}{\cos(dx+c)+1}+6\right|}{\left|4\sqrt{2}-\frac{2(\cos(dx+c)-1)}{\cos(dx+c)+1}+6\right|}\right)}{4\,d}$$

input
```
integrate(1/(csc(d*x+c)+sin(d*x+c)),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*log(abs(-4*sqrt(2) - 2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) +
 6)/abs(4*sqrt(2) - 2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 6))/d
```

3.215.9 Mupad [B] (verification not implemented)

Time = 23.38 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \frac{1}{\csc(c+dx)+\sin(c+dx)}\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{2\sqrt{2}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1}\right)}{2\,d}$$

input
```
int(1/(sin(c + d*x) + 1/sin(c + d*x)),x)
```

output
```
(2^(1/2)*atanh((2*2^(1/2)*sin(c/2 + (d*x)/2)^2)/(2*sin(c/2 + (d*x)/2)^2 +
1)))/(2*d)
```

3.216 $\int \frac{\sin(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx$

3.216.1 Optimal result

Integrand size = 22, antiderivative size = 51

$$\int \frac{\sin(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx = x-\frac{x}{\sqrt{2}}-\frac{\arctan\left(\frac{\cos(c+dx)\sin(c+dx)}{1+\sqrt{2}+\sin^2(c+dx)}\right)}{\sqrt{2}d}$$

output
```
x-1/2*x*2^(1/2)-1/2*arctan(cos(d*x+c)*sin(d*x+c)/(1+sin(d*x+c)^2+2^(1/2)))
/d*2^(1/2)
```

3.216.2 Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.59

$$\int \frac{\sin(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx = \frac{c}{d}+x-\frac{\arctan\left(\sqrt{2}\tan(c+dx)\right)}{\sqrt{2}d}$$

input
```
Integrate[Sin[c + d*x]/(Csc[c + d*x] + Sin[c + d*x]),x]
```

output
```
c/d + x - ArcTan[Sqrt[2]*Tan[c + d*x]]/(Sqrt[2]*d)
```

3.216.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.63, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3042, 4889, 1450, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(c+dx)}{\sin(c+dx)+\csc(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(c+dx)}{\sin(c+dx)+\csc(c+dx)}dx$$

$$\Big\downarrow \text{ 4889}$$

$$\frac{\int \frac{\tan^2(c+dx)}{2\tan^4(c+dx)+3\tan^2(c+dx)+1}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{ 1450}$$

$$\frac{2\int \frac{1}{2\tan^2(c+dx)+2}d\tan(c+dx)-\int \frac{1}{2\tan^2(c+dx)+1}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\arctan(\tan(c+dx))-\frac{\arctan\left(\sqrt{2}\tan(c+dx)\right)}{\sqrt{2}}}{d}$$

input
```
Int[Sin[c + d*x]/(Csc[c + d*x] + Sin[c + d*x]),x]
```

output
```
(ArcTan[Tan[c + d*x]] - ArcTan[Sqrt[2]*Tan[c + d*x]]/Sqrt[2])/d
```

3.216.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1450
```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^2/2)*(b/q + 1)   Int[(d*x)^(m - 2)/(b/
2 + q/2 + c*x^2), x], x] - Simp[(d^2/2)*(b/q - 1)   Int[(d*x)^(m - 2)/(b/2
- q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] &&
 GeQ[m, 2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.216.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.57

method	result	size
derivativedivides	$\frac{\arctan(\tan(dx+c))-\frac{\sqrt{2}\arctan\left(\tan(dx+c)\sqrt{2}\right)}{2}}{d}$	29
default	$\frac{\arctan(\tan(dx+c))-\frac{\sqrt{2}\arctan\left(\tan(dx+c)\sqrt{2}\right)}{2}}{d}$	29
risch	$x-\frac{i\sqrt{2}\ln\left(e^{2i(dx+c)}-2\sqrt{2}-3\right)}{4d}+\frac{i\sqrt{2}\ln\left(e^{2i(dx+c)}+2\sqrt{2}-3\right)}{4d}$	55

input
```
int(sin(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(arctan(tan(d*x+c))-1/2*2^(1/2)*arctan(tan(d*x+c)*2^(1/2)))
```

3.216.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.02

$$\int \frac{\sin(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx = \frac{4\,dx+\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(dx+c)^2-2\sqrt{2}}{4\cos(dx+c)\sin(dx+c)}\right)}{4\,d}$$

input
```
integrate(sin(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*(4*d*x + sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(d*x + c)^2 - 2*sqrt(2))/(co
s(d*x + c)*sin(d*x + c))))/d
```

3.216.6 Sympy [F]

$$\int \frac{\sin(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx = \int \frac{\sin{(c+dx)}}{\sin{(c+dx)}+\csc{(c+dx)}}\,dx$$

input
```
integrate(sin(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x)
```

output
```
Integral(sin(c + d*x)/(sin(c + d*x) + csc(c + d*x)), x)
```

3.216.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. 2(45) = 90.

Time = 0.34 (sec) , antiderivative size = 252, normalized size of antiderivative = 4.94

$$\int \frac{\sin(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx = \frac{4\,dx-\sqrt{2}\arctan\left(\frac{2\sqrt{2}\sin(dx+c)}{2\left(\sqrt{2}+1\right)\cos(dx+c)+\cos(dx+c)^2+\sin(dx+c)^2+2\sqrt{2}+3},\frac{\cos(dx+c)^2+\sin(dx+c)^2+2\cos(dx+c)-1}{2\left(\sqrt{2}+1\right)\cos(dx+c)+\cos(dx+c)^2+\sin(dx+c)^2+2\sqrt{2}+3}\right)}{}$$

input
```
integrate(sin(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="maxima")
```

output
```
1/4*(4*d*x - sqrt(2)*arctan2(2*sqrt(2)*sin(d*x + c)/(2*(sqrt(2) + 1)*cos(d
*x + c) + cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sqrt(2) + 3), (cos(d*x + c)^
2 + sin(d*x + c)^2 + 2*cos(d*x + c) - 1)/(2*(sqrt(2) + 1)*cos(d*x + c) + c
os(d*x + c)^2 + sin(d*x + c)^2 + 2*sqrt(2) + 3)) + sqrt(2)*arctan2(2*sqrt(
2)*sin(d*x + c)/(2*(sqrt(2) - 1)*cos(d*x + c) + cos(d*x + c)^2 + sin(d*x +
 c)^2 - 2*sqrt(2) + 3), (cos(d*x + c)^2 + sin(d*x + c)^2 - 2*cos(d*x + c)
- 1)/(2*(sqrt(2) - 1)*cos(d*x + c) + cos(d*x + c)^2 + sin(d*x + c)^2 - 2*s
qrt(2) + 3)) + 4*c)/d
```

3.216.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.61

$$\int \frac{\sin(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx = \frac{2\,dx-\sqrt{2}\left(dx+c+\arctan\left(-\frac{\sqrt{2}\sin(2\,dx+2\,c)-2\,\sin(2\,dx+2\,c)}{\sqrt{2}\cos(2\,dx+2\,c)+\sqrt{2}-2\,\cos(2\,dx+2\,c)+2}\right)\right)+2\,c}{2\,d}$$

input
```
integrate(sin(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(2*d*x - sqrt(2)*(d*x + c + arctan(-(sqrt(2)*sin(2*d*x + 2*c) - 2*sin(
2*d*x + 2*c))/(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2) - 2*cos(2*d*x + 2*c) + 2
))) + 2*c)/d
```

3.216.9 Mupad [B] (verification not implemented)

Time = 23.80 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.22

$$\int \frac{\sin(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx = x-\frac{\sqrt{2}\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4}+\frac{7\sqrt{2}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}\right)\right)}{4\,d}$$

input
```
int(sin(c + d*x)/(sin(c + d*x) + 1/sin(c + d*x)),x)
```

output
```
x - (2^(1/2)*(2*atan((7*2^(1/2)*tan(c/2 + (d*x)/2))/4 + (2^(1/2)*tan(c/2 +
 (d*x)/2)^3)/4) + 2*atan((2^(1/2)*tan(c/2 + (d*x)/2))/4)))/(4*d)
```

3.217 $\int \frac{\cos(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx$

3.217.1 Optimal result

Integrand size = 22, antiderivative size = 18

$$\int \frac{\cos(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx = \frac{\log\left(1+\sin^2(c+dx)\right)}{2d}$$

output
```
1/2*ln(1+sin(d*x+c)^2)/d
```

3.217.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\cos(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx = \frac{\log(3-\cos(2(c+dx)))}{2d}$$

input
```
Integrate[Cos[c + d*x]/(Csc[c + d*x] + Sin[c + d*x]),x]
```

output
```
Log[3 - Cos[2*(c + d*x)]]/(2*d)
```

3.217.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3042, 4834, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{\sin(c+dx)+\csc(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(c+dx)}{\sin(c+dx)+\csc(c+dx)}dx$$

$$\Big\downarrow \text{4834}$$

$$\frac{\int \frac{\sin(c+dx)}{\sin^2(c+dx)+1}d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{240}$$

$$\frac{\log\left(\sin^2(c+dx)+1\right)}{2d}$$

input
```
Int[Cos[c + d*x]/(Csc[c + d*x] + Sin[c + d*x]),x]
```

output
```
Log[1 + Sin[c + d*x]^2]/(2*d)
```

3.217.3.1 Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4834
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

3.217.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\frac{\ln\left(\cos(dx+c)^{2}-2\right)}{2d}$	17
default	$\frac{\ln\left(\cos(dx+c)^{2}-2\right)}{2d}$	17
risch	$-ix-\frac{2ic}{d}+\frac{\ln(\mathrm{e}^{4i(dx+c)}-6\,\mathrm{e}^{2i(dx+c)}+1)}{2d}$	41
parallelrisch	$\frac{-\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)+\ln\left(\sqrt{-4+4\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}\right)}{d}$	49
norman	$-\frac{\ln\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}{d}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}+6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1\right)}{2d}$	53

input
```
int(cos(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*ln(cos(d*x+c)^2-2)
```

3.217.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\cos(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx=\frac{\log\left(-\cos\left(dx+c\right)^{2}+2\right)}{2\,d}$$

input
```
integrate(cos(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*log(-cos(d*x + c)^2 + 2)/d
```

3.217.6 Sympy [F]

$$\int \frac{\cos(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx = \int \frac{\cos{(c+dx)}}{\sin{(c+dx)}+\csc{(c+dx)}}\,dx$$

input
```
integrate(cos(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x)
```

output
```
Integral(cos(c + d*x)/(sin(c + d*x) + csc(c + d*x)), x)
```

3.217.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{\cos(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx = \frac{\log\left(\sin{(dx+c)}^2+1\right)}{2\,d}$$

input
```
integrate(cos(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*log(sin(d*x + c)^2 + 1)/d
```

3.217.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{\cos(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx = \frac{\log\left(\sin{(dx+c)}^2+1\right)}{2\,d}$$

input
```
integrate(cos(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="giac")
```

output
```
1/2*log(sin(d*x + c)^2 + 1)/d
```

3.217.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{\cos(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx=\frac{\ln\left(\sin\left(c+d\,x\right)^2+1\right)}{2\,d}$$

input
```
int(cos(c + d*x)/(sin(c + d*x) + 1/sin(c + d*x)),x)
```

output
```
log(sin(c + d*x)^2 + 1)/(2*d)
```

3.218 $\int \frac{\tan(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx$

3.218.1 Optimal result

Integrand size = 22, antiderivative size = 29

$$\int \frac{\tan(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx = -\frac{\arctan(\sin(c+dx))}{2d}+\frac{\text{arctanh}(\sin(c+dx))}{2d}$$

output
```
-1/2*arctan(sin(d*x+c))/d+1/2*arctanh(sin(d*x+c))/d
```

3.218.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{\tan(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx = \frac{-\arctan(\sin(c+dx))+\text{arctanh}(\sin(c+dx))}{2d}$$

input
```
Integrate[Tan[c + d*x]/(Csc[c + d*x] + Sin[c + d*x]),x]
```

output
```
(-ArcTan[Sin[c + d*x]] + ArcTanh[Sin[c + d*x]])/(2*d)
```

3.218.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {3042, 4878, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(c+dx)}{\sin(c+dx)+\csc(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)}{\sin(c+dx)+\csc(c+dx)}dx$$

$$\Big\downarrow \text{4878}$$

$$\frac{\int \frac{\sin^2(c+dx)}{1-\sin^4(c+dx)}d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{827}$$

$$\frac{\frac{1}{2}\int \frac{1}{1-\sin^2(c+dx)}d\sin(c+dx)-\frac{1}{2}\int \frac{1}{\sin^2(c+dx)+1}d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{1}{2}\int \frac{1}{1-\sin^2(c+dx)}d\sin(c+dx)-\frac{1}{2}\arctan(\sin(c+dx))}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{2}\text{arctanh}(\sin(c+dx))-\frac{1}{2}\arctan(\sin(c+dx))}{d}$$

input
```
Int[Tan[c + d*x]/(Csc[c + d*x] + Sin[c + d*x]),x]
```

output
```
(-1/2*ArcTan[Sin[c + d*x]] + ArcTanh[Sin[c + d*x]]/2)/d
```

3.218.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

3.218.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.28

method	result	size
derivativedivides	$\frac{\frac{\ln(\sin(dx+c)+1)}{4}-\frac{\ln(\sin(dx+c)-1)}{4}-\frac{\arctan(\sin(dx+c))}{2}}{d}$	37
default	$\frac{\frac{\ln(\sin(dx+c)+1)}{4}-\frac{\ln(\sin(dx+c)-1)}{4}-\frac{\arctan(\sin(dx+c))}{2}}{d}$	37
risch	$\frac{\ln(i+e^{i(dx+c)})}{2d}-\frac{\ln(e^{i(dx+c)}-i)}{2d}-\frac{i\ln(e^{2i(dx+c)}-2\,e^{i(dx+c)}-1)}{4d}+\frac{i\ln(e^{2i(dx+c)}+2\,e^{i(dx+c)}-1)}{4d}$	96

input
```
int(tan(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/4*ln(sin(d*x+c)+1)-1/4*ln(sin(d*x+c)-1)-1/2*arctan(sin(d*x+c)))
```

3.218.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.28

$$\int \frac{\tan(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx = -\frac{2\arctan(\sin(dx+c)) - \log(\sin(dx+c)+1) + \log(-\sin(dx+c)+1)}{4d}$$

input
```
integrate(tan(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/4*(2*arctan(sin(d*x + c)) - log(sin(d*x + c) + 1) + log(-sin(d*x + c) +
 1))/d
```

3.218.6 Sympy [F]

$$\int \frac{\tan(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx = \int \frac{\tan(c+dx)}{\sin(c+dx)+\csc(c+dx)}\,dx$$

input
```
integrate(tan(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x)
```

output
```
Integral(tan(c + d*x)/(sin(c + d*x) + csc(c + d*x)), x)
```

3.218.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21

$$\int \frac{\tan(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx = -\frac{2\arctan(\sin(dx+c)) - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1)}{4d}$$

input
```
integrate(tan(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/4*(2*arctan(sin(d*x + c)) - log(sin(d*x + c) + 1) + log(sin(d*x + c) -
1))/d
```

3.218.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.28

$$\int \frac{\tan(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx = -\frac{2\arctan\left(\sin\left(dx+c\right)\right)-\log\left(\left|\sin\left(dx+c\right)+1\right|\right)+\log\left(\left|\sin\left(dx+c\right)-1\right|\right)}{4\,d}$$

input
```
integrate(tan(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="giac")
```

output
```
-1/4*(2*arctan(sin(d*x + c)) - log(abs(sin(d*x + c) + 1)) + log(abs(sin(d*
x + c) - 1)))/d
```

3.218.9 Mupad [B] (verification not implemented)

Time = 23.92 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.10

$$\int \frac{\tan(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx = \frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} - \frac{\mathrm{atan}\left(\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{2}+\frac{5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)-\mathrm{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)}{2\,d}$$

input
```
int(tan(c + d*x)/(sin(c + d*x) + 1/sin(c + d*x)),x)
```

output
```
atanh(tan(c/2 + (d*x)/2))/d - (atan((5*tan(c/2 + (d*x)/2))/2 + tan(c/2 + (
d*x)/2)^3/2) - atan(tan(c/2 + (d*x)/2)/2))/(2*d)
```

3.219 $\int \frac{\cot(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx$

3.219.1 Optimal result

Integrand size = 22, antiderivative size = 11

$$\int \frac{\cot(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx = \frac{\arctan(\sin(c+dx))}{d}$$

output
```
arctan(sin(d*x+c))/d
```

3.219.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\cot(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx = \frac{\arctan(\sin(c+dx))}{d}$$

input
```
Integrate[Cot[c + d*x]/(Csc[c + d*x] + Sin[c + d*x]),x]
```

output
```
ArcTan[Sin[c + d*x]]/d
```

3.219.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3042, 4838, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(c+dx)}{\sin(c+dx)+\csc(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cot(c+dx)}{\sin(c+dx)+\csc(c+dx)}dx$$

$$\Big\downarrow \text{ 4838}$$

$$\frac{\int \frac{1}{\sin^2(c+dx)+1}d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\arctan(\sin(c+dx))}{d}$$

input
```
Int[Cot[c + d*x]/(Csc[c + d*x] + Sin[c + d*x]),x]
```

output
```
ArcTan[Sin[c + d*x]]/d
```

3.219.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4838
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[1/(b*c)   Subst[Int[SubstFor[1/x, Sin[c*(a +
 b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

3.219.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$\frac{\arctan(\sin(dx+c))}{d}$	12
default	$\frac{\arctan(\sin(dx+c))}{d}$	12
risch	$-\frac{i\ln\left(\mathrm{e}^{2i(dx+c)}+2\,\mathrm{e}^{i(dx+c)}-1\right)}{2d}+\frac{i\ln\left(\mathrm{e}^{2i(dx+c)}-2\,\mathrm{e}^{i(dx+c)}-1\right)}{2d}$	60

input
```
int(cot(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
arctan(sin(d*x+c))/d
```

3.219.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\cot(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx = \frac{\arctan\left(\sin\left(dx+c\right)\right)}{d}$$

input
```
integrate(cot(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="fricas")
```

output
```
arctan(sin(d*x + c))/d
```

3.219.6 Sympy [F]

$$\int \frac{\cot(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx = \int \frac{\cot(c+dx)}{\sin(c+dx)+\csc(c+dx)}\,dx$$

input
```
integrate(cot(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x)
```

output
```
Integral(cot(c + d*x)/(sin(c + d*x) + csc(c + d*x)), x)
```

3.219.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\cot(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx = \frac{\arctan(\sin(dx+c))}{d}$$

input
```
integrate(cot(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="maxima")
```

output
```
arctan(sin(d*x + c))/d
```

3.219.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\cot(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx = \frac{\arctan(\sin(dx+c))}{d}$$

input
```
integrate(cot(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="giac")
```

output
```
arctan(sin(d*x + c))/d
```

3.219.9 Mupad [B] (verification not implemented)

Time = 23.94 (sec) , antiderivative size = 45, normalized size of antiderivative = 4.09

$$\int \frac{\cot(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx = \frac{\operatorname{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}+\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)-\operatorname{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)}{d}$$

input
```
int(cot(c + d*x)/(sin(c + d*x) + 1/sin(c + d*x)),x)
```

output
```
(atan((5*tan(c/2 + (d*x)/2))/2 + tan(c/2 + (d*x)/2)^3/2) - atan(tan(c/2 +
(d*x)/2)/2))/d
```

3.220 $\int \frac{\sec(c+dx)}{\csc(c+dx)+\sin(c+dx)} dx$

3.220.1 Optimal result

Integrand size = 22, antiderivative size = 16

$$\int \frac{\sec(c+dx)}{\csc(c+dx)+\sin(c+dx)} dx = \frac{\operatorname{arctanh}(\sin^2(c+dx))}{2d}$$

output
```
1/2*arctanh(sin(d*x+c)^2)/d
```

3.220.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.88

$$\int \frac{\sec(c+dx)}{\csc(c+dx)+\sin(c+dx)} dx = \frac{-2\log(\cos(c+dx))+\log\left(2-\cos^2(c+dx)\right)}{4d}$$

input
```
Integrate[Sec[c + d*x]/(Csc[c + d*x] + Sin[c + d*x]),x]
```

output
```
(-2*Log[Cos[c + d*x]] + Log[2 - Cos[c + d*x]^2])/(4*d)
```

3.220.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3042, 4878, 807, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{\sin(c+dx)+\csc(c+dx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sec(c+dx)}{\sin(c+dx)+\csc(c+dx)}dx$$

$$\Big\downarrow \text{ 4878}$$

$$\frac{\int \frac{\sin(c+dx)}{1-\sin^4(c+dx)}d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{ 807}$$

$$\frac{\int \frac{1}{1-\sin^4(c+dx)}d\sin^2(c+dx)}{2d}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\operatorname{arctanh}\left(\sin^2(c+dx)\right)}{2d}$$

input
```
Int[Sec[c + d*x]/(Csc[c + d*x] + Sin[c + d*x]),x]
```

output
```
ArcTanh[Sin[c + d*x]^2]/(2*d)
```

3.220.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

3.220.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

method	result	size
derivativedivides	$\frac{\ln\left(2\sec(dx+c)^2-1\right)}{4d}$	19
default	$\frac{\ln\left(2\sec(dx+c)^2-1\right)}{4d}$	19
risch	$-\frac{\ln(e^{2i(dx+c)}+1)}{2d}+\frac{\ln(e^{4i(dx+c)}-6\,e^{2i(dx+c)}+1)}{4d}$	47
parallelrisch	$\frac{-2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\ln\left(-4+4\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^4\right)}{4d}$	62
norman	$-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2d}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)}{4d}$	68

input
```
int(sec(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/4/d*ln(2*sec(d*x+c)^2-1)
```

3.220.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. 2(14) = 28.

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.88

$$\int \frac{\sec(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx = \frac{\log\left(-\cos\left(dx+c\right)^2+2\right)-2\log\left(-\cos\left(dx+c\right)\right)}{4\,d}$$

input
```
integrate(sec(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*(log(-cos(d*x + c)^2 + 2) - 2*log(-cos(d*x + c)))/d
```

3.220.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx = \int \frac{\sec\left(c+dx\right)}{\sin\left(c+dx\right)+\csc\left(c+dx\right)}\,dx$$

input
```
integrate(sec(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)/(sin(c + d*x) + csc(c + d*x)), x)
```

3.220.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(14) = 28.

Time = 0.31 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.44

$$\int \frac{\sec(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx = \frac{\log\left(\sin\left(dx+c\right)^2+1\right)-\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)}{4\,d}$$

input
```
integrate(sec(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="maxima")
```

output
```
1/4*(log(sin(d*x + c)^2 + 1) - log(sin(d*x + c) + 1) - log(sin(d*x + c) -
1))/d
```

3.220.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(14) = 28$.

Time = 0.32 (sec) , antiderivative size = 79, normalized size of antiderivative = 4.94

$$\int \frac{\sec(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx = -\frac{2\,\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)-\log\left(\left|-\frac{6\,(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+1\right|\right)}{4\,d}$$

input
```
integrate(sec(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="giac")
```

output
```
-1/4*(2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) - log(abs(-6*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + (cos(d*x + c) - 1)^2/(cos(d*x + c)
 + 1)^2 + 1)))/d
```

3.220.9 Mupad [B] (verification not implemented)

Time = 23.74 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\sec(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx = \frac{\operatorname{atanh}\left(\sin\left(c+d\,x\right)^2\right)}{2\,d}$$

input
```
int(1/(cos(c + d*x)*(sin(c + d*x) + 1/sin(c + d*x))),x)
```

output
```
atanh(sin(c + d*x)^2)/(2*d)
```

3.221 $\int \frac{\csc(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx$

3.221.1 Optimal result

Integrand size = 22, antiderivative size = 48

$$\int \frac{\csc(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx = \frac{x}{\sqrt{2}}+\frac{\arctan\left(\frac{\cos(c+dx)\sin(c+dx)}{1+\sqrt{2}+\sin^2(c+dx)}\right)}{\sqrt{2}d}$$

output
```
1/2*x*2^(1/2)+1/2*arctan(cos(d*x+c)*sin(d*x+c)/(1+sin(d*x+c)^2+2^(1/2)))/d
*2^(1/2)
```

3.221.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.46

$$\int \frac{\csc(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx = \frac{\arctan\left(\sqrt{2}\tan(c+dx)\right)}{\sqrt{2}d}$$

input
```
Integrate[Csc[c + d*x]/(Csc[c + d*x] + Sin[c + d*x]),x]
```

output
```
ArcTan[Sqrt[2]*Tan[c + d*x]]/(Sqrt[2]*d)
```

3.221.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.46, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3042, 4889, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(c+dx)}{\sin(c+dx)+\csc(c+dx)}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\csc(c+dx)}{\sin(c+dx)+\csc(c+dx)}dx$$

$$\Big\downarrow \ 4889$$

$$\frac{\int \frac{1}{2\tan^2(c+dx)+1}d\tan(c+dx)}{d}$$

$$\Big\downarrow \ 216$$

$$\frac{\arctan\left(\sqrt{2}\tan(c+dx)\right)}{\sqrt{2}d}$$

input
```
Int[Csc[c + d*x]/(Csc[c + d*x] + Sin[c + d*x]),x]
```

output
```
ArcTan[Sqrt[2]*Tan[c + d*x]]/(Sqrt[2]*d)
```

3.221.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.221.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.42

method	result	size
derivativedivides	$\frac{\sqrt{2}\,\arctan\left(\tan(dx+c)\sqrt{2}\right)}{2d}$	20
default	$\frac{\sqrt{2}\,\arctan\left(\tan(dx+c)\sqrt{2}\right)}{2d}$	20
risch	$\frac{i\sqrt{2}\,\ln\left(e^{2i(dx+c)}-2\sqrt{2}-3\right)}{4d}-\frac{i\sqrt{2}\,\ln\left(e^{2i(dx+c)}+2\sqrt{2}-3\right)}{4d}$	54

input
```
int(csc(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*2^(1/2)*arctan(tan(d*x+c)*2^(1/2))
```

3.221.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int \frac{\csc(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx = -\frac{\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(dx+c)^2-2\sqrt{2}}{4\cos(dx+c)\sin(dx+c)}\right)}{4\,d}$$

input
```
integrate(csc(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/4*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(d*x + c)^2 - 2*sqrt(2))/(cos(d*x +
c)*sin(d*x + c)))/d
```

3.221.6 Sympy [F]

$$\int \frac{\csc(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx = \int \frac{\csc(c+dx)}{\sin(c+dx)+\csc(c+dx)}\,dx$$

input
```
integrate(csc(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x)
```

output
```
Integral(csc(c + d*x)/(sin(c + d*x) + csc(c + d*x)), x)
```

3.221.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(44) = 88$.

Time = 0.34 (sec) , antiderivative size = 245, normalized size of antiderivative = 5.10

$$\int \frac{\csc(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx = \frac{\sqrt{2}\arctan\left(\frac{2\sqrt{2}\sin(dx+c)}{2\left(\sqrt{2}+1\right)\cos(dx+c)+\cos(dx+c)^2+\sin(dx+c)^2+2\sqrt{2}+3}, \frac{\cos(dx+c)^2+\sin(dx+c)^2+2\cos(dx+c)-1}{2\left(\sqrt{2}+1\right)\cos(dx+c)+\cos(dx+c)^2+\sin(dx+c)^2+2\sqrt{2}+3}\right)-\sqrt{2}}{4d}$$

input
```
integrate(csc(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="maxima")
```

output
```
1/4*(sqrt(2)*arctan2(2*sqrt(2)*sin(d*x + c)/(2*(sqrt(2) + 1)*cos(d*x + c)
+ cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sqrt(2) + 3), (cos(d*x + c)^2 + sin(
d*x + c)^2 + 2*cos(d*x + c) - 1)/(2*(sqrt(2) + 1)*cos(d*x + c) + cos(d*x +
 c)^2 + sin(d*x + c)^2 + 2*sqrt(2) + 3)) - sqrt(2)*arctan2(2*sqrt(2)*sin(d
*x + c)/(2*(sqrt(2) - 1)*cos(d*x + c) + cos(d*x + c)^2 + sin(d*x + c)^2 -
2*sqrt(2) + 3), (cos(d*x + c)^2 + sin(d*x + c)^2 - 2*cos(d*x + c) - 1)/(2*
(sqrt(2) - 1)*cos(d*x + c) + cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sqrt(2) +
 3)))/d
```

3.221.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.50

$$\int \frac{\csc(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx = \frac{\sqrt{2}\left(dx+c+\arctan\left(-\frac{\sqrt{2}\sin(2\,dx+2\,c)-2\,\sin(2\,dx+2\,c)}{\sqrt{2}\cos(2\,dx+2\,c)+\sqrt{2}-2\,\cos(2\,dx+2\,c)+2}\right)\right)}{2\,d}$$

input
```
integrate(csc(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*(d*x + c + arctan(-(sqrt(2)*sin(2*d*x + 2*c) - 2*sin(2*d*x + 2
*c))/(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2) - 2*cos(2*d*x + 2*c) + 2)))/d
```

3.221.9 Mupad [B] (verification not implemented)

Time = 24.87 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.17

$$\int \frac{\csc(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx = \frac{\sqrt{2}\left(\mathrm{atan}\left(\frac{\sqrt{2}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4}+\frac{7\,\sqrt{2}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}\right)+\mathrm{atan}\left(\frac{\sqrt{2}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}\right)\right)}{2\,d}$$

input
```
int(1/(sin(c + d*x)*(sin(c + d*x) + 1/sin(c + d*x))),x)
```

output
```
(2^(1/2)*(atan((7*2^(1/2)*tan(c/2 + (d*x)/2))/4 + (2^(1/2)*tan(c/2 + (d*x)
/2)^3)/4) + atan((2^(1/2)*tan(c/2 + (d*x)/2))/4)))/(2*d)
```

3.222 $\int \frac{1}{\csc(c+dx)-\sin(c+dx)}\,dx$

3.222.1 Optimal result

Integrand size = 17, antiderivative size = 10

$$\int \frac{1}{\csc(c+dx)-\sin(c+dx)}\,dx = \frac{\sec(c+dx)}{d}$$

output
```
sec(d*x+c)/d
```

3.222.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{\csc(c+dx)-\sin(c+dx)}\,dx = \frac{\sec(c+dx)}{d}$$

input
```
Integrate[(Csc[c + d*x] - Sin[c + d*x])^(-1),x]
```

output
```
Sec[c + d*x]/d
```

3.222.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3042, 4897, 3042, 3086, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\csc(c+dx)-\sin(c+dx)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\csc(c+dx)-\sin(c+dx)}dx$$
$$\Big\downarrow \; 4897$$
$$\int \tan(c+dx)\sec(c+dx)dx$$
$$\Big\downarrow \; 3042$$
$$\int \tan(c+dx)\sec(c+dx)dx$$
$$\Big\downarrow \; 3086$$
$$\frac{\int 1d\sec(c+dx)}{d}$$
$$\Big\downarrow \; 24$$
$$\frac{\sec(c+dx)}{d}$$

input
```
Int[(Csc[c + d*x] - Sin[c + d*x])^(-1),x]
```

output
```
Sec[c + d*x]/d
```

3.222.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.222.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.30

method	result	size
derivativedivides	$\frac{1}{d\cos(dx+c)}$	13
default	$\frac{1}{d\cos(dx+c)}$	13
norman	$-\frac{2}{d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)}$	21
parallelrisch	$-\frac{2}{d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)}$	21
risch	$\frac{2\,\mathrm{e}^{i(dx+c)}}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)}$	28

input
```
int(1/(csc(d*x+c)-sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d/cos(d*x+c)
```

3.222.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{\csc(c+dx)-\sin(c+dx)}\,dx = \frac{1}{d\cos(dx+c)}$$

input
```
integrate(1/(csc(d*x+c)-sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/(d*cos(d*x + c))
```

3.222.6 Sympy [F]

$$\int \frac{1}{\csc(c+dx)-\sin(c+dx)}\,dx = \int \frac{1}{-\sin(c+dx)+\csc(c+dx)}\,dx$$

input
```
integrate(1/(csc(d*x+c)-sin(d*x+c)),x)
```

output
```
Integral(1/(-sin(c + d*x) + csc(c + d*x)), x)
```

3.222.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. 2(10) = 20.

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.80

$$\int \frac{1}{\csc(c+dx)-\sin(c+dx)}\,dx = -\frac{2}{d\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)}$$

input
```
integrate(1/(csc(d*x+c)-sin(d*x+c)),x, algorithm="maxima")
```

output
```
-2/(d*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1))
```

3.222.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. 2(10) = 20.

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.80

$$\int \frac{1}{\csc(c+dx)-\sin(c+dx)}\,dx = \frac{2}{d\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)}$$

input
```
integrate(1/(csc(d*x+c)-sin(d*x+c)),x, algorithm="giac")
```

output
```
2/(d*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))
```

3.222.9 Mupad [B] (verification not implemented)

Time = 24.59 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.00

$$\int \frac{1}{\csc(c+dx)-\sin(c+dx)}\,dx = -\frac{2}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)}$$

input
```
int(-1/(sin(c + d*x) - 1/sin(c + d*x)),x)
```

output
```
-2/(d*(tan(c/2 + (d*x)/2)^2 - 1))
```

3.223 $\int \frac{\sin(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx$

3.223.1 Optimal result

Integrand size = 24, antiderivative size = 14

$$\int \frac{\sin(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx = -x+\frac{\tan(c+dx)}{d}$$

output
```
-x+tan(d*x+c)/d
```

3.223.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.64

$$\int \frac{\sin(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx = -\frac{\arctan(\tan(c+dx))}{d}+\frac{\tan(c+dx)}{d}$$

input
```
Integrate[Sin[c + d*x]/(Csc[c + d*x] - Sin[c + d*x]),x]
```

output
```
-(ArcTan[Tan[c + d*x]]/d) + Tan[c + d*x]/d
```

3.223.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 4889, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(c+dx)}{\csc(c+dx)-\sin(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(c+dx)}{\csc(c+dx)-\sin(c+dx)}dx$$

$$\Big\downarrow \text{4889}$$

$$\frac{\int \frac{\tan^2(c+dx)}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{262}$$

$$\frac{\tan(c+dx)-\int \frac{1}{\tan^2(c+dx)+1}d\tan(c+dx)}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{\tan(c+dx)-\arctan(\tan(c+dx))}{d}$$

input
```
Int[Sin[c + d*x]/(Csc[c + d*x] - Sin[c + d*x]),x]
```

output
```
(-ArcTan[Tan[c + d*x]] + Tan[c + d*x])/d
```

3.223.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.223.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.50

method	result	size
derivativedivides	$\frac{\tan(dx+c)-\arctan(\tan(dx+c))}{d}$	21
default	$\frac{\tan(dx+c)-\arctan(\tan(dx+c))}{d}$	21
risch	$-x+\frac{2i}{d(\mathrm{e}^{2i(dx+c)}+1)}$	24
parallelrisch	$\frac{-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 xd+dx-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)}$	50
norman	$\frac{x-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}-x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)}$	78

input
```
int(sin(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(tan(d*x+c)-arctan(tan(d*x+c)))
```

3.223.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(14) = 28$.

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.21

$$\int \frac{\sin(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx = -\frac{dx\cos(dx+c)-\sin(dx+c)}{d\cos(dx+c)}$$

input
```
integrate(sin(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="fricas")
```

output
```
-(d*x*cos(d*x + c) - sin(d*x + c))/(d*cos(d*x + c))
```

3.223.6 Sympy [F]

$$\int \frac{\sin(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx = \int \frac{\sin(c+dx)}{-\sin(c+dx)+\csc(c+dx)}\,dx$$

input
```
integrate(sin(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x)
```

output
```
Integral(sin(c + d*x)/(-sin(c + d*x) + csc(c + d*x)), x)
```

3.223.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(14) = 28$.

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 4.57

$$\int \frac{\sin(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx = -\frac{2\left(\frac{\sin(dx+c)}{\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)(\cos(dx+c)+1)}+\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)\right)}{d}$$

input
```
integrate(sin(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="maxima")
```

output
```
-2*(sin(d*x + c)/((sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)*(cos(d*x + c)
+ 1)) + arctan(sin(d*x + c)/(cos(d*x + c) + 1)))/d
```

3.223.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \frac{\sin(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx = -\frac{dx+c-\tan(dx+c)}{d}$$

input
```
integrate(sin(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="giac")
```

output
```
-(d*x + c - tan(d*x + c))/d
```

3.223.9 Mupad [B] (verification not implemented)

Time = 23.70 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.36

$$\int \frac{\sin(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx = -x-\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)}$$

input
```
int(-sin(c + d*x)/(sin(c + d*x) - 1/sin(c + d*x)),x)
```

output
```
- x - (2*tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)/2)^2 - 1))
```

3.224 $\int \frac{\cos(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx$

3.224.1 Optimal result

Integrand size = 24, antiderivative size = 12

$$\int \frac{\cos(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx = -\frac{\log(\cos(c+dx))}{d}$$

output
```
-ln(cos(d*x+c))/d
```

3.224.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\cos(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx = -\frac{\log(\cos(c+dx))}{d}$$

input
```
Integrate[Cos[c + d*x]/(Csc[c + d*x] - Sin[c + d*x]),x]
```

output
```
-(Log[Cos[c + d*x]]/d)
```

3.224.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3042, 4834, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{\cos(c+dx)}{\csc(c+dx)-\sin(c+dx)}dx$$
$$\Big\downarrow \ 4834$$
$$\frac{\int \frac{\sin(c+dx)}{1-\sin^2(c+dx)}d\sin(c+dx)}{d}$$
$$\Big\downarrow \ 240$$
$$-\frac{\log\left(1-\sin^2(c+dx)\right)}{2d}$$

input
```
Int[Cos[c + d*x]/(Csc[c + d*x] - Sin[c + d*x]),x]
```

output
```
-1/2*Log[1 - Sin[c + d*x]^2]/d
```

3.224.3.1 Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4834
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

3.224.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$-\frac{\ln(\cos(dx+c))}{d}$	13
default	$-\frac{\ln(\cos(dx+c))}{d}$	13
risch	$ix+\frac{2ic}{d}-\frac{\ln(\mathrm{e}^{2i(dx+c)}+1)}{d}$	30
parallelrisch	$\frac{-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\ln\left(\sec\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}{d}$	46
norman	$\frac{\ln\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}{d}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}$	54

input
```
int(cos(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-ln(cos(d*x+c))/d
```

3.224.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cos(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx=-\frac{\log\left(-\cos\left(dx+c\right)\right)}{d}$$

input
```
integrate(cos(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="fricas")
```

output
```
-log(-cos(d*x + c))/d
```

3.224.6 Sympy [F]

$$\int \frac{\cos(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx = \int \frac{\cos(c+dx)}{-\sin(c+dx)+\csc(c+dx)}\,dx$$

input
```
integrate(cos(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x)
```

output
```
Integral(cos(c + d*x)/(-sin(c + d*x) + csc(c + d*x)), x)
```

3.224.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.00

$$\int \frac{\cos(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx = -\frac{\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)}{2\,d}$$

input
```
integrate(cos(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/2*(log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1))/d
```

3.224.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(12) = 24.

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.17

$$\int \frac{\cos(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx = -\frac{\log(|\sin(dx+c)+1|)+\log(|\sin(dx+c)-1|)}{2\,d}$$

input
```
integrate(cos(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="giac")
```

output
```
-1/2*(log(abs(sin(d*x + c) + 1)) + log(abs(sin(d*x + c) - 1)))/d
```

3.224.9 Mupad [B] (verification not implemented)

Time = 22.69 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cos(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx = -\frac{\ln\left(\cos\left(c+d\,x\right)^2\right)}{2\,d}$$

input
```
int(-cos(c + d*x)/(sin(c + d*x) - 1/sin(c + d*x)),x)
```

output
```
-log(cos(c + d*x)^2)/(2*d)
```

3.225 $\int \frac{\tan(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx$

3.225.1 Optimal result

Integrand size = 24, antiderivative size = 34

$$\int \frac{\tan(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx = -\frac{\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{\sec(c+dx)\tan(c+dx)}{2d}$$

output
```
-1/2*arctanh(sin(d*x+c))/d+1/2*sec(d*x+c)*tan(d*x+c)/d
```

3.225.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\tan(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx = -\frac{\operatorname{arctanh}(\sin(c+dx))}{2d}+\frac{\sec(c+dx)\tan(c+dx)}{2d}$$

input
```
Integrate[Tan[c + d*x]/(Csc[c + d*x] - Sin[c + d*x]),x]
```

output
```
-1/2*ArcTanh[Sin[c + d*x]]/d + (Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

3.225.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 4878, 252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(c+dx)}{\csc(c+dx)-\sin(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\tan(c+dx)}{\csc(c+dx)-\sin(c+dx)}dx$$

$$\Big\downarrow \text{ 4878}$$

$$\frac{\int \frac{\sin^2(c+dx)}{(1-\sin^2(c+dx))^2}d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{ 252}$$

$$\frac{\frac{\sin(c+dx)}{2(1-\sin^2(c+dx))}-\frac{1}{2}\int \frac{1}{1-\sin^2(c+dx)}d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{\sin(c+dx)}{2(1-\sin^2(c+dx))}-\frac{1}{2}\text{arctanh}(\sin(c+dx))}{d}$$

input
```
Int[Tan[c + d*x]/(Csc[c + d*x] - Sin[c + d*x]),x]
```

output
```
(-1/2*ArcTanh[Sin[c + d*x]] + Sin[c + d*x]/(2*(1 - Sin[c + d*x]^2)))/d
```

3.225.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

3.225.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.53

method	result	size
derivativedivides	$\frac{-\frac{1}{4(\sin(dx+c)-1)}+\frac{\ln(\sin(dx+c)-1)}{4}-\frac{1}{4(\sin(dx+c)+1)}-\frac{\ln(\sin(dx+c)+1)}{4}}{d}$	52
default	$\frac{-\frac{1}{4(\sin(dx+c)-1)}+\frac{\ln(\sin(dx+c)-1)}{4}-\frac{1}{4(\sin(dx+c)+1)}-\frac{\ln(\sin(dx+c)+1)}{4}}{d}$	52
risch	$-\frac{i\left(e^{3i(dx+c)}-e^{i(dx+c)}\right)}{d\left(e^{2i(dx+c)}+1\right)^2}-\frac{\ln\left(i+e^{i(dx+c)}\right)}{2d}+\frac{\ln\left(e^{i(dx+c)}-i\right)}{2d}$	78

input
```
int(tan(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/4/(sin(d*x+c)-1)+1/4*ln(sin(d*x+c)-1)-1/4/(sin(d*x+c)+1)-1/4*ln(si
n(d*x+c)+1))
```

3.225.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(30) = 60$.

Time = 0.24 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.79

$$\int \frac{\tan(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx = -\frac{\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)-2\sin\left(dx+c\right)}{4\,d\cos\left(dx+c\right)^2}$$

input
```
integrate(tan(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/4*(cos(d*x + c)^2*log(sin(d*x + c) + 1) - cos(d*x + c)^2*log(-sin(d*x +
 c) + 1) - 2*sin(d*x + c))/(d*cos(d*x + c)^2)
```

3.225.6 Sympy [F]

$$\int \frac{\tan(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx = \int \frac{\tan\left(c+dx\right)}{-\sin\left(c+dx\right)+\csc\left(c+dx\right)}\,dx$$

input
```
integrate(tan(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x)
```

output
```
Integral(tan(c + d*x)/(-sin(c + d*x) + csc(c + d*x)), x)
```

3.225.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

$$\int \frac{\tan(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx = -\frac{\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}+\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)}{4\,d}$$

input
```
integrate(tan(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - log(si
n(d*x + c) - 1))/d
```

3.225.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.41

$$\int \frac{\tan(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx = -\frac{\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} + \log\left(|\sin\left(dx+c\right)+1|\right) - \log\left(|\sin\left(dx+c\right)-1|\right)}{4\,d}$$

input
```
integrate(tan(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="giac")
```

output
```
-1/4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(abs(sin(d*x + c) + 1)) - l
og(abs(sin(d*x + c) - 1)))/d
```

3.225.9 Mupad [B] (verification not implemented)

Time = 23.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.03

$$\int \frac{\tan(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)} - \frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

input
```
int(-tan(c + d*x)/(sin(c + d*x) - 1/sin(c + d*x)),x)
```

output
```
(tan(c/2 + (d*x)/2) + tan(c/2 + (d*x)/2)^3)/(d*(tan(c/2 + (d*x)/2)^4 - 2*t
an(c/2 + (d*x)/2)^2 + 1)) - atanh(tan(c/2 + (d*x)/2))/d
```

3.226 $\int \frac{\cot(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx$

3.226.1 Optimal result

Integrand size = 24, antiderivative size = 11

$$\int \frac{\cot(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{d}$$

output
```
arctanh(sin(d*x+c))/d
```

3.226.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\cot(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{d}$$

input
```
Integrate[Cot[c + d*x]/(Csc[c + d*x] - Sin[c + d*x]),x]
```

output
```
ArcTanh[Sin[c + d*x]]/d
```

3.226.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3042, 4838, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(c+dx)}{\csc(c+dx)-\sin(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cot(c+dx)}{\csc(c+dx)-\sin(c+dx)}dx$$

$$\Big\downarrow \text{ 4838}$$

$$\frac{\int \frac{1}{1-\sin^2(c+dx)}d\sin(c+dx)}{d}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\text{arctanh}(\sin(c+dx))}{d}$$

input
```
Int[Cot[c + d*x]/(Csc[c + d*x] - Sin[c + d*x]),x]
```

output
```
ArcTanh[Sin[c + d*x]]/d
```

3.226.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4838
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[1/(b*c)   Subst[Int[SubstFor[1/x, Sin[c*(a +
 b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

3.226.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$\frac{\operatorname{arctanh}(\sin(dx+c))}{d}$	12
default	$\frac{\operatorname{arctanh}(\sin(dx+c))}{d}$	12
risch	$\frac{\ln\left(i+e^{i(dx+c)}\right)}{d}-\frac{\ln\left(e^{i(dx+c)}-i\right)}{d}$	37

input
```
int(cot(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
arctanh(sin(d*x+c))/d
```

3.226.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(11) = 22$.

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.55

$$\int \frac{\cot(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx = \frac{\log\left(\sin\left(dx+c\right)+1\right)-\log\left(-\sin\left(dx+c\right)+1\right)}{2\,d}$$

input
```
integrate(cot(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(log(sin(d*x + c) + 1) - log(-sin(d*x + c) + 1))/d
```

3.226.6 Sympy [F]

$$\int \frac{\cot(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx = \int \frac{\cot(c+dx)}{-\sin(c+dx)+\csc(c+dx)}\,dx$$

input
```
integrate(cot(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x)
```

output
```
Integral(cot(c + d*x)/(-sin(c + d*x) + csc(c + d*x)), x)
```

3.226.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(11) = 22$.

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.36

$$\int \frac{\cot(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx = \frac{\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)}{2\,d}$$

input
```
integrate(cot(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1))/d
```

3.226.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(11) = 22$.

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.55

$$\int \frac{\cot(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx = \frac{\log(|\sin(dx+c)+1|)-\log(|\sin(dx+c)-1|)}{2\,d}$$

input
```
integrate(cot(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(log(abs(sin(d*x + c) + 1)) - log(abs(sin(d*x + c) - 1)))/d
```

3.226.9 Mupad [B] (verification not implemented)

Time = 22.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \frac{\cot(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx = \frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}$$

input
```
int(-cot(c + d*x)/(sin(c + d*x) - 1/sin(c + d*x)),x)
```

output
```
(2*atanh(tan(c/2 + (d*x)/2)))/d
```

3.227 $\int \frac{\sec(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx$

3.227.1 Optimal result

Integrand size = 24, antiderivative size = 15

$$\int \frac{\sec(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx=\frac{\sec^2(c+dx)}{2d}$$

output
```
1/2*sec(d*x+c)^2/d
```

3.227.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\sec(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx=\frac{\sec^2(c+dx)}{2d}$$

input
```
Integrate[Sec[c + d*x]/(Csc[c + d*x] - Sin[c + d*x]),x]
```

output
```
Sec[c + d*x]^2/(2*d)
```

3.227.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.40, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3042, 4878, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{\csc(c+dx)-\sin(c+dx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sec(c+dx)}{\csc(c+dx)-\sin(c+dx)}dx$$
$$\Big\downarrow \text{4878}$$
$$\frac{\int \frac{\sin(c+dx)}{\left(1-\sin^2(c+dx)\right)^2}d\sin(c+dx)}{d}$$
$$\Big\downarrow \text{241}$$
$$\frac{1}{2d\left(1-\sin^2(c+dx)\right)}$$

input
```
Int[Sec[c + d*x]/(Csc[c + d*x] - Sin[c + d*x]),x]
```

output
```
1/(2*d*(1 - Sin[c + d*x]^2))
```

3.227.3.1 Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

3.227.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\sec(dx+c)^2}{2d}$	14
default	$\frac{\sec(dx+c)^2}{2d}$	14
risch	$\frac{2\,\mathrm{e}^{2i(dx+c)}}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)^2}$	28
norman	$\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)^2}$	32
parallelrisch	$\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}$	43

input
```
int(sec(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/2*sec(d*x+c)^2/d
```

3.227.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{\sec(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx = \frac{1}{2\,d\cos\left(dx+c\right)^2}$$

input
```
integrate(sec(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/2/(d*cos(d*x + c)^2)
```

3.227.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx = \int \frac{\sec(c+dx)}{-\sin(c+dx)+\csc(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)/(-sin(c + d*x) + csc(c + d*x)), x)
```

3.227.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\sec(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx = -\frac{1}{2\left(\sin(dx+c)^2-1\right)d}$$

input
```
integrate(sec(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/2/((sin(d*x + c)^2 - 1)*d)
```

3.227.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(13) = 26.

Time = 0.31 (sec) , antiderivative size = 46, normalized size of antiderivative = 3.07

$$\int \frac{\sec(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx = -\frac{2(\cos(dx+c)-1)}{d\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^2(\cos(dx+c)+1)}$$

input
```
integrate(sec(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="giac")
```

output
```
-2*(cos(d*x + c) - 1)/(d*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^2*(co
s(d*x + c) + 1))
```

3.227.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{\sec(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx = \frac{1}{2\,d\cos\left(c+d\,x\right)^2}$$

input
```
int(-1/(cos(c + d*x)*(sin(c + d*x) - 1/sin(c + d*x))),x)
```

output
```
1/(2*d*cos(c + d*x)^2)
```

3.228 $\int \frac{\csc(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx$

3.228.1 Optimal result

Integrand size = 24, antiderivative size = 10

$$\int \frac{\csc(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx=\frac{\tan(c+dx)}{d}$$

output
```
tan(d*x+c)/d
```

3.228.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\csc(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx=\frac{\tan(c+dx)}{d}$$

input
```
Integrate[Csc[c + d*x]/(Csc[c + d*x] - Sin[c + d*x]),x]
```

output
```
Tan[c + d*x]/d
```

3.228.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3042, 4889, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc(c+dx)}{\csc(c+dx)-\sin(c+dx)}dx$$

$$\Big\downarrow \; 4889$$

$$\frac{\int 1d\tan(c+dx)}{d}$$

$$\Big\downarrow \; 24$$

$$\frac{\tan(c+dx)}{d}$$

input
```
Int[Csc[c + d*x]/(Csc[c + d*x] - Sin[c + d*x]),x]
```

output
```
Tan[c + d*x]/d
```

3.228.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.228.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$\frac{\tan(dx+c)}{d}$	11
default	$\frac{\tan(dx+c)}{d}$	11
risch	$\frac{2i}{d(e^{2i(dx+c)}+1)}$	20
norman	$-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)}$	30
parallelrisch	$-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)}$	30

input
```
int(csc(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
tan(d*x+c)/d
```

3.228.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

$$\int \frac{\csc(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx = \frac{\sin(dx+c)}{d\cos(dx+c)}$$

input
```
integrate(csc(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="fricas")
```

output
```
sin(d*x + c)/(d*cos(d*x + c))
```

3.228.6 Sympy [F]

$$\int \frac{\csc(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx = \int \frac{\csc(c+dx)}{-\sin(c+dx)+\csc(c+dx)}\,dx$$

input
```
integrate(csc(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x)
```

output
```
Integral(csc(c + d*x)/(-sin(c + d*x) + csc(c + d*x)), x)
```

3.228.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(10) = 20$.

Time = 0.20 (sec) , antiderivative size = 44, normalized size of antiderivative = 4.40

$$\int \frac{\csc(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx = -\frac{2\sin(dx+c)}{d\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)(\cos(dx+c)+1)}$$

input
```
integrate(csc(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="maxima")
```

output
```
-2*sin(d*x + c)/(d*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)*(cos(d*x + c)
 + 1))
```

3.228.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\csc(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx = \frac{\tan(dx+c)}{d}$$

input
```
integrate(csc(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="giac")
```

output
```
tan(d*x + c)/d
```

3.228.9 Mupad [B] (verification not implemented)

Time = 22.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.90

$$\int \frac{\csc(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx = -\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)}$$

input
```
int(-1/(sin(c + d*x)*(sin(c + d*x) - 1/sin(c + d*x))),x)
```

output
```
-(2*tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)/2)^2 - 1))
```

3.229 $\int \cos^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx$

3.229.1 Optimal result

Integrand size = 26, antiderivative size = 33

$$\int \cos^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx=-\frac{b\cos^3(c+dx)}{3d}-\frac{a\cos^4(c+dx)}{4d}$$

output
```
-1/3*b*cos(d*x+c)^3/d-1/4*a*cos(d*x+c)^4/d
```

3.229.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \cos^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx=-\frac{b\cos^3(c+dx)}{3d}-\frac{a\cos^4(c+dx)}{4d}$$

input
```
Integrate[Cos[c + d*x]^3*(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

output
```
-1/3*(b*Cos[c + d*x]^3)/d - (a*Cos[c + d*x]^4)/(4*d)
```

3.229.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3042, 4877, 27, 3042, 3045, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(c+dx)^3(a\sin(c+dx)+b\tan(c+dx))dx$$

$$\Big\downarrow \text{4877}$$

$$a\int \cos^3(c+dx)\sin(c+dx)dx+\int b\cos^2(c+dx)\sin(c+dx)dx$$

$$\Big\downarrow \text{27}$$

$$a\int \cos^3(c+dx)\sin(c+dx)dx+b\int \cos^2(c+dx)\sin(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \cos(c+dx)^3\sin(c+dx)dx+b\int \cos(c+dx)^2\sin(c+dx)dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{a\int \cos^3(c+dx)d\cos(c+dx)}{d}-\frac{b\int \cos^2(c+dx)d\cos(c+dx)}{d}$$

$$\Big\downarrow \text{15}$$

$$-\frac{a\cos^4(c+dx)}{4d}-\frac{b\cos^3(c+dx)}{3d}$$

input
```
Int[Cos[c + d*x]^3*(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

output
```
-1/3*(b*Cos[c + d*x]^3)/d - (a*Cos[c + d*x]^4)/(4*d)
```

3.229.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 4877
```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Simp[d   Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[
c*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Intege
rQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

3.229.4 Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$-\frac{\frac{a\cos(dx+c)^4}{4}+\frac{b\cos(dx+c)^3}{3}}{d}$	29
default	$-\frac{\frac{a\cos(dx+c)^4}{4}+\frac{b\cos(dx+c)^3}{3}}{d}$	29
risch	$-\frac{b\cos(dx+c)}{4d}-\frac{a\cos(4dx+4c)}{32d}-\frac{b\cos(3dx+3c)}{12d}-\frac{a\cos(2dx+2c)}{8d}$	59

input
```
int(cos(d*x+c)^3*(sin(d*x+c)*a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/d*(1/4*a*cos(d*x+c)^4+1/3*b*cos(d*x+c)^3)
```

3.229.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int \cos^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx=-\frac{3\,a\cos\left(dx+c\right)^4+4\,b\cos\left(dx+c\right)^3}{12\,d}$$

input
```
integrate(cos(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-1/12*(3*a*cos(d*x + c)^4 + 4*b*cos(d*x + c)^3)/d
```

3.229.6 Sympy [F]

$$\int \cos^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx$$
$$=\int\left(a\sin\left(c+dx\right)+b\tan\left(c+dx\right)\right)\cos^3\left(c+dx\right)dx$$

input
```
integrate(cos(d*x+c)**3*(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

output
```
Integral((a*sin(c + d*x) + b*tan(c + d*x))*cos(c + d*x)**3, x)
```

3.229.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int \cos^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx=-\frac{3\,a\cos\left(dx+c\right)^4+4\,b\cos\left(dx+c\right)^3}{12\,d}$$

input
```
integrate(cos(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
-1/12*(3*a*cos(d*x + c)^4 + 4*b*cos(d*x + c)^3)/d
```

3.229.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11588 vs. $2(29) = 58$.

Time = 2.14 (sec) , antiderivative size = 11588, normalized size of antiderivative = 351.15

$$\int \cos^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
-1/32*a*cos(4*d*x + 4*c)/d - 1/8*a*cos(2*d*x + 2*c)/d + 1/96*(3*pi*b*sgn(t
an(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2
- tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan
(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) -
1)*tan(1/2*d*x)^6*tan(1/2*c)^6 + 3*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 -
2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c)
 - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(
1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^
6 - 3*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 -
 tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/
2*c)^6 + 3*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c
)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*t
an(1/2*c)^6 - 6*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*
tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 +
9*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan
(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*
c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan
(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^4 + 9*pi*b*sgn(tan(1/2*d*x)^2*tan
(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 -
2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(...
```

3.229.9 Mupad [B] (verification not implemented)

Time = 23.39 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \cos^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx = -\frac{a\cos\left(c+d\,x\right)^4}{4\,d}-\frac{b\cos\left(c+d\,x\right)^3}{3\,d}$$

input
```
int(cos(c + d*x)^3*(a*sin(c + d*x) + b*tan(c + d*x)),x)
```

output
```
- (a*cos(c + d*x)^4)/(4*d) - (b*cos(c + d*x)^3)/(3*d)
```

3.230 $\int \cos^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx$

3.230.1 Optimal result

Integrand size = 26, antiderivative size = 33

$$\int \cos^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx = -\frac{a\cos^3(c+dx)}{3d}+\frac{b\sin^2(c+dx)}{2d}$$

output
```
-1/3*a*cos(d*x+c)^3/d+1/2*b*sin(d*x+c)^2/d
```

3.230.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.15

$$\int \cos^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx$$
$$= -\frac{3a\cos(c+dx)+3b\cos(2(c+dx))+a\cos(3(c+dx))}{12d}$$

input
```
Integrate[Cos[c + d*x]^2*(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

output
```
-1/12*(3*a*Cos[c + d*x] + 3*b*Cos[2*(c + d*x)] + a*Cos[3*(c + d*x)])/d
```

3.230.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 4877, 27, 3042, 3044, 15, 3045, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \cos(c+dx)^2(a\sin(c+dx)+b\tan(c+dx))dx$$

$$\Big\downarrow \; 4877$$

$$a\int \cos^2(c+dx)\sin(c+dx)dx+\int b\cos(c+dx)\sin(c+dx)dx$$

$$\Big\downarrow \; 27$$

$$a\int \cos^2(c+dx)\sin(c+dx)dx+b\int \cos(c+dx)\sin(c+dx)dx$$

$$\Big\downarrow \; 3042$$

$$a\int \cos(c+dx)^2\sin(c+dx)dx+b\int \cos(c+dx)\sin(c+dx)dx$$

$$\Big\downarrow \; 3044$$

$$a\int \cos(c+dx)^2\sin(c+dx)dx+\frac{b\int \sin(c+dx)d\sin(c+dx)}{d}$$

$$\Big\downarrow \; 15$$

$$a\int \cos(c+dx)^2\sin(c+dx)dx+\frac{b\sin^2(c+dx)}{2d}$$

$$\Big\downarrow \; 3045$$

$$\frac{b\sin^2(c+dx)}{2d}-\frac{a\int \cos^2(c+dx)d\cos(c+dx)}{d}$$

$$\Big\downarrow \; 15$$

$$\frac{b\sin^2(c+dx)}{2d}-\frac{a\cos^3(c+dx)}{3d}$$

input
```
Int[Cos[c + d*x]^2*(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

output
```
-1/3*(a*Cos[c + d*x]^3)/d + (b*Sin[c + d*x]^2)/(2*d)
```

3.230.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 4877
```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Simp[d   Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[
c*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Intege
rQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

3.230.4 Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$-\frac{\frac{\cos(dx+c)^3 a}{3}+\frac{\cos(dx+c)^2 b}{2}}{d}$	29
default	$-\frac{\frac{\cos(dx+c)^3 a}{3}+\frac{\cos(dx+c)^2 b}{2}}{d}$	29
risch	$-\frac{a\cos(dx+c)}{4d}-\frac{a\cos(3dx+3c)}{12d}-\frac{b\cos(2dx+2c)}{4d}$	44

input
```
int(cos(d*x+c)^2*(sin(d*x+c)*a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/d*(1/3*cos(d*x+c)^3*a+1/2*cos(d*x+c)^2*b)
```

3.230.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int \cos^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx=-\frac{2\,a\cos\left(dx+c\right)^3+3\,b\cos\left(dx+c\right)^2}{6\,d}$$

input
```
integrate(cos(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-1/6*(2*a*cos(d*x + c)^3 + 3*b*cos(d*x + c)^2)/d
```

3.230.6 Sympy [F]

$$\int \cos^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx$$
$$=\int\left(a\sin\left(c+dx\right)+b\tan\left(c+dx\right)\right)\cos^2\left(c+dx\right)\,dx$$

input
```
integrate(cos(d*x+c)**2*(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

output
```
Integral((a*sin(c + d*x) + b*tan(c + d*x))*cos(c + d*x)**2, x)
```

3.230.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int \cos^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx = -\frac{2\,a\cos\left(dx+c\right)^3-3\,b\sin\left(dx+c\right)^2}{6\,d}$$

input
```
integrate(cos(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
-1/6*(2*a*cos(d*x + c)^3 - 3*b*sin(d*x + c)^2)/d
```

3.230.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(29) = 58.

Time = 0.36 (sec) , antiderivative size = 99, normalized size of antiderivative = 3.00

$$\int \cos^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx$$
$$= -\frac{a\cos\left(3\,dx+3\,c\right)}{12\,d}-\frac{a\cos\left(dx+c\right)}{4\,d} - \frac{b\tan\left(dx\right)^2\tan\left(c\right)^2-b\tan\left(dx\right)^2-4\,b\tan\left(dx\right)\tan\left(c\right)-b\tan\left(c\right)^2+b}{4\left(d\tan\left(dx\right)^2\tan\left(c\right)^2+d\tan\left(dx\right)^2+d\tan\left(c\right)^2+d\right)}$$

input
```
integrate(cos(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
-1/12*a*cos(3*d*x + 3*c)/d - 1/4*a*cos(d*x + c)/d - 1/4*(b*tan(d*x)^2*tan(
c)^2 - b*tan(d*x)^2 - 4*b*tan(d*x)*tan(c) - b*tan(c)^2 + b)/(d*tan(d*x)^2*
tan(c)^2 + d*tan(d*x)^2 + d*tan(c)^2 + d)
```

3.230.9 Mupad [B] (verification not implemented)

Time = 22.96 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.48

$$\int \cos^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx$$
$$= -\frac{\left(\cos\left(c+d\,x\right)+1\right)\,\left(2\,a-3\,b-2\,a\,\cos\left(c+d\,x\right)+3\,b\,\cos\left(c+d\,x\right)+2\,a\cos\left(c+d\,x\right)^2\right)}{6\,d}$$

input
```
int(cos(c + d*x)^2*(a*sin(c + d*x) + b*tan(c + d*x)),x)
```

output
```
-((cos(c + d*x) + 1)*(2*a - 3*b - 2*a*cos(c + d*x) + 3*b*cos(c + d*x) + 2*
a*cos(c + d*x)^2))/(6*d)
```

3.230. $\int \cos^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx$

3.231 $\int \cos(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx$

3.231.1 Optimal result

Integrand size = 24, antiderivative size = 22

$$\int \cos(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx=-\frac{(b+a\cos(c+dx))^2}{2ad}$$

output
```
-1/2*(b+a*cos(d*x+c))^2/a/d
```

3.231.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

$$\int \cos(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx =-\frac{b\cos(c)\cos(dx)}{d}-\frac{a\cos^2(c+dx)}{2d}+\frac{b\sin(c)\sin(dx)}{d}$$

input
```
Integrate[Cos[c + d*x]*(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

output
```
-((b*Cos[c]*Cos[d*x])/d) - (a*Cos[c + d*x]^2)/(2*d) + (b*Sin[c]*Sin[d*x])/
d
```

3.231.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.32, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {3042, 4877, 27, 3042, 3044, 15, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(c+dx)(a\sin(c+dx)+b\tan(c+dx))dx$$

$$\Big\downarrow \text{4877}$$

$$a\int \cos(c+dx)\sin(c+dx)dx+\int b\sin(c+dx)dx$$

$$\Big\downarrow \text{27}$$

$$a\int \cos(c+dx)\sin(c+dx)dx+b\int \sin(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \cos(c+dx)\sin(c+dx)dx+b\int \sin(c+dx)dx$$

$$\Big\downarrow \text{3044}$$

$$\frac{a\int \sin(c+dx)d\sin(c+dx)}{d}+b\int \sin(c+dx)dx$$

$$\Big\downarrow \text{15}$$

$$b\int \sin(c+dx)dx+\frac{a\sin^2(c+dx)}{2d}$$

$$\Big\downarrow \text{3118}$$

$$\frac{a\sin^2(c+dx)}{2d}-\frac{b\cos(c+dx)}{d}$$

input
```
Int[Cos[c + d*x]*(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

output
```
-((b*Cos[c + d*x])/d) + (a*Sin[c + d*x]^2)/(2*d)
```

3.231.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 4877
```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Simp[d   Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[
c*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Intege
rQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

3.231.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

method	result	size
derivativedivides	$\frac{-\frac{a\cos(dx+c)^2}{2}-\cos(dx+c)b}{d}$	26
default	$\frac{-\frac{a\cos(dx+c)^2}{2}-\cos(dx+c)b}{d}$	26
risch	$-\frac{b\cos(dx+c)}{d}-\frac{a\cos(2dx+2c)}{4d}$	29

input
```
int(cos(d*x+c)*(sin(d*x+c)*a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2*a*cos(d*x+c)^2-cos(d*x+c)*b)
```

3.231.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \cos(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx = -\frac{a\cos{(dx+c)}^2+2\,b\cos{(dx+c)}}{2\,d}$$

input
```
integrate(cos(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2*(a*cos(d*x + c)^2 + 2*b*cos(d*x + c))/d
```

3.231.6 Sympy [F]

$$\int \cos(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx$$
$$= \int (a\sin{(c+dx)}+b\tan{(c+dx)})\cos{(c+dx)}\,dx$$

input
```
integrate(cos(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

output
```
Integral((a*sin(c + d*x) + b*tan(c + d*x))*cos(c + d*x), x)
```

3.231.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \cos(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx = -\frac{a\cos{(dx+c)}^2+2\,b\cos{(dx+c)}}{2\,d}$$

input
```
integrate(cos(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
-1/2*(a*cos(d*x + c)^2 + 2*b*cos(d*x + c))/d
```

3.231.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(20) = 40$.

Time = 0.34 (sec) , antiderivative size = 102, normalized size of antiderivative = 4.64

$$\int \cos(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx$$
$$= -\frac{a\cos(2\,dx+2\,c)}{4\,d} - \frac{b\tan\left(\frac{1}{2}\,dx\right)^2\tan\left(\frac{1}{2}\,c\right)^2 - b\tan\left(\frac{1}{2}\,dx\right)^2 - 4\,b\tan\left(\frac{1}{2}\,dx\right)\tan\left(\frac{1}{2}\,c\right) - b\tan\left(\frac{1}{2}\,c\right)^2 + b}{d\tan\left(\frac{1}{2}\,dx\right)^2\tan\left(\frac{1}{2}\,c\right)^2 + d\tan\left(\frac{1}{2}\,dx\right)^2 + d\tan\left(\frac{1}{2}\,c\right)^2 + d}$$

input
```
integrate(cos(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
-1/4*a*cos(2*d*x + 2*c)/d - (b*tan(1/2*d*x)^2*tan(1/2*c)^2 - b*tan(1/2*d*x
)^2 - 4*b*tan(1/2*d*x)*tan(1/2*c) - b*tan(1/2*c)^2 + b)/(d*tan(1/2*d*x)^2*
tan(1/2*c)^2 + d*tan(1/2*d*x)^2 + d*tan(1/2*c)^2 + d)
```

3.231.9 Mupad [B] (verification not implemented)

Time = 22.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27

$$\int \cos(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx$$
$$= -\frac{(\cos(c+d\,x)+1)\,(2\,b-a+a\,\cos(c+d\,x))}{2\,d}$$

input
```
int(cos(c + d*x)*(a*sin(c + d*x) + b*tan(c + d*x)),x)
```

output
```
-((cos(c + d*x) + 1)*(2*b - a + a*cos(c + d*x)))/(2*d)
```

3.232 $\int (a\sin(c+dx)+b\tan(c+dx))\,dx$

3.232.1 Optimal result

Integrand size = 17, antiderivative size = 26

$$\int (a\sin(c+dx)+b\tan(c+dx))\,dx=-\frac{a\cos(c+dx)}{d}-\frac{b\log(\cos(c+dx))}{d}$$

output
```
-a*cos(d*x+c)/d-b*ln(cos(d*x+c))/d
```

3.232.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.42

$$\int (a\sin(c+dx)+b\tan(c+dx))\,dx=-\frac{a\cos(c)\cos(dx)}{d}-\frac{b\log(\cos(c+dx))}{d}+\frac{a\sin(c)\sin(dx)}{d}$$

input
```
Integrate[a*Sin[c + d*x] + b*Tan[c + d*x],x]
```

output
```
-((a*Cos[c]*Cos[d*x])/d) - (b*Log[Cos[c + d*x]])/d + (a*Sin[c]*Sin[d*x])/d
```

3.232.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(c+dx)+b\tan(c+dx))\,dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a\cos(c+dx)}{d}-\frac{b\log(\cos(c+dx))}{d}$$

input
```
Int[a*Sin[c + d*x] + b*Tan[c + d*x],x]
```

output
```
-((a*Cos[c + d*x])/d) - (b*Log[Cos[c + d*x]])/d
```

3.232.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.232.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{-\cos(dx+c)a-b\ln(\cos(dx+c))}{d}$	25
parallelrisch	$\frac{-\cos(dx+c)a+b\ln\left(\sqrt{\sec(dx+c)^2}\right)+a}{d}$	29
default	$\frac{b\ln\left(1+\tan(dx+c)^2\right)}{2d}-\frac{a\cos(dx+c)}{d}$	31
parts	$\frac{b\ln\left(1+\tan(dx+c)^2\right)}{2d}-\frac{a\cos(dx+c)}{d}$	31
risch	$ibx+\frac{2ibc}{d}-\frac{b\ln(e^{2i(dx+c)}+1)}{d}-\frac{a\cos(dx+c)}{d}$	45
norman	$\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)}+\frac{b\ln\left(1+\tan(dx+c)^2\right)}{2d}$	51

input
```
int(sin(d*x+c)*a+b*tan(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-cos(d*x+c)*a-b*ln(cos(d*x+c)))
```

3.232.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int (a\sin(c+dx)+b\tan(c+dx))\,dx=-\frac{a\cos(dx+c)+b\log(-\cos(dx+c))}{d}$$

input
```
integrate(a*sin(d*x+c)+b*tan(d*x+c),x, algorithm="fricas")
```

output
```
-(a*cos(d*x + c) + b*log(-cos(d*x + c)))/d
```

3.232.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.42

$$\int (a\sin(c+dx)+b\tan(c+dx))\,dx=a\left(\begin{cases}-\frac{\cos(c+dx)}{d} & \text{for } d\neq 0\\ x\sin(c) & \text{otherwise}\end{cases}\right)+b\left(\begin{cases}\frac{\log(\tan^2(c+dx)+1)}{2d} & \text{for } d\neq 0\\ x\tan(c) & \text{otherwise}\end{cases}\right)$$

input
```
integrate(a*sin(d*x+c)+b*tan(d*x+c),x)
```

output
```
a*Piecewise((-cos(c + d*x)/d, Ne(d, 0)), (x*sin(c), True)) + b*Piecewise((
log(tan(c + d*x)**2 + 1)/(2*d), Ne(d, 0)), (x*tan(c), True))
```

3.232.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int (a\sin(c+dx)+b\tan(c+dx))\,dx = -\frac{a\cos(dx+c)}{d}+\frac{b\log(\sec(dx+c))}{d}$$

input
```
integrate(a*sin(d*x+c)+b*tan(d*x+c),x, algorithm="maxima")
```

output
```
-a*cos(d*x + c)/d + b*log(sec(d*x + c))/d
```

3.232.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

$$\int (a\sin(c+dx)+b\tan(c+dx))\,dx = -\frac{a\cos(dx+c)}{d}-\frac{b\log(|\cos(dx+c)|)}{d}$$

input
```
integrate(a*sin(d*x+c)+b*tan(d*x+c),x, algorithm="giac")
```

output
```
-a*cos(d*x + c)/d - b*log(abs(cos(d*x + c)))/d
```

3.232.9 Mupad [B] (verification not implemented)

Time = 22.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.54

$$\int (a\sin(c+dx)+b\tan(c+dx))\,dx = \frac{2\,b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d}-\frac{2\,a}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

input
```
int(a*sin(c + d*x) + b*tan(c + d*x),x)
```

output
```
(2*b*atanh(tan(c/2 + (d*x)/2)^2))/d - (2*a)/(d*(tan(c/2 + (d*x)/2)^2 + 1))
```

3.233 $\int \sec(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx$

3.233.1 Optimal result

Integrand size = 24, antiderivative size = 25

$$\int \sec(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx=-\frac{a\log(\cos(c+dx))}{d}+\frac{b\sec(c+dx)}{d}$$

output
```
-a*ln(cos(d*x+c))/d+b*sec(d*x+c)/d
```

3.233.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \sec(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx=-\frac{a\log(\cos(c+dx))}{d}+\frac{b\sec(c+dx)}{d}$$

input
```
Integrate[Sec[c + d*x]*(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

output
```
-((a*Log[Cos[c + d*x]])/d) + (b*Sec[c + d*x])/d
```

3.233.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {3042, 4877, 27, 3042, 3086, 24, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)(a\sin(c+dx)+b\tan(c+dx))dx$$

$$\Big\downarrow \text{4877}$$

$$a\int \tan(c+dx)dx+\int b\sec(c+dx)\tan(c+dx)dx$$

$$\Big\downarrow \text{27}$$

$$a\int \tan(c+dx)dx+b\int \sec(c+dx)\tan(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \tan(c+dx)dx+b\int \sec(c+dx)\tan(c+dx)dx$$

$$\Big\downarrow \text{3086}$$

$$a\int \tan(c+dx)dx+\frac{b\int 1d\sec(c+dx)}{d}$$

$$\Big\downarrow \text{24}$$

$$a\int \tan(c+dx)dx+\frac{b\sec(c+dx)}{d}$$

$$\Big\downarrow \text{3956}$$

$$\frac{b\sec(c+dx)}{d}-\frac{a\log(\cos(c+dx))}{d}$$

input
```
Int[Sec[c + d*x]*(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

output
```
-((a*Log[Cos[c + d*x]])/d) + (b*Sec[c + d*x])/d
```

3.233.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4877
```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Simp[d   Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[
c*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Intege
rQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

3.233.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{\sec(dx+c)b+a\ln(\sec(dx+c))}{d}$	23
default	$\frac{\sec(dx+c)b+a\ln(\sec(dx+c))}{d}$	23
risch	$iax+\frac{2iac}{d}+\frac{2b\,e^{i(dx+c)}}{d\left(e^{2i(dx+c)}+1\right)}-\frac{a\ln\left(e^{2i(dx+c)}+1\right)}{d}$	61

input
```
int(sec(d*x+c)*(sin(d*x+c)*a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(sec(d*x+c)*b+a*ln(sec(d*x+c)))
```

3.233.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.36

$$\int \sec(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx = -\frac{a\cos(dx+c)\log(-\cos(dx+c))-b}{d\cos(dx+c)}$$

input
```
integrate(sec(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-(a*cos(d*x + c)*log(-cos(d*x + c)) - b)/(d*cos(d*x + c))
```

3.233.6 Sympy [F]

$$\int \sec(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx$$
$$= \int (a\sin(c+dx)+b\tan(c+dx))\sec(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

output
```
Integral((a*sin(c + d*x) + b*tan(c + d*x))*sec(c + d*x), x)
```

3.233.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

$$\int \sec(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx = -\frac{a\log\left(-\sin(dx+c)^2+1\right)-\frac{2b}{\cos(dx+c)}}{2d}$$

input
```
integrate(sec(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
-1/2*(a*log(-sin(d*x + c)^2 + 1) - 2*b/cos(d*x + c))/d
```

3.233.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(25) = 50$.

Time = 0.36 (sec) , antiderivative size = 107, normalized size of antiderivative = 4.28

$$\int \sec(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx = \frac{a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{a+2\,b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1}}{d}$$

input
```
integrate(sec(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
(a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - a*log(abs(-(cos(
d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (a + 2*b + a*(cos(d*x + c) - 1)/(
cos(d*x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/d
```

3.233.9 Mupad [B] (verification not implemented)

Time = 22.64 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.60

$$\int \sec(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx = \frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d}-\frac{2\,b}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

input
```
int((a*sin(c + d*x) + b*tan(c + d*x))/cos(c + d*x),x)
```

output
```
(2*a*atanh(tan(c/2 + (d*x)/2)^2))/d - (2*b)/(d*(tan(c/2 + (d*x)/2)^2 - 1))
```

3.234 $\int \sec^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx$

3.234.1 Optimal result

Integrand size = 26, antiderivative size = 28

$$\int \sec^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx=\frac{a\sec(c+dx)}{d}+\frac{b\sec^2(c+dx)}{2d}$$

output
```
a*sec(d*x+c)/d+1/2*b*sec(d*x+c)^2/d
```

3.234.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \sec^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx=\frac{a\sec(c+dx)}{d}+\frac{b\sec^2(c+dx)}{2d}$$

input
```
Integrate[Sec[c + d*x]^2*(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

output
```
(a*Sec[c + d*x])/d + (b*Sec[c + d*x]^2)/(2*d)
```

3.234.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {3042, 4877, 27, 3042, 3086, 15, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^2(a\sin(c+dx)+b\tan(c+dx))dx$$

$$\Big\downarrow \text{4877}$$

$$a\int \sec(c+dx)\tan(c+dx)dx+\int b\sec^2(c+dx)\tan(c+dx)dx$$

$$\Big\downarrow \text{27}$$

$$a\int \sec(c+dx)\tan(c+dx)dx+b\int \sec^2(c+dx)\tan(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \sec(c+dx)\tan(c+dx)dx+b\int \sec(c+dx)^2\tan(c+dx)dx$$

$$\Big\downarrow \text{3086}$$

$$\frac{a\int 1d\sec(c+dx)}{d}+\frac{b\int \sec(c+dx)d\sec(c+dx)}{d}$$

$$\Big\downarrow \text{15}$$

$$\frac{a\int 1d\sec(c+dx)}{d}+\frac{b\sec^2(c+dx)}{2d}$$

$$\Big\downarrow \text{24}$$

$$\frac{a\sec(c+dx)}{d}+\frac{b\sec^2(c+dx)}{2d}$$

input
```
Int[Sec[c + d*x]^2*(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

output
```
(a*Sec[c + d*x])/d + (b*Sec[c + d*x]^2)/(2*d)
```

3.234.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4877
```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Simp[d   Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[
c*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Intege
rQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

3.234.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{\frac{b\sec(dx+c)^2}{2}+\sec(dx+c)a}{d}$	25
default	$\frac{\frac{b\sec(dx+c)^2}{2}+\sec(dx+c)a}{d}$	25
risch	$\frac{2a\,e^{3i(dx+c)}+2b\,e^{2i(dx+c)}+2a\,e^{i(dx+c)}}{d\left(e^{2i(dx+c)}+1\right)^2}$	53

input
```
int(sec(d*x+c)^2*(sin(d*x+c)*a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2*b*sec(d*x+c)^2+sec(d*x+c)*a)
```

3.234.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \sec^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx = \frac{2\,a\cos\left(dx+c\right)+b}{2\,d\cos\left(dx+c\right)^2}$$

input
```
integrate(sec(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(2*a*cos(d*x + c) + b)/(d*cos(d*x + c)^2)
```

3.234.6 Sympy [F]

$$\int \sec^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx$$
$$= \int \left(a\sin\left(c+dx\right)+b\tan\left(c+dx\right)\right)\sec^2\left(c+dx\right)dx$$

input
```
integrate(sec(d*x+c)**2*(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

output
```
Integral((a*sin(c + d*x) + b*tan(c + d*x))*sec(c + d*x)**2, x)
```

3.234.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \sec^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx = \frac{b\tan\left(dx+c\right)^2+\frac{2\,a}{\cos\left(dx+c\right)}}{2\,d}$$

input
```
integrate(sec(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*(b*tan(d*x + c)^2 + 2*a/cos(d*x + c))/d
```

3.234.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(26) = 52$.

Time = 0.35 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.54

$$\int \sec^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx = \frac{2\left(a+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{d\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^2}$$

input
```
integrate(sec(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
2*(a + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos
(d*x + c) + 1))/(d*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^2)
```

3.234.9 Mupad [B] (verification not implemented)

Time = 22.61 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \sec^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx = \frac{\frac{b}{2}+a\,\cos\left(c+d\,x\right)}{d\cos\left(c+d\,x\right)^2}$$

input
```
int((a*sin(c + d*x) + b*tan(c + d*x))/cos(c + d*x)^2,x)
```

output
```
(b/2 + a*cos(c + d*x))/(d*cos(c + d*x)^2)
```

3.235 $\int \sec^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx$

3.235.1 Optimal result

Integrand size = 26, antiderivative size = 33

$$\int \sec^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx=\frac{a\sec^2(c+dx)}{2d}+\frac{b\sec^3(c+dx)}{3d}$$

output
```
1/2*a*sec(d*x+c)^2/d+1/3*b*sec(d*x+c)^3/d
```

3.235.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \sec^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx=\frac{a\sec^2(c+dx)}{2d}+\frac{b\sec^3(c+dx)}{3d}$$

input
```
Integrate[Sec[c + d*x]^3*(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

output
```
(a*Sec[c + d*x]^2)/(2*d) + (b*Sec[c + d*x]^3)/(3*d)
```

3.235.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 4877, 27, 3042, 3086, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^3(a\sin(c+dx)+b\tan(c+dx))dx$$

$$\Big\downarrow \text{4877}$$

$$a\int \sec^2(c+dx)\tan(c+dx)dx+\int b\sec^3(c+dx)\tan(c+dx)dx$$

$$\Big\downarrow \text{27}$$

$$a\int \sec^2(c+dx)\tan(c+dx)dx+b\int \sec^3(c+dx)\tan(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \sec(c+dx)^2\tan(c+dx)dx+b\int \sec(c+dx)^3\tan(c+dx)dx$$

$$\Big\downarrow \text{3086}$$

$$\frac{a\int \sec(c+dx)d\sec(c+dx)}{d}+\frac{b\int \sec^2(c+dx)d\sec(c+dx)}{d}$$

$$\Big\downarrow \text{15}$$

$$\frac{a\sec^2(c+dx)}{2d}+\frac{b\sec^3(c+dx)}{3d}$$

input
```
Int[Sec[c + d*x]^3*(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

output
```
(a*Sec[c + d*x]^2)/(2*d) + (b*Sec[c + d*x]^3)/(3*d)
```

3.235.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4877
```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Simp[d   Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[
c*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Intege
rQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

3.235.4 Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{\frac{b\sec(dx+c)^3}{3}+\frac{a\sec(dx+c)^2}{2}}{d}$	28
default	$\frac{\frac{b\sec(dx+c)^3}{3}+\frac{a\sec(dx+c)^2}{2}}{d}$	28
risch	$\frac{2a\,e^{4i(dx+c)}+\frac{8b\,e^{3i(dx+c)}}{3}+2\,e^{2i(dx+c)}a}{d\left(e^{2i(dx+c)}+1\right)^3}$	56

input
```
int(sec(d*x+c)^3*(sin(d*x+c)*a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/3*b*sec(d*x+c)^3+1/2*a*sec(d*x+c)^2)
```

3.235.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int \sec^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx=\frac{3\,a\cos\left(dx+c\right)+2\,b}{6\,d\cos\left(dx+c\right)^3}$$

input
```
integrate(sec(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/6*(3*a*cos(d*x + c) + 2*b)/(d*cos(d*x + c)^3)
```

3.235.6 Sympy [F]

$$\int \sec^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx$$
$$=\int\left(a\sin\left(c+dx\right)+b\tan\left(c+dx\right)\right)\sec^3\left(c+dx\right)\,dx$$

input
```
integrate(sec(d*x+c)**3*(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

output
```
Integral((a*sin(c + d*x) + b*tan(c + d*x))*sec(c + d*x)**3, x)
```

3.235.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \sec^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx=-\frac{\frac{3\,a}{\sin(dx+c)^2-1}-\frac{2\,b}{\cos(dx+c)^3}}{6\,d}$$

input
```
integrate(sec(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
-1/6*(3*a/(sin(d*x + c)^2 - 1) - 2*b/cos(d*x + c)^3)/d
```

3.235.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(29) = 58$.

Time = 0.37 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.94

$$\int \sec^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx = \frac{2\left(b-\frac{3\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{3\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{3\,b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)}{3\,d\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^3}$$

input
```
integrate(sec(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
2/3*(b - 3*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 3*a*(cos(d*x + c) - 1
)^2/(cos(d*x + c) + 1)^2 + 3*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/
(d*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^3)
```

3.235.9 Mupad [B] (verification not implemented)

Time = 22.44 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \sec^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx = \frac{a}{2\,d\cos{(c+d\,x)}^2}+\frac{b}{3\,d\cos{(c+d\,x)}^3}$$

input
```
int((a*sin(c + d*x) + b*tan(c + d*x))/cos(c + d*x)^3,x)
```

output
```
a/(2*d*cos(c + d*x)^2) + b/(3*d*cos(c + d*x)^3)
```

3.236 $\int \cos^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$

3.236.1 Optimal result

Integrand size = 28, antiderivative size = 106

$$\int \cos^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx = \frac{abx}{4}-\frac{ab\cos(c+dx)\sin(c+dx)}{4d}+\frac{(4a^2+b^2)\sin^3(c+dx)}{30d}+\frac{b(b+a\cos(c+dx))\sin^3(c+dx)}{10d}+\frac{(b+a\cos(c+dx))^2\sin^3(c+dx)}{5d}$$

output
```
1/4*a*b*x-1/4*a*b*cos(d*x+c)*sin(d*x+c)/d+1/30*(4*a^2+b^2)*sin(d*x+c)^3/d+
1/10*b*(b+a*cos(d*x+c))*sin(d*x+c)^3/d+1/5*(b+a*cos(d*x+c))^2*sin(d*x+c)^3
/d
```

3.236.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.73

$$\int \cos^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx = \frac{30(a^2+2b^2)\sin(c+dx)-5(a^2+4b^2)\sin(3(c+dx))-3a(-20b(c+dx)+5b\sin(4(c+dx))+a\sin(5(c}{240d}$$

input
```
Integrate[Cos[c + d*x]^3*(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

output
```
(30*(a^2 + 2*b^2)*Sin[c + d*x] - 5*(a^2 + 4*b^2)*Sin[3*(c + d*x)] - 3*a*(-
20*b*(c + d*x) + 5*b*Sin[4*(c + d*x)] + a*Sin[5*(c + d*x)]))/(240*d)
```

3.236.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.10, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4897, 3042, 3341, 27, 3042, 3341, 3042, 3148, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(c+dx)^3(a\sin(c+dx)+b\tan(c+dx))^2dx$$

$$\Big\downarrow \text{4897}$$

$$\int \sin^2(c+dx)\cos(c+dx)(a\cos(c+dx)+b)^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(c+dx+\frac{\pi}{2}\right)\cos\left(c+dx+\frac{\pi}{2}\right)^2\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+b\right)^2dx$$

$$\Big\downarrow \text{3341}$$

$$\frac{1}{5}\int 2(b+a\cos(c+dx))(a+b\cos(c+dx))\sin^2(c+dx)dx+\frac{\sin^3(c+dx)(a\cos(c+dx)+b)^2}{5d}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{5}\int (b+a\cos(c+dx))(a+b\cos(c+dx))\sin^2(c+dx)dx+\frac{\sin^3(c+dx)(a\cos(c+dx)+b)^2}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}\int \cos\left(c+dx+\frac{\pi}{2}\right)^2\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(a+b\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx+\frac{\sin^3(c+dx)(a\cos(c+dx)+b)^2}{5d}$$

$$\Big\downarrow \text{3341}$$

$$\frac{2}{5}\left(\frac{1}{4}\int \left(5ab+\left(a^2+4b^2\right)\cos(c+dx)\right)\sin^2(c+dx)dx+\frac{a\sin^3(c+dx)(a+b\cos(c+dx))}{4d}\right)+\frac{\sin^3(c+dx)(a\cos(c+dx)+b)^2}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}\left(\frac{1}{4}\int \cos\left(c+dx-\frac{\pi}{2}\right)^2\left(5ab-\left(a^2+4b^2\right)\sin\left(c+dx-\frac{\pi}{2}\right)\right)dx+\frac{a\sin^3(c+dx)(a+b\cos(c+dx))}{4d}\right)+\frac{\sin^3(c+dx)(a\cos(c+dx)+b)^2}{5d}$$

$\Big\downarrow$ 3148

$$\frac{2}{5}\left(\frac{1}{4}\left(5ab\int \sin^2(c+dx)dx+\frac{\left(a^2+4b^2\right)\sin^3(c+dx)}{3d}\right)+\frac{a\sin^3(c+dx)(a+b\cos(c+dx))}{4d}\right)+\frac{\sin^3(c+dx)(a\cos(c+dx)+b)^2}{5d}$$

$\Big\downarrow$ 3042

$$\frac{2}{5}\left(\frac{1}{4}\left(5ab\int \sin(c+dx)^2dx+\frac{\left(a^2+4b^2\right)\sin^3(c+dx)}{3d}\right)+\frac{a\sin^3(c+dx)(a+b\cos(c+dx))}{4d}\right)+\frac{\sin^3(c+dx)(a\cos(c+dx)+b)^2}{5d}$$

$\Big\downarrow$ 3115

$$\frac{2}{5}\left(\frac{1}{4}\left(5ab\left(\frac{\int 1dx}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{\left(a^2+4b^2\right)\sin^3(c+dx)}{3d}\right)+\frac{a\sin^3(c+dx)(a+b\cos(c+dx))}{4d}\right)+\frac{\sin^3(c+dx)(a\cos(c+dx)+b)^2}{5d}$$

$\Big\downarrow$ 24

$$\frac{2}{5}\left(\frac{1}{4}\left(\frac{\left(a^2+4b^2\right)\sin^3(c+dx)}{3d}+5ab\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)\right)+\frac{a\sin^3(c+dx)(a+b\cos(c+dx))}{4d}\right)+\frac{\sin^3(c+dx)(a\cos(c+dx)+b)^2}{5d}$$

input
```
Int[Cos[c + d*x]^3*(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

output
```
((b + a*Cos[c + d*x])^2*Sin[c + d*x]^3)/(5*d) + (2*((a*(a + b*Cos[c + d*x]
)*Sin[c + d*x]^3)/(4*d) + (((a^2 + 4*b^2)*Sin[c + d*x]^3)/(3*d) + 5*a*b*(x
/2 - (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/4))/5
```

3.236. $\int \cos^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$

3.236.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3341
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[1/(m + p + 1)   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Sim
p[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
 &&  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && S
implerQ[c + d*x, a + b*x])
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.236. $\int \cos^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$

3.236.4 Maple [A] (verified)

Time = 9.11 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{a^2\left(-\frac{\cos(dx+c)^4\sin(dx+c)}{5}+\frac{\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{15}\right)+2ab\left(-\frac{\sin(dx+c)\cos(dx+c)^3}{4}+\frac{\sin(dx+c)\cos(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)+b^2}{d}$
default	$\frac{a^2\left(-\frac{\cos(dx+c)^4\sin(dx+c)}{5}+\frac{\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{15}\right)+2ab\left(-\frac{\sin(dx+c)\cos(dx+c)^3}{4}+\frac{\sin(dx+c)\cos(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)+b^2}{d}$
risch	$\frac{xab}{4}+\frac{a^2\sin(dx+c)}{8d}+\frac{b^2\sin(dx+c)}{4d}-\frac{a^2\sin(5dx+5c)}{80d}-\frac{ab\sin(4dx+4c)}{16d}-\frac{\sin(3dx+3c)a^2}{48d}-\frac{\sin(3dx+3c)b^2}{12d}$

input
```
int(cos(d*x+c)^3*(sin(d*x+c)*a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(-1/5*cos(d*x+c)^4*sin(d*x+c)+1/15*(2+cos(d*x+c)^2)*sin(d*x+c))+2
*a*b*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*sin(d*x+c)*cos(d*x+c)+1/8*d*x+1/8*c
)+1/3*b^2*sin(d*x+c)^3)
```

3.236.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.80

$$\int \cos^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$$
$$=\frac{15\,abdx-\left(12\,a^2\cos\left(dx+c\right)^4+30\,ab\cos\left(dx+c\right)^3-15\,ab\cos\left(dx+c\right)-4\left(a^2-5\,b^2\right)\cos\left(dx+c\right)^2-8\,a}{60\,d}$$

input
```
integrate(cos(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas"
)
```

output
```
1/60*(15*a*b*d*x - (12*a^2*cos(d*x + c)^4 + 30*a*b*cos(d*x + c)^3 - 15*a*b
*cos(d*x + c) - 4*(a^2 - 5*b^2)*cos(d*x + c)^2 - 8*a^2 - 20*b^2)*sin(d*x +
 c))/d
```

3.236.6 Sympy [F]

$$\int \cos^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$$
$$=\int (a\sin(c+dx)+b\tan(c+dx))^2\cos^3(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)**3*(a*sin(d*x+c)+b*tan(d*x+c))**2,x)
```

output
```
Integral((a*sin(c + d*x) + b*tan(c + d*x))**2*cos(c + d*x)**3, x)
```

3.236.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.64

$$\int \cos^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$$
$$=\frac{80\,b^2\sin(dx+c)^3-16\left(3\sin(dx+c)^5-5\sin(dx+c)^3\right)a^2+15\,(4\,dx+4\,c-\sin(4\,dx+4\,c))ab}{240\,d}$$

input
```
integrate(cos(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima"
)
```

output
```
1/240*(80*b^2*sin(d*x + c)^3 - 16*(3*sin(d*x + c)^5 - 5*sin(d*x + c)^3)*a^
2 + 15*(4*d*x + 4*c - sin(4*d*x + 4*c))*a*b)/d
```

3.236.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43089 vs. 2(96) = 192.

Time = 154.27 (sec) , antiderivative size = 43089, normalized size of antiderivative = 406.50

$$\int \cos^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx=\text{Too large to display}$$

input
```
integrate(cos(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/80*a^2*sin(5*d*x + 5*c)/d - 1/48*a^2*sin(3*d*x + 3*c)/d + 1/8*a^2*sin(d
*x + c)/d + 1/96*(3*pi*a*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^
2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(1/2
*d*x)^6*tan(1/2*c)^6*tan(c)^4 + 24*a*b*d*x*tan(d*x)^4*tan(1/2*d*x)^6*tan(1
/2*c)^6*tan(c)^4 + 3*pi*a*b*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2
 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^4
+ 6*pi*a*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan
(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(1/2*d*x)^6*tan(1/2*
c)^6*tan(c)^2 + 9*pi*a*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*
tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(1/2*d
*x)^6*tan(1/2*c)^4*tan(c)^4 + 9*pi*a*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(
-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*
x)^4*tan(1/2*d*x)^4*tan(1/2*c)^6*tan(c)^4 + 6*pi*a*b*sgn(2*tan(d*x)^2*tan(
c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*
tan(c))*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^4 + 6*a*b*arctan((ta
n(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*tan(d*x)^4*tan(1/2*d*x)^6*tan(1/2*
c)^6*tan(c)^4 - 6*a*b*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*t
an(d*x)^4*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^4 + 48*a*b*d*x*tan(d*x)^4*tan
(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^2 + 6*pi*a*b*sgn(-2*tan(d*x)^2*tan(c) + 2*
tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(1/2*d*x)^6*ta...
```

3.236.9 Mupad [B] (verification not implemented)

Time = 24.06 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.95

$$\int \cos^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx = \frac{a^2\,\sin\left(c+d\,x\right)}{8\,d}+\frac{b^2\,\sin\left(c+d\,x\right)}{4\,d}+\frac{a\,b\,x}{4}-\frac{a^2\,\sin\left(3\,c+3\,d\,x\right)}{48\,d}-\frac{a^2\,\sin\left(5\,c+5\,d\,x\right)}{80\,d}-\frac{b^2\,\sin\left(3\,c+3\,d\,x\right)}{12\,d}-\frac{a\,b\,\sin\left(4\,c+4\,d\,x\right)}{16\,d}$$

input
```
int(cos(c + d*x)^3*(a*sin(c + d*x) + b*tan(c + d*x))^2,x)
```

output
```
(a^2*sin(c + d*x))/(8*d) + (b^2*sin(c + d*x))/(4*d) + (a*b*x)/4 - (a^2*sin
(3*c + 3*d*x))/(48*d) - (a^2*sin(5*c + 5*d*x))/(80*d) - (b^2*sin(3*c + 3*d
*x))/(12*d) - (a*b*sin(4*c + 4*d*x))/(16*d)
```

3.237 $\int \cos^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$

3.237.1 Optimal result

Integrand size = 28, antiderivative size = 86

$$\int \cos^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx = \frac{1}{8}\left(a^2+4b^2\right)x-\frac{(a^2+4b^2)\cos(c+dx)\sin(c+dx)}{8d} + \frac{5ab\sin^3(c+dx)}{12d}+\frac{a(b+a\cos(c+dx))\sin^3(c+dx)}{4d}$$

output
```
1/8*(a^2+4*b^2)*x-1/8*(a^2+4*b^2)*cos(d*x+c)*sin(d*x+c)/d+5/12*a*b*sin(d*x
+c)^3/d+1/4*a*(b+a*cos(d*x+c))*sin(d*x+c)^3/d
```

3.237.2 Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.95

$$\int \cos^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx = \frac{12a^2c+48b^2c+12a^2dx+48b^2dx+48ab\sin(c+dx)-24b^2\sin(2(c+dx))-16ab\sin(3(c+dx))-3a^2\text{si}}{96d}$$

input
```
Integrate[Cos[c + d*x]^2*(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

output
```
(12*a^2*c + 48*b^2*c + 12*a^2*d*x + 48*b^2*d*x + 48*a*b*Sin[c + d*x] - 24*
b^2*Sin[2*(c + d*x)] - 16*a*b*Sin[3*(c + d*x)] - 3*a^2*Sin[4*(c + d*x)])/(
96*d)
```

3.237.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {3042, 4897, 3042, 3171, 3042, 3148, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(c+dx)^2(a\sin(c+dx)+b\tan(c+dx))^2dx$$

$$\Big\downarrow \text{4897}$$

$$\int \sin^2(c+dx)(a\cos(c+dx)+b)^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos\left(c+dx-\frac{\pi}{2}\right)^2\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{3171}$$

$$\frac{1}{4}\int \left(a^2+5b\cos(c+dx)a+4b^2\right)\sin^2(c+dx)dx+\frac{a\sin^3(c+dx)(a\cos(c+dx)+b)}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \cos\left(c+dx-\frac{\pi}{2}\right)^2\left(a^2-5b\sin\left(c+dx-\frac{\pi}{2}\right)a+4b^2\right)dx+\frac{a\sin^3(c+dx)(a\cos(c+dx)+b)}{4d}$$

$$\Big\downarrow \text{3148}$$

$$\frac{1}{4}\left(\left(a^2+4b^2\right)\int \sin^2(c+dx)dx+\frac{5ab\sin^3(c+dx)}{3d}\right)+\frac{a\sin^3(c+dx)(a\cos(c+dx)+b)}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\left(a^2+4b^2\right)\int \sin(c+dx)^2dx+\frac{5ab\sin^3(c+dx)}{3d}\right)+\frac{a\sin^3(c+dx)(a\cos(c+dx)+b)}{4d}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{4}\left(\left(a^2+4b^2\right)\left(\frac{\int 1dx}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{5ab\sin^3(c+dx)}{3d}\right)+\frac{a\sin^3(c+dx)(a\cos(c+dx)+b)}{4d}$$

$$\Big\downarrow 24$$

$$\frac{1}{4}\left((a^2+4b^2)\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)+\frac{5ab\sin^3(c+dx)}{3d}\right)+\frac{a\sin^3(c+dx)(a\cos(c+dx)+b)}{4d}$$

input
```
Int[Cos[c + d*x]^2*(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

output
```
(a*(b + a*Cos[c + d*x])*Sin[c + d*x]^3)/(4*d) + ((5*a*b*Sin[c + d*x]^3)/(3
*d) + (a^2 + 4*b^2)*(x/2 - (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/4
```

3.237.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 3171
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Simp[1/(m + p)   Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
 && GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.237.4 Maple [A] (verified)

Time = 4.48 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.90

method	result	size
risch	$\frac{a^2 x}{8}+\frac{x\,b^2}{2}+\frac{ab\sin(dx+c)}{2d}-\frac{a^2\sin(4dx+4c)}{32d}-\frac{ab\sin(3dx+3c)}{6d}-\frac{b^2\sin(2dx+2c)}{4d}$	77
derivativedivides	$\frac{a^2\left(-\frac{\sin(dx+c)\cos(dx+c)^3}{4}+\frac{\sin(dx+c)\cos(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)+\frac{2ab\sin(dx+c)^3}{3}+b^2\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	86
default	$\frac{a^2\left(-\frac{\sin(dx+c)\cos(dx+c)^3}{4}+\frac{\sin(dx+c)\cos(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)+\frac{2ab\sin(dx+c)^3}{3}+b^2\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	86

input
```
int(cos(d*x+c)^2*(sin(d*x+c)*a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*a^2*x+1/2*x*b^2+1/2*a*b*sin(d*x+c)/d-1/32*a^2/d*sin(4*d*x+4*c)-1/6*a*b
/d*sin(3*d*x+3*c)-1/4*b^2/d*sin(2*d*x+2*c)
```

3.237.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.86

$$\int \cos^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$$
$$=\frac{3\,(a^2+4\,b^2)dx-\left(6\,a^2\cos{(dx+c)}^3+16\,ab\cos{(dx+c)}^2-16\,ab-3\,(a^2-4\,b^2)\cos{(dx+c)}\right)\sin{(dx+c)}}{24\,d}$$

input
```
integrate(cos(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas"
)
```

output
```
1/24*(3*(a^2 + 4*b^2)*d*x - (6*a^2*cos(d*x + c)^3 + 16*a*b*cos(d*x + c)^2
- 16*a*b - 3*(a^2 - 4*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

3.237.6 Sympy [F]

$$\int \cos^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$$
$$=\int (a\sin(c+dx)+b\tan(c+dx))^2\cos^2(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)**2*(a*sin(d*x+c)+b*tan(d*x+c))**2,x)
```

output
```
Integral((a*sin(c + d*x) + b*tan(c + d*x))**2*cos(c + d*x)**2, x)
```

3.237.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.77

$$\int \cos^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$$
$$=\frac{64\,ab\sin(dx+c)^3+3\,(4\,dx+4\,c-\sin(4\,dx+4\,c))a^2+24\,(2\,dx+2\,c-\sin(2\,dx+2\,c))b^2}{96\,d}$$

input
```
integrate(cos(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima"
)
```

output
```
1/96*(64*a*b*sin(d*x + c)^3 + 3*(4*d*x + 4*c - sin(4*d*x + 4*c))*a^2 + 24*
(2*d*x + 2*c - sin(2*d*x + 2*c))*b^2)/d
```

3.237.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5161 vs. 2(78) = 156.

Time = 2.94 (sec) , antiderivative size = 5161, normalized size of antiderivative = 60.01

$$\int \cos^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx=\text{Too large to display}$$

input
```
integrate(cos(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/8*a^2*x - 1/32*a^2*sin(4*d*x + 4*c)/d + 1/6*(3*b^2*d*x*tan(d*x)^2*tan(1/
2*d*x)^6*tan(1/2*c)^6*tan(c)^2 + 3*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^6*tan(1
/2*c)^6 + 9*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^4*tan(c)^2 + 9*b^
2*d*x*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^6*tan(c)^2 + 3*b^2*d*x*tan(1/2*
d*x)^6*tan(1/2*c)^6*tan(c)^2 + 3*b^2*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^
6*tan(c) + 3*b^2*tan(d*x)*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^2 + 9*b^2*d*x
*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^4 + 9*b^2*d*x*tan(d*x)^2*tan(1/2*d*x
)^4*tan(1/2*c)^6 + 3*b^2*d*x*tan(1/2*d*x)^6*tan(1/2*c)^6 + 9*b^2*d*x*tan(d
*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^2*tan(c)^2 + 27*b^2*d*x*tan(d*x)^2*tan(1/2
*d*x)^4*tan(1/2*c)^4*tan(c)^2 + 9*b^2*d*x*tan(1/2*d*x)^6*tan(1/2*c)^4*tan(
c)^2 + 9*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^6*tan(c)^2 + 9*b^2*d
*x*tan(1/2*d*x)^4*tan(1/2*c)^6*tan(c)^2 - 3*b^2*tan(d*x)*tan(1/2*d*x)^6*ta
n(1/2*c)^6 + 9*b^2*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^4*tan(c) + 9*b^2*t
an(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^6*tan(c) - 3*b^2*tan(1/2*d*x)^6*tan(1/
2*c)^6*tan(c) - 32*a*b*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^3*tan(c)^2 - 9
6*a*b*tan(d*x)^2*tan(1/2*d*x)^5*tan(1/2*c)^4*tan(c)^2 + 9*b^2*tan(d*x)*tan
(1/2*d*x)^6*tan(1/2*c)^4*tan(c)^2 - 96*a*b*tan(d*x)^2*tan(1/2*d*x)^4*tan(1
/2*c)^5*tan(c)^2 - 32*a*b*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^6*tan(c)^2
+ 9*b^2*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^6*tan(c)^2 + 9*b^2*d*x*tan(d*x)
^2*tan(1/2*d*x)^6*tan(1/2*c)^2 + 27*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^4*t...
```

3.237.9 Mupad [B] (verification not implemented)

Time = 22.33 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.88

$$\int \cos^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx = \frac{a^2\,x}{8}+\frac{b^2\,x}{2}-\frac{a^2\,\sin\left(4\,c+4\,d\,x\right)}{32\,d}-\frac{b^2\,\sin\left(2\,c+2\,d\,x\right)}{4\,d} + \frac{a\,b\,\sin\left(c+d\,x\right)}{2\,d}-\frac{a\,b\,\sin\left(3\,c+3\,d\,x\right)}{6\,d}$$

input
```
int(cos(c + d*x)^2*(a*sin(c + d*x) + b*tan(c + d*x))^2,x)
```

output
```
(a^2*x)/8 + (b^2*x)/2 - (a^2*sin(4*c + 4*d*x))/(32*d) - (b^2*sin(2*c + 2*d
*x))/(4*d) + (a*b*sin(c + d*x))/(2*d) - (a*b*sin(3*c + 3*d*x))/(6*d)
```

3.238 $\int \cos(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$

3.238.1 Optimal result

Integrand size = 26, antiderivative size = 87

$$\int \cos(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$$
$$= abx + \frac{b^2\text{arctanh}(\sin(c+dx))}{d} + \frac{(a^2-2b^2)\sin(c+dx)}{3d} - \frac{ab\cos(c+dx)\sin(c+dx)}{3d} - \frac{(b+a\cos(c+dx))^2\sin(c+dx)}{3d}$$

output
```
a*b*x+b^2*arctanh(sin(d*x+c))/d+1/3*(a^2-2*b^2)*sin(d*x+c)/d-1/3*a*b*cos(d
*x+c)*sin(d*x+c)/d-1/3*(b+a*cos(d*x+c))^2*sin(d*x+c)/d
```

3.238.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.34

$$\int \cos(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$$
$$= \frac{12abc + 12abdx - 12b^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right) + 12b^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right) + \sin\left(\frac{1}{2}(c+d\right.\right.}{12d}$$

input
```
Integrate[Cos[c + d*x]*(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

output
```
(12*a*b*c + 12*a*b*d*x - 12*b^2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] +
 12*b^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 3*(a^2 - 4*b^2)*Sin[c +
 d*x] - 6*a*b*Sin[2*(c + d*x)] - a^2*Sin[3*(c + d*x)])/(12*d)
```

3.238.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.03, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {3042, 4897, 3042, 3368, 3042, 3529, 3042, 3512, 27, 3042, 3502, 27, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \cos(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2dx$$

$$\Big\downarrow \; 4897$$

$$\int \sin(c+dx)\tan(c+dx)(a\cos(c+dx)+b)^2dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\cos\left(c+dx+\frac{\pi}{2}\right)^2\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+b\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 3368$$

$$\int \left(1-\cos^2(c+dx)\right)\sec(c+dx)(a\cos(c+dx)+b)^2dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(1-\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+b\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 3529$$

$$\frac{1}{3}\int (b+a\cos(c+dx))\left(-2b\cos^2(c+dx)+a\cos(c+dx)+3b\right)\sec(c+dx)dx-\frac{\sin(c+dx)(a\cos(c+dx)+b)^2}{3d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{3}\int \frac{\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(-2b\sin\left(c+dx+\frac{\pi}{2}\right)^2+a\sin\left(c+dx+\frac{\pi}{2}\right)+3b\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{\sin(c+dx)(a\cos(c+dx)+b)^2}{3d}$$

$$\Big\downarrow \; 3512$$

$$\frac{1}{3}\left(\frac{1}{2}\int 2\left(3b^2+3a\cos(c+dx)b+\left(a^2-2b^2\right)\cos^2(c+dx)\right)\sec(c+dx)dx-\frac{ab\sin(c+dx)\cos(c+dx)}{d}\right)-\frac{\sin(c+dx)(a\cos(c+dx)+b)^2}{3d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\left(\int \left(3b^2+3a\cos(c+dx)b+\left(a^2-2b^2\right)\cos^2(c+dx)\right)\sec(c+dx)dx-\frac{ab\sin(c+dx)\cos(c+dx)}{d}\right)-\frac{\sin(c+dx)(a\cos(c+dx)+b)^2}{3d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{3}\left(\int \frac{3b^2+3a\sin\left(c+dx+\frac{\pi}{2}\right)b+\left(a^2-2b^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{ab\sin(c+dx)\cos(c+dx)}{d}\right)-\frac{\sin(c+dx)(a\cos(c+dx)+b)^2}{3d}$$

$$\Big\downarrow \; 3502$$

$$\frac{1}{3}\left(\int 3\left(b^2+a\cos(c+dx)b\right)\sec(c+dx)dx+\frac{\left(a^2-2b^2\right)\sin(c+dx)}{d}-\frac{ab\sin(c+dx)\cos(c+dx)}{d}\right)-\frac{\sin(c+dx)(a\cos(c+dx)+b)^2}{3d}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\left(3\int \left(b^2+a\cos(c+dx)b\right)\sec(c+dx)dx+\frac{\left(a^2-2b^2\right)\sin(c+dx)}{d}-\frac{ab\sin(c+dx)\cos(c+dx)}{d}\right)-\frac{\sin(c+dx)(a\cos(c+dx)+b)^2}{3d}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{3}\left(3\int \frac{b^2+a\sin\left(c+dx+\frac{\pi}{2}\right)b}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\left(a^2-2b^2\right)\sin(c+dx)}{d}-\frac{ab\sin(c+dx)\cos(c+dx)}{d}\right)-\frac{\sin(c+dx)(a\cos(c+dx)+b)^2}{3d}$$

$$\Big\downarrow \; 3214$$

3.238. $\int \cos(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$

$$\frac{1}{3}\left(3\left(b^2\int \sec(c+dx)dx+abx\right)+\frac{(a^2-2b^2)\sin(c+dx)}{d}-\frac{ab\sin(c+dx)\cos(c+dx)}{d}\right)-\frac{\sin(c+dx)(a\cos(c+dx)+b)^2}{3d}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(3\left(b^2\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+abx\right)+\frac{(a^2-2b^2)\sin(c+dx)}{d}-\frac{ab\sin(c+dx)\cos(c+dx)}{d}\right)-\frac{\sin(c+dx)(a\cos(c+dx)+b)^2}{3d}$$

$\Big\downarrow$ 4257

$$\frac{1}{3}\left(\frac{(a^2-2b^2)\sin(c+dx)}{d}+3\left(abx+\frac{b^2\text{arctanh}(\sin(c+dx))}{d}\right)-\frac{ab\sin(c+dx)\cos(c+dx)}{d}\right)-\frac{\sin(c+dx)(a\cos(c+dx)+b)^2}{3d}$$

input
```
Int[Cos[c + d*x]*(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

output
```
-1/3*((b + a*Cos[c + d*x])^2*Sin[c + d*x])/d + (3*(a*b*x + (b^2*ArcTanh[Si
n[c + d*x]])/d) + ((a^2 - 2*b^2)*Sin[c + d*x])/d - (a*b*Cos[c + d*x]*Sin[c
 + d*x])/d)/3
```

3.238.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3368
```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 3512
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Simp[1/(b*(m + 3))   Int[(a + b*Si
n[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) +
A*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

rule 3529
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n +
1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m + n + 2))   Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(
n + 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*
(a*d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c,
0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.238. $\int \cos(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$

3.238.4 Maple [A] (verified)

Time = 2.20 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{\frac{a^2\sin(dx+c)^3}{3}+2ab\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+b^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))}{d}$
default	$\frac{\frac{a^2\sin(dx+c)^3}{3}+2ab\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+b^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))}{d}$
risch	$xab-\frac{ie^{i(dx+c)}a^2}{8d}+\frac{ie^{i(dx+c)}b^2}{2d}+\frac{ie^{-i(dx+c)}a^2}{8d}-\frac{ie^{-i(dx+c)}b^2}{2d}+\frac{b^2\ln(i+e^{i(dx+c)})}{d}-\frac{b^2\ln(e^{i(dx+c)}-i)}{d}-$

input
```
int(cos(d*x+c)*(sin(d*x+c)*a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/3*a^2*sin(d*x+c)^3+2*a*b*(-1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)
+b^2*(-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c))))
```

3.238.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.95

$$\int\cos(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$$
$$=\frac{6\,abdx+3\,b^2\log\left(\sin\left(dx+c\right)+1\right)-3\,b^2\log\left(-\sin\left(dx+c\right)+1\right)-2\,\left(a^2\cos\left(dx+c\right)^2+3\,ab\cos\left(dx+c\right)\right.}{6\,d}$$

input
```
integrate(cos(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/6*(6*a*b*d*x + 3*b^2*log(sin(d*x + c) + 1) - 3*b^2*log(-sin(d*x + c) + 1
) - 2*(a^2*cos(d*x + c)^2 + 3*a*b*cos(d*x + c) - a^2 + 3*b^2)*sin(d*x + c)
)/d
```

3.238.6 Sympy [F]

$$\int \cos(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$$
$$=\int (a\sin{(c+dx)}+b\tan{(c+dx)})^2\cos{(c+dx)}\,dx$$

input
```
integrate(cos(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))**2,x)
```

output
```
Integral((a*sin(c + d*x) + b*tan(c + d*x))**2*cos(c + d*x), x)
```

3.238.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.87

$$\int \cos(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$$
$$=\frac{2\,a^2\sin{(dx+c)}^3+3\,(2\,dx+2\,c-\sin{(2\,dx+2\,c)})ab+3\,b^2(\log{(\sin{(dx+c)}+1)}-\log{(\sin{(dx+c)}-1)}}{6\,d}$$

input
```
integrate(cos(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/6*(2*a^2*sin(d*x + c)^3 + 3*(2*d*x + 2*c - sin(2*d*x + 2*c))*a*b + 3*b^2
*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*x + c)))/d
```

3.238.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4849 vs. 2(81) = 162.

Time = 2.21 (sec) , antiderivative size = 4849, normalized size of antiderivative = 55.74

$$\int \cos(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx=\text{Too large to display}$$

input
```
integrate(cos(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/12*a^2*sin(3*d*x + 3*c)/d + 1/4*a^2*sin(d*x + c)/d + 1/2*(2*a*b*d*x*tan
(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 - b^2*log(2*(tan(1/2*d*x)^2*t
an(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 +
tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/
2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(d*x)^2*tan
(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 + b^2*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2
 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x
)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*ta
n(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^2
*tan(1/2*c)^2*tan(c)^2 + 2*a*b*d*x*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2
+ 2*a*b*d*x*tan(d*x)^2*tan(1/2*d*x)^2*tan(c)^2 + 2*a*b*d*x*tan(d*x)^2*tan(
1/2*c)^2*tan(c)^2 + 2*a*b*d*x*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 - b^2*l
og(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/
2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*t
an(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^
2 + 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + b^2*log(2*(tan(1/2*d*x)^2
*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2
+ tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(
1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(d*x)^2*t
an(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*b*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)...
```

3.238.9 Mupad [B] (verification not implemented)

Time = 22.84 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.39

$$\int \cos(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$$
$$= \frac{a^2\sin(c+d\,x)}{4\,d} - \frac{b^2\sin(c+d\,x)}{d} + \frac{2\,b^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} - \frac{a^2\sin(3\,c+3\,d\,x)}{12\,d} + \frac{2\,a\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} - \frac{a\,b\,\sin(2\,c+2\,d\,x)}{2\,d}$$

input
```
int(cos(c + d*x)*(a*sin(c + d*x) + b*tan(c + d*x))^2,x)
```

output
```
(a^2*sin(c + d*x))/(4*d) - (b^2*sin(c + d*x))/d + (2*b^2*atanh(sin(c/2 + (
d*x)/2)/cos(c/2 + (d*x)/2)))/d - (a^2*sin(3*c + 3*d*x))/(12*d) + (2*a*b*at
an(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (a*b*sin(2*c + 2*d*x))/(2*d
)
```

3.239 $\int (a \sin(c+dx) + b \tan(c+dx))^2 \, dx$

3.239.1 Optimal result

Integrand size = 19, antiderivative size = 77

$$\int (a \sin(c+dx) + b \tan(c+dx))^2 \, dx = \frac{a^2 x}{2} - b^2 x + \frac{2ab \operatorname{arctanh}(\sin(c+dx))}{d} - \frac{2ab \sin(c+dx)}{d} - \frac{a^2 \cos(c+dx) \sin(c+dx)}{2d} + \frac{b^2 \tan(c+dx)}{d}$$

output
```
1/2*a^2*x-b^2*x+2*a*b*arctanh(sin(d*x+c))/d-2*a*b*sin(d*x+c)/d-1/2*a^2*cos
(d*x+c)*sin(d*x+c)/d+b^2*tan(d*x+c)/d
```

3.239.2 Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.51

$$\int (a \sin(c+dx) + b \tan(c+dx))^2 \, dx = -\frac{-2(a^2-2b^2)(c+dx) + 8ab \log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right) - 8ab \log\left(\cos\left(\frac{1}{2}(c+dx)\right) + \sin\left(\frac{1}{2}\right.\right.}{4d}$$

input
```
Integrate[(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

output
```
-1/4*(-2*(a^2 - 2*b^2)*(c + d*x) + 8*a*b*Log[Cos[(c + d*x)/2] - Sin[(c + d
*x)/2]] - 8*a*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 8*a*b*Sin[c + d
*x] + (a^2 - 4*b^2 + a^2*Cos[2*(c + d*x)])*Tan[c + d*x])/d
```

3.239.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3042, 4897, 3042, 3201, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(c+dx)+b\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (a\sin(c+dx)+b\tan(c+dx))^2dx$$

$$\Big\downarrow \text{ 4897}$$

$$\int \tan^2(c+dx)(a\cos(c+dx)+b)^2dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)^2}{\tan\left(c+dx-\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{ 3201}$$

$$\int \left(a^2\sin^2(c+dx)+2ab\sin(c+dx)\tan(c+dx)+b^2\tan^2(c+dx)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{a^2x}{2}+\frac{2ab\text{arctanh}(\sin(c+dx))}{d}-\frac{2ab\sin(c+dx)}{d}+\frac{b^2\tan(c+dx)}{d}-b^2x$$

input
```
Int[(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

output
```
(a^2*x)/2 - b^2*x + (2*a*b*ArcTanh[Sin[c + d*x]])/d - (2*a*b*Sin[c + d*x])
/d - (a^2*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (b^2*Tan[c + d*x])/d
```

3.239.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3201
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.239.4 Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{a^2\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2ab(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+b^2(\tan(dx+c)-dx-c)}{d}$
default	$\frac{a^2\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2ab(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+b^2(\tan(dx+c)-dx-c)}{d}$
parts	$\frac{a^2\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{b^2(\tan(dx+c)-\arctan(\tan(dx+c)))}{d}+\frac{2ab(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(da}{d}$
risch	$\frac{a^2x}{2}-x\,b^2+\frac{ia^2e^{2i(dx+c)}}{8d}+\frac{iab\,e^{i(dx+c)}}{d}-\frac{iab\,e^{-i(dx+c)}}{d}-\frac{ia^2e^{-2i(dx+c)}}{8d}+\frac{2ib^2}{d(e^{2i(dx+c)}+1)}-\frac{2ab\ln(e^{i(d}{d}$

input
```
int((sin(d*x+c)*a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(-1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)+2*a*b*(-sin(d*x+c)+ln(
sec(d*x+c)+tan(d*x+c)))+b^2*(tan(d*x+c)-d*x-c))
```

3.239.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.40

$$\int (a\sin(c+dx)+b\tan(c+dx))^2\,dx$$
$$=\frac{(a^2-2\,b^2)dx\cos{(dx+c)}+2\,ab\cos{(dx+c)}\log{(\sin{(dx+c)}+1)}-2\,ab\cos{(dx+c)}\log{(-\sin{(dx+c)}+}}{2\,d\cos{(dx+c)}}$$

input
```
integrate((a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/2*((a^2 - 2*b^2)*d*x*cos(d*x + c) + 2*a*b*cos(d*x + c)*log(sin(d*x + c)
+ 1) - 2*a*b*cos(d*x + c)*log(-sin(d*x + c) + 1) - (a^2*cos(d*x + c)^2 + 4
*a*b*cos(d*x + c) - 2*b^2)*sin(d*x + c))/(d*cos(d*x + c))
```

3.239.6 Sympy [F]

$$\int (a\sin(c+dx)+b\tan(c+dx))^2\,dx=\int (a\sin{(c+dx)}+b\tan{(c+dx)})^2\,dx$$

input
```
integrate((a*sin(d*x+c)+b*tan(d*x+c))**2,x)
```

output
```
Integral((a*sin(c + d*x) + b*tan(c + d*x))**2, x)
```

3.239.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.09

$$\int (a\sin(c+dx)+b\tan(c+dx))^2\,dx$$
$$=\frac{(2\,dx+2\,c-\sin{(2\,dx+2\,c)})a^2}{4\,d}-\frac{(dx+c-\tan{(dx+c)})b^2}{d}$$
$$+\frac{ab(\log{(\sin{(dx+c)}+1)}-\log{(\sin{(dx+c)}-1)}-2\,\sin{(dx+c)})}{d}$$

input
```
integrate((a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/4*(2*d*x + 2*c - sin(2*d*x + 2*c))*a^2/d - (d*x + c - tan(d*x + c))*b^2/
d + a*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*x + c))/d
```

3.239.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2320 vs. $2(73) = 146$.

Time = 0.75 (sec) , antiderivative size = 2320, normalized size of antiderivative = 30.13

$$\int (a\sin(c+dx)+b\tan(c+dx))^2\,dx = \text{Too large to display}$$

input
```
integrate((a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/2*a^2*x - 1/4*a^2*sin(2*d*x + 2*c)/d - (b^2*d*x*tan(d*x)*tan(1/2*d*x)^2*
tan(1/2*c)^2*tan(c) + a*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d
*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*
c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + t
an(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2*ta
n(c) - a*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c
) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/
2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 +
 tan(1/2*c)^2 + 1))*tan(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c) - b^2*d*x*
tan(1/2*d*x)^2*tan(1/2*c)^2 + b^2*d*x*tan(d*x)*tan(1/2*d*x)^2*tan(c) + b^2
*d*x*tan(d*x)*tan(1/2*c)^2*tan(c) - a*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2
 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x
)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*ta
n(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)
^2 + a*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c)
- 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*
d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + t
an(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 + b^2*tan(d*x)*tan(1/2*d*x)^
2*tan(1/2*c)^2 + a*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2
*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)...
```

3.239.9 Mupad [B] (verification not implemented)

Time = 22.87 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.86

$$\int (a\sin(c+dx)+b\tan(c+dx))^2\,dx = \frac{a^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} - \frac{2\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{b^2\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)} - \frac{2\,a\,b\,\sin\left(c+d\,x\right)}{d} + \frac{4\,a\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} - \frac{a^2\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{2\,d}$$

input
```
int((a*sin(c + d*x) + b*tan(c + d*x))^2,x)
```

output
```
(a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (2*b^2*atan(sin(c/2
+ (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (b^2*sin(c + d*x))/(d*cos(c + d*x)) -
(2*a*b*sin(c + d*x))/d + (4*a*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2
)))/d - (a^2*cos(c + d*x)*sin(c + d*x))/(2*d)
```

3.240 $\int \sec(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$

3.240.1 Optimal result

Integrand size = 26, antiderivative size = 90

$$\int \sec(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx = -2abx+\frac{(2a^2-b^2)\,\mathrm{arctanh}(\sin(c+dx))}{2d}-\frac{3a^2\sin(c+dx)}{2d}+\frac{ab\tan(c+dx)}{d}+\frac{(b+a\cos(c+dx))^2\sec(c+dx)\tan(c+dx)}{2d}$$

output
```
-2*a*b*x+1/2*(2*a^2-b^2)*arctanh(sin(d*x+c))/d-3/2*a^2*sin(d*x+c)/d+a*b*ta
n(d*x+c)/d+1/2*(b+a*cos(d*x+c))^2*sec(d*x+c)*tan(d*x+c)/d
```

3.240.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.83

$$\int \sec(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx = \frac{-4ab\arctan(\tan(c+dx))+(2a^2-b^2)\,\mathrm{arctanh}(\sin(c+dx))-2a^2\sin(c+dx)+4ab\tan(c+dx)+b^2\sec}{2d}$$

input
```
Integrate[Sec[c + d*x]*(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

output
```
(-4*a*b*ArcTan[Tan[c + d*x]] + (2*a^2 - b^2)*ArcTanh[Sin[c + d*x]] - 2*a^2
*Sin[c + d*x] + 4*a*b*Tan[c + d*x] + b^2*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

3.240.3 Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.01, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.577, Rules used = {3042, 4897, 3042, 3368, 3042, 3527, 3042, 3510, 25, 3042, 3502, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2dx$$

$$\Big\downarrow \text{4897}$$

$$\int \tan^2(c+dx)\sec(c+dx)(a\cos(c+dx)+b)^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos\left(c+dx+\frac{\pi}{2}\right)^2\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+b\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{3368}$$

$$\int \left(1-\cos^2(c+dx)\right)\sec^3(c+dx)(a\cos(c+dx)+b)^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(1-\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+b\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{3527}$$

$$\frac{1}{2}\int (b+a\cos(c+dx))\left(-3a\cos^2(c+dx)-b\cos(c+dx)+2a\right)\sec^2(c+dx)dx+\frac{\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+b)^2}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(-3a\sin\left(c+dx+\frac{\pi}{2}\right)^2-b\sin\left(c+dx+\frac{\pi}{2}\right)+2a\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+b)^2}{2d}$$

$$\Big\downarrow \text{3510}$$

$$\frac{1}{2}\left(\frac{2ab\tan(c+dx)}{d}-\int -\left(\left(-3\cos^2(c+dx)a^2+2a^2-4b\cos(c+dx)a-b^2\right)\sec(c+dx)\right)dx\right)+\frac{\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+b)^2}{2d}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\int \left(-3\cos^2(c+dx)a^2+2a^2-4b\cos(c+dx)a-b^2\right)\sec(c+dx)dx+\frac{2ab\tan(c+dx)}{d}\right)+\frac{\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+b)^2}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\int \frac{-3\sin\left(c+dx+\frac{\pi}{2}\right)^2a^2+2a^2-4b\sin\left(c+dx+\frac{\pi}{2}\right)a-b^2}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{2ab\tan(c+dx)}{d}\right)+\frac{\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+b)^2}{2d}$$

$$\Big\downarrow \text{3502}$$

$$\frac{1}{2}\left(\int \left(2a^2-4b\cos(c+dx)a-b^2\right)\sec(c+dx)dx-\frac{3a^2\sin(c+dx)}{d}+\frac{2ab\tan(c+dx)}{d}\right)+\frac{\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+b)^2}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\int \frac{2a^2-4b\sin\left(c+dx+\frac{\pi}{2}\right)a-b^2}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx-\frac{3a^2\sin(c+dx)}{d}+\frac{2ab\tan(c+dx)}{d}\right)+\frac{\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+b)^2}{2d}$$

$$\Big\downarrow \text{3214}$$

$$\frac{1}{2}\left(\left(2a^2-b^2\right)\int \sec(c+dx)dx-\frac{3a^2\sin(c+dx)}{d}+\frac{2ab\tan(c+dx)}{d}-4abx\right)+\frac{\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+b)^2}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\left(2a^2-b^2\right)\int \csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{3a^2\sin(c+dx)}{d}+\frac{2ab\tan(c+dx)}{d}-4abx\right)+\frac{\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+b)^2}{2d}$$

$$\Big\downarrow \text{4257}$$

3.240. $\int \sec(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$

$$\frac{1}{2}\left(\frac{(2a^2-b^2)\operatorname{arctanh}(\sin(c+dx))}{d}-\frac{3a^2\sin(c+dx)}{d}+\frac{2ab\tan(c+dx)}{d}-4abx\right)+\frac{\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+b)^2}{2d}$$

input
```
Int[Sec[c + d*x]*(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

output
```
((b + a*Cos[c + d*x])^2*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (-4*a*b*x + ((2
*a^2 - b^2)*ArcTanh[Sin[c + d*x]])/d - (3*a^2*Sin[c + d*x])/d + (2*a*b*Tan
[c + d*x])/d)/2
```

3.240.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3368
```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 3510
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - S
imp[1/(b^2*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(
m + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
 + 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))
))*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1]
```

rule 3527
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^
2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*
d*(b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b
*c*(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(
A*d^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.240.4 Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{a^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+2ab(\tan(dx+c)-dx-c)+b^2\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(d\cdots}{2}\cdots\right.}{d}$
default	$\frac{a^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+2ab(\tan(dx+c)-dx-c)+b^2\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(d\cdots}{2}\cdots\right.}{d}$
risch	$-2xab+\frac{i e^{i(dx+c)}a^2}{2d}-\frac{i e^{-i(dx+c)}a^2}{2d}-\frac{ib\left(b\,e^{3i(dx+c)}-4\,e^{2i(dx+c)}a-b\,e^{i(dx+c)}-4a\right)}{d\left(e^{2i(dx+c)}+1\right)^2}+\frac{\ln\left(i+e^{i(dx+c)}\right)a^2}{d}-\cdots$

input
```
int(sec(d*x+c)*(sin(d*x+c)*a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+2*a*b*(tan(d*x+c)-d*x-c)+
b^2*(1/2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*sin(d*x+c)-1/2*ln(sec(d*x+c)+tan(d*
x+c))))
```

3.240.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.40

$$\int \sec(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx = -\frac{8\,abdx\cos\left(dx+c\right)^2-\left(2\,a^2-b^2\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)+\left(2\,a^2-b^2\right)\cos\left(dx+c\right)^2\log\left(-\sin\ldots\right.}{4\,d\cos\left(dx+c\right)^2}$$

input
```
integrate(sec(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/4*(8*a*b*d*x*cos(d*x + c)^2 - (2*a^2 - b^2)*cos(d*x + c)^2*log(sin(d*x
+ c) + 1) + (2*a^2 - b^2)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(2*a^2
*cos(d*x + c)^2 - 4*a*b*cos(d*x + c) - b^2)*sin(d*x + c))/(d*cos(d*x + c)^
2)
```

3.240.6 Sympy [F]

$$\int \sec(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx = \int \left(a\sin\left(c+dx\right)+b\tan\left(c+dx\right)\right)^2\sec\left(c+dx\right)dx$$

input
```
integrate(sec(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))**2,x)
```

output
```
Integral((a*sin(c + d*x) + b*tan(c + d*x))**2*sec(c + d*x), x)
```

3.240.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.13

$$\int \sec(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx =$$

$$-\frac{8\left(dx+c-\tan\left(dx+c\right)\right)ab+b^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}+\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)\right)-2\,a^2(\mathrm{lo}}{4\,d}$$

input
```
integrate(sec(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/4*(8*(d*x + c - tan(d*x + c))*a*b + b^2*(2*sin(d*x + c)/(sin(d*x + c)^2
 - 1) + log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) - 2*a^2*(log(sin(d*
x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*x + c)))/d
```

3.240.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(84) = 168.

Time = 0.94 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.90

$$\int \sec(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx =$$

$$-\frac{4\left(dx+c\right)ab-\left(2\,a^2-b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)+\left(2\,a^2-b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{4\,a^2\,\mathrm{ta}}{\tan(\frac{1}{2}}}{2\,d}$$

input
```
integrate(sec(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/2*(4*(d*x + c)*a*b - (2*a^2 - b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) +
 (2*a^2 - b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 4*a^2*tan(1/2*d*x + 1/
2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1) + 2*(4*a*b*tan(1/2*d*x + 1/2*c)^3 - b^2*
tan(1/2*d*x + 1/2*c)^3 - 4*a*b*tan(1/2*d*x + 1/2*c) - b^2*tan(1/2*d*x + 1/
2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

3.240.9 Mupad [B] (verification not implemented)

Time = 22.20 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.63

$$\int \sec(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$$
$$=\frac{2\,a^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{a^2\,\sin\left(c+d\,x\right)}{d}-\frac{b^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{b^2\,\sin\left(c+d\,x\right)}{2\,d\cos\left(c+d\,x\right)^2}-\frac{4\,a\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,a\,b\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}$$

input
```
int((a*sin(c + d*x) + b*tan(c + d*x))^2/cos(c + d*x),x)
```

output
```
(2*a^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (a^2*sin(c + d*x)
)/d - (b^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (b^2*sin(c +
d*x))/(2*d*cos(c + d*x)^2) - (4*a*b*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x
)/2)))/d + (2*a*b*sin(c + d*x))/(d*cos(c + d*x))
```

3.241 $\int \sec^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$

3.241.1 Optimal result

Integrand size = 28, antiderivative size = 99

$$\int \sec^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx = -a^2x-\frac{ab\,\text{arctanh}(\sin(c+dx))}{d}+\frac{(2a^2-b^2)\tan(c+dx)}{3d} + \frac{ab\sec(c+dx)\tan(c+dx)}{3d}+\frac{(b+a\cos(c+dx))^2\sec^2(c+dx)\tan(c+dx)}{3d}$$

output
```
-a^2*x-a*b*arctanh(sin(d*x+c))/d+1/3*(2*a^2-b^2)*tan(d*x+c)/d+1/3*a*b*sec(
d*x+c)*tan(d*x+c)/d+1/3*(b+a*cos(d*x+c))^2*sec(d*x+c)^2*tan(d*x+c)/d
```

3.241.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.67

$$\int \sec^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx = \frac{-3a^2\arctan(\tan(c+dx))-3ab\,\text{arctanh}(\sin(c+dx))+\tan(c+dx)\left(3a^2+3ab\sec(c+dx)+b^2\tan^2(c+\right.}{3d}$$

input
```
Integrate[Sec[c + d*x]^2*(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

output
```
(-3*a^2*ArcTan[Tan[c + d*x]] - 3*a*b*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*
(3*a^2 + 3*a*b*Sec[c + d*x] + b^2*Tan[c + d*x]^2))/(3*d)
```

3.241.3 Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3042, 4897, 3042, 3368, 3042, 3527, 3042, 3510, 27, 3042, 3500, 27, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^2(a\sin(c+dx)+b\tan(c+dx))^2dx$$

$$\Big\downarrow \text{4897}$$

$$\int \tan^2(c+dx)\sec^2(c+dx)(a\cos(c+dx)+b)^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos\left(c+dx+\frac{\pi}{2}\right)^2\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+b\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{3368}$$

$$\int \left(1-\cos^2(c+dx)\right)\sec^4(c+dx)(a\cos(c+dx)+b)^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(1-\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+b\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{3527}$$

$$\frac{1}{3}\int (b+a\cos(c+dx))\left(-3a\cos^2(c+dx)-b\cos(c+dx)+2a\right)\sec^3(c+dx)dx+\frac{\tan(c+dx)\sec^2(c+dx)(a\cos(c+dx)+b)^2}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(-3a\sin\left(c+dx+\frac{\pi}{2}\right)^2-b\sin\left(c+dx+\frac{\pi}{2}\right)+2a\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx+\frac{\tan(c+dx)\sec^2(c+dx)(a\cos(c+dx)+b)^2}{3d}$$

$$\Big\downarrow \text{3510}$$

$$\frac{1}{3}\left(\frac{ab\tan(c+dx)\sec(c+dx)}{d}-\frac{1}{2}\int -2\left(-3\cos^2(c+dx)a^2+2a^2-3b\cos(c+dx)a-b^2\right)\sec^2(c+dx)dx\right)+\frac{\tan(c+dx)\sec^2(c+dx)(a\cos(c+dx)+b)^2}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\int \left(-3\cos^2(c+dx)a^2+2a^2-3b\cos(c+dx)a-b^2\right)\sec^2(c+dx)dx+\frac{ab\tan(c+dx)\sec(c+dx)}{d}\right)+\frac{\tan(c+dx)\sec^2(c+dx)(a\cos(c+dx)+b)^2}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\int \frac{-3\sin\left(c+dx+\frac{\pi}{2}\right)^2a^2+2a^2-3b\sin\left(c+dx+\frac{\pi}{2}\right)a-b^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx+\frac{ab\tan(c+dx)\sec(c+dx)}{d}\right)+\frac{\tan(c+dx)\sec^2(c+dx)(a\cos(c+dx)+b)^2}{3d}$$

$$\Big\downarrow \text{3500}$$

$$\frac{1}{3}\left(\int -3\left(\cos(c+dx)a^2+ba\right)\sec(c+dx)dx+\frac{\left(2a^2-b^2\right)\tan(c+dx)}{d}+\frac{ab\tan(c+dx)\sec(c+dx)}{d}\right)+\frac{\tan(c+dx)\sec^2(c+dx)(a\cos(c+dx)+b)^2}{3d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(-3\int \left(\cos(c+dx)a^2+ba\right)\sec(c+dx)dx+\frac{\left(2a^2-b^2\right)\tan(c+dx)}{d}+\frac{ab\tan(c+dx)\sec(c+dx)}{d}\right)+\frac{\tan(c+dx)\sec^2(c+dx)(a\cos(c+dx)+b)^2}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(-3\int \frac{\sin\left(c+dx+\frac{\pi}{2}\right)a^2+ba}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx+\frac{\left(2a^2-b^2\right)\tan(c+dx)}{d}+\frac{ab\tan(c+dx)\sec(c+dx)}{d}\right)+\frac{\tan(c+dx)\sec^2(c+dx)(a\cos(c+dx)+b)^2}{3d}$$

$$\Big\downarrow \text{3214}$$

3.241. $\int \sec^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$

$$\frac{1}{3}\left(-3\left(ab\int \sec(c+dx)dx+a^2x\right)+\frac{(2a^2-b^2)\tan(c+dx)}{d}+\frac{ab\tan(c+dx)\sec(c+dx)}{d}\right)+\frac{\tan(c+dx)\sec^2(c+dx)(a\cos(c+dx)+b)^2}{3d}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(-3\left(ab\int \csc\left(c+dx+\frac{\pi}{2}\right)dx+a^2x\right)+\frac{(2a^2-b^2)\tan(c+dx)}{d}+\frac{ab\tan(c+dx)\sec(c+dx)}{d}\right)+\frac{\tan(c+dx)\sec^2(c+dx)(a\cos(c+dx)+b)^2}{3d}$$

$\Big\downarrow$ 4257

$$\frac{1}{3}\left(-3\left(a^2x+\frac{ab\text{arctanh}(\sin(c+dx))}{d}\right)+\frac{(2a^2-b^2)\tan(c+dx)}{d}+\frac{ab\tan(c+dx)\sec(c+dx)}{d}\right)+\frac{\tan(c+dx)\sec^2(c+dx)(a\cos(c+dx)+b)^2}{3d}$$

input
```
Int[Sec[c + d*x]^2*(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

output
```
((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*Tan[c + d*x])/(3*d) + (-3*(a^2*x +
(a*b*ArcTanh[Sin[c + d*x]])/d) + ((2*a^2 - b^2)*Tan[c + d*x])/d + (a*b*Sec
[c + d*x]*Tan[c + d*x])/d)/3
```

3.241.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3368
```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 3510
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - S
imp[1/(b^2*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(
m + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
 + 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))
))*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1]
```

rule 3527
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^
2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*
d*(b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b
*c*(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(
A*d^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.241. $\int \sec^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.241.4 Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{a^2(\tan(dx+c)-dx-c)+2ab\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{b^2\sin(dx+c)^3}{3\cos(dx+c)^3}}{d}$
default	$\frac{a^2(\tan(dx+c)-dx-c)+2ab\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{b^2\sin(dx+c)^3}{3\cos(dx+c)^3}}{d}$
risch	$-a^2x-\frac{2i\left(3ab\,\mathrm{e}^{5i(dx+c)}-3a^2\mathrm{e}^{4i(dx+c)}+3b^2\mathrm{e}^{4i(dx+c)}-6a^2\mathrm{e}^{2i(dx+c)}-3ab\,\mathrm{e}^{i(dx+c)}-3a^2+b^2\right)}{3d\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}+\frac{ab\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{d}$

input
```
int(sec(d*x+c)^2*(sin(d*x+c)*a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(tan(d*x+c)-d*x-c)+2*a*b*(1/2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*sin(d
*x+c)-1/2*ln(sec(d*x+c)+tan(d*x+c)))+1/3*b^2*sin(d*x+c)^3/cos(d*x+c)^3)
```

3.241.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.16

$$\int \sec^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx =$$
$$-\frac{6\,a^2dx\cos{(dx+c)}^3+3\,ab\cos{(dx+c)}^3\log{(\sin{(dx+c)}+1)}-3\,ab\cos{(dx+c)}^3\log{(-\sin{(dx+c)}+}}{6\,d\cos{(dx+c)}^3}$$

input
```
integrate(sec(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas"
)
```

output
```
-1/6*(6*a^2*d*x*cos(d*x + c)^3 + 3*a*b*cos(d*x + c)^3*log(sin(d*x + c) + 1
) - 3*a*b*cos(d*x + c)^3*log(-sin(d*x + c) + 1) - 2*(3*a*b*cos(d*x + c) +
(3*a^2 - b^2)*cos(d*x + c)^2 + b^2)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

3.241.6 Sympy [F]

$$\int \sec^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx = \int (a\sin(c+dx)+b\tan(c+dx))^2\sec^2(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**2*(a*sin(d*x+c)+b*tan(d*x+c))**2,x)
```

output
```
Integral((a*sin(c + d*x) + b*tan(c + d*x))**2*sec(c + d*x)**2, x)
```

3.241.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.83

$$\int \sec^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx = \frac{2\,b^2\tan(dx+c)^3-6\,(dx+c-\tan(dx+c))a^2-3\,ab\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}+\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)\right)}{6\,d}$$

input
```
integrate(sec(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima"
)
```

output
```
1/6*(2*b^2*tan(d*x + c)^3 - 6*(d*x + c - tan(d*x + c))*a^2 - 3*a*b*(2*sin(
d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - log(sin(d*x + c) -
 1)))/d
```

3.241.8 Giac [A] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.60

$$\int \sec^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx = -\frac{3\,(dx+c)a^2+3\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-\cdots\right.}{\cdots}}{3\,d}$$

input
```
integrate(sec(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/3*(3*(d*x + c)*a^2 + 3*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a*b*l
og(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(3*a^2*tan(1/2*d*x + 1/2*c)^5 - 3*a*
b*tan(1/2*d*x + 1/2*c)^5 - 6*a^2*tan(1/2*d*x + 1/2*c)^3 + 4*b^2*tan(1/2*d*
x + 1/2*c)^3 + 3*a^2*tan(1/2*d*x + 1/2*c) + 3*a*b*tan(1/2*d*x + 1/2*c))/(t
an(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.241.9 Mupad [B] (verification not implemented)

Time = 22.51 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.29

$$\int \sec^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx =$$

$$-\frac{\frac{b^2\sin(3c+3dx)}{12} - \frac{b^2\sin(c+dx)}{4} - \frac{a^2\sin(3c+3dx)}{4} - \frac{a^2\sin(c+dx)}{4} + \frac{3a^2\cos(c+dx)\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{2} + \frac{a^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right.}{2}}{d\left(\frac{3\cos(c+dx)}{4}+\cos(3\right.}$$

input
```
int((a*sin(c + d*x) + b*tan(c + d*x))^2/cos(c + d*x)^2,x)
```

output
```
-((b^2*sin(3*c + 3*d*x))/12 - (b^2*sin(c + d*x))/4 - (a^2*sin(3*c + 3*d*x)
)/4 - (a^2*sin(c + d*x))/4 + (3*a^2*cos(c + d*x)*atan(sin(c/2 + (d*x)/2)/c
os(c/2 + (d*x)/2)))/2 + (a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*c
os(3*c + 3*d*x))/2 - (a*b*sin(2*c + 2*d*x))/2 + (3*a*b*cos(c + d*x)*atanh(
sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/2 + (a*b*atanh(sin(c/2 + (d*x)/2)/
cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x))/2)/(d*((3*cos(c + d*x))/4 + cos(3*c
+ 3*d*x)/4))
```

3.242 $\int \sec^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$

3.242.1 Optimal result

Integrand size = 28, antiderivative size = 125

$$\int \sec^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx = -\frac{(4a^2+b^2)\,\text{arctanh}(\sin(c+dx))}{8d}-\frac{2ab\tan(c+dx)}{3d}+\frac{(2a^2-b^2)\sec(c+dx)\tan(c+dx)}{8d}+\frac{ab\sec^2(c+dx)\tan(c+dx)}{6d}+\frac{(b+a\cos(c+dx))^2\sec^3(c+dx)\tan(c+dx)}{4d}$$

output
```
-1/8*(4*a^2+b^2)*arctanh(sin(d*x+c))/d-2/3*a*b*tan(d*x+c)/d+1/8*(2*a^2-b^2
)*sec(d*x+c)*tan(d*x+c)/d+1/6*a*b*sec(d*x+c)^2*tan(d*x+c)/d+1/4*(b+a*cos(d
*x+c))^2*sec(d*x+c)^3*tan(d*x+c)/d
```

3.242.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.62

$$\int \sec^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx = \frac{-3(4a^2+b^2)\,\text{arctanh}(\sin(c+dx))+\tan(c+dx)\left(3(4a^2-b^2)\sec(c+dx)+6b^2\sec^3(c+dx)+16ab\tan^2(\right.}{24d}$$

input
```
Integrate[Sec[c + d*x]^3*(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

output
```
(-3*(4*a^2 + b^2)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(3*(4*a^2 - b^2)*Se
c[c + d*x] + 6*b^2*Sec[c + d*x]^3 + 16*a*b*Tan[c + d*x]^2))/(24*d)
```

3.242.3 Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.09, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {3042, 4897, 3042, 3368, 3042, 3527, 3042, 3510, 25, 3042, 3500, 25, 3042, 3227, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^3(a\sin(c+dx)+b\tan(c+dx))^2dx$$

$$\Big\downarrow \text{4897}$$

$$\int \tan^2(c+dx)\sec^3(c+dx)(a\cos(c+dx)+b)^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos\left(c+dx+\frac{\pi}{2}\right)^2\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+b\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{3368}$$

$$\int \left(1-\cos^2(c+dx)\right)\sec^5(c+dx)(a\cos(c+dx)+b)^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(1-\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+b\right)^2}{\sin\left(c+dx+\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{3527}$$

$$\frac{1}{4}\int (b+a\cos(c+dx))\left(-3a\cos^2(c+dx)-b\cos(c+dx)+2a\right)\sec^4(c+dx)dx+\frac{\tan(c+dx)\sec^3(c+dx)(a\cos(c+dx)+b)^2}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \frac{\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(-3a\sin\left(c+dx+\frac{\pi}{2}\right)^2-b\sin\left(c+dx+\frac{\pi}{2}\right)+2a\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx+\frac{\tan(c+dx)\sec^3(c+dx)(a\cos(c+dx)+b)^2}{4d}$$

$\downarrow$ 3510

$$\frac{1}{4}\left(\frac{2ab\tan(c+dx)\sec^2(c+dx)}{3d}-\frac{1}{3}\int -\left(\left(-9a^2\cos^2(c+dx)-8ab\cos(c+dx)+3\left(2a^2-b^2\right)\right)\sec^3(c+dx)\right)dx\right)$$
$$\frac{\tan(c+dx)\sec^3(c+dx)(a\cos(c+dx)+b)^2}{4d}$$

$\downarrow$ 25

$$\frac{1}{4}\left(\frac{1}{3}\int \left(-9a^2\cos^2(c+dx)-8ab\cos(c+dx)+3\left(2a^2-b^2\right)\right)\sec^3(c+dx)dx+\frac{2ab\tan(c+dx)\sec^2(c+dx)}{3d}\right)+$$
$$\frac{\tan(c+dx)\sec^3(c+dx)(a\cos(c+dx)+b)^2}{4d}$$

$\downarrow$ 3042

$$\frac{1}{4}\left(\frac{1}{3}\int \frac{-9a^2\sin\left(c+dx+\frac{\pi}{2}\right)^2-8ab\sin\left(c+dx+\frac{\pi}{2}\right)+3\left(2a^2-b^2\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^3}dx+\frac{2ab\tan(c+dx)\sec^2(c+dx)}{3d}\right)+$$
$$\frac{\tan(c+dx)\sec^3(c+dx)(a\cos(c+dx)+b)^2}{4d}$$

$\downarrow$ 3500

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\int -\left(\left(16ab+3\left(4a^2+b^2\right)\cos(c+dx)\right)\sec^2(c+dx)\right)dx+\frac{3\left(2a^2-b^2\right)\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{2ab\,\mathrm{t}}{}\right.$$
$$\frac{\tan(c+dx)\sec^3(c+dx)(a\cos(c+dx)+b)^2}{4d}$$

$\downarrow$ 25

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{3\left(2a^2-b^2\right)\tan(c+dx)\sec(c+dx)}{2d}-\frac{1}{2}\int \left(16ab+3\left(4a^2+b^2\right)\cos(c+dx)\right)\sec^2(c+dx)dx\right)+\frac{2ab\tan(c}{}\right.$$
$$\frac{\tan(c+dx)\sec^3(c+dx)(a\cos(c+dx)+b)^2}{4d}$$

$\downarrow$ 3042

3.242. $\int \sec^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{3(2a^2-b^2)\tan(c+dx)\sec(c+dx)}{2d}-\frac{1}{2}\int\frac{16ab+3(4a^2+b^2)\sin\left(c+dx+\frac{\pi}{2}\right)}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx\right)+\frac{2ab\tan(c+dx)\sec}{3d}\right.$$
$$\frac{\tan(c+dx)\sec^3(c+dx)(a\cos(c+dx)+b)^2}{4d}$$

$$\Big\downarrow \text{3227}$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(-3(4a^2+b^2)\int\sec(c+dx)dx-16ab\int\sec^2(c+dx)dx\right)+\frac{3(2a^2-b^2)\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{2a}{}\right.$$
$$\frac{\tan(c+dx)\sec^3(c+dx)(a\cos(c+dx)+b)^2}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(-3(4a^2+b^2)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx-16ab\int\csc\left(c+dx+\frac{\pi}{2}\right)^2dx\right)+\frac{3(2a^2-b^2)\tan(c+dx)\sec(c}{2d}\right.\right.$$
$$\frac{\tan(c+dx)\sec^3(c+dx)(a\cos(c+dx)+b)^2}{4d}$$

$$\Big\downarrow \text{4254}$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(\frac{16ab\int 1d(-\tan(c+dx))}{d}-3(4a^2+b^2)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx\right)+\frac{3(2a^2-b^2)\tan(c+dx)\sec(c+dx}{2d}\right.\right.$$
$$\frac{\tan(c+dx)\sec^3(c+dx)(a\cos(c+dx)+b)^2}{4d}$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(-3(4a^2+b^2)\int\csc\left(c+dx+\frac{\pi}{2}\right)dx-\frac{16ab\tan(c+dx)}{d}\right)+\frac{3(2a^2-b^2)\tan(c+dx)\sec(c+dx)}{2d}\right)+\right.$$
$$\frac{\tan(c+dx)\sec^3(c+dx)(a\cos(c+dx)+b)^2}{4d}$$

$$\Big\downarrow \text{4257}$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\left(-\frac{3(4a^2+b^2)\text{arctanh}(\sin(c+dx))}{d}-\frac{16ab\tan(c+dx)}{d}\right)+\frac{3(2a^2-b^2)\tan(c+dx)\sec(c+dx)}{2d}\right)+\frac{2a}{}\right.$$
$$\frac{\tan(c+dx)\sec^3(c+dx)(a\cos(c+dx)+b)^2}{4d}$$

input
```
Int[Sec[c + d*x]^3*(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

3.242. $\int\sec^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$

output
```
((b + a*Cos[c + d*x])^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + ((2*a*b*Sec[c
 + d*x]^2*Tan[c + d*x])/(3*d) + ((3*(2*a^2 - b^2)*Sec[c + d*x]*Tan[c + d*x
])/(2*d) + ((-3*(4*a^2 + b^2)*ArcTanh[Sin[c + d*x]])/d - (16*a*b*Tan[c + d
*x])/d)/2)/3)/4
```

3.242.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3368
```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

rule 3500
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Simp[1/(b*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A
*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

rule 3510
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - S
imp[1/(b^2*(m + 1)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(
m + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
 + 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))
))*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1]
```

rule 3527
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(d*(n + 1)*(c^2 - d^
2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*
d*(b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b
*c*(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(
A*d^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.242. $\int \sec^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$

3.242.4 Maple [A] (verified)

Time = 1.97 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{a^2\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{2ab\sin(dx+c)^3}{3\cos(dx+c)^3}+b^2\left(\frac{\sin(dx+c)^3}{4\cos(dx+c)^4}+\frac{\sin(dx+c)^3}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\ln(\right.}{d}$
default	$\frac{a^2\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{2ab\sin(dx+c)^3}{3\cos(dx+c)^3}+b^2\left(\frac{\sin(dx+c)^3}{4\cos(dx+c)^4}+\frac{\sin(dx+c)^3}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\ln(\right.}{d}$
risch	$-\frac{i(12a^2e^{7i(dx+c)}-3b^2e^{7i(dx+c)}+48ab\,e^{6i(dx+c)}+12a^2e^{5i(dx+c)}+21b^2e^{5i(dx+c)}+48ab\,e^{4i(dx+c)}-12a^2e^{3i(dx+c)}-21b^2}{12d\left(e^{2i(dx+c)}+1\right)^4}$

input
```
int(sec(d*x+c)^3*(sin(d*x+c)*a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(1/2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*sin(d*x+c)-1/2*ln(sec(d*x+c)+t
an(d*x+c)))+2/3*a*b*sin(d*x+c)^3/cos(d*x+c)^3+b^2*(1/4*sin(d*x+c)^3/cos(d*
x+c)^4+1/8*sin(d*x+c)^3/cos(d*x+c)^2+1/8*sin(d*x+c)-1/8*ln(sec(d*x+c)+tan(
d*x+c))))
```

3.242.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.03

$$\int \sec^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx =$$
$$-\frac{3\,(4\,a^2+b^2)\cos(dx+c)^4\log(\sin(dx+c)+1)-3\,(4\,a^2+b^2)\cos(dx+c)^4\log(-\sin(dx+c)+1)+2}{48\,d\cos(dx+c)}$$

input
```
integrate(sec(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas"
)
```

output
```
-1/48*(3*(4*a^2 + b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(4*a^2 + b
^2)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(16*a*b*cos(d*x + c)^3 - 16*
a*b*cos(d*x + c) - 3*(4*a^2 - b^2)*cos(d*x + c)^2 - 6*b^2)*sin(d*x + c))/(
d*cos(d*x + c)^4)
```

3.242.6 Sympy [F]

$$\int \sec^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$$
$$=\int (a\sin(c+dx)+b\tan(c+dx))^2\sec^3(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**3*(a*sin(d*x+c)+b*tan(d*x+c))**2,x)
```

output
```
Integral((a*sin(c + d*x) + b*tan(c + d*x))**2*sec(c + d*x)**3, x)
```

3.242.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.03

$$\int \sec^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$$
$$=\frac{32\,ab\tan(dx+c)^3+3\,b^2\left(\frac{2\left(\sin(dx+c)^3+\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)-12\,a}{48\,d}$$

input
```
integrate(sec(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima"
)
```

output
```
1/48*(32*a*b*tan(d*x + c)^3 + 3*b^2*(2*(sin(d*x + c)^3 + sin(d*x + c))/(si
n(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - log(sin(d*x + c) + 1) + log(sin(d*x
 + c) - 1)) - 12*a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x +
c) + 1) - log(sin(d*x + c) - 1)))/d
```

3.242.8 Giac [A] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.81

$$\int \sec^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx=$$
$$-\frac{3\,(4\,a^2+b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(4\,a^2+b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(12\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right.}{}}{}$$

input
```
integrate(sec(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/24*(3*(4*a^2 + b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(4*a^2 + b^2
)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(12*a^2*tan(1/2*d*x + 1/2*c)^7 +
3*b^2*tan(1/2*d*x + 1/2*c)^7 - 12*a^2*tan(1/2*d*x + 1/2*c)^5 - 64*a*b*tan(
1/2*d*x + 1/2*c)^5 + 21*b^2*tan(1/2*d*x + 1/2*c)^5 - 12*a^2*tan(1/2*d*x +
1/2*c)^3 + 64*a*b*tan(1/2*d*x + 1/2*c)^3 + 21*b^2*tan(1/2*d*x + 1/2*c)^3 +
 12*a^2*tan(1/2*d*x + 1/2*c) + 3*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x +
1/2*c)^2 - 1)^4)/d
```

3.242.9 Mupad [B] (verification not implemented)

Time = 25.26 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.42

$$\int \sec^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$$
$$= \frac{\left(a^2+\frac{b^2}{4}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\left(-a^2-\frac{16\,a\,b}{3}+\frac{7\,b^2}{4}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\left(-a^2+\frac{16\,a\,b}{3}+\frac{7\,b^2}{4}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+\left(a\right.}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}$$
$$-\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(a^2+\frac{b^2}{4}\right)}{d}$$

input
```
int((a*sin(c + d*x) + b*tan(c + d*x))^2/cos(c + d*x)^3,x)
```

output
```
(tan(c/2 + (d*x)/2)^3*((16*a*b)/3 - a^2 + (7*b^2)/4) + tan(c/2 + (d*x)/2)*
(a^2 + b^2/4) + tan(c/2 + (d*x)/2)^7*(a^2 + b^2/4) - tan(c/2 + (d*x)/2)^5*
((16*a*b)/3 + a^2 - (7*b^2)/4))/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (
d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) - (atanh(t
an(c/2 + (d*x)/2))*(a^2 + b^2/4))/d
```

3.243 $\int \cos^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx$

3.243.1 Optimal result

Integrand size = 28, antiderivative size = 77

$$\int \cos^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx = -\frac{(a^2-b^2)\,(b+a\cos(c+dx))^4}{4a^3d}-\frac{2b(b+a\cos(c+dx))^5}{5a^3d}+\frac{(b+a\cos(c+dx))^6}{6a^3d}$$

output
```
-1/4*(a^2-b^2)*(b+a*cos(d*x+c))^4/a^3/d-2/5*b*(b+a*cos(d*x+c))^5/a^3/d+1/6
*(b+a*cos(d*x+c))^6/a^3/d
```

3.243.2 Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.48

$$\int \cos^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx = \frac{-360b(a^2+2b^2)\cos(c+dx)-45(a^3+8ab^2)\cos(2(c+dx))-60a^2b\cos(3(c+dx))+80b^3\cos(3(c+dx)}{960d}$$

input
```
Integrate[Cos[c + d*x]^3*(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

output
```
(-360*b*(a^2 + 2*b^2)*Cos[c + d*x] - 45*(a^3 + 8*a*b^2)*Cos[2*(c + d*x)] -
 60*a^2*b*Cos[3*(c + d*x)] + 80*b^3*Cos[3*(c + d*x)] + 90*a*b^2*Cos[4*(c +
 d*x)] + 36*a^2*b*Cos[5*(c + d*x)] + 5*a^3*Cos[6*(c + d*x)])/(960*d)
```

3.243.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.87, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 4897, 3042, 3147, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(c+dx)^3(a\sin(c+dx)+b\tan(c+dx))^3dx$$

$$\Big\downarrow \text{4897}$$

$$\int \sin^3(c+dx)(a\cos(c+dx)+b)^3dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos\left(c+dx-\frac{\pi}{2}\right)^3\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{3147}$$

$$-\frac{\int (b+a\cos(c+dx))^3\left(a^2-a^2\cos^2(c+dx)\right)d(a\cos(c+dx))}{a^3d}$$

$$\Big\downarrow \text{476}$$

$$-\frac{\int \left(-(b+a\cos(c+dx))^5+2b(b+a\cos(c+dx))^4+\left(a^2-b^2\right)(b+a\cos(c+dx))^3\right)d(a\cos(c+dx))}{a^3d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{1}{4}\left(a^2-b^2\right)(a\cos(c+dx)+b)^4-\frac{1}{6}(a\cos(c+dx)+b)^6+\frac{2}{5}b(a\cos(c+dx)+b)^5}{a^3d}$$

input
```
Int[Cos[c + d*x]^3*(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

output
```
-((((a^2 - b^2)*(b + a*Cos[c + d*x])^4)/4 + (2*b*(b + a*Cos[c + d*x])^5)/5
 - (b + a*Cos[c + d*x])^6/6)/(a^3*d))
```

3.243.3.1 Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.243.4 Maple [A] (verified)

Time = 30.63 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.30

method	result
derivativedivides	$\frac{\frac{a^3\cos(dx+c)^6}{6}+\frac{3a^2b\cos(dx+c)^5}{5}+\frac{\left(-a^3+3a\,b^2\right)\cos(dx+c)^4}{4}+\frac{\left(-3a^2b+b^3\right)\cos(dx+c)^3}{3}-\frac{3a\,b^2\cos(dx+c)^2}{2}-\cos(dx+c)b^3}{d}$
default	$\frac{\frac{a^3\cos(dx+c)^6}{6}+\frac{3a^2b\cos(dx+c)^5}{5}+\frac{\left(-a^3+3a\,b^2\right)\cos(dx+c)^4}{4}+\frac{\left(-3a^2b+b^3\right)\cos(dx+c)^3}{3}-\frac{3a\,b^2\cos(dx+c)^2}{2}-\cos(dx+c)b^3}{d}$
risch	$-\frac{3a^2b\cos(dx+c)}{8d}-\frac{3b^3\cos(dx+c)}{4d}+\frac{a^3\cos(6dx+6c)}{192d}+\frac{3b\,a^2\cos(5dx+5c)}{80d}+\frac{3a\,b^2\cos(4dx+4c)}{32d}-\frac{b\cos(3dx+3}{16d}$

input
```
int(cos(d*x+c)^3*(sin(d*x+c)*a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/6*a^3*cos(d*x+c)^6+3/5*a^2*b*cos(d*x+c)^5+1/4*(-a^3+3*a*b^2)*cos(d*
x+c)^4+1/3*(-3*a^2*b+b^3)*cos(d*x+c)^3-3/2*a*b^2*cos(d*x+c)^2-cos(d*x+c)*b
^3)
```

3.243.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.30

$$\int \cos^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx$$
$$=\frac{10\,a^3\cos\left(dx+c\right)^6+36\,a^2b\cos\left(dx+c\right)^5-90\,ab^2\cos\left(dx+c\right)^2-15\,\left(a^3-3\,ab^2\right)\cos\left(dx+c\right)^4-60\,b^3\cos}{60\,d}$$

input
```
integrate(cos(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas"
)
```

output
```
1/60*(10*a^3*cos(d*x + c)^6 + 36*a^2*b*cos(d*x + c)^5 - 90*a*b^2*cos(d*x +
 c)^2 - 15*(a^3 - 3*a*b^2)*cos(d*x + c)^4 - 60*b^3*cos(d*x + c) - 20*(3*a^
2*b - b^3)*cos(d*x + c)^3)/d
```

3.243.6 Sympy [F]

$$\int \cos^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx$$
$$=\int\left(a\sin\left(c+dx\right)+b\tan\left(c+dx\right)\right)^3\cos^3\left(c+dx\right)\,dx$$

input
```
integrate(cos(d*x+c)**3*(a*sin(d*x+c)+b*tan(d*x+c))**3,x)
```

output
```
Integral((a*sin(c + d*x) + b*tan(c + d*x))**3*cos(c + d*x)**3, x)
```

3.243.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.23

$$\int \cos^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx$$
$$=\frac{45\,ab^2\sin\left(dx+c\right)^4-5\,\left(2\,\sin\left(dx+c\right)^6-3\,\sin\left(dx+c\right)^4\right)a^3+12\,\left(3\,\cos\left(dx+c\right)^5-5\,\cos\left(dx+c\right)^3\right)a^2b-}{60\,d}$$

input
```
integrate(cos(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima"
)
```

output
```
1/60*(45*a*b^2*sin(d*x + c)^4 - 5*(2*sin(d*x + c)^6 - 3*sin(d*x + c)^4)*a^
3 + 12*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a^2*b + 20*(cos(d*x + c)^3 -
3*cos(d*x + c))*b^3)/d
```

3.243.8 Giac [F(-1)]

Timed out.

$$\int \cos^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
Timed out
```

3.243.9 Mupad [B] (verification not implemented)

Time = 22.71 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.94

$$\begin{aligned}\int \cos^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx &= \frac{32\,a^3}{3\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^6}+\frac{4\,(a-b)^3}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^2}-\frac{32\,a^2\,(5\,a-3\,b)}{5\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^5}\\&\quad-\frac{8\,(a-b)^2\,(7\,a-b)}{3\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^3}+\frac{12\,a\,(3\,a^2-4\,a\,b+b^2)}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^4}\end{aligned}$$

input
```
int(cos(c + d*x)^3*(a*sin(c + d*x) + b*tan(c + d*x))^3,x)
```

output
```
(32*a^3)/(3*d*(tan(c/2 + (d*x)/2)^2 + 1)^6) + (4*(a - b)^3)/(d*(tan(c/2 +
(d*x)/2)^2 + 1)^2) - (32*a^2*(5*a - 3*b))/(5*d*(tan(c/2 + (d*x)/2)^2 + 1)^
5) - (8*(a - b)^2*(7*a - b))/(3*d*(tan(c/2 + (d*x)/2)^2 + 1)^3) + (12*a*(3
*a^2 - 4*a*b + b^2))/(d*(tan(c/2 + (d*x)/2)^2 + 1)^4)
```

3.244 $\int \cos^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx$

3.244.1 Optimal result

Integrand size = 28, antiderivative size = 120

$$\int \cos^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx = -\frac{3ab^2\cos(c+dx)}{d}-\frac{b(3a^2-b^2)\cos^2(c+dx)}{2d}-\frac{a(a^2-3b^2)\cos^3(c+dx)}{3d}+\frac{3a^2b\cos^4(c+dx)}{4d}+\frac{a^3\cos^5(c+dx)}{5d}-\frac{b^3\log(\cos(c+dx))}{d}$$

output
```
-3*a*b^2*cos(d*x+c)/d-1/2*b*(3*a^2-b^2)*cos(d*x+c)^2/d-1/3*a*(a^2-3*b^2)*c
os(d*x+c)^3/d+3/4*a^2*b*cos(d*x+c)^4/d+1/5*a^3*cos(d*x+c)^5/d-b^3*ln(cos(d
*x+c))/d
```

3.244.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.88

$$\int \cos^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx = -\frac{3ab^2\cos(c+dx)+\frac{1}{2}b(3a^2-b^2)\cos^2(c+dx)+\frac{1}{3}a(a^2-3b^2)\cos^3(c+dx)-\frac{3}{4}a^2b\cos^4(c+dx)-\frac{1}{5}a^3\,\mathrm{co}}{d}$$

input
```
Integrate[Cos[c + d*x]^2*(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

output
```
-((3*a*b^2*Cos[c + d*x] + (b*(3*a^2 - b^2)*Cos[c + d*x]^2)/2 + (a*(a^2 - 3
*b^2)*Cos[c + d*x]^3)/3 - (3*a^2*b*Cos[c + d*x]^4)/4 - (a^3*Cos[c + d*x]^5
)/5 + b^3*Log[Cos[c + d*x]])/d)
```

3.244.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 4897, 3042, 25, 3316, 27, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(c+dx)^2(a\sin(c+dx)+b\tan(c+dx))^3dx$$

$$\Big\downarrow \text{4897}$$

$$\int \sin^2(c+dx)\tan(c+dx)(a\cos(c+dx)+b)^3dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\cos\left(c+dx+\frac{\pi}{2}\right)^3\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+b\right)^3}{\sin\left(c+dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\cos\left(\frac{1}{2}(2c+\pi)+dx\right)^3\left(b+a\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^3}{\sin\left(\frac{1}{2}(2c+\pi)+dx\right)}dx$$

$$\Big\downarrow \text{3316}$$

$$-\frac{\int (b+a\cos(c+dx))^3\left(a^2-a^2\cos^2(c+dx)\right)\sec(c+dx)d(a\cos(c+dx))}{a^3d}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{(b+a\cos(c+dx))^3\left(a^2-a^2\cos^2(c+dx)\right)\sec(c+dx)}{a}d(a\cos(c+dx))}{a^2d}$$

$$\Big\downarrow \text{522}$$

$$-\frac{\int \left(-a^4\cos^4(c+dx)-3a^3b\cos^3(c+dx)+a^2\left(a^2-3b^2\right)\cos^2(c+dx)+ab\left(3a^2-b^2\right)\cos(c+dx)+3a^2b^2+ab^3\mathrm{s}\right.}{a^2d}$$

$\big\downarrow$ 2009

$$-\frac{-\frac{1}{5}a^5\cos^5(c+dx)-\frac{3}{4}a^4b\cos^4(c+dx)+3a^3b^2\cos(c+dx)+a^2b^3\log(a\cos(c+dx))+\frac{1}{2}a^2b\left(3a^2-b^2\right)\cos^2(c+\right.}{a^2d}$$

input
```
Int[Cos[c + d*x]^2*(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

output
```
-((3*a^3*b^2*Cos[c + d*x] + (a^2*b*(3*a^2 - b^2)*Cos[c + d*x]^2)/2 + (a^3*
(a^2 - 3*b^2)*Cos[c + d*x]^3)/3 - (3*a^4*b*Cos[c + d*x]^4)/4 - (a^5*Cos[c
+ d*x]^5)/5 + a^2*b^3*Log[a*Cos[c + d*x]])/(a^2*d))
```

3.244.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.244.4 Maple [A] (verified)

Time = 17.40 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{a^3\left(-\frac{\cos(dx+c)^3\sin(dx+c)^2}{5}-\frac{2\cos(dx+c)^3}{15}\right)+\frac{3a^2b\sin(dx+c)^4}{4}-a\,b^2\left(2+\sin(dx+c)^2\right)\cos(dx+c)+b^3\left(-\frac{\sin(dx+c)^2}{2}-\ln(\cos\right.}{d}$
default	$\frac{a^3\left(-\frac{\cos(dx+c)^3\sin(dx+c)^2}{5}-\frac{2\cos(dx+c)^3}{15}\right)+\frac{3a^2b\sin(dx+c)^4}{4}-a\,b^2\left(2+\sin(dx+c)^2\right)\cos(dx+c)+b^3\left(-\frac{\sin(dx+c)^2}{2}-\ln(\cos\right.}{d}$
risch	$ix\,b^3-\frac{3b\,e^{2i(dx+c)}a^2}{16d}+\frac{b^3e^{2i(dx+c)}}{8d}-\frac{3b\,e^{-2i(dx+c)}a^2}{16d}+\frac{b^3e^{-2i(dx+c)}}{8d}+\frac{2ib^3c}{d}-\frac{b^3\ln\left(e^{2i(dx+c)}+1\right)}{d}-\frac{a^3}{}$

input
```
int(cos(d*x+c)^2*(sin(d*x+c)*a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(-1/5*cos(d*x+c)^3*sin(d*x+c)^2-2/15*cos(d*x+c)^3)+3/4*a^2*b*sin(
d*x+c)^4-a*b^2*(2+sin(d*x+c)^2)*cos(d*x+c)+b^3*(-1/2*sin(d*x+c)^2-ln(cos(d
*x+c))))
```

3.244.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.84

$$\int \cos^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx$$
$$=\frac{12\,a^3\cos(dx+c)^5+45\,a^2b\cos(dx+c)^4-180\,ab^2\cos(dx+c)-20\,(a^3-3\,ab^2)\cos(dx+c)^3-60\,b^3\log}{60\,d}$$

input
```
integrate(cos(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas"
)
```

output
```
1/60*(12*a^3*cos(d*x + c)^5 + 45*a^2*b*cos(d*x + c)^4 - 180*a*b^2*cos(d*x
+ c) - 20*(a^3 - 3*a*b^2)*cos(d*x + c)^3 - 60*b^3*log(-cos(d*x + c)) - 30*
(3*a^2*b - b^3)*cos(d*x + c)^2)/d
```

3.244.6 Sympy [F]

$$\int \cos^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx = \int (a\sin(c+dx)+b\tan(c+dx))^3\cos^2(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)**2*(a*sin(d*x+c)+b*tan(d*x+c))**3,x)
```

output
```
Integral((a*sin(c + d*x) + b*tan(c + d*x))**3*cos(c + d*x)**2, x)
```

3.244.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.78

$$\int \cos^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx = \frac{45\,a^2b\sin(dx+c)^4+4\left(3\cos(dx+c)^5-5\cos(dx+c)^3\right)a^3+60\left(\cos(dx+c)^3-3\cos(dx+c)\right)ab^2-3}{60\,d}$$

input
```
integrate(cos(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima"
)
```

output
```
1/60*(45*a^2*b*sin(d*x + c)^4 + 4*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a^
3 + 60*(cos(d*x + c)^3 - 3*cos(d*x + c))*a*b^2 - 30*(sin(d*x + c)^2 + log(
sin(d*x + c)^2 - 1))*b^3)/d
```

3.244.8 Giac [F(-2)]

Exception generated.

$$\int \cos^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Modgcd: no suitable evaluation poin
tindex.cc index_m operator + Error: Bad Argument ValueDone
```

3.244.9 Mupad [B] (verification not implemented)

Time = 23.73 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.98

$$\int \cos^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx$$
$$= \frac{40\,a^3\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{3\,d}-\frac{4\,a^3\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{d}-\frac{16\,a^3\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8}{d}+\frac{32\,a^3\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{5\,d}-\frac{2\,b^3\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{d}+\frac{2\,b^3\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{d}+\frac{2\,b^3\,\mathrm{atanh}\left(\frac{1}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^2}-1\right)}{d}-\frac{12\,a\,b^2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{d}+\frac{12\,a^2\,b\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{d}+\frac{8\,a\,b^2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{d}-\frac{24\,a^2\,b\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{d}+\frac{12\,a^2\,b\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8}{d}$$

input
```
int(cos(c + d*x)^2*(a*sin(c + d*x) + b*tan(c + d*x))^3,x)
```

output
```
(40*a^3*cos(c/2 + (d*x)/2)^6)/(3*d) - (4*a^3*cos(c/2 + (d*x)/2)^4)/d - (16
*a^3*cos(c/2 + (d*x)/2)^8)/d + (32*a^3*cos(c/2 + (d*x)/2)^10)/(5*d) - (2*b
^3*cos(c/2 + (d*x)/2)^2)/d + (2*b^3*cos(c/2 + (d*x)/2)^4)/d + (2*b^3*atanh
(1/cos(c/2 + (d*x)/2)^2 - 1))/d - (12*a*b^2*cos(c/2 + (d*x)/2)^4)/d + (12*
a^2*b*cos(c/2 + (d*x)/2)^4)/d + (8*a*b^2*cos(c/2 + (d*x)/2)^6)/d - (24*a^2
*b*cos(c/2 + (d*x)/2)^6)/d + (12*a^2*b*cos(c/2 + (d*x)/2)^8)/d
```

3.245 $\int \cos(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx$

3.245.1 Optimal result

Integrand size = 26, antiderivative size = 112

$$\int \cos(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx = -\frac{b(3a^2-b^2)\cos(c+dx)}{d}-\frac{a(a^2-3b^2)\cos^2(c+dx)}{2d}+\frac{a^2b\cos^3(c+dx)}{d}+\frac{a^3\cos^4(c+dx)}{4d}-\frac{3ab^2\log(\cos(c+dx))}{d}+\frac{b^3\sec(c+dx)}{d}$$

output
```
-b*(3*a^2-b^2)*cos(d*x+c)/d-1/2*a*(a^2-3*b^2)*cos(d*x+c)^2/d+a^2*b*cos(d*x
+c)^3/d+1/4*a^3*cos(d*x+c)^4/d-3*a*b^2*ln(cos(d*x+c))/d+b^3*sec(d*x+c)/d
```

3.245.2 Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.88

$$\int \cos(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx = \frac{8b(-9a^2+4b^2)\cos(c+dx)-4(a^3-6ab^2)\cos(2(c+dx))+8a^2b\cos(3(c+dx))+a^3\cos(4(c+dx))-96}{32d}$$

input
```
Integrate[Cos[c + d*x]*(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

output
```
(8*b*(-9*a^2 + 4*b^2)*Cos[c + d*x] - 4*(a^3 - 6*a*b^2)*Cos[2*(c + d*x)] +
8*a^2*b*Cos[3*(c + d*x)] + a^3*Cos[4*(c + d*x)] - 96*a*b^2*Log[Cos[c + d*x
]] + 32*b^3*Sec[c + d*x])/(32*d)
```

3.245.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 4897, 3042, 25, 3316, 27, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3dx$$

$$\Big\downarrow \text{4897}$$

$$\int \sin(c+dx)\tan^2(c+dx)(a\cos(c+dx)+b)^3dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\cos\left(c+dx+\frac{\pi}{2}\right)^3\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+b\right)^3}{\sin\left(c+dx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\cos\left(\frac{1}{2}(2c+\pi)+dx\right)^3\left(b+a\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^3}{\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2}dx$$

$$\Big\downarrow \text{3316}$$

$$-\frac{\int (b+a\cos(c+dx))^3\left(a^2-a^2\cos^2(c+dx)\right)\sec^2(c+dx)d(a\cos(c+dx))}{a^3d}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{(b+a\cos(c+dx))^3\left(a^2-a^2\cos^2(c+dx)\right)\sec^2(c+dx)}{a^2}d(a\cos(c+dx))}{ad}$$

$$\Big\downarrow \text{522}$$

$$-\frac{\int \left(\sec^2(c+dx)b^3+3a\sec(c+dx)b^2-3a^2\cos^2(c+dx)b+3a^2\left(1-\frac{b^2}{3a^2}\right)b-a^3\cos^3(c+dx)+a\left(a^2-3b^2\right)\cos(\right.}{ad}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{1}{4}a^4\cos^4(c+dx)-a^3b\cos^3(c+dx)+\frac{1}{2}a^2\left(a^2-3b^2\right)\cos^2(c+dx)+ab\left(3a^2-b^2\right)\cos(c+dx)+3a^2b^2\log(a\cos}{ad}$$

input
```
Int[Cos[c + d*x]*(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

output
```
-((a*b*(3*a^2 - b^2)*Cos[c + d*x] + (a^2*(a^2 - 3*b^2)*Cos[c + d*x]^2)/2 -
 a^3*b*Cos[c + d*x]^3 - (a^4*Cos[c + d*x]^4)/4 + 3*a^2*b^2*Log[a*Cos[c + d
*x]] - a*b^3*Sec[c + d*x])/(a*d))
```

3.245.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.245. $\int \cos(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx$

3.245.4 Maple [A] (verified)

Time = 9.20 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{\frac{a^3 \sin(dx+c)^4}{4} - a^2 b\left(2+\sin(dx+c)^2\right)\cos(dx+c) + 3a\,b^2\left(-\frac{\sin(dx+c)^2}{2} - \ln(\cos(dx+c))\right) + b^3\left(\frac{\sin(dx+c)^4}{\cos(dx+c)} + \left(2+\sin(dx+c)^2\right.\right.}{d}$
default	$\frac{\frac{a^3 \sin(dx+c)^4}{4} - a^2 b\left(2+\sin(dx+c)^2\right)\cos(dx+c) + 3a\,b^2\left(-\frac{\sin(dx+c)^2}{2} - \ln(\cos(dx+c))\right) + b^3\left(\frac{\sin(dx+c)^4}{\cos(dx+c)} + \left(2+\sin(dx+c)^2\right.\right.}{d}$
risch	$3ixa\,b^2 - \frac{a^3 e^{2i(dx+c)}}{16d} + \frac{3a\,e^{2i(dx+c)} b^2}{8d} - \frac{9b\,e^{i(dx+c)} a^2}{8d} + \frac{b^3 e^{i(dx+c)}}{2d} - \frac{9b\,e^{-i(dx+c)} a^2}{8d} + \frac{b^3 e^{-i(dx+c)}}{2d} -$

input
```
int(cos(d*x+c)*(sin(d*x+c)*a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/4*a^3*sin(d*x+c)^4-a^2*b*(2+sin(d*x+c)^2)*cos(d*x+c)+3*a*b^2*(-1/2*
sin(d*x+c)^2-ln(cos(d*x+c)))+b^3*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)
*cos(d*x+c)))
```

3.245.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.14

$$\int \cos(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx = \frac{8\,a^3\cos(dx+c)^5 + 32\,a^2 b\cos(dx+c)^4 - 96\,ab^2\cos(dx+c)\log(-\cos(dx+c)) - 16\,(a^3-3\,ab^2)\cos(dx}{32\,d\cos(dx+c)}$$

input
```
integrate(cos(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/32*(8*a^3*cos(d*x + c)^5 + 32*a^2*b*cos(d*x + c)^4 - 96*a*b^2*cos(d*x +
c)*log(-cos(d*x + c)) - 16*(a^3 - 3*a*b^2)*cos(d*x + c)^3 + 32*b^3 - 32*(3
*a^2*b - b^3)*cos(d*x + c)^2 + (5*a^3 - 24*a*b^2)*cos(d*x + c))/(d*cos(d*x
 + c))
```

3.245.6 Sympy [F]

$$\int \cos(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx = \int (a\sin(c+dx)+b\tan(c+dx))^3\cos(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))**3,x)
```

output
```
Integral((a*sin(c + d*x) + b*tan(c + d*x))**3*cos(c + d*x), x)
```

3.245.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.78

$$\int \cos(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx = \frac{a^3\sin(dx+c)^4+4\left(\cos(dx+c)^3-3\cos(dx+c)\right)a^2b-6\left(\sin(dx+c)^2+\log\left(\sin(dx+c)^2-1\right)\right)ab^2+}{4\,d}$$

input
```
integrate(cos(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/4*(a^3*sin(d*x + c)^4 + 4*(cos(d*x + c)^3 - 3*cos(d*x + c))*a^2*b - 6*(s
in(d*x + c)^2 + log(sin(d*x + c)^2 - 1))*a*b^2 + 4*b^3*(1/cos(d*x + c) + c
os(d*x + c)))/d
```

3.245.8 Giac [F(-2)]

Exception generated.

$$\int \cos(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Modgcd: no suitable evaluation poin
tindex.cc index_m operator + Error: Bad Argument ValueDone
```

3.245.9 Mupad [B] (verification not implemented)

Time = 25.90 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.01

$$\int \cos(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx$$
$$= \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 (4\,a^3+4\,a^2\,b-6\,a\,b^2+12\,b^3)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 (12\,a^2\,b+6\,a\,b^2-12\,b^3)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 (-4}{d\left(-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+2\tan\left(\frac{c}{2}+\right.\right.}$$
$$+\frac{6\,a\,b^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}{d}$$

input
```
int(cos(c + d*x)*(a*sin(c + d*x) + b*tan(c + d*x))^3,x)
```

output
```
(tan(c/2 + (d*x)/2)^4*(4*a^2*b - 6*a*b^2 + 4*a^3 + 12*b^3) - tan(c/2 + (d*
x)/2)^2*(6*a*b^2 + 12*a^2*b - 12*b^3) + tan(c/2 + (d*x)/2)^6*(6*a*b^2 + 12
*a^2*b - 4*a^3 + 4*b^3) - 4*a^2*b + 4*b^3 + 6*a*b^2*tan(c/2 + (d*x)/2)^8)/
(d*(3*tan(c/2 + (d*x)/2)^2 + 2*tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)
^6 - 3*tan(c/2 + (d*x)/2)^8 - tan(c/2 + (d*x)/2)^10 + 1)) + (6*a*b^2*atanh
(tan(c/2 + (d*x)/2)^2))/d
```

3.246 $\int (a\sin(c+dx)+b\tan(c+dx))^3\,dx$

3.246.1 Optimal result

Integrand size = 19, antiderivative size = 116

$$\int (a\sin(c+dx)+b\tan(c+dx))^3\,dx = -\frac{a(a^2-3b^2)\cos(c+dx)}{d}+\frac{3a^2b\cos^2(c+dx)}{2d}+\frac{a^3\cos^3(c+dx)}{3d}-\frac{b(3a^2-b^2)\log(\cos(c+dx))}{d}+\frac{3ab^2\sec(c+dx)}{d}+\frac{b^3\sec^2(c+dx)}{2d}$$

output
```
-a*(a^2-3*b^2)*cos(d*x+c)/d+3/2*a^2*b*cos(d*x+c)^2/d+1/3*a^3*cos(d*x+c)^3/
d-b*(3*a^2-b^2)*ln(cos(d*x+c))/d+3*a*b^2*sec(d*x+c)/d+1/2*b^3*sec(d*x+c)^2
/d
```

3.246.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.88

$$\int (a\sin(c+dx)+b\tan(c+dx))^3\,dx = \frac{-9a(a^2-4b^2)\cos(c+dx)+9a^2b\cos(2(c+dx))+a^3\cos(3(c+dx))-36a^2b\log(\cos(c+dx))+12b^3\log(}{12d}$$

input
```
Integrate[(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

output
```
(-9*a*(a^2 - 4*b^2)*Cos[c + d*x] + 9*a^2*b*Cos[2*(c + d*x)] + a^3*Cos[3*(c
 + d*x)] - 36*a^2*b*Log[Cos[c + d*x]] + 12*b^3*Log[Cos[c + d*x]] + 36*a*b^
2*Sec[c + d*x] + 6*b^3*Sec[c + d*x]^2)/(12*d)
```

3.246.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.89, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3042, 4897, 3042, 25, 3200, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(c+dx)+b\tan(c+dx))^3\,dx$$

$$\Big\downarrow \ 3042$$

$$\int (a\sin(c+dx)+b\tan(c+dx))^3dx$$

$$\Big\downarrow \ 4897$$

$$\int \tan^3(c+dx)(a\cos(c+dx)+b)^3dx$$

$$\Big\downarrow \ 3042$$

$$\int -\frac{\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}{\tan\left(c+dx-\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \ 25$$

$$-\int \frac{\left(b-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)^3}{\tan\left(\frac{1}{2}(2c-\pi)+dx\right)^3}dx$$

$$\Big\downarrow \ 3200$$

$$-\frac{\int \frac{(b+a\cos(c+dx))^3(a^2-a^2\cos^2(c+dx))\sec^3(c+dx)}{a^3}d(a\cos(c+dx))}{d}$$

$$\Big\downarrow \ 522$$

$$-\frac{\int \left(\frac{b^3\sec^3(c+dx)}{a}+3b^2\sec^2(c+dx)+\frac{(3a^2b-b^3)\sec(c+dx)}{a}-a^2\cos^2(c+dx)+a^2\left(1-\frac{3b^2}{a^2}\right)-3ab\cos(c+dx)\right)d(a\cos(c+dx))}{d}$$

$$\Big\downarrow \ 2009$$

$$-\frac{-\frac{1}{3}a^3\cos^3(c+dx)+a(a^2-3b^2)\cos(c+dx)+b(3a^2-b^2)\log(a\cos(c+dx))-\frac{3}{2}a^2b\cos^2(c+dx)-3ab^2\sec(c+}{d}$$

input
```
Int[(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

output
```
-((a*(a^2 - 3*b^2)*Cos[c + d*x] - (3*a^2*b*Cos[c + d*x]^2)/2 - (a^3*Cos[c
+ d*x]^3)/3 + b*(3*a^2 - b^2)*Log[a*Cos[c + d*x]] - 3*a*b^2*Sec[c + d*x] -
 (b^3*Sec[c + d*x]^2)/2)/d)
```

3.246.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.246.4 Maple [A] (verified)

Time = 5.01 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{-\frac{a^3\left(2+\sin(dx+c)^2\right)\cos(dx+c)}{3}+3a^2b\left(-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)+3a\,b^2\left(\frac{\sin(dx+c)^4}{\cos(dx+c)}+\left(2+\sin(dx+c)^2\right)\cos(dx+c)\right)+}{d}$
default	$\frac{-\frac{a^3\left(2+\sin(dx+c)^2\right)\cos(dx+c)}{3}+3a^2b\left(-\frac{\sin(dx+c)^2}{2}-\ln(\cos(dx+c))\right)+3a\,b^2\left(\frac{\sin(dx+c)^4}{\cos(dx+c)}+\left(2+\sin(dx+c)^2\right)\cos(dx+c)\right)+}{d}$
parts	$-\frac{a^3\left(2+\sin(dx+c)^2\right)\cos(dx+c)}{3d}+\frac{b^3\left(\frac{\tan(dx+c)^2}{2}-\frac{\ln\left(1+\tan(dx+c)^2\right)}{2}\right)}{d}+\frac{3a\,b^2\left(\frac{\sin(dx+c)^4}{\cos(dx+c)}+\left(2+\sin(dx+c)^2\right)\cos(\right.}{d}$
risch	$-ix\,b^3-\frac{2ib^3c}{d}+\frac{a^3e^{3i(dx+c)}}{24d}+\frac{3b\,e^{2i(dx+c)}a^2}{8d}-\frac{3a^3e^{i(dx+c)}}{8d}+\frac{3a\,e^{i(dx+c)}b^2}{2d}-\frac{3a^3e^{-i(dx+c)}}{8d}+\frac{3a\,e^{-i(d}}{2d}$

input
```
int((sin(d*x+c)*a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/3*a^3*(2+sin(d*x+c)^2)*cos(d*x+c)+3*a^2*b*(-1/2*sin(d*x+c)^2-ln(co
s(d*x+c)))+3*a*b^2*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+b
^3*(1/2*tan(d*x+c)^2+ln(cos(d*x+c))))
```

3.246.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.06

$$\int (a\sin(c+dx)+b\tan(c+dx))^3\,dx$$
$$=\frac{4\,a^3\cos(dx+c)^5+18\,a^2b\cos(dx+c)^4-9\,a^2b\cos(dx+c)^2+36\,ab^2\cos(dx+c)-12\,(a^3-3\,ab^2)\cos(d}{12\,d\cos(dx+c)^2}$$

input
```
integrate((a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/12*(4*a^3*cos(d*x + c)^5 + 18*a^2*b*cos(d*x + c)^4 - 9*a^2*b*cos(d*x + c
)^2 + 36*a*b^2*cos(d*x + c) - 12*(a^3 - 3*a*b^2)*cos(d*x + c)^3 - 12*(3*a^
2*b - b^3)*cos(d*x + c)^2*log(-cos(d*x + c)) + 6*b^3)/(d*cos(d*x + c)^2)
```

3.246.6 Sympy [F]

$$\int (a\sin(c+dx)+b\tan(c+dx))^3\,dx = \int (a\sin(c+dx)+b\tan(c+dx))^3\,dx$$

input
```
integrate((a*sin(d*x+c)+b*tan(d*x+c))**3,x)
```

output
```
Integral((a*sin(c + d*x) + b*tan(c + d*x))**3, x)
```

3.246.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.97

$$\int (a\sin(c+dx)+b\tan(c+dx))^3\,dx = \frac{(\cos(dx+c)^3 - 3\cos(dx+c))a^3}{3d} - \frac{3(\sin(dx+c)^2 + \log(\sin(dx+c)^2 - 1))a^2b}{2d} - \frac{b^3\left(\frac{1}{\sin(dx+c)^2-1} - \log(\sin(dx+c)^2 - 1)\right)}{2d} + \frac{3ab^2\left(\frac{1}{\cos(dx+c)} + \cos(dx+c)\right)}{d}$$

input
```
integrate((a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/3*(cos(d*x + c)^3 - 3*cos(d*x + c))*a^3/d - 3/2*(sin(d*x + c)^2 + log(si
n(d*x + c)^2 - 1))*a^2*b/d - 1/2*b^3*(1/(sin(d*x + c)^2 - 1) - log(sin(d*x
 + c)^2 - 1))/d + 3*a*b^2*(1/cos(d*x + c) + cos(d*x + c))/d
```

3.246.8 Giac [F(-2)]

Exception generated.

$$\int (a\sin(c+dx)+b\tan(c+dx))^3\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Modgcd: no suitable evaluation poin
tindex.cc index_m operator + Error: Bad Argument ValueDone
```

3.246.9 Mupad [B] (verification not implemented)

Time = 26.13 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.89

$$\int (a\sin(c+dx)+b\tan(c+dx))^3\,dx$$
$$= \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(-\frac{4a^3}{3}-6a^2b+12ab^2+2b^3\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6(4a^3-6a^2b+12ab^2-6b^3)+\tan\left(\frac{c}{2}+\frac{d}{2}\right.}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)^2\left(\tan\left(\frac{c}{2}+\frac{}{}\right.\right.}$$
$$-\frac{2b^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)-6a^2b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}{d}$$

input
```
int((a*sin(c + d*x) + b*tan(c + d*x))^3,x)
```

output
```
(tan(c/2 + (d*x)/2)^2*(12*a*b^2 - 6*a^2*b - (4*a^3)/3 + 2*b^3) - tan(c/2 +
 (d*x)/2)^6*(12*a*b^2 - 6*a^2*b + 4*a^3 - 6*b^3) + tan(c/2 + (d*x)/2)^4*(6
*a^2*b - 12*a*b^2 + (20*a^3)/3 + 6*b^3) + 12*a*b^2 - tan(c/2 + (d*x)/2)^8*
(6*a^2*b - 2*b^3) - (4*a^3)/3)/(d*(tan(c/2 + (d*x)/2)^2 - 1)^2*(tan(c/2 +
(d*x)/2)^2 + 1)^3) - (2*b^3*atanh(tan(c/2 + (d*x)/2)^2) - 6*a^2*b*atanh(ta
n(c/2 + (d*x)/2)^2))/d
```

3.247 $\int \sec(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx$

3.247.1 Optimal result

Integrand size = 26, antiderivative size = 115

$$\begin{aligned}\int \sec(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx &= \frac{3a^2b\cos(c+dx)}{d}+\frac{a^3\cos^2(c+dx)}{2d}-\frac{a(a^2-3b^2)\log(\cos(c+dx))}{d}\\&\quad+\frac{b(3a^2-b^2)\sec(c+dx)}{d}+\frac{3ab^2\sec^2(c+dx)}{2d}+\frac{b^3\sec^3(c+dx)}{3d}\end{aligned}$$

output
```
3*a^2*b*cos(d*x+c)/d+1/2*a^3*cos(d*x+c)^2/d-a*(a^2-3*b^2)*ln(cos(d*x+c))/d
+b*(3*a^2-b^2)*sec(d*x+c)/d+3/2*a*b^2*sec(d*x+c)^2/d+1/3*b^3*sec(d*x+c)^3/
d
```

3.247.2 Mathematica [A] (verified)

Time = 1.40 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.87

$$\int \sec(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx = \frac{36a^2b\cos(c+dx)+3a^3\cos(2(c+dx))+2(-6a(a^2-3b^2)\log(\cos(c+dx))-6b(-3a^2+b^2)\sec(c+dx)-}{12d}$$

input
```
Integrate[Sec[c + d*x]*(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

output
```
(36*a^2*b*Cos[c + d*x] + 3*a^3*Cos[2*(c + d*x)] + 2*(-6*a*(a^2 - 3*b^2)*Lo
g[Cos[c + d*x]] - 6*b*(-3*a^2 + b^2)*Sec[c + d*x] + 9*a*b^2*Sec[c + d*x]^2
 + 2*b^3*Sec[c + d*x]^3))/(12*d)
```

3.247.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.93, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 4897, 3042, 25, 3316, 27, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sec(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3dx$$

$$\Big\downarrow \ 4897$$

$$\int \tan^3(c+dx)\sec(c+dx)(a\cos(c+dx)+b)^3dx$$

$$\Big\downarrow \ 3042$$

$$\int -\frac{\cos\left(c+dx+\frac{\pi}{2}\right)^3\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+b\right)^3}{\sin\left(c+dx+\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \ 25$$

$$-\int \frac{\cos\left(\frac{1}{2}(2c+\pi)+dx\right)^3\left(b+a\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^3}{\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^4}dx$$

$$\Big\downarrow \ 3316$$

$$-\frac{\int (b+a\cos(c+dx))^3\left(a^2-a^2\cos^2(c+dx)\right)\sec^4(c+dx)d(a\cos(c+dx))}{a^3d}$$

$$\Big\downarrow \ 27$$

$$-\frac{a\int \frac{(b+a\cos(c+dx))^3\left(a^2-a^2\cos^2(c+dx)\right)\sec^4(c+dx)}{a^4}d(a\cos(c+dx))}{d}$$

$$\Big\downarrow \ 522$$

$$-\frac{a\int\left(\frac{b^3\sec^4(c+dx)}{a^2}+\frac{3b^2\sec^3(c+dx)}{a}+\frac{(3a^2b-b^3)\sec^2(c+dx)}{a^2}+\frac{(a^2-3b^2)\sec(c+dx)}{a}-3b-a\cos(c+dx)\right)d(a\cos(c+dx))}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a\left(-\frac{b(3a^2-b^2)\sec(c+dx)}{a}+(a^2-3b^2)\log(a\cos(c+dx))-\frac{1}{2}a^2\cos^2(c+dx)-\frac{b^3\sec^3(c+dx)}{3a}-3ab\cos(c+dx)-\frac{3}{2}b^2\right.}{d}$$

input
```
Int[Sec[c + d*x]*(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

output
```
-((a*(-3*a*b*Cos[c + d*x] - (a^2*Cos[c + d*x]^2)/2 + (a^2 - 3*b^2)*Log[a*C
os[c + d*x]] - (b*(3*a^2 - b^2)*Sec[c + d*x])/a - (3*b^2*Sec[c + d*x]^2)/2
 - (b^3*Sec[c + d*x]^3)/(3*a)))/d)
```

3.247.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.247.4 Maple [A] (verified)

Time = 8.56 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{\frac{b^3\sec(dx+c)^3}{3}+\frac{3a\,b^2\sec(dx+c)^2}{2}+3a^2b\sec(dx+c)-\sec(dx+c)b^3+\frac{3a^2b}{\sec(dx+c)}+\frac{a^3}{2\sec(dx+c)^2}+a(a^2-3b^2)\ln(\sec(dx+c))}{d}$
default	$\frac{\frac{b^3\sec(dx+c)^3}{3}+\frac{3a\,b^2\sec(dx+c)^2}{2}+3a^2b\sec(dx+c)-\sec(dx+c)b^3+\frac{3a^2b}{\sec(dx+c)}+\frac{a^3}{2\sec(dx+c)^2}+a(a^2-3b^2)\ln(\sec(dx+c))}{d}$
risch	$ia^3x-3ixa\,b^2+\frac{a^3e^{2i(dx+c)}}{8d}+\frac{3b\,e^{i(dx+c)}a^2}{2d}+\frac{3b\,e^{-i(dx+c)}a^2}{2d}+\frac{a^3e^{-2i(dx+c)}}{8d}+\frac{2ia^3c}{d}-\frac{6ia\,b^2c}{d}+\frac{2b}{}$

input
```
int(sec(d*x+c)*(sin(d*x+c)*a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/3*b^3*sec(d*x+c)^3+3/2*a*b^2*sec(d*x+c)^2+3*a^2*b*sec(d*x+c)-sec(d*
x+c)*b^3+3*a^2*b/sec(d*x+c)+1/2*a^3/sec(d*x+c)^2+a*(a^2-3*b^2)*ln(sec(d*x+
c)))
```

3.247.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.06

$$\int \sec(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx$$
$$=\frac{6\,a^3\cos{(dx+c)}^5+36\,a^2b\cos{(dx+c)}^4-3\,a^3\cos{(dx+c)}^3-12\,(a^3-3\,ab^2)\cos{(dx+c)}^3\log{(-\cos{(dx}}{12\,d\cos{(dx+c)}^3}$$

input
```
integrate(sec(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/12*(6*a^3*cos(d*x + c)^5 + 36*a^2*b*cos(d*x + c)^4 - 3*a^3*cos(d*x + c)^
3 - 12*(a^3 - 3*a*b^2)*cos(d*x + c)^3*log(-cos(d*x + c)) + 18*a*b^2*cos(d*
x + c) + 4*b^3 + 12*(3*a^2*b - b^3)*cos(d*x + c)^2)/(d*cos(d*x + c)^3)
```

3.247.6 Sympy [F]

$$\int \sec(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx$$
$$=\int (a\sin(c+dx)+b\tan(c+dx))^3\sec(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))**3,x)
```

output
```
Integral((a*sin(c + d*x) + b*tan(c + d*x))**3*sec(c + d*x), x)
```

3.247.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.95

$$\int \sec(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx =$$
$$-\frac{3\left(\sin(dx+c)^2+\log\left(\sin(dx+c)^2-1\right)\right)a^3+9\,ab^2\left(\frac{1}{\sin(dx+c)^2-1}-\log\left(\sin(dx+c)^2-1\right)\right)-18\,a^2b\left(\frac{}{\mathrm{co}}\right.}{6\,d}$$

input
```
integrate(sec(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/6*(3*(sin(d*x + c)^2 + log(sin(d*x + c)^2 - 1))*a^3 + 9*a*b^2*(1/(sin(d
*x + c)^2 - 1) - log(sin(d*x + c)^2 - 1)) - 18*a^2*b*(1/cos(d*x + c) + cos
(d*x + c)) + 2*(3*cos(d*x + c)^2 - 1)*b^3/cos(d*x + c)^3)/d
```

3.247.8 Giac [F(-2)]

Exception generated.

$$\int \sec(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Modgcd: no suitable evaluation poin
tindex.cc index_m operator + Error: Bad Argument ValueDone
```

3.247.9 Mupad [B] (verification not implemented)

Time = 26.18 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.90

$$\int \sec(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx$$
$$= \frac{2a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)-6ab^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}{d} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(-2a^3-12a^2b+6ab^2+\frac{4b^3}{3}\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(6a^3-12a^2b+6ab^2-4b^3\right)+\tan\left(\frac{c}{2}+\cdots\right.}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)^3\left(\tan\left(\frac{c}{2}\cdots\right.\right.}$$

input
```
int((a*sin(c + d*x) + b*tan(c + d*x))^3/cos(c + d*x),x)
```

output
```
(2*a^3*atanh(tan(c/2 + (d*x)/2)^2) - 6*a*b^2*atanh(tan(c/2 + (d*x)/2)^2))/
d - (tan(c/2 + (d*x)/2)^2*(6*a*b^2 - 12*a^2*b - 2*a^3 + (4*b^3)/3) - tan(c
/2 + (d*x)/2)^6*(6*a*b^2 - 12*a^2*b + 6*a^3 - 4*b^3) + tan(c/2 + (d*x)/2)^
4*(6*a*b^2 - 12*a^2*b + 6*a^3 + (20*b^3)/3) + 12*a^2*b - tan(c/2 + (d*x)/2
)^8*(6*a*b^2 - 2*a^3) - (4*b^3)/3)/(d*(tan(c/2 + (d*x)/2)^2 - 1)^3*(tan(c/
2 + (d*x)/2)^2 + 1)^2)
```

3.248 $\int \sec^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx$

3.248.1 Optimal result

Integrand size = 28, antiderivative size = 111

$$\int \sec^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx = \frac{a^3\cos(c+dx)}{d}+\frac{3a^2b\log(\cos(c+dx))}{d}+\frac{a(a^2-3b^2)\sec(c+dx)}{d}+\frac{b(3a^2-b^2)\sec^2(c+dx)}{2d}+\frac{ab^2\sec^3(c+dx)}{d}+\frac{b^3\sec^4(c+dx)}{4d}$$

output
```
a^3*cos(d*x+c)/d+3*a^2*b*ln(cos(d*x+c))/d+a*(a^2-3*b^2)*sec(d*x+c)/d+1/2*b
*(3*a^2-b^2)*sec(d*x+c)^2/d+a*b^2*sec(d*x+c)^3/d+1/4*b^3*sec(d*x+c)^4/d
```

3.248.2 Mathematica [A] (verified)

Time = 2.06 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.87

$$\int \sec^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx = \frac{4a^3\cos(c+dx)+12a^2b\log(\cos(c+dx))+4a(a^2-3b^2)\sec(c+dx)+(6a^2b-2b^3)\sec^2(c+dx)+4ab^2\mathrm{se}}{4d}$$

input
```
Integrate[Sec[c + d*x]^2*(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

output
```
(4*a^3*Cos[c + d*x] + 12*a^2*b*Log[Cos[c + d*x]] + 4*a*(a^2 - 3*b^2)*Sec[c
 + d*x] + (6*a^2*b - 2*b^3)*Sec[c + d*x]^2 + 4*a*b^2*Sec[c + d*x]^3 + b^3*
Sec[c + d*x]^4)/(4*d)
```

3.248.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 4897, 3042, 25, 3316, 27, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^2(a\sin(c+dx)+b\tan(c+dx))^3dx$$

$$\Big\downarrow \text{4897}$$

$$\int \tan^3(c+dx)\sec^2(c+dx)(a\cos(c+dx)+b)^3dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\cos\left(c+dx+\frac{\pi}{2}\right)^3\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+b\right)^3}{\sin\left(c+dx+\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\cos\left(\frac{1}{2}(2c+\pi)+dx\right)^3\left(b+a\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^3}{\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^5}dx$$

$$\Big\downarrow \text{3316}$$

$$-\frac{\int (b+a\cos(c+dx))^3\left(a^2-a^2\cos^2(c+dx)\right)\sec^5(c+dx)d(a\cos(c+dx))}{a^3d}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^2\int \frac{(b+a\cos(c+dx))^3(a^2-a^2\cos^2(c+dx))\sec^5(c+dx)}{a^5}d(a\cos(c+dx))}{d}$$

$$\Big\downarrow \text{522}$$

$$-\frac{a^2\int \left(\frac{b^3\sec^5(c+dx)}{a^3}+\frac{3b^2\sec^4(c+dx)}{a^2}+\frac{(3a^2b-b^3)\sec^3(c+dx)}{a^3}+\frac{(a^2-3b^2)\sec^2(c+dx)}{a^2}-\frac{3b\sec(c+dx)}{a}-1\right)d(a\cos(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2\left(-\frac{b^3\sec^4(c+dx)}{4a^2}-\frac{b(3a^2-b^2)\sec^2(c+dx)}{2a^2}-\frac{(a^2-3b^2)\sec(c+dx)}{a}-\frac{b^2\sec^3(c+dx)}{a}-3b\log(a\cos(c+dx))-a\cos(c+dx)\right)}{d}$$

input
```
Int[Sec[c + d*x]^2*(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

output
```
-((a^2*(-(a*Cos[c + d*x]) - 3*b*Log[a*Cos[c + d*x]] - ((a^2 - 3*b^2)*Sec[c
 + d*x])/a - (b*(3*a^2 - b^2)*Sec[c + d*x]^2)/(2*a^2) - (b^2*Sec[c + d*x]^
3)/a - (b^3*Sec[c + d*x]^4)/(4*a^2)))/d)
```

3.248.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.248.4 Maple [A] (verified)

Time = 8.52 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{\frac{b^3\sec(dx+c)^4}{4}+a\,b^2\sec(dx+c)^3+\frac{3a^2b\sec(dx+c)^2}{2}-\frac{b^3\sec(dx+c)^2}{2}+\sec(dx+c)a^3-3\sec(dx+c)a\,b^2+\frac{a^3}{\sec(dx+c)}-3a^2b\ln(\sec}{d}$
default	$\frac{\frac{b^3\sec(dx+c)^4}{4}+a\,b^2\sec(dx+c)^3+\frac{3a^2b\sec(dx+c)^2}{2}-\frac{b^3\sec(dx+c)^2}{2}+\sec(dx+c)a^3-3\sec(dx+c)a\,b^2+\frac{a^3}{\sec(dx+c)}-3a^2b\ln(\sec}{d}$
risch	$-3ia^2bx+\frac{a^3e^{i(dx+c)}}{2d}+\frac{a^3e^{-i(dx+c)}}{2d}-\frac{6ib\,a^2c}{d}+\frac{2a^3e^{7i(dx+c)}-6a\,b^2e^{7i(dx+c)}+6a^2b\,e^{6i(dx+c)}-2b^3e^{6i(dx+c)}}{}$

input
```
int(sec(d*x+c)^2*(sin(d*x+c)*a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/4*b^3*sec(d*x+c)^4+a*b^2*sec(d*x+c)^3+3/2*a^2*b*sec(d*x+c)^2-1/2*b^
3*sec(d*x+c)^2+sec(d*x+c)*a^3-3*sec(d*x+c)*a*b^2+a^3/sec(d*x+c)-3*a^2*b*ln
(sec(d*x+c)))
```

3.248.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.96

$$\int \sec^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx$$
$$=\frac{4\,a^3\cos\left(dx+c\right)^5+12\,a^2b\cos\left(dx+c\right)^4\log\left(-\cos\left(dx+c\right)\right)+4\,ab^2\cos\left(dx+c\right)+4\left(a^3-3\,ab^2\right)\cos\left(dx+\right.}{4\,d\cos\left(dx+c\right)^4}$$

input
```
integrate(sec(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas"
)
```

output
```
1/4*(4*a^3*cos(d*x + c)^5 + 12*a^2*b*cos(d*x + c)^4*log(-cos(d*x + c)) + 4
*a*b^2*cos(d*x + c) + 4*(a^3 - 3*a*b^2)*cos(d*x + c)^3 + b^3 + 2*(3*a^2*b
- b^3)*cos(d*x + c)^2)/(d*cos(d*x + c)^4)
```

3.248.6 Sympy [F]

$$\int \sec^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx = \int (a\sin(c+dx)+b\tan(c+dx))^3\sec^2(c+dx)\,dx$$

input
```
integrate(sec(d*x+c)**2*(a*sin(d*x+c)+b*tan(d*x+c))**3,x)
```

output
```
Integral((a*sin(c + d*x) + b*tan(c + d*x))**3*sec(c + d*x)**2, x)
```

3.248.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.86

$$\int \sec^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx = \frac{b^3\tan(dx+c)^4 - 6a^2b\left(\frac{1}{\sin(dx+c)^2-1} - \log\left(\sin(dx+c)^2-1\right)\right) + 4a^3\left(\frac{1}{\cos(dx+c)} + \cos(dx+c)\right) - \frac{4\left(3\cos(d\cdots\right.}{\cos(\cdots}}{4d}$$

input
```
integrate(sec(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima"
)
```

output
```
1/4*(b^3*tan(d*x + c)^4 - 6*a^2*b*(1/(sin(d*x + c)^2 - 1) - log(sin(d*x +
c)^2 - 1)) + 4*a^3*(1/cos(d*x + c) + cos(d*x + c)) - 4*(3*cos(d*x + c)^2 -
 1)*a*b^2/cos(d*x + c)^3)/d
```

3.248.8 Giac [F(-2)]

Exception generated.

$$\int \sec^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Modgcd: no suitable evaluation poin
tindex.cc index_m operator + Error: Bad Argument ValueDone
```

3.248.9 Mupad [B] (verification not implemented)

Time = 26.51 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.01

$$\int \sec^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx$$
$$= \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(-12\,a^3+6\,a^2\,b+12\,a\,b^2\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(12\,a^3-6\,a^2\,b+4\,a\,b^2+4\,b^3\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6(4}{d\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+2\tan\left(\frac{c}{2}+\right.\right.}$$
$$-\frac{6\,a^2\,b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}{d}$$

input
```
int((a*sin(c + d*x) + b*tan(c + d*x))^3/cos(c + d*x)^2,x)
```

output
```
(tan(c/2 + (d*x)/2)^2*(12*a*b^2 + 6*a^2*b - 12*a^3) + tan(c/2 + (d*x)/2)^4
*(4*a*b^2 - 6*a^2*b + 12*a^3 + 4*b^3) - tan(c/2 + (d*x)/2)^6*(12*a*b^2 + 6
*a^2*b + 4*a^3 - 4*b^3) - 4*a*b^2 + 4*a^3 + 6*a^2*b*tan(c/2 + (d*x)/2)^8)/
(d*(2*tan(c/2 + (d*x)/2)^4 - 3*tan(c/2 + (d*x)/2)^2 + 2*tan(c/2 + (d*x)/2)
^6 - 3*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 + 1)) - (6*a^2*b*atanh
(tan(c/2 + (d*x)/2)^2))/d
```

3.249 $\int \sec^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx$

3.249.1 Optimal result

Integrand size = 28, antiderivative size = 119

$$\int \sec^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx = \frac{a^3\log(\cos(c+dx))}{d}-\frac{3a^2b\sec(c+dx)}{d}+\frac{a(a^2-3b^2)\sec^2(c+dx)}{2d}+\frac{b(3a^2-b^2)\sec^3(c+dx)}{3d}+\frac{3ab^2\sec^4(c+dx)}{4d}+\frac{b^3\sec^5(c+dx)}{5d}$$

output
```
a^3*ln(cos(d*x+c))/d-3*a^2*b*sec(d*x+c)/d+1/2*a*(a^2-3*b^2)*sec(d*x+c)^2/d
+1/3*b*(3*a^2-b^2)*sec(d*x+c)^3/d+3/4*a*b^2*sec(d*x+c)^4/d+1/5*b^3*sec(d*x
+c)^5/d
```

3.249.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.83

$$\int \sec^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx = \frac{60a^3\log(\cos(c+dx))-180a^2b\sec(c+dx)+30a(a^2-3b^2)\sec^2(c+dx)-20b(-3a^2+b^2)\sec^3(c+dx)+}{60d}$$

input
```
Integrate[Sec[c + d*x]^3*(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

output
```
(60*a^3*Log[Cos[c + d*x]] - 180*a^2*b*Sec[c + d*x] + 30*a*(a^2 - 3*b^2)*Se
c[c + d*x]^2 - 20*b*(-3*a^2 + b^2)*Sec[c + d*x]^3 + 45*a*b^2*Sec[c + d*x]^
4 + 12*b^3*Sec[c + d*x]^5)/(60*d)
```

3.249.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4897, 3042, 25, 3316, 27, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(c+dx)^3(a\sin(c+dx)+b\tan(c+dx))^3dx$$

$$\Big\downarrow \text{4897}$$

$$\int \tan^3(c+dx)\sec^3(c+dx)(a\cos(c+dx)+b)^3dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\cos\left(c+dx+\frac{\pi}{2}\right)^3\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+b\right)^3}{\sin\left(c+dx+\frac{\pi}{2}\right)^6}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\cos\left(\frac{1}{2}(2c+\pi)+dx\right)^3\left(b+a\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^3}{\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^6}dx$$

$$\Big\downarrow \text{3316}$$

$$-\frac{\int (b+a\cos(c+dx))^3\left(a^2-a^2\cos^2(c+dx)\right)\sec^6(c+dx)d(a\cos(c+dx))}{a^3d}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^3\int \frac{(b+a\cos(c+dx))^3\left(a^2-a^2\cos^2(c+dx)\right)\sec^6(c+dx)}{a^6}d(a\cos(c+dx))}{d}$$

$$\Big\downarrow \text{522}$$

$$-\frac{a^3 \int \left(\frac{b^3 \sec^6(c+dx)}{a^4}+\frac{3b^2 \sec^5(c+dx)}{a^3}+\frac{(3a^2b-b^3)\sec^4(c+dx)}{a^4}+\frac{(a^2-3b^2)\sec^3(c+dx)}{a^3}-\frac{3b\sec^2(c+dx)}{a^2}-\frac{\sec(c+dx)}{a}\right)d(a\cos(c+}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3\left(-\frac{b^3\sec^5(c+dx)}{5a^3}-\frac{3b^2\sec^4(c+dx)}{4a^2}-\frac{(a^2-3b^2)\sec^2(c+dx)}{2a^2}-\frac{b(3a^2-b^2)\sec^3(c+dx)}{3a^3}+\frac{3b\sec(c+dx)}{a}-\log(a\cos(c+dx))\right)}{d}$$

input
```
Int[Sec[c + d*x]^3*(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

output
```
-((a^3*(-Log[a*Cos[c + d*x]] + (3*b*Sec[c + d*x])/a - ((a^2 - 3*b^2)*Sec[c
 + d*x]^2)/(2*a^2) - (b*(3*a^2 - b^2)*Sec[c + d*x]^3)/(3*a^3) - (3*b^2*Sec
[c + d*x]^4)/(4*a^2) - (b^3*Sec[c + d*x]^5)/(5*a^3)))/d)
```

3.249.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.249.4 Maple [A] (verified)

Time = 3.87 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{\frac{b^3\sec(dx+c)^5}{5}+\frac{3a\,b^2\sec(dx+c)^4}{4}+a^2b\sec(dx+c)^3-\frac{b^3\sec(dx+c)^3}{3}+\frac{a^3\sec(dx+c)^2}{2}-\frac{3a\,b^2\sec(dx+c)^2}{2}-3a^2b\sec(dx+c)-a^3\ln}{d}$
default	$\frac{\frac{b^3\sec(dx+c)^5}{5}+\frac{3a\,b^2\sec(dx+c)^4}{4}+a^2b\sec(dx+c)^3-\frac{b^3\sec(dx+c)^3}{3}+\frac{a^3\sec(dx+c)^2}{2}-\frac{3a\,b^2\sec(dx+c)^2}{2}-3a^2b\sec(dx+c)-a^3\ln}{d}$
risch	$-ia^3x-\frac{2ia^3c}{d}+\frac{-6a^2b\,e^{9i(dx+c)}+2a^3e^{8i(dx+c)}-6a\,b^2e^{8i(dx+c)}-16a^2b\,e^{7i(dx+c)}-\frac{8b^3e^{7i(dx+c)}}{3}+6a^3e^{6i(dx+c)}-6}{}$

input
```
int(sec(d*x+c)^3*(sin(d*x+c)*a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/5*b^3*sec(d*x+c)^5+3/4*a*b^2*sec(d*x+c)^4+a^2*b*sec(d*x+c)^3-1/3*b^
3*sec(d*x+c)^3+1/2*a^3*sec(d*x+c)^2-3/2*a*b^2*sec(d*x+c)^2-3*a^2*b*sec(d*x
+c)-a^3*ln(sec(d*x+c)))
```

3.249.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.92

$$\int \sec^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx$$
$$=\frac{60\,a^3\cos(dx+c)^5\log(-\cos(dx+c))-180\,a^2b\cos(dx+c)^4+45\,ab^2\cos(dx+c)+30\,(a^3-3\,ab^2)\cos(}{60\,d\cos(dx+c)^5}$$

input
```
integrate(sec(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas"
)
```

output
```
1/60*(60*a^3*cos(d*x + c)^5*log(-cos(d*x + c)) - 180*a^2*b*cos(d*x + c)^4
+ 45*a*b^2*cos(d*x + c) + 30*(a^3 - 3*a*b^2)*cos(d*x + c)^3 + 12*b^3 + 20*
(3*a^2*b - b^3)*cos(d*x + c)^2)/(d*cos(d*x + c)^5)
```

3.249.6 Sympy [F]

$$\int \sec^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx$$
$$=\int (a\sin{(c+dx)}+b\tan{(c+dx)})^3\sec^3{(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**3*(a*sin(d*x+c)+b*tan(d*x+c))**3,x)
```

output
```
Integral((a*sin(c + d*x) + b*tan(c + d*x))**3*sec(c + d*x)**3, x)
```

3.249.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.08

$$\int \sec^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx =$$
$$-\frac{30\,a^3\left(\frac{1}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)^2-1\right)\right)-\frac{45\left(2\sin(dx+c)^2-1\right)ab^2}{\sin(dx+c)^4-2\sin(dx+c)^2+1}+\frac{60\left(3\cos(dx+c)^2-1\right)a^2b}{\cos(dx+c)^3}+\frac{4\left(5\cos(dx\cdots\right.}{\cos(dx\cdots}}{60\,d}$$

input
```
integrate(sec(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima"
)
```

output
```
-1/60*(30*a^3*(1/(sin(d*x + c)^2 - 1) - log(sin(d*x + c)^2 - 1)) - 45*(2*s
in(d*x + c)^2 - 1)*a*b^2/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) + 60*(3*c
os(d*x + c)^2 - 1)*a^2*b/cos(d*x + c)^3 + 4*(5*cos(d*x + c)^2 - 3)*b^3/cos
(d*x + c)^5)/d
```

3.249.8 Giac [F(-2)]

Exception generated.

$$\int \sec^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Modgcd: no suitable evaluation poin
tindex.cc index_m operator + Error: Bad Argument ValueDone
```

3.249.9 Mupad [B] (verification not implemented)

Time = 26.30 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.85

$$\int \sec^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx = -\frac{2\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d}$$
$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\left(6\,a^3+12\,a^2\,b-12\,a\,b^2+4\,b^3\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(-6\,a^3-28\,a^2\,b+12\,a\,b^2+\frac{4\,b^3}{3}\right)-2\,a^3\,\mathrm{t}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\tan\left(\frac{c}{2}\right.\right.}$$

input
```
int((a*sin(c + d*x) + b*tan(c + d*x))^3/cos(c + d*x)^3,x)
```

output
```
- (2*a^3*atanh(tan(c/2 + (d*x)/2)^2))/d - (tan(c/2 + (d*x)/2)^6*(12*a^2*b
- 12*a*b^2 + 6*a^3 + 4*b^3) + tan(c/2 + (d*x)/2)^4*(12*a*b^2 - 28*a^2*b -
6*a^3 + (4*b^3)/3) - 2*a^3*tan(c/2 + (d*x)/2)^8 - 4*a^2*b + tan(c/2 + (d*x
)/2)^2*(20*a^2*b + 2*a^3 + (4*b^3)/3) - (4*b^3)/15)/(d*(5*tan(c/2 + (d*x)/
2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*
x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

3.250 $\int \frac{\cos^3(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx$

3.250.1 Optimal result

Integrand size = 28, antiderivative size = 113

$$\int \frac{\cos^3(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx = -\frac{b\cos(c+dx)}{a^2d}+\frac{\cos^2(c+dx)}{2ad}+\frac{\log(1-\cos(c+dx))}{2(a+b)d}+\frac{\log(1+\cos(c+dx))}{2(a-b)d}-\frac{b^4\log(b+a\cos(c+dx))}{a^3(a^2-b^2)\,d}$$

output
```
-b*cos(d*x+c)/a^2/d+1/2*cos(d*x+c)^2/a/d+1/2*ln(1-cos(d*x+c))/(a+b)/d+1/2*
ln(1+cos(d*x+c))/(a-b)/d-b^4*ln(b+a*cos(d*x+c))/a^3/(a^2-b^2)/d
```

3.250.2 Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.88

$$\int \frac{\cos^3(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx = \frac{-\frac{4b\cos(c+dx)}{a^2}+\frac{\cos(2(c+dx))}{a}+4\left(\frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{a-b}+\frac{b^4\log(b+a\cos(c+dx))}{a^3(-a^2+b^2)}+\frac{\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a+b}\right)}{4d}$$

input
```
Integrate[Cos[c + d*x]^3/(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

output
```
((-4*b*Cos[c + d*x])/a^2 + Cos[2*(c + d*x)]/a + 4*(Log[Cos[(c + d*x)/2]]/(
a - b) + (b^4*Log[b + a*Cos[c + d*x]])/(a^3*(-a^2 + b^2)) + Log[Sin[(c + d
*x)/2]]/(a + b)))/(4*d)
```

3.250.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {3042, 4897, 3042, 3316, 27, 604, 27, 2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(c+dx)^3}{a\sin(c+dx)+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{ 4897}$$

$$\int \frac{\cos^3(c+dx)\cot(c+dx)}{a\cos(c+dx)+b}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin\left(c+dx-\frac{\pi}{2}\right)^4}{\cos\left(c+dx-\frac{\pi}{2}\right)\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{ 3316}$$

$$-\frac{a\int \frac{\cos^4(c+dx)}{(b+a\cos(c+dx))(a^2-a^2\cos^2(c+dx))}d(a\cos(c+dx))}{d}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int \frac{a^4\cos^4(c+dx)}{(b+a\cos(c+dx))(a^2-a^2\cos^2(c+dx))}d(a\cos(c+dx))}{a^3d}$$

$$\Big\downarrow \text{ 604}$$

$$-\frac{-\frac{1}{2}\int -\frac{2\left(-2b\cos^3(c+dx)a^3+2b\cos(c+dx)a^3+b^2a^2+(a^2-b^2)\cos^2(c+dx)a^2\right)}{(b+a\cos(c+dx))(a^2-a^2\cos^2(c+dx))}d(a\cos(c+dx))-\frac{1}{2}(a\cos(c+dx)+b)^2}{a^3d}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int \frac{-2b\cos^3(c+dx)a^3+2b\cos(c+dx)a^3+b^2a^2+(a^2-b^2)\cos^2(c+dx)a^2}{(b+a\cos(c+dx))(a^2-a^2\cos^2(c+dx))}d(a\cos(c+dx))-\frac{1}{2}(a\cos(c+dx)+b)^2}{a^3d}$$

$$\Big\downarrow \text{ 2160}$$

$$-\frac{\int \left(\frac{b^4}{(a-b)(a+b)(b+a\cos(c+dx))}+2b+\frac{a^3}{2(a+b)(a-a\cos(c+dx))}-\frac{a^3}{2(a-b)(\cos(c+dx)a+a)}\right)d(a\cos(c+dx))-\frac{1}{2}(a\cos(c+dx)}{a^3d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{a^3\log(a-a\cos(c+dx))}{2(a+b)}-\frac{a^3\log(a\cos(c+dx)+a)}{2(a-b)}+\frac{b^4\log(a\cos(c+dx)+b)}{a^2-b^2}+2ab\cos(c+dx)-\frac{1}{2}(a\cos(c+dx)+b)^2}{a^3d}$$

input
```
Int[Cos[c + d*x]^3/(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

output
```
-((2*a*b*Cos[c + d*x] - (b + a*Cos[c + d*x])^2/2 - (a^3*Log[a - a*Cos[c +
d*x]])/(2*(a + b)) - (a^3*Log[a + a*Cos[c + d*x]])/(2*(a - b)) + (b^4*Log[
b + a*Cos[c + d*x]])/(a^2 - b^2))/(a^3*d))
```

3.250.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 604
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(c + d*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*d^(m - 1)*(m + n +
2*p + 1))), x] + Simp[1/(b*d^m*(m + n + 2*p + 1))   Int[(c + d*x)^n*(a + b*
x^2)^p*ExpandToSum[b*d^m*(m + n + 2*p + 1)*x^m - b*(m + n + 2*p + 1)*(c + d
*x)^m - (c + d*x)^(m - 2)*(a*d^2*(m + n - 1) - b*c^2*(m + n + 2*p + 1) - 2*
b*c*d*(m + n + p)*x), x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && IGtQ[m,
 1] && NeQ[m + n + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.250.4 Maple [A] (verified)

Time = 2.41 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{\frac{\frac{a\cos(dx+c)^2}{2}-\cos(dx+c)b}{a^2}+\frac{\ln(\cos(dx+c)-1)}{2a+2b}+\frac{\ln(\cos(dx+c)+1)}{2a-2b}-\frac{b^4\ln(b+\cos(dx+c)a)}{a^3(a+b)(a-b)}}{d}$
default	$\frac{\frac{\frac{a\cos(dx+c)^2}{2}-\cos(dx+c)b}{a^2}+\frac{\ln(\cos(dx+c)-1)}{2a+2b}+\frac{\ln(\cos(dx+c)+1)}{2a-2b}-\frac{b^4\ln(b+\cos(dx+c)a)}{a^3(a+b)(a-b)}}{d}$
risch	$\frac{ix}{a}+\frac{ix\,b^2}{a^3}+\frac{e^{2i(dx+c)}}{8ad}-\frac{b\,e^{i(dx+c)}}{2a^2d}-\frac{b\,e^{-i(dx+c)}}{2a^2d}+\frac{e^{-2i(dx+c)}}{8ad}-\frac{ix}{a+b}-\frac{ic}{d(a+b)}-\frac{ix}{a-b}-\frac{ic}{d(a-b)}+$

input
```
int(cos(d*x+c)^3/(sin(d*x+c)*a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/a^2*(1/2*a*cos(d*x+c)^2-cos(d*x+c)*b)+1/(2*a+2*b)*ln(cos(d*x+c)-1)+
1/(2*a-2*b)*ln(cos(d*x+c)+1)-1/a^3*b^4/(a+b)/(a-b)*ln(b+cos(d*x+c)*a))
```

3.250.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.09

$$\int \frac{\cos^3(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx =$$
$$-\frac{2\,b^4\log\left(a\cos\left(dx+c\right)+b\right)-\left(a^4-a^2b^2\right)\cos\left(dx+c\right)^2+2\left(a^3b-ab^3\right)\cos\left(dx+c\right)-\left(a^4+a^3b\right)\log\left(\frac{1}{2}\right.}{2\left(a^5-a^3b^2\right)d}$$

input
```
integrate(cos(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2*(2*b^4*log(a*cos(d*x + c) + b) - (a^4 - a^2*b^2)*cos(d*x + c)^2 + 2*(
a^3*b - a*b^3)*cos(d*x + c) - (a^4 + a^3*b)*log(1/2*cos(d*x + c) + 1/2) -
(a^4 - a^3*b)*log(-1/2*cos(d*x + c) + 1/2))/((a^5 - a^3*b^2)*d)
```

3.250.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3/(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

output
```
Timed out
```

3.250.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.66

$$\int \frac{\cos^3(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx = -\frac{\frac{b^4\log\left(a+b-\frac{(a-b)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^5-a^3b^2}+\frac{2\left(b+\frac{(a+b)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^2+\frac{2a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a+b}+\frac{(a^2+b^2)\log\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)}{a^3}}{d}$$

input
```
integrate(cos(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
-(b^4*log(a + b - (a - b)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/(a^5 - a^3*
b^2) + 2*(b + (a + b)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/(a^2 + 2*a^2*si
n(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^
4) - log(sin(d*x + c)/(cos(d*x + c) + 1))/(a + b) + (a^2 + b^2)*log(sin(d*
x + c)^2/(cos(d*x + c) + 1)^2 + 1)/a^3)/d
```

3.250.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(107) = 214$.

Time = 0.37 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.68

$$\int \frac{\cos^3(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx = -\frac{\frac{2b^4\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^5-a^3b^2}-\frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a+b}+\frac{2(a^2+b^2)\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^3}-\frac{3a^2-4ab+3b^2-2a\cdots}{\cdots}}{2d}$$

input
```
integrate(cos(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
-1/2*(2*b^4*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(
cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^5 - a^3*b^2) - log(abs(-cos(d*x
+ c) + 1)/abs(cos(d*x + c) + 1))/(a + b) + 2*(a^2 + b^2)*log(abs(-(cos(d*x
 + c) - 1)/(cos(d*x + c) + 1) + 1))/a^3 - (3*a^2 - 4*a*b + 3*b^2 - 2*a^2*(
cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 4*a*b*(cos(d*x + c) - 1)/(cos(d*x +
 c) + 1) - 6*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 3*a^2*(cos(d*x +
c) - 1)^2/(cos(d*x + c) + 1)^2 + 3*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c)
+ 1)^2)/(a^3*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^2))/d
```

3.250.9 Mupad [B] (verification not implemented)

Time = 24.40 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.42

$$\int \frac{\cos^3(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx = \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d\,(a+b)} - \frac{\frac{2b}{a^2}+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(a+b)}{a^2}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)} - \frac{b^4\ln\left(a+b-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}{d\,(a^5-a^3b^2)} - \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)(a^2+b^2)}{a^3\,d}$$

input
```
int(cos(c + d*x)^3/(a*sin(c + d*x) + b*tan(c + d*x)),x)
```

output
```
log(tan(c/2 + (d*x)/2))/(d*(a + b)) - ((2*b)/a^2 + (2*tan(c/2 + (d*x)/2)^2
*(a + b))/a^2)/(d*(2*tan(c/2 + (d*x)/2)^2 + tan(c/2 + (d*x)/2)^4 + 1)) - (
b^4*log(a + b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2))/(d*(a^5
- a^3*b^2)) - (log(tan(c/2 + (d*x)/2)^2 + 1)*(a^2 + b^2))/(a^3*d)
```

3.250. $\int \frac{\cos^3(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx$

3.251 $\int \frac{\cos^2(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx$

3.251.1 Optimal result

Integrand size = 28, antiderivative size = 92

$$\int \frac{\cos^2(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx = \frac{\cos(c+dx)}{ad}+\frac{\log(1-\cos(c+dx))}{2(a+b)d} - \frac{\log(1+\cos(c+dx))}{2(a-b)d}+\frac{b^3\log(b+a\cos(c+dx))}{a^2(a^2-b^2)\,d}$$

output
```
cos(d*x+c)/a/d+1/2*ln(1-cos(d*x+c))/(a+b)/d-1/2*ln(1+cos(d*x+c))/(a-b)/d+b
^3*ln(b+a*cos(d*x+c))/a^2/(a^2-b^2)/d
```

3.251.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.87

$$\int \frac{\cos^2(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx = \frac{\frac{\cos(c+dx)}{a}+\frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{-a+b}+\frac{b^3\log(b+a\cos(c+dx))}{a^4-a^2b^2}+\frac{\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a+b}}{d}$$

input
```
Integrate[Cos[c + d*x]^2/(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

output
```
(Cos[c + d*x]/a + Log[Cos[(c + d*x)/2]]/(-a + b) + (b^3*Log[b + a*Cos[c +
d*x]])/(a^4 - a^2*b^2) + Log[Sin[(c + d*x)/2]]/(a + b))/d
```

3.251.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {3042, 4897, 3042, 25, 3316, 25, 27, 604, 25, 2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(c+dx)^2}{a\sin(c+dx)+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \frac{\cos^2(c+dx)\cot(c+dx)}{a\cos(c+dx)+b}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sin\left(c+dx-\frac{\pi}{2}\right)^3}{\cos\left(c+dx-\frac{\pi}{2}\right)\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sin\left(\frac{1}{2}(2c-\pi)+dx\right)^3}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)\left(b-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)}dx$$

$$\Big\downarrow \text{3316}$$

$$\frac{a\int -\frac{\cos^3(c+dx)}{(b+a\cos(c+dx))(a^2-a^2\cos^2(c+dx))}d(a\cos(c+dx))}{d}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a\int \frac{\cos^3(c+dx)}{(b+a\cos(c+dx))(a^2-a^2\cos^2(c+dx))}d(a\cos(c+dx))}{d}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{a^3\cos^3(c+dx)}{(b+a\cos(c+dx))(a^2-a^2\cos^2(c+dx))}d(a\cos(c+dx))}{a^2d}$$

$$\Big\downarrow \text{604}$$

$$-\frac{-\int -\frac{\cos(c+dx)a^3-b\cos^2(c+dx)a^2+ba^2}{(b+a\cos(c+dx))(a^2-a^2\cos^2(c+dx))}d(a\cos(c+dx))-a\cos(c+dx)-b}{a^2d}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int \frac{\cos(c+dx)a^3-b\cos^2(c+dx)a^2+ba^2}{(b+a\cos(c+dx))(a^2-a^2\cos^2(c+dx))}d(a\cos(c+dx))-a\cos(c+dx)-b}{a^2d}$$

$$\Big\downarrow \ 2160$$

$$-\frac{\int \left(\frac{b^3}{(b-a)(a+b)(b+a\cos(c+dx))}+\frac{a^2}{2(a+b)(a-a\cos(c+dx))}+\frac{a^2}{2(a-b)(\cos(c+dx)a+a)}\right)d(a\cos(c+dx))-a\cos(c+dx)-b}{a^2d}$$

$$\Big\downarrow \ 2009$$

$$-\frac{-\frac{b^3\log(a\cos(c+dx)+b)}{a^2-b^2}-\frac{a^2\log(a-a\cos(c+dx))}{2(a+b)}+\frac{a^2\log(a\cos(c+dx)+a)}{2(a-b)}-a\cos(c+dx)-b}{a^2d}$$

input
```
Int[Cos[c + d*x]^2/(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

output
```
-((-b - a*Cos[c + d*x] - (a^2*Log[a - a*Cos[c + d*x]])/(2*(a + b)) + (a^2*
Log[a + a*Cos[c + d*x]])/(2*(a - b)) - (b^3*Log[b + a*Cos[c + d*x]])/(a^2
- b^2))/(a^2*d))
```

3.251.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 604
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(c + d*x)^(m + n - 1)*((a + b*x^2)^(p + 1)/(b*d^(m - 1)*(m + n +
2*p + 1))), x] + Simp[1/(b*d^m*(m + n + 2*p + 1))   Int[(c + d*x)^n*(a + b*
x^2)^p*ExpandToSum[b*d^m*(m + n + 2*p + 1)*x^m - b*(m + n + 2*p + 1)*(c + d
*x)^m - (c + d*x)^(m - 2)*(a*d^2*(m + n - 1) - b*c^2*(m + n + 2*p + 1) - 2*
b*c*d*(m + n + p)*x), x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && IGtQ[m,
 1] && NeQ[m + n + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.251. $\int \frac{\cos^2(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx$

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.251.4 Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{\frac{\cos(dx+c)}{a}+\frac{b^3\ln(b+\cos(dx+c)a)}{a^2(a+b)(a-b)}+\frac{\ln(\cos(dx+c)-1)}{2a+2b}-\frac{\ln(\cos(dx+c)+1)}{2a-2b}}{d}$
default	$\frac{\frac{\cos(dx+c)}{a}+\frac{b^3\ln(b+\cos(dx+c)a)}{a^2(a+b)(a-b)}+\frac{\ln(\cos(dx+c)-1)}{2a+2b}-\frac{\ln(\cos(dx+c)+1)}{2a-2b}}{d}$
risch	$-\frac{ixb}{a^2}+\frac{e^{i(dx+c)}}{2ad}+\frac{e^{-i(dx+c)}}{2ad}+\frac{ix}{a-b}+\frac{ic}{d(a-b)}-\frac{ix}{a+b}-\frac{ic}{d(a+b)}-\frac{2ib^3x}{a^2(a^2-b^2)}-\frac{2ib^3c}{a^2d(a^2-b^2)}-\frac{\ln(e^{i(dx}}{d(a}$

input
```
int(cos(d*x+c)^2/(sin(d*x+c)*a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(cos(d*x+c)/a+1/a^2*b^3/(a+b)/(a-b)*ln(b+cos(d*x+c)*a)+1/(2*a+2*b)*ln(
cos(d*x+c)-1)-1/(2*a-2*b)*ln(cos(d*x+c)+1))
```

3.251.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.07

$$\int \frac{\cos^2(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx = \frac{2\,b^3\log\left(a\cos\left(dx+c\right)+b\right)+2\left(a^3-ab^2\right)\cos\left(dx+c\right)-\left(a^3+a^2b\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+\left(a^3-a^2b\right)\mathrm{l}}{2\left(a^4-a^2b^2\right)d}$$

input
```
integrate(cos(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(2*b^3*log(a*cos(d*x + c) + b) + 2*(a^3 - a*b^2)*cos(d*x + c) - (a^3 +
 a^2*b)*log(1/2*cos(d*x + c) + 1/2) + (a^3 - a^2*b)*log(-1/2*cos(d*x + c)
+ 1/2))/((a^4 - a^2*b^2)*d)
```

3.251.6 Sympy [F]

$$\int \frac{\cos^2(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx = \int \frac{\cos^2\left(c+dx\right)}{a\sin\left(c+dx\right)+b\tan\left(c+dx\right)}\,dx$$

input
```
integrate(cos(d*x+c)**2/(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

output
```
Integral(cos(c + d*x)**2/(a*sin(c + d*x) + b*tan(c + d*x)), x)
```

3.251.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.40

$$\int \frac{\cos^2(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx = \frac{\frac{b^3\log\left(a+b-\frac{(a-b)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^4-a^2b^2}+\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a+b}+\frac{b\log\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)}{a^2}+\frac{2}{a+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}}{d}$$

input
```
integrate(cos(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
(b^3*log(a + b - (a - b)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/(a^4 - a^2*b
^2) + log(sin(d*x + c)/(cos(d*x + c) + 1))/(a + b) + b*log(sin(d*x + c)^2/
(cos(d*x + c) + 1)^2 + 1)/a^2 + 2/(a + a*sin(d*x + c)^2/(cos(d*x + c) + 1)
^2))/d
```

3.251.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(88) = 176$.

Time = 0.37 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.07

$$\int \frac{\cos^2(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx = \frac{\frac{2\,b^3\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^4-a^2b^2}+\frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a+b}+\frac{2\,b\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^2}-\frac{2\left(2\,a-b+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{a^2\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)}}{2\,d}$$

input
```
integrate(cos(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(2*b^3*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(c
os(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^4 - a^2*b^2) + log(abs(-cos(d*x +
 c) + 1)/abs(cos(d*x + c) + 1))/(a + b) + 2*b*log(abs(-(cos(d*x + c) - 1)/
(cos(d*x + c) + 1) + 1))/a^2 - 2*(2*a - b + b*(cos(d*x + c) - 1)/(cos(d*x
+ c) + 1))/(a^2*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)))/d
```

3.251.9 Mupad [B] (verification not implemented)

Time = 23.64 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.27

$$\int \frac{\cos^2(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx = \frac{2}{a\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d\,(a+b)} + \frac{b\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{a^2\,d} + \frac{b^3\,\ln\left(a+b-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{a^2\,d\,(a^2-b^2)}$$

input
```
int(cos(c + d*x)^2/(a*sin(c + d*x) + b*tan(c + d*x)),x)
```

output
```
2/(a*d*(tan(c/2 + (d*x)/2)^2 + 1)) + log(tan(c/2 + (d*x)/2))/(d*(a + b)) +
 (b*log(tan(c/2 + (d*x)/2)^2 + 1))/(a^2*d) + (b^3*log(a + b - a*tan(c/2 +
(d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2))/(a^2*d*(a^2 - b^2))
```

3.251. $\int \frac{\cos^2(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx$

3.252 $\int \frac{\cos(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx$

3.252.1 Optimal result

Integrand size = 26, antiderivative size = 80

$$\int \frac{\cos(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx = \frac{\log(1-\cos(c+dx))}{2(a+b)d} + \frac{\log(1+\cos(c+dx))}{2(a-b)d} - \frac{b^2\log(b+a\cos(c+dx))}{a(a^2-b^2)d}$$

output
```
1/2*ln(1-cos(d*x+c))/(a+b)/d+1/2*ln(1+cos(d*x+c))/(a-b)/d-b^2*ln(b+a*cos(d
*x+c))/a/(a^2-b^2)/d
```

3.252.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.88

$$\int \frac{\cos(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx = \frac{a(a+b)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) - b^2\log(b+a\cos(c+dx)) + a(a-b)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a(a-b)(a+b)d}$$

input
```
Integrate[Cos[c + d*x]/(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

output
```
(a*(a + b)*Log[Cos[(c + d*x)/2]] - b^2*Log[b + a*Cos[c + d*x]] + a*(a - b)
*Log[Sin[(c + d*x)/2]])/(a*(a - b)*(a + b)*d)
```

3.252.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {3042, 4897, 3042, 3316, 27, 615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \frac{\cos(c+dx)\cot(c+dx)}{a\cos(c+dx)+b}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx-\frac{\pi}{2}\right)^2}{\cos\left(c+dx-\frac{\pi}{2}\right)\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3316}$$

$$-\frac{a\int \frac{\cos^2(c+dx)}{(b+a\cos(c+dx))(a^2-a^2\cos^2(c+dx))}d(a\cos(c+dx))}{d}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{a^2\cos^2(c+dx)}{(b+a\cos(c+dx))(a^2-a^2\cos^2(c+dx))}d(a\cos(c+dx))}{ad}$$

$$\Big\downarrow \text{615}$$

$$-\frac{\int \left(\frac{b^2}{(a-b)(a+b)(b+a\cos(c+dx))}+\frac{a}{2(a+b)(a-a\cos(c+dx))}-\frac{a}{2(a-b)(\cos(c+dx)a+a)}\right)d(a\cos(c+dx))}{ad}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{b^2\log(a\cos(c+dx)+b)}{a^2-b^2}-\frac{a\log(a-a\cos(c+dx))}{2(a+b)}-\frac{a\log(a\cos(c+dx)+a)}{2(a-b)}}{ad}$$

input
```
Int[Cos[c + d*x]/(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

output
```
-((-1/2*(a*Log[a - a*Cos[c + d*x]])/(a + b) - (a*Log[a + a*Cos[c + d*x]])/
(2*(a - b)) + (b^2*Log[b + a*Cos[c + d*x]])/(a^2 - b^2))/(a*d))
```

3.252.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.252.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{\frac{\ln(\cos(dx+c)-1)}{2a+2b}+\frac{\ln(\cos(dx+c)+1)}{2a-2b}-\frac{b^2\ln(b+\cos(dx+c)a)}{(a+b)(a-b)a}}{d}$
default	$\frac{\frac{\ln(\cos(dx+c)-1)}{2a+2b}+\frac{\ln(\cos(dx+c)+1)}{2a-2b}-\frac{b^2\ln(b+\cos(dx+c)a)}{(a+b)(a-b)a}}{d}$
risch	$\frac{ix}{a}-\frac{ix}{a+b}-\frac{ic}{d(a+b)}-\frac{ix}{a-b}-\frac{ic}{d(a-b)}+\frac{2ib^2x}{a(a^2-b^2)}+\frac{2ib^2c}{ad(a^2-b^2)}+\frac{\ln(e^{i(dx+c)}-1)}{d(a+b)}+\frac{\ln(e^{i(dx+c)}+1)}{d(a-b)}-$

input
```
int(cos(d*x+c)/(sin(d*x+c)*a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/(2*a+2*b)*ln(cos(d*x+c)-1)+1/(2*a-2*b)*ln(cos(d*x+c)+1)-b^2/(a+b)/(
a-b)/a*ln(b+cos(d*x+c)*a))
```

3.252.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94

$$\int \frac{\cos(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx = -\frac{2\,b^2\log\left(a\cos\left(dx+c\right)+b\right)-\left(a^2+ab\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-\left(a^2-ab\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\left(a^3-ab^2\right)d}$$

input
```
integrate(cos(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2*(2*b^2*log(a*cos(d*x + c) + b) - (a^2 + a*b)*log(1/2*cos(d*x + c) + 1
/2) - (a^2 - a*b)*log(-1/2*cos(d*x + c) + 1/2))/((a^3 - a*b^2)*d)
```

3.252.6 Sympy [F]

$$\int \frac{\cos(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx = \int \frac{\cos{\left(c+dx \right)}}{a\sin{\left(c+dx \right)}+b\tan{\left(c+dx \right)}}\,dx$$

input
```
integrate(cos(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

output
```
Integral(cos(c + d*x)/(a*sin(c + d*x) + b*tan(c + d*x)), x)
```

3.252.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.28

$$\int \frac{\cos(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx = -\frac{\frac{b^2\log\left(a+b-\frac{(a-b)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^3-ab^2}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a+b}+\frac{\log\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)}{a}}{d}$$

input
```
integrate(cos(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
-(b^2*log(a + b - (a - b)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/(a^3 - a*b^
2) - log(sin(d*x + c)/(cos(d*x + c) + 1))/(a + b) + log(sin(d*x + c)^2/(co
s(d*x + c) + 1)^2 + 1)/a)/d
```

3.252.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(76) = 152$.

Time = 0.36 (sec) , antiderivative size = 257, normalized size of antiderivative = 3.21

$$\int \frac{\cos(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx = -\frac{\frac{a\log\left(\left|-a-b+\frac{2\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right|\right)}{a^2-b^2}-\frac{(a^2-2\,b^2)\log\left(\frac{\left|-2\,b-\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{2\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}-2\,|a|\right|}{\left|-2\,b-\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{2\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+2\,|a|\right|}\right)}{(a^2-b^2)|a|} -}{2\,d}$$

input
```
integrate(cos(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
-1/2*(a*log(abs(-a - b + 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + a*(co
s(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - b*(cos(d*x + c) - 1)^2/(cos(d*x +
 c) + 1)^2))/(a^2 - b^2) - (a^2 - 2*b^2)*log(abs(-2*b - 2*a*(cos(d*x + c)
- 1)/(cos(d*x + c) + 1) + 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*ab
s(a))/abs(-2*b - 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2*b*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1) + 2*abs(a)))/((a^2 - b^2)*abs(a)) - log(abs(-
cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a + b))/d
```

3.252.9 Mupad [B] (verification not implemented)

Time = 22.35 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.16

$$\int \frac{\cos(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx = \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d\,(a+b)} - \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{a\,d} + \frac{b^2\,\ln\left(a+b-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}{d\,(a\,b^2-a^3)}$$

input
```
int(cos(c + d*x)/(a*sin(c + d*x) + b*tan(c + d*x)),x)
```

output
```
log(tan(c/2 + (d*x)/2))/(d*(a + b)) - log(tan(c/2 + (d*x)/2)^2 + 1)/(a*d)
+ (b^2*log(a + b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2))/(d*(a
*b^2 - a^3))
```

3.253 $\int \frac{1}{a\sin(c+dx)+b\tan(c+dx)}\,dx$

3.253.1 Optimal result

Integrand size = 19, antiderivative size = 74

$$\int \frac{1}{a\sin(c+dx)+b\tan(c+dx)}\,dx = \frac{\log(1-\cos(c+dx))}{2(a+b)d}-\frac{\log(1+\cos(c+dx))}{2(a-b)d}+\frac{b\log(b+a\cos(c+dx))}{(a^2-b^2)\,d}$$

output
```
1/2*ln(1-cos(d*x+c))/(a+b)/d-1/2*ln(1+cos(d*x+c))/(a-b)/d+b*ln(b+a*cos(d*x
+c))/(a^2-b^2)/d
```

3.253.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.85

$$\int \frac{1}{a\sin(c+dx)+b\tan(c+dx)}\,dx = \frac{-\left((a+b)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)+b\log(b+a\cos(c+dx))+(a-b)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{(a-b)(a+b)d}$$

input
```
Integrate[(a*Sin[c + d*x] + b*Tan[c + d*x])^(-1),x]
```

output
```
(-((a + b)*Log[Cos[(c + d*x)/2]]) + b*Log[b + a*Cos[c + d*x]] + (a - b)*Lo
g[Sin[(c + d*x)/2]])/((a - b)*(a + b)*d)
```

3.253.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {3042, 4897, 3042, 25, 3200, 587, 16, 452, 219, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a\sin(c+dx)+b\tan(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{a\sin(c+dx)+b\tan(c+dx)}dx$$

$$\Big\downarrow \; 4897$$

$$\int \frac{\cot(c+dx)}{a\cos(c+dx)+b}dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{\tan\left(c+dx-\frac{\pi}{2}\right)}{b-a\sin\left(c+dx-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 25$$

$$-\int \frac{\tan\left(\frac{1}{2}(2c-\pi)+dx\right)}{b-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}dx$$

$$\Big\downarrow \; 3200$$

$$-\frac{\int \frac{a\cos(c+dx)}{(b+a\cos(c+dx))(a^2-a^2\cos^2(c+dx))}d(a\cos(c+dx))}{d}$$

$$\Big\downarrow \; 587$$

$$-\frac{\frac{\int \frac{a^2-ab\cos(c+dx)}{a^2-a^2\cos^2(c+dx)}d(a\cos(c+dx))}{a^2-b^2}-\frac{b\int \frac{1}{b+a\cos(c+dx)}d(a\cos(c+dx))}{a^2-b^2}}{d}$$

$$\Big\downarrow \; 16$$

$$-\frac{\frac{\int \frac{a^2-ab\cos(c+dx)}{a^2-a^2\cos^2(c+dx)}d(a\cos(c+dx))}{a^2-b^2}-\frac{b\log(a\cos(c+dx)+b)}{a^2-b^2}}{d}$$

$$\Big\downarrow \; 452$$

$$-\frac{\frac{a^2\int \frac{1}{a^2-a^2\cos^2(c+dx)}d(a\cos(c+dx))-b\int \frac{a\cos(c+dx)}{a^2-a^2\cos^2(c+dx)}d(a\cos(c+dx))}{a^2-b^2}-\frac{b\log(a\cos(c+dx)+b)}{a^2-b^2}}{d}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{\frac{a\text{arctanh}(\cos(c+dx))-b\int \frac{a\cos(c+dx)}{a^2-a^2\cos^2(c+dx)}d(a\cos(c+dx))}{a^2-b^2}-\frac{b\log(a\cos(c+dx)+b)}{a^2-b^2}}{d}$$

$$\Big\downarrow \text{ 240}$$

$$-\frac{\frac{\frac{1}{2}b\log(a^2-a^2\cos^2(c+dx))+a\text{arctanh}(\cos(c+dx))}{a^2-b^2}-\frac{b\log(a\cos(c+dx)+b)}{a^2-b^2}}{d}$$

input
```
Int[(a*Sin[c + d*x] + b*Tan[c + d*x])^(-1),x]
```

output
```
-((-((b*Log[b + a*Cos[c + d*x]])/(a^2 - b^2)) + (a*ArcTanh[Cos[c + d*x]] +
 (b*Log[a^2 - a^2*Cos[c + d*x]^2])/2)/(a^2 - b^2))/d)
```

3.253.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

3.253. $\int \frac{1}{a\sin(c+dx)+b\tan(c+dx)}\,dx$

rule 587
```
Int[(x_.)/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)), x_Symbol] :> Simp[(-
c)*(d/(b*c^2 + a*d^2))   Int[1/(c + d*x), x], x] + Simp[1/(b*c^2 + a*d^2)
 Int[(a*d + b*c*x)/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
^2 + a*d^2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.253.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{\frac{\ln(\cos(dx+c)-1)}{2a+2b}+\frac{b\ln(b+\cos(dx+c)a)}{(a+b)(a-b)}-\frac{\ln(\cos(dx+c)+1)}{2a-2b}}{d}$
default	$\frac{\frac{\ln(\cos(dx+c)-1)}{2a+2b}+\frac{b\ln(b+\cos(dx+c)a)}{(a+b)(a-b)}-\frac{\ln(\cos(dx+c)+1)}{2a-2b}}{d}$
risch	$\frac{ix}{a-b}+\frac{ic}{d(a-b)}-\frac{ix}{a+b}-\frac{ic}{d(a+b)}-\frac{2ibx}{a^2-b^2}-\frac{2ibc}{(a^2-b^2)d}-\frac{\ln(e^{i(dx+c)}+1)}{d(a-b)}+\frac{\ln(e^{i(dx+c)}-1)}{d(a+b)}+\frac{b\ln\left(e^{2i(dx+ \right.}{(}$

input
```
int(1/(sin(d*x+c)*a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/(2*a+2*b)*ln(cos(d*x+c)-1)+b/(a+b)/(a-b)*ln(b+cos(d*x+c)*a)-1/(2*a-
2*b)*ln(cos(d*x+c)+1))
```

3.253.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int \frac{1}{a\sin(c+dx)+b\tan(c+dx)}\,dx = \frac{2\,b\log\left(a\cos\left(dx+c\right)+b\right)-(a+b)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+(a-b)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\,(a^2-b^2)d}$$

input
```
integrate(1/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(2*b*log(a*cos(d*x + c) + b) - (a + b)*log(1/2*cos(d*x + c) + 1/2) + (
a - b)*log(-1/2*cos(d*x + c) + 1/2))/((a^2 - b^2)*d)
```

3.253.6 Sympy [F]

$$\int \frac{1}{a\sin(c+dx)+b\tan(c+dx)}\,dx = \int \frac{1}{a\sin\left(c+dx\right)+b\tan\left(c+dx\right)}\,dx$$

input
```
integrate(1/(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

output
```
Integral(1/(a*sin(c + d*x) + b*tan(c + d*x)), x)
```

3.253.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.96

$$\int \frac{1}{a\sin(c+dx)+b\tan(c+dx)}\,dx = \frac{\frac{b\log\left(a+b-\frac{(a-b)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^2-b^2}+\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a+b}}{d}$$

input
```
integrate(1/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
(b*log(a + b - (a - b)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/(a^2 - b^2) +
log(sin(d*x + c)/(cos(d*x + c) + 1))/(a + b))/d
```

3.253.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.35

$$\int \frac{1}{a\sin(c+dx)+b\tan(c+dx)}\,dx = \frac{\frac{2\,b\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^2-b^2}+\frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a+b}}{2\,d}$$

input
```
integrate(1/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(2*b*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos
(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^2 - b^2) + log(abs(-cos(d*x + c) +
1)/abs(cos(d*x + c) + 1))/(a + b))/d
```

3.253.9 Mupad [B] (verification not implemented)

Time = 22.62 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.91

$$\int \frac{1}{a\sin(c+dx)+b\tan(c+dx)}\,dx = \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d\,(a+b)} + \frac{b\,\ln\left(a+b-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d\,(a^2-b^2)}$$

input
```
int(1/(a*sin(c + d*x) + b*tan(c + d*x)),x)
```

output
```
log(tan(c/2 + (d*x)/2))/(d*(a + b)) + (b*log(a + b - a*tan(c/2 + (d*x)/2)^
2 + b*tan(c/2 + (d*x)/2)^2))/(d*(a^2 - b^2))
```

3.254 $\int \frac{\sec(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx$

3.254.1 Optimal result

Integrand size = 26, antiderivative size = 75

$$\int \frac{\sec(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx = \frac{\log(1-\cos(c+dx))}{2(a+b)d}+\frac{\log(1+\cos(c+dx))}{2(a-b)d} - \frac{a\log(b+a\cos(c+dx))}{(a^2-b^2)\,d}$$

output
```
1/2*ln(1-cos(d*x+c))/(a+b)/d+1/2*ln(1+cos(d*x+c))/(a-b)/d-a*ln(b+a*cos(d*x
+c))/(a^2-b^2)/d
```

3.254.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.85

$$\int \frac{\sec(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx = \frac{(a-b)\log(1-\cos(c+dx))+(a+b)\log(1+\cos(c+dx))-2a\log(b+a\cos(c+dx))}{2(a-b)(a+b)d}$$

input
```
Integrate[Sec[c + d*x]/(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

output
```
((a - b)*Log[1 - Cos[c + d*x]] + (a + b)*Log[1 + Cos[c + d*x]] - 2*a*Log[b
 + a*Cos[c + d*x]])/(2*(a - b)*(a + b)*d)
```

3.254.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3042, 4897, 3042, 3147, 477, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sec(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}dx$$

$$\Big\downarrow \ 4897$$

$$\int \frac{\csc(c+dx)}{a\cos(c+dx)+b}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\cos\left(c+dx-\frac{\pi}{2}\right)\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \ 3147$$

$$-\frac{a\int \frac{1}{(b+a\cos(c+dx))(a^2-a^2\cos^2(c+dx))}d(a\cos(c+dx))}{d}$$

$$\Big\downarrow \ 477$$

$$-\frac{\int \left(\frac{a^2}{(a^2-b^2)(b+a\cos(c+dx))}+\frac{a}{2(a+b)(a-a\cos(c+dx))}-\frac{a}{2(a-b)(\cos(c+dx)a+a)}\right)d(a\cos(c+dx))}{ad}$$

$$\Big\downarrow \ 2009$$

$$-\frac{\frac{a^2\log(a\cos(c+dx)+b)}{a^2-b^2}-\frac{a\log(a-a\cos(c+dx))}{2(a+b)}-\frac{a\log(a\cos(c+dx)+a)}{2(a-b)}}{ad}$$

input
```
Int[Sec[c + d*x]/(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

output
```
-((-1/2*(a*Log[a - a*Cos[c + d*x]])/(a + b) - (a*Log[a + a*Cos[c + d*x]])/
(2*(a - b)) + (a^2*Log[b + a*Cos[c + d*x]])/(a^2 - b^2))/(a*d))
```

3.254.3.1 Defintions of rubi rules used

rule 477
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ExpandIntegrand[(c + d*x)^n*(1 - Rt[-b/a, 2]*x)^p*(1 + Rt[-b/a, 2
]*x)^p, x], x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[n] &
& NiceSqrtQ[-b/a] &&  !FractionalPowerFactorQ[Rt[-b/a, 2]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.254.4 Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{-\frac{a\ln(b+\cos(dx+c)a)}{(a+b)(a-b)}+\frac{\ln(\cos(dx+c)-1)}{2a+2b}+\frac{\ln(\cos(dx+c)+1)}{2a-2b}}{d}$
default	$\frac{-\frac{a\ln(b+\cos(dx+c)a)}{(a+b)(a-b)}+\frac{\ln(\cos(dx+c)-1)}{2a+2b}+\frac{\ln(\cos(dx+c)+1)}{2a-2b}}{d}$
risch	$-\frac{ix}{a-b}-\frac{ic}{d(a-b)}-\frac{ix}{a+b}-\frac{ic}{d(a+b)}+\frac{2iax}{a^2-b^2}+\frac{2iac}{(a^2-b^2)d}+\frac{\ln(e^{i(dx+c)}+1)}{d(a-b)}+\frac{\ln(e^{i(dx+c)}-1)}{d(a+b)}-\frac{a\ln\left(e^{2i(d}\right.}{}$

input
```
int(sec(d*x+c)/(sin(d*x+c)*a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a/(a+b)/(a-b)*ln(b+cos(d*x+c)*a)+1/(2*a+2*b)*ln(cos(d*x+c)-1)+1/(2*a
-2*b)*ln(cos(d*x+c)+1))
```

3.254.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.87

$$\int \frac{\sec(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx = -\frac{2\,a\log\left(a\cos\left(dx+c\right)+b\right)-(a+b)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-(a-b)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\,(a^2-b^2)d}$$

input
```
integrate(sec(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2*(2*a*log(a*cos(d*x + c) + b) - (a + b)*log(1/2*cos(d*x + c) + 1/2) -
(a - b)*log(-1/2*cos(d*x + c) + 1/2))/((a^2 - b^2)*d)
```

3.254.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx = \int \frac{\sec\left(c+dx\right)}{a\sin\left(c+dx\right)+b\tan\left(c+dx\right)}\,dx$$

input
```
integrate(sec(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)/(a*sin(c + d*x) + b*tan(c + d*x)), x)
```

3.254.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int \frac{\sec(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx = -\frac{\frac{a\log\left(a+b-\frac{(a-b)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^2-b^2}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a+b}}{d}$$

input
```
integrate(sec(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
-(a*log(a + b - (a - b)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/(a^2 - b^2) -
 log(sin(d*x + c)/(cos(d*x + c) + 1))/(a + b))/d
```

3.254.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.35

$$\int \frac{\sec(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx = -\frac{\frac{2\,a\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^2-b^2}-\frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a+b}}{2\,d}$$

input
```
integrate(sec(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
-1/2*(2*a*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(co
s(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^2 - b^2) - log(abs(-cos(d*x + c) +
 1)/abs(cos(d*x + c) + 1))/(a + b))/d
```

3.254.9 Mupad [B] (verification not implemented)

Time = 22.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.91

$$\int \frac{\sec(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx = \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d\,(a+b)} - \frac{a\,\ln\left(a+b-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d\,(a^2-b^2)}$$

input
```
int(1/(cos(c + d*x)*(a*sin(c + d*x) + b*tan(c + d*x))),x)
```

output
```
log(tan(c/2 + (d*x)/2))/(d*(a + b)) - (a*log(a + b - a*tan(c/2 + (d*x)/2)^
2 + b*tan(c/2 + (d*x)/2)^2))/(d*(a^2 - b^2))
```

3.255 $\int \frac{\sec^2(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx$

3.255.1 Optimal result

Integrand size = 28, antiderivative size = 94

$$\int \frac{\sec^2(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx = \frac{\log(1-\cos(c+dx))}{2(a+b)d} - \frac{\log(\cos(c+dx))}{bd} - \frac{\log(1+\cos(c+dx))}{2(a-b)d} + \frac{a^2\log(b+a\cos(c+dx))}{b(a^2-b^2)d}$$

output
```
1/2*ln(1-cos(d*x+c))/(a+b)/d-ln(cos(d*x+c))/b/d-1/2*ln(1+cos(d*x+c))/(a-b)
/d+a^2*ln(b+a*cos(d*x+c))/b/(a^2-b^2)/d
```

3.255.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.10

$$\int \frac{\sec^2(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx = 2\left(\frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2(-a+b)d} - \frac{\log(\cos(c+dx))}{2bd} - \frac{a^2\log(b+a\cos(c+dx))}{2b(-a^2+b^2)d} + \frac{\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2(a+b)d}\right)$$

input
```
Integrate[Sec[c + d*x]^2/(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

output
```
2*(Log[Cos[(c + d*x)/2]]/(2*(-a + b)*d) - Log[Cos[c + d*x]]/(2*b*d) - (a^2
*Log[b + a*Cos[c + d*x]])/(2*b*(-a^2 + b^2)*d) + Log[Sin[(c + d*x)/2]]/(2*
(a + b)*d))
```

3.255.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {3042, 4897, 3042, 25, 3316, 25, 27, 615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^2}{a\sin(c+dx)+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \frac{\csc(c+dx)\sec(c+dx)}{a\cos(c+dx)+b}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\sin\left(c+dx-\frac{\pi}{2}\right)\cos\left(c+dx-\frac{\pi}{2}\right)\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\left(b-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)}dx$$

$$\Big\downarrow \text{3316}$$

$$\frac{a\int -\frac{\sec(c+dx)}{(b+a\cos(c+dx))(a^2-a^2\cos^2(c+dx))}d(a\cos(c+dx))}{d}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a\int \frac{\sec(c+dx)}{(b+a\cos(c+dx))(a^2-a^2\cos^2(c+dx))}d(a\cos(c+dx))}{d}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^2\int \frac{\sec(c+dx)}{a(b+a\cos(c+dx))(a^2-a^2\cos^2(c+dx))}d(a\cos(c+dx))}{d}$$

$$\Big\downarrow \text{615}$$

$$-\frac{a^2\int \left(\frac{\sec(c+dx)}{a^3b}+\frac{1}{2a^2(a+b)(a-a\cos(c+dx))}+\frac{1}{2a^2(a-b)(\cos(c+dx)a+a)}+\frac{1}{b(b-a)(a+b)(b+a\cos(c+dx))}\right)d(a\cos(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2\left(-\frac{\log(a\cos(c+dx)+b)}{b(a^2-b^2)}+\frac{\log(a\cos(c+dx))}{a^2b}-\frac{\log(a-a\cos(c+dx))}{2a^2(a+b)}+\frac{\log(a\cos(c+dx)+a)}{2a^2(a-b)}\right)}{d}$$

input
```
Int[Sec[c + d*x]^2/(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

output
```
-((a^2*(Log[a*Cos[c + d*x]]/(a^2*b) - Log[a - a*Cos[c + d*x]]/(2*a^2*(a +
b)) + Log[a + a*Cos[c + d*x]]/(2*a^2*(a - b)) - Log[b + a*Cos[c + d*x]]/(b
*(a^2 - b^2))))/d)
```

3.255.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.255. $\int \frac{\sec^2(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx$

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.255.4 Maple [A] (verified)

Time = 1.91 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{-\frac{\ln(\cos(dx+c))}{b}+\frac{a^2\ln(b+\cos(dx+c)a)}{(a+b)(a-b)b}+\frac{\ln(\cos(dx+c)-1)}{2a+2b}-\frac{\ln(\cos(dx+c)+1)}{2a-2b}}{d}$
default	$\frac{-\frac{\ln(\cos(dx+c))}{b}+\frac{a^2\ln(b+\cos(dx+c)a)}{(a+b)(a-b)b}+\frac{\ln(\cos(dx+c)-1)}{2a+2b}-\frac{\ln(\cos(dx+c)+1)}{2a-2b}}{d}$
risch	$\frac{ix}{a-b}+\frac{ic}{d(a-b)}-\frac{ix}{a+b}-\frac{ic}{d(a+b)}-\frac{2ia^2x}{b(a^2-b^2)}-\frac{2ia^2c}{bd(a^2-b^2)}+\frac{2ix}{b}+\frac{2ic}{bd}-\frac{\ln(e^{i(dx+c)}+1)}{d(a-b)}+\frac{\ln(e^{i(dx+c)}-}{d(a+b)}$

input
```
int(sec(d*x+c)^2/(sin(d*x+c)*a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/b*ln(cos(d*x+c))+a^2/(a+b)/(a-b)/b*ln(b+cos(d*x+c)*a)+1/(2*a+2*b)*
ln(cos(d*x+c)-1)-1/(2*a-2*b)*ln(cos(d*x+c)+1))
```

3.255.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.02

$$\int \frac{\sec^2(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx$$
$$=\frac{2a^2\log(a\cos(dx+c)+b)-2(a^2-b^2)\log(-\cos(dx+c))-(ab+b^2)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+(ab-}{2(a^2b-b^3)d}$$

input
```
integrate(sec(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(2*a^2*log(a*cos(d*x + c) + b) - 2*(a^2 - b^2)*log(-cos(d*x + c)) - (a
*b + b^2)*log(1/2*cos(d*x + c) + 1/2) + (a*b - b^2)*log(-1/2*cos(d*x + c)
+ 1/2))/((a^2*b - b^3)*d)
```

3.255.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx=\int \frac{\sec^2(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**2/(a*sin(c + d*x) + b*tan(c + d*x)), x)
```

3.255.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.33

$$\int \frac{\sec^2(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx = \frac{\frac{a^2\log\left(a+b-\frac{(a-b)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^2b-b^3}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{b}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{b}+\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a+b}}{d}$$

input
```
integrate(sec(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
(a^2*log(a + b - (a - b)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/(a^2*b - b^3
) - log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/b - log(sin(d*x + c)/(cos(d*x
 + c) + 1) - 1)/b + log(sin(d*x + c)/(cos(d*x + c) + 1))/(a + b))/d
```

3.255.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(90) = 180.

Time = 0.36 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.69

$$\int \frac{\sec^2(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx = \frac{\frac{b\log\left(\left|a+b+\frac{2a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right|\right)}{a^2-b^2}+\frac{(2a^2-b^2)\log\left(\frac{\left|-2a-\frac{2a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{2b(\cos(dx+c)-1)}{\cos(dx+c)+1}-2|b|\right|}{\left|-2a-\frac{2a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{2b(\cos(dx+c)-1)}{\cos(dx+c)+1}+2|b|\right|}\right)}{(a^2-b^2)|b|}+\frac{\log}{}}{2d}$$

input
```
integrate(sec(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(b*log(abs(a + b + 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + a*(cos(
d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - b*(cos(d*x + c) - 1)^2/(cos(d*x + c
) + 1)^2))/(a^2 - b^2) + (2*a^2 - b^2)*log(abs(-2*a - 2*a*(cos(d*x + c) -
1)/(cos(d*x + c) + 1) + 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*abs(
b))/abs(-2*a - 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2*b*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) + 2*abs(b)))/((a^2 - b^2)*abs(b)) + log(abs(-co
s(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a + b))/d
```

3.255.9 Mupad [B] (verification not implemented)

Time = 22.52 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.99

$$\int \frac{\sec^2(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx = \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d\,(a+b)}-\frac{\ln\left({\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}^2-1\right)}{b\,d}+\frac{a^2\,\ln\left(a+b-a\,{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}^2+b\,{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}^2\right)}{d\,(a^2\,b-b^3)}$$

input
```
int(1/(cos(c + d*x)^2*(a*sin(c + d*x) + b*tan(c + d*x))),x)
```

output
```
log(tan(c/2 + (d*x)/2))/(d*(a + b)) - log(tan(c/2 + (d*x)/2)^2 - 1)/(b*d)
+ (a^2*log(a + b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2))/(d*(a
^2*b - b^3))
```

3.256 $\int \frac{\sec^3(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx$

3.256.1 Optimal result

Integrand size = 28, antiderivative size = 108

$$\int \frac{\sec^3(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx = \frac{\log(1-\cos(c+dx))}{2(a+b)d}+\frac{a\log(\cos(c+dx))}{b^2d}+\frac{\log(1+\cos(c+dx))}{2(a-b)d}-\frac{a^3\log(b+a\cos(c+dx))}{b^2(a^2-b^2)\,d}+\frac{\sec(c+dx)}{bd}$$

output
```
1/2*ln(1-cos(d*x+c))/(a+b)/d+a*ln(cos(d*x+c))/b^2/d+1/2*ln(1+cos(d*x+c))/(
a-b)/d-a^3*ln(b+a*cos(d*x+c))/b^2/(a^2-b^2)/d+sec(d*x+c)/b/d
```

3.256.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.85

$$\int \frac{\sec^3(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx = \frac{\frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{a-b}+\frac{a\log(\cos(c+dx))}{b^2}+\frac{a^3\log(b+a\cos(c+dx))}{-a^2b^2+b^4}+\frac{\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a+b}+\frac{\sec(c+dx)}{b}}{d}$$

input
```
Integrate[Sec[c + d*x]^3/(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

output
```
(Log[Cos[(c + d*x)/2]]/(a - b) + (a*Log[Cos[c + d*x]])/b^2 + (a^3*Log[b +
a*Cos[c + d*x]])/(-(a^2*b^2) + b^4) + Log[Sin[(c + d*x)/2]]/(a + b) + Sec[
c + d*x]/b)/d
```

3.256.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 4897, 3042, 3316, 27, 615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^3}{a\sin(c+dx)+b\tan(c+dx)}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \frac{\csc(c+dx)\sec^2(c+dx)}{a\cos(c+dx)+b}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(c+dx-\frac{\pi}{2}\right)^2\cos\left(c+dx-\frac{\pi}{2}\right)\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3316}$$

$$-\frac{a\int \frac{\sec^2(c+dx)}{(b+a\cos(c+dx))(a^2-a^2\cos^2(c+dx))}d(a\cos(c+dx))}{d}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a^3\int \frac{\sec^2(c+dx)}{a^2(b+a\cos(c+dx))(a^2-a^2\cos^2(c+dx))}d(a\cos(c+dx))}{d}$$

$$\Big\downarrow \text{615}$$

$$-\frac{a^3\int \left(\frac{\sec^2(c+dx)}{a^4b}-\frac{\sec(c+dx)}{a^3b^2}+\frac{1}{2a^3(a+b)(a-a\cos(c+dx))}-\frac{1}{2a^3(a-b)(\cos(c+dx)a+a)}-\frac{1}{b^2(b-a)(a+b)(b+a\cos(c+dx))}\right)d(a\cos(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3\left(-\frac{\sec(c+dx)}{a^3b}-\frac{\log(a-a\cos(c+dx))}{2a^3(a+b)}-\frac{\log(a\cos(c+dx)+a)}{2a^3(a-b)}-\frac{\log(a\cos(c+dx))}{a^2b^2}+\frac{\log(a\cos(c+dx)+b)}{b^2(a^2-b^2)}\right)}{d}$$

input
```
Int[Sec[c + d*x]^3/(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

output
```
-((a^3*(-(Log[a*Cos[c + d*x]]/(a^2*b^2)) - Log[a - a*Cos[c + d*x]]/(2*a^3*
(a + b)) - Log[a + a*Cos[c + d*x]]/(2*a^3*(a - b)) + Log[b + a*Cos[c + d*x
]]/(b^2*(a^2 - b^2)) - Sec[c + d*x]/(a^3*b)))/d)
```

3.256.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.256. $\int \frac{\sec^3(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx$

3.256.4 Maple [A] (verified)

Time = 3.94 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{\frac{\ln(\cos(dx+c)-1)}{2a+2b}-\frac{a^3\ln(b+\cos(dx+c)a)}{(a+b)(a-b)b^2}+\frac{a\ln(\cos(dx+c))}{b^2}+\frac{1}{b\cos(dx+c)}+\frac{\ln(\cos(dx+c)+1)}{2a-2b}}{d}$
default	$\frac{\frac{\ln(\cos(dx+c)-1)}{2a+2b}-\frac{a^3\ln(b+\cos(dx+c)a)}{(a+b)(a-b)b^2}+\frac{a\ln(\cos(dx+c))}{b^2}+\frac{1}{b\cos(dx+c)}+\frac{\ln(\cos(dx+c)+1)}{2a-2b}}{d}$
risch	$-\frac{ix}{a+b}-\frac{ic}{d(a+b)}-\frac{ix}{a-b}-\frac{ic}{d(a-b)}-\frac{2iax}{b^2}-\frac{2iac}{b^2d}+\frac{2ia^3x}{b^2(a^2-b^2)}+\frac{2ia^3c}{b^2d(a^2-b^2)}+\frac{2\,e^{i(dx+c)}}{db(e^{2i(dx+c)}+1)}+\frac{\ln(e$

input
```
int(sec(d*x+c)^3/(sin(d*x+c)*a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/(2*a+2*b)*ln(cos(d*x+c)-1)-a^3/(a+b)/(a-b)/b^2*ln(b+cos(d*x+c)*a)+a
/b^2*ln(cos(d*x+c))+1/b/cos(d*x+c)+1/(2*a-2*b)*ln(cos(d*x+c)+1))
```

3.256.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.36

$$\int \frac{\sec^3(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx = -\frac{2\,a^3\cos(dx+c)\log(a\cos(dx+c)+b)-2\,a^2b+2\,b^3-2\,(a^3-ab^2)\cos(dx+c)\log(-\cos(dx+c))-}{2\,(a^2b^2-b^4)d\,c}$$

input
```
integrate(sec(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2*(2*a^3*cos(d*x + c)*log(a*cos(d*x + c) + b) - 2*a^2*b + 2*b^3 - 2*(a^
3 - a*b^2)*cos(d*x + c)*log(-cos(d*x + c)) - (a*b^2 + b^3)*cos(d*x + c)*lo
g(1/2*cos(d*x + c) + 1/2) - (a*b^2 - b^3)*cos(d*x + c)*log(-1/2*cos(d*x +
c) + 1/2))/((a^2*b^2 - b^4)*d*cos(d*x + c))
```

3.256.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx=\int \frac{\sec^3{(c+dx)}}{a\sin{(c+dx)}+b\tan{(c+dx)}}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

output
```
Integral(sec(c + d*x)**3/(a*sin(c + d*x) + b*tan(c + d*x)), x)
```

3.256.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.46

$$\int \frac{\sec^3(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx=-\frac{\frac{a^3\log\left(a+b-\frac{(a-b)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^2b^2-b^4}-\frac{a\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{b^2}-\frac{a\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{b^2}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a+b}-\frac{2}{b-\frac{b\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}}{d}$$

input
```
integrate(sec(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
-(a^3*log(a + b - (a - b)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/(a^2*b^2 -
b^4) - a*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/b^2 - a*log(sin(d*x + c)
/(cos(d*x + c) + 1) - 1)/b^2 - log(sin(d*x + c)/(cos(d*x + c) + 1))/(a + b
) - 2/(b - b*sin(d*x + c)^2/(cos(d*x + c) + 1)^2))/d
```

3.256.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.76

$$\int \frac{\sec^3(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx=-\frac{\frac{2a^3\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^2b^2-b^4}-\frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a+b}-\frac{2a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{b^2}+\frac{2\left(a-2b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{b^2\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)}}{2d}$$

input
```
integrate(sec(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
-1/2*(2*a^3*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(
cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^2*b^2 - b^4) - log(abs(-cos(d*x
+ c) + 1)/abs(cos(d*x + c) + 1))/(a + b) - 2*a*log(abs(-(cos(d*x + c) - 1)
/(cos(d*x + c) + 1) - 1))/b^2 + 2*(a - 2*b + a*(cos(d*x + c) - 1)/(cos(d*x
 + c) + 1))/(b^2*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)))/d
```

3.256.9 Mupad [B] (verification not implemented)

Time = 22.26 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.09

$$\int \frac{\sec^3(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx = \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d\,\left(a+b\right)} - \frac{2}{b\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)} + \frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}{b^2\,d} - \frac{a^3\,\ln\left(a+b-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{b^2\,d\,\left(a^2-b^2\right)}$$

input
```
int(1/(cos(c + d*x)^3*(a*sin(c + d*x) + b*tan(c + d*x))),x)
```

output
```
log(tan(c/2 + (d*x)/2))/(d*(a + b)) - 2/(b*d*(tan(c/2 + (d*x)/2)^2 - 1)) +
 (a*log(tan(c/2 + (d*x)/2)^2 - 1))/(b^2*d) - (a^3*log(a + b - a*tan(c/2 +
(d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2))/(b^2*d*(a^2 - b^2))
```

3.257 $\int \frac{\cos^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$

3.257.1 Optimal result

Integrand size = 28, antiderivative size = 243

$$\int \frac{\cos^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx = \frac{2bx}{a^3}+\frac{2b^6\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3(a-b)^{5/2}(a+b)^{5/2}d}+\frac{2b^4(5a^2-3b^2)\,\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3(a-b)^{5/2}(a+b)^{5/2}d}-\frac{\sin(c+dx)}{a^2d}-\frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+dx))}-\frac{\sin(c+dx)}{2(a-b)^2d(1+\cos(c+dx))}-\frac{b^5\sin(c+dx)}{a^2\left(a^2-b^2\right)^2d(b+a\cos(c+dx))}$$

output
```
2*b*x/a^3+2*b^6*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^3/(a
-b)^(5/2)/(a+b)^(5/2)/d+2*b^4*(5*a^2-3*b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*
x+1/2*c)/(a+b)^(1/2))/a^3/(a-b)^(5/2)/(a+b)^(5/2)/d-sin(d*x+c)/a^2/d-1/2*s
in(d*x+c)/(a+b)^2/d/(1-cos(d*x+c))-1/2*sin(d*x+c)/(a-b)^2/d/(1+cos(d*x+c))
-b^5*sin(d*x+c)/a^2/(a^2-b^2)^2/d/(b+a*cos(d*x+c))
```

3.257.2 Mathematica [A] (verified)

Time = 3.99 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.67

$$\int \frac{\cos^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx =$$
$$-\frac{-\frac{4b(c+dx)}{a^3}+\frac{4b^4(5a^2-2b^2)\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3(a^2-b^2)^{5/2}}+\frac{\cot\left(\frac{1}{2}(c+dx)\right)}{(a+b)^2}+\frac{2\sin(c+dx)}{a^2}+\frac{2b^5\sin(c+dx)}{a^2(a-b)^2(a+b)^2(b+a\cos(c+dx))}+}{2d}$$

input
```
Integrate[Cos[c + d*x]^3/(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

output
```
-1/2*((-4*b*(c + d*x))/a^3 + (4*b^4*(5*a^2 - 2*b^2)*ArcTanh[((-a + b)*Tan[
(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^3*(a^2 - b^2)^(5/2)) + Cot[(c + d*x)/2]
/(a + b)^2 + (2*Sin[c + d*x])/a^2 + (2*b^5*Sin[c + d*x])/(a^2*(a - b)^2*(a
 + b)^2*(b + a*Cos[c + d*x])) + Tan[(c + d*x)/2]/(a - b)^2)/d
```

3.257.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 4897, 3042, 25, 3376, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos(c+dx)^3}{(a\sin(c+dx)+b\tan(c+dx))^2}dx$$
$$\Big\downarrow \text{4897}$$
$$\int \frac{\cos^3(c+dx)\cot^2(c+dx)}{(a\cos(c+dx)+b)^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int -\frac{\sin\left(c+dx-\frac{\pi}{2}\right)^5}{\cos\left(c+dx-\frac{\pi}{2}\right)^2\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \; 25$$

$$-\int \frac{\sin\left(\frac{1}{2}(2c-\pi)+dx\right)^5}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^2\left(b-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)^2}dx$$

$$\Big\downarrow \; 3376$$

$$-\int \left(\frac{b^5}{a^3\left(a^2-b^2\right)\left(-b-a\cos(c+dx)\right)^2}-\frac{2b}{a^3}+\frac{\cos(c+dx)}{a^2}-\frac{1}{2(a-b)^2(-\cos(c+dx)-1)}-\frac{1}{2(a+b)^2(1-\cos$$

$$\Big\downarrow \; 2009$$

$$\frac{2b^6\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{5/2}(a+b)^{5/2}}+\frac{2bx}{a^3}-\frac{b^5\sin(c+dx)}{a^2d\left(a^2-b^2\right)^2\left(a\cos(c+dx)+b\right)}-\frac{\sin(c+dx)}{a^2d}+\frac{2b^4\left(5a^2-3b^2\right)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{5/2}(a+b)^{5/2}}-\frac{\sin(c+dx)}{2d(a+b)^2(1-\cos(c+dx))}-\frac{\sin(c+dx)}{2d(a-b)^2(\cos(c+dx)+1)}$$

input
```
Int[Cos[c + d*x]^3/(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

output
```
(2*b*x)/a^3 + (2*b^6*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/
(a^3*(a - b)^(5/2)*(a + b)^(5/2)*d) + (2*b^4*(5*a^2 - 3*b^2)*ArcTanh[(Sqrt
[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^3*(a - b)^(5/2)*(a + b)^(5/2)*d
) - Sin[c + d*x]/(a^2*d) - Sin[c + d*x]/(2*(a + b)^2*d*(1 - Cos[c + d*x]))
 - Sin[c + d*x]/(2*(a - b)^2*d*(1 + Cos[c + d*x])) - (b^5*Sin[c + d*x])/(a
^2*(a^2 - b^2)^2*d*(b + a*Cos[c + d*x]))
```

3.257.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.257. $\int \frac{\cos^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$

rule 3376
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.257.4 Maple [A] (verified)

Time = 5.34 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2\left(a^2-2ab+b^2\right)}-\frac{2b^4\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b}-\frac{\left(5a^2-2b^2\right)\operatorname{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)^2(a-b)^2a^3}-\frac{1}{2(a+b)^2\tan\left(\frac{dx}{2}\right.}}{d}$
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2\left(a^2-2ab+b^2\right)}-\frac{2b^4\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b}-\frac{\left(5a^2-2b^2\right)\operatorname{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)^2(a-b)^2a^3}-\frac{1}{2(a+b)^2\tan\left(\frac{dx}{2}\right.}}{d}$
risch	$\frac{2xb}{a^3}+\frac{ie^{i(dx+c)}}{2a^2d}-\frac{ie^{-i(dx+c)}}{2a^2d}-\frac{2i\left(a^6e^{3i(dx+c)}+a^4b^2e^{3i(dx+c)}+b^6e^{3i(dx+c)}+2a^3b^3e^{2i(dx+c)}+a\,b^5e^{2i(dx+c)}+a^6e^{i}\right.}{a^3(a^2-b^2)^2d\left(a\,e^{4i(dx+c)}+2b\,e^{3i(dx+c)}-2b\,e^{i}\right.}$

input
```
int(cos(d*x+c)^3/(sin(d*x+c)*a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2*tan(1/2*d*x+1/2*c)/(a^2-2*a*b+b^2)-2*b^4/(a+b)^2/(a-b)^2/a^3*(-a
*b*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)-
(5*a^2-2*b^2)/((a+b)*(a-b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a+b)*
(a-b))^(1/2)))-1/2/(a+b)^2/tan(1/2*d*x+1/2*c)+2/a^3*(-tan(1/2*d*x+1/2*c)/(
1+tan(1/2*d*x+1/2*c)^2)*a+2*b*arctan(tan(1/2*d*x+1/2*c))))
```

3.257.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 857, normalized size of antiderivative = 3.53

$$\int \frac{\cos^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$$
$$= \left[-\frac{4\,a^7b-6\,a^5b^3+6\,a^3b^5-4\,ab^7-2\left(a^8-3\,a^6b^2+3\,a^4b^4-a^2b^6\right)\cos\left(dx+c\right)^3+\left(5\,a^2b^5-2\,b^7+\left(5\,a^3b^4\right.\right.}{}\right.$$
$$-\frac{2\,a^7b-3\,a^5b^3+3\,a^3b^5-2\,ab^7-\left(a^8-3\,a^6b^2+3\,a^4b^4-a^2b^6\right)\cos\left(dx+c\right)^3-\left(5\,a^2b^5-2\,b^7+\left(5\,a^3b^4-\right.\right.}{}$$

input
```
integrate(cos(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas"
)
```

output
```
[-1/2*(4*a^7*b - 6*a^5*b^3 + 6*a^3*b^5 - 4*a*b^7 - 2*(a^8 - 3*a^6*b^2 + 3*
a^4*b^4 - a^2*b^6)*cos(d*x + c)^3 + (5*a^2*b^5 - 2*b^7 + (5*a^3*b^4 - 2*a*
b^6)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)
*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*
a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) -
 2*(3*a^7*b - 5*a^5*b^3 + 4*a^3*b^5 - 2*a*b^7)*cos(d*x + c)^2 + 2*(2*a^8 -
 5*a^6*b^2 + 4*a^4*b^4 - a^2*b^6)*cos(d*x + c) - 4*((a^7*b - 3*a^5*b^3 + 3
*a^3*b^5 - a*b^7)*d*x*cos(d*x + c) + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^
8)*d*x)*sin(d*x + c))/(((a^10 - 3*a^8*b^2 + 3*a^6*b^4 - a^4*b^6)*d*cos(d*x
 + c) + (a^9*b - 3*a^7*b^3 + 3*a^5*b^5 - a^3*b^7)*d)*sin(d*x + c)), -(2*a^
7*b - 3*a^5*b^3 + 3*a^3*b^5 - 2*a*b^7 - (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2
*b^6)*cos(d*x + c)^3 - (5*a^2*b^5 - 2*b^7 + (5*a^3*b^4 - 2*a*b^6)*cos(d*x
+ c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2
 - b^2)*sin(d*x + c)))*sin(d*x + c) - (3*a^7*b - 5*a^5*b^3 + 4*a^3*b^5 - 2
*a*b^7)*cos(d*x + c)^2 + (2*a^8 - 5*a^6*b^2 + 4*a^4*b^4 - a^2*b^6)*cos(d*x
 + c) - 2*((a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*x*cos(d*x + c) + (a^6
*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*x)*sin(d*x + c))/(((a^10 - 3*a^8*b^2
 + 3*a^6*b^4 - a^4*b^6)*d*cos(d*x + c) + (a^9*b - 3*a^7*b^3 + 3*a^5*b^5 -
a^3*b^7)*d)*sin(d*x + c))]
```

3.257.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3/(a*sin(d*x+c)+b*tan(d*x+c))**2,x)
```

output
```
Timed out
```

3.257.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima"
)
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.257.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1362 vs. 2(219) = 438.

Time = 0.77 (sec) , antiderivative size = 1362, normalized size of antiderivative = 5.60

$$\int \frac{\cos^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/2*(2*((2*a^4*b - 2*a^3*b^2 - 4*a^2*b^3 - a*b^4 + 2*b^5)*sqrt(-a^2 + b^2
)*abs(a^7 - 2*a^5*b^2 + a^3*b^4)*abs(a - b) - (2*a^11*b - 2*a^10*b^2 - 8*a
^9*b^3 + 13*a^8*b^4 + 12*a^7*b^5 - 24*a^6*b^6 - 8*a^5*b^7 + 17*a^4*b^8 + 2
*a^3*b^9 - 4*a^2*b^10)*sqrt(-a^2 + b^2)*abs(a - b))*(pi*floor(1/2*(d*x + c
)/pi + 1/2) + arctan(tan(1/2*d*x + 1/2*c)/sqrt(-(a^6*b - 2*a^4*b^3 + a^2*b
^5 + sqrt((a^7 + a^6*b - 2*a^5*b^2 - 2*a^4*b^3 + a^3*b^4 + a^2*b^5)*(a^7 -
 a^6*b - 2*a^5*b^2 + 2*a^4*b^3 + a^3*b^4 - a^2*b^5) + (a^6*b - 2*a^4*b^3 +
 a^2*b^5)^2))/(a^7 - a^6*b - 2*a^5*b^2 + 2*a^4*b^3 + a^3*b^4 - a^2*b^5))))
/((a^7 - 2*a^5*b^2 + a^3*b^4)^2*(a^2 - 2*a*b + b^2) + (a^8*b - 2*a^7*b^2 -
 a^6*b^3 + 4*a^5*b^4 - a^4*b^5 - 2*a^3*b^6 + a^2*b^7)*abs(a^7 - 2*a^5*b^2
+ a^3*b^4)) + 2*(2*a^11*b - 2*a^10*b^2 - 8*a^9*b^3 + 13*a^8*b^4 + 12*a^7*b
^5 - 24*a^6*b^6 - 8*a^5*b^7 + 17*a^4*b^8 + 2*a^3*b^9 - 4*a^2*b^10 + 2*a^4*
b*abs(a^7 - 2*a^5*b^2 + a^3*b^4) - 2*a^3*b^2*abs(a^7 - 2*a^5*b^2 + a^3*b^4
) - 4*a^2*b^3*abs(a^7 - 2*a^5*b^2 + a^3*b^4) - a*b^4*abs(a^7 - 2*a^5*b^2 +
 a^3*b^4) + 2*b^5*abs(a^7 - 2*a^5*b^2 + a^3*b^4))*(pi*floor(1/2*(d*x + c)/
pi + 1/2) + arctan(tan(1/2*d*x + 1/2*c)/sqrt(-(a^6*b - 2*a^4*b^3 + a^2*b^5
 - sqrt((a^7 + a^6*b - 2*a^5*b^2 - 2*a^4*b^3 + a^3*b^4 + a^2*b^5)*(a^7 - a
^6*b - 2*a^5*b^2 + 2*a^4*b^3 + a^3*b^4 - a^2*b^5) + (a^6*b - 2*a^4*b^3 + a
^2*b^5)^2))/(a^7 - a^6*b - 2*a^5*b^2 + 2*a^4*b^3 + a^3*b^4 - a^2*b^5))))/(
a^6*b*abs(a^7 - 2*a^5*b^2 + a^3*b^4) - 2*a^4*b^3*abs(a^7 - 2*a^5*b^2 + ...
```

3.257.9 Mupad [B] (verification not implemented)

Time = 29.44 (sec) , antiderivative size = 7329, normalized size of antiderivative = 30.16

$$\int \frac{\cos^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(cos(c + d*x)^3/(a*sin(c + d*x) + b*tan(c + d*x))^2,x)
```

output
```
((a^2 - 2*a*b + b^2)/(a + b) + (2*tan(c/2 + (d*x)/2)^2*(2*a*b^4 + 3*a^4*b
+ 2*a^5 + 4*b^5 - 3*a^2*b^3 - 6*a^3*b^2))/(a^2*(a + b)^2) - (tan(c/2 + (d*
x)/2)^4*(4*a*b^4 - 7*a^4*b + 5*a^5 - 8*b^5 + 7*a^2*b^3 - 5*a^3*b^2))/(a^2*
(a + b)^2))/(d*(tan(c/2 + (d*x)/2)^5*(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) -
 tan(c/2 + (d*x)/2)^3*(4*a^2*b - 8*a*b^2 + 4*b^3) + tan(c/2 + (d*x)/2)*(2*
a*b^2 + 2*a^2*b - 2*a^3 - 2*b^3))) - tan(c/2 + (d*x)/2)/(2*d*(a - b)^2) +
(4*b*atan((1920*a^7*b^22*tan(c/2 + (d*x)/2))/(1920*a^7*b^22 - 1920*a^8*b^2
1 - 16640*a^9*b^20 + 16640*a^10*b^19 + 62080*a^11*b^18 - 62080*a^12*b^17 -
 131072*a^13*b^16 + 131072*a^14*b^15 + 172672*a^15*b^14 - 172672*a^16*b^13
 - 147200*a^17*b^12 + 147200*a^18*b^11 + 81280*a^19*b^10 - 81280*a^20*b^9
- 28160*a^21*b^8 + 28160*a^22*b^7 + 5632*a^23*b^6 - 5632*a^24*b^5 - 512*a^
25*b^4 + 512*a^26*b^3) - (1920*a^8*b^21*tan(c/2 + (d*x)/2))/(1920*a^7*b^22
 - 1920*a^8*b^21 - 16640*a^9*b^20 + 16640*a^10*b^19 + 62080*a^11*b^18 - 62
080*a^12*b^17 - 131072*a^13*b^16 + 131072*a^14*b^15 + 172672*a^15*b^14 - 1
72672*a^16*b^13 - 147200*a^17*b^12 + 147200*a^18*b^11 + 81280*a^19*b^10 -
81280*a^20*b^9 - 28160*a^21*b^8 + 28160*a^22*b^7 + 5632*a^23*b^6 - 5632*a^
24*b^5 - 512*a^25*b^4 + 512*a^26*b^3) - (16640*a^9*b^20*tan(c/2 + (d*x)/2)
)/(1920*a^7*b^22 - 1920*a^8*b^21 - 16640*a^9*b^20 + 16640*a^10*b^19 + 6208
0*a^11*b^18 - 62080*a^12*b^17 - 131072*a^13*b^16 + 131072*a^14*b^15 + 1726
72*a^15*b^14 - 172672*a^16*b^13 - 147200*a^17*b^12 + 147200*a^18*b^11 +...
```

3.257. $\int \frac{\cos^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$

3.258 $\int \frac{\cos^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$

3.258.1 Optimal result

Integrand size = 28, antiderivative size = 227

$$\int \frac{\cos^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx = -\frac{x}{a^2} - \frac{2b^5\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{5/2}(a+b)^{5/2}d} - \frac{4b^3(2a^2-b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{5/2}(a+b)^{5/2}d} - \frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+dx))} + \frac{\sin(c+dx)}{2(a-b)^2d(1+\cos(c+dx))} + \frac{b^4\sin(c+dx)}{a\left(a^2-b^2\right)^2d(b+a\cos(c+dx))}$$

output
```
-x/a^2-2*b^5*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^2/(a-b)
^(5/2)/(a+b)^(5/2)/d-4*b^3*(2*a^2-b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2
*c)/(a+b)^(1/2))/a^2/(a-b)^(5/2)/(a+b)^(5/2)/d-1/2*sin(d*x+c)/(a+b)^2/d/(1
-cos(d*x+c))+1/2*sin(d*x+c)/(a-b)^2/d/(1+cos(d*x+c))+b^4*sin(d*x+c)/a/(a^2
-b^2)^2/d/(b+a*cos(d*x+c))
```

3.258.2 Mathematica [A] (verified)

Time = 2.59 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.67

$$\int \frac{\cos^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx = \frac{-\frac{2(c+dx)}{a^2}-\frac{4b^3(-4a^2+b^2)\text{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)^{5/2}}-\frac{\cot\left(\frac{1}{2}(c+dx)\right)}{(a+b)^2}+\frac{2b^4\sin(c+dx)}{a(a-b)^2(a+b)^2(b+a\cos(c+dx))}+\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{(a-b)^2}}{2d}$$

input
```
Integrate[Cos[c + d*x]^2/(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

output
```
((-2*(c + d*x))/a^2 - (4*b^3*(-4*a^2 + b^2)*ArcTanh[((-a + b)*Tan[(c + d*x
)/2])/Sqrt[a^2 - b^2]])/(a^2*(a^2 - b^2)^(5/2)) - Cot[(c + d*x)/2]/(a + b)
^2 + (2*b^4*Sin[c + d*x])/(a*(a - b)^2*(a + b)^2*(b + a*Cos[c + d*x])) + T
an[(c + d*x)/2]/(a - b)^2)/(2*d)
```

3.258.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3042, 4897, 3042, 3376, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(c+dx)^2}{(a\sin(c+dx)+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{ 4897}$$

$$\int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(a\cos(c+dx)+b)^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin\left(c+dx-\frac{\pi}{2}\right)^4}{\cos\left(c+dx-\frac{\pi}{2}\right)^2\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3376}$$

$$\int \left(\frac{b^4}{a^2\left(a^2-b^2\right)(-a\cos(c+dx)-b)^2}+\frac{2\left(2a^2b^3-b^5\right)}{a^2\left(a^2-b^2\right)^2(-a\cos(c+dx)-b)}-\frac{1}{a^2}-\frac{1}{2(a-b)^2(-\cos(c+dx)-1)}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2b^5\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{5/2}(a+b)^{5/2}}-\frac{4b^3\left(2a^2-b^2\right)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{5/2}(a+b)^{5/2}}+\frac{b^4\sin(c+dx)}{ad\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}-\frac{x}{a^2}-\frac{\sin(c+dx)}{2d(a+b)^2(1-\cos(c+dx))}+\frac{\sin(c+dx)}{2d(a-b)^2(\cos(c+dx)+1)}$$

input
```
Int[Cos[c + d*x]^2/(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

output
```
-(x/a^2) - (2*b^5*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^
2*(a - b)^(5/2)*(a + b)^(5/2)*d) - (4*b^3*(2*a^2 - b^2)*ArcTanh[(Sqrt[a -
b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2*(a - b)^(5/2)*(a + b)^(5/2)*d) - S
in[c + d*x]/(2*(a + b)^2*d*(1 - Cos[c + d*x])) + Sin[c + d*x]/(2*(a - b)^2
*d*(1 + Cos[c + d*x])) + (b^4*Sin[c + d*x])/(a*(a^2 - b^2)^2*d*(b + a*Cos[
c + d*x]))
```

3.258.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3376
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.258.4 Maple [A] (verified)

Time = 1.43 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.81

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2-4ab+2b^2}-\frac{1}{2(a+b)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}+\frac{2b^3\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b}-\frac{\left(4a^2-b^2\right)\operatorname{arctanh}}{\sqrt{(a}}\right.}{(a+b)^2(a-b)^2a^2}}{d}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2-4ab+2b^2}-\frac{1}{2(a+b)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}+\frac{2b^3\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b}-\frac{\left(4a^2-b^2\right)\operatorname{arctanh}}{\sqrt{(a}}\right.}{(a+b)^2(a-b)^2a^2}}{d}$
risch	$-\frac{x}{a^2}-\frac{2i\left(-2a^4b\,e^{3i(dx+c)}-b^5e^{3i(dx+c)}+a^5e^{2i(dx+c)}-3a^3b^2e^{2i(dx+c)}-a\,b^4e^{2i(dx+c)}+2a^2b^3e^{i(dx+c)}+b^5e^{i(dx+c)}+a^5\right.}{\left(e^{2i(dx+c)}a+2b\,e^{i(dx+c)}+a\right)\left(a^2-b^2\right)^2a^2\left(e^{2i(dx+c)}-1\right)d}$

input
```
int(cos(d*x+c)^2/(sin(d*x+c)*a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2*tan(1/2*d*x+1/2*c)/(a^2-2*a*b+b^2)-1/2/(a+b)^2/tan(1/2*d*x+1/2*c)
-2/a^2*arctan(tan(1/2*d*x+1/2*c))+2*b^3/(a+b)^2/(a-b)^2/a^2*(-a*b*tan(1/2*
d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)-(4*a^2-b^2)
/((a+b)*(a-b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a+b)*(a-b))^(1/2))
))
```

3.258.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 705, normalized size of antiderivative = 3.11

$$\int \frac{\cos^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$$
$$=\left[\frac{4\,a^5b^2-2\,a^3b^4-2\,ab^6-\left(4\,a^2b^4-b^6+\left(4\,a^3b^3-ab^5\right)\cos\left(dx+c\right)\right)\sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos(}{a^2\cos}\right.}{}\right.$$

input
```
integrate(cos(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas"
)
```

output
```
[1/2*(4*a^5*b^2 - 2*a^3*b^4 - 2*a*b^6 - (4*a^2*b^4 - b^6 + (4*a^3*b^3 - a*
b^5)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)
*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*
a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) -
 2*(a^7 - a*b^6)*cos(d*x + c)^2 + 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*cos(d*x
+ c) - 2*((a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*x*cos(d*x + c) + (a^6*b
- 3*a^4*b^3 + 3*a^2*b^5 - b^7)*d*x)*sin(d*x + c))/(((a^9 - 3*a^7*b^2 + 3*a
^5*b^4 - a^3*b^6)*d*cos(d*x + c) + (a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^
7)*d)*sin(d*x + c)), (2*a^5*b^2 - a^3*b^4 - a*b^6 - (4*a^2*b^4 - b^6 + (4*
a^3*b^3 - a*b^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(
b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))*sin(d*x + c) - (a^7 - a*b^
6)*cos(d*x + c)^2 + (a^6*b - 2*a^4*b^3 + a^2*b^5)*cos(d*x + c) - ((a^7 - 3
*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*x*cos(d*x + c) + (a^6*b - 3*a^4*b^3 + 3*a^
2*b^5 - b^7)*d*x)*sin(d*x + c))/(((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*
d*cos(d*x + c) + (a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*d)*sin(d*x + c)
)]
```

3.258.6 Sympy [F]

$$\int \frac{\cos^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx=\int \frac{\cos^2{(c+dx)}}{(a\sin{(c+dx)}+b\tan{(c+dx)})^2}\,dx$$

input
```
integrate(cos(d*x+c)**2/(a*sin(d*x+c)+b*tan(d*x+c))**2,x)
```

output
```
Integral(cos(c + d*x)**2/(a*sin(c + d*x) + b*tan(c + d*x))**2, x)
```

3.258.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima"
)
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.258.8 Giac [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.46

$$\int \frac{\cos^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$$
$$= \frac{\frac{4\,(4\,a^2b^3-b^5)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^6-2\,a^4b^2+a^2b^4)\sqrt{-a^2+b^2}}+\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2-2\,ab+b^2}-\frac{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,a^3b\tan\left(\frac{1}{2}\right.\cdots}{(a^5-2\,a^3b^2+\cdots}}{2\,d}$$

input
```
integrate(cos(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/2*(4*(4*a^2*b^3 - b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b)
+ arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2
)))/((a^6 - 2*a^4*b^2 + a^2*b^4)*sqrt(-a^2 + b^2)) + tan(1/2*d*x + 1/2*c)/
(a^2 - 2*a*b + b^2) - (a^4*tan(1/2*d*x + 1/2*c)^2 - 3*a^3*b*tan(1/2*d*x +
1/2*c)^2 + 3*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 - a*b^3*tan(1/2*d*x + 1/2*c)^2
 + 4*b^4*tan(1/2*d*x + 1/2*c)^2 - a^4 + a^3*b + a^2*b^2 - a*b^3)/((a^5 - 2
*a^3*b^2 + a*b^4)*(a*tan(1/2*d*x + 1/2*c)^3 - b*tan(1/2*d*x + 1/2*c)^3 - a
*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))) - 2*(d*x + c)/a^2)/d
```

3.258.9 Mupad [B] (verification not implemented)

Time = 27.94 (sec) , antiderivative size = 6093, normalized size of antiderivative = 26.84

$$\int \frac{\cos^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(cos(c + d*x)^2/(a*sin(c + d*x) + b*tan(c + d*x))^2,x)
```

output
```
((a^2 - 2*a*b + b^2)/(a + b) - (tan(c/2 + (d*x)/2)^2*(a^4 - 3*a^3*b - a*b^
3 + 4*b^4 + 3*a^2*b^2))/(a*(a + b)^2))/(d*(tan(c/2 + (d*x)/2)^3*(6*a*b^2 -
 6*a^2*b + 2*a^3 - 2*b^3) + tan(c/2 + (d*x)/2)*(2*a*b^2 + 2*a^2*b - 2*a^3
- 2*b^3))) - (2*atan(-((tan(c/2 + (d*x)/2)*(32*a^26 - 96*a^25*b - 64*a^3*b
^23 + 128*a^4*b^22 + 672*a^5*b^21 - 1376*a^6*b^20 - 3008*a^7*b^19 + 6528*a
^8*b^18 + 7072*a^9*b^17 - 17632*a^10*b^16 - 8480*a^11*b^15 + 29600*a^12*b^
14 + 2176*a^13*b^13 - 31744*a^14*b^12 + 8224*a^15*b^11 + 21344*a^16*b^10 -
 12992*a^17*b^9 - 8128*a^18*b^8 + 9568*a^19*b^7 + 992*a^20*b^6 - 4000*a^21
*b^5 + 480*a^22*b^4 + 928*a^23*b^3 - 224*a^24*b^2) - ((32*a^28 - 32*a^27*b
 + 32*a^6*b^22 - 416*a^8*b^20 + 224*a^9*b^19 + 2080*a^10*b^18 - 1824*a^11*
b^17 - 5472*a^12*b^16 + 6528*a^13*b^15 + 8256*a^14*b^14 - 13440*a^15*b^13
- 6720*a^16*b^12 + 17472*a^17*b^11 + 1344*a^18*b^10 - 14784*a^19*b^9 + 288
0*a^20*b^8 + 8064*a^21*b^7 - 3168*a^22*b^6 - 2688*a^23*b^5 + 1504*a^24*b^4
 + 480*a^25*b^3 - 352*a^26*b^2 - (tan(c/2 + (d*x)/2)*(128*a^8*b^22 - 64*a^
7*b^23 - 64*a^29*b + 576*a^9*b^21 - 1280*a^10*b^20 - 2240*a^11*b^19 + 5760
*a^12*b^18 + 4800*a^13*b^17 - 15360*a^14*b^16 - 5760*a^15*b^15 + 26880*a^1
6*b^14 + 2688*a^17*b^13 - 32256*a^18*b^12 + 2688*a^19*b^11 + 26880*a^20*b^
10 - 5760*a^21*b^9 - 15360*a^22*b^8 + 4800*a^23*b^7 + 5760*a^24*b^6 - 2240
*a^25*b^5 - 1280*a^26*b^4 + 576*a^27*b^3 + 128*a^28*b^2)*1i)/a^2)*1i)/a^2)
/a^2 + (tan(c/2 + (d*x)/2)*(32*a^26 - 96*a^25*b - 64*a^3*b^23 + 128*a^4...
```

3.258. $\int \frac{\cos^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$

3.259 $\int \frac{\cos(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$

3.259.1 Optimal result

Integrand size = 26, antiderivative size = 219

$$\int \frac{\cos(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx = \frac{2b^4\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a(a-b)^{5/2}(a+b)^{5/2}d} + \frac{2b^2(3a^2-b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a(a-b)^{5/2}(a+b)^{5/2}d} - \frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+dx))} - \frac{\sin(c+dx)}{2(a-b)^2d(1+\cos(c+dx))} - \frac{b^3\sin(c+dx)}{(a^2-b^2)^2\,d(b+a\cos(c+dx))}$$

output
```
2*b^4*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a/(a-b)^(5/2)/(a
+b)^(5/2)/d+2*b^2*(3*a^2-b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)
^(1/2))/a/(a-b)^(5/2)/(a+b)^(5/2)/d-1/2*sin(d*x+c)/(a+b)^2/d/(1-cos(d*x+c)
)-1/2*sin(d*x+c)/(a-b)^2/d/(1+cos(d*x+c))-b^3*sin(d*x+c)/(a^2-b^2)^2/d/(b+
a*cos(d*x+c))
```

3.259.2 Mathematica [A] (verified)

Time = 1.83 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.60

$$\int \frac{\cos(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx = \frac{-\frac{12ab^2\text{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{(-3b^3-2a(a^2-b^2)\cos(c+dx)+(2a^2b+b^3)\cos(2(c+dx)))\csc(c+dx)}{b+a\cos(c+dx)}}{2(a-b)^2(a+b)^2d}$$

input
```
Integrate[Cos[c + d*x]/(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

output
```
((-12*a*b^2*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2
 - b^2] + ((-3*b^3 - 2*a*(a^2 - b^2)*Cos[c + d*x] + (2*a^2*b + b^3)*Cos[2*
(c + d*x)])*Csc[c + d*x])/(b + a*Cos[c + d*x]))/(2*(a - b)^2*(a + b)^2*d)
```

3.259.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3042, 4897, 3042, 25, 3376, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \frac{\cos(c+dx)\cot^2(c+dx)}{(a\cos(c+dx)+b)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sin\left(c+dx-\frac{\pi}{2}\right)^3}{\cos\left(c+dx-\frac{\pi}{2}\right)^2\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sin\left(\frac{1}{2}(2c-\pi)+dx\right)^3}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^2\left(b-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)^2}dx$$

$$\Big\downarrow \text{3376}$$

$$-\int \left(-\frac{b^3}{a\left(b^2-a^2\right)\left(b+a\cos(c+dx)\right)^2}-\frac{1}{2(a-b)^2(-\cos(c+dx)-1)}-\frac{1}{2(a+b)^2(1-\cos(c+dx))}+\frac{}{a\left(a^2-b\right.}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2b^2\left(3a^2-b^2\right)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad(a-b)^{5/2}(a+b)^{5/2}}-\frac{b^3\sin(c+dx)}{d\left(a^2-b^2\right)^2\left(a\cos(c+dx)+b\right)}+\frac{2b^4\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad(a-b)^{5/2}(a+b)^{5/2}}-\frac{\sin(c+dx)}{2d(a+b)^2(1-\cos(c+dx))}-\frac{\sin(c+dx)}{2d(a-b)^2(\cos(c+dx)+1)}$$

input
```
Int[Cos[c + d*x]/(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

output
```
(2*b^4*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*(a - b)^(5/
2)*(a + b)^(5/2)*d) + (2*b^2*(3*a^2 - b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d
*x)/2])/Sqrt[a + b]])/(a*(a - b)^(5/2)*(a + b)^(5/2)*d) - Sin[c + d*x]/(2*
(a + b)^2*d*(1 - Cos[c + d*x])) - Sin[c + d*x]/(2*(a - b)^2*d*(1 + Cos[c +
 d*x])) - (b^3*Sin[c + d*x])/((a^2 - b^2)^2*d*(b + a*Cos[c + d*x]))
```

3.259.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3376
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.259.4 Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.71

method	result
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2\left(a^2-2ab+b^2\right)}-\frac{1}{2(a+b)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2b^2\left(-\frac{b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b}-\frac{3a\,\operatorname{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a-b)^2(a+b)^2}}{d}$
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2\left(a^2-2ab+b^2\right)}-\frac{1}{2(a+b)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2b^2\left(-\frac{b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b}-\frac{3a\,\operatorname{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a-b)^2(a+b)^2}}{d}$
risch	$-\frac{2i\left(a^4e^{3i(dx+c)}+a^2b^2e^{3i(dx+c)}+b^4e^{3i(dx+c)}+3a\,b^3e^{2i(dx+c)}+a^4e^{i(dx+c)}-3a^2b^2e^{i(dx+c)}-b^4e^{i(dx+c)}-2a^3b-a\,b^3\right)}{\left(e^{2i(dx+c)}a+2b\,e^{i(dx+c)}+a\right)\left(a^2-b^2\right)^2a\left(e^{2i(dx+c)}-1\right)d}+$

input
```
int(cos(d*x+c)/(sin(d*x+c)*a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2*tan(1/2*d*x+1/2*c)/(a^2-2*a*b+b^2)-1/2/(a+b)^2/tan(1/2*d*x+1/2*c
)-2*b^2/(a-b)^2/(a+b)^2*(-b*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan
(1/2*d*x+1/2*c)^2*b-a-b)-3*a/((a+b)*(a-b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c
)*(a-b)/((a+b)*(a-b))^(1/2))))
```

3.259.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 518, normalized size of antiderivative = 2.37

$$\int \frac{\cos(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$$

$$= \left[-\frac{2\,a^4b+2\,a^2b^3-4\,b^5-3\left(a^2b^2\cos\left(dx+c\right)+ab^3\right)\sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2+2\,\sqrt{a^2-b^2}(b}{a^2\cos(dx+c)^2+2\,ab\cos(d}\right.}{2\left((a^7-3\,a^5b^2+3\,a^3b^4-ab^6)d\cos\left(dx\right.\right.}\right.$$

$$-\frac{a^4b+a^2b^3-2\,b^5-3\left(a^2b^2\cos\left(dx+c\right)+ab^3\right)\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cos(dx+c)+a)}{(a^2-b^2)\sin(dx+c)}\right)\sin\left(dx+c\right)-}{\left((a^7-3\,a^5b^2+3\,a^3b^4-ab^6)d\cos\left(dx+c\right)+\left(a^6b-3\,a^4b^3+\right.\right.}$$

input
```
integrate(cos(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
[-1/2*(2*a^4*b + 2*a^2*b^3 - 4*b^5 - 3*(a^2*b^2*cos(d*x + c) + a*b^3)*sqrt
(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqr
t(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x
 + c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) - 2*(2*a^4*b - a^2*b^3 -
 b^5)*cos(d*x + c)^2 + 2*(a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x + c))/(((a^7 -
3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c) + (a^6*b - 3*a^4*b^3 + 3*a^2
*b^5 - b^7)*d)*sin(d*x + c)), -(a^4*b + a^2*b^3 - 2*b^5 - 3*(a^2*b^2*cos(d
*x + c) + a*b^3)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c)
 + a)/((a^2 - b^2)*sin(d*x + c)))*sin(d*x + c) - (2*a^4*b - a^2*b^3 - b^5)
*cos(d*x + c)^2 + (a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x + c))/(((a^7 - 3*a^5*b
^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c) + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 -
b^7)*d)*sin(d*x + c))]
```

3.259.6 Sympy [F]

$$\int \frac{\cos(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx=\int \frac{\cos{(c+dx)}}{(a\sin{(c+dx)}+b\tan{(c+dx)})^2}\,dx$$

input
```
integrate(cos(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))**2,x)
```

output
```
Integral(cos(c + d*x)/(a*sin(c + d*x) + b*tan(c + d*x))**2, x)
```

3.259.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(cos(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.259.8 Giac [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.29

$$\int \frac{\cos(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx = -\frac{\frac{12\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)ab^2}{(a^4-2a^2b^2+b^4)\sqrt{-a^2+b^2}}+\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^2-2ab+b^2}+\frac{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-3a^2b\tan\left(\frac{1}{2}dx+\cdots\right.}{(a^4-2a^2b^2+b^4)\left(a\tan\left(\frac{1}{2}dx+\cdots\right.\right.}}{2d}$$

input
```
integrate(cos(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/2*(12*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(
1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*a*b^2/((a^4
- 2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)) + tan(1/2*d*x + 1/2*c)/(a^2 - 2*a*b +
 b^2) + (a^3*tan(1/2*d*x + 1/2*c)^2 - 3*a^2*b*tan(1/2*d*x + 1/2*c)^2 + 3*a
*b^2*tan(1/2*d*x + 1/2*c)^2 - 5*b^3*tan(1/2*d*x + 1/2*c)^2 - a^3 + a^2*b +
 a*b^2 - b^3)/((a^4 - 2*a^2*b^2 + b^4)*(a*tan(1/2*d*x + 1/2*c)^3 - b*tan(1
/2*d*x + 1/2*c)^3 - a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))))/d
```

3.259.9 Mupad [B] (verification not implemented)

Time = 23.40 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.97

$$\int \frac{\cos(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$$
$$= \frac{\frac{a^2-2ab+b^2}{a+b}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(a^3-3a^2b+3ab^2-5b^3)}{(a+b)^2}}{d\left((2a^3-6a^2b+6ab^2-2b^3)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+(-2a^3+2a^2b+2ab^2-2b^3)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}$$
$$-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2d(a-b)^2}+\frac{6ab^2\,\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(a^4-2a^2b^2+b^4)}{(a+b)^{5/2}(a-b)^{3/2}}\right)}{d(a+b)^{5/2}(a-b)^{5/2}}$$

input
```
int(cos(c + d*x)/(a*sin(c + d*x) + b*tan(c + d*x))^2,x)
```

output
```
((a^2 - 2*a*b + b^2)/(a + b) - (tan(c/2 + (d*x)/2)^2*(3*a*b^2 - 3*a^2*b +
a^3 - 5*b^3))/(a + b)^2)/(d*(tan(c/2 + (d*x)/2)^3*(6*a*b^2 - 6*a^2*b + 2*a
^3 - 2*b^3) + tan(c/2 + (d*x)/2)*(2*a*b^2 + 2*a^2*b - 2*a^3 - 2*b^3))) - t
an(c/2 + (d*x)/2)/(2*d*(a - b)^2) + (6*a*b^2*atanh((tan(c/2 + (d*x)/2)*(a^
4 + b^4 - 2*a^2*b^2))/((a + b)^(5/2)*(a - b)^(3/2))))/(d*(a + b)^(5/2)*(a
- b)^(5/2))
```

3.259. $\int \frac{\cos(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$

3.260 $\int \frac{1}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$

3.260.1 Optimal result

Integrand size = 19, antiderivative size = 203

$$\int \frac{1}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx = -\frac{4a^2b\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{2b^3\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+dx))} + \frac{\sin(c+dx)}{2(a-b)^2d(1+\cos(c+dx))} + \frac{ab^2\sin(c+dx)}{(a^2-b^2)^2\,d(b+a\cos(c+dx))}$$

output
```
-4*a^2*b*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2)/(
a+b)^(5/2)/d-2*b^3*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-
b)^(5/2)/(a+b)^(5/2)/d-1/2*sin(d*x+c)/(a+b)^2/d/(1-cos(d*x+c))+1/2*sin(d*x
+c)/(a-b)^2/d/(1+cos(d*x+c))+a*b^2*sin(d*x+c)/(a^2-b^2)^2/d/(b+a*cos(d*x+c
))
```

3.260.2 Mathematica [A] (verified)

Time = 1.46 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.63

$$\int \frac{1}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx = \frac{\frac{4b(2a^2+b^2)\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}-\frac{\cot\left(\frac{1}{2}(c+dx)\right)}{(a+b)^2}+\frac{\frac{2ab^2\sin(c+dx)}{(a+b)^2(b+a\cos(c+dx))}+\tan\left(\frac{1}{2}(c+dx)\right)}{(a-b)^2}}{2d}$$

input
```
Integrate[(a*Sin[c + d*x] + b*Tan[c + d*x])^(-2),x]
```

output
```
((4*b*(2*a^2 + b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/
(a^2 - b^2)^(5/2) - Cot[(c + d*x)/2]/(a + b)^2 + ((2*a*b^2*Sin[c + d*x])/(
(a + b)^2*(b + a*Cos[c + d*x])) + Tan[(c + d*x)/2])/(a - b)^2)/(2*d)
```

3.260.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3042, 4897, 3042, 3210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{(a\sin(c+dx)+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \ 4897$$

$$\int \frac{\cot^2(c+dx)}{(a\cos(c+dx)+b)^2}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\tan\left(c+dx-\frac{\pi}{2}\right)^2}{\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \ 3210$$

$$\int \left(-\frac{2a^2b}{(a^2-b^2)^2\,(a\cos(c+dx)+b)}-\frac{b^2}{(b^2-a^2)\,(a\cos(c+dx)+b)^2}-\frac{1}{2(a+b)^2(\cos(c+dx)-1)}+\frac{}{2(a-b)^2(\text{co}}\right.$$

$$\Big\downarrow \; 2009$$

$$-\frac{4a^2b\,\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}+\frac{ab^2\sin(c+dx)}{d\,(a^2-b^2)^2\,(a\cos(c+dx)+b)}-\frac{2b^3\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{\sin(c+dx)}{2d(a+b)^2(1-\cos(c+dx))}+\frac{\sin(c+dx)}{2d(a-b)^2(\cos(c+dx)+1)}$$

input
```
Int[(a*Sin[c + d*x] + b*Tan[c + d*x])^(-2),x]
```

output
```
(-4*a^2*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5
/2)*(a + b)^(5/2)*d) - (2*b^3*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[
a + b]])/((a - b)^(5/2)*(a + b)^(5/2)*d) - Sin[c + d*x]/(2*(a + b)^2*d*(1
- Cos[c + d*x])) + Sin[c + d*x]/(2*(a - b)^2*d*(1 + Cos[c + d*x])) + (a*b^
2*Sin[c + d*x])/((a^2 - b^2)^2*d*(b + a*Cos[c + d*x]))
```

3.260.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3210
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e + f*x])^m/
(1 - Sin[e + f*x]^2)^(p/2)), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 -
b^2, 0] && IntegersQ[m, p/2]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.260. $\int \frac{1}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$

3.260.4 Maple [A] (verified)

Time = 1.39 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.80

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2-4ab+2b^2}+\frac{2b\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b}-\frac{\left(2a^2+b^2\right)\operatorname{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a-b)^2(a+b)^2}-\frac{1}{2(a+b)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{d}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2-4ab+2b^2}+\frac{2b\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b}-\frac{\left(2a^2+b^2\right)\operatorname{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a-b)^2(a+b)^2}-\frac{1}{2(a+b)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{d}$
risch	$-\frac{2i\left(-2a^2b\,\mathrm{e}^{3i(dx+c)}-b^3\mathrm{e}^{3i(dx+c)}+a^3\mathrm{e}^{2i(dx+c)}-4a\,b^2\mathrm{e}^{2i(dx+c)}+3b^3\mathrm{e}^{i(dx+c)}+a^3+2a\,b^2\right)}{\left(\mathrm{e}^{2i(dx+c)}a+2b\,\mathrm{e}^{i(dx+c)}+a\right)\left(a^2-b^2\right)^2\left(\mathrm{e}^{2i(dx+c)}-1\right)d}+\frac{2b\ln\left(\mathrm{e}^{i(dx+c)}-\frac{ia^2-ib^2}{a\sqrt{}}\right.}{\sqrt{a^2-b^2}\,(a+b)^2(}$

input
```
int(1/(sin(d*x+c)*a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2*tan(1/2*d*x+1/2*c)/(a^2-2*a*b+b^2)+2*b/(a-b)^2/(a+b)^2*(-a*b*tan(
1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)-(2*a^2+
b^2)/((a+b)*(a-b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a+b)*(a-b))^(1
/2)))-1/2/(a+b)^2/tan(1/2*d*x+1/2*c))
```

3.260.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 526, normalized size of antiderivative = 2.59

$$\int \frac{1}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$$
$$= \left[\frac{6\,a^3b^2-6\,ab^4+\left(2\,a^2b^2+b^4+\left(2\,a^3b+ab^3\right)\cos\left(dx+c\right)\right)\sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos(dx+c)^2-2\,\sqrt{}}{a^2\cos(dx+c)^2+2}\right.}{2\left(\left(a^7-3\,a^5b^2+3\,a^3b^4-ab^6\right)d\cos\left(d\right.\right.}\right.$$

input
```
integrate(1/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
[1/2*(6*a^3*b^2 - 6*a*b^4 + (2*a^2*b^2 + b^4 + (2*a^3*b + a*b^3)*cos(d*x +
 c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^
2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^
2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) - 2*(a^5 + a^3*
b^2 - 2*a*b^4)*cos(d*x + c)^2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c))/
(((a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c) + (a^6*b - 3*a^4*b^
3 + 3*a^2*b^5 - b^7)*d)*sin(d*x + c)), (3*a^3*b^2 - 3*a*b^4 - (2*a^2*b^2 +
 b^4 + (2*a^3*b + a*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2
+ b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))*sin(d*x + c) - (a^
5 + a^3*b^2 - 2*a*b^4)*cos(d*x + c)^2 + (a^4*b - 2*a^2*b^3 + b^5)*cos(d*x
+ c))/(((a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c) + (a^6*b - 3*
a^4*b^3 + 3*a^2*b^5 - b^7)*d)*sin(d*x + c))]
```

3.260.6 Sympy [F]

$$\int \frac{1}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx = \int \frac{1}{(a\sin{(c+dx)}+b\tan{(c+dx)})^2}\,dx$$

input
```
integrate(1/(a*sin(d*x+c)+b*tan(d*x+c))**2,x)
```

output
```
Integral((a*sin(c + d*x) + b*tan(c + d*x))**(-2), x)
```

3.260.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.260.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.42

$$\int \frac{1}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$$
$$= \frac{\frac{4\,(2\,a^2b+b^3)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^4-2\,a^2b^2+b^4)\sqrt{-a^2+b^2}}+\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2-2\,ab+b^2}-\frac{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,a^2b\tan\left(\frac{1}{2}\right.}{(a^4-2\,a^2b^2+b^4)\left(a\tan\left(\frac{1}{2}\,d\right.\right.}}{2\,d}$$

input
```
integrate(1/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/2*(4*(2*a^2*b + b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) +
arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2))
)/((a^4 - 2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)) + tan(1/2*d*x + 1/2*c)/(a^2 -
 2*a*b + b^2) - (a^3*tan(1/2*d*x + 1/2*c)^2 - 3*a^2*b*tan(1/2*d*x + 1/2*c)
^2 + 7*a*b^2*tan(1/2*d*x + 1/2*c)^2 - b^3*tan(1/2*d*x + 1/2*c)^2 - a^3 + a
^2*b + a*b^2 - b^3)/((a^4 - 2*a^2*b^2 + b^4)*(a*tan(1/2*d*x + 1/2*c)^3 - b
*tan(1/2*d*x + 1/2*c)^3 - a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))
))/d
```

3.260.9 Mupad [B] (verification not implemented)

Time = 23.43 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.21

$$\int \frac{1}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$$
$$= \frac{\frac{a^2-2\,a\,b+b^2}{a+b}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,(a^3-3\,a^2\,b+7\,a\,b^2-b^3)}{(a+b)^2}}{d\,\left(\left(2\,a^3-6\,a^2\,b+6\,a\,b^2-2\,b^3\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(-2\,a^3+2\,a^2\,b+2\,a\,b^2-2\,b^3\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}$$
$$+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d\,(a-b)^2}+\frac{b\,\mathrm{atan}\left(\frac{1\mathrm{i}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^4-2\mathrm{i}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b^2+1\mathrm{i}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^4}{(a+b)^{5/2}\,(a-b)^{3/2}}\right)\,\left(2\,a^2+b^2\right)\,2\mathrm{i}}{d\,(a+b)^{5/2}\,(a-b)^{5/2}}$$

input
```
int(1/(a*sin(c + d*x) + b*tan(c + d*x))^2,x)
```

output
```
((a^2 - 2*a*b + b^2)/(a + b) - (tan(c/2 + (d*x)/2)^2*(7*a*b^2 - 3*a^2*b +
a^3 - b^3))/(a + b)^2)/(d*(tan(c/2 + (d*x)/2)^3*(6*a*b^2 - 6*a^2*b + 2*a^3
 - 2*b^3) + tan(c/2 + (d*x)/2)*(2*a*b^2 + 2*a^2*b - 2*a^3 - 2*b^3))) + tan
(c/2 + (d*x)/2)/(2*d*(a - b)^2) + (b*atan((a^4*tan(c/2 + (d*x)/2)*1i + b^4
*tan(c/2 + (d*x)/2)*1i - a^2*b^2*tan(c/2 + (d*x)/2)*2i)/((a + b)^(5/2)*(a
- b)^(3/2)))*(2*a^2 + b^2)*2i)/(d*(a + b)^(5/2)*(a - b)^(5/2))
```

3.260. $\int \frac{1}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$

3.261 $\int \frac{\sec(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$

3.261.1 Optimal result

Integrand size = 26, antiderivative size = 136

$$\int \frac{\sec(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx = \frac{2a(a^2+2b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{b\csc(c+dx)}{(a^2-b^2)\,d(b+a\cos(c+dx))} - \frac{(a^2+2b^2-3ab\cos(c+dx))\csc(c+dx)}{(a^2-b^2)^2\,d}$$

output
```
2*a*(a^2+2*b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^
(5/2)/(a+b)^(5/2)/d-b*csc(d*x+c)/(a^2-b^2)/d/(b+a*cos(d*x+c))-(a^2+2*b^2-3
*a*b*cos(d*x+c))*csc(d*x+c)/(a^2-b^2)^2/d
```

3.261.2 Mathematica [A] (verified)

Time = 1.45 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.93

$$\int \frac{\sec(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx = -\frac{\frac{4a(a^2+2b^2)\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} + \frac{\cot\left(\frac{1}{2}(c+dx)\right)}{(a+b)^2} + \frac{\frac{2a^2b\sin(c+dx)}{(a+b)^2(b+a\cos(c+dx))}+\tan\left(\frac{1}{2}(c+dx)\right)}{(a-b)^2}}{2d}$$

input
```
Integrate[Sec[c + d*x]/(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

output
```
-1/2*((4*a*(a^2 + 2*b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^
2]])/(a^2 - b^2)^(5/2) + Cot[(c + d*x)/2]/(a + b)^2 + ((2*a^2*b*Sin[c + d*
x])/((a + b)^2*(b + a*Cos[c + d*x])) + Tan[(c + d*x)/2])/(a - b)^2)/d
```

3.261.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.18, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3042, 4897, 3042, 25, 3343, 25, 3042, 3345, 27, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \frac{\cot(c+dx)\csc(c+dx)}{(a\cos(c+dx)+b)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sin\left(c+dx-\frac{\pi}{2}\right)}{\cos\left(c+dx-\frac{\pi}{2}\right)^2\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^2\left(b-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)^2}dx$$

$$\Big\downarrow \text{3343}$$

$$-\frac{\int -\frac{(a-2b\cos(c+dx))\csc^2(c+dx)}{b+a\cos(c+dx)}dx}{a^2-b^2}-\frac{b\csc(c+dx)}{d\left(a^2-b^2\right)(a\cos(c+dx)+b)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{(a-2b\cos(c+dx))\csc^2(c+dx)}{b+a\cos(c+dx)}dx}{a^2-b^2}-\frac{b\csc(c+dx)}{d(a^2-b^2)(a\cos(c+dx)+b)}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{a+2b\sin\left(c+dx-\frac{\pi}{2}\right)}{\cos\left(c+dx-\frac{\pi}{2}\right)^2\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{a^2-b^2}-\frac{b\csc(c+dx)}{d(a^2-b^2)(a\cos(c+dx)+b)}$$

$\Big\downarrow$ 3345

$$\frac{\frac{\int \frac{a(a^2+2b^2)}{b+a\cos(c+dx)}dx}{a^2-b^2}-\frac{\csc(c+dx)(a^2-3ab\cos(c+dx)+2b^2)}{d(a^2-b^2)}}{a^2-b^2}-\frac{b\csc(c+dx)}{d(a^2-b^2)(a\cos(c+dx)+b)}$$

$\Big\downarrow$ 27

$$\frac{\frac{a(a^2+2b^2)\int \frac{1}{b+a\cos(c+dx)}dx}{a^2-b^2}-\frac{\csc(c+dx)(a^2-3ab\cos(c+dx)+2b^2)}{d(a^2-b^2)}}{a^2-b^2}-\frac{b\csc(c+dx)}{d(a^2-b^2)(a\cos(c+dx)+b)}$$

$\Big\downarrow$ 3042

$$\frac{\frac{a(a^2+2b^2)\int \frac{1}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{\csc(c+dx)(a^2-3ab\cos(c+dx)+2b^2)}{d(a^2-b^2)}}{a^2-b^2}-\frac{b\csc(c+dx)}{d(a^2-b^2)(a\cos(c+dx)+b)}$$

$\Big\downarrow$ 3138

$$\frac{\frac{2a(a^2+2b^2)\int \frac{1}{-\left((a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{d(a^2-b^2)}-\frac{\csc(c+dx)(a^2-3ab\cos(c+dx)+2b^2)}{d(a^2-b^2)}}{a^2-b^2}-\frac{b\csc(c+dx)}{d(a^2-b^2)(a\cos(c+dx)+b)}$$

$\Big\downarrow$ 221

$$\frac{\frac{2a(a^2+2b^2)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}(a^2-b^2)}-\frac{\csc(c+dx)(a^2-3ab\cos(c+dx)+2b^2)}{d(a^2-b^2)}}{a^2-b^2}-\frac{b\csc(c+dx)}{d(a^2-b^2)(a\cos(c+dx)+b)}$$

input
```
Int[Sec[c + d*x]/(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

3.261. $\int \frac{\sec(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}dx$

output
```
-((b*Csc[c + d*x])/((a^2 - b^2)*d*(b + a*Cos[c + d*x]))) + ((2*a*(a^2 + 2*
b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqr
t[a + b]*(a^2 - b^2)*d) - ((a^2 + 2*b^2 - 3*a*b*Cos[c + d*x])*Csc[c + d*x]
)/((a^2 - b^2)*d))/(a^2 - b^2)
```

3.261.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3343
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Simp[1/((a^2 - b^2)*(m + 1))   Int[(g*Cos[e + f*x])^p
*(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ
[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

3.261. $\int \frac{\sec(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$

rule 3345
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*(a^2 - b^2)*(p +
 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
 2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[p, -1] && IntegerQ[2*m]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.261.4 Maple [A] (verified)

Time = 3.59 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.19

method	result
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2\left(a^2-2ab+b^2\right)}-\frac{1}{2(a+b)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2a\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b}-\frac{\left(a^2+2b^2\right)\operatorname{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a-b)^2(a+b)^2}}{d}$
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2\left(a^2-2ab+b^2\right)}-\frac{1}{2(a+b)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2a\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b}-\frac{\left(a^2+2b^2\right)\operatorname{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a-b)^2(a+b)^2}}{d}$
risch	$-\frac{2i\left(a^3e^{3i(dx+c)}+2a\,b^2e^{3i(dx+c)}+a^2b\,e^{2i(dx+c)}+2b^3e^{2i(dx+c)}+a^3e^{i(dx+c)}-4a\,b^2e^{i(dx+c)}-3a^2b\right)}{\left(e^{2i(dx+c)}a+2b\,e^{i(dx+c)}+a\right)\left(-a^2+b^2\right)^2\left(e^{2i(dx+c)}-1\right)d}+\frac{a^3\ln\left(e^{i(dx+c)}+\right.}{\sqrt{a^2-b^2}\,(}$

input
```
int(sec(d*x+c)/(sin(d*x+c)*a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2*tan(1/2*d*x+1/2*c)/(a^2-2*a*b+b^2)-1/2/(a+b)^2/tan(1/2*d*x+1/2*c
)-2*a/(a-b)^2/(a+b)^2*(-a*b*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan
(1/2*d*x+1/2*c)^2*b-a-b)-(a^2+2*b^2)/((a+b)*(a-b))^(1/2)*arctanh(tan(1/2*d
*x+1/2*c)*(a-b)/((a+b)*(a-b))^(1/2))))
```

3.261.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 532, normalized size of antiderivative = 3.91

$$\int \frac{\sec(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$$
$$= \left[-\frac{4\,a^4b-2\,a^2b^3-2\,b^5-(a^3b+2\,ab^3+(a^4+2\,a^2b^2)\cos{(dx+c)})\sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(d}{a^2\cos}\right.}{2\,((a^7-3\,a^5b^2+3\,a^3b^4-ab^6)d\cos}\right.$$
$$-\frac{2\,a^4b-a^2b^3-b^5-(a^3b+2\,ab^3+(a^4+2\,a^2b^2)\cos{(dx+c)})\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cos(dx+c)+a)}{(a^2-b^2)\sin(dx+c)}\right.}{((a^7-3\,a^5b^2+3\,a^3b^4-ab^6)d\cos{(dx+c)}+(a^6b-3\,a^4}$$

input
```
integrate(sec(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas")
```

output
```
[-1/2*(4*a^4*b - 2*a^2*b^3 - 2*b^5 - (a^3*b + 2*a*b^3 + (a^4 + 2*a^2*b^2)*
cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(
d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 -
 b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) - 6*(a
^4*b - a^2*b^3)*cos(d*x + c)^2 + 2*(a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x + c))
/(((a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c) + (a^6*b - 3*a^4*b
^3 + 3*a^2*b^5 - b^7)*d)*sin(d*x + c)), -(2*a^4*b - a^2*b^3 - b^5 - (a^3*b
 + 2*a*b^3 + (a^4 + 2*a^2*b^2)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt
(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))*sin(d*x + c)
 - 3*(a^4*b - a^2*b^3)*cos(d*x + c)^2 + (a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x
+ c))/(((a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c) + (a^6*b - 3*
a^4*b^3 + 3*a^2*b^5 - b^7)*d)*sin(d*x + c))]
```

3.261.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx = \int \frac{\sec{(c+dx)}}{(a\sin{(c+dx)}+b\tan{(c+dx)})^2}\,dx$$

input
```
integrate(sec(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)/(a*sin(c + d*x) + b*tan(c + d*x))**2, x)
```

3.261.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.261.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(128) = 256$.

Time = 0.45 (sec) , antiderivative size = 288, normalized size of antiderivative = 2.12

$$\int \frac{\sec(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx = -\frac{\frac{4\,(a^3+2\,ab^2)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^4-2\,a^2b^2+b^4)\sqrt{-a^2+b^2}}+\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2-2\,ab+b^2}+\frac{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-7\,a^2b\tan}{(a^4-2\,a^2b^2+b^4)\left(a\tan(\right.}}{2\,d}$$

input
```
integrate(sec(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/2*(4*(a^3 + 2*a*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) +
 arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)
))/((a^4 - 2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)) + tan(1/2*d*x + 1/2*c)/(a^2
- 2*a*b + b^2) + (a^3*tan(1/2*d*x + 1/2*c)^2 - 7*a^2*b*tan(1/2*d*x + 1/2*c
)^2 + 3*a*b^2*tan(1/2*d*x + 1/2*c)^2 - b^3*tan(1/2*d*x + 1/2*c)^2 - a^3 +
a^2*b + a*b^2 - b^3)/((a^4 - 2*a^2*b^2 + b^4)*(a*tan(1/2*d*x + 1/2*c)^3 -
b*tan(1/2*d*x + 1/2*c)^3 - a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c)
)))/d
```

3.261.9 Mupad [B] (verification not implemented)

Time = 23.15 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.80

$$\int \frac{\sec(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$$
$$= \frac{\frac{a^2-2\,a\,b+b^2}{a+b}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,(a^3-7\,a^2\,b+3\,a\,b^2-b^3)}{(a+b)^2}}{d\,\left(\left(2\,a^3-6\,a^2\,b+6\,a\,b^2-2\,b^3\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(-2\,a^3+2\,a^2\,b+2\,a\,b^2-2\,b^3\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}$$
$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d\,(a-b)^2}-\frac{a\,\mathrm{atan}\left(\frac{1\mathrm{i}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^4-2\mathrm{i}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^2\,b^2+1\mathrm{i}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)b^4}{(a+b)^{5/2}\,(a-b)^{3/2}}\right)\,\left(a^2+2\,b^2\right)\,2\mathrm{i}}{d\,(a+b)^{5/2}\,(a-b)^{5/2}}$$

input
```
int(1/(cos(c + d*x)*(a*sin(c + d*x) + b*tan(c + d*x))^2),x)
```

output
```
((a^2 - 2*a*b + b^2)/(a + b) - (tan(c/2 + (d*x)/2)^2*(3*a*b^2 - 7*a^2*b +
a^3 - b^3))/(a + b)^2)/(d*(tan(c/2 + (d*x)/2)^3*(6*a*b^2 - 6*a^2*b + 2*a^3
 - 2*b^3) + tan(c/2 + (d*x)/2)*(2*a*b^2 + 2*a^2*b - 2*a^3 - 2*b^3))) - tan
(c/2 + (d*x)/2)/(2*d*(a - b)^2) - (a*atan((a^4*tan(c/2 + (d*x)/2)*1i + b^4
*tan(c/2 + (d*x)/2)*1i - a^2*b^2*tan(c/2 + (d*x)/2)*2i)/((a + b)^(5/2)*(a
- b)^(3/2)))*(a^2 + 2*b^2)*2i)/(d*(a + b)^(5/2)*(a - b)^(5/2))
```

3.262 $\int \frac{\sec^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$

3.262.1 Optimal result

Integrand size = 28, antiderivative size = 131

$$\int \frac{\sec^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx = -\frac{6a^2b\,\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} + \frac{a\csc(c+dx)}{(a^2-b^2)\,d(b+a\cos(c+dx))} + \frac{(3ab-(2a^2+b^2)\cos(c+dx))\csc(c+dx)}{(a^2-b^2)^2\,d}$$

output
```
-6*a^2*b*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2)/(
a+b)^(5/2)/d+a*csc(d*x+c)/(a^2-b^2)/d/(b+a*cos(d*x+c))+(3*a*b-(2*a^2+b^2)*
cos(d*x+c))*csc(d*x+c)/(a^2-b^2)^2/d
```

3.262.2 Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.92

$$\int \frac{\sec^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx = \frac{\frac{12a^2b\,\text{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} - \frac{\cot\left(\frac{1}{2}(c+dx)\right)}{(a+b)^2} + \frac{\frac{2a^3\sin(c+dx)}{(a+b)^2(b+a\cos(c+dx))}+\tan\left(\frac{1}{2}(c+dx)\right)}{(a-b)^2}}{2d}$$

input
```
Integrate[Sec[c + d*x]^2/(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

output
```
((12*a^2*b*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^
2)^(5/2) - Cot[(c + d*x)/2]/(a + b)^2 + ((2*a^3*Sin[c + d*x])/((a + b)^2*(
b + a*Cos[c + d*x])) + Tan[(c + d*x)/2])/(a - b)^2)/(2*d)
```

3.262.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.20, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {3042, 4897, 3042, 3173, 25, 3042, 3345, 27, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^2}{(a\sin(c+dx)+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \frac{\csc^2(c+dx)}{(a\cos(c+dx)+b)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos\left(c+dx-\frac{\pi}{2}\right)^2\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3173}$$

$$\frac{\int -\frac{(b-2a\cos(c+dx))\csc^2(c+dx)}{b+a\cos(c+dx)}dx}{a^2-b^2}+\frac{a\csc(c+dx)}{d\left(a^2-b^2\right)(a\cos(c+dx)+b)}$$

$$\Big\downarrow \text{25}$$

$$\frac{a\csc(c+dx)}{d\left(a^2-b^2\right)(a\cos(c+dx)+b)}-\frac{\int \frac{(b-2a\cos(c+dx))\csc^2(c+dx)}{b+a\cos(c+dx)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\csc(c+dx)}{d\left(a^2-b^2\right)(a\cos(c+dx)+b)}-\frac{\int \frac{b+2a\sin\left(c+dx-\frac{\pi}{2}\right)}{\cos\left(c+dx-\frac{\pi}{2}\right)^2\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{3345}$$

$$\frac{a\csc(c+dx)}{d\left(a^2-b^2\right)(a\cos(c+dx)+b)}-\frac{\frac{\int \frac{3a^2b}{b+a\cos(c+dx)}dx}{a^2-b^2}-\frac{\csc(c+dx)\left(3ab-\left(2a^2+b^2\right)\cos(c+dx)\right)}{d\left(a^2-b^2\right)}}{a^2-b^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\csc(c+dx)}{d\left(a^2-b^2\right)(a\cos(c+dx)+b)}-\frac{\frac{3a^2b\int \frac{1}{b+a\cos(c+dx)}dx}{a^2-b^2}-\frac{\csc(c+dx)\left(3ab-\left(2a^2+b^2\right)\cos(c+dx)\right)}{d\left(a^2-b^2\right)}}{a^2-b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\csc(c+dx)}{d\left(a^2-b^2\right)(a\cos(c+dx)+b)}-\frac{\frac{3a^2b\int \frac{1}{b+a\sin\left(c+dx+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{\csc(c+dx)\left(3ab-\left(2a^2+b^2\right)\cos(c+dx)\right)}{d\left(a^2-b^2\right)}}{a^2-b^2}$$

$$\Big\downarrow \text{3138}$$

$$\frac{a\csc(c+dx)}{d\left(a^2-b^2\right)(a\cos(c+dx)+b)}-\frac{\frac{6a^2b\int \frac{1}{-\left((a-b)\tan^2\left(\frac{1}{2}(c+dx)\right)\right)+a+b}d\tan\left(\frac{1}{2}(c+dx)\right)}{d\left(a^2-b^2\right)}-\frac{\csc(c+dx)\left(3ab-\left(2a^2+b^2\right)\cos(c+dx)\right)}{d\left(a^2-b^2\right)}}{a^2-b^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{a\csc(c+dx)}{d\left(a^2-b^2\right)(a\cos(c+dx)+b)}-\frac{\frac{6a^2b\,\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}\left(a^2-b^2\right)}-\frac{\csc(c+dx)\left(3ab-\left(2a^2+b^2\right)\cos(c+dx)\right)}{d\left(a^2-b^2\right)}}{a^2-b^2}$$

input
```
Int[Sec[c + d*x]^2/(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

output
```
(a*Csc[c + d*x])/((a^2 - b^2)*d*(b + a*Cos[c + d*x])) - ((6*a^2*b*ArcTanh[
(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b]*(a^2
 - b^2)*d) - ((3*a*b - (2*a^2 + b^2)*Cos[c + d*x])*Csc[c + d*x])/((a^2 - b
^2)*d))/(a^2 - b^2)
```

3.262. $\int \frac{\sec^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$

3.262.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3173
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Simp[1/((a^2 - b^2)*(m + 1))
   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m +
p + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b
^2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

rule 3345
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*(a^2 - b^2)*(p +
 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
 2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[p, -1] && IntegerQ[2*m]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.262. $\int \frac{\sec^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$

3.262.4 Maple [A] (verified)

Time = 8.32 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.18

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^{2}-4ab+2b^{2}}-\frac{1}{2(a+b)^{2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2a^{2}\left(-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b-a-b}-\frac{3b\,\mathrm{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a-b)^{2}(a+b)^{2}}}{d}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^{2}-4ab+2b^{2}}-\frac{1}{2(a+b)^{2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2a^{2}\left(-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b-a-b}-\frac{3b\,\mathrm{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a-b)^{2}(a+b)^{2}}}{d}$
risch	$-\frac{2i\left(-3a^{2}b\,\mathrm{e}^{3i(dx+c)}-3a\,b^{2}\mathrm{e}^{2i(dx+c)}+a^{2}b\,\mathrm{e}^{i(dx+c)}+2b^{3}\mathrm{e}^{i(dx+c)}+2a^{3}+a\,b^{2}\right)}{\left(\mathrm{e}^{2i(dx+c)}a+2b\,\mathrm{e}^{i(dx+c)}+a\right)\left(a^{2}-b^{2}\right)^{2}\left(\mathrm{e}^{2i(dx+c)}-1\right)d}+\frac{3b\ln\left(\mathrm{e}^{i(dx+c)}-\frac{ia^{2}-ib^{2}-\sqrt{a^{2}-b^{2}}\,b}{a\sqrt{a^{2}-b^{2}}}\right)\cdots}{\sqrt{a^{2}-b^{2}}\,(a+b)^{2}(a-b)^{2}d}$

input
```
int(sec(d*x+c)^2/(sin(d*x+c)*a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2*tan(1/2*d*x+1/2*c)/(a^2-2*a*b+b^2)-1/2/(a+b)^2/tan(1/2*d*x+1/2*c)
+2*a^2/(a-b)^2/(a+b)^2*(-a*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(
1/2*d*x+1/2*c)^2*b-a-b)-3*b/((a+b)*(a-b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)
*(a-b)/((a+b)*(a-b))^(1/2))))
```

3.262.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 516, normalized size of antiderivative = 3.94

$$\int \frac{\sec^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx = \left[\frac{2\,a^5+2\,a^3b^2-4\,ab^4+3\,(a^3b\cos(dx+c)+a^2b^2)\sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2-2\,\sqrt{a^2-b^2}(b\cos\cdots}{a^2\cos(dx+c)^2+2\,ab\cos(dx+\cdots}\right.\cdots}{2\,((a^7-3\,a^5b^2+3\,a^3b^4-ab^6)d\cos(dx+\cdots}\right.$$

input
```
integrate(sec(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas"
)
```

output
```
[1/2*(2*a^5 + 2*a^3*b^2 - 4*a*b^4 + 3*(a^3*b*cos(d*x + c) + a^2*b^2)*sqrt(
a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt
(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x
+ c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) - 2*(2*a^5 - a^3*b^2 - a*
b^4)*cos(d*x + c)^2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c))/(((a^7 - 3
*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c) + (a^6*b - 3*a^4*b^3 + 3*a^2*
b^5 - b^7)*d)*sin(d*x + c)), (a^5 + a^3*b^2 - 2*a*b^4 - 3*(a^3*b*cos(d*x +
 c) + a^2*b^2)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) +
 a)/((a^2 - b^2)*sin(d*x + c)))*sin(d*x + c) - (2*a^5 - a^3*b^2 - a*b^4)*c
os(d*x + c)^2 + (a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c))/(((a^7 - 3*a^5*b^2
 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c) + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^
7)*d)*sin(d*x + c))]
```

3.262.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx=\int \frac{\sec^2{(c+dx)}}{(a\sin{(c+dx)}+b\tan{(c+dx)})^2}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a*sin(d*x+c)+b*tan(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)**2/(a*sin(c + d*x) + b*tan(c + d*x))**2, x)
```

3.262.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima"
)
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.262.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(122) = 244$.

Time = 0.50 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.17

$$\int \frac{\sec^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$$
$$= \frac{\frac{12\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)a^2b}{(a^4-2a^2b^2+b^4)\sqrt{-a^2+b^2}}+\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^2-2ab+b^2}-\frac{5a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-3a^2b\tan\left(\frac{1}{2}dx+\right.}{(a^4-2a^2b^2+b^4)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.\right.}}{2d}$$

input
```
integrate(sec(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/2*(12*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1
/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*a^2*b/((a^4 -
 2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)) + tan(1/2*d*x + 1/2*c)/(a^2 - 2*a*b +
b^2) - (5*a^3*tan(1/2*d*x + 1/2*c)^2 - 3*a^2*b*tan(1/2*d*x + 1/2*c)^2 + 3*
a*b^2*tan(1/2*d*x + 1/2*c)^2 - b^3*tan(1/2*d*x + 1/2*c)^2 - a^3 + a^2*b +
a*b^2 - b^3)/((a^4 - 2*a^2*b^2 + b^4)*(a*tan(1/2*d*x + 1/2*c)^3 - b*tan(1/
2*d*x + 1/2*c)^3 - a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))))/d
```

3.262.9 Mupad [B] (verification not implemented)

Time = 22.74 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.64

$$\int \frac{\sec^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2d\,(a-b)^2}$$
$$+\frac{\frac{a^2-2ab+b^2}{a+b}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(5a^3-3a^2b+3ab^2-b^3)}{(a+b)^2}}{d\left((2a^3-6a^2b+6ab^2-2b^3)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+(-2a^3+2a^2b+2ab^2-2b^3)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}$$
$$-\frac{6a^2b\,\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(a^4-2a^2b^2+b^4)}{(a+b)^{5/2}(a-b)^{3/2}}\right)}{d\,(a+b)^{5/2}(a-b)^{5/2}}$$

input
```
int(1/(cos(c + d*x)^2*(a*sin(c + d*x) + b*tan(c + d*x))^2),x)
```

output
```
tan(c/2 + (d*x)/2)/(2*d*(a - b)^2) + ((a^2 - 2*a*b + b^2)/(a + b) - (tan(c
/2 + (d*x)/2)^2*(3*a*b^2 - 3*a^2*b + 5*a^3 - b^3))/(a + b)^2)/(d*(tan(c/2
+ (d*x)/2)^3*(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) + tan(c/2 + (d*x)/2)*(2*a
*b^2 + 2*a^2*b - 2*a^3 - 2*b^3))) - (6*a^2*b*atanh((tan(c/2 + (d*x)/2)*(a^
4 + b^4 - 2*a^2*b^2))/((a + b)^(5/2)*(a - b)^(3/2))))/(d*(a + b)^(5/2)*(a
- b)^(5/2))
```

3.262. $\int \frac{\sec^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$

3.263 $\int \frac{\sec^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$

3.263.1 Optimal result

Integrand size = 28, antiderivative size = 231

$$\int \frac{\sec^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{b^2d} + \frac{2a^3\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{2a^3(a^2-3b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}b^2(a+b)^{5/2}d} - \frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+dx))} - \frac{\sin(c+dx)}{2(a-b)^2d(1+\cos(c+dx))} - \frac{a^4\sin(c+dx)}{b\left(a^2-b^2\right)^2d(b+a\cos(c+dx))}$$

output
```
arctanh(sin(d*x+c))/b^2/d+2*a^3*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+
b)^(1/2))/(a-b)^(5/2)/(a+b)^(5/2)/d-2*a^3*(a^2-3*b^2)*arctanh((a-b)^(1/2)*
tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2)/b^2/(a+b)^(5/2)/d-1/2*sin(d*x+
c)/(a+b)^2/d/(1-cos(d*x+c))-1/2*sin(d*x+c)/(a-b)^2/d/(1+cos(d*x+c))-a^4*si
n(d*x+c)/b/(a^2-b^2)^2/d/(b+a*cos(d*x+c))
```

3.263.2 Mathematica [A] (verified)

Time = 2.68 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.85

$$\int \frac{\sec^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx = -\frac{-\frac{4(a^5-4a^3b^2)\,\text{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2(a^2-b^2)^{5/2}}+\frac{\cot\left(\frac{1}{2}(c+dx)\right)}{(a+b)^2}+\frac{2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{b^2}-\frac{2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{b^2}$$

$$2d$$

input
```
Integrate[Sec[c + d*x]^3/(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

output
```
-1/2*((-4*(a^5 - 4*a^3*b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 -
 b^2]])/(b^2*(a^2 - b^2)^(5/2)) + Cot[(c + d*x)/2]/(a + b)^2 + (2*Log[Cos[
(c + d*x)/2] - Sin[(c + d*x)/2]])/b^2 - (2*Log[Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2]])/b^2 + (2*a^4*Sin[c + d*x])/((a - b)^2*b*(a + b)^2*(b + a*Cos[c
+ d*x])) + Tan[(c + d*x)/2]/(a - b)^2)/d
```

3.263.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 4897, 3042, 25, 3376, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^3}{(a\sin(c+dx)+b\tan(c+dx))^2}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \frac{\csc^2(c+dx)\sec(c+dx)}{(a\cos(c+dx)+b)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\sin\left(c+dx-\frac{\pi}{2}\right)\cos\left(c+dx-\frac{\pi}{2}\right)^2\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^2\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\left(b-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)^2}dx$$

$$\Big\downarrow \text{3376}$$

$$-\int \left(\frac{a^3}{b\left(a^2-b^2\right)(-b-a\cos(c+dx))^2}-\frac{\sec(c+dx)}{b^2}-\frac{1}{2(a-b)^2(-\cos(c+dx)-1)}-\frac{1}{2(a+b)^2(1-\cos(c+dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^3\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{a^4\sin(c+dx)}{bd\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}-\frac{2a^3\left(a^2-3b^2\right)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d(a-b)^{5/2}(a+b)^{5/2}}-\frac{\sin(c+dx)}{2d(a+b)^2(1-\cos(c+dx))}-\frac{\sin(c+dx)}{2d(a-b)^2(\cos(c+dx)+1)}+\frac{\text{arctanh}(\sin(c+dx))}{b^2d}$$

input
```
Int[Sec[c + d*x]^3/(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

output
```
ArcTanh[Sin[c + d*x]]/(b^2*d) + (2*a^3*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/
2])/Sqrt[a + b]])/((a - b)^(5/2)*(a + b)^(5/2)*d) - (2*a^3*(a^2 - 3*b^2)*A
rcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b^2*(a
+ b)^(5/2)*d) - Sin[c + d*x]/(2*(a + b)^2*d*(1 - Cos[c + d*x])) - Sin[c +
d*x]/(2*(a - b)^2*d*(1 + Cos[c + d*x])) - (a^4*Sin[c + d*x])/(b*(a^2 - b^2
)^2*d*(b + a*Cos[c + d*x]))
```

3.263.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.263. $\int \frac{\sec^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$

rule 3376
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.263.4 Maple [A] (verified)

Time = 17.05 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2\left(a^2-2ab+b^2\right)}+\frac{2a^3\left(\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b}-\frac{\left(a^2-4b^2\right)\operatorname{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a-b)^2(a+b)^2b^2}-\frac{1}{2(a+b)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right.}}{d}$
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2\left(a^2-2ab+b^2\right)}+\frac{2a^3\left(\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2b-a-b}-\frac{\left(a^2-4b^2\right)\operatorname{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a-b)^2(a+b)^2b^2}-\frac{1}{2(a+b)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right.}}{d}$
risch	$-\frac{2i\left(2a^3b\,e^{3i(dx+c)}+a\,b^3e^{3i(dx+c)}+a^4e^{2i(dx+c)}+2b^4e^{2i(dx+c)}-3a\,b^3e^{i(dx+c)}-a^4-2a^2b^2\right)}{\left(e^{2i(dx+c)}a+2b\,e^{i(dx+c)}+a\right)b\left(-a^2+b^2\right)^2\left(e^{2i(dx+c)}-1\right)d}+\frac{a^5\ln\left(e^{i(dx+c)}-\frac{ia^2-ib^2}{a\sqrt{}}\right.}{\sqrt{a^2-b^2}\,(a+b)^2(a-}$

input
```
int(sec(d*x+c)^3/(sin(d*x+c)*a+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2*tan(1/2*d*x+1/2*c)/(a^2-2*a*b+b^2)+2*a^3/(a-b)^2/(a+b)^2/b^2*(a*
b*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)-(
a^2-4*b^2)/((a+b)*(a-b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a+b)*(a-
b))^(1/2)))-1/2/(a+b)^2/tan(1/2*d*x+1/2*c)-1/b^2*ln(tan(1/2*d*x+1/2*c)-1)+
1/b^2*ln(tan(1/2*d*x+1/2*c)+1))
```

3.263.5 Fricas [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 864, normalized size of antiderivative = 3.74

$$\int \frac{\sec^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$$
$$= \left[-\frac{2\,a^6b-2\,b^7+(a^5b-4\,a^3b^3+(a^6-4\,a^4b^2)\cos(dx+c))\sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2+2\,\sqrt{}}{a^2\cos(dx+c)^2+2}\right.}{}\right.$$
$$-\frac{2\,a^6b-2\,b^7+2\,(a^5b-4\,a^3b^3+(a^6-4\,a^4b^2)\cos(dx+c))\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cos(dx+c)+a)}{(a^2-b^2)\sin(dx+c)}\right)\mathrm{si}}{}$$

input
```
integrate(sec(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas"
)
```

output
```
[-1/2*(2*a^6*b - 2*b^7 + (a^5*b - 4*a^3*b^3 + (a^6 - 4*a^4*b^2)*cos(d*x +
c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2
 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2
*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) - 2*(a^6*b + a^4
*b^3 - 2*a^2*b^5)*cos(d*x + c)^2 - (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7 +
(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cos(d*x + c))*log(sin(d*x + c) + 1)*
sin(d*x + c) + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7 + (a^7 - 3*a^5*b^2 + 3
*a^3*b^4 - a*b^6)*cos(d*x + c))*log(-sin(d*x + c) + 1)*sin(d*x + c) + 2*(a
^5*b^2 - 2*a^3*b^4 + a*b^6)*cos(d*x + c))/(((a^7*b^2 - 3*a^5*b^4 + 3*a^3*b
^6 - a*b^8)*d*cos(d*x + c) + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*d)*si
n(d*x + c)), -1/2*(2*a^6*b - 2*b^7 + 2*(a^5*b - 4*a^3*b^3 + (a^6 - 4*a^4*b
^2)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c
) + a)/((a^2 - b^2)*sin(d*x + c)))*sin(d*x + c) - 2*(a^6*b + a^4*b^3 - 2*a
^2*b^5)*cos(d*x + c)^2 - (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7 + (a^7 - 3*a
^5*b^2 + 3*a^3*b^4 - a*b^6)*cos(d*x + c))*log(sin(d*x + c) + 1)*sin(d*x +
c) + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7 + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 -
 a*b^6)*cos(d*x + c))*log(-sin(d*x + c) + 1)*sin(d*x + c) + 2*(a^5*b^2 - 2
*a^3*b^4 + a*b^6)*cos(d*x + c))/(((a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8
)*d*cos(d*x + c) + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*d)*sin(d*x + c)
)]
```

3.263.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx = \int \frac{\sec^3\left(c+dx\right)}{\left(a\sin\left(c+dx\right)+b\tan\left(c+dx\right)\right)^2}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a*sin(d*x+c)+b*tan(d*x+c))**2,x)
```

output
```
Integral(sec(c + d*x)**3/(a*sin(c + d*x) + b*tan(c + d*x))**2, x)
```

3.263.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima"
)
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.263.8 Giac [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.53

$$\int \frac{\sec^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx = \frac{\frac{4\,(a^5-4\,a^3b^2)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^4b^2-2\,a^2b^4+b^6)\sqrt{-a^2+b^2}} - \frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2-2\,ab+b^2} + \frac{4\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a^3b\tan\left(\frac{1}{2}\right.}{(a^4b-2\,a^2b^3}}{}$$

input
```
integrate(sec(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
1/2*(4*(a^5 - 4*a^3*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b)
+ arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2
)))/((a^4*b^2 - 2*a^2*b^4 + b^6)*sqrt(-a^2 + b^2)) - tan(1/2*d*x + 1/2*c)/
(a^2 - 2*a*b + b^2) + (4*a^4*tan(1/2*d*x + 1/2*c)^2 - a^3*b*tan(1/2*d*x +
1/2*c)^2 + 3*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 - 3*a*b^3*tan(1/2*d*x + 1/2*c)
^2 + b^4*tan(1/2*d*x + 1/2*c)^2 + a^3*b - a^2*b^2 - a*b^3 + b^4)/((a^4*b -
 2*a^2*b^3 + b^5)*(a*tan(1/2*d*x + 1/2*c)^3 - b*tan(1/2*d*x + 1/2*c)^3 - a
*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))) + 2*log(abs(tan(1/2*d*x +
 1/2*c) + 1))/b^2 - 2*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^2)/d
```

3.263.9 Mupad [B] (verification not implemented)

Time = 27.60 (sec) , antiderivative size = 6056, normalized size of antiderivative = 26.22

$$\int \frac{\sec^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(1/(cos(c + d*x)^3*(a*sin(c + d*x) + b*tan(c + d*x))^2),x)
```

output
```
((a^2 - 2*a*b + b^2)/(a + b) + (tan(c/2 + (d*x)/2)^2*(4*a^4 - a^3*b - 3*a*
b^3 + b^4 + 3*a^2*b^2))/(b*(a + b)^2))/(d*(tan(c/2 + (d*x)/2)^3*(6*a*b^2 -
 6*a^2*b + 2*a^3 - 2*b^3) + tan(c/2 + (d*x)/2)*(2*a*b^2 + 2*a^2*b - 2*a^3
- 2*b^3))) - (atan(-(((tan(c/2 + (d*x)/2)*(32*b^26 - 96*a*b^25 - 224*a^2*b
^24 + 928*a^3*b^23 + 480*a^4*b^22 - 4000*a^5*b^21 + 992*a^6*b^20 + 9568*a^
7*b^19 - 8128*a^8*b^18 - 12992*a^9*b^17 + 21344*a^10*b^16 + 8224*a^11*b^15
 - 31744*a^12*b^14 + 2176*a^13*b^13 + 29600*a^14*b^12 - 8480*a^15*b^11 - 1
7632*a^16*b^10 + 7072*a^17*b^9 + 6528*a^18*b^8 - 3008*a^19*b^7 - 1376*a^20
*b^6 + 672*a^21*b^5 + 128*a^22*b^4 - 64*a^23*b^3) + (32*b^28 - 32*a*b^27 -
 352*a^2*b^26 + 480*a^3*b^25 + 1504*a^4*b^24 - 2688*a^5*b^23 - 3168*a^6*b^
22 + 8064*a^7*b^21 + 2880*a^8*b^20 - 14784*a^9*b^19 + 1344*a^10*b^18 + 174
72*a^11*b^17 - 6720*a^12*b^16 - 13440*a^13*b^15 + 8256*a^14*b^14 + 6528*a^
15*b^13 - 5472*a^16*b^12 - 1824*a^17*b^11 + 2080*a^18*b^10 + 224*a^19*b^9
- 416*a^20*b^8 + 32*a^22*b^6 - (tan(c/2 + (d*x)/2)*(128*a^2*b^28 - 64*a*b^
29 + 576*a^3*b^27 - 1280*a^4*b^26 - 2240*a^5*b^25 + 5760*a^6*b^24 + 4800*a
^7*b^23 - 15360*a^8*b^22 - 5760*a^9*b^21 + 26880*a^10*b^20 + 2688*a^11*b^1
9 - 32256*a^12*b^18 + 2688*a^13*b^17 + 26880*a^14*b^16 - 5760*a^15*b^15 -
15360*a^16*b^14 + 4800*a^17*b^13 + 5760*a^18*b^12 - 2240*a^19*b^11 - 1280*
a^20*b^10 + 576*a^21*b^9 + 128*a^22*b^8 - 64*a^23*b^7))/b^2)/b^2)*1i)/b^2
+ ((tan(c/2 + (d*x)/2)*(32*b^26 - 96*a*b^25 - 224*a^2*b^24 + 928*a^3*b^...
```

3.263. $\int \frac{\sec^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$

3.264 $\int \frac{\cos^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$

3.264.1 Optimal result

Integrand size = 28, antiderivative size = 248

$$\begin{aligned}\int \frac{\cos^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx &= \frac{b^6}{2a^3(a^2-b^2)^2 d(b+a\cos(c+dx))^2}-\frac{2b^5(3a^2-b^2)}{a^3(a^2-b^2)^3 d(b+a\cos(c+dx))}\\&\quad-\frac{(a(a^2+3b^2)-b(3a^2+b^2)\cos(c+dx))\csc^2(c+dx)}{2(a^2-b^2)^3 d}-\frac{(2a+5b)\log(1-\cos(c+dx))}{4(a+b)^4 d}\\&\quad-\frac{(2a-5b)\log(1+\cos(c+dx))}{4(a-b)^4 d}-\frac{b^4(15a^4-4a^2b^2+b^4)\log(b+a\cos(c+dx))}{a^3(a^2-b^2)^4 d}\end{aligned}$$

output
```
1/2*b^6/a^3/(a^2-b^2)^2/d/(b+a*cos(d*x+c))^2-2*b^5*(3*a^2-b^2)/a^3/(a^2-b^
2)^3/d/(b+a*cos(d*x+c))-1/2*(a*(a^2+3*b^2)-b*(3*a^2+b^2)*cos(d*x+c))*csc(d
*x+c)^2/(a^2-b^2)^3/d-1/4*(2*a+5*b)*ln(1-cos(d*x+c))/(a+b)^4/d-1/4*(2*a-5*
b)*ln(1+cos(d*x+c))/(a-b)^4/d-b^4*(15*a^4-4*a^2*b^2+b^4)*ln(b+a*cos(d*x+c)
)/a^3/(a^2-b^2)^4/d
```

3.264.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.14 (sec) , antiderivative size = 713, normalized size of antiderivative = 2.88

$$\int \frac{\cos^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$$
$$= \frac{b^6(b+a\cos(c+dx))\tan^3(c+dx)}{2a^3(-a+b)^2(a+b)^2d(a\sin(c+dx)+b\tan(c+dx))^3}$$
$$- \frac{2b^5(-3a^2+b^2)(b+a\cos(c+dx))^2\tan^3(c+dx)}{a^3(-a+b)^3(a+b)^3d(a\sin(c+dx)+b\tan(c+dx))^3}$$
$$- \frac{2i(a^5-4a^3b^2-9ab^4)(c+dx)(b+a\cos(c+dx))^3\tan^3(c+dx)}{(a-b)^4(a+b)^4d(a\sin(c+dx)+b\tan(c+dx))^3}$$
$$- \frac{i(-2a-5b)\arctan(\tan(c+dx))(b+a\cos(c+dx))^3\tan^3(c+dx)}{2(a+b)^4d(a\sin(c+dx)+b\tan(c+dx))^3}$$
$$- \frac{i(-2a+5b)\arctan(\tan(c+dx))(b+a\cos(c+dx))^3\tan^3(c+dx)}{2(-a+b)^4d(a\sin(c+dx)+b\tan(c+dx))^3}$$
$$- \frac{(b+a\cos(c+dx))^3\csc^2\left(\frac{1}{2}(c+dx)\right)\tan^3(c+dx)}{8(a+b)^3d(a\sin(c+dx)+b\tan(c+dx))^3}$$
$$+ \frac{(-2a+5b)(b+a\cos(c+dx))^3\log\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\right)\tan^3(c+dx)}{4(-a+b)^4d(a\sin(c+dx)+b\tan(c+dx))^3}$$
$$+ \frac{(-15a^4b^4+4a^2b^6-b^8)(b+a\cos(c+dx))^3\log(b+a\cos(c+dx))\tan^3(c+dx)}{a^3(-a^2+b^2)^4d(a\sin(c+dx)+b\tan(c+dx))^3}$$
$$+ \frac{(-2a-5b)(b+a\cos(c+dx))^3\log\left(\sin^2\left(\frac{1}{2}(c+dx)\right)\right)\tan^3(c+dx)}{4(a+b)^4d(a\sin(c+dx)+b\tan(c+dx))^3}$$
$$+ \frac{(b+a\cos(c+dx))^3\sec^2\left(\frac{1}{2}(c+dx)\right)\tan^3(c+dx)}{8(-a+b)^3d(a\sin(c+dx)+b\tan(c+dx))^3}$$

input
```
Integrate[Cos[c + d*x]^3/(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

output
```
(b^6*(b + a*Cos[c + d*x])*Tan[c + d*x]^3)/(2*a^3*(-a + b)^2*(a + b)^2*d*(a
*Sin[c + d*x] + b*Tan[c + d*x])^3) - (2*b^5*(-3*a^2 + b^2)*(b + a*Cos[c +
d*x])^2*Tan[c + d*x]^3)/(a^3*(-a + b)^3*(a + b)^3*d*(a*Sin[c + d*x] + b*Ta
n[c + d*x])^3) - ((2*I)*(a^5 - 4*a^3*b^2 - 9*a*b^4)*(c + d*x)*(b + a*Cos[c
 + d*x])^3*Tan[c + d*x]^3)/((a - b)^4*(a + b)^4*d*(a*Sin[c + d*x] + b*Tan[
c + d*x])^3) - ((I/2)*(-2*a - 5*b)*ArcTan[Tan[c + d*x]]*(b + a*Cos[c + d*x
])^3*Tan[c + d*x]^3)/((a + b)^4*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3) - (
(I/2)*(-2*a + 5*b)*ArcTan[Tan[c + d*x]]*(b + a*Cos[c + d*x])^3*Tan[c + d*x
]^3)/((-a + b)^4*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3) - ((b + a*Cos[c +
d*x])^3*Csc[(c + d*x)/2]^2*Tan[c + d*x]^3)/(8*(a + b)^3*d*(a*Sin[c + d*x]
+ b*Tan[c + d*x])^3) + ((-2*a + 5*b)*(b + a*Cos[c + d*x])^3*Log[Cos[(c + d
*x)/2]^2]*Tan[c + d*x]^3)/(4*(-a + b)^4*d*(a*Sin[c + d*x] + b*Tan[c + d*x]
)^3) + ((-15*a^4*b^4 + 4*a^2*b^6 - b^8)*(b + a*Cos[c + d*x])^3*Log[b + a*C
os[c + d*x]]*Tan[c + d*x]^3)/(a^3*(-a^2 + b^2)^4*d*(a*Sin[c + d*x] + b*Tan
[c + d*x])^3) + ((-2*a - 5*b)*(b + a*Cos[c + d*x])^3*Log[Sin[(c + d*x)/2]^
2]*Tan[c + d*x]^3)/(4*(a + b)^4*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3) + (
(b + a*Cos[c + d*x])^3*Sec[(c + d*x)/2]^2*Tan[c + d*x]^3)/(8*(-a + b)^3*d*
(a*Sin[c + d*x] + b*Tan[c + d*x])^3)
```

3.264.3 Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {3042, 4897, 3042, 3316, 27, 601, 25, 2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(c+dx)^3}{(a\sin(c+dx)+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \frac{\cos^3(c+dx)\cot^3(c+dx)}{(a\cos(c+dx)+b)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(c+dx-\frac{\pi}{2}\right)^6}{\cos\left(c+dx-\frac{\pi}{2}\right)^3\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$\downarrow$ 3316

$$-\frac{a^3\int \frac{\cos^6(c+dx)}{(b+a\cos(c+dx))^3(a^2-a^2\cos^2(c+dx))^2}d(a\cos(c+dx))}{d}$$

$\downarrow$ 27

$$-\frac{\int \frac{a^6\cos^6(c+dx)}{(b+a\cos(c+dx))^3(a^2-a^2\cos^2(c+dx))^2}d(a\cos(c+dx))}{a^3d}$$

$\downarrow$ 601

$$-\frac{\frac{a^4\left(a^2\left(a^2+3b^2\right)-ab\left(3a^2+b^2\right)\cos(c+dx)\right)}{2\left(a^2-b^2\right)^3\left(a^2-a^2\cos^2(c+dx)\right)}-\frac{\int -\frac{-\frac{b\left(3a^2+b^2\right)\cos^3(c+dx)a^9}{\left(a^2-b^2\right)^3}+\frac{b^3\left(7a^2-3b^2\right)\cos(c+dx)a^7}{\left(a^2-b^2\right)^3}-2\cos^4(c+dx)a^6+\frac{b^2\left(3a^4-9b^2a^2+2b^4\right)\cos^2(c\cdots}{\left(a^2-b^2\right)^3}\cdots}{(b+a\cos(c+dx))^3\left(a^2-a^2\cos^2(c+dx)\right)}\cdots}{2a^2}\cdots}{a^3d}$$

$\downarrow$ 25

$$-\frac{\frac{\int \frac{-\frac{b\left(3a^2+b^2\right)\cos^3(c+dx)a^9}{\left(a^2-b^2\right)^3}+\frac{b^3\left(7a^2-3b^2\right)\cos(c+dx)a^7}{\left(a^2-b^2\right)^3}-2\cos^4(c+dx)a^6+\frac{b^2\left(3a^4-9b^2a^2+2b^4\right)\cos^2(c+dx)a^6}{\left(a^2-b^2\right)^3}+\frac{b^4\left(3a^2+b^2\right)a^6}{\left(a^2-b^2\right)^3}}{(b+a\cos(c+dx))^3\left(a^2-a^2\cos^2(c+dx)\right)}d(a\cos(c+dx))}{2a^2}+\frac{a^4(a\cdots}{\cdots}}{a^3d}$$

$\downarrow$ 2160

$$-\frac{\frac{\int\left(\frac{2a^2b^6}{(a-b)^2(a+b)^2(b+a\cos(c+dx))^3}-\frac{4a^2\left(3a^2-b^2\right)b^5}{(a-b)^3(a+b)^3(b+a\cos(c+dx))^2}+\frac{2a^2\left(15a^4-4b^2a^2+b^4\right)b^4}{(a-b)^4(a+b)^4(b+a\cos(c+dx))}-\frac{a^5(2a+5b)}{2(a+b)^4(a-a\cos(c+dx))}+\frac{a^5(2a-5b)}{2(a-b)^4(\cos(c+dx)a+a)}\right)\cdots}{2a^2}\cdots}{a^3d}$$

$\downarrow$ 2009

$$-\frac{\frac{a^4\left(a^2\left(a^2+3b^2\right)-ab\left(3a^2+b^2\right)\cos(c+dx)\right)}{2\left(a^2-b^2\right)^3\left(a^2-a^2\cos^2(c+dx)\right)}+\frac{\frac{a^5(2a+5b)\log(a-a\cos(c+dx))}{2(a+b)^4}+\frac{a^5(2a-5b)\log(a\cos(c+dx)+a)}{2(a-b)^4}-\frac{a^2b^6}{\left(a^2-b^2\right)^2(a\cos(c+dx)+b)^2}+\frac{4a^2b^5}{\left(a^2-b^2\right)^3(\cdots}\cdots}{2a^2}\cdots}{a^3d}$$

input
```
Int[Cos[c + d*x]^3/(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

3.264. $\int \frac{\cos^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$

output
```
-(((a^4*(a^2*(a^2 + 3*b^2) - a*b*(3*a^2 + b^2)*Cos[c + d*x]))/(2*(a^2 - b^
2)^3*(a^2 - a^2*Cos[c + d*x]^2)) + (-((a^2*b^6)/((a^2 - b^2)^2*(b + a*Cos[
c + d*x])^2)) + (4*a^2*b^5*(3*a^2 - b^2))/((a^2 - b^2)^3*(b + a*Cos[c + d*
x])) + (a^5*(2*a + 5*b)*Log[a - a*Cos[c + d*x]])/(2*(a + b)^4) + (a^5*(2*a
 - 5*b)*Log[a + a*Cos[c + d*x]])/(2*(a - b)^4) + (2*a^2*b^4*(15*a^4 - 4*a^
2*b^2 + b^4)*Log[b + a*Cos[c + d*x]])/(a^2 - b^2)^4)/(2*a^2))/(a^3*d))
```

3.264.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 601
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c
+ d*x)^n*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*(p + 1)*Qx)/(c + d*x)^n + (e*
(2*p + 3))/(c + d*x)^n, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 1]
 && LtQ[p, -1] && ILtQ[n, 0] && NeQ[b*c^2 + a*d^2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.264. $\int \frac{\cos^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.264.4 Maple [A] (verified)

Time = 5.24 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{\frac{1}{4(a+b)^3(\cos(dx+c)-1)}+\frac{(-2a-5b)\ln(\cos(dx+c)-1)}{4(a+b)^4}+\frac{b^6}{2a^3(a+b)^2(a-b)^2(b+\cos(dx+c)a)^2}-\frac{2b^5(3a^2-b^2)}{a^3(a+b)^3(a-b)^3(b+\cos(dx+c)a)}-\frac{b^4(}{}}{d}$
default	$\frac{\frac{1}{4(a+b)^3(\cos(dx+c)-1)}+\frac{(-2a-5b)\ln(\cos(dx+c)-1)}{4(a+b)^4}+\frac{b^6}{2a^3(a+b)^2(a-b)^2(b+\cos(dx+c)a)^2}-\frac{2b^5(3a^2-b^2)}{a^3(a+b)^3(a-b)^3(b+\cos(dx+c)a)}-\frac{b^4(}{}}{d}$
risch	Expression too large to display

input
```
int(cos(d*x+c)^3/(sin(d*x+c)*a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/4/(a+b)^3/(cos(d*x+c)-1)+1/4/(a+b)^4*(-2*a-5*b)*ln(cos(d*x+c)-1)+1/
2*b^6/a^3/(a+b)^2/(a-b)^2/(b+cos(d*x+c)*a)^2-2/a^3*b^5*(3*a^2-b^2)/(a+b)^3
/(a-b)^3/(b+cos(d*x+c)*a)-b^4*(15*a^4-4*a^2*b^2+b^4)/(a+b)^4/(a-b)^4/a^3*l
n(b+cos(d*x+c)*a)-1/4/(a-b)^3/(cos(d*x+c)+1)+1/4/(a-b)^4*(-2*a+5*b)*ln(cos
(d*x+c)+1))
```

3.264.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1180 vs. 2(240) = 480.

Time = 0.60 (sec) , antiderivative size = 1180, normalized size of antiderivative = 4.76

$$\int \frac{\cos^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas"
)
```

output
```
1/4*(2*a^8*b^2 + 4*a^6*b^4 + 16*a^4*b^6 - 28*a^2*b^8 + 6*b^10 - 2*(3*a^9*b
 - 2*a^7*b^3 + 11*a^5*b^5 - 16*a^3*b^7 + 4*a*b^9)*cos(d*x + c)^3 + 2*(a^10
 - 4*a^8*b^2 + a^6*b^4 - 9*a^4*b^6 + 14*a^2*b^8 - 3*b^10)*cos(d*x + c)^2 +
 2*(2*a^9*b + a^7*b^3 + 8*a^5*b^5 - 15*a^3*b^7 + 4*a*b^9)*cos(d*x + c) + 4
*(15*a^4*b^6 - 4*a^2*b^8 + b^10 - (15*a^6*b^4 - 4*a^4*b^6 + a^2*b^8)*cos(d
*x + c)^4 - 2*(15*a^5*b^5 - 4*a^3*b^7 + a*b^9)*cos(d*x + c)^3 + (15*a^6*b^
4 - 19*a^4*b^6 + 5*a^2*b^8 - b^10)*cos(d*x + c)^2 + 2*(15*a^5*b^5 - 4*a^3*
b^7 + a*b^9)*cos(d*x + c))*log(a*cos(d*x + c) + b) + (2*a^8*b^2 + 3*a^7*b^
3 - 8*a^6*b^4 - 22*a^5*b^5 - 18*a^4*b^6 - 5*a^3*b^7 - (2*a^10 + 3*a^9*b -
8*a^8*b^2 - 22*a^7*b^3 - 18*a^6*b^4 - 5*a^5*b^5)*cos(d*x + c)^4 - 2*(2*a^9
*b + 3*a^8*b^2 - 8*a^7*b^3 - 22*a^6*b^4 - 18*a^5*b^5 - 5*a^4*b^6)*cos(d*x
+ c)^3 + (2*a^10 + 3*a^9*b - 10*a^8*b^2 - 25*a^7*b^3 - 10*a^6*b^4 + 17*a^5
*b^5 + 18*a^4*b^6 + 5*a^3*b^7)*cos(d*x + c)^2 + 2*(2*a^9*b + 3*a^8*b^2 - 8
*a^7*b^3 - 22*a^6*b^4 - 18*a^5*b^5 - 5*a^4*b^6)*cos(d*x + c))*log(1/2*cos(
d*x + c) + 1/2) + (2*a^8*b^2 - 3*a^7*b^3 - 8*a^6*b^4 + 22*a^5*b^5 - 18*a^4
*b^6 + 5*a^3*b^7 - (2*a^10 - 3*a^9*b - 8*a^8*b^2 + 22*a^7*b^3 - 18*a^6*b^4
 + 5*a^5*b^5)*cos(d*x + c)^4 - 2*(2*a^9*b - 3*a^8*b^2 - 8*a^7*b^3 + 22*a^6
*b^4 - 18*a^5*b^5 + 5*a^4*b^6)*cos(d*x + c)^3 + (2*a^10 - 3*a^9*b - 10*a^8
*b^2 + 25*a^7*b^3 - 10*a^6*b^4 - 17*a^5*b^5 + 18*a^4*b^6 - 5*a^3*b^7)*cos(
d*x + c)^2 + 2*(2*a^9*b - 3*a^8*b^2 - 8*a^7*b^3 + 22*a^6*b^4 - 18*a^5*b...
```

3.264.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cos(d*x+c)**3/(a*sin(d*x+c)+b*tan(d*x+c))**3,x)
```

output
```
Timed out
```

3.264.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 684 vs. $2(240) = 480$.

Time = 0.34 (sec) , antiderivative size = 684, normalized size of antiderivative = 2.76

$$\int \frac{\cos^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx =$$
$$-\frac{\frac{8\,(15\,a^4b^4-4\,a^2b^6+b^8)\log\left(a+b-\frac{(a-b)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^{11}-4\,a^9b^2+6\,a^7b^4-4\,a^5b^6+a^3b^8}+\frac{4\,(2\,a+5\,b)\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}+\frac{a^8-2\,a^7b-a^6b^2+4\,a^5b^3-a^4b^4-2\cdots}{\frac{(a^{11}+a^{10}b-4\,a^9b^2-4\,a^8b^3+6\,a^7b^4+6\,a^6b^5-4\,a^5b^6-4\cdots}{(\cos(dx+c)+1)^2}}}{\cdots}$$

input
```
integrate(cos(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima"
)
```

output
```
-1/8*(8*(15*a^4*b^4 - 4*a^2*b^6 + b^8)*log(a + b - (a - b)*sin(d*x + c)^2/
(cos(d*x + c) + 1)^2)/(a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)
 + 4*(2*a + 5*b)*log(sin(d*x + c)/(cos(d*x + c) + 1))/(a^4 + 4*a^3*b + 6*a
^2*b^2 + 4*a*b^3 + b^4) + (a^8 - 2*a^7*b - a^6*b^2 + 4*a^5*b^3 - a^4*b^4 -
 2*a^3*b^5 + a^2*b^6 - 2*(a^8 - 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 44*a^3*b
^5 - 49*a^2*b^6 + 8*a*b^7 + 8*b^8)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + (
a^8 - 6*a^7*b + 15*a^6*b^2 - 20*a^5*b^3 + 15*a^4*b^4 - 102*a^3*b^5 + 81*a^
2*b^6 + 32*a*b^7 - 16*b^8)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)/((a^11 + a
^10*b - 4*a^9*b^2 - 4*a^8*b^3 + 6*a^7*b^4 + 6*a^6*b^5 - 4*a^5*b^6 - 4*a^4*
b^7 + a^3*b^8 + a^2*b^9)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 2*(a^11 - a
^10*b - 4*a^9*b^2 + 4*a^8*b^3 + 6*a^7*b^4 - 6*a^6*b^5 - 4*a^5*b^6 + 4*a^4*
b^7 + a^3*b^8 - a^2*b^9)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + (a^11 - 3*a
^10*b + 8*a^8*b^3 - 6*a^7*b^4 - 6*a^6*b^5 + 8*a^5*b^6 - 3*a^3*b^8 + a^2*b^
9)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + sin(d*x + c)^2/((a^3 - 3*a^2*b +
 3*a*b^2 - b^3)*(cos(d*x + c) + 1)^2) - 8*log(sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + 1)/a^3)/d
```

3.264.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 848 vs. $2(240) = 480$.

Time = 0.95 (sec) , antiderivative size = 848, normalized size of antiderivative = 3.42

$$\int \frac{\cos^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/8*(2*(2*a + 5*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^4
 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + 8*(15*a^4*b^4 - 4*a^2*b^6 + b^8)
*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c
) - 1)/(cos(d*x + c) + 1)))/(a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^
3*b^8) - (a + b + 4*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 10*b*(cos(d*
x + c) - 1)/(cos(d*x + c) + 1))*(cos(d*x + c) + 1)/((a^4 + 4*a^3*b + 6*a^2
*b^2 + 4*a*b^3 + b^4)*(cos(d*x + c) - 1)) - (cos(d*x + c) - 1)/((a^3 - 3*a
^2*b + 3*a*b^2 - b^3)*(cos(d*x + c) + 1)) - 4*(45*a^6*b^4 + 66*a^5*b^5 - 1
5*a^4*b^6 - 44*a^3*b^7 - a^2*b^8 + 10*a*b^9 + 3*b^10 + 90*a^6*b^4*(cos(d*x
 + c) - 1)/(cos(d*x + c) + 1) - 24*a^5*b^5*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1) - 118*a^4*b^6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 28*a^3*b^7*(c
os(d*x + c) - 1)/(cos(d*x + c) + 1) + 34*a^2*b^8*(cos(d*x + c) - 1)/(cos(d
*x + c) + 1) - 4*a*b^9*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 6*b^10*(cos
(d*x + c) - 1)/(cos(d*x + c) + 1) + 45*a^6*b^4*(cos(d*x + c) - 1)^2/(cos(d
*x + c) + 1)^2 - 90*a^5*b^5*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 33
*a^4*b^6*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 24*a^3*b^7*(cos(d*x +
 c) - 1)^2/(cos(d*x + c) + 1)^2 - 9*a^2*b^8*(cos(d*x + c) - 1)^2/(cos(d*x
+ c) + 1)^2 - 6*a*b^9*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 3*b^10*(
cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/((a^11 - 4*a^9*b^2 + 6*a^7*b^4 -
 4*a^5*b^6 + a^3*b^8)*(a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) ...
```

3.264.9 Mupad [B] (verification not implemented)

Time = 25.34 (sec) , antiderivative size = 527, normalized size of antiderivative = 2.12

$$\int \frac{\cos^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$$

$$= \frac{\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(-a^7+5a^6b-10a^5b^2+10a^4b^3-5a^3b^4+97a^2b^5+16ab^6-16b^7\right)}{2a^2(a+b)(a^2+2ab+b^2)}-\frac{a^3-3a^2}{2}}{d\left(\left(4a^5-20a^4b+40a^3b^2-40a^2b^3+20ab^4-4b^5\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+\left(-8a^5+24a^4b-16a^3b^2-16a\right.\right.}$$

$$-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8d(a-b)^3}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{a^3d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)(2a+5b)}{d\left(2a^4+8a^3b+12a^2b^2+8ab^3+2b^4\right)}$$

$$-\frac{b^4\ln\left(a+b-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)\left(15a^4-4a^2b^2+b^4\right)}{a^3d\left(a^2-b^2\right)^4}$$

input
```
int(cos(c + d*x)^3/(a*sin(c + d*x) + b*tan(c + d*x))^3,x)
```

output
```
((tan(c/2 + (d*x)/2)^4*(16*a*b^6 + 5*a^6*b - a^7 - 16*b^7 + 97*a^2*b^5 - 5
*a^3*b^4 + 10*a^4*b^3 - 10*a^5*b^2))/(2*a^2*(a + b)*(2*a*b + a^2 + b^2)) -
 (3*a*b^2 - 3*a^2*b + a^3 - b^3)/(2*(a + b)) + (tan(c/2 + (d*x)/2)^2*(a^7
- 5*a^6*b + 8*b^7 - 49*a^2*b^5 + 5*a^3*b^4 - 10*a^4*b^3 + 10*a^5*b^2))/(a^
2*(a + b)^2*(a - b)))/(d*(tan(c/2 + (d*x)/2)^2*(4*a*b^4 - 4*a^4*b + 4*a^5
- 4*b^5 + 8*a^2*b^3 - 8*a^3*b^2) - tan(c/2 + (d*x)/2)^4*(8*a^5 - 24*a^4*b
- 24*a*b^4 + 8*b^5 + 16*a^2*b^3 + 16*a^3*b^2) + tan(c/2 + (d*x)/2)^6*(20*a
*b^4 - 20*a^4*b + 4*a^5 - 4*b^5 - 40*a^2*b^3 + 40*a^3*b^2))) - tan(c/2 + (
d*x)/2)^2/(8*d*(a - b)^3) + log(tan(c/2 + (d*x)/2)^2 + 1)/(a^3*d) - (log(t
an(c/2 + (d*x)/2))*(2*a + 5*b))/(d*(8*a*b^3 + 8*a^3*b + 2*a^4 + 2*b^4 + 12
*a^2*b^2)) - (b^4*log(a + b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2
)^2)*(15*a^4 + b^4 - 4*a^2*b^2))/(a^3*d*(a^2 - b^2)^4)
```

3.264. $\int \frac{\cos^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$

3.265 $\int \frac{\cos^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$

3.265.1 Optimal result

Integrand size = 28, antiderivative size = 232

$$\begin{aligned}\int \frac{\cos^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx &= -\frac{b^5}{2a^2(a^2-b^2)^2 d(b+a\cos(c+dx))^2}+\frac{b^4(5a^2-b^2)}{a^2(a^2-b^2)^3 d(b+a\cos(c+dx))}\\ &\quad+\frac{(b(3a^2+b^2)-a(a^2+3b^2)\cos(c+dx))\csc^2(c+dx)}{2(a^2-b^2)^3 d}-\frac{(a+4b)\log(1-\cos(c+dx))}{4(a+b)^4 d}\\ &\quad+\frac{(a-4b)\log(1+\cos(c+dx))}{4(a-b)^4 d}+\frac{2b^3(5a^2+b^2)\log(b+a\cos(c+dx))}{(a^2-b^2)^4 d}\end{aligned}$$

output
```
-1/2*b^5/a^2/(a^2-b^2)^2/d/(b+a*cos(d*x+c))^2+b^4*(5*a^2-b^2)/a^2/(a^2-b^2
)^3/d/(b+a*cos(d*x+c))+1/2*(b*(3*a^2+b^2)-a*(a^2+3*b^2)*cos(d*x+c))*csc(d*
x+c)^2/(a^2-b^2)^3/d-1/4*(a+4*b)*ln(1-cos(d*x+c))/(a+b)^4/d+1/4*(a-4*b)*ln
(1+cos(d*x+c))/(a-b)^4/d+2*b^3*(5*a^2+b^2)*ln(b+a*cos(d*x+c))/(a^2-b^2)^4/
d
```

3.265.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 477 vs. $2(232) = 464$.

Time = 7.05 (sec) , antiderivative size = 477, normalized size of antiderivative = 2.06

$$\begin{aligned}
&\int \frac{\cos^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx \\
&= -\frac{b^5(b+a\cos(c+dx))\tan^3(c+dx)}{2a^2(-a+b)^2(a+b)^2d(a\sin(c+dx)+b\tan(c+dx))^3} \\
&\quad+\frac{b^4(-5a^2+b^2)\,(b+a\cos(c+dx))^2\tan^3(c+dx)}{a^2(-a+b)^3(a+b)^3d(a\sin(c+dx)+b\tan(c+dx))^3} \\
&\quad-\frac{(b+a\cos(c+dx))^3\csc^2\left(\frac{1}{2}(c+dx)\right)\tan^3(c+dx)}{8(a+b)^3d(a\sin(c+dx)+b\tan(c+dx))^3} \\
&\quad+\frac{(a-4b)(b+a\cos(c+dx))^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\tan^3(c+dx)}{2(-a+b)^4d(a\sin(c+dx)+b\tan(c+dx))^3} \\
&\quad+\frac{2(5a^2b^3+b^5)\,(b+a\cos(c+dx))^3\log(b+a\cos(c+dx))\tan^3(c+dx)}{(-a^2+b^2)^4\,d(a\sin(c+dx)+b\tan(c+dx))^3} \\
&\quad+\frac{(-a-4b)(b+a\cos(c+dx))^3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\tan^3(c+dx)}{2(a+b)^4d(a\sin(c+dx)+b\tan(c+dx))^3} \\
&\quad-\frac{(b+a\cos(c+dx))^3\sec^2\left(\frac{1}{2}(c+dx)\right)\tan^3(c+dx)}{8(-a+b)^3d(a\sin(c+dx)+b\tan(c+dx))^3}
\end{aligned}$$

input
```
Integrate[Cos[c + d*x]^2/(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

output
```
-1/2*(b^5*(b + a*Cos[c + d*x])*Tan[c + d*x]^3)/(a^2*(-a + b)^2*(a + b)^2*d
*(a*Sin[c + d*x] + b*Tan[c + d*x])^3) + (b^4*(-5*a^2 + b^2)*(b + a*Cos[c +
 d*x])^2*Tan[c + d*x]^3)/(a^2*(-a + b)^3*(a + b)^3*d*(a*Sin[c + d*x] + b*T
an[c + d*x])^3) - ((b + a*Cos[c + d*x])^3*Csc[(c + d*x)/2]^2*Tan[c + d*x]^
3)/(8*(a + b)^3*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3) + ((a - 4*b)*(b + a
*Cos[c + d*x])^3*Log[Cos[(c + d*x)/2]]*Tan[c + d*x]^3)/(2*(-a + b)^4*d*(a*
Sin[c + d*x] + b*Tan[c + d*x])^3) + (2*(5*a^2*b^3 + b^5)*(b + a*Cos[c + d*
x])^3*Log[b + a*Cos[c + d*x]]*Tan[c + d*x]^3)/((-a^2 + b^2)^4*d*(a*Sin[c +
 d*x] + b*Tan[c + d*x])^3) + ((-a - 4*b)*(b + a*Cos[c + d*x])^3*Log[Sin[(c
 + d*x)/2]]*Tan[c + d*x]^3)/(2*(a + b)^4*d*(a*Sin[c + d*x] + b*Tan[c + d*x
])^3) - ((b + a*Cos[c + d*x])^3*Sec[(c + d*x)/2]^2*Tan[c + d*x]^3)/(8*(-a
+ b)^3*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3)
```

3.265.3 Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3042, 4897, 3042, 25, 3316, 25, 27, 601, 2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \textbf{3042}$$

$$\int \frac{\cos(c+dx)^2}{(a\sin(c+dx)+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \textbf{4897}$$

$$\int \frac{\cos^2(c+dx)\cot^3(c+dx)}{(a\cos(c+dx)+b)^3}dx$$

$$\Big\downarrow \textbf{3042}$$

$$\int -\frac{\sin\left(c+dx-\frac{\pi}{2}\right)^5}{\cos\left(c+dx-\frac{\pi}{2}\right)^3\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \textbf{25}$$

$$-\int \frac{\sin\left(\frac{1}{2}(2c-\pi)+dx\right)^5}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^3\left(b-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)^3}dx$$

$$\Big\downarrow \textbf{3316}$$

$$\frac{a^3\int -\frac{\cos^5(c+dx)}{(b+a\cos(c+dx))^3(a^2-a^2\cos^2(c+dx))^2}d(a\cos(c+dx))}{d}$$

$$\Big\downarrow \textbf{25}$$

$$-\frac{a^3\int \frac{\cos^5(c+dx)}{(b+a\cos(c+dx))^3(a^2-a^2\cos^2(c+dx))^2}d(a\cos(c+dx))}{d}$$

$$\Big\downarrow \textbf{27}$$

$$-\frac{\int \frac{a^5\cos^5(c+dx)}{(b+a\cos(c+dx))^3(a^2-a^2\cos^2(c+dx))^2}d(a\cos(c+dx))}{a^2d}$$

$$\Big\downarrow \textbf{601}$$

$$-\frac{-\frac{\int \frac{\frac{b(3a^2-7b^2)\cos^2(c+dx)a^8}{(a^2-b^2)^3}+\frac{b^3(a^2+3b^2)a^6}{(a^2-b^2)^3}+\frac{(a^6-9b^2a^4+6b^4a^2-2b^6)\cos^3(c+dx)a^5}{(a^2-b^2)^3}+\frac{b^2(3a^4+3b^2a^2-2b^4)\cos(c+dx)a^5}{(a^2-b^2)^3}}{(b+a\cos(c+dx))^3\left(a^2-a^2\cos^2(c+dx)\right)}d(a\cos(c+dx))}{2a^2}-\frac{a^4(b(3a\ldots}{2(a\ldots}}{a^2d}$$

$\downarrow$ 2160

$$-\frac{-\frac{\int\left(\frac{2a^2b^5}{(a^2-b^2)^2(b+a\cos(c+dx))^3}-\frac{2a^2(5a^2-b^2)b^4}{(a^2-b^2)^3(b+a\cos(c+dx))^2}+\frac{4a^4(5a^2+b^2)b^3}{(a^2-b^2)^4(b+a\cos(c+dx))}+\frac{a^4(a+4b)}{2(a+b)^4(a-a\cos(c+dx))}+\frac{a^4(a-4b)}{2(a-b)^4(\cos(c+dx)a+a)}\right)d(a\cos\ldots}{2a^2}}{a^2d}$$

$\downarrow$ 2009

$$-\frac{-\frac{a^4(b(3a^2+b^2)-a(a^2+3b^2)\cos(c+dx))}{2(a^2-b^2)^3(a^2-a^2\cos^2(c+dx))}-\frac{-\frac{a^4(a+4b)\log(a-a\cos(c+dx))}{2(a+b)^4}+\frac{a^4(a-4b)\log(a\cos(c+dx)+a)}{2(a-b)^4}-\frac{a^2b^5}{(a^2-b^2)^2(a\cos(c+dx)+b)^2}+\frac{2a^2b^4(\ldots}{(a^2-b^2)^3(\ldots}}{2a^2}}{a^2d}$$

input
```
Int[Cos[c + d*x]^2/(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

output
```
-((-1/2*(a^4*(b*(3*a^2 + b^2) - a*(a^2 + 3*b^2)*Cos[c + d*x]))/((a^2 - b^2
)^3*(a^2 - a^2*Cos[c + d*x]^2)) - (-((a^2*b^5)/((a^2 - b^2)^2*(b + a*Cos[c
 + d*x])^2)) + (2*a^2*b^4*(5*a^2 - b^2))/((a^2 - b^2)^3*(b + a*Cos[c + d*x
])) - (a^4*(a + 4*b)*Log[a - a*Cos[c + d*x]])/(2*(a + b)^4) + (a^4*(a - 4*
b)*Log[a + a*Cos[c + d*x]])/(2*(a - b)^4) + (4*a^4*b^3*(5*a^2 + b^2)*Log[b
 + a*Cos[c + d*x]])/(a^2 - b^2)^4)/(2*a^2))/(a^2*d))
```

3.265.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 601
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c
+ d*x)^n*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*(p + 1)*Qx)/(c + d*x)^n + (e*
(2*p + 3))/(c + d*x)^n, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 1]
 && LtQ[p, -1] && ILtQ[n, 0] && NeQ[b*c^2 + a*d^2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.265.4 Maple [A] (verified)

Time = 5.15 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{-\frac{b^5}{2a^2(a+b)^2(a-b)^2(b+\cos(dx+c)a)^2}+\frac{2b^3(5a^2+b^2)\ln(b+\cos(dx+c)a)}{(a+b)^4(a-b)^4}+\frac{b^4(5a^2-b^2)}{(a+b)^3(a-b)^3a^2(b+\cos(dx+c)a)}+\frac{1}{4(a+b)^3(\cos(dx+c)-1}}{d}$
default	$\frac{-\frac{b^5}{2a^2(a+b)^2(a-b)^2(b+\cos(dx+c)a)^2}+\frac{2b^3(5a^2+b^2)\ln(b+\cos(dx+c)a)}{(a+b)^4(a-b)^4}+\frac{b^4(5a^2-b^2)}{(a+b)^3(a-b)^3a^2(b+\cos(dx+c)a)}+\frac{1}{4(a+b)^3(\cos(dx+c)-1}}{d}$
risch	Expression too large to display

3.265. $\int \frac{\cos^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$

input
```
int(cos(d*x+c)^2/(sin(d*x+c)*a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2*b^5/a^2/(a+b)^2/(a-b)^2/(b+cos(d*x+c)*a)^2+2*b^3*(5*a^2+b^2)/(a+
b)^4/(a-b)^4*ln(b+cos(d*x+c)*a)+b^4*(5*a^2-b^2)/(a+b)^3/(a-b)^3/a^2/(b+cos
(d*x+c)*a)+1/4/(a+b)^3/(cos(d*x+c)-1)+1/4/(a+b)^4*(-a-4*b)*ln(cos(d*x+c)-1
)+1/4/(a-b)^3/(cos(d*x+c)+1)+1/4*(a-4*b)/(a-b)^4*ln(cos(d*x+c)+1))
```

3.265.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1045 vs. $2(224) = 448$.

Time = 0.46 (sec) , antiderivative size = 1045, normalized size of antiderivative = 4.50

$$\int \frac{\cos^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(cos(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas"
)
```

output
```
-1/4*(6*a^6*b^3 + 14*a^4*b^5 - 22*a^2*b^7 + 2*b^9 - 2*(a^9 + 2*a^7*b^2 + 7
*a^5*b^4 - 12*a^3*b^6 + 2*a*b^8)*cos(d*x + c)^3 + 2*(a^8*b - 6*a^6*b^3 - 4
*a^4*b^5 + 10*a^2*b^7 - b^9)*cos(d*x + c)^2 + 2*(5*a^7*b^2 + 4*a^5*b^4 - 1
1*a^3*b^6 + 2*a*b^8)*cos(d*x + c) + 8*(5*a^4*b^5 + a^2*b^7 - (5*a^6*b^3 +
a^4*b^5)*cos(d*x + c)^4 - 2*(5*a^5*b^4 + a^3*b^6)*cos(d*x + c)^3 + (5*a^6*
b^3 - 4*a^4*b^5 - a^2*b^7)*cos(d*x + c)^2 + 2*(5*a^5*b^4 + a^3*b^6)*cos(d*
x + c))*log(a*cos(d*x + c) + b) + (a^7*b^2 - 10*a^5*b^4 - 20*a^4*b^5 - 15*
a^3*b^6 - 4*a^2*b^7 - (a^9 - 10*a^7*b^2 - 20*a^6*b^3 - 15*a^5*b^4 - 4*a^4*
b^5)*cos(d*x + c)^4 - 2*(a^8*b - 10*a^6*b^3 - 20*a^5*b^4 - 15*a^4*b^5 - 4*
a^3*b^6)*cos(d*x + c)^3 + (a^9 - 11*a^7*b^2 - 20*a^6*b^3 - 5*a^5*b^4 + 16*
a^4*b^5 + 15*a^3*b^6 + 4*a^2*b^7)*cos(d*x + c)^2 + 2*(a^8*b - 10*a^6*b^3 -
 20*a^5*b^4 - 15*a^4*b^5 - 4*a^3*b^6)*cos(d*x + c))*log(1/2*cos(d*x + c) +
 1/2) - (a^7*b^2 - 10*a^5*b^4 + 20*a^4*b^5 - 15*a^3*b^6 + 4*a^2*b^7 - (a^9
 - 10*a^7*b^2 + 20*a^6*b^3 - 15*a^5*b^4 + 4*a^4*b^5)*cos(d*x + c)^4 - 2*(a
^8*b - 10*a^6*b^3 + 20*a^5*b^4 - 15*a^4*b^5 + 4*a^3*b^6)*cos(d*x + c)^3 +
(a^9 - 11*a^7*b^2 + 20*a^6*b^3 - 5*a^5*b^4 - 16*a^4*b^5 + 15*a^3*b^6 - 4*a
^2*b^7)*cos(d*x + c)^2 + 2*(a^8*b - 10*a^6*b^3 + 20*a^5*b^4 - 15*a^4*b^5 +
 4*a^3*b^6)*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/((a^12 - 4*a^10*b^
2 + 6*a^8*b^4 - 4*a^6*b^6 + a^4*b^8)*d*cos(d*x + c)^4 + 2*(a^11*b - 4*a^9*
b^3 + 6*a^7*b^5 - 4*a^5*b^7 + a^3*b^9)*d*cos(d*x + c)^3 - (a^12 - 5*a^1...
```

3.265.6 Sympy [F]

$$\int \frac{\cos^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx = \int \frac{\cos^2{(c+dx)}}{\left(a\sin{(c+dx)}+b\tan{(c+dx)}\right)^3}\,dx$$

input
```
integrate(cos(d*x+c)**2/(a*sin(d*x+c)+b*tan(d*x+c))**3,x)
```

output
```
Integral(cos(c + d*x)**2/(a*sin(c + d*x) + b*tan(c + d*x))**3, x)
```

3.265.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 589 vs. $2(224) = 448$.

Time = 0.27 (sec) , antiderivative size = 589, normalized size of antiderivative = 2.54

$$\int \frac{\cos^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$$
$$= \frac{\frac{16\left(5a^2b^3+b^5\right)\log\left(a+b-\frac{(a-b)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8} - \frac{4(a+4b)\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4+4a^3b+6a^2b^2+4ab^3+b^4} - \frac{a^6-2a^5b-a^4b^2+4a^3b^3-a^2b^4-2a}{\frac{\left(a^9+a^8b-4a^7b^2-4a^6b^3+6a^5b^4+6a^4b^5-4a^3b^6-4a^2b^7+ab^8+b^9\right)s}{(\cos(dx+c)+1)^2}}}{}$$

input
```
integrate(cos(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima"
)
```

output
```
1/8*(16*(5*a^2*b^3 + b^5)*log(a + b - (a - b)*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2)/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) - 4*(a + 4*b)*log
(sin(d*x + c)/(cos(d*x + c) + 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b
^4) - (a^6 - 2*a^5*b - a^4*b^2 + 4*a^3*b^3 - a^2*b^4 - 2*a*b^5 + b^6 - 2*(
a^6 - 4*a^5*b + 5*a^4*b^2 + 35*a^2*b^4 + 44*a*b^5 - b^6)*sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 + (a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 95*a^2*b^4
 - 70*a*b^5 - 15*b^6)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)/((a^9 + a^8*b -
 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b^7 + a
*b^8 + b^9)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 2*(a^9 - a^8*b - 4*a^7*b
^2 + 4*a^6*b^3 + 6*a^5*b^4 - 6*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8 - b
^9)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + (a^9 - 3*a^8*b + 8*a^6*b^3 - 6*a
^5*b^4 - 6*a^4*b^5 + 8*a^3*b^6 - 3*a*b^8 + b^9)*sin(d*x + c)^6/(cos(d*x +
c) + 1)^6) + sin(d*x + c)^2/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(cos(d*x + c)
 + 1)^2))/d
```

3.265.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 676 vs. $2(224) = 448$.

Time = 0.88 (sec) , antiderivative size = 676, normalized size of antiderivative = 2.91

$$\int \frac{\cos^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx =$$

$$-\frac{\frac{2\,(a+4\,b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4} - \frac{16\,(5\,a^2b^3+b^5)\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8} - \frac{\left(a+b+\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{8\,b(\cos(dx+c)\cdots}{\cos(dx+c)+\cdots}\right.}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)(c\cdots}}{\cdots}$$

input

```
integrate(cos(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")
```

output

```
-1/8*(2*(a + 4*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^4 +
 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - 16*(5*a^2*b^3 + b^5)*log(abs(-a -
b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c) - 1)/(cos(d*
x + c) + 1)))/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) - (a + b + 2
*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 8*b*(cos(d*x + c) - 1)/(cos(d*x
 + c) + 1))*(cos(d*x + c) + 1)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4
)*(cos(d*x + c) - 1)) + (cos(d*x + c) - 1)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3
)*(cos(d*x + c) + 1)) + 8*(15*a^4*b^3 + 20*a^3*b^4 - 2*a^2*b^5 - 4*a*b^6 +
 3*b^7 + 30*a^4*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 10*a^3*b^4*(co
s(d*x + c) - 1)/(cos(d*x + c) + 1) - 26*a^2*b^5*(cos(d*x + c) - 1)/(cos(d*
x + c) + 1) + 10*a*b^6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 4*b^7*(cos(
d*x + c) - 1)/(cos(d*x + c) + 1) + 15*a^4*b^3*(cos(d*x + c) - 1)^2/(cos(d*
x + c) + 1)^2 - 30*a^3*b^4*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 18*
a^2*b^5*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 6*a*b^6*(cos(d*x + c)
- 1)^2/(cos(d*x + c) + 1)^2 + 3*b^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1
)^2)/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(a + b + a*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))^2)
)/d
```

3.265.9 Mupad [B] (verification not implemented)

Time = 22.89 (sec) , antiderivative size = 491, normalized size of antiderivative = 2.12

$$\int \frac{\cos^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8\,d\,(a-b)^3}$$

$$-\frac{\frac{a^3-3\,a^2\,b+3\,a\,b^2-b^3}{2\,(a+b)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,(a^5-5\,a^4\,b+10\,a^3\,b^2-10\,a^2\,b^3+85}{2\,(a+b)\,(a^2+2\,a\,b+b^2)}}{d\,\left((4\,a^5-20\,a^4\,b+40\,a^3\,b^2-40\,a^2\,b^3+20\,a\,b^4-4\,b^5)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+(-8\,a^5+24\,a^4\,b-16\,a^3\,b^2-1}$$

$$-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,(a+4\,b)}{d\,(2\,a^4+8\,a^3\,b+12\,a^2\,b^2+8\,a\,b^3+2\,b^4)}$$

$$+\frac{\ln\left(a+b-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)\,(10\,a^2\,b^3+2\,b^5)}{d\,(a^8-4\,a^6\,b^2+6\,a^4\,b^4-4\,a^2\,b^6+b^8)}$$

input
```
int(cos(c + d*x)^2/(a*sin(c + d*x) + b*tan(c + d*x))^3,x)
```

output
```
tan(c/2 + (d*x)/2)^2/(8*d*(a - b)^3) - ((3*a*b^2 - 3*a^2*b + a^3 - b^3)/(2
*(a + b)) + (tan(c/2 + (d*x)/2)^4*(85*a*b^4 - 5*a^4*b + a^5 + 15*b^5 - 10*
a^2*b^3 + 10*a^3*b^2))/(2*(a + b)*(2*a*b + a^2 + b^2)) - (tan(c/2 + (d*x)/
2)^2*(45*a*b^4 - 5*a^4*b + a^5 - b^5 - 10*a^2*b^3 + 10*a^3*b^2))/((a - b)*
(2*a*b + a^2 + b^2)))/(d*(tan(c/2 + (d*x)/2)^2*(4*a*b^4 - 4*a^4*b + 4*a^5
- 4*b^5 + 8*a^2*b^3 - 8*a^3*b^2) - tan(c/2 + (d*x)/2)^4*(8*a^5 - 24*a^4*b
- 24*a*b^4 + 8*b^5 + 16*a^2*b^3 + 16*a^3*b^2) + tan(c/2 + (d*x)/2)^6*(20*a
*b^4 - 20*a^4*b + 4*a^5 - 4*b^5 - 40*a^2*b^3 + 40*a^3*b^2))) - (log(tan(c/
2 + (d*x)/2))*(a + 4*b))/(d*(8*a*b^3 + 8*a^3*b + 2*a^4 + 2*b^4 + 12*a^2*b^
2)) + (log(a + b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2)*(2*b^5
 + 10*a^2*b^3))/(d*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2))
```

3.266 $\int \frac{\cos(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$

3.266.1 Optimal result

Integrand size = 26, antiderivative size = 211

$$\begin{aligned}\int \frac{\cos(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx &= \frac{b^4}{2a\left(a^2-b^2\right)^2d(b+a\cos(c+dx))^2}-\frac{4ab^3}{\left(a^2-b^2\right)^3d(b+a\cos(c+dx))}\\ &\quad-\frac{(a(a^2+3b^2)-b(3a^2+b^2)\cos(c+dx))\csc^2(c+dx)}{2\left(a^2-b^2\right)^3d}-\frac{3b\log(1-\cos(c+dx))}{4(a+b)^4d}\\ &\quad+\frac{3b\log(1+\cos(c+dx))}{4(a-b)^4d}-\frac{6ab^2(a^2+b^2)\log(b+a\cos(c+dx))}{\left(a^2-b^2\right)^4d}\end{aligned}$$

output
```
1/2*b^4/a/(a^2-b^2)^2/d/(b+a*cos(d*x+c))^2-4*a*b^3/(a^2-b^2)^3/d/(b+a*cos(
d*x+c))-1/2*(a*(a^2+3*b^2)-b*(3*a^2+b^2)*cos(d*x+c))*csc(d*x+c)^2/(a^2-b^2
)^3/d-3/4*b*ln(1-cos(d*x+c))/(a+b)^4/d+3/4*b*ln(1+cos(d*x+c))/(a-b)^4/d-6*
a*b^2*(a^2+b^2)*ln(b+a*cos(d*x+c))/(a^2-b^2)^4/d
```

3.266.2 Mathematica [A] (verified)

Time = 6.29 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.87

$$\int \frac{\cos(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$$
$$= \frac{\frac{4b^4}{a(a-b)^2(a+b)^2(b+a\cos(c+dx))^2}+\frac{32ab^3}{(-a+b)^3(a+b)^3(b+a\cos(c+dx))}-\frac{\csc^2\left(\frac{1}{2}(c+dx)\right)}{(a+b)^3}+\frac{12b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{(a-b)^4}-\frac{48ab^2(a^2+b^2)\log(b+}{(a^2-b^2)^4}}{8d}$$

input
```
Integrate[Cos[c + d*x]/(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

output
```
((4*b^4)/(a*(a - b)^2*(a + b)^2*(b + a*Cos[c + d*x])^2) + (32*a*b^3)/((-a
+ b)^3*(a + b)^3*(b + a*Cos[c + d*x])) - Csc[(c + d*x)/2]^2/(a + b)^3 + (1
2*b*Log[Cos[(c + d*x)/2]])/(a - b)^4 - (48*a*b^2*(a^2 + b^2)*Log[b + a*Cos
[c + d*x]])/(a^2 - b^2)^4 - (12*b*Log[Sin[(c + d*x)/2]])/(a + b)^4 + Sec[(
c + d*x)/2]^2/(-a + b)^3)/(8*d)
```

3.266.3 Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {3042, 4897, 3042, 3316, 27, 601, 25, 2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\cos(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}dx$$
$$\Big\downarrow \; 4897$$
$$\int \frac{\cos(c+dx)\cot^3(c+dx)}{(a\cos(c+dx)+b)^3}dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\sin\left(c+dx-\frac{\pi}{2}\right)^4}{\cos\left(c+dx-\frac{\pi}{2}\right)^3\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{ 3316}$$

$$-\frac{a^3\int \frac{\cos^4(c+dx)}{(b+a\cos(c+dx))^3(a^2-a^2\cos^2(c+dx))^2}d(a\cos(c+dx))}{d}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int \frac{a^4\cos^4(c+dx)}{(b+a\cos(c+dx))^3(a^2-a^2\cos^2(c+dx))^2}d(a\cos(c+dx))}{ad}$$

$$\Big\downarrow \text{ 601}$$

$$-\frac{\frac{a^2(a^2(a^2+3b^2)-ab(3a^2+b^2)\cos(c+dx))}{2(a^2-b^2)^3(a^2-a^2\cos^2(c+dx))}-\frac{\int -\frac{-\frac{b(3a^2+b^2)\cos^3(c+dx)a^7}{(a^2-b^2)^3}+\frac{b^3(7a^2-3b^2)\cos(c+dx)a^5}{(a^2-b^2)^3}+\frac{b^2(3a^4-9b^2a^2+2b^4)\cos^2(c+dx)a^4}{(a^2-b^2)^3}+\frac{b^4(3a^2}{(a^2-}}{(b+a\cos(c+dx))^3(a^2-a^2\cos^2(c+dx))}}{2a^2}}{ad}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\frac{\int \frac{-\frac{b(3a^2+b^2)\cos^3(c+dx)a^7}{(a^2-b^2)^3}+\frac{b^3(7a^2-3b^2)\cos(c+dx)a^5}{(a^2-b^2)^3}+\frac{b^2(3a^4-9b^2a^2+2b^4)\cos^2(c+dx)a^4}{(a^2-b^2)^3}+\frac{b^4(3a^2+b^2)a^4}{(a^2-b^2)^3}}{(b+a\cos(c+dx))^3(a^2-a^2\cos^2(c+dx))}d(a\cos(c+dx))}{2a^2}+\frac{a^2(a^2(a^2+3b^2)-ab}{2(a^2-b^2)^3(a^2-}}{ad}$$

$$\Big\downarrow \text{ 2160}$$

$$-\frac{\frac{\int \left(\frac{12b^2(a^2+b^2)a^4}{(a^2-b^2)^4(b+a\cos(c+dx))}-\frac{8b^3a^4}{(a^2-b^2)^3(b+a\cos(c+dx))^2}-\frac{3ba^3}{2(a+b)^4(a-a\cos(c+dx))}-\frac{3ba^3}{2(a-b)^4(\cos(c+dx)a+a)}+\frac{2b^4a^2}{(a^2-b^2)^2(b+a\cos(c+dx))^3}\right)d(a\cos(c}{2a^2}}{ad}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\frac{a^2(a^2(a^2+3b^2)-ab(3a^2+b^2)\cos(c+dx))}{2(a^2-b^2)^3(a^2-a^2\cos^2(c+dx))}+\frac{\frac{3a^3b\log(a-a\cos(c+dx))}{2(a+b)^4}-\frac{3a^3b\log(a\cos(c+dx)+a)}{2(a-b)^4}-\frac{a^2b^4}{(a^2-b^2)^2(a\cos(c+dx)+b)^2}+\frac{12a^4b^2(a^2+b^2)\log(a\,c}{(a^2-b^2)^4}}{2a^2}}{ad}$$

input
```
Int[Cos[c + d*x]/(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

3.266. $\int \frac{\cos(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$

output
```
-(((a^2*(a^2*(a^2 + 3*b^2) - a*b*(3*a^2 + b^2)*Cos[c + d*x]))/(2*(a^2 - b^
2)^3*(a^2 - a^2*Cos[c + d*x]^2)) + (-((a^2*b^4)/((a^2 - b^2)^2*(b + a*Cos[
c + d*x])^2)) + (8*a^4*b^3)/((a^2 - b^2)^3*(b + a*Cos[c + d*x])) + (3*a^3*
b*Log[a - a*Cos[c + d*x]])/(2*(a + b)^4) - (3*a^3*b*Log[a + a*Cos[c + d*x]
])/(2*(a - b)^4) + (12*a^4*b^2*(a^2 + b^2)*Log[b + a*Cos[c + d*x]])/(a^2 -
 b^2)^4)/(2*a^2))/(a*d))
```

3.266.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 601
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c
+ d*x)^n*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*(p + 1)*Qx)/(c + d*x)^n + (e*
(2*p + 3))/(c + d*x)^n, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 1]
 && LtQ[p, -1] && ILtQ[n, 0] && NeQ[b*c^2 + a*d^2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.266.4 Maple [A] (verified)

Time = 5.15 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{\frac{b^4}{2(a+b)^2(a-b)^2a(b+\cos(dx+c)a)^2}-\frac{4a\,b^3}{(a+b)^3(a-b)^3(b+\cos(dx+c)a)}-\frac{6a\,b^2\left(a^2+b^2\right)\ln(b+\cos(dx+c)a)}{(a+b)^4(a-b)^4}-\frac{1}{4(a-b)^3(\cos(dx+c)+1)}+\frac{3b}{}}{d}$
default	$\frac{\frac{b^4}{2(a+b)^2(a-b)^2a(b+\cos(dx+c)a)^2}-\frac{4a\,b^3}{(a+b)^3(a-b)^3(b+\cos(dx+c)a)}-\frac{6a\,b^2\left(a^2+b^2\right)\ln(b+\cos(dx+c)a)}{(a+b)^4(a-b)^4}-\frac{1}{4(a-b)^3(\cos(dx+c)+1)}+\frac{3b}{}}{d}$
risch	$-\frac{3ibx}{2(a^4-4a^3b+6a^2b^2-4a\,b^3+b^4)}-\frac{3ibc}{2d(a^4-4a^3b+6a^2b^2-4a\,b^3+b^4)}+\frac{3ibx}{2(a^4+4a^3b+6a^2b^2+4a\,b^3+b^4)}+\frac{}{2d(a^4+4a}$

input
```
int(cos(d*x+c)/(sin(d*x+c)*a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2*b^4/(a+b)^2/(a-b)^2/a/(b+cos(d*x+c)*a)^2-4*a*b^3/(a+b)^3/(a-b)^3/
(b+cos(d*x+c)*a)-6*a*b^2*(a^2+b^2)/(a+b)^4/(a-b)^4*ln(b+cos(d*x+c)*a)-1/4/
(a-b)^3/(cos(d*x+c)+1)+3/4*b/(a-b)^4*ln(cos(d*x+c)+1)+1/4/(a+b)^3/(cos(d*x
+c)-1)-3/4*b/(a+b)^4*ln(cos(d*x+c)-1))
```

3.266.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 994 vs. $2(203) = 406$.

Time = 0.42 (sec) , antiderivative size = 994, normalized size of antiderivative = 4.71

$$\int \frac{\cos(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx = \frac{2\,a^6b^2+18\,a^4b^4-18\,a^2b^6-2\,b^8-6\,(a^7b+2\,a^5b^3-3\,a^3b^5)\cos{(dx+c)}^3+2\,(a^8-4\,a^6b^2-6\,a^4b^4+8\,a^2b^6}{}$$

input
```
integrate(cos(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/4*(2*a^6*b^2 + 18*a^4*b^4 - 18*a^2*b^6 - 2*b^8 - 6*(a^7*b + 2*a^5*b^3 -
3*a^3*b^5)*cos(d*x + c)^3 + 2*(a^8 - 4*a^6*b^2 - 6*a^4*b^4 + 8*a^2*b^6 + b
^8)*cos(d*x + c)^2 + 2*(2*a^7*b + 9*a^5*b^3 - 12*a^3*b^5 + a*b^7)*cos(d*x
+ c) + 24*(a^4*b^4 + a^2*b^6 - (a^6*b^2 + a^4*b^4)*cos(d*x + c)^4 - 2*(a^5
*b^3 + a^3*b^5)*cos(d*x + c)^3 + (a^6*b^2 - a^2*b^6)*cos(d*x + c)^2 + 2*(a
^5*b^3 + a^3*b^5)*cos(d*x + c))*log(a*cos(d*x + c) + b) - 3*(a^5*b^3 + 4*a
^4*b^4 + 6*a^3*b^5 + 4*a^2*b^6 + a*b^7 - (a^7*b + 4*a^6*b^2 + 6*a^5*b^3 +
4*a^4*b^4 + a^3*b^5)*cos(d*x + c)^4 - 2*(a^6*b^2 + 4*a^5*b^3 + 6*a^4*b^4 +
 4*a^3*b^5 + a^2*b^6)*cos(d*x + c)^3 + (a^7*b + 4*a^6*b^2 + 5*a^5*b^3 - 5*
a^3*b^5 - 4*a^2*b^6 - a*b^7)*cos(d*x + c)^2 + 2*(a^6*b^2 + 4*a^5*b^3 + 6*a
^4*b^4 + 4*a^3*b^5 + a^2*b^6)*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2) +
3*(a^5*b^3 - 4*a^4*b^4 + 6*a^3*b^5 - 4*a^2*b^6 + a*b^7 - (a^7*b - 4*a^6*b^
2 + 6*a^5*b^3 - 4*a^4*b^4 + a^3*b^5)*cos(d*x + c)^4 - 2*(a^6*b^2 - 4*a^5*b
^3 + 6*a^4*b^4 - 4*a^3*b^5 + a^2*b^6)*cos(d*x + c)^3 + (a^7*b - 4*a^6*b^2
+ 5*a^5*b^3 - 5*a^3*b^5 + 4*a^2*b^6 - a*b^7)*cos(d*x + c)^2 + 2*(a^6*b^2 -
 4*a^5*b^3 + 6*a^4*b^4 - 4*a^3*b^5 + a^2*b^6)*cos(d*x + c))*log(-1/2*cos(d
*x + c) + 1/2))/((a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*co
s(d*x + c)^4 + 2*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*
cos(d*x + c)^3 - (a^11 - 5*a^9*b^2 + 10*a^7*b^4 - 10*a^5*b^6 + 5*a^3*b^8 -
 a*b^10)*d*cos(d*x + c)^2 - 2*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b...
```

3.266.6 Sympy [F]

$$\int \frac{\cos(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx = \int \frac{\cos{(c+dx)}}{(a\sin{(c+dx)}+b\tan{(c+dx)})^3}\,dx$$

input
```
integrate(cos(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))**3,x)
```

output
```
Integral(cos(c + d*x)/(a*sin(c + d*x) + b*tan(c + d*x))**3, x)
```

3.266.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 593 vs. $2(203) = 406$.

Time = 0.25 (sec) , antiderivative size = 593, normalized size of antiderivative = 2.81

$$\int \frac{\cos(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx =$$

$$-\frac{\frac{48\,(a^3b^2+ab^4)\log\left(a+b-\frac{(a-b)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8}+\frac{12\,b\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}+\frac{a^6-2\,a^5b-a^4b^2+4\,a^3b^3-a^2b^4-2\,ab}{\frac{(a^9+a^8b-4\,a^7b^2-4\,a^6b^3+6\,a^5b^4+6\,a^4b^5-4\,a^3b^6-4\,a^2b^7+ab^8+b^9}{(\cos(dx+c)+1)^2}}}{}$$

input
```
integrate(cos(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/8*(48*(a^3*b^2 + a*b^4)*log(a + b - (a - b)*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2)/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) + 12*b*log(sin(d
*x + c)/(cos(d*x + c) + 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) +
(a^6 - 2*a^5*b - a^4*b^2 + 4*a^3*b^3 - a^2*b^4 - 2*a*b^5 + b^6 - 2*(a^6 -
4*a^5*b + 5*a^4*b^2 - 32*a^3*b^3 - 37*a^2*b^4 - 4*a*b^5 - 9*b^6)*sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 + (a^6 - 6*a^5*b + 15*a^4*b^2 - 84*a^3*b^3 + 63
*a^2*b^4 - 6*a*b^5 + 17*b^6)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)/((a^9 +
a^8*b - 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*
b^7 + a*b^8 + b^9)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 2*(a^9 - a^8*b -
4*a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 - 6*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*
b^8 - b^9)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + (a^9 - 3*a^8*b + 8*a^6*b^
3 - 6*a^5*b^4 - 6*a^4*b^5 + 8*a^3*b^6 - 3*a*b^8 + b^9)*sin(d*x + c)^6/(cos
(d*x + c) + 1)^6) + sin(d*x + c)^2/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(cos(d
*x + c) + 1)^2))/d
```

3.266.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 690 vs. $2(203) = 406$.

Time = 0.87 (sec) , antiderivative size = 690, normalized size of antiderivative = 3.27

$$\int \frac{\cos(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx =$$

$$-\frac{\frac{6\,b\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}+\frac{48\,(a^3b^2+ab^4)\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8}-\frac{\left(a+b+\frac{6\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)(\cos(dx+c)-}}{}$$

input
```
integrate(cos(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/8*(6*b*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^4 + 4*a^3*b
 + 6*a^2*b^2 + 4*a*b^3 + b^4) + 48*(a^3*b^2 + a*b^4)*log(abs(-a - b - a*(c
os(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c) - 1)/(cos(d*x + c) +
 1)))/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) - (a + b + 6*b*(cos(
d*x + c) - 1)/(cos(d*x + c) + 1))*(cos(d*x + c) + 1)/((a^4 + 4*a^3*b + 6*a
^2*b^2 + 4*a*b^3 + b^4)*(cos(d*x + c) - 1)) - (cos(d*x + c) - 1)/((a^3 - 3
*a^2*b + 3*a*b^2 - b^3)*(cos(d*x + c) + 1)) - 8*(9*a^5*b^2 + 10*a^4*b^3 +
2*a^3*b^4 + 8*a^2*b^5 + 5*a*b^6 - 2*b^7 + 18*a^5*b^2*(cos(d*x + c) - 1)/(c
os(d*x + c) + 1) - 8*a^4*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*a^3
*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 6*a^2*b^5*(cos(d*x + c) - 1)/
(cos(d*x + c) + 1) - 16*a*b^6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2*b^
7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*a^5*b^2*(cos(d*x + c) - 1)^2/(
cos(d*x + c) + 1)^2 - 18*a^4*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2
 + 18*a^3*b^4*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 18*a^2*b^5*(cos(
d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 9*a*b^6*(cos(d*x + c) - 1)^2/(cos(d
*x + c) + 1)^2)/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(a + b +
a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x +
c) + 1))^2))/d
```

3.266.9 Mupad [B] (verification not implemented)

Time = 22.77 (sec) , antiderivative size = 490, normalized size of antiderivative = 2.32

$$\int \frac{\cos(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$$
$$= \frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,(a^5-5\,a^4\,b+10\,a^3\,b^2-42\,a^2\,b^3+5\,a\,b^4-9\,b^5)}{(a-b)\,(a^2+2\,a\,b+b^2)}-\frac{a^3-3\,a^2\,b+3\,a\,b}{2\,(a+b)}}{d\,\left(\left(4\,a^5-20\,a^4\,b+40\,a^3\,b^2-40\,a^2\,b^3+20\,a\,b^4-4\,b^5\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\left(-8\,a^5+24\,a^4\,b-16\,a^3\,b^2-16\,a\right.\right.}$$
$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d\,{(a-b)}^3}-\frac{\ln\left(a+b-a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)\,\left(6\,a^3\,b^2+6\,a\,b^4\right)}{d\,\left(a^8-4\,a^6\,b^2+6\,a^4\,b^4-4\,a^2\,b^6+b^8\right)}$$
$$-\frac{3\,b\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d\,\left(2\,a^4+8\,a^3\,b+12\,a^2\,b^2+8\,a\,b^3+2\,b^4\right)}$$

input
```
int(cos(c + d*x)/(a*sin(c + d*x) + b*tan(c + d*x))^3,x)
```

output
```
((tan(c/2 + (d*x)/2)^2*(5*a*b^4 - 5*a^4*b + a^5 - 9*b^5 - 42*a^2*b^3 + 10*
a^3*b^2))/((a - b)*(2*a*b + a^2 + b^2)) - (3*a*b^2 - 3*a^2*b + a^3 - b^3)/
(2*(a + b)) + (tan(c/2 + (d*x)/2)^4*(11*a*b^4 + 5*a^4*b - a^5 + 17*b^5 + 7
4*a^2*b^3 - 10*a^3*b^2))/(2*(a + b)*(2*a*b + a^2 + b^2)))/(d*(tan(c/2 + (d
*x)/2)^2*(4*a*b^4 - 4*a^4*b + 4*a^5 - 4*b^5 + 8*a^2*b^3 - 8*a^3*b^2) - tan
(c/2 + (d*x)/2)^4*(8*a^5 - 24*a^4*b - 24*a*b^4 + 8*b^5 + 16*a^2*b^3 + 16*a
^3*b^2) + tan(c/2 + (d*x)/2)^6*(20*a*b^4 - 20*a^4*b + 4*a^5 - 4*b^5 - 40*a
^2*b^3 + 40*a^3*b^2))) - tan(c/2 + (d*x)/2)^2/(8*d*(a - b)^3) - (log(a + b
 - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2)*(6*a*b^4 + 6*a^3*b^2))
/(d*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2)) - (3*b*log(tan(c/2 +
(d*x)/2)))/(d*(8*a*b^3 + 8*a^3*b + 2*a^4 + 2*b^4 + 12*a^2*b^2))
```

3.266. $\int \frac{\cos(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$

3.267 $\int \frac{1}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$

3.267.1 Optimal result

Integrand size = 19, antiderivative size = 229

$$\begin{aligned}\int \frac{1}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx &= -\frac{b^3}{2\left(a^2-b^2\right)^2 d(b+a\cos(c+dx))^2}+\frac{b^2(3a^2+b^2)}{\left(a^2-b^2\right)^3 d(b+a\cos(c+dx))} \\ &\quad+\frac{(b(3a^2+b^2)-a(a^2+3b^2)\cos(c+dx))\csc^2(c+dx)}{2\left(a^2-b^2\right)^3 d}+\frac{(a-2b)\log(1-\cos(c+dx))}{4(a+b)^4 d} \\ &\quad-\frac{(a+2b)\log(1+\cos(c+dx))}{4(a-b)^4 d}+\frac{b(3a^4+8a^2b^2+b^4)\log(b+a\cos(c+dx))}{\left(a^2-b^2\right)^4 d}\end{aligned}$$

output
```
-1/2*b^3/(a^2-b^2)^2/d/(b+a*cos(d*x+c))^2+b^2*(3*a^2+b^2)/(a^2-b^2)^3/d/(b
+a*cos(d*x+c))+1/2*(b*(3*a^2+b^2)-a*(a^2+3*b^2)*cos(d*x+c))*csc(d*x+c)^2/(
a^2-b^2)^3/d+1/4*(a-2*b)*ln(1-cos(d*x+c))/(a+b)^4/d-1/4*(a+2*b)*ln(1+cos(d
*x+c))/(a-b)^4/d+b*(3*a^4+8*a^2*b^2+b^4)*ln(b+a*cos(d*x+c))/(a^2-b^2)^4/d
```

3.267.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.51 (sec) , antiderivative size = 696, normalized size of antiderivative = 3.04

$$
\begin{aligned}
&\int \frac{1}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx \\
&= -\frac{b^3(b+a\cos(c+dx))\tan^3(c+dx)}{2(-a+b)^2(a+b)^2d(a\sin(c+dx)+b\tan(c+dx))^3} \\
&\quad - \frac{b^2(3a^2+b^2)(b+a\cos(c+dx))^2\tan^3(c+dx)}{(-a+b)^3(a+b)^3d(a\sin(c+dx)+b\tan(c+dx))^3} \\
&\quad - \frac{2i(3a^4b+8a^2b^3+b^5)(c+dx)(b+a\cos(c+dx))^3\tan^3(c+dx)}{(a-b)^4(a+b)^4d(a\sin(c+dx)+b\tan(c+dx))^3} \\
&\quad - \frac{i(-a-2b)\arctan(\tan(c+dx))(b+a\cos(c+dx))^3\tan^3(c+dx)}{2(-a+b)^4d(a\sin(c+dx)+b\tan(c+dx))^3} \\
&\quad - \frac{i(a-2b)\arctan(\tan(c+dx))(b+a\cos(c+dx))^3\tan^3(c+dx)}{2(a+b)^4d(a\sin(c+dx)+b\tan(c+dx))^3} \\
&\quad - \frac{(b+a\cos(c+dx))^3\csc^2\left(\frac{1}{2}(c+dx)\right)\tan^3(c+dx)}{8(a+b)^3d(a\sin(c+dx)+b\tan(c+dx))^3} \\
&\quad + \frac{(-a-2b)(b+a\cos(c+dx))^3\log\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\right)\tan^3(c+dx)}{4(-a+b)^4d(a\sin(c+dx)+b\tan(c+dx))^3} \\
&\quad + \frac{(3a^4b+8a^2b^3+b^5)(b+a\cos(c+dx))^3\log(b+a\cos(c+dx))\tan^3(c+dx)}{(-a^2+b^2)^4\,d(a\sin(c+dx)+b\tan(c+dx))^3} \\
&\quad + \frac{(a-2b)(b+a\cos(c+dx))^3\log\left(\sin^2\left(\frac{1}{2}(c+dx)\right)\right)\tan^3(c+dx)}{4(a+b)^4d(a\sin(c+dx)+b\tan(c+dx))^3} \\
&\quad - \frac{(b+a\cos(c+dx))^3\sec^2\left(\frac{1}{2}(c+dx)\right)\tan^3(c+dx)}{8(-a+b)^3d(a\sin(c+dx)+b\tan(c+dx))^3}
\end{aligned}
$$

input
```
Integrate[(a*Sin[c + d*x] + b*Tan[c + d*x])^(-3),x]
```

output
```
-1/2*(b^3*(b + a*Cos[c + d*x])*Tan[c + d*x]^3)/((-a + b)^2*(a + b)^2*d*(a*
Sin[c + d*x] + b*Tan[c + d*x])^3) - (b^2*(3*a^2 + b^2)*(b + a*Cos[c + d*x]
)^2*Tan[c + d*x]^3)/((-a + b)^3*(a + b)^3*d*(a*Sin[c + d*x] + b*Tan[c + d*
x])^3) - ((2*I)*(3*a^4*b + 8*a^2*b^3 + b^5)*(c + d*x)*(b + a*Cos[c + d*x])
^3*Tan[c + d*x]^3)/((a - b)^4*(a + b)^4*d*(a*Sin[c + d*x] + b*Tan[c + d*x]
)^3) - ((I/2)*(-a - 2*b)*ArcTan[Tan[c + d*x]]*(b + a*Cos[c + d*x])^3*Tan[c
 + d*x]^3)/((-a + b)^4*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3) - ((I/2)*(a
- 2*b)*ArcTan[Tan[c + d*x]]*(b + a*Cos[c + d*x])^3*Tan[c + d*x]^3)/((a + b
)^4*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3) - ((b + a*Cos[c + d*x])^3*Csc[(
c + d*x)/2]^2*Tan[c + d*x]^3)/(8*(a + b)^3*d*(a*Sin[c + d*x] + b*Tan[c + d
*x])^3) + ((-a - 2*b)*(b + a*Cos[c + d*x])^3*Log[Cos[(c + d*x)/2]^2]*Tan[c
 + d*x]^3)/(4*(-a + b)^4*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3) + ((3*a^4*
b + 8*a^2*b^3 + b^5)*(b + a*Cos[c + d*x])^3*Log[b + a*Cos[c + d*x]]*Tan[c
+ d*x]^3)/((-a^2 + b^2)^4*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3) + ((a - 2
*b)*(b + a*Cos[c + d*x])^3*Log[Sin[(c + d*x)/2]^2]*Tan[c + d*x]^3)/(4*(a +
 b)^4*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3) - ((b + a*Cos[c + d*x])^3*Sec
[(c + d*x)/2]^2*Tan[c + d*x]^3)/(8*(-a + b)^3*d*(a*Sin[c + d*x] + b*Tan[c
+ d*x])^3)
```

3.267.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {3042, 4897, 3042, 25, 3200, 601, 2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a \sin(c+dx)+b \tan(c+dx))^3} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a \sin(c+dx)+b \tan(c+dx))^3}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \frac{\cot^3(c+dx)}{(a \cos(c+dx)+b)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\tan\left(c+dx-\frac{\pi}{2}\right)^3}{\left(b-a \sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \ 25$$

$$-\int \frac{\tan\left(\frac{1}{2}(2c-\pi)+dx\right)^3}{\left(b-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)^3}dx$$

$$\Big\downarrow \ 3200$$

$$-\frac{\int \frac{a^3\cos^3(c+dx)}{(b+a\cos(c+dx))^3(a^2-a^2\cos^2(c+dx))^2}d(a\cos(c+dx))}{d}$$

$$\Big\downarrow \ 601$$

$$-\frac{-\frac{\int \frac{-\frac{(a^2+3b^2)\cos^3(c+dx)a^7}{(a^2-b^2)^3}+\frac{b(3a^2-7b^2)\cos^2(c+dx)a^6}{(a^2-b^2)^3}+\frac{b^3(a^2+3b^2)a^4}{(a^2-b^2)^3}+\frac{b^2(3a^4+3b^2a^2-2b^4)\cos(c+dx)a^3}{(a^2-b^2)^3}}{(b+a\cos(c+dx))^3(a^2-a^2\cos^2(c+dx))}d(a\cos(c+dx))}{2a^2}-\frac{a^2(b(3a^2+b^2)-a(a}{2(a^2-b^2)^3(a^2-}}{d}$$

$$\Big\downarrow \ 2160$$

$$-\frac{-\frac{\int\left(\frac{2a^2b^3}{(a^2-b^2)^2(b+a\cos(c+dx))^3}-\frac{2a^2(3a^2+b^2)b^2}{(a^2-b^2)^3(b+a\cos(c+dx))^2}+\frac{2a^2(3a^4+8b^2a^2+b^4)b}{(a^2-b^2)^4(b+a\cos(c+dx))}-\frac{a^2(a-2b)}{2(a+b)^4(a-a\cos(c+dx))}-\frac{a^2(a+2b)}{2(a-b)^4(\cos(c+dx)a+a)}\right)d(a\cos}{2a^2}}{d}$$

$$\Big\downarrow \ 2009$$

$$-\frac{-\frac{a^2(b(3a^2+b^2)-a(a^2+3b^2)\cos(c+dx))}{2(a^2-b^2)^3(a^2-a^2\cos^2(c+dx))}-\frac{\frac{2a^2b^2(3a^2+b^2)}{(a^2-b^2)^3(a\cos(c+dx)+b)}-\frac{a^2b^3}{(a^2-b^2)^2(a\cos(c+dx)+b)^2}+\frac{a^2(a-2b)\log(a-a\cos(c+dx))}{2(a+b)^4}-\frac{a^2(a+2b)\log(a\,c}{2(a-b}}{2a^2}}{d}$$

input

```
Int[(a*Sin[c + d*x] + b*Tan[c + d*x])^(-3),x]
```

output

```
-((-1/2*(a^2*(b*(3*a^2 + b^2) - a*(a^2 + 3*b^2)*Cos[c + d*x]))/((a^2 - b^2
)^3*(a^2 - a^2*Cos[c + d*x]^2)) - (-((a^2*b^3)/((a^2 - b^2)^2*(b + a*Cos[c
 + d*x])^2)) + (2*a^2*b^2*(3*a^2 + b^2))/((a^2 - b^2)^3*(b + a*Cos[c + d*x
])) + (a^2*(a - 2*b)*Log[a - a*Cos[c + d*x]])/(2*(a + b)^4) - (a^2*(a + 2*
b)*Log[a + a*Cos[c + d*x]])/(2*(a - b)^4) + (2*a^2*b*(3*a^4 + 8*a^2*b^2 +
b^4)*Log[b + a*Cos[c + d*x]])/(a^2 - b^2)^4)/(2*a^2))/d)
```

3.267. $\int \frac{1}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$

3.267.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 601
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c
+ d*x)^n*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*(p + 1)*Qx)/(c + d*x)^n + (e*
(2*p + 3))/(c + d*x)^n, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 1]
 && LtQ[p, -1] && ILtQ[n, 0] && NeQ[b*c^2 + a*d^2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.267. $\int \frac{1}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$

3.267.4 Maple [A] (verified)

Time = 5.07 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{\frac{1}{4(a-b)^3(\cos(dx+c)+1)}+\frac{(-a-2b)\ln(\cos(dx+c)+1)}{4(a-b)^4}+\frac{1}{4(a+b)^3(\cos(dx+c)-1)}+\frac{(a-2b)\ln(\cos(dx+c)-1)}{4(a+b)^4}-\frac{b^3}{2(a+b)^2(a-b)^2(b+\cos(dx}}{d}$
default	$\frac{\frac{1}{4(a-b)^3(\cos(dx+c)+1)}+\frac{(-a-2b)\ln(\cos(dx+c)+1)}{4(a-b)^4}+\frac{1}{4(a+b)^3(\cos(dx+c)-1)}+\frac{(a-2b)\ln(\cos(dx+c)-1)}{4(a+b)^4}-\frac{b^3}{2(a+b)^2(a-b)^2(b+\cos(dx}}{d}$
risch	Expression too large to display

input
```
int(1/(sin(d*x+c)*a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/4/(a-b)^3/(cos(d*x+c)+1)+1/4/(a-b)^4*(-a-2*b)*ln(cos(d*x+c)+1)+1/4/
(a+b)^3/(cos(d*x+c)-1)+1/4*(a-2*b)/(a+b)^4*ln(cos(d*x+c)-1)-1/2*b^3/(a+b)^
2/(a-b)^2/(b+cos(d*x+c)*a)^2+b*(3*a^4+8*a^2*b^2+b^4)/(a+b)^4/(a-b)^4*ln(b+
cos(d*x+c)*a)+b^2*(3*a^2+b^2)/(a+b)^3/(a-b)^3/(b+cos(d*x+c)*a))
```

3.267.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1071 vs. 2(221) = 442.

Time = 0.42 (sec) , antiderivative size = 1071, normalized size of antiderivative = 4.68

$$\int \frac{1}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/4*(16*a^4*b^3 - 8*a^2*b^5 - 8*b^7 - 2*(a^7 + 8*a^5*b^2 - 7*a^3*b^4 - 2*
a*b^6)*cos(d*x + c)^3 + 2*(a^6*b - 11*a^4*b^3 + 7*a^2*b^5 + 3*b^7)*cos(d*x
 + c)^2 + 2*(11*a^5*b^2 - 10*a^3*b^4 - a*b^6)*cos(d*x + c) + 4*(3*a^4*b^3
+ 8*a^2*b^5 + b^7 - (3*a^6*b + 8*a^4*b^3 + a^2*b^5)*cos(d*x + c)^4 - 2*(3*
a^5*b^2 + 8*a^3*b^4 + a*b^6)*cos(d*x + c)^3 + (3*a^6*b + 5*a^4*b^3 - 7*a^2
*b^5 - b^7)*cos(d*x + c)^2 + 2*(3*a^5*b^2 + 8*a^3*b^4 + a*b^6)*cos(d*x + c
))*log(a*cos(d*x + c) + b) - (a^5*b^2 + 6*a^4*b^3 + 14*a^3*b^4 + 16*a^2*b^
5 + 9*a*b^6 + 2*b^7 - (a^7 + 6*a^6*b + 14*a^5*b^2 + 16*a^4*b^3 + 9*a^3*b^4
 + 2*a^2*b^5)*cos(d*x + c)^4 - 2*(a^6*b + 6*a^5*b^2 + 14*a^4*b^3 + 16*a^3*
b^4 + 9*a^2*b^5 + 2*a*b^6)*cos(d*x + c)^3 + (a^7 + 6*a^6*b + 13*a^5*b^2 +
10*a^4*b^3 - 5*a^3*b^4 - 14*a^2*b^5 - 9*a*b^6 - 2*b^7)*cos(d*x + c)^2 + 2*
(a^6*b + 6*a^5*b^2 + 14*a^4*b^3 + 16*a^3*b^4 + 9*a^2*b^5 + 2*a*b^6)*cos(d*
x + c))*log(1/2*cos(d*x + c) + 1/2) + (a^5*b^2 - 6*a^4*b^3 + 14*a^3*b^4 -
16*a^2*b^5 + 9*a*b^6 - 2*b^7 - (a^7 - 6*a^6*b + 14*a^5*b^2 - 16*a^4*b^3 +
9*a^3*b^4 - 2*a^2*b^5)*cos(d*x + c)^4 - 2*(a^6*b - 6*a^5*b^2 + 14*a^4*b^3
- 16*a^3*b^4 + 9*a^2*b^5 - 2*a*b^6)*cos(d*x + c)^3 + (a^7 - 6*a^6*b + 13*a
^5*b^2 - 10*a^4*b^3 - 5*a^3*b^4 + 14*a^2*b^5 - 9*a*b^6 + 2*b^7)*cos(d*x +
c)^2 + 2*(a^6*b - 6*a^5*b^2 + 14*a^4*b^3 - 16*a^3*b^4 + 9*a^2*b^5 - 2*a*b^
6)*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/((a^10 - 4*a^8*b^2 + 6*a^6*
b^4 - 4*a^4*b^6 + a^2*b^8)*d*cos(d*x + c)^4 + 2*(a^9*b - 4*a^7*b^3 + 6*...
```

3.267.6 Sympy [F]

$$\int \frac{1}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx = \int \frac{1}{\left(a\sin\left(c+dx\right)+b\tan\left(c+dx\right)\right)^3}\,dx$$

input
```
integrate(1/(a*sin(d*x+c)+b*tan(d*x+c))**3,x)
```

output
```
Integral((a*sin(c + d*x) + b*tan(c + d*x))**(-3), x)
```

3.267.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 601 vs. $2(221) = 442$.

Time = 0.25 (sec) , antiderivative size = 601, normalized size of antiderivative = 2.62

$$\int \frac{1}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$$
$$= \frac{\frac{8\left(3a^4b+8a^2b^3+b^5\right)\log\left(a+b-\frac{(a-b)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8}+\frac{4(a-2b)\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4+4a^3b+6a^2b^2+4ab^3+b^4}-\frac{a^6-2a^5b-a^4b^2+4a^3b^3-a^2b^4-2\cdots}{\frac{\left(a^9+a^8b-4a^7b^2-4a^6b^3+6a^5b^4+6a^4b^5-4a^3b^6-4a^2b^7+ab^8\cdots\right.}{(\cos(dx+c)+1)^2}}\cdots}{\cdots}$$

input
```
integrate(1/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/8*(8*(3*a^4*b + 8*a^2*b^3 + b^5)*log(a + b - (a - b)*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2)/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) + 4*(a -
 2*b)*log(sin(d*x + c)/(cos(d*x + c) + 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*
a*b^3 + b^4) - (a^6 - 2*a^5*b - a^4*b^2 + 4*a^3*b^3 - a^2*b^4 - 2*a*b^5 +
b^6 - 2*(a^6 - 4*a^5*b + 29*a^4*b^2 + 24*a^3*b^3 + 11*a^2*b^4 + 20*a*b^5 -
 b^6)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + (a^6 - 6*a^5*b + 63*a^4*b^2 -
52*a^3*b^3 + 31*a^2*b^4 - 38*a*b^5 + b^6)*sin(d*x + c)^4/(cos(d*x + c) + 1
)^4)/((a^9 + a^8*b - 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3
*b^6 - 4*a^2*b^7 + a*b^8 + b^9)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 2*(a
^9 - a^8*b - 4*a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 - 6*a^4*b^5 - 4*a^3*b^6 + 4
*a^2*b^7 + a*b^8 - b^9)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + (a^9 - 3*a^8
*b + 8*a^6*b^3 - 6*a^5*b^4 - 6*a^4*b^5 + 8*a^3*b^6 - 3*a*b^8 + b^9)*sin(d*
x + c)^6/(cos(d*x + c) + 1)^6) + sin(d*x + c)^2/((a^3 - 3*a^2*b + 3*a*b^2
- b^3)*(cos(d*x + c) + 1)^2))/d
```

3.267.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 800 vs. $2(221) = 442$.

Time = 0.39 (sec) , antiderivative size = 800, normalized size of antiderivative = 3.49

$$\int \frac{1}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/8*(2*(a - 2*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^4 +
4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + 8*(3*a^4*b + 8*a^2*b^3 + b^5)*log(a
bs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c) - 1)
/(cos(d*x + c) + 1)))/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) + (a
 + b - 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 4*b*(cos(d*x + c) - 1)/
(cos(d*x + c) + 1))*(cos(d*x + c) + 1)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b
^3 + b^4)*(cos(d*x + c) - 1)) - (cos(d*x + c) - 1)/((a^3 - 3*a^2*b + 3*a*b
^2 - b^3)*(cos(d*x + c) + 1)) - 4*(9*a^6*b + 6*a^5*b^2 + 9*a^4*b^3 + 28*a^
3*b^4 + 11*a^2*b^5 - 2*a*b^6 + 3*b^7 + 18*a^6*b*(cos(d*x + c) - 1)/(cos(d*
x + c) + 1) - 12*a^5*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 26*a^4*b^
3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 4*a^3*b^4*(cos(d*x + c) - 1)/(co
s(d*x + c) + 1) - 38*a^2*b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 8*a*b
^6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 6*b^7*(cos(d*x + c) - 1)/(cos(d
*x + c) + 1) + 9*a^6*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 18*a^5*
b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 33*a^4*b^3*(cos(d*x + c) -
 1)^2/(cos(d*x + c) + 1)^2 - 48*a^3*b^4*(cos(d*x + c) - 1)^2/(cos(d*x + c)
 + 1)^2 + 27*a^2*b^5*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 6*a*b^6*(
cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 3*b^7*(cos(d*x + c) - 1)^2/(cos
(d*x + c) + 1)^2)/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(a + b
+ a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d...
```

3.267.9 Mupad [B] (verification not implemented)

Time = 23.24 (sec) , antiderivative size = 494, normalized size of antiderivative = 2.16

$$\int \frac{1}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8\,d\,(a-b)^3} - \frac{\frac{a^3-3\,a^2\,b+3\,a\,b^2-b^3}{2\,(a+b)} + \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,(a^5-5\,a^4\,b+58\,a^3\,b^2+6\,a^2\,b^3+37}{2\,(a+b)\,(a^2+2\,a\,b+b^2)}}{d\,\left(\left(4\,a^5-20\,a^4\,b+40\,a^3\,b^2-40\,a^2\,b^3+20\,a\,b^4-4\,b^5\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\left(-8\,a^5+24\,a^4\,b-16\,a^3\,b^2-1\right.}$$
$$+ \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,(a-2\,b)}{d\,\left(2\,a^4+8\,a^3\,b+12\,a^2\,b^2+8\,a\,b^3+2\,b^4\right)} + \frac{\ln\left(a+b-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)\,\left(3\,a^4\,b+8\,a^2\,b^3+b^5\right)}{d\,\left(a^8-4\,a^6\,b^2+6\,a^4\,b^4-4\,a^2\,b^6+b^8\right)}$$

input
```
int(1/(a*sin(c + d*x) + b*tan(c + d*x))^3,x)
```

output
```
tan(c/2 + (d*x)/2)^2/(8*d*(a - b)^3) - ((3*a*b^2 - 3*a^2*b + a^3 - b^3)/(2
*(a + b)) + (tan(c/2 + (d*x)/2)^4*(37*a*b^4 - 5*a^4*b + a^5 - b^5 + 6*a^2*
b^3 + 58*a^3*b^2))/(2*(a + b)*(2*a*b + a^2 + b^2)) - (tan(c/2 + (d*x)/2)^2
*(21*a*b^4 - 5*a^4*b + a^5 - b^5 - 10*a^2*b^3 + 34*a^3*b^2))/((a - b)*(2*a
*b + a^2 + b^2)))/(d*(tan(c/2 + (d*x)/2)^2*(4*a*b^4 - 4*a^4*b + 4*a^5 - 4*
b^5 + 8*a^2*b^3 - 8*a^3*b^2) - tan(c/2 + (d*x)/2)^4*(8*a^5 - 24*a^4*b - 24
*a*b^4 + 8*b^5 + 16*a^2*b^3 + 16*a^3*b^2) + tan(c/2 + (d*x)/2)^6*(20*a*b^4
 - 20*a^4*b + 4*a^5 - 4*b^5 - 40*a^2*b^3 + 40*a^3*b^2))) + (log(tan(c/2 +
(d*x)/2))*(a - 2*b))/(d*(8*a*b^3 + 8*a^3*b + 2*a^4 + 2*b^4 + 12*a^2*b^2))
+ (log(a + b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2)*(3*a^4*b +
 b^5 + 8*a^2*b^3))/(d*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2))
```

3.267. $\int \frac{1}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$

3.268 $\int \frac{\sec(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$

3.268.1 Optimal result

Integrand size = 26, antiderivative size = 231

$$\begin{aligned}\int \frac{\sec(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx &= \frac{ab^2}{2(a^2-b^2)^2 d(b+a\cos(c+dx))^2} - \frac{2ab(a^2+b^2)}{(a^2-b^2)^3 d(b+a\cos(c+dx))} \\ &\quad - \frac{(a(a^2+3b^2)-b(3a^2+b^2)\cos(c+dx))\csc^2(c+dx)}{2(a^2-b^2)^3 d} + \frac{(2a-b)\log(1-\cos(c+dx))}{4(a+b)^4 d} \\ &\quad + \frac{(2a+b)\log(1+\cos(c+dx))}{4(a-b)^4 d} - \frac{a(a^4+8a^2b^2+3b^4)\log(b+a\cos(c+dx))}{(a^2-b^2)^4 d}\end{aligned}$$

output
```
1/2*a*b^2/(a^2-b^2)^2/d/(b+a*cos(d*x+c))^2-2*a*b*(a^2+b^2)/(a^2-b^2)^3/d/(
b+a*cos(d*x+c))-1/2*(a*(a^2+3*b^2)-b*(3*a^2+b^2)*cos(d*x+c))*csc(d*x+c)^2/
(a^2-b^2)^3/d+1/4*(2*a-b)*ln(1-cos(d*x+c))/(a+b)^4/d+1/4*(2*a+b)*ln(1+cos(
d*x+c))/(a-b)^4/d-a*(a^4+8*a^2*b^2+3*b^4)*ln(b+a*cos(d*x+c))/(a^2-b^2)^4/d
```

3.268.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.88 (sec) , antiderivative size = 703, normalized size of antiderivative = 3.04

$$
\begin{aligned}
&\int \frac{\sec(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx \\
&= \frac{ab^2(b+a\cos(c+dx))\tan^3(c+dx)}{2(-a+b)^2(a+b)^2d(a\sin(c+dx)+b\tan(c+dx))^3} \\
&\quad + \frac{2ab(-ia+b)(ia+b)(b+a\cos(c+dx))^2\tan^3(c+dx)}{(-a+b)^3(a+b)^3d(a\sin(c+dx)+b\tan(c+dx))^3} \\
&\quad + \frac{2i(a^5+8a^3b^2+3ab^4)\,(c+dx)(b+a\cos(c+dx))^3\tan^3(c+dx)}{(a-b)^4(a+b)^4d(a\sin(c+dx)+b\tan(c+dx))^3} \\
&\quad - \frac{i(2a-b)\arctan(\tan(c+dx))(b+a\cos(c+dx))^3\tan^3(c+dx)}{2(a+b)^4d(a\sin(c+dx)+b\tan(c+dx))^3} \\
&\quad - \frac{i(2a+b)\arctan(\tan(c+dx))(b+a\cos(c+dx))^3\tan^3(c+dx)}{2(-a+b)^4d(a\sin(c+dx)+b\tan(c+dx))^3} \\
&\quad - \frac{(b+a\cos(c+dx))^3\csc^2\left(\frac{1}{2}(c+dx)\right)\tan^3(c+dx)}{8(a+b)^3d(a\sin(c+dx)+b\tan(c+dx))^3} \\
&\quad + \frac{(2a+b)(b+a\cos(c+dx))^3\log\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\right)\tan^3(c+dx)}{4(-a+b)^4d(a\sin(c+dx)+b\tan(c+dx))^3} \\
&\quad + \frac{(-a^5-8a^3b^2-3ab^4)\,(b+a\cos(c+dx))^3\log(b+a\cos(c+dx))\tan^3(c+dx)}{(-a^2+b^2)^4\,d(a\sin(c+dx)+b\tan(c+dx))^3} \\
&\quad + \frac{(2a-b)(b+a\cos(c+dx))^3\log\left(\sin^2\left(\frac{1}{2}(c+dx)\right)\right)\tan^3(c+dx)}{4(a+b)^4d(a\sin(c+dx)+b\tan(c+dx))^3} \\
&\quad + \frac{(b+a\cos(c+dx))^3\sec^2\left(\frac{1}{2}(c+dx)\right)\tan^3(c+dx)}{8(-a+b)^3d(a\sin(c+dx)+b\tan(c+dx))^3}
\end{aligned}
$$

input
```
Integrate[Sec[c + d*x]/(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

output
```
(a*b^2*(b + a*Cos[c + d*x])*Tan[c + d*x]^3)/(2*(-a + b)^2*(a + b)^2*d*(a*S
in[c + d*x] + b*Tan[c + d*x])^3) + (2*a*b*((-I)*a + b)*(I*a + b)*(b + a*Co
s[c + d*x])^2*Tan[c + d*x]^3)/((-a + b)^3*(a + b)^3*d*(a*Sin[c + d*x] + b*
Tan[c + d*x])^3) + ((2*I)*(a^5 + 8*a^3*b^2 + 3*a*b^4)*(c + d*x)*(b + a*Cos
[c + d*x])^3*Tan[c + d*x]^3)/((a - b)^4*(a + b)^4*d*(a*Sin[c + d*x] + b*Ta
n[c + d*x])^3) - ((I/2)*(2*a - b)*ArcTan[Tan[c + d*x]]*(b + a*Cos[c + d*x]
)^3*Tan[c + d*x]^3)/((a + b)^4*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3) - ((
I/2)*(2*a + b)*ArcTan[Tan[c + d*x]]*(b + a*Cos[c + d*x])^3*Tan[c + d*x]^3)
/((-a + b)^4*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3) - ((b + a*Cos[c + d*x]
)^3*Csc[(c + d*x)/2]^2*Tan[c + d*x]^3)/(8*(a + b)^3*d*(a*Sin[c + d*x] + b*
Tan[c + d*x])^3) + ((2*a + b)*(b + a*Cos[c + d*x])^3*Log[Cos[(c + d*x)/2]^
2]*Tan[c + d*x]^3)/(4*(-a + b)^4*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3) +
((-a^5 - 8*a^3*b^2 - 3*a*b^4)*(b + a*Cos[c + d*x])^3*Log[b + a*Cos[c + d*x
]]*Tan[c + d*x]^3)/((-a^2 + b^2)^4*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3)
+ ((2*a - b)*(b + a*Cos[c + d*x])^3*Log[Sin[(c + d*x)/2]^2]*Tan[c + d*x]^3
)/(4*(a + b)^4*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3) + ((b + a*Cos[c + d*
x])^3*Sec[(c + d*x)/2]^2*Tan[c + d*x]^3)/(8*(-a + b)^3*d*(a*Sin[c + d*x] +
 b*Tan[c + d*x])^3)
```

3.268.3 Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {3042, 4897, 3042, 3316, 27, 601, 25, 2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sec(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{ 4897}$$

$$\int \frac{\cot^2(c+dx)\csc(c+dx)}{(a\cos(c+dx)+b)^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin\left(c+dx-\frac{\pi}{2}\right)^2}{\cos\left(c+dx-\frac{\pi}{2}\right)^3\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \; 3316$$

$$-\frac{a^3\int \frac{\cos^2(c+dx)}{(b+a\cos(c+dx))^3(a^2-a^2\cos^2(c+dx))^2}d(a\cos(c+dx))}{d}$$

$$\Big\downarrow \; 27$$

$$-\frac{a\int \frac{a^2\cos^2(c+dx)}{(b+a\cos(c+dx))^3(a^2-a^2\cos^2(c+dx))^2}d(a\cos(c+dx))}{d}$$

$$\Big\downarrow \; 601$$

$$-\frac{a\left(\frac{a^2(a^2+3b^2)-ab(3a^2+b^2)\cos(c+dx)}{2(a^2-b^2)^3(a^2-a^2\cos^2(c+dx))}-\frac{\int -\frac{-\frac{b(3a^2+b^2)\cos^3(c+dx)a^5}{(a^2-b^2)^3}+\frac{(2a^4-3b^2a^2-3b^4)\cos^2(c+dx)a^4}{(a^2-b^2)^3}+\frac{b^3(7a^2-3b^2)\cos(c+dx)a^3}{(a^2-b^2)^3}+\frac{b^4(3a^2+}{(a^2-b^}}{(b+a\cos(c+dx))^3(a^2-a^2\cos^2(c+dx))}}{2a^2}\right.}{d}$$

$$\Big\downarrow \; 25$$

$$-\frac{a\left(\frac{\int \frac{-\frac{b(3a^2+b^2)\cos^3(c+dx)a^5}{(a^2-b^2)^3}+\frac{(2a^4-3b^2a^2-3b^4)\cos^2(c+dx)a^4}{(a^2-b^2)^3}+\frac{b^3(7a^2-3b^2)\cos(c+dx)a^3}{(a^2-b^2)^3}+\frac{b^4(3a^2+b^2)a^2}{(a^2-b^2)^3}}{(b+a\cos(c+dx))^3(a^2-a^2\cos^2(c+dx))}d(a\cos(c+dx))}{2a^2}+\frac{a^2(a^2+3b^2)-ab(3}{2(a^2-b^2)^3(a^2-}\right.}{d}$$

$$\Big\downarrow \; 2160$$

$$-\frac{a\left(\frac{\int\left(-\frac{4b(a^2+b^2)a^2}{(a^2-b^2)^3(b+a\cos(c+dx))^2}+\frac{2b^2a^2}{(a^2-b^2)^2(b+a\cos(c+dx))^3}+\frac{(2a-b)a}{2(a+b)^4(a-a\cos(c+dx))}-\frac{(2a+b)a}{2(a-b)^4(\cos(c+dx)a+a)}+\frac{2(a^6+8b^2a^4+3b^4a^2)}{(a^2-b^2)^4(b+a\cos(c+dx))}\right)d(a}{2a^2}\right.}{d}$$

$$\Big\downarrow \; 2009$$

$$-\frac{a\left(\frac{a^2(a^2+3b^2)-ab(3a^2+b^2)\cos(c+dx)}{2(a^2-b^2)^3(a^2-a^2\cos^2(c+dx))}+\frac{\frac{4a^2b(a^2+b^2)}{(a^2-b^2)^3(a\cos(c+dx)+b)}-\frac{a^2b^2}{(a^2-b^2)^2(a\cos(c+dx)+b)^2}+\frac{2a^2(a^4+8a^2b^2+3b^4)\log(a\cos(c+dx)+b)}{(a^2-b^2)^4}-\frac{a(2a}{}}{2a^2}\right.}{d}$$

input
```
Int[Sec[c + d*x]/(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

3.268. $\int \frac{\sec(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$

output
```
-((a*((a^2*(a^2 + 3*b^2) - a*b*(3*a^2 + b^2)*Cos[c + d*x])/(2*(a^2 - b^2)^
3*(a^2 - a^2*Cos[c + d*x]^2)) + (-((a^2*b^2)/((a^2 - b^2)^2*(b + a*Cos[c +
 d*x])^2)) + (4*a^2*b*(a^2 + b^2))/((a^2 - b^2)^3*(b + a*Cos[c + d*x])) -
(a*(2*a - b)*Log[a - a*Cos[c + d*x]])/(2*(a + b)^4) - (a*(2*a + b)*Log[a +
 a*Cos[c + d*x]])/(2*(a - b)^4) + (2*a^2*(a^4 + 8*a^2*b^2 + 3*b^4)*Log[b +
 a*Cos[c + d*x]])/(a^2 - b^2)^4)/(2*a^2)))/d)
```

3.268.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 601
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c
+ d*x)^n*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*(p + 1)*Qx)/(c + d*x)^n + (e*
(2*p + 3))/(c + d*x)^n, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 1]
 && LtQ[p, -1] && ILtQ[n, 0] && NeQ[b*c^2 + a*d^2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.268. $\int \frac{\sec(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.268.4 Maple [A] (verified)

Time = 14.86 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{-\frac{a\left(a^{4}+8a^{2}b^{2}+3b^{4}\right)\ln\left(b+\cos\left(dx+c\right)a\right)}{\left(a+b\right)^{4}\left(a-b\right)^{4}}+\frac{b^{2}a}{2\left(a+b\right)^{2}\left(a-b\right)^{2}\left(b+\cos\left(dx+c\right)a\right)^{2}}-\frac{2ab\left(a^{2}+b^{2}\right)}{\left(a+b\right)^{3}\left(a-b\right)^{3}\left(b+\cos\left(dx+c\right)a\right)}+\frac{1}{4\left(a+b\right)^{3}\left(\cos\left(dx+c\right)-1\right.}}{d}$
default	$\frac{-\frac{a\left(a^{4}+8a^{2}b^{2}+3b^{4}\right)\ln\left(b+\cos\left(dx+c\right)a\right)}{\left(a+b\right)^{4}\left(a-b\right)^{4}}+\frac{b^{2}a}{2\left(a+b\right)^{2}\left(a-b\right)^{2}\left(b+\cos\left(dx+c\right)a\right)^{2}}-\frac{2ab\left(a^{2}+b^{2}\right)}{\left(a+b\right)^{3}\left(a-b\right)^{3}\left(b+\cos\left(dx+c\right)a\right)}+\frac{1}{4\left(a+b\right)^{3}\left(\cos\left(dx+c\right)-1\right.}}{d}$
risch	Expression too large to display

input
```
int(sec(d*x+c)/(sin(d*x+c)*a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a*(a^4+8*a^2*b^2+3*b^4)/(a+b)^4/(a-b)^4*ln(b+cos(d*x+c)*a)+1/2*b^2/(
a+b)^2*a/(a-b)^2/(b+cos(d*x+c)*a)^2-2*a*b*(a^2+b^2)/(a+b)^3/(a-b)^3/(b+cos
(d*x+c)*a)+1/4/(a+b)^3/(cos(d*x+c)-1)+1/4*(2*a-b)/(a+b)^4*ln(cos(d*x+c)-1)
-1/4/(a-b)^3/(cos(d*x+c)+1)+1/4*(2*a+b)/(a-b)^4*ln(cos(d*x+c)+1))
```

3.268.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1076 vs. 2(223) = 446.

Time = 0.42 (sec) , antiderivative size = 1076, normalized size of antiderivative = 4.66

$$\int \frac{\sec(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/4*(8*a^5*b^2 + 8*a^3*b^4 - 16*a*b^6 - 2*(7*a^6*b - 2*a^4*b^3 - 5*a^2*b^5
)*cos(d*x + c)^3 + 2*(a^7 - 7*a^5*b^2 - a^3*b^4 + 7*a*b^6)*cos(d*x + c)^2
+ 2*(6*a^6*b + a^4*b^3 - 8*a^2*b^5 + b^7)*cos(d*x + c) + 4*(a^5*b^2 + 8*a^
3*b^4 + 3*a*b^6 - (a^7 + 8*a^5*b^2 + 3*a^3*b^4)*cos(d*x + c)^4 - 2*(a^6*b
+ 8*a^4*b^3 + 3*a^2*b^5)*cos(d*x + c)^3 + (a^7 + 7*a^5*b^2 - 5*a^3*b^4 - 3
*a*b^6)*cos(d*x + c)^2 + 2*(a^6*b + 8*a^4*b^3 + 3*a^2*b^5)*cos(d*x + c))*l
og(a*cos(d*x + c) + b) - (2*a^5*b^2 + 9*a^4*b^3 + 16*a^3*b^4 + 14*a^2*b^5
+ 6*a*b^6 + b^7 - (2*a^7 + 9*a^6*b + 16*a^5*b^2 + 14*a^4*b^3 + 6*a^3*b^4 +
 a^2*b^5)*cos(d*x + c)^4 - 2*(2*a^6*b + 9*a^5*b^2 + 16*a^4*b^3 + 14*a^3*b^
4 + 6*a^2*b^5 + a*b^6)*cos(d*x + c)^3 + (2*a^7 + 9*a^6*b + 14*a^5*b^2 + 5*
a^4*b^3 - 10*a^3*b^4 - 13*a^2*b^5 - 6*a*b^6 - b^7)*cos(d*x + c)^2 + 2*(2*a
^6*b + 9*a^5*b^2 + 16*a^4*b^3 + 14*a^3*b^4 + 6*a^2*b^5 + a*b^6)*cos(d*x +
c))*log(1/2*cos(d*x + c) + 1/2) - (2*a^5*b^2 - 9*a^4*b^3 + 16*a^3*b^4 - 14
*a^2*b^5 + 6*a*b^6 - b^7 - (2*a^7 - 9*a^6*b + 16*a^5*b^2 - 14*a^4*b^3 + 6*
a^3*b^4 - a^2*b^5)*cos(d*x + c)^4 - 2*(2*a^6*b - 9*a^5*b^2 + 16*a^4*b^3 -
14*a^3*b^4 + 6*a^2*b^5 - a*b^6)*cos(d*x + c)^3 + (2*a^7 - 9*a^6*b + 14*a^5
*b^2 - 5*a^4*b^3 - 10*a^3*b^4 + 13*a^2*b^5 - 6*a*b^6 + b^7)*cos(d*x + c)^2
 + 2*(2*a^6*b - 9*a^5*b^2 + 16*a^4*b^3 - 14*a^3*b^4 + 6*a^2*b^5 - a*b^6)*c
os(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/((a^10 - 4*a^8*b^2 + 6*a^6*b^4
- 4*a^4*b^6 + a^2*b^8)*d*cos(d*x + c)^4 + 2*(a^9*b - 4*a^7*b^3 + 6*a^5*...
```

3.268.6 Sympy [F]

$$\int \frac{\sec(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx=\int \frac{\sec{(c+dx)}}{(a\sin{(c+dx)}+b\tan{(c+dx)})^3}\,dx$$

input
```
integrate(sec(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)/(a*sin(c + d*x) + b*tan(c + d*x))**3, x)
```

3.268.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 602 vs. $2(223) = 446$.

Time = 0.26 (sec) , antiderivative size = 602, normalized size of antiderivative = 2.61

$$\int \frac{\sec(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx =$$

$$-\frac{\frac{8\,(a^5+8\,a^3b^2+3\,ab^4)\log\left(a+b-\frac{(a-b)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8}-\frac{4\,(2\,a-b)\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}+\frac{a^6-2\,a^5b-a^4b^2+4\,a^3b^3-a^2b\cdots}{\frac{\left(a^9+a^8b-4\,a^7b^2-4\,a^6b^3+6\,a^5b^4+6\,a^4b^5-4\,a^3b^6-4\,a^2b^7+\cdots\right.}{(\cos(dx+c)+1)^2}\cdots}}{\cdots}$$

input
```
integrate(sec(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/8*(8*(a^5 + 8*a^3*b^2 + 3*a*b^4)*log(a + b - (a - b)*sin(d*x + c)^2/(co
s(d*x + c) + 1)^2)/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) - 4*(2*
a - b)*log(sin(d*x + c)/(cos(d*x + c) + 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4
*a*b^3 + b^4) + (a^6 - 2*a^5*b - a^4*b^2 + 4*a^3*b^3 - a^2*b^4 - 2*a*b^5 +
 b^6 - 2*(a^6 - 20*a^5*b - 11*a^4*b^2 - 24*a^3*b^3 - 29*a^2*b^4 + 4*a*b^5
- b^6)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + (a^6 - 38*a^5*b + 31*a^4*b^2
- 52*a^3*b^3 + 63*a^2*b^4 - 6*a*b^5 + b^6)*sin(d*x + c)^4/(cos(d*x + c) +
1)^4)/((a^9 + a^8*b - 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^
3*b^6 - 4*a^2*b^7 + a*b^8 + b^9)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 2*(
a^9 - a^8*b - 4*a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 - 6*a^4*b^5 - 4*a^3*b^6 +
4*a^2*b^7 + a*b^8 - b^9)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + (a^9 - 3*a^
8*b + 8*a^6*b^3 - 6*a^5*b^4 - 6*a^4*b^5 + 8*a^3*b^6 - 3*a*b^8 + b^9)*sin(d
*x + c)^6/(cos(d*x + c) + 1)^6) + sin(d*x + c)^2/((a^3 - 3*a^2*b + 3*a*b^2
 - b^3)*(cos(d*x + c) + 1)^2))/d
```

3.268.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 801 vs. $2(223) = 446$.

Time = 0.67 (sec) , antiderivative size = 801, normalized size of antiderivative = 3.47

$$\int \frac{\sec(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sec(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/8*(2*(2*a - b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^4 +
4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - 8*(a^5 + 8*a^3*b^2 + 3*a*b^4)*log(a
bs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c) - 1)
/(cos(d*x + c) + 1)))/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) + (a
 + b - 4*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2*b*(cos(d*x + c) - 1)/
(cos(d*x + c) + 1))*(cos(d*x + c) + 1)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b
^3 + b^4)*(cos(d*x + c) - 1)) + (cos(d*x + c) - 1)/((a^3 - 3*a^2*b + 3*a*b
^2 - b^3)*(cos(d*x + c) + 1)) + 4*(3*a^7 - 2*a^6*b + 11*a^5*b^2 + 28*a^4*b
^3 + 9*a^3*b^4 + 6*a^2*b^5 + 9*a*b^6 + 6*a^7*(cos(d*x + c) - 1)/(cos(d*x +
 c) + 1) - 8*a^6*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 38*a^5*b^2*(cos
(d*x + c) - 1)/(cos(d*x + c) + 1) - 4*a^4*b^3*(cos(d*x + c) - 1)/(cos(d*x
+ c) + 1) - 26*a^3*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 12*a^2*b^5*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 18*a*b^6*(cos(d*x + c) - 1)/(cos(d
*x + c) + 1) + 3*a^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 6*a^6*b*(
cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 27*a^5*b^2*(cos(d*x + c) - 1)^2
/(cos(d*x + c) + 1)^2 - 48*a^4*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)
^2 + 33*a^3*b^4*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 18*a^2*b^5*(co
s(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 9*a*b^6*(cos(d*x + c) - 1)^2/(cos
(d*x + c) + 1)^2)/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(a + b
+ a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d...
```

3.268.9 Mupad [B] (verification not implemented)

Time = 23.58 (sec) , antiderivative size = 496, normalized size of antiderivative = 2.15

$$\int \frac{\sec(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$$
$$= \frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(-a^5+37\,a^4\,b+6\,a^3\,b^2+58\,a^2\,b^3-5\,a\,b^4+b^5\right)}{2\,(a+b)\,\left(a^2+2\,a\,b+b^2\right)}-\frac{a^3-3\,a^2\,b+3\,\cdots}{2\,(a+b\cdots}}{d\,\left(\left(4\,a^5-20\,a^4\,b+40\,a^3\,b^2-40\,a^2\,b^3+20\,a\,b^4-4\,b^5\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\left(-8\,a^5+24\,a^4\,b-16\,a^3\,b^2-16\,a\cdots\right.\right.}$$
$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d\,(a-b)^3}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(2\,a-b\right)}{d\,\left(2\,a^4+8\,a^3\,b+12\,a^2\,b^2+8\,a\,b^3+2\,b^4\right)}$$
$$-\frac{\ln\left(a+b-a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)\,\left(a^5+8\,a^3\,b^2+3\,a\,b^4\right)}{d\,\left(a^8-4\,a^6\,b^2+6\,a^4\,b^4-4\,a^2\,b^6+b^8\right)}$$

input
```
int(1/(cos(c + d*x)*(a*sin(c + d*x) + b*tan(c + d*x))^3),x)
```

output
```
((tan(c/2 + (d*x)/2)^4*(37*a^4*b - 5*a*b^4 - a^5 + b^5 + 58*a^2*b^3 + 6*a^
3*b^2))/(2*(a + b)*(2*a*b + a^2 + b^2)) - (3*a*b^2 - 3*a^2*b + a^3 - b^3)/
(2*(a + b)) + (tan(c/2 + (d*x)/2)^2*(5*a*b^4 - 21*a^4*b + a^5 - b^5 - 34*a
^2*b^3 + 10*a^3*b^2))/((a - b)*(2*a*b + a^2 + b^2)))/(d*(tan(c/2 + (d*x)/2
)^2*(4*a*b^4 - 4*a^4*b + 4*a^5 - 4*b^5 + 8*a^2*b^3 - 8*a^3*b^2) - tan(c/2
+ (d*x)/2)^4*(8*a^5 - 24*a^4*b - 24*a*b^4 + 8*b^5 + 16*a^2*b^3 + 16*a^3*b^
2) + tan(c/2 + (d*x)/2)^6*(20*a*b^4 - 20*a^4*b + 4*a^5 - 4*b^5 - 40*a^2*b^
3 + 40*a^3*b^2))) - tan(c/2 + (d*x)/2)^2/(8*d*(a - b)^3) + (log(tan(c/2 +
(d*x)/2))*(2*a - b))/(d*(8*a*b^3 + 8*a^3*b + 2*a^4 + 2*b^4 + 12*a^2*b^2))
- (log(a + b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2)*(3*a*b^4 +
 a^5 + 8*a^3*b^2))/(d*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2))
```

3.268. $\int \frac{\sec(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$

3.269 $\int \frac{\sec^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$

3.269.1 Optimal result

Integrand size = 28, antiderivative size = 212

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx &= -\frac{3a^2b}{2\left(a^2-b^2\right)^2d(b+a\cos(c+dx))^2} \\ &\quad+\frac{3a^2(a^2+3b^2)}{2\left(a^2-b^2\right)^3d(b+a\cos(c+dx))} \\ &\quad+\frac{(b-a\cos(c+dx))\csc^2(c+dx)}{2\left(a^2-b^2\right)d(b+a\cos(c+dx))^2} \\ &\quad+\frac{3a\log(1-\cos(c+dx))}{4(a+b)^4d}-\frac{3a\log(1+\cos(c+dx))}{4(a-b)^4d} \\ &\quad+\frac{6a^2b(a^2+b^2)\log(b+a\cos(c+dx))}{\left(a^2-b^2\right)^4d}\end{aligned}$$

output
```
-3/2*a^2*b/(a^2-b^2)^2/d/(b+a*cos(d*x+c))^2+3/2*a^2*(a^2+3*b^2)/(a^2-b^2)^
3/d/(b+a*cos(d*x+c))+1/2*(b-a*cos(d*x+c))*csc(d*x+c)^2/(a^2-b^2)/d/(b+a*co
s(d*x+c))^2+3/4*a*ln(1-cos(d*x+c))/(a+b)^4/d-3/4*a*ln(1+cos(d*x+c))/(a-b)^
4/d+6*a^2*b*(a^2+b^2)*ln(b+a*cos(d*x+c))/(a^2-b^2)^4/d
```

3.269.2 Mathematica [A] (verified)

Time = 6.64 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.02

$$\int \frac{\sec^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx = -\frac{a^2b}{2(-a+b)^2(a+b)^2d(b+a\cos(c+dx))^2} - \frac{a^2(a^2+3b^2)}{(-a+b)^3(a+b)^3d(b+a\cos(c+dx))} - \frac{\csc^2\left(\frac{1}{2}(c+dx)\right)}{8(a+b)^3d} - \frac{3a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2(-a+b)^4d} + \frac{6(a^4b+a^2b^3)\log(b+a\cos(c+dx))}{(-a^2+b^2)^4\,d} + \frac{3a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2(a+b)^4d} - \frac{\sec^2\left(\frac{1}{2}(c+dx)\right)}{8(-a+b)^3d}$$

input
```
Integrate[Sec[c + d*x]^2/(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

output
```
-1/2*(a^2*b)/((-a + b)^2*(a + b)^2*d*(b + a*Cos[c + d*x])^2) - (a^2*(a^2 +
 3*b^2))/((-a + b)^3*(a + b)^3*d*(b + a*Cos[c + d*x])) - Csc[(c + d*x)/2]^
2/(8*(a + b)^3*d) - (3*a*Log[Cos[(c + d*x)/2]])/(2*(-a + b)^4*d) + (6*(a^4
*b + a^2*b^3)*Log[b + a*Cos[c + d*x]])/((-a^2 + b^2)^4*d) + (3*a*Log[Sin[(
c + d*x)/2]])/(2*(a + b)^4*d) - Sec[(c + d*x)/2]^2/(8*(-a + b)^3*d)
```

3.269.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {3042, 4897, 3042, 25, 3316, 25, 27, 593, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^2}{(a\sin(c+dx)+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \frac{\cot(c+dx)\csc^2(c+dx)}{(a\cos(c+dx)+b)^3}dx$$

$\Big\downarrow$ 3042

$$\int -\frac{\sin\left(c+dx-\frac{\pi}{2}\right)}{\cos\left(c+dx-\frac{\pi}{2}\right)^3\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$\Big\downarrow$ 25

$$-\int \frac{\sin\left(\frac{1}{2}(2c-\pi)+dx\right)}{\cos\left(\frac{1}{2}(2c-\pi)+dx\right)^3\left(b-a\sin\left(\frac{1}{2}(2c-\pi)+dx\right)\right)^3}dx$$

$\Big\downarrow$ 3316

$$\frac{a^3\int -\frac{\cos(c+dx)}{(b+a\cos(c+dx))^3(a^2-a^2\cos^2(c+dx))^2}d(a\cos(c+dx))}{d}$$

$\Big\downarrow$ 25

$$-\frac{a^3\int \frac{\cos(c+dx)}{(b+a\cos(c+dx))^3(a^2-a^2\cos^2(c+dx))^2}d(a\cos(c+dx))}{d}$$

$\Big\downarrow$ 27

$$-\frac{a^2\int \frac{a\cos(c+dx)}{(b+a\cos(c+dx))^3(a^2-a^2\cos^2(c+dx))^2}d(a\cos(c+dx))}{d}$$

$\Big\downarrow$ 593

$$-\frac{a^2\left(\frac{\int -\frac{3(b-a\cos(c+dx))}{(b+a\cos(c+dx))^3\left(a^2-a^2\cos^2(c+dx)\right)}d(a\cos(c+dx))}{2(a^2-b^2)}-\frac{b-a\cos(c+dx)}{2(a^2-b^2)(a^2-a^2\cos^2(c+dx))(a\cos(c+dx)+b)^2}\right)}{d}$$

$\Big\downarrow$ 27

$$-\frac{a^2\left(-\frac{3\int \frac{b-a\cos(c+dx)}{(b+a\cos(c+dx))^3\left(a^2-a^2\cos^2(c+dx)\right)}d(a\cos(c+dx))}{2(a^2-b^2)}-\frac{b-a\cos(c+dx)}{2(a^2-b^2)(a^2-a^2\cos^2(c+dx))(a\cos(c+dx)+b)^2}\right)}{d}$$

$\Big\downarrow$ 657

$$-\frac{a^2\left(-\frac{3\int\left(\frac{-a-b}{2a(a-b)^3(\cos(c+dx)a+a)}+\frac{b-a}{2a(a+b)^3(a-a\cos(c+dx))}+\frac{4b\left(a^2+b^2\right)}{(a-b)^3(a+b)^3(b+a\cos(c+dx))}+\frac{-a^2-3b^2}{(a-b)^2(a+b)^2(b+a\cos(c+dx))^2}+\frac{2b}{(a-b)(a+b)(b+a\cos\cdots}\cdots\right.}{2(a^2-b^2)}\cdots\right.}{d}$$

$\Big\downarrow$ 2009

3.269. $\int \frac{\sec^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$

$$-\frac{a^2\left(-\frac{b-a\cos(c+dx)}{2(a^2-b^2)(a^2-a^2\cos^2(c+dx))(a\cos(c+dx)+b)^2}-\frac{3\left(-\frac{b}{(a^2-b^2)(a\cos(c+dx)+b)^2}+\frac{a^2+3b^2}{(a^2-b^2)^2(a\cos(c+dx)+b)}+\frac{4b(a^2+b^2)\log(a\cos(c+dx)+}{(a^2-b^2)^3}\right.}{2(a^2-b^2)}\right.}{d}$$

input
```
Int[Sec[c + d*x]^2/(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

output
```
-((a^2*(-1/2*(b - a*Cos[c + d*x])/((a^2 - b^2)*(b + a*Cos[c + d*x])^2*(a^2
 - a^2*Cos[c + d*x]^2)) - (3*(-(b/((a^2 - b^2)*(b + a*Cos[c + d*x])^2)) +
(a^2 + 3*b^2)/((a^2 - b^2)^2*(b + a*Cos[c + d*x])) + ((a - b)*Log[a - a*Co
s[c + d*x]])/(2*a*(a + b)^3) - ((a + b)*Log[a + a*Cos[c + d*x]])/(2*a*(a -
 b)^3) + (4*b*(a^2 + b^2)*Log[b + a*Cos[c + d*x]])/(a^2 - b^2)^3))/(2*(a^2
 - b^2))))/d)
```

3.269.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 593
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[(c + d*x)^(n + 1)*(c - d*x)*((a + b*x^2)^(p + 1)/(2*(p + 1)*(b*c^2 +
a*d^2))), x] - Simp[d/(2*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a + b*
x^2)^(p + 1)*(c*n - d*(n + 2*p + 4)*x), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && LtQ[p, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.269. $\int \frac{\sec^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.269.4 Maple [A] (verified)

Time = 31.82 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{-\frac{b\,a^2}{2(a+b)^2(a-b)^2(b+\cos(dx+c)a)^2}+\frac{a^2\left(a^2+3b^2\right)}{(a+b)^3(a-b)^3(b+\cos(dx+c)a)}+\frac{6a^2b\left(a^2+b^2\right)\ln(b+\cos(dx+c)a)}{(a+b)^4(a-b)^4}+\frac{1}{4(a-b)^3(\cos(dx+c)+1)}-\frac{3a}{}}{d}$
default	$\frac{-\frac{b\,a^2}{2(a+b)^2(a-b)^2(b+\cos(dx+c)a)^2}+\frac{a^2\left(a^2+3b^2\right)}{(a+b)^3(a-b)^3(b+\cos(dx+c)a)}+\frac{6a^2b\left(a^2+b^2\right)\ln(b+\cos(dx+c)a)}{(a+b)^4(a-b)^4}+\frac{1}{4(a-b)^3(\cos(dx+c)+1)}-\frac{3a}{}}{d}$
risch	$-\frac{3iax}{2(a^4+4a^3b+6a^2b^2+4a\,b^3+b^4)}-\frac{3iac}{2d(a^4+4a^3b+6a^2b^2+4a\,b^3+b^4)}+\frac{3iax}{2(a^4-4a^3b+6a^2b^2-4a\,b^3+b^4)}+\frac{}{2d(a^4-4a^3}$

input
```
int(sec(d*x+c)^2/(sin(d*x+c)*a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2*b*a^2/(a+b)^2/(a-b)^2/(b+cos(d*x+c)*a)^2+a^2*(a^2+3*b^2)/(a+b)^3
/(a-b)^3/(b+cos(d*x+c)*a)+6*a^2*b*(a^2+b^2)/(a+b)^4/(a-b)^4*ln(b+cos(d*x+c
)*a)+1/4/(a-b)^3/(cos(d*x+c)+1)-3/4/(a-b)^4*a*ln(cos(d*x+c)+1)+1/4/(a+b)^3
/(cos(d*x+c)-1)+3/4/(a+b)^4*a*ln(cos(d*x+c)-1))
```

3.269.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 939 vs. $2(203) = 406$.

Time = 0.43 (sec) , antiderivative size = 939, normalized size of antiderivative = 4.43

$$\int \frac{\sec^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx = -\frac{2\,a^6b+18\,a^4b^3-18\,a^2b^5-2\,b^7-6\,(a^7+2\,a^5b^2-3\,a^3b^4)\cos{(dx+c)}^3-24\,(a^4b^3-a^2b^5)\cos{(dx+c)}^2}{}$$

input
```
integrate(sec(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas"
)
```

output
```
-1/4*(2*a^6*b + 18*a^4*b^3 - 18*a^2*b^5 - 2*b^7 - 6*(a^7 + 2*a^5*b^2 - 3*a
^3*b^4)*cos(d*x + c)^3 - 24*(a^4*b^3 - a^2*b^5)*cos(d*x + c)^2 + 2*(2*a^7
+ 9*a^5*b^2 - 12*a^3*b^4 + a*b^6)*cos(d*x + c) + 24*(a^4*b^3 + a^2*b^5 - (
a^6*b + a^4*b^3)*cos(d*x + c)^4 - 2*(a^5*b^2 + a^3*b^4)*cos(d*x + c)^3 + (
a^6*b - a^2*b^5)*cos(d*x + c)^2 + 2*(a^5*b^2 + a^3*b^4)*cos(d*x + c))*log(
a*cos(d*x + c) + b) - 3*(a^5*b^2 + 4*a^4*b^3 + 6*a^3*b^4 + 4*a^2*b^5 + a*b
^6 - (a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*cos(d*x + c)^4 - 2*
(a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)*cos(d*x + c)^3 + (a^
7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*cos(d*x + c)^2 +
2*(a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)*cos(d*x + c))*log(
1/2*cos(d*x + c) + 1/2) + 3*(a^5*b^2 - 4*a^4*b^3 + 6*a^3*b^4 - 4*a^2*b^5 +
 a*b^6 - (a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*cos(d*x + c)^4
- 2*(a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*cos(d*x + c)^3 +
 (a^7 - 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 + 4*a^2*b^5 - a*b^6)*cos(d*x + c)^
2 + 2*(a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*cos(d*x + c))*
log(-1/2*cos(d*x + c) + 1/2))/((a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 +
 a^2*b^8)*d*cos(d*x + c)^4 + 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7
+ a*b^9)*d*cos(d*x + c)^3 - (a^10 - 5*a^8*b^2 + 10*a^6*b^4 - 10*a^4*b^6 +
5*a^2*b^8 - b^10)*d*cos(d*x + c)^2 - 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*
a^3*b^7 + a*b^9)*d*cos(d*x + c) - (a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4*...
```

3.269.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx=\int \frac{\sec^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a*sin(d*x+c)+b*tan(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)**2/(a*sin(c + d*x) + b*tan(c + d*x))**3, x)
```

3.269.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 596 vs. $2(203) = 406$.

Time = 0.25 (sec) , antiderivative size = 596, normalized size of antiderivative = 2.81

$$\int \frac{\sec^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx = \frac{\frac{48\,(a^4b+a^2b^3)\log\left(a+b-\frac{(a-b)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8}+\frac{12\,a\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}-\frac{a^6-2\,a^5b-a^4b^2+4\,a^3b^3-a^2b^4-2\,ab^5+}{\frac{\left(a^9+a^8b-4\,a^7b^2-4\,a^6b^3+6\,a^5b^4+6\,a^4b^5-4\,a^3b^6-4\,a^2b^7+ab^8+b^9\right)\mathrm{si}}{(\cos(dx+c)+1)^2}}}{}$$

input
```
integrate(sec(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima"
)
```

output
```
1/8*(48*(a^4*b + a^2*b^3)*log(a + b - (a - b)*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2)/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) + 12*a*log(sin(d*
x + c)/(cos(d*x + c) + 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - (
a^6 - 2*a^5*b - a^4*b^2 + 4*a^3*b^3 - a^2*b^4 - 2*a*b^5 + b^6 - 2*(9*a^6 +
 4*a^5*b + 37*a^4*b^2 + 32*a^3*b^3 - 5*a^2*b^4 + 4*a*b^5 - b^6)*sin(d*x +
c)^2/(cos(d*x + c) + 1)^2 + (17*a^6 - 6*a^5*b + 63*a^4*b^2 - 84*a^3*b^3 +
15*a^2*b^4 - 6*a*b^5 + b^6)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)/((a^9 + a
^8*b - 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b
^7 + a*b^8 + b^9)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 2*(a^9 - a^8*b - 4
*a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 - 6*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*b
^8 - b^9)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + (a^9 - 3*a^8*b + 8*a^6*b^3
 - 6*a^5*b^4 - 6*a^4*b^5 + 8*a^3*b^6 - 3*a*b^8 + b^9)*sin(d*x + c)^6/(cos(
d*x + c) + 1)^6) + sin(d*x + c)^2/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(cos(d*
x + c) + 1)^2))/d
```

3.269.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 689 vs. $2(203) = 406$.

Time = 0.73 (sec) , antiderivative size = 689, normalized size of antiderivative = 3.25

$$\int \frac{\sec^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$$

$$= \frac{\frac{6\,a\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4} + \frac{48\,(a^4b+a^2b^3)\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8} + \frac{\left(a+b-\frac{6\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)(\cos(dx+c)-1)}}{}$$

input

```
integrate(sec(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")
```

output

```
1/8*(6*a*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^4 + 4*a^3*b
+ 6*a^2*b^2 + 4*a*b^3 + b^4) + 48*(a^4*b + a^2*b^3)*log(abs(-a - b - a*(co
s(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c) - 1)/(cos(d*x + c) +
1)))/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) + (a + b - 6*a*(cos(d
*x + c) - 1)/(cos(d*x + c) + 1))*(cos(d*x + c) + 1)/((a^4 + 4*a^3*b + 6*a^
2*b^2 + 4*a*b^3 + b^4)*(cos(d*x + c) - 1)) - (cos(d*x + c) - 1)/((a^3 - 3*
a^2*b + 3*a*b^2 - b^3)*(cos(d*x + c) + 1)) + 8*(2*a^7 - 5*a^6*b - 8*a^5*b^
2 - 2*a^4*b^3 - 10*a^3*b^4 - 9*a^2*b^5 + 2*a^7*(cos(d*x + c) - 1)/(cos(d*x
 + c) + 1) - 16*a^6*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 6*a^5*b^2*(c
os(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*a^4*b^3*(cos(d*x + c) - 1)/(cos(d*
x + c) + 1) - 8*a^3*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 18*a^2*b^5
*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 9*a^6*b*(cos(d*x + c) - 1)^2/(cos
(d*x + c) + 1)^2 + 18*a^5*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 -
18*a^4*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 18*a^3*b^4*(cos(d*x
 + c) - 1)^2/(cos(d*x + c) + 1)^2 - 9*a^2*b^5*(cos(d*x + c) - 1)^2/(cos(d*
x + c) + 1)^2)/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(a + b + a
*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1))^2))/d
```

3.269.9 Mupad [B] (verification not implemented)

Time = 23.15 (sec) , antiderivative size = 492, normalized size of antiderivative = 2.32

$$\int \frac{\sec^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8\,d\,(a-b)^3}$$
$$-\frac{\frac{a^3-3\,a^2\,b+3\,a\,b^2-b^3}{2\,(a+b)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,(9\,a^5-5\,a^4\,b+42\,a^3\,b^2-10\,a^2\,b^3+5}{(a-b)\,(a^2+2\,a\,b+b^2)}}{d\,\left((4\,a^5-20\,a^4\,b+40\,a^3\,b^2-40\,a^2\,b^3+20\,a\,b^4-4\,b^5)\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+(-8\,a^5+24\,a^4\,b-16\,a^3\,b^2-1\right.}$$
$$+\frac{\ln\left(a+b-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)\,(6\,a^4\,b+6\,a^2\,b^3)}{d\,(a^8-4\,a^6\,b^2+6\,a^4\,b^4-4\,a^2\,b^6+b^8)}$$
$$+\frac{3\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d\,(2\,a^4+8\,a^3\,b+12\,a^2\,b^2+8\,a\,b^3+2\,b^4)}$$

input
```
int(1/(cos(c + d*x)^2*(a*sin(c + d*x) + b*tan(c + d*x))^3),x)
```

output
```
tan(c/2 + (d*x)/2)^2/(8*d*(a - b)^3) - ((3*a*b^2 - 3*a^2*b + a^3 - b^3)/(2
*(a + b)) - (tan(c/2 + (d*x)/2)^2*(5*a*b^4 - 5*a^4*b + 9*a^5 - b^5 - 10*a^
2*b^3 + 42*a^3*b^2))/((a - b)*(2*a*b + a^2 + b^2)) + (tan(c/2 + (d*x)/2)^4
*(5*a*b^4 + 11*a^4*b + 17*a^5 - b^5 - 10*a^2*b^3 + 74*a^3*b^2))/(2*(a + b)
*(2*a*b + a^2 + b^2)))/(d*(tan(c/2 + (d*x)/2)^2*(4*a*b^4 - 4*a^4*b + 4*a^5
 - 4*b^5 + 8*a^2*b^3 - 8*a^3*b^2) - tan(c/2 + (d*x)/2)^4*(8*a^5 - 24*a^4*b
 - 24*a*b^4 + 8*b^5 + 16*a^2*b^3 + 16*a^3*b^2) + tan(c/2 + (d*x)/2)^6*(20*
a*b^4 - 20*a^4*b + 4*a^5 - 4*b^5 - 40*a^2*b^3 + 40*a^3*b^2))) + (log(a + b
 - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2)*(6*a^4*b + 6*a^2*b^3))
/(d*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2)) + (3*a*log(tan(c/2 +
(d*x)/2)))/(d*(8*a*b^3 + 8*a^3*b + 2*a^4 + 2*b^4 + 12*a^2*b^2))
```

3.270 $\int \frac{\sec^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$

3.270.1 Optimal result

Integrand size = 28, antiderivative size = 228

$$\int \frac{\sec^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx = \frac{a(2a^2+b^2)}{2(a^2-b^2)^2 d(b+a\cos(c+dx))^2} - \frac{ab(11a^2+b^2)}{2(a^2-b^2)^3 d(b+a\cos(c+dx))} - \frac{(a-b\cos(c+dx))\csc^2(c+dx)}{2(a^2-b^2)d(b+a\cos(c+dx))^2} + \frac{(4a+b)\log(1-\cos(c+dx))}{4(a+b)^4 d} + \frac{(4a-b)\log(1+\cos(c+dx))}{4(a-b)^4 d} - \frac{2a^3(a^2+5b^2)\log(b+a\cos(c+dx))}{(a^2-b^2)^4 d}$$

output
```
1/2*a*(2*a^2+b^2)/(a^2-b^2)^2/d/(b+a*cos(d*x+c))^2-1/2*a*b*(11*a^2+b^2)/(a
^2-b^2)^3/d/(b+a*cos(d*x+c))-1/2*(a-b*cos(d*x+c))*csc(d*x+c)^2/(a^2-b^2)/d
/(b+a*cos(d*x+c))^2+1/4*(4*a+b)*ln(1-cos(d*x+c))/(a+b)^4/d+1/4*(4*a-b)*ln(
1+cos(d*x+c))/(a-b)^4/d-2*a^3*(a^2+5*b^2)*ln(b+a*cos(d*x+c))/(a^2-b^2)^4/d
```

3.270.2 Mathematica [A] (verified)

Time = 6.75 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.95

$$\int \frac{\sec^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx = \frac{a^3}{2(-a+b)^2(a+b)^2d(b+a\cos(c+dx))^2} + \frac{4a^3b}{(-a+b)^3(a+b)^3d(b+a\cos(c+dx))} - \frac{\csc^2\left(\frac{1}{2}(c+dx)\right)}{8(a+b)^3d} + \frac{(4a-b)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2(-a+b)^4d} - \frac{2(a^5+5a^3b^2)\log(b+a\cos(c+dx))}{(-a^2+b^2)^4\,d} + \frac{(4a+b)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2(a+b)^4d} + \frac{\sec^2\left(\frac{1}{2}(c+dx)\right)}{8(-a+b)^3d}$$

input
```
Integrate[Sec[c + d*x]^3/(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

output
```
a^3/(2*(-a + b)^2*(a + b)^2*d*(b + a*Cos[c + d*x])^2) + (4*a^3*b)/((-a + b
)^3*(a + b)^3*d*(b + a*Cos[c + d*x])) - Csc[(c + d*x)/2]^2/(8*(a + b)^3*d)
 + ((4*a - b)*Log[Cos[(c + d*x)/2]])/(2*(-a + b)^4*d) - (2*(a^5 + 5*a^3*b^
2)*Log[b + a*Cos[c + d*x]])/((-a^2 + b^2)^4*d) + ((4*a + b)*Log[Sin[(c + d
*x)/2]])/(2*(a + b)^4*d) + Sec[(c + d*x)/2]^2/(8*(-a + b)^3*d)
```

3.270.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 4897, 3042, 3147, 477, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(c+dx)^3}{(a\sin(c+dx)+b\tan(c+dx))^3}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \frac{\csc^3(c+dx)}{(a\cos(c+dx)+b)^3}dx$$

$\big\downarrow$ 3042

$$\int \frac{1}{\cos\left(c+dx-\frac{\pi}{2}\right)^3\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)^3}dx$$

$\big\downarrow$ 3147

$$-\frac{a^3\int \frac{1}{(b+a\cos(c+dx))^3(a^2-a^2\cos^2(c+dx))^2}d(a\cos(c+dx))}{d}$$

$\big\downarrow$ 477

$$-\frac{\int \left(\frac{2(a^2+5b^2)a^4}{(a^2-b^2)^4(b+a\cos(c+dx))}-\frac{4ba^4}{(a^2-b^2)^3(b+a\cos(c+dx))^2}+\frac{a^4}{(a^2-b^2)^2(b+a\cos(c+dx))^3}+\frac{a^2}{4(a+b)^3(a-a\cos(c+dx))^2}-\frac{a^2}{4(a-b)^3(\cos(c\cdots}{ad}$$

$\big\downarrow$ 2009

$$-\frac{\frac{a^2}{4(a+b)^3(a-a\cos(c+dx))}+\frac{a^2}{4(a-b)^3(a\cos(c+dx)+a)}+\frac{4a^4b}{(a^2-b^2)^3(a\cos(c+dx)+b)}-\frac{a^4}{2(a^2-b^2)^2(a\cos(c+dx)+b)^2}+\frac{2a^4(a^2+5b^2)\log(a\cos\cdots}{(a^2-b^2)^4}\cdots}{ad}$$

input

```
Int[Sec[c + d*x]^3/(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

output

```
-((a^2/(4*(a + b)^3*(a - a*Cos[c + d*x])) + a^2/(4*(a - b)^3*(a + a*Cos[c
+ d*x])) - a^4/(2*(a^2 - b^2)^2*(b + a*Cos[c + d*x])^2) + (4*a^4*b)/((a^2
- b^2)^3*(b + a*Cos[c + d*x])) - (a*(4*a + b)*Log[a - a*Cos[c + d*x]])/(4*
(a + b)^4) - (a*(4*a - b)*Log[a + a*Cos[c + d*x]])/(4*(a - b)^4) + (2*a^4*
(a^2 + 5*b^2)*Log[b + a*Cos[c + d*x]])/(a^2 - b^2)^4)/(a*d))
```

3.270.3.1 Defintions of rubi rules used

rule 477

```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_^2))^(p_), x_Symbol] :> Simp[
a^p   Int[ExpandIntegrand[(c + d*x)^n*(1 - Rt[-b/a, 2]*x)^p*(1 + Rt[-b/a, 2
]*x)^p, x], x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[n] &
& NiceSqrtQ[-b/a] &&  !FractionalPowerFactorQ[Rt[-b/a, 2]]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.270. $\int \frac{\sec^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.270.4 Maple [A] (verified)

Time = 54.58 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.81

method	result
derivativedivides	$\frac{\frac{1}{4(a+b)^3(\cos(dx+c)-1)}+\frac{(4a+b)\ln(\cos(dx+c)-1)}{4(a+b)^4}-\frac{1}{4(a-b)^3(\cos(dx+c)+1)}+\frac{(4a-b)\ln(\cos(dx+c)+1)}{4(a-b)^4}+\frac{a^3}{2(a+b)^2(a-b)^2(b+\cos(dx+}}{d}$
default	$\frac{\frac{1}{4(a+b)^3(\cos(dx+c)-1)}+\frac{(4a+b)\ln(\cos(dx+c)-1)}{4(a+b)^4}-\frac{1}{4(a-b)^3(\cos(dx+c)+1)}+\frac{(4a-b)\ln(\cos(dx+c)+1)}{4(a-b)^4}+\frac{a^3}{2(a+b)^2(a-b)^2(b+\cos(dx+}}{d}$
risch	Expression too large to display

input
```
int(sec(d*x+c)^3/(sin(d*x+c)*a+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/4/(a+b)^3/(cos(d*x+c)-1)+1/4*(4*a+b)/(a+b)^4*ln(cos(d*x+c)-1)-1/4/(
a-b)^3/(cos(d*x+c)+1)+1/4*(4*a-b)/(a-b)^4*ln(cos(d*x+c)+1)+1/2*a^3/(a+b)^2
/(a-b)^2/(b+cos(d*x+c)*a)^2-4*a^3*b/(a+b)^3/(a-b)^3/(b+cos(d*x+c)*a)-2*a^3
*(a^2+5*b^2)/(a+b)^4/(a-b)^4*ln(b+cos(d*x+c)*a))
```

3.270.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 971 vs. $2(219) = 438$.

Time = 0.42 (sec) , antiderivative size = 971, normalized size of antiderivative = 4.26

$$\int \frac{\sec^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx = -\frac{2\,a^7 - 22\,a^5b^2 + 14\,a^3b^4 + 6\,ab^6 + 2\,(11\,a^6b - 10\,a^4b^3 - a^2b^5)\cos\left(dx+c\right)^3 - 4\,(a^7 - 7\,a^5b^2 + 5\,a^3b^4 + a}{}$$

input
```
integrate(sec(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas"
)
```

output
```
-1/4*(2*a^7 - 22*a^5*b^2 + 14*a^3*b^4 + 6*a*b^6 + 2*(11*a^6*b - 10*a^4*b^3
 - a^2*b^5)*cos(d*x + c)^3 - 4*(a^7 - 7*a^5*b^2 + 5*a^3*b^4 + a*b^6)*cos(d
*x + c)^2 - 2*(10*a^6*b - 7*a^4*b^3 - 4*a^2*b^5 + b^7)*cos(d*x + c) - 8*(a
^5*b^2 + 5*a^3*b^4 - (a^7 + 5*a^5*b^2)*cos(d*x + c)^4 - 2*(a^6*b + 5*a^4*b
^3)*cos(d*x + c)^3 + (a^7 + 4*a^5*b^2 - 5*a^3*b^4)*cos(d*x + c)^2 + 2*(a^6
*b + 5*a^4*b^3)*cos(d*x + c))*log(a*cos(d*x + c) + b) + (4*a^5*b^2 + 15*a^
4*b^3 + 20*a^3*b^4 + 10*a^2*b^5 - b^7 - (4*a^7 + 15*a^6*b + 20*a^5*b^2 + 1
0*a^4*b^3 - a^2*b^5)*cos(d*x + c)^4 - 2*(4*a^6*b + 15*a^5*b^2 + 20*a^4*b^3
 + 10*a^3*b^4 - a*b^6)*cos(d*x + c)^3 + (4*a^7 + 15*a^6*b + 16*a^5*b^2 - 5
*a^4*b^3 - 20*a^3*b^4 - 11*a^2*b^5 + b^7)*cos(d*x + c)^2 + 2*(4*a^6*b + 15
*a^5*b^2 + 20*a^4*b^3 + 10*a^3*b^4 - a*b^6)*cos(d*x + c))*log(1/2*cos(d*x
+ c) + 1/2) + (4*a^5*b^2 - 15*a^4*b^3 + 20*a^3*b^4 - 10*a^2*b^5 + b^7 - (4
*a^7 - 15*a^6*b + 20*a^5*b^2 - 10*a^4*b^3 + a^2*b^5)*cos(d*x + c)^4 - 2*(4
*a^6*b - 15*a^5*b^2 + 20*a^4*b^3 - 10*a^3*b^4 + a*b^6)*cos(d*x + c)^3 + (4
*a^7 - 15*a^6*b + 16*a^5*b^2 + 5*a^4*b^3 - 20*a^3*b^4 + 11*a^2*b^5 - b^7)*
cos(d*x + c)^2 + 2*(4*a^6*b - 15*a^5*b^2 + 20*a^4*b^3 - 10*a^3*b^4 + a*b^6
)*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/((a^10 - 4*a^8*b^2 + 6*a^6*b
^4 - 4*a^4*b^6 + a^2*b^8)*d*cos(d*x + c)^4 + 2*(a^9*b - 4*a^7*b^3 + 6*a^5*
b^5 - 4*a^3*b^7 + a*b^9)*d*cos(d*x + c)^3 - (a^10 - 5*a^8*b^2 + 10*a^6*b^4
 - 10*a^4*b^6 + 5*a^2*b^8 - b^10)*d*cos(d*x + c)^2 - 2*(a^9*b - 4*a^7*b...
```

3.270.6 Sympy [F]

$$\int \frac{\sec^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx = \int \frac{\sec^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$$

input
```
integrate(sec(d*x+c)**3/(a*sin(d*x+c)+b*tan(d*x+c))**3,x)
```

output
```
Integral(sec(c + d*x)**3/(a*sin(c + d*x) + b*tan(c + d*x))**3, x)
```

3.270.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 591 vs. $2(219) = 438$.

Time = 0.24 (sec) , antiderivative size = 591, normalized size of antiderivative = 2.59

$$\int \frac{\sec^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx = -\frac{\frac{16\,(a^5+5\,a^3b^2)\log\left(a+b-\frac{(a-b)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8} - \frac{4\,(4\,a+b)\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4} + \frac{a^6-2\,a^5b-a^4b^2+4\,a^3b^3-a^2b^4-\cdots}{\frac{\left(a^9+a^8b-4\,a^7b^2-4\,a^6b^3+6\,a^5b^4+6\,a^4b^5-4\,a^3b^6-4\,a^2b^7+ab^8+b\cdots\right.}{(\cos(dx+c)+1)^2}}}{\cdots}$$

input
```
integrate(sec(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima"
)
```

output
```
-1/8*(16*(a^5 + 5*a^3*b^2)*log(a + b - (a - b)*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2)/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) - 4*(4*a + b)*lo
g(sin(d*x + c)/(cos(d*x + c) + 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 +
b^4) + (a^6 - 2*a^5*b - a^4*b^2 + 4*a^3*b^3 - a^2*b^4 - 2*a*b^5 + b^6 - 2*
(a^6 - 44*a^5*b - 35*a^4*b^2 - 5*a^2*b^4 + 4*a*b^5 - b^6)*sin(d*x + c)^2/(
cos(d*x + c) + 1)^2 - (15*a^6 + 70*a^5*b - 95*a^4*b^2 + 20*a^3*b^3 - 15*a^
2*b^4 + 6*a*b^5 - b^6)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)/((a^9 + a^8*b
- 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b^7 +
a*b^8 + b^9)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 2*(a^9 - a^8*b - 4*a^7*
b^2 + 4*a^6*b^3 + 6*a^5*b^4 - 6*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8 -
b^9)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + (a^9 - 3*a^8*b + 8*a^6*b^3 - 6*
a^5*b^4 - 6*a^4*b^5 + 8*a^3*b^6 - 3*a*b^8 + b^9)*sin(d*x + c)^6/(cos(d*x +
 c) + 1)^6) + sin(d*x + c)^2/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(cos(d*x + c
) + 1)^2))/d
```

3.270.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 675 vs. $2(219) = 438$.

Time = 0.86 (sec) , antiderivative size = 675, normalized size of antiderivative = 2.96

$$\int \frac{\sec^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$$
$$= \frac{\frac{2(4a+b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^4+4a^3b+6a^2b^2+4ab^3+b^4} - \frac{16(a^5+5a^3b^2)\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8} + \frac{\left(a+b-\frac{8a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{2b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right.}{(a^4+4a^3b+6a^2b^2+4ab^3+b^4)(\cos(}}{}$$

input
```
integrate(sec(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
1/8*(2*(4*a + b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^4 +
4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - 16*(a^5 + 5*a^3*b^2)*log(abs(-a - b
 - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c) - 1)/(cos(d*x
 + c) + 1)))/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) + (a + b - 8*
a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*b*(cos(d*x + c) - 1)/(cos(d*x
+ c) + 1))*(cos(d*x + c) + 1)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)
*(cos(d*x + c) - 1)) + (cos(d*x + c) - 1)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)
*(cos(d*x + c) + 1)) + 8*(3*a^7 - 4*a^6*b - 2*a^5*b^2 + 20*a^4*b^3 + 15*a^
3*b^4 + 4*a^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 10*a^6*b*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) + 26*a^5*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) +
 1) + 10*a^4*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 30*a^3*b^4*(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) + 3*a^7*(cos(d*x + c) - 1)^2/(cos(d*x + c)
 + 1)^2 - 6*a^6*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 18*a^5*b^2*(
cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 30*a^4*b^3*(cos(d*x + c) - 1)^2
/(cos(d*x + c) + 1)^2 + 15*a^3*b^4*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)
^2)/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(a + b + a*(cos(d*x +
 c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))^2))
/d
```

3.270.9 Mupad [B] (verification not implemented)

Time = 22.81 (sec) , antiderivative size = 490, normalized size of antiderivative = 2.15

$$\int \frac{\sec^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$$

$$= \frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,(15\,a^5+85\,a^4\,b-10\,a^3\,b^2+10\,a^2\,b^3-5\,a\,b^4+b^5)}{2\,(a+b)\,(a^2+2\,a\,b+b^2)} - \frac{a^3-3\,a^2\,b+3}{2\,(a+}}{d\,\left((4\,a^5-20\,a^4\,b+40\,a^3\,b^2-40\,a^2\,b^3+20\,a\,b^4-4\,b^5)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+(-8\,a^5+24\,a^4\,b-16\,a^3\,b^2-16\,a\right.}$$

$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d\,(a-b)^3}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,(4\,a+b)}{d\,(2\,a^4+8\,a^3\,b+12\,a^2\,b^2+8\,a\,b^3+2\,b^4)}$$

$$-\frac{\ln\left(a+b-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)\,(2\,a^5+10\,a^3\,b^2)}{d\,(a^8-4\,a^6\,b^2+6\,a^4\,b^4-4\,a^2\,b^6+b^8)}$$

input
```
int(1/(cos(c + d*x)^3*(a*sin(c + d*x) + b*tan(c + d*x))^3),x)
```

output
```
((tan(c/2 + (d*x)/2)^4*(85*a^4*b - 5*a*b^4 + 15*a^5 + b^5 + 10*a^2*b^3 - 1
0*a^3*b^2))/(2*(a + b)*(2*a*b + a^2 + b^2)) - (3*a*b^2 - 3*a^2*b + a^3 - b
^3)/(2*(a + b)) + (tan(c/2 + (d*x)/2)^2*(5*a*b^4 - 45*a^4*b + a^5 - b^5 -
10*a^2*b^3 + 10*a^3*b^2))/((a - b)*(2*a*b + a^2 + b^2)))/(d*(tan(c/2 + (d*
x)/2)^2*(4*a*b^4 - 4*a^4*b + 4*a^5 - 4*b^5 + 8*a^2*b^3 - 8*a^3*b^2) - tan(
c/2 + (d*x)/2)^4*(8*a^5 - 24*a^4*b - 24*a*b^4 + 8*b^5 + 16*a^2*b^3 + 16*a^
3*b^2) + tan(c/2 + (d*x)/2)^6*(20*a*b^4 - 20*a^4*b + 4*a^5 - 4*b^5 - 40*a^
2*b^3 + 40*a^3*b^2))) - tan(c/2 + (d*x)/2)^2/(8*d*(a - b)^3) + (log(tan(c/
2 + (d*x)/2))*(4*a + b))/(d*(8*a*b^3 + 8*a^3*b + 2*a^4 + 2*b^4 + 12*a^2*b^
2)) - (log(a + b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2)*(2*a^5
 + 10*a^3*b^2))/(d*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2))
```

3.271 $\int \cos^m(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx$

3.271.1 Optimal result

Integrand size = 28, antiderivative size = 155

$$\int \cos^m(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx = \frac{b^3\cos^{-2+m}(c+dx)}{d(2-m)}+\frac{3ab^2\cos^{-1+m}(c+dx)}{d(1-m)}-\frac{b(3a^2-b^2)\cos^m(c+dx)}{dm} -\frac{a(a^2-3b^2)\cos^{1+m}(c+dx)}{d(1+m)}+\frac{3a^2b\cos^{2+m}(c+dx)}{d(2+m)}+\frac{a^3\cos^{3+m}(c+dx)}{d(3+m)}$$

output
```
b^3*cos(d*x+c)^(-2+m)/d/(2-m)+3*a*b^2*cos(d*x+c)^(-1+m)/d/(1-m)-b*(3*a^2-b
^2)*cos(d*x+c)^m/d/m-a*(a^2-3*b^2)*cos(d*x+c)^(1+m)/d/(1+m)+3*a^2*b*cos(d*
x+c)^(2+m)/d/(2+m)+a^3*cos(d*x+c)^(3+m)/d/(3+m)
```

3.271.2 Mathematica [A] (verified)

Time = 6.86 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.59

$$\int \cos^m(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx = \frac{\cos^{1+m}(c+dx)\left(-4b^3m(-6-5m+5m^2+5m^3+m^4)-12ab^2m(-12-16m-m^2+4m^3+m^4)\cos(c+\right.}{}$$

input
```
Integrate[Cos[c + d*x]^m*(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

output
```
(Cos[c + d*x]^(1 + m)*(-4*b^3*m*(-6 - 5*m + 5*m^2 + 5*m^3 + m^4) - 12*a*b^
2*m*(-12 - 16*m - m^2 + 4*m^3 + m^4)*Cos[c + d*x] - a*m*(4 - 4*m - m^2 + m
^3)*(-12*b^2*(3 + m) + a^2*(9 + m))*Cos[c + d*x]^3 + (2 - m - 2*m^2 + m^3)
*Cos[c + d*x]^2*(2*b*(3 + m)*(2*b^2*(2 + m) - 3*a^2*(4 + m)) + 6*a^2*b*m*(
3 + m)*Cos[2*(c + d*x)] + a^3*m*(2 + m)*Cos[3*(c + d*x)]))*(a + b*Sec[c +
d*x])^3)/(4*d*(-2 + m)*(-1 + m)*m*(1 + m)*(2 + m)*(3 + m)*(b + a*Cos[c + d
*x])^3)
```

3.271.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 4897, 3042, 25, 3316, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^m(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \cos(c+dx)^m(a\sin(c+dx)+b\tan(c+dx))^3dx$$

$$\Big\downarrow \text{ 4897}$$

$$\int \sin^3(c+dx)\cos^{m-3}(c+dx)(a\cos(c+dx)+b)^3dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\cos\left(c+dx+\frac{\pi}{2}\right)^3\sin\left(c+dx+\frac{\pi}{2}\right)^{m-3}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+b\right)^3dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \cos\left(\frac{1}{2}(2c+\pi)+dx\right)^3\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^{m-3}\left(b+a\sin\left(\frac{1}{2}(2c+\pi)+dx\right)\right)^3dx$$

$$\Big\downarrow \text{ 3316}$$

$$-\frac{\int \cos^{m-3}(c+dx)(b+a\cos(c+dx))^3\left(a^2-a^2\cos^2(c+dx)\right)d(a\cos(c+dx))}{a^3d}$$

$$\Big\downarrow \text{ 522}$$

$$-\frac{\int \left(a^2b^3\cos^{m-3}(c+dx)+3a^3b^2\cos^{m-2}(c+dx)+a^2b\left(3a^2-b^2\right)\cos^{m-1}(c+dx)+a^3\left(a^2-3b^2\right)\cos^{m}(c+dx)-\right.}{a^3d}$$

$\Big\downarrow$ 2009

$$-\frac{-\frac{a^6\cos^{m+3}(c+dx)}{m+3}-\frac{3a^5b\cos^{m+2}(c+dx)}{m+2}-\frac{3a^4b^2\cos^{m-1}(c+dx)}{1-m}-\frac{a^3b^3\cos^{m-2}(c+dx)}{2-m}+\frac{a^4\left(a^2-3b^2\right)\cos^{m+1}(c+dx)}{m+1}+\frac{a^3b\left(3a^2-b^2\right)}{m}}{a^3d}$$

input
```
Int[Cos[c + d*x]^m*(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

output
```
-((-((a^3*b^3*Cos[c + d*x]^(-2 + m))/(2 - m)) - (3*a^4*b^2*Cos[c + d*x]^(-
1 + m))/(1 - m) + (a^3*b*(3*a^2 - b^2)*Cos[c + d*x]^m)/m + (a^4*(a^2 - 3*b
^2)*Cos[c + d*x]^(1 + m))/(1 + m) - (3*a^5*b*Cos[c + d*x]^(2 + m))/(2 + m)
 - (a^6*Cos[c + d*x]^(3 + m))/(3 + m))/(a^3*d))
```

3.271.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.271.4 Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.37

$$-\frac{a^{3} \cos \left(d x+c\right)^{1+m}}{d \left(1+m\right)}+\frac{a^{3} \cos \left(d x+c\right)^{3} {\mathrm e}^{m \ln \left(\cos \left(d x+c\right)\right)}}{d \left(3+m\right)}+\frac{b^{3} \cos \left(d x+c\right)^{m}}{m d}-\frac{b^{3} {\mathrm e}^{m \ln \left(\cos \left(d x+c\right)\right)}}{d \left(-2+m\right) \cos \left(d x+c\right)^{2}}-\frac{3 a^{2} b \cos}{m}$$

input
```
int(cos(d*x+c)^m*(sin(d*x+c)*a+b*tan(d*x+c))^3,x)
```

output
```
-a^3/d*cos(d*x+c)^(1+m)/(1+m)+a^3/d/(3+m)*cos(d*x+c)^3*exp(m*ln(cos(d*x+c)
))+b^3/m/d*cos(d*x+c)^m-b^3/d/(-2+m)*exp(m*ln(cos(d*x+c)))/cos(d*x+c)^2-3*
a^2*b/m/d*cos(d*x+c)^m+3*a^2*b/d/(2+m)*cos(d*x+c)^2*exp(m*ln(cos(d*x+c)))+
3*a*b^2/d*cos(d*x+c)^(1+m)/(1+m)-3*a*b^2/d/(-1+m)*exp(m*ln(cos(d*x+c)))/co
s(d*x+c)
```

3.271.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 412 vs. 2(152) = 304.

Time = 0.29 (sec) , antiderivative size = 412, normalized size of antiderivative = 2.66

$$\int \cos^m(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx =$$
$$-\frac{\left(b^{3} m^{5}+5 \, b^{3} m^{4}+5 \, b^{3} m^{3}-\left(a^{3} m^{5}-5 \, a^{3} m^{3}+4 \, a^{3} m\right) \cos \left(d x+c\right)^{5}-5 \, b^{3} m^{2}-3 \, \left(a^{2} b m^{5}+a^{2} b m^{4}-7 \, a\right.}{}$$

input
```
integrate(cos(d*x+c)^m*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas"
)
```

output
```
-(b^3*m^5 + 5*b^3*m^4 + 5*b^3*m^3 - (a^3*m^5 - 5*a^3*m^3 + 4*a^3*m)*cos(d*
x + c)^5 - 5*b^3*m^2 - 3*(a^2*b*m^5 + a^2*b*m^4 - 7*a^2*b*m^3 - a^2*b*m^2
+ 6*a^2*b*m)*cos(d*x + c)^4 - 6*b^3*m + ((a^3 - 3*a*b^2)*m^5 + 2*(a^3 - 3*
a*b^2)*m^4 - 7*(a^3 - 3*a*b^2)*m^3 - 8*(a^3 - 3*a*b^2)*m^2 + 12*(a^3 - 3*a
*b^2)*m)*cos(d*x + c)^3 + ((3*a^2*b - b^3)*m^5 + 3*(3*a^2*b - b^3)*m^4 - 5
*(3*a^2*b - b^3)*m^3 + 36*a^2*b - 12*b^3 - 15*(3*a^2*b - b^3)*m^2 + 4*(3*a
^2*b - b^3)*m)*cos(d*x + c)^2 + 3*(a*b^2*m^5 + 4*a*b^2*m^4 - a*b^2*m^3 - 1
6*a*b^2*m^2 - 12*a*b^2*m)*cos(d*x + c))*cos(d*x + c)^m/((d*m^6 + 3*d*m^5 -
 5*d*m^4 - 15*d*m^3 + 4*d*m^2 + 12*d*m)*cos(d*x + c)^2)
```

3.271.6 Sympy [F]

$$\int \cos^m(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx = \int (a\sin(c+dx)+b\tan(c+dx))^3\cos^m(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)**m*(a*sin(d*x+c)+b*tan(d*x+c))**3,x)
```

output
```
Integral((a*sin(c + d*x) + b*tan(c + d*x))**3*cos(c + d*x)**m, x)
```

3.271.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.16

$$\int \cos^m(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx = \frac{\frac{\left((m+1)\cos(dx+c)^3-(m+3)\cos(dx+c)\right)a^3\cos(dx+c)^m}{m^2+4m+3}+\frac{3\left(m\cos(dx+c)^2-m-2\right)a^2b\cos(dx+c)^m}{m^2+2m}+\frac{3\left((m-1)\cos(dx+c)^2-m-1\right)ab^2\cos}{(m^2-1)\cos(dx+c)}}{d}$$

input
```
integrate(cos(d*x+c)^m*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima"
)
```

output
```
(((m + 1)*cos(d*x + c)^3 - (m + 3)*cos(d*x + c))*a^3*cos(d*x + c)^m/(m^2 +
 4*m + 3) + 3*(m*cos(d*x + c)^2 - m - 2)*a^2*b*cos(d*x + c)^m/(m^2 + 2*m)
+ 3*((m - 1)*cos(d*x + c)^2 - m - 1)*a*b^2*cos(d*x + c)^m/((m^2 - 1)*cos(d
*x + c)) + ((m - 2)*cos(d*x + c)^2 - m)*b^3*cos(d*x + c)^m/((m^2 - 2*m)*co
s(d*x + c)^2))/d
```

3.271.8 Giac [F(-2)]

Exception generated.

$$\int \cos^m(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cos(d*x+c)^m*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Modgcd: no suitable evaluation poin
tindex.cc index_m operator + Error: Bad Argument ValueDone
```

3.271.9 Mupad [B] (verification not implemented)

Time = 33.54 (sec) , antiderivative size = 861, normalized size of antiderivative = 5.55

$$\int \cos^m(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx$$
$$= \frac{\left(\frac{1}{2}\right)^m \left(\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}+\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\right)^m \left(\frac{a^3\left(\frac{m^4}{8}-\frac{5\,m^2}{8}+\frac{1}{2}\right)}{d\,(m^5+3\,m^4-5\,m^3-15\,m^2+4\,m+12)}+\frac{a^3\,\mathrm{e}^{c\,10\mathrm{i}+d\,x\,10\mathrm{i}}\,(m^4-5\,m^2+4)}{8\,d\,(m^5+3\,m^4-5\,m^3-15\,m^2+4\,m+12)}-\frac{a\,\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,(}{8}\right.}{}$$

input
```
int(cos(c + d*x)^m*(a*sin(c + d*x) + b*tan(c + d*x))^3,x)
```

output
```
((1/2)^m*(exp(- c*1i - d*x*1i) + exp(c*1i + d*x*1i))^m*((a^3*(m^4/8 - (5*m
^2)/8 + 1/2))/(d*(4*m - 15*m^2 - 5*m^3 + 3*m^4 + m^5 + 12)) + (a^3*exp(c*1
0i + d*x*10i)*(m^4 - 5*m^2 + 4))/(8*d*(4*m - 15*m^2 - 5*m^3 + 3*m^4 + m^5
+ 12)) - (a*exp(c*2i + d*x*2i)*(4*m + m^2 - m^3 - 4)*(a^2*m + 12*b^2*m - 7
*a^2 + 36*b^2))/(8*d*(4*m - 15*m^2 - 5*m^3 + 3*m^4 + m^5 + 12)) - (a*exp(c
*8i + d*x*8i)*(4*m + m^2 - m^3 - 4)*(a^2*m + 12*b^2*m - 7*a^2 + 36*b^2))/(
8*d*(4*m - 15*m^2 - 5*m^3 + 3*m^4 + m^5 + 12)) + (3*a^2*b*exp(c*1i + d*x*1
i)*(m^3 - 7*m^2 - m + m^4 + 6))/(4*d*(4*m - 15*m^2 - 5*m^3 + 3*m^4 + m^5 +
 12)) + (3*a^2*b*exp(c*9i + d*x*9i)*(m^3 - 7*m^2 - m + m^4 + 6))/(4*d*(4*m
 - 15*m^2 - 5*m^3 + 3*m^4 + m^5 + 12)) - (a*exp(c*4i + d*x*4i)*(m^2 - 4)*(
12*a^2*m + 60*b^2*m - 13*a^2 + 126*b^2 + a^2*m^2 + 6*b^2*m^2))/(4*d*(4*m -
 15*m^2 - 5*m^3 + 3*m^4 + m^5 + 12)) - (a*exp(c*6i + d*x*6i)*(m^2 - 4)*(12
*a^2*m + 60*b^2*m - 13*a^2 + 126*b^2 + a^2*m^2 + 6*b^2*m^2))/(4*d*(4*m - 1
5*m^2 - 5*m^3 + 3*m^4 + m^5 + 12)) + (b*exp(c*3i + d*x*3i)*(b^2*m - 6*a^2
+ 2*b^2)*(m^3 - 7*m^2 - m + m^4 + 6))/(d*m*(4*m - 15*m^2 - 5*m^3 + 3*m^4 +
 m^5 + 12)) + (b*exp(c*7i + d*x*7i)*(b^2*m - 6*a^2 + 2*b^2)*(m^3 - 7*m^2 -
 m + m^4 + 6))/(d*m*(4*m - 15*m^2 - 5*m^3 + 3*m^4 + m^5 + 12)) + (b*exp(c*
5i + d*x*5i)*(m - 3*m^2 - m^3 + 3)*(18*a^2*m + 16*b^2*m - 48*a^2 + 16*b^2
+ 3*a^2*m^2 + 4*b^2*m^2))/(2*d*m*(4*m - 15*m^2 - 5*m^3 + 3*m^4 + m^5 + 12)
)))/(exp(c*3i + d*x*3i) + 2*exp(c*5i + d*x*5i) + exp(c*7i + d*x*7i))
```

3.271. $\int \cos^m(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx$

3.272 $\int \cos^m(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$

3.272.1 Optimal result

Integrand size = 28, antiderivative size = 264

$$\int \cos^m(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx = \frac{(a^2-2b^2)\cos^{-1+m}(c+dx)\sin(c+dx)}{dm(2+m)}$$
$$-\frac{2ab\cos^m(c+dx)\sin(c+dx)}{d\,(2+3m+m^2)}-\frac{\cos^{-1+m}(c+dx)(b+a\cos(c+dx))^2\sin(c+dx)}{d(2+m)}$$
$$-\frac{(a^2(1-m)-b^2(2+m))\cos^{-1+m}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-1+m),\frac{1+m}{2},\cos^2(c+dx)\right)\sin(c}{d(1-m)m(2+m)\sqrt{\sin^2(c+dx)}}$$
$$-\frac{2ab\cos^m(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m}{2},\frac{2+m}{2},\cos^2(c+dx)\right)\sin(c+dx)}{dm(1+m)\sqrt{\sin^2(c+dx)}}$$

output
```
(a^2-2*b^2)*cos(d*x+c)^(-1+m)*sin(d*x+c)/d/m/(2+m)-2*a*b*cos(d*x+c)^m*sin(
d*x+c)/d/(m^2+3*m+2)-cos(d*x+c)^(-1+m)*(b+a*cos(d*x+c))^2*sin(d*x+c)/d/(2+
m)-(a^2*(1-m)-b^2*(2+m))*cos(d*x+c)^(-1+m)*hypergeom([1/2, -1/2+1/2*m],[1/
2+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(1-m)/m/(2+m)/(sin(d*x+c)^2)^(1/2)-2*a
*b*cos(d*x+c)^m*hypergeom([1/2, 1/2*m],[1+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/
d/m/(1+m)/(sin(d*x+c)^2)^(1/2)
```

3.272.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 47.88 (sec) , antiderivative size = 6639, normalized size of antiderivative = 25.15

$$\int \cos^m(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx = \text{Result too large to show}$$

input
```
Integrate[Cos[c + d*x]^m*(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

output
```
Result too large to show
```

3.272.3 Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.01, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.536$, Rules used = {3042, 4897, 3042, 3368, 3042, 3529, 3042, 3512, 3042, 3502, 25, 3042, 3227, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^m(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \cos(c+dx)^m(a\sin(c+dx)+b\tan(c+dx))^2dx$$

$$\Big\downarrow \ 4897$$

$$\int \sin^2(c+dx)\cos^{m-2}(c+dx)(a\cos(c+dx)+b)^2dx$$

$$\Big\downarrow \ 3042$$

$$\int \cos\left(c+dx+\frac{\pi}{2}\right)^2\sin\left(c+dx+\frac{\pi}{2}\right)^{m-2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+b\right)^2dx$$

$$\Big\downarrow \ 3368$$

$$\int \left(1-\cos^2(c+dx)\right)\cos^{m-2}(c+dx)(a\cos(c+dx)+b)^2dx$$

$$\Big\downarrow \ 3042$$

$$\int \left(1-\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)\sin\left(c+dx+\frac{\pi}{2}\right)^{m-2}\left(a\sin\left(c+dx+\frac{\pi}{2}\right)+b\right)^2dx$$

$$\Big\downarrow \text{3529}$$

$$\frac{\int \cos^{m-2}(c+dx)(b+a\cos(c+dx))\left(-2b\cos^2(c+dx)+a\cos(c+dx)+3b\right)dx}{m+2}-\frac{\sin(c+dx)\cos^{m-1}(c+dx)(a\cos(c+dx)+b)^2}{d(m+2)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m-2}\left(b+a\sin\left(c+dx+\frac{\pi}{2}\right)\right)\left(-2b\sin\left(c+dx+\frac{\pi}{2}\right)^2+a\sin\left(c+dx+\frac{\pi}{2}\right)+3b\right)dx}{m+2}-\frac{\sin(c+dx)\cos^{m-1}(c+dx)(a\cos(c+dx)+b)^2}{d(m+2)}$$

$$\Big\downarrow \text{3512}$$

$$\frac{\frac{\int \cos^{m-2}(c+dx)\left(3(m+1)b^2+2a(m+2)\cos(c+dx)b+\left(a^2-2b^2\right)(m+1)\cos^2(c+dx)\right)dx}{m+1}-\frac{2ab\sin(c+dx)\cos^m(c+dx)}{d(m+1)}}{m+2}-\frac{\sin(c+dx)\cos^{m-1}(c+dx)(a\cos(c+dx)+b)^2}{d(m+2)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m-2}\left(3(m+1)b^2+2a(m+2)\sin\left(c+dx+\frac{\pi}{2}\right)b+\left(a^2-2b^2\right)(m+1)\sin\left(c+dx+\frac{\pi}{2}\right)^2\right)dx}{m+1}-\frac{2ab\sin(c+dx)\cos^m(c+dx)}{d(m+1)}}{m+2}-\frac{\sin(c+dx)\cos^{m-1}(c+dx)(a\cos(c+dx)+b)^2}{d(m+2)}$$

$$\Big\downarrow \text{3502}$$

$$\frac{\frac{\frac{\int -\cos^{m-2}(c+dx)\left((m+1)\left(a^2(1-m)-b^2(m+2)\right)-2abm(m+2)\cos(c+dx)\right)dx}{m}+\frac{(m+1)\left(a^2-2b^2\right)\sin(c+dx)\cos^{m-1}(c+dx)}{dm}}{m+1}-\frac{2ab\sin(c+dx)\cos^m(c+dx)}{d(m+1)}}{m+2}-\frac{\sin(c+dx)\cos^{m-1}(c+dx)(a\cos(c+dx)+b)^2}{d(m+2)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\frac{(m+1)\left(a^2-2b^2\right)\sin(c+dx)\cos^{m-1}(c+dx)}{dm}-\frac{\int \cos^{m-2}(c+dx)\left((m+1)\left(a^2(1-m)-b^2(m+2)\right)-2abm(m+2)\cos(c+dx)\right)dx}{m}}{m+1}-\frac{2ab\sin(c+dx)\cos^m(c+dx)}{d(m+1)}}{m+2}-\frac{\sin(c+dx)\cos^{m-1}(c+dx)(a\cos(c+dx)+b)^2}{d(m+2)}$$

3.272. $\int \cos^m(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{(m+1)\left(a^2-2b^2\right)\sin(c+dx)\cos^{m-1}(c+dx)}{dm}-\frac{\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m-2}\left((m+1)\left(a^2(1-m)-b^2(m+2)\right)-2abm(m+2)\sin\left(c+dx+\frac{\pi}{2}\right)\right)dx}{m}}{m+1}-\frac{2ab\sin(c+dx)\cos^m(c+a}{d(m+1)}}{m+2}$$

$$\frac{\sin(c+dx)\cos^{m-1}(c+dx)(a\cos(c+dx)+b)^2}{d(m+2)}$$

$$\Big\downarrow \text{3227}$$

$$\frac{\frac{\frac{(m+1)\left(a^2-2b^2\right)\sin(c+dx)\cos^{m-1}(c+dx)}{dm}-\frac{(m+1)\left(a^2(1-m)-b^2(m+2)\right)\int \cos^{m-2}(c+dx)dx-2abm(m+2)\int \cos^{m-1}(c+dx)dx}{m}}{m+1}-\frac{2ab\sin(c+dx)\cos^m(c+dx}{d(m+1)}}{m+2}$$

$$\frac{\sin(c+dx)\cos^{m-1}(c+dx)(a\cos(c+dx)+b)^2}{d(m+2)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{(m+1)\left(a^2-2b^2\right)\sin(c+dx)\cos^{m-1}(c+dx)}{dm}-\frac{(m+1)\left(a^2(1-m)-b^2(m+2)\right)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m-2}dx-2abm(m+2)\int \sin\left(c+dx+\frac{\pi}{2}\right)^{m-1}dx}{m}}{m+1}-\frac{2ab\sin(c+dx)\cos}{d(m+1)}}{m+2}$$

$$\frac{\sin(c+dx)\cos^{m-1}(c+dx)(a\cos(c+dx)+b)^2}{d(m+2)}$$

$$\Big\downarrow \text{3122}$$

$$\frac{\frac{\frac{(m+1)\left(a^2-2b^2\right)\sin(c+dx)\cos^{m-1}(c+dx)}{dm}-\frac{\frac{(m+1)\left(a^2(1-m)-b^2(m+2)\right)\sin(c+dx)\cos^{m-1}(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m-1}{2},\frac{m+1}{2},\cos^2(c+dx)\right)}{d(1-m)\sqrt{\sin^2(c+dx)}}+\frac{2ab(m+2)}{}}{m}}{m+1}}{m+2}$$

$$\frac{\sin(c+dx)\cos^{m-1}(c+dx)(a\cos(c+dx)+b)^2}{d(m+2)}$$

input
```
Int[Cos[c + d*x]^m*(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

output
```
-((Cos[c + d*x]^(-1 + m)*(b + a*Cos[c + d*x])^2*Sin[c + d*x])/(d*(2 + m)))
 + ((-2*a*b*Cos[c + d*x]^m*Sin[c + d*x])/(d*(1 + m)) + (((a^2 - 2*b^2)*(1
+ m)*Cos[c + d*x]^(-1 + m)*Sin[c + d*x])/(d*m) - (((1 + m)*(a^2*(1 - m) -
b^2*(2 + m))*Cos[c + d*x]^(-1 + m)*Hypergeometric2F1[1/2, (-1 + m)/2, (1 +
 m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 - m)*Sqrt[Sin[c + d*x]^2]) + (2
*a*b*(2 + m)*Cos[c + d*x]^m*Hypergeometric2F1[1/2, m/2, (2 + m)/2, Cos[c +
 d*x]^2]*Sin[c + d*x])/(d*Sqrt[Sin[c + d*x]^2]))/m)/(1 + m))/(2 + m)
```

3.272. $\int \cos^m(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$

3.272.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3368
```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 3512
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Simp[1/(b*(m + 3))   Int[(a + b*Si
n[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) +
A*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d,
0] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

rule 3529
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n +
1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m + n + 2))   Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(
n + 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*
(a*d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c,
0])))
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.272.4 Maple [F]

$$\int \cos\left(dx+c\right)^{m} \left(\sin\left(dx+c\right)a+b\tan\left(dx+c\right)\right)^{2}dx$$

input
```
int(cos(d*x+c)^m*(sin(d*x+c)*a+b*tan(d*x+c))^2,x)
```

output
```
int(cos(d*x+c)^m*(sin(d*x+c)*a+b*tan(d*x+c))^2,x)
```

3.272.5 Fricas [F]

$$\int \cos^m(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$$
$$=\int \left(a\sin\left(dx+c\right)+b\tan\left(dx+c\right)\right)^{2}\cos\left(dx+c\right)^{m}\,dx$$

input
```
integrate(cos(d*x+c)^m*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas"
)
```

output
```
integral(-(a^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c)*tan(d*x + c) - b^2*tan(
d*x + c)^2 - a^2)*cos(d*x + c)^m, x)
```

3.272.6 Sympy [F]

$$\int \cos^m(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx = \int (a\sin(c+dx)+b\tan(c+dx))^2\cos^m(c+dx)\,dx$$

input
```
integrate(cos(d*x+c)**m*(a*sin(d*x+c)+b*tan(d*x+c))**2,x)
```

output
```
Integral((a*sin(c + d*x) + b*tan(c + d*x))**2*cos(c + d*x)**m, x)
```

3.272.7 Maxima [F]

$$\int \cos^m(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx = \int (a\sin(dx+c)+b\tan(dx+c))^2\cos(dx+c)^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima"
)
```

output
```
integrate((a*sin(d*x + c) + b*tan(d*x + c))^2*cos(d*x + c)^m, x)
```

3.272.8 Giac [F]

$$\int \cos^m(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx = \int (a\sin(dx+c)+b\tan(dx+c))^2\cos(dx+c)^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate((a*sin(d*x + c) + b*tan(d*x + c))^2*cos(d*x + c)^m, x)
```

3.272.9 Mupad [F(-1)]

Timed out.

$$\int \cos^m(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$$
$$=\int \cos(c+d\,x)^m\,(a\,\sin(c+d\,x)+b\tan(c+d\,x))^2\,dx$$

input
```
int(cos(c + d*x)^m*(a*sin(c + d*x) + b*tan(c + d*x))^2,x)
```

output
```
int(cos(c + d*x)^m*(a*sin(c + d*x) + b*tan(c + d*x))^2, x)
```

3.273 $\int \cos^m(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx$

3.273.1 Optimal result

Integrand size = 26, antiderivative size = 39

$$\int \cos^m(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx=-\frac{b\cos^m(c+dx)}{dm}-\frac{a\cos^{1+m}(c+dx)}{d(1+m)}$$

output
```
-b*cos(d*x+c)^m/d/m-a*cos(d*x+c)^(1+m)/d/(1+m)
```

3.273.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \cos^m(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx=-\frac{\cos^m(c+dx)(b+bm+am\cos(c+dx))}{dm(1+m)}$$

input
```
Integrate[Cos[c + d*x]^m*(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

output
```
-((Cos[c + d*x]^m*(b + b*m + a*m*Cos[c + d*x]))/(d*m*(1 + m)))
```

3.273.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 4877, 27, 3042, 3045, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^m(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(c+dx)^m(a\sin(c+dx)+b\tan(c+dx))dx$$

$$\Big\downarrow \text{4877}$$

$$a\int \cos^m(c+dx)\sin(c+dx)dx+\int b\cos^{m-1}(c+dx)\sin(c+dx)dx$$

$$\Big\downarrow \text{27}$$

$$a\int \cos^m(c+dx)\sin(c+dx)dx+b\int \cos^{m-1}(c+dx)\sin(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \cos(c+dx)^m\sin(c+dx)dx+b\int \cos(c+dx)^{m-1}\sin(c+dx)dx$$

$$\Big\downarrow \text{3045}$$

$$-\frac{a\int \cos^m(c+dx)d\cos(c+dx)}{d}-\frac{b\int \cos^{m-1}(c+dx)d\cos(c+dx)}{d}$$

$$\Big\downarrow \text{15}$$

$$-\frac{a\cos^{m+1}(c+dx)}{d(m+1)}-\frac{b\cos^m(c+dx)}{dm}$$

input
```
Int[Cos[c + d*x]^m*(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

output
```
-((b*Cos[c + d*x]^m)/(d*m)) - (a*Cos[c + d*x]^(1 + m))/(d*(1 + m))
```

3.273.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 4877
```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Simp[d   Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[
c*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Intege
rQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

3.273.4 Maple [A] (verified)

Time = 7.97 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

method	result
parts	$-\frac{b\cos(dx+c)^{m}}{dm}-\frac{a\cos(dx+c)^{1+m}}{d(1+m)}$
default	$-\frac{b\,\mathrm{e}^{m\ln(\cos(dx+c))}}{dm}-\frac{a\cos(dx+c)\mathrm{e}^{m\ln(\cos(dx+c))}}{d(1+m)}$
risch	$-\frac{a\left(\frac{1}{2}\right)^{m}\left(\mathrm{e}^{i(dx+c)}\right)^{-m}\left(\mathrm{e}^{2i(dx+c)}+1\right)^{m}\mathrm{e}^{-\frac{i\left(m\pi\,\mathrm{csgn}(i\cos(dx+c))^{3}-m\pi\,\mathrm{csgn}(i\cos(dx+c))^{2}\,\mathrm{csgn}\left(i\mathrm{e}^{-i(dx+c)}\right)-m\pi\,\mathrm{csgn}(i\cos(dx+c))^{2}\,\mathrm{csgn}\left(i\right.}{2}}}{2d(1+m)}$

input
```
int(cos(d*x+c)^m*(sin(d*x+c)*a+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-b*cos(d*x+c)^m/d/m-a*cos(d*x+c)^(1+m)/d/(1+m)
```

3.273.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \cos^m(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx=-\frac{(am\cos{(dx+c)}+bm+b)\cos{(dx+c)}^m}{dm^2+dm}$$

input
```
integrate(cos(d*x+c)^m*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
-(a*m*cos(d*x + c) + b*m + b)*cos(d*x + c)^m/(d*m^2 + d*m)
```

3.273.6 Sympy [F]

$$\int \cos^m(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx$$
$$=\int (a\sin{(c+dx)}+b\tan{(c+dx)})\cos^m{(c+dx)}\,dx$$

input
```
integrate(cos(d*x+c)**m*(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

output
```
Integral((a*sin(c + d*x) + b*tan(c + d*x))*cos(c + d*x)**m, x)
```

3.273.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \cos^m(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx=-\frac{\frac{a\cos(dx+c)^{m+1}}{m+1}+\frac{b\cos(dx+c)^m}{m}}{d}$$

input
```
integrate(cos(d*x+c)^m*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
-(a*cos(d*x + c)^(m + 1)/(m + 1) + b*cos(d*x + c)^m/m)/d
```

3.273.8 Giac [F]

$$\int \cos^m(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx = \int (a\sin(dx+c)+b\tan(dx+c))\cos(dx+c)^m\,dx$$

input
```
integrate(cos(d*x+c)^m*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
integrate((a*sin(d*x + c) + b*tan(d*x + c))*cos(d*x + c)^m, x)
```

3.273.9 Mupad [B] (verification not implemented)

Time = 23.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \cos^m(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx = -\frac{\cos(c+d\,x)^m\,(b+b\,m+a\,m\,\cos(c+d\,x))}{d\,m\,(m+1)}$$

input
```
int(cos(c + d*x)^m*(a*sin(c + d*x) + b*tan(c + d*x)),x)
```

output
```
-(cos(c + d*x)^m*(b + b*m + a*m*cos(c + d*x)))/(d*m*(m + 1))
```

3.274 $\int \frac{\cos^m(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx$

3.274.1 Optimal result

Integrand size = 28, antiderivative size = 144

$$\begin{aligned}\int \frac{\cos^m(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx &= \frac{\cos^{2+m}(c+dx)\,\text{Hypergeometric2F1}(1,2+m,3+m,-\cos(c+dx))}{2(a-b)d(2+m)}\\ &\quad-\frac{\cos^{2+m}(c+dx)\,\text{Hypergeometric2F1}(1,2+m,3+m,\cos(c+dx))}{2(a+b)d(2+m)}\\ &\quad-\frac{a^2\cos^{2+m}(c+dx)\,\text{Hypergeometric2F1}\left(1,2+m,3+m,-\frac{a\cos(c+dx)}{b}\right)}{b\,(a^2-b^2)\,d(2+m)}\end{aligned}$$

output
```
1/2*cos(d*x+c)^(2+m)*hypergeom([1, 2+m],[3+m],-cos(d*x+c))/(a-b)/d/(2+m)-1
/2*cos(d*x+c)^(2+m)*hypergeom([1, 2+m],[3+m],cos(d*x+c))/(a+b)/d/(2+m)-a^2
*cos(d*x+c)^(2+m)*hypergeom([1, 2+m],[3+m],-a*cos(d*x+c)/b)/b/(a^2-b^2)/d/
(2+m)
```

3.274.2 Mathematica [A] (verified)

Time = 2.31 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.74

$$\int \frac{\cos^m(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx$$
$$= \frac{\cos^{2+m}(c+dx)\left(b(a+b)\,\text{Hypergeometric2F1}(1,2+m,3+m,-\cos(c+dx))-(a-b)b\,\text{Hypergeometric}\right.}{2(a-b)b(a+b)d(}$$

input
```
Integrate[Cos[c + d*x]^m/(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

output
```
(Cos[c + d*x]^(2 + m)*(b*(a + b)*Hypergeometric2F1[1, 2 + m, 3 + m, -Cos[c
 + d*x]] - (a - b)*b*Hypergeometric2F1[1, 2 + m, 3 + m, Cos[c + d*x]] - 2*
a^2*Hypergeometric2F1[1, 2 + m, 3 + m, -((a*Cos[c + d*x])/b)]))/(2*(a - b)
*b*(a + b)*d*(2 + m))
```

3.274.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 4897, 3042, 3316, 615, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^m(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{\cos(c+dx)^m}{a\sin(c+dx)+b\tan(c+dx)}dx$$
$$\Big\downarrow \ 4897$$
$$\int \frac{\csc(c+dx)\cos^{m+1}(c+dx)}{a\cos(c+dx)+b}dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{\left(-\sin\left(c+dx-\frac{\pi}{2}\right)\right)^{m+1}}{\cos\left(c+dx-\frac{\pi}{2}\right)\left(b-a\sin\left(c+dx-\frac{\pi}{2}\right)\right)}dx$$
$$\Big\downarrow \ 3316$$

$$-\frac{a\int \frac{\cos^{m+1}(c+dx)}{(b+a\cos(c+dx))(a^2-a^2\cos^2(c+dx))}d(a\cos(c+dx))}{d}$$

$$\Big\downarrow \; 615$$

$$-\frac{a\int \left(\frac{\cos^{m+1}(c+dx)}{2a(a+b)(a-a\cos(c+dx))}-\frac{\cos^{m+1}(c+dx)}{2a(a-b)(\cos(c+dx)a+a)}+\frac{\cos^{m+1}(c+dx)}{(a-b)(a+b)(b+a\cos(c+dx))}\right)d(a\cos(c+dx))}{d}$$

$$\Big\downarrow \; 2009$$

$$-\frac{a\left(\frac{a\cos^{m+2}(c+dx)\text{Hypergeometric2F1}\left(1,m+2,m+3,-\frac{a\cos(c+dx)}{b}\right)}{b(m+2)(a^2-b^2)}-\frac{\cos^{m+2}(c+dx)\text{Hypergeometric2F1}(1,m+2,m+3,-\cos(c+dx))}{2a(m+2)(a-b)}+\text{c}\right.}{d}$$

input
```
Int[Cos[c + d*x]^m/(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

output
```
-((a*(-1/2*(Cos[c + d*x]^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, -Cos[c
 + d*x]])/(a*(a - b)*(2 + m)) + (Cos[c + d*x]^(2 + m)*Hypergeometric2F1[1,
 2 + m, 3 + m, Cos[c + d*x]])/(2*a*(a + b)*(2 + m)) + (a*Cos[c + d*x]^(2 +
 m)*Hypergeometric2F1[1, 2 + m, 3 + m, -((a*Cos[c + d*x])/b)])/(b*(a^2 - b
^2)*(2 + m))))/d)
```

3.274.3.1 Defintions of rubi rules used

rule 615
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_),
 x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x], x]
 /; FreeQ[{a, b, c, d, e, m, n}, x] && ILtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3316
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b^p*
f)   Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)
/2] && NeQ[a^2 - b^2, 0]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.274.4 Maple [F]

$$\int \frac{\cos(dx+c)^m}{\sin(dx+c)a+b\tan(dx+c)}dx$$

input
```
int(cos(d*x+c)^m/(sin(d*x+c)*a+b*tan(d*x+c)),x)
```

output
```
int(cos(d*x+c)^m/(sin(d*x+c)*a+b*tan(d*x+c)),x)
```

3.274.5 Fricas [F]

$$\int \frac{\cos^m(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx = \int \frac{\cos(dx+c)^m}{a\sin(dx+c)+b\tan(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^m/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
```

output
```
integral(cos(d*x + c)^m/(a*sin(d*x + c) + b*tan(d*x + c)), x)
```

3.274.6 Sympy [F]

$$\int \frac{\cos^m(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx = \int \frac{\cos^m(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx$$

input
```
integrate(cos(d*x+c)**m/(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

output
```
Integral(cos(c + d*x)**m/(a*sin(c + d*x) + b*tan(c + d*x)), x)
```

3.274.7 Maxima [F]

$$\int \frac{\cos^m(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx = \int \frac{\cos(dx+c)^m}{a\sin(dx+c)+b\tan(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^m/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(cos(d*x + c)^m/(a*sin(d*x + c) + b*tan(d*x + c)), x)
```

3.274.8 Giac [F]

$$\int \frac{\cos^m(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx = \int \frac{\cos(dx+c)^m}{a\sin(dx+c)+b\tan(dx+c)}\,dx$$

input
```
integrate(cos(d*x+c)^m/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
```

output
```
integrate(cos(d*x + c)^m/(a*sin(d*x + c) + b*tan(d*x + c)), x)
```

3.274.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^m(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx = \int \frac{\cos(c+d\,x)\,\cos(c+d\,x)^m}{\sin(c+d\,x)\,\left(b+a\,\cos(c+d\,x)\right)}\,dx$$

input
```
int(cos(c + d*x)^m/(a*sin(c + d*x) + b*tan(c + d*x)),x)
```

output
```
int((cos(c + d*x)*cos(c + d*x)^m)/(sin(c + d*x)*(b + a*cos(c + d*x))), x)
```

3.275 $\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx$

3.275.1 Optimal result

Integrand size = 16, antiderivative size = 65

$$\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx=\frac{ab\,\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}-\frac{a\cos(x)}{a^2+b^2}+\frac{b\sin(x)}{a^2+b^2}$$

output
```
a*b*arctanh((b*cos(x)-a*sin(x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)-a*cos(x)/
(a^2+b^2)+b*sin(x)/(a^2+b^2)
```

3.275.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx=-\frac{2ab\,\text{arctanh}\left(\frac{-b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{-a\cos(x)+b\sin(x)}{a^2+b^2}$$

input
```
Integrate[(Cos[x]*Sin[x])/(a*Cos[x] + b*Sin[x]),x]
```

output
```
(-2*a*b*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2) + (-
(a*Cos[x]) + b*Sin[x])/(a^2 + b^2)
```

3.275.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3042, 3588, 3042, 3117, 3118, 3553, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)\cos(x)}{a\cos(x)+b\sin(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)\cos(x)}{a\cos(x)+b\sin(x)}dx$$

$$\Big\downarrow \text{3588}$$

$$\frac{a\int \sin(x)dx}{a^2+b^2}+\frac{b\int \cos(x)dx}{a^2+b^2}-\frac{ab\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \sin(x)dx}{a^2+b^2}+\frac{b\int \sin\left(x+\frac{\pi}{2}\right)dx}{a^2+b^2}-\frac{ab\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3117}$$

$$\frac{a\int \sin(x)dx}{a^2+b^2}-\frac{ab\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\sin(x)}{a^2+b^2}$$

$$\Big\downarrow \text{3118}$$

$$-\frac{ab\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\sin(x)}{a^2+b^2}-\frac{a\cos(x)}{a^2+b^2}$$

$$\Big\downarrow \text{3553}$$

$$\frac{ab\int \frac{1}{a^2+b^2-(b\cos(x)-a\sin(x))^2}d(b\cos(x)-a\sin(x))}{a^2+b^2}+\frac{b\sin(x)}{a^2+b^2}-\frac{a\cos(x)}{a^2+b^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{a\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{b\sin(x)}{a^2+b^2}-\frac{a\cos(x)}{a^2+b^2}$$

input
```
Int[(Cos[x]*Sin[x])/(a*Cos[x] + b*Sin[x]),x]
```

output
```
(a*b*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2) - (
a*Cos[x])/(a^2 + b^2) + (b*Sin[x])/(a^2 + b^2)
```

3.275.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3588
```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b
/(a^2 + b^2)   Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Simp[a/(a
^2 + b^2)   Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Simp[a*(b/(a^
2 + b^2))   Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x]
+ b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
&& IGtQ[m, 0] && IGtQ[n, 0]
```

3.275.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.26

method	result	size
default	$-\frac{4ab\,\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{(2a^2+2b^2)\sqrt{a^2+b^2}}+\frac{2b\tan\left(\frac{x}{2}\right)-2a}{(a^2+b^2)\left(1+\tan\left(\frac{x}{2}\right)^2\right)}$	82
risch	$-\frac{\mathrm{e}^{ix}}{2(-ib+a)}-\frac{\mathrm{e}^{-ix}}{2(ib+a)}+\frac{iba\ln\left(\mathrm{e}^{ix}+\frac{ib+a}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}\,(a^2+b^2)}-\frac{iba\ln\left(\mathrm{e}^{ix}-\frac{ib+a}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}\,(a^2+b^2)}$	141

input
```
int(cos(x)*sin(x)/(a*cos(x)+b*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
-4*a*b/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*x)-2*b)/(a^2
+b^2)^(1/2))+2/(a^2+b^2)*(b*tan(1/2*x)-a)/(1+tan(1/2*x)^2)
```

3.275.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. 2(61) = 122.

Time = 0.27 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.18

$$\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx$$
$$=\frac{\sqrt{a^2+b^2}ab\log\left(\frac{2\,ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2-2\,a^2-b^2-2\,\sqrt{a^2+b^2}(b\cos(x)-a\sin(x))}{2\,ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2+b^2}\right)-2\,(a^3+ab^2)\cos(x)+2\,(a^2b\cdots}{2\,(a^4+2\,a^2b^2+b^4)}$$

input
```
integrate(cos(x)*sin(x)/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

output
```
1/2*(sqrt(a^2 + b^2)*a*b*log((2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 -
 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos(x)*sin(
x) + (a^2 - b^2)*cos(x)^2 + b^2)) - 2*(a^3 + a*b^2)*cos(x) + 2*(a^2*b + b^
3)*sin(x))/(a^4 + 2*a^2*b^2 + b^4)
```

3.275.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 67.47 (sec) , antiderivative size = 699, normalized size of antiderivative = 10.75

$$\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx = \begin{cases} \tilde{\infty}\sin(x) \\ \frac{\sin(x)}{b} \\ \frac{i\sin^2(x)}{3ib\sin(x)+3b\cos(x)} + \frac{\sin(x)\cos(x)}{3ib\sin(x)+3b\cos(x)} - \frac{i\cos^2(x)}{3ib\sin(x)+3b\cos(x)} \\ -\frac{i\sin^2(x)}{-3ib\sin(x)+3b\cos(x)} + \frac{\sin(x)\cos(x)}{-3ib\sin(x)+3b\cos(x)} + \frac{i\cos^2(x)}{-3ib\sin(x)+3b\cos(x)} \\ \frac{ab\log\left(\tan\left(\frac{x}{2}\right)-\frac{b}{a}-\frac{\sqrt{a^2+b^2}}{a}\right)\tan^2\left(\frac{x}{2}\right)}{a^2\sqrt{a^2+b^2}\tan^2\left(\frac{x}{2}\right)+a^2\sqrt{a^2+b^2}+b^2\sqrt{a^2+b^2}\tan^2\left(\frac{x}{2}\right)+b^2\sqrt{a^2+b^2}} + \frac{ab\log\left(\tan\left(\frac{x}{2}\right)-\frac{b}{a}-\frac{\sqrt{a^2+b^2}}{a}\right)}{a^2\sqrt{a^2+b^2}\tan^2\left(\frac{x}{2}\right)+a^2\sqrt{a^2+b^2}+b^2\sqrt{a^2+b^2}\tan^2\left(\frac{x}{2}\right)+b^2\sqrt{a^2+b^2}} - \end{cases}$$

input
```
integrate(cos(x)*sin(x)/(a*cos(x)+b*sin(x)),x)
```

output
```
Piecewise((zoo*sin(x), Eq(a, 0) & Eq(b, 0)), (sin(x)/b, Eq(a, 0)), (I*sin(
x)**2/(3*I*b*sin(x) + 3*b*cos(x)) + sin(x)*cos(x)/(3*I*b*sin(x) + 3*b*cos(
x)) - I*cos(x)**2/(3*I*b*sin(x) + 3*b*cos(x)), Eq(a, -I*b)), (-I*sin(x)**2
/(-3*I*b*sin(x) + 3*b*cos(x)) + sin(x)*cos(x)/(-3*I*b*sin(x) + 3*b*cos(x))
 + I*cos(x)**2/(-3*I*b*sin(x) + 3*b*cos(x)), Eq(a, I*b)), (a*b*log(tan(x/2
) - b/a - sqrt(a**2 + b**2)/a)*tan(x/2)**2/(a**2*sqrt(a**2 + b**2)*tan(x/2
)**2 + a**2*sqrt(a**2 + b**2) + b**2*sqrt(a**2 + b**2)*tan(x/2)**2 + b**2*
sqrt(a**2 + b**2)) + a*b*log(tan(x/2) - b/a - sqrt(a**2 + b**2)/a)/(a**2*s
qrt(a**2 + b**2)*tan(x/2)**2 + a**2*sqrt(a**2 + b**2) + b**2*sqrt(a**2 + b
**2)*tan(x/2)**2 + b**2*sqrt(a**2 + b**2)) - a*b*log(tan(x/2) - b/a + sqrt
(a**2 + b**2)/a)*tan(x/2)**2/(a**2*sqrt(a**2 + b**2)*tan(x/2)**2 + a**2*sq
rt(a**2 + b**2) + b**2*sqrt(a**2 + b**2)*tan(x/2)**2 + b**2*sqrt(a**2 + b*
*2)) - a*b*log(tan(x/2) - b/a + sqrt(a**2 + b**2)/a)/(a**2*sqrt(a**2 + b**
2)*tan(x/2)**2 + a**2*sqrt(a**2 + b**2) + b**2*sqrt(a**2 + b**2)*tan(x/2)*
*2 + b**2*sqrt(a**2 + b**2)) - 2*a*sqrt(a**2 + b**2)/(a**2*sqrt(a**2 + b**
2)*tan(x/2)**2 + a**2*sqrt(a**2 + b**2) + b**2*sqrt(a**2 + b**2)*tan(x/2)*
*2 + b**2*sqrt(a**2 + b**2)) + 2*b*sqrt(a**2 + b**2)*tan(x/2)/(a**2*sqrt(a
**2 + b**2)*tan(x/2)**2 + a**2*sqrt(a**2 + b**2) + b**2*sqrt(a**2 + b**2)*
tan(x/2)**2 + b**2*sqrt(a**2 + b**2)), True))
```

3.275.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.62

$$\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{ab\log\left(\frac{b-\frac{a\sin(x)}{\cos(x)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(x)}{\cos(x)+1}-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}} - \frac{2\left(a-\frac{b\sin(x)}{\cos(x)+1}\right)}{a^2+b^2+\frac{(a^2+b^2)\sin(x)^2}{(\cos(x)+1)^2}}$$

input
```
integrate(cos(x)*sin(x)/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

output
```
a*b*log((b - a*sin(x)/(cos(x) + 1) + sqrt(a^2 + b^2))/(b - a*sin(x)/(cos(x
) + 1) - sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(a - b*sin(x)/(cos(x) + 1
))/(a^2 + b^2 + (a^2 + b^2)*sin(x)^2/(cos(x) + 1)^2)
```

3.275.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.45

$$\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{ab\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^2+b^2)^{\frac{3}{2}}} + \frac{2\left(b\tan\left(\frac{1}{2}\,x\right)-a\right)}{(a^2+b^2)\left(\tan\left(\frac{1}{2}\,x\right)^2+1\right)}$$

input
```
integrate(cos(x)*sin(x)/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

output
```
a*b*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*x) -
 2*b + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) + 2*(b*tan(1/2*x) - a)/((a^2
+ b^2)*(tan(1/2*x)^2 + 1))
```

3.275.9 Mupad [B] (verification not implemented)

Time = 22.41 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.43

$$\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{2\,a\,b\,\mathrm{atanh}\left(\frac{2\,a^2\,b+2\,b^3-2\,a\tan\left(\frac{x}{2}\right)\,(a^2+b^2)}{2\,(a^2+b^2)^{3/2}}\right)}{(a^2+b^2)^{3/2}} - \frac{\frac{2\,a}{a^2+b^2}-\frac{2\,b\tan\left(\frac{x}{2}\right)}{a^2+b^2}}{\tan\left(\frac{x}{2}\right)^2+1}$$

input
```
int((cos(x)*sin(x))/(a*cos(x) + b*sin(x)),x)
```

output
```
(2*a*b*atanh((2*a^2*b + 2*b^3 - 2*a*tan(x/2)*(a^2 + b^2))/(2*(a^2 + b^2)^(
3/2))))/(a^2 + b^2)^(3/2) - ((2*a)/(a^2 + b^2) - (2*b*tan(x/2))/(a^2 + b^2
))/(tan(x/2)^2 + 1)
```

3.275. $\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx$

3.276 $\int \frac{\cos(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx$

3.276.1 Optimal result

Integrand size = 18, antiderivative size = 92

$$\int \frac{\cos(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx = -\frac{ab^2x}{(a^2+b^2)^2}+\frac{ax}{2(a^2+b^2)}+\frac{a^2b\log(a\cos(x)+b\sin(x))}{(a^2+b^2)^2} - \frac{a\cos(x)\sin(x)}{2(a^2+b^2)}+\frac{b\sin^2(x)}{2(a^2+b^2)}$$

output
```
-a*b^2*x/(a^2+b^2)^2+1/2*a*x/(a^2+b^2)+a^2*b*ln(a*cos(x)+b*sin(x))/(a^2+b^
2)^2-1/2*a*cos(x)*sin(x)/(a^2+b^2)+1/2*b*sin(x)^2/(a^2+b^2)
```

3.276.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.66

$$\int \frac{\cos(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx = -\frac{-2a^3x-6ia^2bx+6ab^2x+2ib^3x-2ib(-3a^2+b^2)\arctan(\tan(x))+2b(a^2+b^2)\cos(2x)-2(a^2+b^2)(}{}$$

input
```
Integrate[(Cos[x]*Sin[x]^2)/(a*Cos[x] + b*Sin[x]),x]
```

output
```
-1/8*(-2*a^3*x - (6*I)*a^2*b*x + 6*a*b^2*x + (2*I)*b^3*x - (2*I)*b*(-3*a^2
 + b^2)*ArcTan[Tan[x]] + 2*b*(a^2 + b^2)*Cos[2*x] - 2*(a^2 + b^2)*(a*x + b
*Log[a*Cos[x] + b*Sin[x]]) - 3*a^2*b*Log[(a*Cos[x] + b*Sin[x])^2] + b^3*Lo
g[(a*Cos[x] + b*Sin[x])^2] + 2*a^3*Sin[2*x] + 2*a*b^2*Sin[2*x])/(a^2 + b^2
)^2
```

3.276.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {3042, 3588, 3042, 3044, 15, 3115, 24, 3576, 3042, 3612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(x)\cos(x)}{a\cos(x)+b\sin(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)^2\cos(x)}{a\cos(x)+b\sin(x)}dx$$

$$\Big\downarrow \text{3588}$$

$$\frac{a\int \sin^2(x)dx}{a^2+b^2}+\frac{b\int \cos(x)\sin(x)dx}{a^2+b^2}-\frac{ab\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \sin(x)^2dx}{a^2+b^2}+\frac{b\int \cos(x)\sin(x)dx}{a^2+b^2}-\frac{ab\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3044}$$

$$\frac{a\int \sin(x)^2dx}{a^2+b^2}+\frac{b\int \sin(x)d\sin(x)}{a^2+b^2}-\frac{ab\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{15}$$

$$\frac{a\int \sin(x)^2dx}{a^2+b^2}-\frac{ab\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\sin^2(x)}{2(a^2+b^2)}$$

$$\Big\downarrow \text{3115}$$

$$\frac{a\left(\frac{\int 1dx}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}-\frac{ab\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\sin^2(x)}{2(a^2+b^2)}$$

$$\Big\downarrow \ 24$$

$$-\frac{ab\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\sin^2(x)}{2\left(a^2+b^2\right)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}$$

$$\Big\downarrow \ 3576$$

$$-\frac{ab\left(\frac{bx}{a^2+b^2}-\frac{a\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\sin^2(x)}{2\left(a^2+b^2\right)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}$$

$$\Big\downarrow \ 3042$$

$$-\frac{ab\left(\frac{bx}{a^2+b^2}-\frac{a\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\sin^2(x)}{2\left(a^2+b^2\right)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}$$

$$\Big\downarrow \ 3612$$

$$\frac{b\sin^2(x)}{2\left(a^2+b^2\right)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}-\frac{ab\left(\frac{bx}{a^2+b^2}-\frac{a\log(a\cos(x)+b\sin(x))}{a^2+b^2}\right)}{a^2+b^2}$$

input
```
Int[(Cos[x]*Sin[x]^2)/(a*Cos[x] + b*Sin[x]),x]
```

output
```
-((a*b*((b*x)/(a^2 + b^2) - (a*Log[a*Cos[x] + b*Sin[x]])/(a^2 + b^2)))/(a^
2 + b^2)) + (b*Sin[x]^2)/(2*(a^2 + b^2)) + (a*(x/2 - (Cos[x]*Sin[x])/2))/(
a^2 + b^2)
```

3.276.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3576
```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(x/(a^2 + b^2)), x] - Simp[a/(a^2 + b
^2)   Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x
]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3588
```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b
/(a^2 + b^2)   Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Simp[a/(a
^2 + b^2)   Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Simp[a*(b/(a^
2 + b^2))   Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x]
+ b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
&& IGtQ[m, 0] && IGtQ[n, 0]
```

rule 3612
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

3.276. $\int \frac{\cos(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx$

3.276.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.07

method	result
default	$\frac{a^{2} b \ln\left(a+b\tan\left(x\right)\right)}{\left(a^{2}+b^{2}\right)^{2}}+\frac{\frac{\left(-\frac{1}{2}a^{3}-\frac{1}{2}a\,b^{2}\right)\tan\left(x\right)-\frac{a^{2}b}{2}-\frac{b^{3}}{2}}{1+\tan\left(x\right)^{2}}+\frac{a\left(-ab\ln\left(1+\tan\left(x\right)^{2}\right)+\left(a^{2}-b^{2}\right)\arctan\left(\tan\left(x\right)\right)\right)}{2}}{\left(a^{2}+b^{2}\right)^{2}}$
parallelrisch	$\frac{-a^{2}b\cos\left(2x\right)-b^{3}\cos\left(2x\right)-a^{3}\sin\left(2x\right)-a\,b^{2}\sin\left(2x\right)-4a^{2}b\ln\left(\frac{1}{\cos\left(x\right)+1}\right)+4a^{2}b\ln\left(\frac{-a\cos\left(x\right)-b\sin\left(x\right)}{\cos\left(x\right)+1}\right)+2a^{3}x-2a\,b^{2}x+a^{2}b+b^{3}}{4\left(a^{2}+b^{2}\right)^{2}}$
risch	$-\frac{ax}{2\left(2iba-a^{2}+b^{2}\right)}+\frac{i\,\mathrm{e}^{2ix}}{-8ib+8a}-\frac{i\,\mathrm{e}^{-2ix}}{8\left(ib+a\right)}-\frac{2ia^{2}bx}{a^{4}+2a^{2}b^{2}+b^{4}}+\frac{a^{2}b\ln\left(\mathrm{e}^{2ix}-\frac{ib+a}{ib-a}\right)}{a^{4}+2a^{2}b^{2}+b^{4}}$
norman	$\frac{\frac{a\tan\left(\frac{x}{2}\right)^{5}}{a^{2}+b^{2}}+\frac{2b\tan\left(\frac{x}{2}\right)^{2}}{a^{2}+b^{2}}+\frac{2b\tan\left(\frac{x}{2}\right)^{4}}{a^{2}+b^{2}}-\frac{a\tan\left(\frac{x}{2}\right)}{a^{2}+b^{2}}+\frac{a\left(a^{2}-b^{2}\right)x}{2a^{4}+4a^{2}b^{2}+2b^{4}}+\frac{3a\left(a^{2}-b^{2}\right)x\tan\left(\frac{x}{2}\right)^{2}}{2\left(a^{4}+2a^{2}b^{2}+b^{4}\right)}+\frac{3a\left(a^{2}-b^{2}\right)x\tan\left(\frac{x}{2}\right)^{4}}{2\left(a^{4}+2a^{2}b^{2}+b^{4}\right)}+\frac{a\left(a^{2}-b^{2}\right)x\tan}{2a^{4}+4a^{2}b^{2}+}}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)^{3}}$

input
```
int(cos(x)*sin(x)^2/(a*cos(x)+b*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
a^2*b/(a^2+b^2)^2*ln(a+b*tan(x))+1/(a^2+b^2)^2*(((-1/2*a^3-1/2*a*b^2)*tan(
x)-1/2*a^2*b-1/2*b^3)/(1+tan(x)^2)+1/2*a*(-a*b*ln(1+tan(x)^2)+(a^2-b^2)*ar
ctan(tan(x))))
```

3.276.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.02

$$\int \frac{\cos(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx$$
$$=\frac{a^2b\log\left(2\,ab\cos\left(x\right)\sin\left(x\right)+\left(a^2-b^2\right)\cos\left(x\right)^2+b^2\right)-\left(a^2b+b^3\right)\cos\left(x\right)^2-\left(a^3+ab^2\right)\cos\left(x\right)\sin\left(x\right)+(a}{2\left(a^4+2\,a^2b^2+b^4\right)}$$

input
```
integrate(cos(x)*sin(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

output
```
1/2*(a^2*b*log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) - (a^2*b
+ b^3)*cos(x)^2 - (a^3 + a*b^2)*cos(x)*sin(x) + (a^3 - a*b^2)*x)/(a^4 + 2*
a^2*b^2 + b^4)
```

3.276.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx = \text{Timed out}$$

input
```
integrate(cos(x)*sin(x)**2/(a*cos(x)+b*sin(x)),x)
```

output
```
Timed out
```

3.276.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(86) = 172$.

Time = 0.31 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.29

$$\int \frac{\cos(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{a^2b\log\left(-a-\frac{2\,b\sin(x)}{\cos(x)+1}+\frac{a\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^4+2\,a^2b^2+b^4} - \frac{a^2b\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{a^4+2\,a^2b^2+b^4} + \frac{(a^3-ab^2)\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^4+2\,a^2b^2+b^4} - \frac{\frac{a\sin(x)}{\cos(x)+1}-\frac{2\,b\sin(x)^2}{(\cos(x)+1)^2}-\frac{a\sin(x)^3}{(\cos(x)+1)^3}}{a^2+b^2+\frac{2\,(a^2+b^2)\sin(x)^2}{(\cos(x)+1)^2}+\frac{(a^2+b^2)\sin(x)^4}{(\cos(x)+1)^4}}$$

input
```
integrate(cos(x)*sin(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

output
```
a^2*b*log(-a - 2*b*sin(x)/(cos(x) + 1) + a*sin(x)^2/(cos(x) + 1)^2)/(a^4 +
 2*a^2*b^2 + b^4) - a^2*b*log(sin(x)^2/(cos(x) + 1)^2 + 1)/(a^4 + 2*a^2*b^
2 + b^4) + (a^3 - a*b^2)*arctan(sin(x)/(cos(x) + 1))/(a^4 + 2*a^2*b^2 + b^
4) - (a*sin(x)/(cos(x) + 1) - 2*b*sin(x)^2/(cos(x) + 1)^2 - a*sin(x)^3/(co
s(x) + 1)^3)/(a^2 + b^2 + 2*(a^2 + b^2)*sin(x)^2/(cos(x) + 1)^2 + (a^2 + b
^2)*sin(x)^4/(cos(x) + 1)^4)
```

3.276.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.65

$$\int \frac{\cos(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{a^2b^2\log\left(|b\tan\left(x\right)+a|\right)}{a^4b+2\,a^2b^3+b^5} - \frac{a^2b\log\left(\tan\left(x\right)^2+1\right)}{2\,(a^4+2\,a^2b^2+b^4)} + \frac{(a^3-ab^2)x}{2\,(a^4+2\,a^2b^2+b^4)} + \frac{a^2b\tan\left(x\right)^2-a^3\tan\left(x\right)-ab^2\tan\left(x\right)-b^3}{2\,(a^4+2\,a^2b^2+b^4)\left(\tan\left(x\right)^2+1\right)}$$

input
```
integrate(cos(x)*sin(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

output
```
a^2*b^2*log(abs(b*tan(x) + a))/(a^4*b + 2*a^2*b^3 + b^5) - 1/2*a^2*b*log(t
an(x)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + 1/2*(a^3 - a*b^2)*x/(a^4 + 2*a^2*b^
2 + b^4) + 1/2*(a^2*b*tan(x)^2 - a^3*tan(x) - a*b^2*tan(x) - b^3)/((a^4 +
2*a^2*b^2 + b^4)*(tan(x)^2 + 1))
```

3.276.9 Mupad [B] (verification not implemented)

Time = 30.05 (sec) , antiderivative size = 3401, normalized size of antiderivative = 36.97

$$\int \frac{\cos(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx = \text{Too large to display}$$

input
```
int((cos(x)*sin(x)^2)/(a*cos(x) + b*sin(x)),x)
```

output
```
((a*tan(x/2)^3)/(a^2 + b^2) - (a*tan(x/2))/(a^2 + b^2) + (2*b*tan(x/2)^2)/
(a^2 + b^2))/(2*tan(x/2)^2 + tan(x/2)^4 + 1) + (a^2*b*log(a + 2*b*tan(x/2)
 - a*tan(x/2)^2))/(a^4 + b^4 + 2*a^2*b^2) - (4*a^2*b*log(1/(cos(x) + 1)))/
(4*a^4 + 4*b^4 + 8*a^2*b^2) - (a*atan((tan(x/2)*((((4*a^2*b*((a*(a + b)*((
8*(12*a^9*b + 12*a^5*b^5 + 24*a^7*b^3))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2
) - (32*a^2*b*(12*a*b^10 + 48*a^3*b^8 + 72*a^5*b^6 + 48*a^7*b^4 + 12*a^9*b
^2))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))*(a
 - b))/(2*(a^4 + b^4 + 2*a^2*b^2)) - (16*a^3*b*(a + b)*(a - b)*(12*a*b^10
+ 48*a^3*b^8 + 72*a^5*b^6 + 48*a^7*b^4 + 12*a^9*b^2))/((4*a^4 + 4*b^4 + 8*
a^2*b^2)*(a^4 + b^4 + 2*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(4
*a^4 + 4*b^4 + 8*a^2*b^2) - (a*((8*(a^9 + 2*a^3*b^6 - 7*a^5*b^4 - 8*a^7*b^
2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (4*a^2*b*((8*(12*a^9*b + 12*a^5*
b^5 + 24*a^7*b^3))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (32*a^2*b*(12*a*b
^10 + 48*a^3*b^8 + 72*a^5*b^6 + 48*a^7*b^4 + 12*a^9*b^2))/((4*a^4 + 4*b^4
+ 8*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(4*a^4 + 4*b^4 + 8*a^2
*b^2))*(a + b)*(a - b))/(2*(a^4 + b^4 + 2*a^2*b^2)) + (a^3*(a + b)^3*(a -
b)^3*(12*a*b^10 + 48*a^3*b^8 + 72*a^5*b^6 + 48*a^7*b^4 + 12*a^9*b^2))/((a^
4 + b^4 + 2*a^2*b^2)^3*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))*(a^6 - b^6 +
35*a^2*b^4 - 35*a^4*b^2))/(a^6 + b^6 + 15*a^2*b^4 + 15*a^4*b^2)^2 - (2*a*b
*(5*a^4 + 5*b^4 - 26*a^2*b^2)*((8*(a^7*b + 2*a^5*b^3))/(a^6 + b^6 + 3*a...
```

3.276. $\int \frac{\cos(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx$

3.277 $\int \frac{\cos(x)\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx$

3.277.1 Optimal result

Integrand size = 18, antiderivative size = 122

$$\int \frac{\cos(x)\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{a^3b\,\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} + \frac{ab^2\cos(x)}{(a^2+b^2)^2} - \frac{a\cos(x)}{a^2+b^2} + \frac{a\cos^3(x)}{3(a^2+b^2)} + \frac{a^2b\sin(x)}{(a^2+b^2)^2} + \frac{b\sin^3(x)}{3(a^2+b^2)}$$

output
```
a^3*b*arctanh((b*cos(x)-a*sin(x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2)+a*b^2*c
os(x)/(a^2+b^2)^2-a*cos(x)/(a^2+b^2)+1/3*a*cos(x)^3/(a^2+b^2)+a^2*b*sin(x)
/(a^2+b^2)^2+1/3*b*sin(x)^3/(a^2+b^2)
```

3.277.2 Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.93

$$\int \frac{\cos(x)\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx = -\frac{2a^3b\,\text{arctanh}\left(\frac{-b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} + \frac{(-9a^3+3ab^2)\cos(x)+a(a^2+b^2)\cos(3x)-2b(-7a^2-b^2+(a^2+b^2)\cos(2x))\sin(x)}{12(a^2+b^2)^2}$$

input
```
Integrate[(Cos[x]*Sin[x]^3)/(a*Cos[x] + b*Sin[x]),x]
```

output
```
(-2*a^3*b*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) +
((-9*a^3 + 3*a*b^2)*Cos[x] + a*(a^2 + b^2)*Cos[3*x] - 2*b*(-7*a^2 - b^2 +
(a^2 + b^2)*Cos[2*x])*Sin[x])/(12*(a^2 + b^2)^2)
```

3.277.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.01, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 3588, 3042, 3044, 15, 3113, 2009, 3578, 3042, 3118, 3553, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(x)\cos(x)}{a\cos(x)+b\sin(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)^3\cos(x)}{a\cos(x)+b\sin(x)}dx$$

$$\Big\downarrow \text{3588}$$

$$\frac{a\int \sin^3(x)dx}{a^2+b^2}+\frac{b\int \cos(x)\sin^2(x)dx}{a^2+b^2}-\frac{ab\int \frac{\sin^2(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \sin(x)^3dx}{a^2+b^2}+\frac{b\int \cos(x)\sin(x)^2dx}{a^2+b^2}-\frac{ab\int \frac{\sin(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3044}$$

$$\frac{b\int \sin^2(x)d\sin(x)}{a^2+b^2}+\frac{a\int \sin(x)^3dx}{a^2+b^2}-\frac{ab\int \frac{\sin(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{15}$$

$$\frac{a\int \sin(x)^3dx}{a^2+b^2}-\frac{ab\int \frac{\sin(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\sin^3(x)}{3(a^2+b^2)}$$

$$\Big\downarrow \text{3113}$$

$$-\frac{a\int \left(1-\cos^2(x)\right)d\cos(x)}{a^2+b^2}-\frac{ab\int \frac{\sin(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\sin^3(x)}{3\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{ab\int \frac{\sin(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\sin^3(x)}{3\left(a^2+b^2\right)}-\frac{a\left(\cos(x)-\frac{\cos^3(x)}{3}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{ 3578}$$

$$-\frac{ab\left(\frac{b\int \sin(x)dx}{a^2+b^2}+\frac{a^2\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}-\frac{a\sin(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\sin^3(x)}{3\left(a^2+b^2\right)}-\frac{a\left(\cos(x)-\frac{\cos^3(x)}{3}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{ab\left(\frac{b\int \sin(x)dx}{a^2+b^2}+\frac{a^2\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}-\frac{a\sin(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\sin^3(x)}{3\left(a^2+b^2\right)}-\frac{a\left(\cos(x)-\frac{\cos^3(x)}{3}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{ 3118}$$

$$-\frac{ab\left(\frac{a^2\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}-\frac{a\sin(x)}{a^2+b^2}-\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\sin^3(x)}{3\left(a^2+b^2\right)}-\frac{a\left(\cos(x)-\frac{\cos^3(x)}{3}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{ 3553}$$

$$-\frac{ab\left(-\frac{a^2\int \frac{1}{a^2+b^2-(b\cos(x)-a\sin(x))^2}d(b\cos(x)-a\sin(x))}{a^2+b^2}-\frac{a\sin(x)}{a^2+b^2}-\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\sin^3(x)}{3\left(a^2+b^2\right)}-\frac{a\left(\cos(x)-\frac{\cos^3(x)}{3}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{ab\left(-\frac{a^2\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}-\frac{a\sin(x)}{a^2+b^2}-\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\sin^3(x)}{3\left(a^2+b^2\right)}-\frac{a\left(\cos(x)-\frac{\cos^3(x)}{3}\right)}{a^2+b^2}$$

input
```
Int[(Cos[x]*Sin[x]^3)/(a*Cos[x] + b*Sin[x]),x]
```

output
```
-((a*(Cos[x] - Cos[x]^3/3))/(a^2 + b^2)) + (b*Sin[x]^3)/(3*(a^2 + b^2)) -
(a*b*(-((a^2*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(
3/2)) - (b*Cos[x])/(a^2 + b^2) - (a*Sin[x])/(a^2 + b^2)))/(a^2 + b^2)
```

3.277. $\int \frac{\cos(x)\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx$

3.277.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

3.277. $\int \frac{\cos(x)\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx$

rule 3578
```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-a)*(Sin[c + d*x]^(m - 1)/(d*(a^2
 + b^2)*(m - 1))), x] + (Simp[a^2/(a^2 + b^2)   Int[Sin[c + d*x]^(m - 2)/(a
*Cos[c + d*x] + b*Sin[c + d*x]), x], x] + Simp[b/(a^2 + b^2)   Int[Sin[c +
d*x]^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ
[m, 1]
```

rule 3588
```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b
/(a^2 + b^2)   Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Simp[a/(a
^2 + b^2)   Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Simp[a*(b/(a^
2 + b^2))   Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x]
+ b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
&& IGtQ[m, 0] && IGtQ[n, 0]
```

3.277.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.34

method	result
default	$-\frac{16a^3b\,\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{(8a^4+16a^2b^2+8b^4)\sqrt{a^2+b^2}}+\frac{2a^2b\tan\left(\frac{x}{2}\right)^5+2a\,b^2\tan\left(\frac{x}{2}\right)^4+2\left(\frac{10}{3}a^2b+\frac{4}{3}b^3\right)\tan\left(\frac{x}{2}\right)^3-4\tan\left(\frac{x}{2}\right)^2a^3+2a^2b\tan\left(\frac{x}{2}\right)-\frac{4a^3}{3}+2}{(a^4+2a^2b^2+b^4)\left(1+\tan\left(\frac{x}{2}\right)^2\right)^3}$
risch	$\frac{ie^{ix}b}{-16iba+8a^2-8b^2}-\frac{3\,e^{ix}a}{8(-2iba+a^2-b^2)}-\frac{ie^{-ix}b}{8(ib+a)^2}-\frac{3\,e^{-ix}a}{8(ib+a)^2}-\frac{ib\,a^3\ln\left(e^{ix}-\frac{ib+a}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}\,(a^2+b^2)^2}+\frac{ib\,a^3\ln\left(e^{ix}+\frac{ib+a}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}\,(a^2+b^2)^2}-$

input
```
int(cos(x)*sin(x)^3/(a*cos(x)+b*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
-16*a^3*b/(8*a^4+16*a^2*b^2+8*b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/
2*x)-2*b)/(a^2+b^2)^(1/2))+2/(a^4+2*a^2*b^2+b^4)*(a^2*b*tan(1/2*x)^5+a*b^2
*tan(1/2*x)^4+(10/3*a^2*b+4/3*b^3)*tan(1/2*x)^3-2*tan(1/2*x)^2*a^3+a^2*b*t
an(1/2*x)-2/3*a^3+1/3*a*b^2)/(1+tan(1/2*x)^2)^3
```

3.277.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.72

$$\int \frac{\cos(x)\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx$$
$$= \frac{3\sqrt{a^2+b^2}a^3b\log\left(\frac{2\,ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2-2\,a^2-b^2-2\,\sqrt{a^2+b^2}(b\cos(x)-a\sin(x))}{2\,ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2+b^2}\right)+2\,(a^5+2\,a^3b^2+ab^4)\cos(x}{6\,(a^6+3\,a^4b^2+3\,a^2b^4}$$

input
```
integrate(cos(x)*sin(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

output
```
1/6*(3*sqrt(a^2 + b^2)*a^3*b*log((2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)
^2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos(x)*
sin(x) + (a^2 - b^2)*cos(x)^2 + b^2)) + 2*(a^5 + 2*a^3*b^2 + a*b^4)*cos(x)
^3 - 6*(a^5 + a^3*b^2)*cos(x) + 2*(4*a^4*b + 5*a^2*b^3 + b^5 - (a^4*b + 2*
a^2*b^3 + b^5)*cos(x)^2)*sin(x))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
```

3.277.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(x)\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx = \text{Timed out}$$

input
```
integrate(cos(x)*sin(x)**3/(a*cos(x)+b*sin(x)),x)
```

output
```
Timed out
```

3.277.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(114) = 228$.

Time = 0.32 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.28

$$\int \frac{\cos(x)\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx$$
$$=\frac{a^3b\log\left(\frac{b-\frac{a\sin(x)}{\cos(x)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(x)}{\cos(x)+1}-\sqrt{a^2+b^2}}\right)}{(a^4+2a^2b^2+b^4)\sqrt{a^2+b^2}}-\frac{2\left(2a^3-ab^2-\frac{3a^2b\sin(x)}{\cos(x)+1}+\frac{6a^3\sin(x)^2}{(\cos(x)+1)^2}-\frac{3ab^2\sin(x)^4}{(\cos(x)+1)^4}-\frac{3a^2b\sin(x)^5}{(\cos(x)+1)^5}-\frac{2(5a^2b+2b^3)\sin(x)^3}{(\cos(x)+1)^3}\right)}{3\left(a^4+2a^2b^2+b^4+\frac{3(a^4+2a^2b^2+b^4)\sin(x)^2}{(\cos(x)+1)^2}+\frac{3(a^4+2a^2b^2+b^4)\sin(x)^4}{(\cos(x)+1)^4}+\frac{(a^4+2a^2b^2+b^4)\sin(x)^6}{(\cos(x)+1)^6}\right)}$$

input
```
integrate(cos(x)*sin(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

output
```
a^3*b*log((b - a*sin(x)/(cos(x) + 1) + sqrt(a^2 + b^2))/(b - a*sin(x)/(cos
(x) + 1) - sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) - 2
/3*(2*a^3 - a*b^2 - 3*a^2*b*sin(x)/(cos(x) + 1) + 6*a^3*sin(x)^2/(cos(x) +
 1)^2 - 3*a*b^2*sin(x)^4/(cos(x) + 1)^4 - 3*a^2*b*sin(x)^5/(cos(x) + 1)^5
- 2*(5*a^2*b + 2*b^3)*sin(x)^3/(cos(x) + 1)^3)/(a^4 + 2*a^2*b^2 + b^4 + 3*
(a^4 + 2*a^2*b^2 + b^4)*sin(x)^2/(cos(x) + 1)^2 + 3*(a^4 + 2*a^2*b^2 + b^4
)*sin(x)^4/(cos(x) + 1)^4 + (a^4 + 2*a^2*b^2 + b^4)*sin(x)^6/(cos(x) + 1)^
6)
```

3.277.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.56

$$\int \frac{\cos(x)\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx=\frac{a^3b\log\left(\frac{\left|2a\tan\left(\frac{1}{2}x\right)-2b-2\sqrt{a^2+b^2}\right|}{\left|2a\tan\left(\frac{1}{2}x\right)-2b+2\sqrt{a^2+b^2}\right|}\right)}{(a^4+2a^2b^2+b^4)\sqrt{a^2+b^2}}$$
$$+\frac{2\left(3a^2b\tan\left(\frac{1}{2}x\right)^5+3ab^2\tan\left(\frac{1}{2}x\right)^4+10a^2b\tan\left(\frac{1}{2}x\right)^3+4b^3\tan\left(\frac{1}{2}x\right)^3-6a^3\tan\left(\frac{1}{2}x\right)^2+3a^2b\tan\left(\right.\right.}{3(a^4+2a^2b^2+b^4)\left(\tan\left(\frac{1}{2}x\right)^2+1\right)^3}$$

input
```
integrate(cos(x)*sin(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

output
```
a^3*b*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*x)
 - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) + 2
/3*(3*a^2*b*tan(1/2*x)^5 + 3*a*b^2*tan(1/2*x)^4 + 10*a^2*b*tan(1/2*x)^3 +
4*b^3*tan(1/2*x)^3 - 6*a^3*tan(1/2*x)^2 + 3*a^2*b*tan(1/2*x) - 2*a^3 + a*b
^2)/((a^4 + 2*a^2*b^2 + b^4)*(tan(1/2*x)^2 + 1)^3)
```

3.277.9 Mupad [B] (verification not implemented)

Time = 23.40 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.34

$$\int \frac{\cos(x)\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{\frac{2\,(a\,b^2-2\,a^3)}{3\,(a^4+2\,a^2\,b^2+b^4)}+\frac{4\tan\left(\frac{x}{2}\right)^3\,(5\,a^2\,b+2\,b^3)}{3\,(a^4+2\,a^2\,b^2+b^4)}-\frac{4\,a^3\tan\left(\frac{x}{2}\right)^2}{a^4+2\,a^2\,b^2+b^4}+\frac{2\,a\,b^2\tan\left(\frac{x}{2}\right)^4}{a^4+2\,a^2\,b^2+b^4}+\frac{2\,a^2\,b\tan\left(\frac{x}{2}\right)^5}{a^4+2\,a^2\,b^2+b^4}+\frac{2\,a^2\,b\tan\left(\frac{x}{2}\right)}{a^4+2\,a^2\,b^2+b^4}}{\tan\left(\frac{x}{2}\right)^6+3\tan\left(\frac{x}{2}\right)^4+3\tan\left(\frac{x}{2}\right)^2+1} + \frac{2\,a^3\,b\,\mathrm{atanh}\left(\frac{2\,a^4\,b+2\,b^5+4\,a^2\,b^3-2\,a\tan\left(\frac{x}{2}\right)\,(a^4+2\,a^2\,b^2+b^4)}{2\,(a^2+b^2)^{5/2}}\right)}{(a^2+b^2)^{5/2}}$$

input
```
int((cos(x)*sin(x)^3)/(a*cos(x) + b*sin(x)),x)
```

output
```
((2*(a*b^2 - 2*a^3))/(3*(a^4 + b^4 + 2*a^2*b^2)) + (4*tan(x/2)^3*(5*a^2*b
+ 2*b^3))/(3*(a^4 + b^4 + 2*a^2*b^2)) - (4*a^3*tan(x/2)^2)/(a^4 + b^4 + 2*
a^2*b^2) + (2*a*b^2*tan(x/2)^4)/(a^4 + b^4 + 2*a^2*b^2) + (2*a^2*b*tan(x/2
)^5)/(a^4 + b^4 + 2*a^2*b^2) + (2*a^2*b*tan(x/2))/(a^4 + b^4 + 2*a^2*b^2))
/(3*tan(x/2)^2 + 3*tan(x/2)^4 + tan(x/2)^6 + 1) + (2*a^3*b*atanh((2*a^4*b
+ 2*b^5 + 4*a^2*b^3 - 2*a*tan(x/2)*(a^4 + b^4 + 2*a^2*b^2))/(2*(a^2 + b^2)
^(5/2))))/(a^2 + b^2)^(5/2)
```

3.278 $\int \frac{\cos^2(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx$

3.278.1 Optimal result

Integrand size = 18, antiderivative size = 93

$$\int \frac{\cos^2(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx=-\frac{a^2bx}{(a^2+b^2)^2}+\frac{bx}{2(a^2+b^2)}-\frac{ab^2\log(a\cos(x)+b\sin(x))}{(a^2+b^2)^2}+\frac{b\cos(x)\sin(x)}{2(a^2+b^2)}+\frac{a\sin^2(x)}{2(a^2+b^2)}$$

output
```
-a^2*b*x/(a^2+b^2)^2+1/2*b*x/(a^2+b^2)-a*b^2*ln(a*cos(x)+b*sin(x))/(a^2+b^
2)^2+1/2*b*cos(x)*sin(x)/(a^2+b^2)+1/2*a*sin(x)^2/(a^2+b^2)
```

3.278.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.88

$$\int \frac{\cos^2(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx$$
$$=\frac{4iab^2\arctan(\tan(x))-a(a^2+b^2)\cos(2x)-2b((a+ib)^2x+ab\log((a\cos(x)+b\sin(x))^2))+b(a^2+b^2)\mathrm{s}}{4(a^2+b^2)^2}$$

input
```
Integrate[(Cos[x]^2*Sin[x])/(a*Cos[x] + b*Sin[x]),x]
```

output
```
((4*I)*a*b^2*ArcTan[Tan[x]] - a*(a^2 + b^2)*Cos[2*x] - 2*b*((a + I*b)^2*x
+ a*b*Log[(a*Cos[x] + b*Sin[x])^2]) + b*(a^2 + b^2)*Sin[2*x])/(4*(a^2 + b^
2)^2)
```

3.278.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {3042, 3588, 3042, 3044, 15, 3115, 24, 3577, 3042, 3612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)\cos^2(x)}{a\cos(x)+b\sin(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)\cos(x)^2}{a\cos(x)+b\sin(x)}dx$$

$$\Big\downarrow \text{3588}$$

$$\frac{b\int \cos^2(x)dx}{a^2+b^2}+\frac{a\int \cos(x)\sin(x)dx}{a^2+b^2}-\frac{ab\int \frac{\cos(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\int \sin\left(x+\frac{\pi}{2}\right)^2dx}{a^2+b^2}+\frac{a\int \cos(x)\sin(x)dx}{a^2+b^2}-\frac{ab\int \frac{\cos(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3044}$$

$$\frac{b\int \sin\left(x+\frac{\pi}{2}\right)^2dx}{a^2+b^2}+\frac{a\int \sin(x)d\sin(x)}{a^2+b^2}-\frac{ab\int \frac{\cos(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{15}$$

$$\frac{b\int \sin\left(x+\frac{\pi}{2}\right)^2dx}{a^2+b^2}-\frac{ab\int \frac{\cos(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^2(x)}{2(a^2+b^2)}$$

$$\Big\downarrow \text{3115}$$

$$\frac{b\left(\frac{\int 1dx}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}-\frac{ab\int \frac{\cos(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^2(x)}{2(a^2+b^2)}$$

$$\Big\downarrow \text{24}$$

$$-\frac{ab\int \frac{\cos(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^2(x)}{2(a^2+b^2)}+\frac{b\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}$$

$$\Big\downarrow \text{3577}$$

$$-\frac{ab\left(\frac{b\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{ax}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^2(x)}{2(a^2+b^2)}+\frac{b\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{ab\left(\frac{b\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{ax}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^2(x)}{2(a^2+b^2)}+\frac{b\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}$$

$$\Big\downarrow \text{3612}$$

$$\frac{a\sin^2(x)}{2(a^2+b^2)}+\frac{b\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}-\frac{ab\left(\frac{ax}{a^2+b^2}+\frac{b\log(a\cos(x)+b\sin(x))}{a^2+b^2}\right)}{a^2+b^2}$$

input
```
Int[(Cos[x]^2*Sin[x])/(a*Cos[x] + b*Sin[x]),x]
```

output
```
-((a*b*((a*x)/(a^2 + b^2) + (b*Log[a*Cos[x] + b*Sin[x]])/(a^2 + b^2)))/(a^
2 + b^2)) + (a*Sin[x]^2)/(2*(a^2 + b^2)) + (b*(x/2 + (Cos[x]*Sin[x])/2))/(
a^2 + b^2)
```

3.278.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3577
```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[a*(x/(a^2 + b^2)), x] + Simp[b/(a^2 + b
^2)   Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x
]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3588
```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b
/(a^2 + b^2)   Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Simp[a/(a
^2 + b^2)   Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Simp[a*(b/(a^
2 + b^2))   Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x]
+ b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
&& IGtQ[m, 0] && IGtQ[n, 0]
```

rule 3612
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

3.278. $\int \frac{\cos^2(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx$

3.278.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.05

method	result
default	$-\frac{a\,b^{2}\ln\left(a+b\tan\left(x\right)\right)}{\left(a^{2}+b^{2}\right)^{2}}+\frac{\frac{\left(\frac{1}{2}a^{2}b+\frac{1}{2}b^{3}\right)\tan\left(x\right)-\frac{a^{3}}{2}-\frac{a\,b^{2}}{2}}{1+\tan\left(x\right)^{2}}+\frac{b\left(a b\ln\left(1+\tan\left(x\right)^{2}\right)+\left(-a^{2}+b^{2}\right)\arctan\left(\tan\left(x\right)\right)\right)}{2}}{\left(a^{2}+b^{2}\right)^{2}}$
parallelrisch	$\frac{-a^{3}\cos\left(2x\right)-a\,b^{2}\cos\left(2x\right)+a^{2}b\sin\left(2x\right)+b^{3}\sin\left(2x\right)-4a\,b^{2}\ln\left(\frac{-a\cos\left(x\right)-b\sin\left(x\right)}{\cos\left(x\right)+1}\right)+4a\,b^{2}\ln\left(\frac{1}{\cos\left(x\right)+1}\right)-2x\,a^{2}b+2x\,b^{3}+a^{3}+a\,b^{2}}{4\left(a^{2}+b^{2}\right)^{2}}$
risch	$\frac{x b}{4iba-2a^{2}+2b^{2}}-\frac{{\mathrm e}^{2 i x}}{8\left(-ib+a\right)}-\frac{{\mathrm e}^{-2 i x}}{8\left(ib+a\right)}+\frac{2i a\,b^{2}x}{a^{4}+2a^{2}b^{2}+b^{4}}-\frac{a\,b^{2}\ln\left({\mathrm e}^{2 i x}-\frac{ib+a}{ib-a}\right)}{a^{4}+2a^{2}b^{2}+b^{4}}$
norman	$\frac{\frac{b\tan\left(\frac{x}{2}\right)}{a^{2}+b^{2}}+\frac{2a\tan\left(\frac{x}{2}\right)^{2}}{a^{2}+b^{2}}+\frac{2a\tan\left(\frac{x}{2}\right)^{4}}{a^{2}+b^{2}}-\frac{b\tan\left(\frac{x}{2}\right)^{5}}{a^{2}+b^{2}}-\frac{b\left(a^{2}-b^{2}\right)x}{2\left(a^{4}+2a^{2}b^{2}+b^{4}\right)}-\frac{3b\left(a^{2}-b^{2}\right)x\tan\left(\frac{x}{2}\right)^{2}}{2\left(a^{4}+2a^{2}b^{2}+b^{4}\right)}-\frac{3b\left(a^{2}-b^{2}\right)x\tan\left(\frac{x}{2}\right)^{4}}{2\left(a^{4}+2a^{2}b^{2}+b^{4}\right)}-\frac{b\left(a^{2}-b^{2}\right)x\tan}{2\left(a^{4}+2a^{2}b^{2}\right.}}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)^{3}}$

input
```
int(cos(x)^2*sin(x)/(a*cos(x)+b*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
-a*b^2/(a^2+b^2)^2*ln(a+b*tan(x))+1/(a^2+b^2)^2*(((1/2*a^2*b+1/2*b^3)*tan(
x)-1/2*a^3-1/2*a*b^2)/(1+tan(x)^2)+1/2*b*(a*b*ln(1+tan(x)^2)+(-a^2+b^2)*ar
ctan(tan(x))))
```

3.278.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.01

$$\int \frac{\cos^2(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx = -\frac{ab^2\log\left(2\,ab\cos\left(x\right)\sin\left(x\right)+\left(a^2-b^2\right)\cos\left(x\right)^2+b^2\right)+\left(a^3+ab^2\right)\cos\left(x\right)^2-\left(a^2b+b^3\right)\cos\left(x\right)\sin\left(x\right)+}{2\left(a^4+2\,a^2b^2+b^4\right)}$$

input
```
integrate(cos(x)^2*sin(x)/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

output
```
-1/2*(a*b^2*log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) + (a^3 +
 a*b^2)*cos(x)^2 - (a^2*b + b^3)*cos(x)*sin(x) + (a^2*b - b^3)*x)/(a^4 + 2
*a^2*b^2 + b^4)
```

3.278.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx=\text{Timed out}$$

input
```
integrate(cos(x)**2*sin(x)/(a*cos(x)+b*sin(x)),x)
```

output
```
Timed out
```

3.278.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(87) = 174$.

Time = 0.33 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.28

$$\int \frac{\cos^2(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx=-\frac{ab^2\log\left(-a-\frac{2\,b\sin(x)}{\cos(x)+1}+\frac{a\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^4+2\,a^2b^2+b^4}+\frac{ab^2\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{a^4+2\,a^2b^2+b^4}-\frac{(a^2b-b^3)\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^4+2\,a^2b^2+b^4}+\frac{\frac{b\sin(x)}{\cos(x)+1}+\frac{2\,a\sin(x)^2}{(\cos(x)+1)^2}-\frac{b\sin(x)^3}{(\cos(x)+1)^3}}{a^2+b^2+\frac{2\,(a^2+b^2)\sin(x)^2}{(\cos(x)+1)^2}+\frac{(a^2+b^2)\sin(x)^4}{(\cos(x)+1)^4}}$$

input
```
integrate(cos(x)^2*sin(x)/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

output
```
-a*b^2*log(-a - 2*b*sin(x)/(cos(x) + 1) + a*sin(x)^2/(cos(x) + 1)^2)/(a^4
+ 2*a^2*b^2 + b^4) + a*b^2*log(sin(x)^2/(cos(x) + 1)^2 + 1)/(a^4 + 2*a^2*b
^2 + b^4) - (a^2*b - b^3)*arctan(sin(x)/(cos(x) + 1))/(a^4 + 2*a^2*b^2 + b
^4) + (b*sin(x)/(cos(x) + 1) + 2*a*sin(x)^2/(cos(x) + 1)^2 - b*sin(x)^3/(c
os(x) + 1)^3)/(a^2 + b^2 + 2*(a^2 + b^2)*sin(x)^2/(cos(x) + 1)^2 + (a^2 +
b^2)*sin(x)^4/(cos(x) + 1)^4)
```

3.278.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.68

$$\int \frac{\cos^2(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx = -\frac{ab^3\log\left(|b\tan\left(x\right)+a|\right)}{a^4b+2\,a^2b^3+b^5} + \frac{ab^2\log\left(\tan\left(x\right)^2+1\right)}{2\left(a^4+2\,a^2b^2+b^4\right)} - \frac{(a^2b-b^3)x}{2\left(a^4+2\,a^2b^2+b^4\right)} - \frac{ab^2\tan\left(x\right)^2-a^2b\tan\left(x\right)-b^3\tan\left(x\right)+a^3+2\,ab^2}{2\left(a^4+2\,a^2b^2+b^4\right)\left(\tan\left(x\right)^2+1\right)}$$

input
```
integrate(cos(x)^2*sin(x)/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

output
```
-a*b^3*log(abs(b*tan(x) + a))/(a^4*b + 2*a^2*b^3 + b^5) + 1/2*a*b^2*log(ta
n(x)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) - 1/2*(a^2*b - b^3)*x/(a^4 + 2*a^2*b^2
 + b^4) - 1/2*(a*b^2*tan(x)^2 - a^2*b*tan(x) - b^3*tan(x) + a^3 + 2*a*b^2)
/((a^4 + 2*a^2*b^2 + b^4)*(tan(x)^2 + 1))
```

3.278.9 Mupad [B] (verification not implemented)

Time = 29.67 (sec) , antiderivative size = 3419, normalized size of antiderivative = 36.76

$$\int \frac{\cos^2(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx = \text{Too large to display}$$

input
```
int((cos(x)^2*sin(x))/(a*cos(x) + b*sin(x)),x)
```

output
```
((b*tan(x/2))/(a^2 + b^2) + (2*a*tan(x/2)^2)/(a^2 + b^2) - (b*tan(x/2)^3)/
(a^2 + b^2))/(2*tan(x/2)^2 + tan(x/2)^4 + 1) - (a*b^2*log(a + 2*b*tan(x/2)
 - a*tan(x/2)^2))/(a^4 + b^4 + 2*a^2*b^2) + (4*a*b^2*log(1/(cos(x) + 1)))/
(4*a^4 + 4*b^4 + 8*a^2*b^2) - (b*atan((tan(x/2)*((((4*a*b^2*((b*(a + b)*(a
 - b)*((8*(12*a^4*b^6 + 24*a^6*b^4 + 12*a^8*b^2))/(a^6 + b^6 + 3*a^2*b^4 +
 3*a^4*b^2) - (32*a*b^2*(12*a*b^10 + 48*a^3*b^8 + 72*a^5*b^6 + 48*a^7*b^4
+ 12*a^9*b^2))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4
*b^2))))/(2*(a^4 + b^4 + 2*a^2*b^2)) - (16*a*b^3*(a + b)*(a - b)*(12*a*b^1
0 + 48*a^3*b^8 + 72*a^5*b^6 + 48*a^7*b^4 + 12*a^9*b^2))/((4*a^4 + 4*b^4 +
8*a^2*b^2)*(a^4 + b^4 + 2*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/
(4*a^4 + 4*b^4 + 8*a^2*b^2) - (b*(a + b)*((8*(2*a*b^8 - 7*a^3*b^6 - 8*a^5*
b^4 + a^7*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (4*a*b^2*((8*(12*a^4
*b^6 + 24*a^6*b^4 + 12*a^8*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (32
*a*b^2*(12*a*b^10 + 48*a^3*b^8 + 72*a^5*b^6 + 48*a^7*b^4 + 12*a^9*b^2))/((
4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(4*a^4 +
 4*b^4 + 8*a^2*b^2))*(a - b))/(2*(a^4 + b^4 + 2*a^2*b^2)) + (b^3*(a + b)^3
*(a - b)^3*(12*a*b^10 + 48*a^3*b^8 + 72*a^5*b^6 + 48*a^7*b^4 + 12*a^9*b^2)
)/((a^4 + b^4 + 2*a^2*b^2)^3*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))*(a^6 -
b^6 + 35*a^2*b^4 - 35*a^4*b^2))/(a^6 + b^6 + 15*a^2*b^4 + 15*a^4*b^2)^2 -
(2*a*b*(5*a^4 + 5*b^4 - 26*a^2*b^2)*((8*(2*a^2*b^6 + a^4*b^4))/(a^6 + b...
```

3.278. $\int \frac{\cos^2(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx$

3.279 $\int \frac{\cos^2(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx$

3.279.1 Optimal result

Integrand size = 20, antiderivative size = 112

$$\int \frac{\cos^2(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx=-\frac{a^2b^2\operatorname{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}+\frac{a^2b\cos(x)}{(a^2+b^2)^2}-\frac{b\cos^3(x)}{3(a^2+b^2)}-\frac{ab^2\sin(x)}{(a^2+b^2)^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}$$

output
```
-a^2*b^2*arctanh((b*cos(x)-a*sin(x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2)+a^2*
b*cos(x)/(a^2+b^2)^2-1/3*b*cos(x)^3/(a^2+b^2)-a*b^2*sin(x)/(a^2+b^2)^2+1/3
*a*sin(x)^3/(a^2+b^2)
```

3.279.2 Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.03

$$\int \frac{\cos^2(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx=\frac{2a^2b^2\operatorname{arctanh}\left(\frac{-b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}-\frac{(-9a^2b+3b^3)\cos(x)+b(a^2+b^2)\cos(3x)+2a(-a^2+5b^2+(a^2+b^2)\cos(2x))\sin(x)}{12(a^2+b^2)^2}$$

input
```
Integrate[(Cos[x]^2*Sin[x]^2)/(a*Cos[x] + b*Sin[x]),x]
```

output
```
(2*a^2*b^2*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) -
 ((-9*a^2*b + 3*b^3)*Cos[x] + b*(a^2 + b^2)*Cos[3*x] + 2*a*(-a^2 + 5*b^2 +
 (a^2 + b^2)*Cos[2*x])*Sin[x])/(12*(a^2 + b^2)^2)
```

3.279.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.650$, Rules used = {3042, 3588, 3042, 3044, 15, 3045, 15, 3588, 3042, 3117, 3118, 3553, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(x)\cos^2(x)}{a\cos(x)+b\sin(x)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin(x)^2\cos(x)^2}{a\cos(x)+b\sin(x)}dx$$

$$\Big\downarrow \; 3588$$

$$\frac{b\int \cos^2(x)\sin(x)dx}{a^2+b^2}+\frac{a\int \cos(x)\sin^2(x)dx}{a^2+b^2}-\frac{ab\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{b\int \cos(x)^2\sin(x)dx}{a^2+b^2}+\frac{a\int \cos(x)\sin(x)^2dx}{a^2+b^2}-\frac{ab\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \; 3044$$

$$\frac{a\int \sin^2(x)d\sin(x)}{a^2+b^2}+\frac{b\int \cos(x)^2\sin(x)dx}{a^2+b^2}-\frac{ab\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \; 15$$

$$\frac{b\int \cos(x)^2\sin(x)dx}{a^2+b^2}-\frac{ab\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}$$

$$\Big\downarrow \; 3045$$

$$-\frac{b\int \cos^2(x)d\cos(x)}{a^2+b^2}-\frac{ab\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}$$

$\Big\downarrow$ 15

$$-\frac{ab\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{b\cos^3(x)}{3(a^2+b^2)}$$

$\Big\downarrow$ 3588

$$-\frac{ab\left(\frac{a\int \sin(x)dx}{a^2+b^2}+\frac{b\int \cos(x)dx}{a^2+b^2}-\frac{ab\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{b\cos^3(x)}{3(a^2+b^2)}$$

$\Big\downarrow$ 3042

$$-\frac{ab\left(\frac{a\int \sin(x)dx}{a^2+b^2}+\frac{b\int \sin(x+\frac{\pi}{2})dx}{a^2+b^2}-\frac{ab\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{b\cos^3(x)}{3(a^2+b^2)}$$

$\Big\downarrow$ 3117

$$-\frac{ab\left(\frac{a\int \sin(x)dx}{a^2+b^2}-\frac{ab\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\sin(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{b\cos^3(x)}{3(a^2+b^2)}$$

$\Big\downarrow$ 3118

$$-\frac{ab\left(-\frac{ab\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\sin(x)}{a^2+b^2}-\frac{a\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{b\cos^3(x)}{3(a^2+b^2)}$$

$\Big\downarrow$ 3553

$$-\frac{ab\left(\frac{ab\int \frac{1}{a^2+b^2-(b\cos(x)-a\sin(x))^2}d(b\cos(x)-a\sin(x))}{a^2+b^2}+\frac{b\sin(x)}{a^2+b^2}-\frac{a\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{b\cos^3(x)}{3(a^2+b^2)}$$

$\Big\downarrow$ 219

$$-\frac{ab\left(\frac{ab\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{b\sin(x)}{a^2+b^2}-\frac{a\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{b\cos^3(x)}{3(a^2+b^2)}$$

input
```
Int[(Cos[x]^2*Sin[x]^2)/(a*Cos[x] + b*Sin[x]),x]
```

3.279. $\int \frac{\cos^2(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx$

output
```
-1/3*(b*Cos[x]^3)/(a^2 + b^2) + (a*Sin[x]^3)/(3*(a^2 + b^2)) - (a*b*((a*b*
ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2) - (a*Cos
[x])/(a^2 + b^2) + (b*Sin[x])/(a^2 + b^2)))/(a^2 + b^2)
```

3.279.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

3.279. $\int \frac{\cos^2(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx$

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3588
```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b
/(a^2 + b^2)   Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Simp[a/(a
^2 + b^2)   Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Simp[a*(b/(a^
2 + b^2))   Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x]
+ b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
&& IGtQ[m, 0] && IGtQ[n, 0]
```

3.279.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.47

method	result
default	$-\frac{2\left(a\,b^{2}\tan\left(\frac{x}{2}\right)^{5}+b^{3}\tan\left(\frac{x}{2}\right)^{4}+\left(-\frac{4}{3}a^{3}+\frac{2}{3}a\,b^{2}\right)\tan\left(\frac{x}{2}\right)^{3}-2\tan\left(\frac{x}{2}\right)^{2}a^{2}b+\tan\left(\frac{x}{2}\right)a\,b^{2}-\frac{2a^{2}b}{3}+\frac{b^{3}}{3}\right)}{\left(a^{4}+2a^{2}b^{2}+b^{4}\right)\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)^{3}}+\frac{8a^{2}b^{2}\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{x}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{\left(4a^{4}+8a^{2}b^{2}+4b^{4}\right)\sqrt{a^{2}+b^{2}}}$
risch	$\frac{\mathrm{e}^{ix}b}{-16iba+8a^{2}-8b^{2}}-\frac{i\mathrm{e}^{ix}a}{8\left(-2iba+a^{2}-b^{2}\right)}+\frac{\mathrm{e}^{-ix}b}{8\left(ib+a\right)^{2}}+\frac{i\mathrm{e}^{-ix}a}{8\left(ib+a\right)^{2}}-\frac{b^{2}a^{2}\ln\left(\mathrm{e}^{ix}-\frac{ia^{5}+2ia^{3}b^{2}+ia\,b^{4}-a^{4}b-2a^{2}b^{3}-b^{5}}{\left(a^{2}+b^{2}\right)^{\frac{5}{2}}}\right)}{\left(a^{2}+b^{2}\right)^{\frac{5}{2}}}+\frac{b^{2}a^{2}\ln}{}$

input
```
int(cos(x)^2*sin(x)^2/(a*cos(x)+b*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
-2/(a^4+2*a^2*b^2+b^4)*(a*b^2*tan(1/2*x)^5+b^3*tan(1/2*x)^4+(-4/3*a^3+2/3*
a*b^2)*tan(1/2*x)^3-2*tan(1/2*x)^2*a^2*b+tan(1/2*x)*a*b^2-2/3*a^2*b+1/3*b^
3)/(1+tan(1/2*x)^2)^3+8*a^2*b^2/(4*a^4+8*a^2*b^2+4*b^4)/(a^2+b^2)^(1/2)*ar
ctanh(1/2*(2*a*tan(1/2*x)-2*b)/(a^2+b^2)^(1/2))
```

3.279.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(104) = 208$.

Time = 0.26 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.92

$$\int \frac{\cos^2(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx$$
$$= \frac{3\sqrt{a^2+b^2}a^2b^2\log\left(-\frac{2\,ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}(b\cos(x)-a\sin(x))}{2\,ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2+b^2}\right)-2\left(a^4b+2\,a^2b^3+b^5\right)\cos}{6\left(a^6+3\,a^4b^2+3\,a^2b^4\right.}$$

input
```
integrate(cos(x)^2*sin(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

output
```
1/6*(3*sqrt(a^2 + b^2)*a^2*b^2*log(-(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos
(x)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos(
x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2)) - 2*(a^4*b + 2*a^2*b^3 + b^5)*cos
(x)^3 + 6*(a^4*b + a^2*b^3)*cos(x) + 2*(a^5 - a^3*b^2 - 2*a*b^4 - (a^5 + 2
*a^3*b^2 + a*b^4)*cos(x)^2)*sin(x))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
```

3.279.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx = \text{Timed out}$$

input
```
integrate(cos(x)**2*sin(x)**2/(a*cos(x)+b*sin(x)),x)
```

output
```
Timed out
```

3.279.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 281 vs. $2(104) = 208$.

Time = 0.30 (sec) , antiderivative size = 281, normalized size of antiderivative = 2.51

$$\int \frac{\cos^2(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx$$
$$= -\frac{a^2b^2\log\left(\frac{b-\frac{a\sin(x)}{\cos(x)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(x)}{\cos(x)+1}-\sqrt{a^2+b^2}}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}}$$
$$+\frac{2\left(2\,a^2b-b^3-\frac{3\,ab^2\sin(x)}{\cos(x)+1}+\frac{6\,a^2b\sin(x)^2}{(\cos(x)+1)^2}-\frac{3\,b^3\sin(x)^4}{(\cos(x)+1)^4}-\frac{3\,ab^2\sin(x)^5}{(\cos(x)+1)^5}+\frac{2\,(2\,a^3-ab^2)\sin(x)^3}{(\cos(x)+1)^3}\right)}{3\left(a^4+2\,a^2b^2+b^4+\frac{3\,(a^4+2\,a^2b^2+b^4)\sin(x)^2}{(\cos(x)+1)^2}+\frac{3\,(a^4+2\,a^2b^2+b^4)\sin(x)^4}{(\cos(x)+1)^4}+\frac{(a^4+2\,a^2b^2+b^4)\sin(x)^6}{(\cos(x)+1)^6}\right)}$$

input
```
integrate(cos(x)^2*sin(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

output
```
-a^2*b^2*log((b - a*sin(x)/(cos(x) + 1) + sqrt(a^2 + b^2))/(b - a*sin(x)/(
cos(x) + 1) - sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2))
+ 2/3*(2*a^2*b - b^3 - 3*a*b^2*sin(x)/(cos(x) + 1) + 6*a^2*b*sin(x)^2/(cos
(x) + 1)^2 - 3*b^3*sin(x)^4/(cos(x) + 1)^4 - 3*a*b^2*sin(x)^5/(cos(x) + 1)
^5 + 2*(2*a^3 - a*b^2)*sin(x)^3/(cos(x) + 1)^3)/(a^4 + 2*a^2*b^2 + b^4 + 3
*(a^4 + 2*a^2*b^2 + b^4)*sin(x)^2/(cos(x) + 1)^2 + 3*(a^4 + 2*a^2*b^2 + b^
4)*sin(x)^4/(cos(x) + 1)^4 + (a^4 + 2*a^2*b^2 + b^4)*sin(x)^6/(cos(x) + 1)
^6)
```

3.279.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.71

$$\int \frac{\cos^2(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx = -\frac{a^2b^2\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}}$$
$$-\frac{2\left(3\,ab^2\tan\left(\frac{1}{2}\,x\right)^5+3\,b^3\tan\left(\frac{1}{2}\,x\right)^4-4\,a^3\tan\left(\frac{1}{2}\,x\right)^3+2\,ab^2\tan\left(\frac{1}{2}\,x\right)^3-6\,a^2b\tan\left(\frac{1}{2}\,x\right)^2+3\,ab^2\tan\left(\frac{1}{2}\right.\right.}{3\,(a^4+2\,a^2b^2+b^4)\left(\tan\left(\frac{1}{2}\,x\right)^2+1\right)^3}$$

input
```
integrate(cos(x)^2*sin(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

output
```
-a^2*b^2*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2
*x) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2))
- 2/3*(3*a*b^2*tan(1/2*x)^5 + 3*b^3*tan(1/2*x)^4 - 4*a^3*tan(1/2*x)^3 + 2*
a*b^2*tan(1/2*x)^3 - 6*a^2*b*tan(1/2*x)^2 + 3*a*b^2*tan(1/2*x) - 2*a^2*b +
 b^3)/((a^4 + 2*a^2*b^2 + b^4)*(tan(1/2*x)^2 + 1)^3)
```

3.279.9 Mupad [B] (verification not implemented)

Time = 23.26 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.47

$$\int \frac{\cos^2(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx = -\frac{\frac{4\tan\left(\frac{x}{2}\right)^3(a\,b^2-2\,a^3)}{3\,(a^4+2\,a^2\,b^2+b^4)}-\frac{2\,b\,(2\,a^2-b^2)}{3\,(a^2+b^2)^2}+\frac{2\,b^3\tan\left(\frac{x}{2}\right)^4}{a^4+2\,a^2\,b^2+b^4}-\frac{4\,a^2\,b\tan\left(\frac{x}{2}\right)^2}{a^4+2\,a^2\,b^2+b^4}+\frac{2\,a\,b^2\tan\left(\frac{x}{2}\right)^5}{a^4+2\,a^2\,b^2+b^4}+\frac{2\,a\,b^2\tan\left(\frac{x}{2}\right)}{a^4+2\,a^2\,b^2+b^4}}{\tan\left(\frac{x}{2}\right)^6+3\tan\left(\frac{x}{2}\right)^4+3\tan\left(\frac{x}{2}\right)^2+1} - \frac{2\,a^2\,b^2\,\mathrm{atanh}\left(\frac{2\,a^4\,b+2\,b^5+4\,a^2\,b^3-2\,a\tan\left(\frac{x}{2}\right)(a^4+2\,a^2\,b^2+b^4)}{2\,(a^2+b^2)^{5/2}}\right)}{(a^2+b^2)^{5/2}}$$

input
```
int((cos(x)^2*sin(x)^2)/(a*cos(x) + b*sin(x)),x)
```

output
```
- ((4*tan(x/2)^3*(a*b^2 - 2*a^3))/(3*(a^4 + b^4 + 2*a^2*b^2)) - (2*b*(2*a^
2 - b^2))/(3*(a^2 + b^2)^2) + (2*b^3*tan(x/2)^4)/(a^4 + b^4 + 2*a^2*b^2) -
 (4*a^2*b*tan(x/2)^2)/(a^4 + b^4 + 2*a^2*b^2) + (2*a*b^2*tan(x/2)^5)/(a^4
+ b^4 + 2*a^2*b^2) + (2*a*b^2*tan(x/2))/(a^4 + b^4 + 2*a^2*b^2))/(3*tan(x/
2)^2 + 3*tan(x/2)^4 + tan(x/2)^6 + 1) - (2*a^2*b^2*atanh((2*a^4*b + 2*b^5
+ 4*a^2*b^3 - 2*a*tan(x/2)*(a^4 + b^4 + 2*a^2*b^2))/(2*(a^2 + b^2)^(5/2)))
)/(a^2 + b^2)^(5/2)
```

3.280 $\int \frac{\cos^2(x)\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx$

3.280.1 Optimal result

Integrand size = 20, antiderivative size = 176

$$\int \frac{\cos^2(x)\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{a^2b^3x}{(a^2+b^2)^3} - \frac{a^2bx}{2(a^2+b^2)^2} + \frac{bx}{8(a^2+b^2)} - \frac{a^3b^2\log(a\cos(x)+b\sin(x))}{(a^2+b^2)^3} + \frac{a^2b\cos(x)\sin(x)}{2(a^2+b^2)^2} + \frac{b\cos(x)\sin(x)}{8(a^2+b^2)} - \frac{b\cos^3(x)\sin(x)}{4(a^2+b^2)} - \frac{ab^2\sin^2(x)}{2(a^2+b^2)^2} + \frac{a\sin^4(x)}{4(a^2+b^2)}$$

output
```
a^2*b^3*x/(a^2+b^2)^3-1/2*a^2*b*x/(a^2+b^2)^2+1/8*b*x/(a^2+b^2)-a^3*b^2*ln
(a*cos(x)+b*sin(x))/(a^2+b^2)^3+1/2*a^2*b*cos(x)*sin(x)/(a^2+b^2)^2+1/8*b*
cos(x)*sin(x)/(a^2+b^2)-1/4*b*cos(x)^3*sin(x)/(a^2+b^2)-1/2*a*b^2*sin(x)^2
/(a^2+b^2)^2+1/4*a*sin(x)^4/(a^2+b^2)
```

3.280.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.80 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.01

$$\int \frac{\cos^2(x)\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{-12a^4bx - 32ia^3b^2x + 24a^2b^3x + 4b^5x + 32ia^3b^2\arctan(\tan(x)) - 4a(a^4-b^4)\cos(2x) + a^5\cos(4x) + 2a}{}$$

input
```
Integrate[(Cos[x]^2*Sin[x]^3)/(a*Cos[x] + b*Sin[x]),x]
```

output
```
(-12*a^4*b*x - (32*I)*a^3*b^2*x + 24*a^2*b^3*x + 4*b^5*x + (32*I)*a^3*b^2*
ArcTan[Tan[x]] - 4*a*(a^4 - b^4)*Cos[2*x] + a^5*Cos[4*x] + 2*a^3*b^2*Cos[4
*x] + a*b^4*Cos[4*x] - 16*a^3*b^2*Log[(a*Cos[x] + b*Sin[x])^2] + 8*a^4*b*S
in[2*x] + 8*a^2*b^3*Sin[2*x] - a^4*b*Sin[4*x] - 2*a^2*b^3*Sin[4*x] - b^5*S
in[4*x])/(32*(a^2 + b^2)^3)
```

3.280.3 Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.93, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {3042, 3588, 3042, 3044, 15, 3048, 3042, 3115, 24, 3588, 3042, 3044, 15, 3115, 24, 3576, 3042, 3612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(x)\cos^2(x)}{a\cos(x)+b\sin(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(x)^3\cos(x)^2}{a\cos(x)+b\sin(x)}dx$$

$$\Big\downarrow \text{ 3588}$$

$$\frac{b\int \cos^2(x)\sin^2(x)dx}{a^2+b^2}+\frac{a\int \cos(x)\sin^3(x)dx}{a^2+b^2}-\frac{ab\int \frac{\cos(x)\sin^2(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b\int \cos(x)^2\sin(x)^2dx}{a^2+b^2}+\frac{a\int \cos(x)\sin(x)^3dx}{a^2+b^2}-\frac{ab\int \frac{\cos(x)\sin(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{ 3044}$$

$$\frac{a\int \sin^3(x)d\sin(x)}{a^2+b^2}+\frac{b\int \cos(x)^2\sin(x)^2dx}{a^2+b^2}-\frac{ab\int \frac{\cos(x)\sin(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{b\int \cos(x)^2\sin(x)^2dx}{a^2+b^2}-\frac{ab\int \frac{\cos(x)\sin(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^4(x)}{4(a^2+b^2)}$$

$$\downarrow 3048$$

$$\frac{b\left(\frac{1}{4}\int \cos^2(x)dx-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}-\frac{ab\int \frac{\cos(x)\sin(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^4(x)}{4\left(a^2+b^2\right)}$$

$$\downarrow 3042$$

$$\frac{b\left(\frac{1}{4}\int \sin\left(x+\frac{\pi}{2}\right)^2dx-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}-\frac{ab\int \frac{\cos(x)\sin(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^4(x)}{4\left(a^2+b^2\right)}$$

$$\downarrow 3115$$

$$\frac{b\left(\frac{1}{4}\left(\frac{\int 1dx}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}-\frac{ab\int \frac{\cos(x)\sin(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^4(x)}{4\left(a^2+b^2\right)}$$

$$\downarrow 24$$

$$-\frac{ab\int \frac{\cos(x)\sin(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^4(x)}{4\left(a^2+b^2\right)}+\frac{b\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}$$

$$\downarrow 3588$$

$$-\frac{ab\left(\frac{a\int \sin^2(x)dx}{a^2+b^2}+\frac{b\int \cos(x)\sin(x)dx}{a^2+b^2}-\frac{ab\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^4(x)}{4\left(a^2+b^2\right)}+\frac{b\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}$$

$$\downarrow 3042$$

$$-\frac{ab\left(\frac{a\int \sin(x)^2dx}{a^2+b^2}+\frac{b\int \cos(x)\sin(x)dx}{a^2+b^2}-\frac{ab\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^4(x)}{4\left(a^2+b^2\right)}+\frac{b\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}$$

$$\downarrow 3044$$

$$-\frac{ab\left(\frac{a\int \sin(x)^2dx}{a^2+b^2}+\frac{b\int \sin(x)d\sin(x)}{a^2+b^2}-\frac{ab\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^4(x)}{4\left(a^2+b^2\right)}+\frac{b\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}$$

$$\downarrow 15$$

3.280. $\int \frac{\cos^2(x)\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx$

$$-\frac{ab\left(\frac{a\int \sin(x)^2dx}{a^2+b^2}-\frac{ab\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\sin^2(x)}{2(a^2+b^2)}\right)}{a^2+b^2}+\frac{a\sin^4(x)}{4(a^2+b^2)}+\frac{b\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}$$

$\downarrow$ 3115

$$-\frac{ab\left(\frac{a\left(\frac{\int 1dx}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}-\frac{ab\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\sin^2(x)}{2(a^2+b^2)}\right)}{a^2+b^2}+\frac{a\sin^4(x)}{4(a^2+b^2)}+\frac{b\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}$$

$\downarrow$ 24

$$-\frac{ab\left(-\frac{ab\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\sin^2(x)}{2(a^2+b^2)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^4(x)}{4(a^2+b^2)}+\frac{b\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}$$

$\downarrow$ 3576

$$-\frac{ab\left(-\frac{ab\left(\frac{bx}{a^2+b^2}-\frac{a\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\sin^2(x)}{2(a^2+b^2)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^4(x)}{4(a^2+b^2)}+\frac{b\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}$$

$\downarrow$ 3042

$$-\frac{ab\left(-\frac{ab\left(\frac{bx}{a^2+b^2}-\frac{a\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\sin^2(x)}{2(a^2+b^2)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^4(x)}{4(a^2+b^2)}+\frac{b\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}$$

$\downarrow$ 3612

3.280. $\int \frac{\cos^2(x)\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx$

$$\frac{a\sin^4(x)}{4(a^2+b^2)}+\frac{b\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}-\frac{ab\left(\frac{b\sin^2(x)}{2(a^2+b^2)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}-\frac{ab\left(\frac{bx}{a^2+b^2}-\frac{a\log(a\cos(x)+b\sin(x))}{a^2+b^2}\right)}{a^2+b^2}\right)}{a^2+b^2}$$

input
```
Int[(Cos[x]^2*Sin[x]^3)/(a*Cos[x] + b*Sin[x]),x]
```

output
```
(a*Sin[x]^4)/(4*(a^2 + b^2)) - (a*b*(-((a*b*((b*x)/(a^2 + b^2) - (a*Log[a*
Cos[x] + b*Sin[x]])/(a^2 + b^2)))/(a^2 + b^2)) + (b*Sin[x]^2)/(2*(a^2 + b^
2)) + (a*(x/2 - (Cos[x]*Sin[x])/2))/(a^2 + b^2)))/(a^2 + b^2) + (b*(-1/4*(
Cos[x]^3*Sin[x]) + (x/2 + (Cos[x]*Sin[x])/2)/4))/(a^2 + b^2)
```

3.280.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3576
```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(x/(a^2 + b^2)), x] - Simp[a/(a^2 + b
^2)   Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x
]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3588
```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b
/(a^2 + b^2)   Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Simp[a/(a
^2 + b^2)   Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Simp[a*(b/(a^
2 + b^2))   Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x]
+ b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
&& IGtQ[m, 0] && IGtQ[n, 0]
```

rule 3612
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

3.280.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.93

method	result
default	$\frac{\frac{\left(\frac{3}{4}a^2b^3+\frac{1}{8}b^5+\frac{5}{8}a^4b\right)\tan(x)^3+\left(-\frac{1}{2}a^5-\frac{1}{2}a^3b^2\right)\tan(x)^2+\left(\frac{3}{8}a^4b+\frac{1}{4}a^2b^3-\frac{1}{8}b^5\right)\tan(x)-\frac{a^5}{4}+\frac{a\,b^4}{4}}{\left(1+\tan(x)^2\right)^2}+\frac{b\left(4a^3b\ln\left(1+\tan(x)^2\right)+\left(-3a^4+6a^2b\right.\right.}{8}}{(a^2+b^2)^3}$
parallelrisch	$\frac{-32a^3b^2\ln\left(\frac{-a\cos(x)-b\sin(x)}{\cos(x)+1}\right)+32a^3b^2\ln\left(\frac{1}{\cos(x)+1}\right)+a\left(a^2+b^2\right)^2\cos(4x)-b\left(a^2+b^2\right)^2\sin(4x)+\left(-4a^5+4a\,b^4\right)\cos(2x)+\left(8a^4\right.}{32(a^2+b^2)^3}$
risch	$-\frac{3ixba}{8(ia^3-3ia\,b^2+3a^2b-b^3)}-\frac{x\,b^2}{8(ia^3-3ia\,b^2+3a^2b-b^3)}-\frac{a\,e^{2ix}}{16(-2iba+a^2-b^2)}-\frac{a\,e^{-2ix}}{16(ib+a)^2}+\frac{2ia^3b^2x}{a^6+3a^4b^2+3a^2b^4+b^6}-$
norman	$-\frac{2a\,b^2\tan\left(\frac{x}{2}\right)^2}{a^4+2a^2b^2+b^4}-\frac{2a\,b^2\tan\left(\frac{x}{2}\right)^8}{a^4+2a^2b^2+b^4}+\frac{2\left(2a^3-a\,b^2\right)\tan\left(\frac{x}{2}\right)^4}{a^4+2a^2b^2+b^4}+\frac{2\left(2a^3-a\,b^2\right)\tan\left(\frac{x}{2}\right)^6}{a^4+2a^2b^2+b^4}+\frac{\left(3a^2-b^2\right)b\tan\left(\frac{x}{2}\right)}{4a^4+8a^2b^2+4b^4}-\frac{\left(3a^2-b^2\right)b\tan\left(\frac{x}{2}\right)^9}{4\left(a^4+2a^2b^2+b^4\right)}+\frac{(7a^2-}{2a^4}$

input
```
int(cos(x)^2*sin(x)^3/(a*cos(x)+b*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
1/(a^2+b^2)^3*(((3/4*a^2*b^3+1/8*b^5+5/8*a^4*b)*tan(x)^3+(-1/2*a^5-1/2*a^3
*b^2)*tan(x)^2+(3/8*a^4*b+1/4*a^2*b^3-1/8*b^5)*tan(x)-1/4*a^5+1/4*a*b^4)/(
1+tan(x)^2)^2+1/8*b*(4*a^3*b*ln(1+tan(x)^2)+(-3*a^4+6*a^2*b^2+b^4)*arctan(
tan(x))))-b^2/(a^2+b^2)^3*a^3*ln(a+b*tan(x))
```

3.280.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.99

$$\int \frac{\cos^2(x)\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx =$$
$$-\frac{4\,a^3b^2\log\left(2\,ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2+b^2\right)-2\left(a^5+2\,a^3b^2+ab^4\right)\cos(x)^4+4\left(a^5+a^3b^2\right)c}{8\left(a^6+3\,a^4b^2+\right.}$$

input
```
integrate(cos(x)^2*sin(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

output
```
-1/8*(4*a^3*b^2*log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) - 2*
(a^5 + 2*a^3*b^2 + a*b^4)*cos(x)^4 + 4*(a^5 + a^3*b^2)*cos(x)^2 + (3*a^4*b
 - 6*a^2*b^3 - b^5)*x + (2*(a^4*b + 2*a^2*b^3 + b^5)*cos(x)^3 - (5*a^4*b +
 6*a^2*b^3 + b^5)*cos(x))*sin(x))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
```

3.280.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(x)\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx = \text{Timed out}$$

input
```
integrate(cos(x)**2*sin(x)**3/(a*cos(x)+b*sin(x)),x)
```

output
```
Timed out
```

3.280.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 431 vs. $2(162) = 324$.

Time = 0.34 (sec) , antiderivative size = 431, normalized size of antiderivative = 2.45

$$\int \frac{\cos^2(x)\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx = -\frac{a^3b^2\log\left(-a-\frac{2\,b\sin(x)}{\cos(x)+1}+\frac{a\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{a^3b^2\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} - \frac{(3\,a^4b-6\,a^2b^3-b^5)\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{4\,(a^6+3\,a^4b^2+3\,a^2b^4+b^6)} - \frac{\frac{8\,ab^2\sin(x)^2}{(\cos(x)+1)^2}-\frac{16\,a^3\sin(x)^4}{(\cos(x)+1)^4}+\frac{8\,ab^2\sin(x)^6}{(\cos(x)+1)^6}-\frac{(3\,a^2b-b^3)\sin(x)}{\cos(x)+1}-\frac{(11\,a^2b+7\,b^3)\sin(x)^3}{(\cos(x)+1)^3}+\frac{(11\,a^2b+7\,b^3)\sin(x)^5}{(\cos(x)+1)^5}+\frac{(3\,a^2b-b^3)\sin}{(\cos(x)+1)}}{4\left(a^4+2\,a^2b^2+b^4+\frac{4\,(a^4+2\,a^2b^2+b^4)\sin(x)^2}{(\cos(x)+1)^2}+\frac{6\,(a^4+2\,a^2b^2+b^4)\sin(x)^4}{(\cos(x)+1)^4}+\frac{4\,(a^4+2\,a^2b^2+b^4)\sin(x)^6}{(\cos(x)+1)^6}+\frac{(a^4+2\,a^2b^2+b^4)\sin(x}{(\cos(x)+1)^8}\right.}$$

input
```
integrate(cos(x)^2*sin(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

output
```
-a^3*b^2*log(-a - 2*b*sin(x)/(cos(x) + 1) + a*sin(x)^2/(cos(x) + 1)^2)/(a^
6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + a^3*b^2*log(sin(x)^2/(cos(x) + 1)^2 + 1
)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 1/4*(3*a^4*b - 6*a^2*b^3 - b^5)*ar
ctan(sin(x)/(cos(x) + 1))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 1/4*(8*a*b
^2*sin(x)^2/(cos(x) + 1)^2 - 16*a^3*sin(x)^4/(cos(x) + 1)^4 + 8*a*b^2*sin(
x)^6/(cos(x) + 1)^6 - (3*a^2*b - b^3)*sin(x)/(cos(x) + 1) - (11*a^2*b + 7*
b^3)*sin(x)^3/(cos(x) + 1)^3 + (11*a^2*b + 7*b^3)*sin(x)^5/(cos(x) + 1)^5
+ (3*a^2*b - b^3)*sin(x)^7/(cos(x) + 1)^7)/(a^4 + 2*a^2*b^2 + b^4 + 4*(a^4
 + 2*a^2*b^2 + b^4)*sin(x)^2/(cos(x) + 1)^2 + 6*(a^4 + 2*a^2*b^2 + b^4)*si
n(x)^4/(cos(x) + 1)^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*sin(x)^6/(cos(x) + 1)^6
+ (a^4 + 2*a^2*b^2 + b^4)*sin(x)^8/(cos(x) + 1)^8)
```

3.280.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.56

$$\int \frac{\cos^2(x)\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx$$
$$= -\frac{a^3b^3\log\left(|b\tan(x)+a|\right)}{a^6b+3a^4b^3+3a^2b^5+b^7} + \frac{a^3b^2\log\left(\tan(x)^2+1\right)}{2\left(a^6+3a^4b^2+3a^2b^4+b^6\right)} - \frac{(3a^4b-6a^2b^3-b^5)x}{8\left(a^6+3a^4b^2+3a^2b^4+b^6\right)} - \frac{6a^3b^2\tan(x)^4-5a^4b\tan(x)^3-6a^2b^3\tan(x)^3-b^5\tan(x)^3+4a^5\tan(x)^2+16a^3b^2\tan(x)^2-3a^4b\tan}{8\left(a^6+3a^4b^2+3a^2b^4+b^6\right)\left(\tan(x)^2+1\right)^2}$$

input
```
integrate(cos(x)^2*sin(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

output
```
-a^3*b^3*log(abs(b*tan(x) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7) + 1/
2*a^3*b^2*log(tan(x)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 1/8*(3*a
^4*b - 6*a^2*b^3 - b^5)*x/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 1/8*(6*a^3
*b^2*tan(x)^4 - 5*a^4*b*tan(x)^3 - 6*a^2*b^3*tan(x)^3 - b^5*tan(x)^3 + 4*a
^5*tan(x)^2 + 16*a^3*b^2*tan(x)^2 - 3*a^4*b*tan(x) - 2*a^2*b^3*tan(x) + b^
5*tan(x) + 2*a^5 + 6*a^3*b^2 - 2*a*b^4)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^
6)*(tan(x)^2 + 1)^2)
```

3.280.9 Mupad [B] (verification not implemented)

Time = 34.76 (sec) , antiderivative size = 5902, normalized size of antiderivative = 33.53

$$\int \frac{\cos^2(x)\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx = \text{Too large to display}$$

input
```
int((cos(x)^2*sin(x)^3)/(a*cos(x) + b*sin(x)),x)
```

output
```
(64*a^3*b^2*log(1/(cos(x) + 1)))/(64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*
b^2) - (b*atan((tan(x/2)*((((64*a^3*b^2*((b*((448*a^8*b^8 - 96*a^4*b^12 -
48*a^6*b^10 - 16*a^2*b^14 + 912*a^10*b^6 + 672*a^12*b^4 + 176*a^14*b^2)/(2
*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10
*b^2)) - (32*a^3*b^2*(192*a*b^16 + 1344*a^3*b^14 + 4032*a^5*b^12 + 6720*a^
7*b^10 + 6720*a^9*b^8 + 4032*a^11*b^6 + 1344*a^13*b^4 + 192*a^15*b^2))/((6
4*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15
*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(b^4 - 3*a^4 + 6*a^2*b^
2))/(8*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (4*a^3*b^3*(b^4 - 3*a^4 + 6*
a^2*b^2)*(192*a*b^16 + 1344*a^3*b^14 + 4032*a^5*b^12 + 6720*a^7*b^10 + 672
0*a^9*b^8 + 4032*a^11*b^6 + 1344*a^13*b^4 + 192*a^15*b^2))/((64*a^6 + 64*b
^6 + 192*a^2*b^4 + 192*a^4*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)*(a^12
+ b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))
)/(64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2) - (b*((2*a*b^14 + 27*a^3*b
^12 + 129*a^5*b^10 + 62*a^7*b^8 - 156*a^9*b^6 - 105*a^11*b^4 + 9*a^13*b^2)
/(2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a
^10*b^2)) - (64*a^3*b^2*((448*a^8*b^8 - 96*a^4*b^12 - 48*a^6*b^10 - 16*a^2
*b^14 + 912*a^10*b^6 + 672*a^12*b^4 + 176*a^14*b^2)/(2*(a^12 + b^12 + 6*a^
2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)) - (32*a^3*b^2
*(192*a*b^16 + 1344*a^3*b^14 + 4032*a^5*b^12 + 6720*a^7*b^10 + 6720*a^9...
```

3.280. $\int \frac{\cos^2(x)\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx$

3.281 $\int \frac{\cos^3(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx$

3.281.1 Optimal result

Integrand size = 18, antiderivative size = 123

$$\int \frac{\cos^3(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{ab^3\operatorname{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} - \frac{ab^2\cos(x)}{(a^2+b^2)^2} - \frac{a\cos^3(x)}{3(a^2+b^2)} - \frac{a^2b\sin(x)}{(a^2+b^2)^2} + \frac{b\sin(x)}{a^2+b^2} - \frac{b\sin^3(x)}{3(a^2+b^2)}$$

output
```
a*b^3*arctanh((b*cos(x)-a*sin(x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2)-a*b^2*c
os(x)/(a^2+b^2)^2-1/3*a*cos(x)^3/(a^2+b^2)-a^2*b*sin(x)/(a^2+b^2)^2+b*sin(
x)/(a^2+b^2)-1/3*b*sin(x)^3/(a^2+b^2)
```

3.281.2 Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.91

$$\int \frac{\cos^3(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx = -\frac{2ab^3\operatorname{arctanh}\left(\frac{-b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} - \frac{3a(a^2+5b^2)\cos(x)+a(a^2+b^2)\cos(3x)-2b(-a^2+5b^2+(a^2+b^2)\cos(2x))\sin(x)}{12(a^2+b^2)^2}$$

input
```
Integrate[(Cos[x]^3*Sin[x])/(a*Cos[x] + b*Sin[x]),x]
```

output
```
(-2*a*b^3*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) -
(3*a*(a^2 + 5*b^2)*Cos[x] + a*(a^2 + b^2)*Cos[3*x] - 2*b*(-a^2 + 5*b^2 + (
a^2 + b^2)*Cos[2*x])*Sin[x])/(12*(a^2 + b^2)^2)
```

3.281.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 3588, 3042, 3045, 15, 3113, 2009, 3579, 3042, 3117, 3553, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)\cos^3(x)}{a\cos(x)+b\sin(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)\cos(x)^3}{a\cos(x)+b\sin(x)}dx$$

$$\Big\downarrow \text{3588}$$

$$\frac{b\int \cos^3(x)dx}{a^2+b^2}+\frac{a\int \cos^2(x)\sin(x)dx}{a^2+b^2}-\frac{ab\int \frac{\cos^2(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\int \sin\left(x+\frac{\pi}{2}\right)^3dx}{a^2+b^2}+\frac{a\int \cos(x)^2\sin(x)dx}{a^2+b^2}-\frac{ab\int \frac{\cos(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3045}$$

$$\frac{b\int \sin\left(x+\frac{\pi}{2}\right)^3dx}{a^2+b^2}-\frac{a\int \cos^2(x)d\cos(x)}{a^2+b^2}-\frac{ab\int \frac{\cos(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{15}$$

$$\frac{b\int \sin\left(x+\frac{\pi}{2}\right)^3dx}{a^2+b^2}-\frac{ab\int \frac{\cos(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}-\frac{a\cos^3(x)}{3(a^2+b^2)}$$

$$\Big\downarrow \text{3113}$$

$$-\frac{b\int\left(1-\sin^2(x)\right)d(-\sin(x))}{a^2+b^2}-\frac{ab\int\frac{\cos(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}-\frac{a\cos^3(x)}{3\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ab\int\frac{\cos(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}-\frac{b\left(\frac{\sin^3(x)}{3}-\sin(x)\right)}{a^2+b^2}-\frac{a\cos^3(x)}{3\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{3579}$$

$$-\frac{ab\left(\frac{a\int\cos(x)dx}{a^2+b^2}+\frac{b^2\int\frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}-\frac{b\left(\frac{\sin^3(x)}{3}-\sin(x)\right)}{a^2+b^2}-\frac{a\cos^3(x)}{3\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{ab\left(\frac{a\int\sin\left(x+\frac{\pi}{2}\right)dx}{a^2+b^2}+\frac{b^2\int\frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}-\frac{b\left(\frac{\sin^3(x)}{3}-\sin(x)\right)}{a^2+b^2}-\frac{a\cos^3(x)}{3\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{3117}$$

$$-\frac{ab\left(\frac{b^2\int\frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin(x)}{a^2+b^2}+\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}-\frac{b\left(\frac{\sin^3(x)}{3}-\sin(x)\right)}{a^2+b^2}-\frac{a\cos^3(x)}{3\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{3553}$$

$$-\frac{ab\left(-\frac{b^2\int\frac{1}{a^2+b^2-(b\cos(x)-a\sin(x))^2}d(b\cos(x)-a\sin(x))}{a^2+b^2}+\frac{a\sin(x)}{a^2+b^2}+\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}-\frac{b\left(\frac{\sin^3(x)}{3}-\sin(x)\right)}{a^2+b^2}-\frac{a\cos^3(x)}{3\left(a^2+b^2\right)}$$

$$\Big\downarrow \text{219}$$

$$-\frac{ab\left(-\frac{b^2\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}+\frac{a\sin(x)}{a^2+b^2}+\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}-\frac{b\left(\frac{\sin^3(x)}{3}-\sin(x)\right)}{a^2+b^2}-\frac{a\cos^3(x)}{3\left(a^2+b^2\right)}$$

input
```
Int[(Cos[x]^3*Sin[x])/(a*Cos[x] + b*Sin[x]),x]
```

output
```
-1/3*(a*Cos[x]^3)/(a^2 + b^2) - (a*b*(-((b^2*ArcTanh[(b*Cos[x] - a*Sin[x])
/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2)) + (b*Cos[x])/(a^2 + b^2) + (a*Sin[x]
)/(a^2 + b^2)))/(a^2 + b^2) - (b*(-Sin[x] + Sin[x]^3/3))/(a^2 + b^2)
```

3.281. $\int\frac{\cos^3(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx$

3.281.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

3.281. $\int \frac{\cos^3(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx$

rule 3579
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(Cos[c + d*x]^(m - 1)/(d*(a^2 +
b^2)*(m - 1))), x] + (Simp[a/(a^2 + b^2)   Int[Cos[c + d*x]^(m - 1), x], x]
 + Simp[b^2/(a^2 + b^2)   Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[
c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m,
 1]
```

rule 3588
```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b
/(a^2 + b^2)   Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Simp[a/(a
^2 + b^2)   Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Simp[a*(b/(a^
2 + b^2))   Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x]
+ b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
&& IGtQ[m, 0] && IGtQ[n, 0]
```

3.281.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.38

method	result
default	$-\frac{4 a\, b^{3} \operatorname{arctanh}\left(\frac{2 a \tan\left(\frac{x}{2}\right)-2 b}{2 \sqrt{a^{2}+b^{2}}}\right)}{\left(2 a^{4}+4 a^{2} b^{2}+2 b^{4}\right) \sqrt{a^{2}+b^{2}}}+\frac{2 b^{3} \tan\left(\frac{x}{2}\right)^{5}+2\left(-a^{3}-2 a\, b^{2}\right) \tan\left(\frac{x}{2}\right)^{4}+2\left(-\frac{4}{3} a^{2} b+\frac{2}{3} b^{3}\right) \tan\left(\frac{x}{2}\right)^{3}-4 \tan\left(\frac{x}{2}\right)^{2} a\, b^{2}+2 \tan\left(\frac{x}{2}\right) b^{3}-}{\left(a^{4}+2 a^{2} b^{2}+b^{4}\right)\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)^{3}}$
risch	$\frac{3 i \mathrm{e}^{i x} b}{8\left(-2 i b a+a^{2}-b^{2}\right)}-\frac{\mathrm{e}^{i x} a}{8\left(-2 i b a+a^{2}-b^{2}\right)}-\frac{3 i \mathrm{e}^{-i x} b}{8(i b+a)^{2}}-\frac{\mathrm{e}^{-i x} a}{8(i b+a)^{2}}+\frac{i b^{3} a \ln\left(\mathrm{e}^{i x}+\frac{i b+a}{\sqrt{-a^{2}-b^{2}}}\right)}{\sqrt{-a^{2}-b^{2}}\left(a^{2}+b^{2}\right)^{2}}-\frac{i b^{3} a \ln\left(\mathrm{e}^{i x}-\frac{i b+a}{\sqrt{-a^{2}-b^{2}}}\right)}{\sqrt{-a^{2}-b^{2}}\left(a^{2}+b^{2}\right)^{2}}+$

input
```
int(cos(x)^3*sin(x)/(a*cos(x)+b*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
-4*a*b^3/(2*a^4+4*a^2*b^2+2*b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*
x)-2*b)/(a^2+b^2)^(1/2))+2/(a^4+2*a^2*b^2+b^4)*(b^3*tan(1/2*x)^5+(-a^3-2*a
*b^2)*tan(1/2*x)^4+(-4/3*a^2*b+2/3*b^3)*tan(1/2*x)^3-2*tan(1/2*x)^2*a*b^2+
tan(1/2*x)*b^3-1/3*a^3-4/3*a*b^2)/(1+tan(1/2*x)^2)^3
```

3.281.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.73

$$\int \frac{\cos^3(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{3\sqrt{a^2+b^2}ab^3\log\left(\frac{2\,ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2-2\,a^2-b^2-2\,\sqrt{a^2+b^2}(b\cos(x)-a\sin(x))}{2\,ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2+b^2}\right)-2\left(a^5+2\,a^3b^2+ab^4\right)\cos(x}{6\left(a^6+3\,a^4b^2+3\,a^2b^4\right.}$$

input
```
integrate(cos(x)^3*sin(x)/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

output
```
1/6*(3*sqrt(a^2 + b^2)*a*b^3*log((2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)
^2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos(x)*
sin(x) + (a^2 - b^2)*cos(x)^2 + b^2)) - 2*(a^5 + 2*a^3*b^2 + a*b^4)*cos(x)
^3 - 6*(a^3*b^2 + a*b^4)*cos(x) - 2*(a^4*b - a^2*b^3 - 2*b^5 - (a^4*b + 2*
a^2*b^3 + b^5)*cos(x)^2)*sin(x))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
```

3.281.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx = \text{Timed out}$$

input
```
integrate(cos(x)**3*sin(x)/(a*cos(x)+b*sin(x)),x)
```

output
```
Timed out
```

3.281.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 281 vs. 2(115) = 230.

Time = 0.30 (sec) , antiderivative size = 281, normalized size of antiderivative = 2.28

$$\int \frac{\cos^3(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{ab^3\log\left(\frac{b-\frac{a\sin(x)}{\cos(x)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(x)}{\cos(x)+1}-\sqrt{a^2+b^2}}\right)}{(a^4+2a^2b^2+b^4)\sqrt{a^2+b^2}} - \frac{2\left(a^3+4ab^2-\frac{3b^3\sin(x)}{\cos(x)+1}+\frac{6ab^2\sin(x)^2}{(\cos(x)+1)^2}-\frac{3b^3\sin(x)^5}{(\cos(x)+1)^5}+\frac{2(2a^2b-b^3)\sin(x)^3}{(\cos(x)+1)^3}+\frac{3(a^3+2ab^2)\sin(x)^4}{(\cos(x)+1)^4}\right)}{3\left(a^4+2a^2b^2+b^4+\frac{3(a^4+2a^2b^2+b^4)\sin(x)^2}{(\cos(x)+1)^2}+\frac{3(a^4+2a^2b^2+b^4)\sin(x)^4}{(\cos(x)+1)^4}+\frac{(a^4+2a^2b^2+b^4)\sin(x)^6}{(\cos(x)+1)^6}\right)}$$

input
```
integrate(cos(x)^3*sin(x)/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

output
```
a*b^3*log((b - a*sin(x)/(cos(x) + 1) + sqrt(a^2 + b^2))/(b - a*sin(x)/(cos
(x) + 1) - sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) - 2
/3*(a^3 + 4*a*b^2 - 3*b^3*sin(x)/(cos(x) + 1) + 6*a*b^2*sin(x)^2/(cos(x) +
 1)^2 - 3*b^3*sin(x)^5/(cos(x) + 1)^5 + 2*(2*a^2*b - b^3)*sin(x)^3/(cos(x)
 + 1)^3 + 3*(a^3 + 2*a*b^2)*sin(x)^4/(cos(x) + 1)^4)/(a^4 + 2*a^2*b^2 + b^
4 + 3*(a^4 + 2*a^2*b^2 + b^4)*sin(x)^2/(cos(x) + 1)^2 + 3*(a^4 + 2*a^2*b^2
 + b^4)*sin(x)^4/(cos(x) + 1)^4 + (a^4 + 2*a^2*b^2 + b^4)*sin(x)^6/(cos(x)
 + 1)^6)
```

3.281.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.63

$$\int \frac{\cos^3(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{ab^3\log\left(\frac{\left|2a\tan\left(\frac{1}{2}x\right)-2b-2\sqrt{a^2+b^2}\right|}{\left|2a\tan\left(\frac{1}{2}x\right)-2b+2\sqrt{a^2+b^2}\right|}\right)}{(a^4+2a^2b^2+b^4)\sqrt{a^2+b^2}} + \frac{2\left(3b^3\tan\left(\frac{1}{2}x\right)^5-3a^3\tan\left(\frac{1}{2}x\right)^4-6ab^2\tan\left(\frac{1}{2}x\right)^4-4a^2b\tan\left(\frac{1}{2}x\right)^3+2b^3\tan\left(\frac{1}{2}x\right)^3-6ab^2\tan\left(\frac{1}{2}\right.\right.}{3(a^4+2a^2b^2+b^4)\left(\tan\left(\frac{1}{2}x\right)^2+1\right)^3}$$

input
```
integrate(cos(x)^3*sin(x)/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

output
```
a*b^3*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*x)
 - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) + 2
/3*(3*b^3*tan(1/2*x)^5 - 3*a^3*tan(1/2*x)^4 - 6*a*b^2*tan(1/2*x)^4 - 4*a^2
*b*tan(1/2*x)^3 + 2*b^3*tan(1/2*x)^3 - 6*a*b^2*tan(1/2*x)^2 + 3*b^3*tan(1/
2*x) - a^3 - 4*a*b^2)/((a^4 + 2*a^2*b^2 + b^4)*(tan(1/2*x)^2 + 1)^3)
```

3.281.9 Mupad [B] (verification not implemented)

Time = 22.97 (sec) , antiderivative size = 291, normalized size of antiderivative = 2.37

$$\int \frac{\cos^3(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{2\,a\,b^3\,\mathrm{atanh}\left(\frac{2\,a^4\,b+2\,b^5+4\,a^2\,b^3-2\,a\tan\left(\frac{x}{2}\right)\,\left(a^4+2\,a^2\,b^2+b^4\right)}{2\,{\left(a^2+b^2\right)}^{5/2}}\right)}{{\left(a^2+b^2\right)}^{5/2}} - \frac{\frac{2\,\left(a^3+4\,a\,b^2\right)}{3\,\left(a^4+2\,a^2\,b^2+b^4\right)}+\frac{4\,{\tan\left(\frac{x}{2}\right)}^3\,\left(2\,a^2\,b-b^3\right)}{3\,\left(a^4+2\,a^2\,b^2+b^4\right)}-\frac{2\,b^3\tan\left(\frac{x}{2}\right)}{a^4+2\,a^2\,b^2+b^4}+\frac{2\,{\tan\left(\frac{x}{2}\right)}^4\,\left(a^3+2\,a\,b^2\right)}{a^4+2\,a^2\,b^2+b^4}-\frac{2\,b^3\,{\tan\left(\frac{x}{2}\right)}^5}{a^4+2\,a^2\,b^2+b^4}+\frac{4\,a\,b^2\,{\tan\left(\frac{x}{2}\right)}^2}{a^4+2\,a^2\,b^2+b^4}}{{\tan\left(\frac{x}{2}\right)}^6+3\,{\tan\left(\frac{x}{2}\right)}^4+3\,{\tan\left(\frac{x}{2}\right)}^2+1}$$

input
```
int((cos(x)^3*sin(x))/(a*cos(x) + b*sin(x)),x)
```

output
```
(2*a*b^3*atanh((2*a^4*b + 2*b^5 + 4*a^2*b^3 - 2*a*tan(x/2)*(a^4 + b^4 + 2*
a^2*b^2))/(2*(a^2 + b^2)^(5/2))))/(a^2 + b^2)^(5/2) - ((2*(4*a*b^2 + a^3))
/(3*(a^4 + b^4 + 2*a^2*b^2)) + (4*tan(x/2)^3*(2*a^2*b - b^3))/(3*(a^4 + b^
4 + 2*a^2*b^2)) - (2*b^3*tan(x/2))/(a^4 + b^4 + 2*a^2*b^2) + (2*tan(x/2)^4
*(2*a*b^2 + a^3))/(a^4 + b^4 + 2*a^2*b^2) - (2*b^3*tan(x/2)^5)/(a^4 + b^4
+ 2*a^2*b^2) + (4*a*b^2*tan(x/2)^2)/(a^4 + b^4 + 2*a^2*b^2))/(3*tan(x/2)^2
 + 3*tan(x/2)^4 + tan(x/2)^6 + 1)
```

3.282 $\int \frac{\cos^3(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx$

3.282.1 Optimal result

Integrand size = 20, antiderivative size = 175

$$\int \frac{\cos^3(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{a^3b^2x}{(a^2+b^2)^3} - \frac{ab^2x}{2(a^2+b^2)^2} + \frac{ax}{8(a^2+b^2)} - \frac{b\cos^4(x)}{4(a^2+b^2)} + \frac{a^2b^3\log(a\cos(x)+b\sin(x))}{(a^2+b^2)^3} - \frac{ab^2\cos(x)\sin(x)}{2(a^2+b^2)^2} + \frac{a\cos(x)\sin(x)}{8(a^2+b^2)} - \frac{a\cos^3(x)\sin(x)}{4(a^2+b^2)} - \frac{a^2b\sin^2(x)}{2(a^2+b^2)^2}$$

output
```
a^3*b^2*x/(a^2+b^2)^3-1/2*a*b^2*x/(a^2+b^2)^2+1/8*a*x/(a^2+b^2)-1/4*b*cos(
x)^4/(a^2+b^2)+a^2*b^3*ln(a*cos(x)+b*sin(x))/(a^2+b^2)^3-1/2*a*b^2*cos(x)*
sin(x)/(a^2+b^2)^2+1/8*a*cos(x)*sin(x)/(a^2+b^2)-1/4*a*cos(x)^3*sin(x)/(a^
2+b^2)-1/2*a^2*b*sin(x)^2/(a^2+b^2)^2
```

3.282.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.04 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.64

$$\int \frac{\cos^3(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx = -\frac{-4a^5x + 4ia^4bx - 24a^3b^2x - 24ia^2b^3x + 12ab^4x + 4ib^5x - 4ib(a^4-6a^2b^2+b^4)\arctan(\tan(x)) + 4b(-}{}$$

input
```
Integrate[(Cos[x]^3*Sin[x]^2)/(a*Cos[x] + b*Sin[x]),x]
```

output
```
-1/32*(-4*a^5*x + (4*I)*a^4*b*x - 24*a^3*b^2*x - (24*I)*a^2*b^3*x + 12*a*b
^4*x + (4*I)*b^5*x - (4*I)*b*(a^4 - 6*a^2*b^2 + b^4)*ArcTan[Tan[x]] + 4*b*
(-a^4 + b^4)*Cos[2*x] + a^4*b*Cos[4*x] + 2*a^2*b^3*Cos[4*x] + b^5*Cos[4*x]
 - 4*a^4*b*Log[a*Cos[x] + b*Sin[x]] - 8*a^2*b^3*Log[a*Cos[x] + b*Sin[x]] -
 4*b^5*Log[a*Cos[x] + b*Sin[x]] + 2*a^4*b*Log[(a*Cos[x] + b*Sin[x])^2] - 1
2*a^2*b^3*Log[(a*Cos[x] + b*Sin[x])^2] + 2*b^5*Log[(a*Cos[x] + b*Sin[x])^2
] + 8*a^3*b^2*Sin[2*x] + 8*a*b^4*Sin[2*x] + a^5*Sin[4*x] + 2*a^3*b^2*Sin[4
*x] + a*b^4*Sin[4*x])/(a^2 + b^2)^3
```

3.282.3 Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.93, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {3042, 3588, 3042, 3045, 15, 3048, 3042, 3115, 24, 3588, 3042, 3044, 15, 3115, 24, 3577, 3042, 3612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(x)\cos^3(x)}{a\cos(x)+b\sin(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)^2\cos(x)^3}{a\cos(x)+b\sin(x)}dx$$

$$\Big\downarrow \text{3588}$$

$$\frac{b\int \cos^3(x)\sin(x)dx}{a^2+b^2}+\frac{a\int \cos^2(x)\sin^2(x)dx}{a^2+b^2}-\frac{ab\int \frac{\cos^2(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\int \cos(x)^3\sin(x)dx}{a^2+b^2}+\frac{a\int \cos(x)^2\sin(x)^2dx}{a^2+b^2}-\frac{ab\int \frac{\cos(x)^2\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3045}$$

$$-\frac{b\int \cos^3(x)d\cos(x)}{a^2+b^2}+\frac{a\int \cos(x)^2\sin(x)^2dx}{a^2+b^2}-\frac{ab\int \frac{\cos(x)^2\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{15}$$

$$\frac{a\int \cos(x)^2\sin(x)^2dx}{a^2+b^2}-\frac{ab\int \frac{\cos(x)^2\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}-\frac{b\cos^4(x)}{4(a^2+b^2)}$$

$$\Big\downarrow \text{ 3048}$$

$$\frac{a\left(\frac{1}{4}\int \cos^2(x)dx-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}-\frac{ab\int \frac{\cos(x)^2\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}-\frac{b\cos^4(x)}{4(a^2+b^2)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{a\left(\frac{1}{4}\int \sin\left(x+\frac{\pi}{2}\right)^2dx-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}-\frac{ab\int \frac{\cos(x)^2\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}-\frac{b\cos^4(x)}{4(a^2+b^2)}$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{a\left(\frac{1}{4}\left(\frac{\int 1dx}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}-\frac{ab\int \frac{\cos(x)^2\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}-\frac{b\cos^4(x)}{4(a^2+b^2)}$$

$$\Big\downarrow \text{ 24}$$

$$-\frac{ab\int \frac{\cos(x)^2\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}-\frac{b\cos^4(x)}{4(a^2+b^2)}+\frac{a\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}$$

$$\Big\downarrow \text{ 3588}$$

$$-\frac{ab\left(\frac{b\int \cos^2(x)dx}{a^2+b^2}+\frac{a\int \cos(x)\sin(x)dx}{a^2+b^2}-\frac{ab\int \frac{\cos(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}-\frac{b\cos^4(x)}{4(a^2+b^2)}+\frac{a\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{ab\left(\frac{b\int \sin\left(x+\frac{\pi}{2}\right)^2dx}{a^2+b^2}+\frac{a\int \cos(x)\sin(x)dx}{a^2+b^2}-\frac{ab\int \frac{\cos(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}-\frac{b\cos^4(x)}{4(a^2+b^2)}+\frac{a\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}$$

$$\Big\downarrow \text{ 3044}$$

$$-\frac{ab\left(\frac{b\int \sin\left(x+\frac{\pi}{2}\right)^2dx}{a^2+b^2}+\frac{a\int \sin(x)d\sin(x)}{a^2+b^2}-\frac{ab\int \frac{\cos(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}-\frac{b\cos^4(x)}{4(a^2+b^2)}+\frac{a\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}$$

$$\Big\downarrow \text{ 15}$$

3.282. $\int \frac{\cos^3(x)\sin^2(x)}{a\cos(x)+b\sin(x)}dx$

$$-\frac{ab\left(\frac{b\int \sin\left(x+\frac{\pi}{2}\right)^2dx}{a^2+b^2}-\frac{ab\int \frac{\cos(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^2(x)}{2(a^2+b^2)}\right)}{a^2+b^2}-\frac{b\cos^4(x)}{4\left(a^2+b^2\right)}+\frac{a\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}$$

$$\Big\downarrow \text{3115}$$

$$-\frac{ab\left(\frac{b\left(\frac{\int 1dx}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}-\frac{ab\int \frac{\cos(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^2(x)}{2(a^2+b^2)}\right)}{a^2+b^2}-\frac{b\cos^4(x)}{4\left(a^2+b^2\right)}+\frac{a\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}$$

$$\Big\downarrow \text{24}$$

$$-\frac{ab\left(-\frac{ab\int \frac{\cos(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^2(x)}{2(a^2+b^2)}+\frac{b\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}-\frac{b\cos^4(x)}{4\left(a^2+b^2\right)}+\frac{a\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}$$

$$\Big\downarrow \text{3577}$$

$$-\frac{ab\left(-\frac{ab\left(\frac{b\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{ax}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^2(x)}{2(a^2+b^2)}+\frac{b\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}-\frac{b\cos^4(x)}{4\left(a^2+b^2\right)}+\frac{a\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{ab\left(-\frac{ab\left(\frac{b\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{ax}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^2(x)}{2(a^2+b^2)}+\frac{b\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}-\frac{b\cos^4(x)}{4\left(a^2+b^2\right)}+\frac{a\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}$$

$$\Big\downarrow \text{3612}$$

3.282. $\int \frac{\cos^3(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx$

$$-\frac{b\cos^4(x)}{4(a^2+b^2)}+\frac{a\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}-\frac{ab\left(\frac{a\sin^2(x)}{2(a^2+b^2)}+\frac{b\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}-\frac{ab\left(\frac{ax}{a^2+b^2}+\frac{b\log(a\cos(x)+b\sin(x))}{a^2+b^2}\right)}{a^2+b^2}\right)}{a^2+b^2}$$

input
```
Int[(Cos[x]^3*Sin[x]^2)/(a*Cos[x] + b*Sin[x]),x]
```

output
```
-1/4*(b*Cos[x]^4)/(a^2 + b^2) + (a*(-1/4*(Cos[x]^3*Sin[x]) + (x/2 + (Cos[x
]*Sin[x])/2)/4))/(a^2 + b^2) - (a*b*(-((a*b*((a*x)/(a^2 + b^2) + (b*Log[a*
Cos[x] + b*Sin[x]])/(a^2 + b^2)))/(a^2 + b^2)) + (a*Sin[x]^2)/(2*(a^2 + b^
2)) + (b*(x/2 + (Cos[x]*Sin[x])/2))/(a^2 + b^2)))/(a^2 + b^2)
```

3.282.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3577
```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[a*(x/(a^2 + b^2)), x] + Simp[b/(a^2 + b
^2)   Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x
]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3588
```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b
/(a^2 + b^2)   Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Simp[a/(a
^2 + b^2)   Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Simp[a*(b/(a^
2 + b^2))   Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x]
+ b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
&& IGtQ[m, 0] && IGtQ[n, 0]
```

rule 3612
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

3.282. $\int \frac{\cos^3(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx$

3.282.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.93

method	result
default	$\frac{\frac{\left(\frac{1}{8}a^5-\frac{1}{4}a^3b^2-\frac{3}{8}ab^4\right)\tan(x)^3+\left(\frac{1}{2}a^4b+\frac{1}{2}a^2b^3\right)\tan(x)^2+\left(-\frac{3}{4}a^3b^2-\frac{5}{8}ab^4-\frac{1}{8}a^5\right)\tan(x)+\frac{a^4b}{4}-\frac{b^5}{4}}{\left(1+\tan(x)^2\right)^2}+\frac{a\left(-4ab^3\ln\left(1+\tan(x)^2\right)+\left(a^4+6a^2\cdots\right.\right.}{8}}{\left(a^2+b^2\right)^3}$
parallelrisch	$\frac{32a^2b^3\ln\left(\frac{-a\cos(x)-b\sin(x)}{\cos(x)+1}\right)-32a^2b^3\ln\left(\frac{1}{\cos(x)+1}\right)-b\left(a^2+b^2\right)^2\cos(4x)-a\left(a^2+b^2\right)^2\sin(4x)+\left(4a^4b-4b^5\right)\cos(2x)+\left(-8a^3b^2\cdots\right.}{32\left(a^2+b^2\right)^3}$
risch	$\frac{3ixab}{4(6iba^2-2ib^3-2a^3+6ab^2)}-\frac{xa^2}{4(6iba^2-2ib^3-2a^3+6ab^2)}-\frac{b\,e^{2ix}}{16(2iba-a^2+b^2)}-\frac{b\,e^{-2ix}}{16(-ia+b)^2}-\frac{2ia^2b^3x}{a^6+3a^4b^2+3a^2b^4+b^6}+\cdots$
norman	$\frac{\frac{2b^3\tan\left(\frac{x}{2}\right)^2}{a^4+2a^2b^2+b^4}+\frac{2b^3\tan\left(\frac{x}{2}\right)^8}{a^4+2a^2b^2+b^4}+\frac{2\left(-2a^2b+b^3\right)\tan\left(\frac{x}{2}\right)^4}{a^4+2a^2b^2+b^4}+\frac{2\left(-2a^2b+b^3\right)\tan\left(\frac{x}{2}\right)^6}{a^4+2a^2b^2+b^4}+\frac{ax\left(a^4+6a^2b^2-3b^4\right)}{8a^6+24a^4b^2+24a^2b^4+8b^6}-\frac{\left(a^2+5b^2\right)a\tan\left(\frac{x}{2}\right)}{4\left(a^4+2a^2b^2+b^4\right)}+\cdots}{\cdots}$

input
```
int(cos(x)^3*sin(x)^2/(a*cos(x)+b*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
1/(a^2+b^2)^3*(((1/8*a^5-1/4*a^3*b^2-3/8*a*b^4)*tan(x)^3+(1/2*a^4*b+1/2*a^
2*b^3)*tan(x)^2+(-3/4*a^3*b^2-5/8*a*b^4-1/8*a^5)*tan(x)+1/4*a^4*b-1/4*b^5)
/(1+tan(x)^2)^2+1/8*a*(-4*a*b^3*ln(1+tan(x)^2)+(a^4+6*a^2*b^2-3*b^4)*arcta
n(tan(x))))+a^2*b^3/(a^2+b^2)^3*ln(a+b*tan(x))
```

3.282.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00

$$\int \frac{\cos^3(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{4\,a^2b^3\log\left(2\,ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2+b^2\right)-2\left(a^4b+2\,a^2b^3+b^5\right)\cos(x)^4+4\left(a^4b+a^2b^3\right)\cos\cdots}{8\left(a^6+3\,a^4b^2+\cdots\right.}$$

input
```
integrate(cos(x)^3*sin(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

output
```
1/8*(4*a^2*b^3*log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) - 2*(
a^4*b + 2*a^2*b^3 + b^5)*cos(x)^4 + 4*(a^4*b + a^2*b^3)*cos(x)^2 + (a^5 +
6*a^3*b^2 - 3*a*b^4)*x - (2*(a^5 + 2*a^3*b^2 + a*b^4)*cos(x)^3 - (a^5 - 2*
a^3*b^2 - 3*a*b^4)*cos(x))*sin(x))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
```

3.282.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx = \text{Timed out}$$

input
```
integrate(cos(x)**3*sin(x)**2/(a*cos(x)+b*sin(x)),x)
```

output
```
Timed out
```

3.282.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 424 vs. $2(161) = 322$.

Time = 0.31 (sec) , antiderivative size = 424, normalized size of antiderivative = 2.42

$$\int \frac{\cos^3(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{a^2b^3\log\left(-a-\frac{2\,b\sin(x)}{\cos(x)+1}+\frac{a\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} - \frac{a^2b^3\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{(a^5+6\,a^3b^2-3\,ab^4)\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{4\,(a^6+3\,a^4b^2+3\,a^2b^4+b^6)} + \frac{\frac{8\,b^3\sin(x)^2}{(\cos(x)+1)^2}-\frac{16\,a^2b\sin(x)^4}{(\cos(x)+1)^4}+\frac{8\,b^3\sin(x)^6}{(\cos(x)+1)^6}-\frac{(a^3+5\,ab^2)\sin(x)}{\cos(x)+1}+\frac{(7\,a^3+3\,ab^2)\sin(x)^3}{(\cos(x)+1)^3}-\frac{(7\,a^3+3\,ab^2)\sin(x)^5}{(\cos(x)+1)^5}+\frac{(a^3+5\,ab^2)\sin(}{(\cos(x)+1)^7}}{4\left(a^4+2\,a^2b^2+b^4+\frac{4\,(a^4+2\,a^2b^2+b^4)\sin(x)^2}{(\cos(x)+1)^2}+\frac{6\,(a^4+2\,a^2b^2+b^4)\sin(x)^4}{(\cos(x)+1)^4}+\frac{4\,(a^4+2\,a^2b^2+b^4)\sin(x)^6}{(\cos(x)+1)^6}+\frac{(a^4+2\,a^2b^2+b^4)\sin(x}{(\cos(x)+1)^8}\right.}$$

input
```
integrate(cos(x)^3*sin(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

output
```
a^2*b^3*log(-a - 2*b*sin(x)/(cos(x) + 1) + a*sin(x)^2/(cos(x) + 1)^2)/(a^6
 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - a^2*b^3*log(sin(x)^2/(cos(x) + 1)^2 + 1)
/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 1/4*(a^5 + 6*a^3*b^2 - 3*a*b^4)*arc
tan(sin(x)/(cos(x) + 1))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 1/4*(8*b^3*
sin(x)^2/(cos(x) + 1)^2 - 16*a^2*b*sin(x)^4/(cos(x) + 1)^4 + 8*b^3*sin(x)^
6/(cos(x) + 1)^6 - (a^3 + 5*a*b^2)*sin(x)/(cos(x) + 1) + (7*a^3 + 3*a*b^2)
*sin(x)^3/(cos(x) + 1)^3 - (7*a^3 + 3*a*b^2)*sin(x)^5/(cos(x) + 1)^5 + (a^
3 + 5*a*b^2)*sin(x)^7/(cos(x) + 1)^7)/(a^4 + 2*a^2*b^2 + b^4 + 4*(a^4 + 2*
a^2*b^2 + b^4)*sin(x)^2/(cos(x) + 1)^2 + 6*(a^4 + 2*a^2*b^2 + b^4)*sin(x)^
4/(cos(x) + 1)^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*sin(x)^6/(cos(x) + 1)^6 + (a^
4 + 2*a^2*b^2 + b^4)*sin(x)^8/(cos(x) + 1)^8)
```

3.282.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.56

$$\int \frac{\cos^3(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx$$
$$= \frac{a^2b^4\log\left(|b\tan\left(x\right)+a|\right)}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7} - \frac{a^2b^3\log\left(\tan\left(x\right)^2+1\right)}{2\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)} + \frac{(a^5+6\,a^3b^2-3\,ab^4)x}{8\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)}$$
$$+ \frac{6\,a^2b^3\tan\left(x\right)^4+a^5\tan\left(x\right)^3-2\,a^3b^2\tan\left(x\right)^3-3\,ab^4\tan\left(x\right)^3+4\,a^4b\tan\left(x\right)^2+16\,a^2b^3\tan\left(x\right)^2-a^5\tan}{8\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)\left(\tan\left(x\right)^2+1\right)^2}$$

input
```
integrate(cos(x)^3*sin(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

output
```
a^2*b^4*log(abs(b*tan(x) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7) - 1/2
*a^2*b^3*log(tan(x)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 1/8*(a^5
+ 6*a^3*b^2 - 3*a*b^4)*x/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 1/8*(6*a^2*
b^3*tan(x)^4 + a^5*tan(x)^3 - 2*a^3*b^2*tan(x)^3 - 3*a*b^4*tan(x)^3 + 4*a^
4*b*tan(x)^2 + 16*a^2*b^3*tan(x)^2 - a^5*tan(x) - 6*a^3*b^2*tan(x) - 5*a*b
^4*tan(x) + 2*a^4*b + 6*a^2*b^3 - 2*b^5)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b
^6)*(tan(x)^2 + 1)^2)
```

3.282.9 Mupad [B] (verification not implemented)

Time = 36.11 (sec) , antiderivative size = 5870, normalized size of antiderivative = 33.54

$$\int \frac{\cos^3(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx = \text{Too large to display}$$

input
```
int((cos(x)^3*sin(x)^2)/(a*cos(x) + b*sin(x)),x)
```

output
```
((tan(x/2)^3*(3*a*b^2 + 7*a^3))/(4*(a^4 + b^4 + 2*a^2*b^2)) - (tan(x/2)^5*
(3*a*b^2 + 7*a^3))/(4*(a^4 + b^4 + 2*a^2*b^2)) - (tan(x/2)*(5*a*b^2 + a^3)
)/(4*(a^4 + b^4 + 2*a^2*b^2)) + (tan(x/2)^7*(5*a*b^2 + a^3))/(4*(a^4 + b^4
 + 2*a^2*b^2)) + (2*b^3*tan(x/2)^2)/(a^4 + b^4 + 2*a^2*b^2) + (2*b^3*tan(x
/2)^6)/(a^4 + b^4 + 2*a^2*b^2) - (4*a^2*b*tan(x/2)^4)/(a^4 + b^4 + 2*a^2*b
^2))/(4*tan(x/2)^2 + 6*tan(x/2)^4 + 4*tan(x/2)^6 + tan(x/2)^8 + 1) - (a*at
an((tan(x/2)*((((64*a^2*b^3*((a*((16*a^15*b + 16*a^3*b^13 + 288*a^5*b^11 +
 1008*a^7*b^9 + 1472*a^9*b^7 + 1008*a^11*b^5 + 288*a^13*b^3)/(2*(a^12 + b^
12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)) - (3
2*a^2*b^3*(192*a*b^16 + 1344*a^3*b^14 + 4032*a^5*b^12 + 6720*a^7*b^10 + 67
20*a^9*b^8 + 4032*a^11*b^6 + 1344*a^13*b^4 + 192*a^15*b^2))/((64*a^6 + 64*
b^6 + 192*a^2*b^4 + 192*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 +
20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(a^4 - 3*b^4 + 6*a^2*b^2))/(8*(a^6
 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (4*a^3*b^3*(a^4 - 3*b^4 + 6*a^2*b^2)*(1
92*a*b^16 + 1344*a^3*b^14 + 4032*a^5*b^12 + 6720*a^7*b^10 + 6720*a^9*b^8 +
 4032*a^11*b^6 + 1344*a^13*b^4 + 192*a^15*b^2))/((64*a^6 + 64*b^6 + 192*a^
2*b^4 + 192*a^4*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)*(a^12 + b^12 + 6*
a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))))/(64*a^6 +
 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2) - (a*((a^15 + 18*a^3*b^12 - 141*a^5*b
^10 - 327*a^7*b^8 - 146*a^9*b^6 + 36*a^11*b^4 + 15*a^13*b^2)/(2*(a^12 +...
```

3.282. $\int \frac{\cos^3(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx$

3.283 $\int \frac{\cos^3(x)\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx$

3.283.1 Optimal result

Integrand size = 20, antiderivative size = 193

$$\int \frac{\cos^3(x)\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{a^3b^3\operatorname{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}} - \frac{a^3b^2\cos(x)}{(a^2+b^2)^3} + \frac{ab^2\cos^3(x)}{3(a^2+b^2)^2} - \frac{a\cos^3(x)}{3(a^2+b^2)} + \frac{a\cos^5(x)}{5(a^2+b^2)} + \frac{a^2b^3\sin(x)}{(a^2+b^2)^3} - \frac{a^2b\sin^3(x)}{3(a^2+b^2)^2} + \frac{b\sin^3(x)}{3(a^2+b^2)} - \frac{b\sin^5(x)}{5(a^2+b^2)}$$

output
```
a^3*b^3*arctanh((b*cos(x)-a*sin(x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(7/2)-a^3*b
^2*cos(x)/(a^2+b^2)^3+1/3*a*b^2*cos(x)^3/(a^2+b^2)^2-1/3*a*cos(x)^3/(a^2+b
^2)+1/5*a*cos(x)^5/(a^2+b^2)+a^2*b^3*sin(x)/(a^2+b^2)^3-1/3*a^2*b*sin(x)^3
/(a^2+b^2)^2+1/3*b*sin(x)^3/(a^2+b^2)-1/5*b*sin(x)^5/(a^2+b^2)
```

3.283.2 Mathematica [A] (verified)

Time = 1.64 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.16

$$\int \frac{\cos^3(x)\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx = -\frac{2a^3b^3\operatorname{arctanh}\left(\frac{-b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}} + \frac{-30a(a^4+8a^2b^2-b^4)\cos(x)-5a(a^4-2a^2b^2-3b^4)\cos(3x)+3a^5\cos(5x)+6a^3b^2\cos(5x)+3ab^4\cos(}{}$$

input
```
Integrate[(Cos[x]^3*Sin[x]^3)/(a*Cos[x] + b*Sin[x]),x]
```

output
```
(-2*a^3*b^3*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(7/2)
+ (-30*a*(a^4 + 8*a^2*b^2 - b^4)*Cos[x] - 5*a*(a^4 - 2*a^2*b^2 - 3*b^4)*Co
s[3*x] + 3*a^5*Cos[5*x] + 6*a^3*b^2*Cos[5*x] + 3*a*b^4*Cos[5*x] - 30*a^4*b
*Sin[x] + 240*a^2*b^3*Sin[x] + 30*b^5*Sin[x] + 15*a^4*b*Sin[3*x] + 10*a^2*
b^3*Sin[3*x] - 5*b^5*Sin[3*x] - 3*a^4*b*Sin[5*x] - 6*a^2*b^3*Sin[5*x] - 3*
b^5*Sin[5*x])/(240*(a^2 + b^2)^3)
```

3.283.3 Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.96, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.050, Rules used = {3042, 3588, 3042, 3044, 244, 2009, 3045, 244, 2009, 3588, 3042, 3044, 15, 3045, 15, 3588, 3042, 3117, 3118, 3553, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(x)\cos^3(x)}{a\cos(x)+b\sin(x)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sin(x)^3\cos(x)^3}{a\cos(x)+b\sin(x)}dx$$

$$\Big\downarrow \ 3588$$

$$\frac{b\int \cos^3(x)\sin^2(x)dx}{a^2+b^2}+\frac{a\int \cos^2(x)\sin^3(x)dx}{a^2+b^2}-\frac{ab\int \frac{\cos^2(x)\sin^2(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{b\int \cos(x)^3\sin(x)^2dx}{a^2+b^2}+\frac{a\int \cos(x)^2\sin(x)^3dx}{a^2+b^2}-\frac{ab\int \frac{\cos(x)^2\sin(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \ 3044$$

$$\frac{b\int \sin^2(x)\left(1-\sin^2(x)\right)d\sin(x)}{a^2+b^2}+\frac{a\int \cos(x)^2\sin(x)^3dx}{a^2+b^2}-\frac{ab\int \frac{\cos(x)^2\sin(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \ 244$$

$$\frac{b\int\left(\sin^2(x)-\sin^4(x)\right)d\sin(x)}{a^2+b^2}+\frac{a\int\cos(x)^2\sin(x)^3dx}{a^2+b^2}-\frac{ab\int\frac{\cos(x)^2\sin(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a\int\cos(x)^2\sin(x)^3dx}{a^2+b^2}-\frac{ab\int\frac{\cos(x)^2\sin(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\left(\frac{\sin^3(x)}{3}-\frac{\sin^5(x)}{5}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{ 3045}$$

$$-\frac{a\int\cos^2(x)\left(1-\cos^2(x)\right)d\cos(x)}{a^2+b^2}-\frac{ab\int\frac{\cos(x)^2\sin(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\left(\frac{\sin^3(x)}{3}-\frac{\sin^5(x)}{5}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{ 244}$$

$$-\frac{a\int\left(\cos^2(x)-\cos^4(x)\right)d\cos(x)}{a^2+b^2}-\frac{ab\int\frac{\cos(x)^2\sin(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\left(\frac{\sin^3(x)}{3}-\frac{\sin^5(x)}{5}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{ab\int\frac{\cos(x)^2\sin(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\left(\frac{\sin^3(x)}{3}-\frac{\sin^5(x)}{5}\right)}{a^2+b^2}-\frac{a\left(\frac{\cos^3(x)}{3}-\frac{\cos^5(x)}{5}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{ 3588}$$

$$-\frac{ab\left(\frac{b\int\cos^2(x)\sin(x)dx}{a^2+b^2}+\frac{a\int\cos(x)\sin^2(x)dx}{a^2+b^2}-\frac{ab\int\frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\left(\frac{\sin^3(x)}{3}-\frac{\sin^5(x)}{5}\right)}{a^2+b^2}-\frac{a\left(\frac{\cos^3(x)}{3}-\frac{\cos^5(x)}{5}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{ab\left(\frac{b\int\cos(x)^2\sin(x)dx}{a^2+b^2}+\frac{a\int\cos(x)\sin(x)^2dx}{a^2+b^2}-\frac{ab\int\frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\left(\frac{\sin^3(x)}{3}-\frac{\sin^5(x)}{5}\right)}{a^2+b^2}-\frac{a\left(\frac{\cos^3(x)}{3}-\frac{\cos^5(x)}{5}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{ 3044}$$

$$-\frac{ab\left(\frac{a\int\sin^2(x)d\sin(x)}{a^2+b^2}+\frac{b\int\cos(x)^2\sin(x)dx}{a^2+b^2}-\frac{ab\int\frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\left(\frac{\sin^3(x)}{3}-\frac{\sin^5(x)}{5}\right)}{a^2+b^2}-\frac{a\left(\frac{\cos^3(x)}{3}-\frac{\cos^5(x)}{5}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{ 15}$$

3.283. $\int\frac{\cos^3(x)\sin^3(x)}{a\cos(x)+b\sin(x)}dx$

$$-\frac{ab\left(\frac{b\int \cos(x)^2\sin(x)dx}{a^2+b^2}-\frac{ab\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}+\frac{b\left(\frac{\sin^3(x)}{3}-\frac{\sin^5(x)}{5}\right)}{a^2+b^2}-\frac{a\left(\frac{\cos^3(x)}{3}-\frac{\cos^5(x)}{5}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{ 3045}$$

$$-\frac{ab\left(-\frac{b\int \cos^2(x)d\cos(x)}{a^2+b^2}-\frac{ab\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}+\frac{b\left(\frac{\sin^3(x)}{3}-\frac{\sin^5(x)}{5}\right)}{a^2+b^2}-\frac{a\left(\frac{\cos^3(x)}{3}-\frac{\cos^5(x)}{5}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{ab\left(-\frac{ab\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{b\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}+\frac{b\left(\frac{\sin^3(x)}{3}-\frac{\sin^5(x)}{5}\right)}{a^2+b^2}-\frac{a\left(\frac{\cos^3(x)}{3}-\frac{\cos^5(x)}{5}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{ 3588}$$

$$-\frac{ab\left(-\frac{ab\left(\frac{a\int \sin(x)dx}{a^2+b^2}+\frac{b\int \cos(x)dx}{a^2+b^2}-\frac{ab\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{b\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}+\frac{b\left(\frac{\sin^3(x)}{3}-\frac{\sin^5(x)}{5}\right)}{a^2+b^2}-\frac{a\left(\frac{\cos^3(x)}{3}-\frac{\cos^5(x)}{5}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{ab\left(-\frac{ab\left(\frac{a\int \sin(x)dx}{a^2+b^2}+\frac{b\int \sin\left(x+\frac{\pi}{2}\right)dx}{a^2+b^2}-\frac{ab\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{b\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}+\frac{b\left(\frac{\sin^3(x)}{3}-\frac{\sin^5(x)}{5}\right)}{a^2+b^2}-\frac{a\left(\frac{\cos^3(x)}{3}-\frac{\cos^5(x)}{5}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{ 3117}$$

$$-\frac{ab\left(-\frac{ab\left(\frac{a\int \sin(x)dx}{a^2+b^2}-\frac{ab\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\sin(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{b\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}+\frac{b\left(\frac{\sin^3(x)}{3}-\frac{\sin^5(x)}{5}\right)}{a^2+b^2}-\frac{a\left(\frac{\cos^3(x)}{3}-\frac{\cos^5(x)}{5}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{ 3118}$$

3.283. $\int \frac{\cos^3(x)\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx$

$$-\frac{ab\left(-\frac{ab\left(-\frac{ab\int\frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\sin(x)}{a^2+b^2}-\frac{a\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{b\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}+\frac{b\left(\frac{\sin^3(x)}{3}-\frac{\sin^5(x)}{5}\right)}{a^2+b^2}-\frac{a\left(\frac{\cos^3(x)}{3}-\frac{\cos^5(x)}{5}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{3553}$$

$$-\frac{ab\left(-\frac{ab\left(\frac{ab\int\frac{1}{a^2+b^2-(b\cos(x)-a\sin(x))^2}d(b\cos(x)-a\sin(x))}{a^2+b^2}+\frac{b\sin(x)}{a^2+b^2}-\frac{a\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{b\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}+\frac{b\left(\frac{\sin^3(x)}{3}-\frac{\sin^5(x)}{5}\right)}{a^2+b^2}-\frac{a\left(\frac{\cos^3(x)}{3}-\frac{\cos^5(x)}{5}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{219}$$

$$-\frac{ab\left(-\frac{ab\left(\frac{ab\operatorname{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{b\sin(x)}{a^2+b^2}-\frac{a\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{b\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}+\frac{b\left(\frac{\sin^3(x)}{3}-\frac{\sin^5(x)}{5}\right)}{a^2+b^2}-\frac{a\left(\frac{\cos^3(x)}{3}-\frac{\cos^5(x)}{5}\right)}{a^2+b^2}$$

input
```
Int[(Cos[x]^3*Sin[x]^3)/(a*Cos[x] + b*Sin[x]),x]
```

output
```
-((a*(Cos[x]^3/3 - Cos[x]^5/5))/(a^2 + b^2)) + (b*(Sin[x]^3/3 - Sin[x]^5/5
))/(a^2 + b^2) - (a*b*(-1/3*(b*Cos[x]^3)/(a^2 + b^2) + (a*Sin[x]^3)/(3*(a^
2 + b^2)) - (a*b*((a*b*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^
2 + b^2)^(3/2) - (a*Cos[x])/(a^2 + b^2) + (b*Sin[x])/(a^2 + b^2)))/(a^2 +
b^2)))/(a^2 + b^2)
```

3.283. $\int \frac{\cos^3(x)\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx$

3.283.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

3.283. $\int \frac{\cos^3(x)\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx$

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3588
```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b
/(a^2 + b^2)   Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Simp[a/(a
^2 + b^2)   Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Simp[a*(b/(a^
2 + b^2))   Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x]
+ b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
&& IGtQ[m, 0] && IGtQ[n, 0]
```

3.283.4 Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.56

method	result
default	$\frac{2a^2b^3\tan\left(\frac{x}{2}\right)^9+2a\,b^4\tan\left(\frac{x}{2}\right)^8+2\left(\frac{16}{3}a^2b^3+\frac{4}{3}b^5\right)\tan\left(\frac{x}{2}\right)^7+2\left(-2a^5-6a^3b^2\right)\tan\left(\frac{x}{2}\right)^6+2\left(-\frac{16}{5}a^4b+\frac{34}{15}a^2b^3-\frac{8}{15}b^5\right)\tan\left(\frac{x}{2}\right)^5+2\left(\frac{2}{3}a^5-\right.}{\left(a^4+2a^2b^2+b^4\right)\left(a^2+b^2\right)\left(1\right.}$
risch	$-\frac{ie^{3ix}b}{96(-2iba+a^2-b^2)}-\frac{e^{3ix}a}{96(-2iba+a^2-b^2)}+\frac{ie^{ix}ba}{-12ib\,a^2+4ib^3+4a^3-12a\,b^2}-\frac{e^{ix}a^2}{16(-3ib\,a^2+ib^3+a^3-3a\,b^2)}+\frac{e^{ix}b^2}{-48ib\,a^2+16ib^3+}$

input
```
int(cos(x)^3*sin(x)^3/(a*cos(x)+b*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
2/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*(a^2*b^3*tan(1/2*x)^9+a*b^4*tan(1/2*x)^8+(
16/3*a^2*b^3+4/3*b^5)*tan(1/2*x)^7+(-2*a^5-6*a^3*b^2)*tan(1/2*x)^6+(-16/5*
a^4*b+34/15*a^2*b^3-8/15*b^5)*tan(1/2*x)^5+(2/3*a^5-10/3*a^3*b^2+2*a*b^4)*
tan(1/2*x)^4+(16/3*a^2*b^3+4/3*b^5)*tan(1/2*x)^3+(-2/3*a^5-14/3*a^3*b^2)*t
an(1/2*x)^2+tan(1/2*x)*a^2*b^3-2/15*a^5-14/15*a^3*b^2+1/5*a*b^4)/(1+tan(1/
2*x)^2)^5-16*a^3*b^3/(8*a^6+24*a^4*b^2+24*a^2*b^4+8*b^6)/(a^2+b^2)^(1/2)*a
rctanh(1/2*(2*a*tan(1/2*x)-2*b)/(a^2+b^2)^(1/2))
```

3.283.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.59

$$\int \frac{\cos^3(x)\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx$$
$$= \frac{15\sqrt{a^2+b^2}a^3b^3\log\left(\frac{2\,ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2-2\,a^2-b^2-2\,\sqrt{a^2+b^2}(b\cos(x)-a\sin(x))}{2\,ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2+b^2}\right)+6\,(a^7+3\,a^5b^2+3\,a^3b^4+}{}$$

input
```
integrate(cos(x)^3*sin(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

output
```
1/30*(15*sqrt(a^2 + b^2)*a^3*b^3*log((2*a*b*cos(x)*sin(x) + (a^2 - b^2)*co
s(x)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos
(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2)) + 6*(a^7 + 3*a^5*b^2 + 3*a^3*b^4
 + a*b^6)*cos(x)^5 - 10*(a^7 + 2*a^5*b^2 + a^3*b^4)*cos(x)^3 - 30*(a^5*b^2
 + a^3*b^4)*cos(x) - 2*(3*a^6*b - 11*a^4*b^3 - 16*a^2*b^5 - 2*b^7 + 3*(a^6
*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cos(x)^4 - (6*a^6*b + 13*a^4*b^3 + 8*a^2
*b^5 + b^7)*cos(x)^2)*sin(x))/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b
^8)
```

3.283.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(x)\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx = \text{Timed out}$$

input
```
integrate(cos(x)**3*sin(x)**3/(a*cos(x)+b*sin(x)),x)
```

output
```
Timed out
```

3.283.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 521 vs. $2(177) = 354$.

Time = 0.32 (sec) , antiderivative size = 521, normalized size of antiderivative = 2.70

$$\int \frac{\cos^3(x)\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{a^3b^3\log\left(\frac{b-\frac{a\sin(x)}{\cos(x)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(x)}{\cos(x)+1}-\sqrt{a^2+b^2}}\right)}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\sqrt{a^2+b^2}} - \frac{2\left(2\,a^5+14\,a^3b^2-3\,ab^4-\frac{15\,a^2b^3\sin(x)}{\cos(x)+1}-\frac{15\,ab^4\sin(x)^8}{(\cos(x)+1)^8}-\frac{15\,a^2b^3\sin(x)^9}{(\cos(x)+1)^9}+\frac{10\,(a^5+7\,a^3b^2)\sin(x)^2}{(\cos(x)+1)^2}-\frac{20\,(4\,a^2b^3+b^5)\sin(}{(\cos(x)+1)^3}\right.}{15\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6+\frac{5\,(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\sin(x)^2}{(\cos(x)+1)^2}+\frac{10\,(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\sin(x)^4}{(\cos(x)+1)^4}+\right.}$$

input
```
integrate(cos(x)^3*sin(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

output
```
a^3*b^3*log((b - a*sin(x)/(cos(x) + 1) + sqrt(a^2 + b^2))/(b - a*sin(x)/(c
os(x) + 1) - sqrt(a^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a
^2 + b^2)) - 2/15*(2*a^5 + 14*a^3*b^2 - 3*a*b^4 - 15*a^2*b^3*sin(x)/(cos(x
) + 1) - 15*a*b^4*sin(x)^8/(cos(x) + 1)^8 - 15*a^2*b^3*sin(x)^9/(cos(x) +
1)^9 + 10*(a^5 + 7*a^3*b^2)*sin(x)^2/(cos(x) + 1)^2 - 20*(4*a^2*b^3 + b^5)
*sin(x)^3/(cos(x) + 1)^3 - 10*(a^5 - 5*a^3*b^2 + 3*a*b^4)*sin(x)^4/(cos(x)
 + 1)^4 + 2*(24*a^4*b - 17*a^2*b^3 + 4*b^5)*sin(x)^5/(cos(x) + 1)^5 + 30*(
a^5 + 3*a^3*b^2)*sin(x)^6/(cos(x) + 1)^6 - 20*(4*a^2*b^3 + b^5)*sin(x)^7/(
cos(x) + 1)^7)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 5*(a^6 + 3*a^4*b^2 + 3
*a^2*b^4 + b^6)*sin(x)^2/(cos(x) + 1)^2 + 10*(a^6 + 3*a^4*b^2 + 3*a^2*b^4
+ b^6)*sin(x)^4/(cos(x) + 1)^4 + 10*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*si
n(x)^6/(cos(x) + 1)^6 + 5*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sin(x)^8/(co
s(x) + 1)^8 + (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sin(x)^10/(cos(x) + 1)^1
0)
```

3.283.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 361 vs. $2(177) = 354$.

Time = 0.32 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.87

$$\int \frac{\cos^3(x)\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{a^3b^3\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\sqrt{a^2+b^2}} + \frac{2\left(15\,a^2b^3\tan\left(\frac{1}{2}\,x\right)^9+15\,ab^4\tan\left(\frac{1}{2}\,x\right)^8+80\,a^2b^3\tan\left(\frac{1}{2}\,x\right)^7+20\,b^5\tan\left(\frac{1}{2}\,x\right)^7-30\,a^5\tan\left(\frac{1}{2}\,x\right)^6-90\,a\right.}{}$$

input
```
integrate(cos(x)^3*sin(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

output
```
a^3*b^3*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*
x) - 2*b + 2*sqrt(a^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a
^2 + b^2)) + 2/15*(15*a^2*b^3*tan(1/2*x)^9 + 15*a*b^4*tan(1/2*x)^8 + 80*a^
2*b^3*tan(1/2*x)^7 + 20*b^5*tan(1/2*x)^7 - 30*a^5*tan(1/2*x)^6 - 90*a^3*b^
2*tan(1/2*x)^6 - 48*a^4*b*tan(1/2*x)^5 + 34*a^2*b^3*tan(1/2*x)^5 - 8*b^5*t
an(1/2*x)^5 + 10*a^5*tan(1/2*x)^4 - 50*a^3*b^2*tan(1/2*x)^4 + 30*a*b^4*tan
(1/2*x)^4 + 80*a^2*b^3*tan(1/2*x)^3 + 20*b^5*tan(1/2*x)^3 - 10*a^5*tan(1/2
*x)^2 - 70*a^3*b^2*tan(1/2*x)^2 + 15*a^2*b^3*tan(1/2*x) - 2*a^5 - 14*a^3*b
^2 + 3*a*b^4)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*(tan(1/2*x)^2 + 1)^5)
```

3.283.9 Mupad [B] (verification not implemented)

Time = 23.58 (sec) , antiderivative size = 600, normalized size of antiderivative = 3.11

$$\int \frac{\cos^3(x)\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{\frac{8\tan\left(\frac{x}{2}\right)^3(4a^2b^3+b^5)}{3(a^6+3a^4b^2+3a^2b^4+b^6)} - \frac{4\tan\left(\frac{x}{2}\right)^2(a^5+7a^3b^2)}{3(a^6+3a^4b^2+3a^2b^4+b^6)} - \frac{4\tan\left(\frac{x}{2}\right)^6(a^5+3a^3b^2)}{a^6+3a^4b^2+3a^2b^4+b^6} - \frac{2(2a^5+14a^3b^2-3ab^4)}{15(a^6+3a^4b^2+3a^2b^4+b^6)} + \frac{4\tan\left(\frac{x}{2}\right)^4(a^5-5a^3b^{\cdots}}{3(a^6+3a^4b^2+3a^2\cdots}}{\tan\left(\frac{x}{2}\right)^{10}+5\tan\left(\frac{x}{2}\right)^8+10\cdots} + \frac{2a^3b^3\,\mathrm{atanh}\left(\frac{2a^6b+2b^7+6a^2b^5+6a^4b^3-2a\tan\left(\frac{x}{2}\right)(a^6+3a^4b^2+3a^2b^4+b^6)}{2(a^2+b^2)^{7/2}}\right)}{(a^2+b^2)^{7/2}}$$

input
```
int((cos(x)^3*sin(x)^3)/(a*cos(x) + b*sin(x)),x)
```

output
```
((8*tan(x/2)^3*(b^5 + 4*a^2*b^3))/(3*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))
- (4*tan(x/2)^2*(a^5 + 7*a^3*b^2))/(3*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))
 - (4*tan(x/2)^6*(a^5 + 3*a^3*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) -
(2*(2*a^5 - 3*a*b^4 + 14*a^3*b^2))/(15*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)
) + (4*tan(x/2)^4*(3*a*b^4 + a^5 - 5*a^3*b^2))/(3*(a^6 + b^6 + 3*a^2*b^4 +
 3*a^4*b^2)) + (8*b^3*tan(x/2)^7*(4*a^2 + b^2))/(3*(a^6 + b^6 + 3*a^2*b^4
+ 3*a^4*b^2)) + (2*a^2*b^3*tan(x/2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) +
 (2*a*b^4*tan(x/2)^8)/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (2*a^2*b^3*tan
(x/2)^9)/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (4*b*tan(x/2)^5*(24*a^4 + 4
*b^4 - 17*a^2*b^2))/(15*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))/(5*tan(x/2)^
2 + 10*tan(x/2)^4 + 10*tan(x/2)^6 + 5*tan(x/2)^8 + tan(x/2)^10 + 1) + (2*a
^3*b^3*atanh((2*a^6*b + 2*b^7 + 6*a^2*b^5 + 6*a^4*b^3 - 2*a*tan(x/2)*(a^6
+ b^6 + 3*a^2*b^4 + 3*a^4*b^2))/(2*(a^2 + b^2)^(7/2))))/(a^2 + b^2)^(7/2)
```

3.284 $\int \frac{\cos(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

3.284.1 Optimal result

Integrand size = 16, antiderivative size = 70

$$\int \frac{\cos(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{2abx}{(a^2+b^2)^2} - \frac{(a^2-b^2)\log(a\cos(x)+b\sin(x))}{(a^2+b^2)^2} - \frac{b\sin(x)}{(a^2+b^2)(a\cos(x)+b\sin(x))}$$

output
```
2*a*b*x/(a^2+b^2)^2-(a^2-b^2)*ln(a*cos(x)+b*sin(x))/(a^2+b^2)^2-b*sin(x)/(
a^2+b^2)/(a*cos(x)+b*sin(x))
```

3.284.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.06

$$\int \frac{\cos(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{a\cos(x)\left(-2i(a+ib)^2x+(-a^2+b^2)\log\left((a\cos(x)+b\sin(x))^2\right)\right)+b(2(a+ib)(a(-1-ix)+b(i+x))+}{2\left(a^2+b^2\right)^2(a\cos(x)}$$

input
```
Integrate[(Cos[x]*Sin[x])/(a*Cos[x] + b*Sin[x])^2,x]
```

output
```
(a*Cos[x]*((-2*I)*(a + I*b)^2*x + (-a^2 + b^2)*Log[(a*Cos[x] + b*Sin[x])^2
]) + b*(2*(a + I*b)*(a*(-1 - I*x) + b*(I + x)) + (-a^2 + b^2)*Log[(a*Cos[x
] + b*Sin[x])^2])*Sin[x] + (2*I)*(a^2 - b^2)*ArcTan[Tan[x]]*(a*Cos[x] + b*
Sin[x]))/(2*(a^2 + b^2)^2*(a*Cos[x] + b*Sin[x]))
```

3.284.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.67, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {3042, 3590, 3042, 3554, 3576, 3042, 3577, 3042, 3612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)\cos(x)}{(a\cos(x)+b\sin(x))^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(x)\cos(x)}{(a\cos(x)+b\sin(x))^2}dx$$

$$\Big\downarrow \text{ 3590}$$

$$-\frac{ab\int \frac{1}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{b\int \frac{\cos(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{ab\int \frac{1}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{b\int \frac{\cos(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{ 3554}$$

$$\frac{b\int \frac{\cos(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}-\frac{b\sin(x)}{(a^2+b^2)(a\cos(x)+b\sin(x))}$$

$$\Big\downarrow \text{ 3576}$$

$$\frac{b\int \frac{\cos(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\left(\frac{bx}{a^2+b^2}-\frac{a\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}-\frac{b\sin(x)}{(a^2+b^2)(a\cos(x)+b\sin(x))}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{b\int \frac{\cos(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\left(\frac{bx}{a^2+b^2}-\frac{a\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}-\frac{b\sin(x)}{(a^2+b^2)(a\cos(x)+b\sin(x))}$$

$$\Big\downarrow \, 3577$$

$$\frac{a\left(\frac{bx}{a^2+b^2}-\frac{a\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\left(\frac{b\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{ax}{a^2+b^2}\right)}{a^2+b^2}-\frac{b\sin(x)}{(a^2+b^2)\,(a\cos(x)+b\sin(x))}$$

$$\Big\downarrow \, 3042$$

$$\frac{a\left(\frac{bx}{a^2+b^2}-\frac{a\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\left(\frac{b\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{ax}{a^2+b^2}\right)}{a^2+b^2}-\frac{b\sin(x)}{(a^2+b^2)\,(a\cos(x)+b\sin(x))}$$

$$\Big\downarrow \, 3612$$

$$-\frac{b\sin(x)}{(a^2+b^2)\,(a\cos(x)+b\sin(x))}+\frac{a\left(\frac{bx}{a^2+b^2}-\frac{a\log(a\cos(x)+b\sin(x))}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\left(\frac{ax}{a^2+b^2}+\frac{b\log(a\cos(x)+b\sin(x))}{a^2+b^2}\right)}{a^2+b^2}$$

input
```
Int[(Cos[x]*Sin[x])/(a*Cos[x] + b*Sin[x])^2,x]
```

output
```
(a*((b*x)/(a^2 + b^2) - (a*Log[a*Cos[x] + b*Sin[x]])/(a^2 + b^2)))/(a^2 +
b^2) + (b*((a*x)/(a^2 + b^2) + (b*Log[a*Cos[x] + b*Sin[x]])/(a^2 + b^2)))/
(a^2 + b^2) - (b*Sin[x])/((a^2 + b^2)*(a*Cos[x] + b*Sin[x]))
```

3.284.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3554
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3576
```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(x/(a^2 + b^2)), x] - Simp[a/(a^2 + b
^2)   Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x
]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3577
```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[a*(x/(a^2 + b^2)), x] + Simp[b/(a^2 + b
^2)   Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x
]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3590
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Sim
p[b/(a^2 + b^2)   Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
 b*Sin[c + d*x])^(p + 1), x], x] + (Simp[a/(a^2 + b^2)   Int[Cos[c + d*x]^(
m - 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - S
imp[a*(b/(a^2 + b^2))   Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Co
s[c + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^
2 + b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

rule 3612
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

3.284.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.20

method	result
default	$\frac{a}{(a^2+b^2)(a+b\tan(x))}-\frac{(a^2-b^2)\ln(a+b\tan(x))}{(a^2+b^2)^2}+\frac{\frac{(a^2-b^2)\ln\left(1+\tan(x)^2\right)}{2}+2ab\arctan(\tan(x))}{(a^2+b^2)^2}$
parallelrisch	$\frac{\left((-a^4+b^4)\cos(2x)-(a-b)^2(a+b)^2\right)\ln\left(\frac{-a\cos(x)-b\sin(x)}{\cos(x)+1}\right)+\left((a^4-b^4)\cos(2x)+(a-b)^2(a+b)^2\right)\ln\left(\frac{1}{\cos(x)+1}\right)+2b\left((a^2+b^2)\right.}{(a^2-b^2+\cos(2x)(a^2+b^2))(a^2+b^2)^2}$
risch	$\frac{ix}{2iba-a^2+b^2}+\frac{2ix\,a^2}{a^4+2a^2b^2+b^4}-\frac{2ix\,b^2}{a^4+2a^2b^2+b^4}-\frac{2iab}{(ib+a)(-ib+a)^2(-ib\,e^{2ix}+a\,e^{2ix}+ib+a)}-\frac{\ln\left(e^{2ix}-\frac{ib+a}{ib-a}\right)a^2}{a^4+2a^2b^2+b^4}+\ln($
norman	$\frac{-\frac{2a^2bx}{a^4+2a^2b^2+b^4}-\frac{4a\,b^2x\tan\left(\frac{x}{2}\right)}{a^4+2a^2b^2+b^4}-\frac{8a\,b^2x\tan\left(\frac{x}{2}\right)^3}{a^4+2a^2b^2+b^4}-\frac{4a\,b^2x\tan\left(\frac{x}{2}\right)^5}{a^4+2a^2b^2+b^4}-\frac{2a^2bx\tan\left(\frac{x}{2}\right)^2}{a^4+2a^2b^2+b^4}+\frac{2a^2bx\tan\left(\frac{x}{2}\right)^4}{a^4+2a^2b^2+b^4}+\frac{2a^2bx\tan\left(\frac{x}{2}\right)^6}{a^4+2a^2b^2+b^4}+\frac{2b\tan\left(\frac{x}{2}\right)}{a^2+b^2}}{\left(1+\tan\left(\frac{x}{2}\right)^2\right)^2\left(\tan\left(\frac{x}{2}\right)^2a-2b\tan\left(\frac{x}{2}\right)-a\right)}$

input
```
int(cos(x)*sin(x)/(a*cos(x)+b*sin(x))^2,x,method=_RETURNVERBOSE)
```

output
```
a/(a^2+b^2)/(a+b*tan(x))-(a^2-b^2)/(a^2+b^2)^2*ln(a+b*tan(x))+1/(a^2+b^2)^
2*(1/2*(a^2-b^2)*ln(1+tan(x)^2)+2*a*b*arctan(tan(x)))
```

3.284.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.97

$$\int \frac{\cos(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx$$
$$= \frac{2\left(2\,a^2bx+ab^2\right)\cos\left(x\right)-\left(\left(a^3-ab^2\right)\cos\left(x\right)+\left(a^2b-b^3\right)\sin\left(x\right)\right)\log\left(2\,ab\cos\left(x\right)\sin\left(x\right)+\left(a^2-b^2\right)\cos\left(x\right.\right.}{2\left(\left(a^5+2\,a^3b^2+ab^4\right)\cos\left(x\right)+\left(a^4b+2\,a^2b^3+b^5\right)\sin\left(x\right)\right)}$$

input
```
integrate(cos(x)*sin(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

output
```
1/2*(2*(2*a^2*b*x + a*b^2)*cos(x) - ((a^3 - a*b^2)*cos(x) + (a^2*b - b^3)*
sin(x))*log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) + 2*(2*a*b^2
*x - a^2*b)*sin(x))/((a^5 + 2*a^3*b^2 + a*b^4)*cos(x) + (a^4*b + 2*a^2*b^3
 + b^5)*sin(x))
```

3.284.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{\cos(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(cos(x)*sin(x)/(a*cos(x)+b*sin(x))**2,x)
```

output
```
Exception raised: AttributeError >> 'NoneType' object has no attribute 'pr
imitive'
```

3.284.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.69

$$\int \frac{\cos(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{2\,abx}{a^4+2\,a^2b^2+b^4} - \frac{(a^2-b^2)\log(b\tan(x)+a)}{a^4+2\,a^2b^2+b^4} + \frac{(a^2-b^2)\log\left(\tan(x)^2+1\right)}{2\,(a^4+2\,a^2b^2+b^4)} + \frac{a}{a^3+ab^2+(a^2b+b^3)\tan(x)}$$

input
```
integrate(cos(x)*sin(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

output
```
2*a*b*x/(a^4 + 2*a^2*b^2 + b^4) - (a^2 - b^2)*log(b*tan(x) + a)/(a^4 + 2*a
^2*b^2 + b^4) + 1/2*(a^2 - b^2)*log(tan(x)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4)
+ a/(a^3 + a*b^2 + (a^2*b + b^3)*tan(x))
```

3.284.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(70) = 140.

Time = 0.29 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.06

$$\int \frac{\cos(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{2\,abx}{a^4+2\,a^2b^2+b^4} + \frac{(a^2-b^2)\log\left(\tan(x)^2+1\right)}{2\,(a^4+2\,a^2b^2+b^4)} - \frac{(a^2b-b^3)\log(|b\tan(x)+a|)}{a^4b+2\,a^2b^3+b^5} + \frac{a^2b\tan(x)-b^3\tan(x)+2\,a^3}{(a^4+2\,a^2b^2+b^4)(b\tan(x)+a)}$$

input
```
integrate(cos(x)*sin(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

output
```
2*a*b*x/(a^4 + 2*a^2*b^2 + b^4) + 1/2*(a^2 - b^2)*log(tan(x)^2 + 1)/(a^4 +
 2*a^2*b^2 + b^4) - (a^2*b - b^3)*log(abs(b*tan(x) + a))/(a^4*b + 2*a^2*b^
3 + b^5) + (a^2*b*tan(x) - b^3*tan(x) + 2*a^3)/((a^4 + 2*a^2*b^2 + b^4)*(b
*tan(x) + a))
```

3.284.9 Mupad [B] (verification not implemented)

Time = 27.76 (sec) , antiderivative size = 1017, normalized size of antiderivative = 14.53

$$\int \frac{\cos(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \text{Too large to display}$$

input
```
int((cos(x)*sin(x))/(a*cos(x) + b*sin(x))^2,x)
```

output
```
-(b^3*sin(x) + a^3*log((a*cos(x) + b*sin(x))/cos(x/2)^2)*cos(x) - b^3*log(
(a*cos(x) + b*sin(x))/cos(x/2)^2)*sin(x) + a^2*b*sin(x) - a^3*log(-(65536*
a^4*b^10 - 131072*a^6*b^8 + 196608*a^8*b^6 - 131072*a^10*b^4 + 65536*a^12*
b^2)/(a^16/2 + b^16/2 + 4*a^2*b^14 + 14*a^4*b^12 + 28*a^6*b^10 + 35*a^8*b^
8 + 28*a^10*b^6 + 14*a^12*b^4 + 4*a^14*b^2 + (a^16*cos(x))/2 + (b^16*cos(x
))/2 + 4*a^2*b^14*cos(x) + 14*a^4*b^12*cos(x) + 28*a^6*b^10*cos(x) + 35*a^
8*b^8*cos(x) + 28*a^10*b^6*cos(x) + 14*a^12*b^4*cos(x) + 4*a^14*b^2*cos(x)
))*cos(x) + b^3*log(-(65536*a^4*b^10 - 131072*a^6*b^8 + 196608*a^8*b^6 - 1
31072*a^10*b^4 + 65536*a^12*b^2)/(a^16/2 + b^16/2 + 4*a^2*b^14 + 14*a^4*b^
12 + 28*a^6*b^10 + 35*a^8*b^8 + 28*a^10*b^6 + 14*a^12*b^4 + 4*a^14*b^2 + (
a^16*cos(x))/2 + (b^16*cos(x))/2 + 4*a^2*b^14*cos(x) + 14*a^4*b^12*cos(x)
+ 28*a^6*b^10*cos(x) + 35*a^8*b^8*cos(x) + 28*a^10*b^6*cos(x) + 14*a^12*b^
4*cos(x) + 4*a^14*b^2*cos(x)))*sin(x) - 4*a^2*b*atan(sin(x/2)/cos(x/2))*co
s(x) - 4*a*b^2*atan(sin(x/2)/cos(x/2))*sin(x) + a*b^2*log(-(65536*a^4*b^10
 - 131072*a^6*b^8 + 196608*a^8*b^6 - 131072*a^10*b^4 + 65536*a^12*b^2)/(a^
16/2 + b^16/2 + 4*a^2*b^14 + 14*a^4*b^12 + 28*a^6*b^10 + 35*a^8*b^8 + 28*a
^10*b^6 + 14*a^12*b^4 + 4*a^14*b^2 + (a^16*cos(x))/2 + (b^16*cos(x))/2 + 4
*a^2*b^14*cos(x) + 14*a^4*b^12*cos(x) + 28*a^6*b^10*cos(x) + 35*a^8*b^8*co
s(x) + 28*a^10*b^6*cos(x) + 14*a^12*b^4*cos(x) + 4*a^14*b^2*cos(x)))*cos(x
) - a^2*b*log(-(65536*a^4*b^10 - 131072*a^6*b^8 + 196608*a^8*b^6 - 1310...
```

3.285 $\int \frac{\cos(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

3.285.1 Optimal result

Integrand size = 18, antiderivative size = 110

$$\int \frac{\cos(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx = -\frac{a(a^2-2b^2)\operatorname{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} - \frac{2ab\cos(x)}{(a^2+b^2)^2} - \frac{(a^2-b^2)\sin(x)}{(a^2+b^2)^2} - \frac{a^2b}{(a^2+b^2)^2(a\cos(x)+b\sin(x))}$$

output
```
-a*(a^2-2*b^2)*arctanh((b*cos(x)-a*sin(x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2
)-2*a*b*cos(x)/(a^2+b^2)^2-(a^2-b^2)*sin(x)/(a^2+b^2)^2-a^2*b/(a^2+b^2)^2/
(a*cos(x)+b*sin(x))
```

3.285.2 Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.01

$$\int \frac{\cos(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{2a(a^2-2b^2)\operatorname{arctanh}\left(\frac{-b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} - \frac{5a^2b-b^3+b(a^2+b^2)\cos(2x)+a(a^2+b^2)\sin(2x)}{2(a^2+b^2)^2(a\cos(x)+b\sin(x))}$$

input
```
Integrate[(Cos[x]*Sin[x]^2)/(a*Cos[x] + b*Sin[x])^2,x]
```

output
```
(2*a*(a^2 - 2*b^2)*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)
^(5/2) - (5*a^2*b - b^3 + b*(a^2 + b^2)*Cos[2*x] + a*(a^2 + b^2)*Sin[2*x])
/(2*(a^2 + b^2)^2*(a*Cos[x] + b*Sin[x]))
```

3.285.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 229 vs. 2(110) = 220.

Time = 1.34 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.08, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 3590, 3042, 3578, 3042, 3118, 3553, 219, 3588, 3042, 3117, 3118, 3553, 219, 3633, 3042, 3553, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(x)\cos(x)}{(a\cos(x)+b\sin(x))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)^2\cos(x)}{(a\cos(x)+b\sin(x))^2}dx$$

$$\Big\downarrow \text{3590}$$

$$\frac{a\int \frac{\sin^2(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}-\frac{ab\int \frac{\sin(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{b\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{ab\int \frac{\sin(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{b\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\int \frac{\sin(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3578}$$

$$-\frac{ab\int \frac{\sin(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{b\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\left(\frac{b\int \sin(x)dx}{a^2+b^2}+\frac{a^2\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}-\frac{a\sin(x)}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{ab\int \frac{\sin(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{a\left(\frac{b\int \sin(x)dx}{a^2+b^2}+\frac{a^2\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}-\frac{a\sin(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3118}$$

$$-\frac{ab\int \frac{\sin(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{a\left(\frac{a^2\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}-\frac{a\sin(x)}{a^2+b^2}-\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3553}$$

$$-\frac{ab\int \frac{\sin(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{b\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\left(-\frac{a^2\int \frac{1}{a^2+b^2-(b\cos(x)-a\sin(x))^2}d(b\cos(x)-a\sin(x))}{a^2+b^2}-\frac{a\sin(x)}{a^2+b^2}-\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{219}$$

$$-\frac{ab\int \frac{\sin(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{b\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\left(-\frac{a^2\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}-\frac{a\sin(x)}{a^2+b^2}-\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{3588}$$

$$-\frac{ab\int \frac{\sin(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{b\left(\frac{a\int \sin(x)dx}{a^2+b^2}+\frac{b\int \cos(x)dx}{a^2+b^2}-\frac{ab\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(-\frac{a^2\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}-\frac{a\sin(x)}{a^2+b^2}-\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{ab\int \frac{\sin(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{b\left(\frac{a\int \sin(x)dx}{a^2+b^2}+\frac{b\int \sin\left(x+\frac{\pi}{2}\right)dx}{a^2+b^2}-\frac{ab\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(-\frac{a^2\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}-\frac{a\sin(x)}{a^2+b^2}-\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{3117}$$

$$-\frac{ab\int \frac{\sin(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{b\left(\frac{a\int \sin(x)dx}{a^2+b^2}-\frac{ab\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\sin(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(-\frac{a^2\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}-\frac{a\sin(x)}{a^2+b^2}-\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}$$

3.285. $\int \frac{\cos(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

$$\Big\downarrow \text{3118}$$

$$-\frac{ab\int \frac{\sin(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{b\left(-\frac{ab\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\sin(x)}{a^2+b^2}-\frac{a\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(-\frac{a^2\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}-\frac{a\sin(x)}{a^2+b^2}-\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{3553}$$

$$-\frac{ab\int \frac{\sin(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{b\left(\frac{ab\int \frac{1}{a^2+b^2-(b\cos(x)-a\sin(x))^2}d(b\cos(x)-a\sin(x))}{a^2+b^2}+\frac{b\sin(x)}{a^2+b^2}-\frac{a\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(-\frac{a^2\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}-\frac{a\sin(x)}{a^2+b^2}-\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{219}$$

$$-\frac{ab\int \frac{\sin(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{a\left(-\frac{a^2\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}-\frac{a\sin(x)}{a^2+b^2}-\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\left(\frac{ab\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{b\sin(x)}{a^2+b^2}-\frac{a\cos(x)}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{3633}$$

$$-\frac{ab\left(\frac{b\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a}{(a^2+b^2)(a\cos(x)+b\sin(x))}\right)}{a^2+b^2}+\frac{a\left(-\frac{a^2\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}-\frac{a\sin(x)}{a^2+b^2}-\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\left(\frac{ab\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{b\sin(x)}{a^2+b^2}-\frac{a\cos(x)}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

3.285. $\int \frac{\cos(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

$$-\frac{ab\left(\frac{b\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a}{(a^2+b^2)(a\cos(x)+b\sin(x))}\right)}{a^2+b^2}+\frac{a\left(-\frac{a^2\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}-\frac{a\sin(x)}{a^2+b^2}-\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\left(\frac{ab\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{b\sin(x)}{a^2+b^2}-\frac{a\cos(x)}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{ 3553}$$

$$-\frac{ab\left(\frac{a}{(a^2+b^2)(a\cos(x)+b\sin(x))}-\frac{b\int \frac{1}{a^2+b^2-(b\cos(x)-a\sin(x))^2}d(b\cos(x)-a\sin(x))}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(-\frac{a^2\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}-\frac{a\sin(x)}{a^2+b^2}-\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\left(\frac{ab\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{b\sin(x)}{a^2+b^2}-\frac{a\cos(x)}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{a\left(-\frac{a^2\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}-\frac{a\sin(x)}{a^2+b^2}-\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\left(\frac{ab\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{b\sin(x)}{a^2+b^2}-\frac{a\cos(x)}{a^2+b^2}\right)}{a^2+b^2}-\frac{ab\left(\frac{a}{(a^2+b^2)(a\cos(x)+b\sin(x))}-\frac{b\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}\right)}{a^2+b^2}$$

input
```
Int[(Cos[x]*Sin[x]^2)/(a*Cos[x] + b*Sin[x])^2,x]
```

3.285. $\int \frac{\cos(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

output
```
(a*(-((a^2*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/
2)) - (b*Cos[x])/(a^2 + b^2) - (a*Sin[x])/(a^2 + b^2)))/(a^2 + b^2) + (b*(
(a*b*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2) - (
a*Cos[x])/(a^2 + b^2) + (b*Sin[x])/(a^2 + b^2)))/(a^2 + b^2) - (a*b*(-((b*
ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2)) + a/((a
^2 + b^2)*(a*Cos[x] + b*Sin[x]))))/(a^2 + b^2)
```

3.285.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3578
```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-a)*(Sin[c + d*x]^(m - 1)/(d*(a^2
 + b^2)*(m - 1))), x] + (Simp[a^2/(a^2 + b^2)   Int[Sin[c + d*x]^(m - 2)/(a
*Cos[c + d*x] + b*Sin[c + d*x]), x], x] + Simp[b/(a^2 + b^2)   Int[Sin[c +
d*x]^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ
[m, 1]
```

3.285. $\int \frac{\cos(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

rule 3588
```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b
/(a^2 + b^2)   Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Simp[a/(a
^2 + b^2)   Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Simp[a*(b/(a^
2 + b^2))   Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x]
+ b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
&& IGtQ[m, 0] && IGtQ[n, 0]
```

rule 3590
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Sim
p[b/(a^2 + b^2)   Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
 b*Sin[c + d*x])^(p + 1), x], x] + (Simp[a/(a^2 + b^2)   Int[Cos[c + d*x]^(
m - 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - S
imp[a*(b/(a^2 + b^2))   Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Co
s[c + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^
2 + b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

rule 3633
```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[-(b*C + (a*C
- c*A)*Cos[d + e*x] + b*A*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
 e*x] + c*Sin[d + e*x])), x] + Simp[(a*A - c*C)/(a^2 - b^2 - c^2)   Int[1/(
a + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, C}
, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - c*C, 0]
```

3.285.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.29

method	result
default	$-\frac{2a\left(\frac{-b^2\tan\left(\frac{x}{2}\right)-ab}{\tan\left(\frac{x}{2}\right)^2 a-2b\tan\left(\frac{x}{2}\right)-a}-\frac{\left(a^2-2b^2\right)\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^2}+\frac{2\left(-a^2+b^2\right)\tan\left(\frac{x}{2}\right)-4ab}{\left(a^4+2a^2b^2+b^4\right)\left(1+\tan\left(\frac{x}{2}\right)^2\right)}$
risch	$\frac{i e^{ix}}{-4iba+2a^2-2b^2}-\frac{i e^{-ix}}{2\left(2iba+a^2-b^2\right)}-\frac{2b\,a^2 e^{ix}}{(ib+a)^2(-ib+a)^2\left(-ib\, e^{2ix}+a\, e^{2ix}+ib+a\right)}-\frac{a^3\ln\left(e^{ix}-\frac{i a^5+2i a^3b^2+i a\, b^4-a^4b-2a^2b^3-b^5}{\left(a^2+b^2\right)^{\frac{5}{2}}}\right.}{\left(a^2+b^2\right)^{\frac{5}{2}}}$

input
```
int(cos(x)*sin(x)^2/(a*cos(x)+b*sin(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-2*a/(a^2+b^2)^2*((-b^2*tan(1/2*x)-a*b)/(tan(1/2*x)^2*a-2*b*tan(1/2*x)-a)-
(a^2-2*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*x)-2*b)/(a^2+b^2)^(1/
2)))+2/(a^4+2*a^2*b^2+b^4)*((-a^2+b^2)*tan(1/2*x)-2*a*b)/(1+tan(1/2*x)^2)
```

3.285.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(106) = 212$.

Time = 0.26 (sec) , antiderivative size = 252, normalized size of antiderivative = 2.29

$$\int \frac{\cos(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx =$$
$$-\frac{4\,a^4b+2\,a^2b^3-2\,b^5+2\,(a^4b+2\,a^2b^3+b^5)\cos{(x)}^2+2\,(a^5+2\,a^3b^2+ab^4)\cos{(x)}\sin{(x)}+\sqrt{a^2+b^2}((\cdots}{2\,((a^7+3\,a^5b^2+3\,a^3b^4+ab^6)\cos{(x)}\cdots}$$

input
```
integrate(cos(x)*sin(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

output
```
-1/2*(4*a^4*b + 2*a^2*b^3 - 2*b^5 + 2*(a^4*b + 2*a^2*b^3 + b^5)*cos(x)^2 +
 2*(a^5 + 2*a^3*b^2 + a*b^4)*cos(x)*sin(x) + sqrt(a^2 + b^2)*((a^4 - 2*a^2
*b^2)*cos(x) + (a^3*b - 2*a*b^3)*sin(x))*log((2*a*b*cos(x)*sin(x) + (a^2 -
 b^2)*cos(x)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2
*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2)))/((a^7 + 3*a^5*b^2 + 3*a
^3*b^4 + a*b^6)*cos(x) + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*sin(x))
```

3.285.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(x)*sin(x)**2/(a*cos(x)+b*sin(x))**2,x)
```

output
```
Timed out
```

3.285.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(106) = 212$.

Time = 0.30 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.41

$$\int \frac{\cos(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx$$
$$= -\frac{(a^2-2\,b^2)a\log\left(\frac{b-\frac{a\sin(x)}{\cos(x)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(x)}{\cos(x)+1}-\sqrt{a^2+b^2}}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}}$$
$$-\frac{2\left(3\,a^2b+\frac{(a^3+4\,ab^2)\sin(x)}{\cos(x)+1}+\frac{(a^2b-2\,b^3)\sin(x)^2}{(\cos(x)+1)^2}-\frac{(a^3-2\,ab^2)\sin(x)^3}{(\cos(x)+1)^3}\right)}{a^5+2\,a^3b^2+ab^4+\frac{2\,(a^4b+2\,a^2b^3+b^5)\sin(x)}{\cos(x)+1}+\frac{2\,(a^4b+2\,a^2b^3+b^5)\sin(x)^3}{(\cos(x)+1)^3}-\frac{(a^5+2\,a^3b^2+ab^4)\sin(x)^4}{(\cos(x)+1)^4}}$$

input
```
integrate(cos(x)*sin(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

output
```
-(a^2 - 2*b^2)*a*log((b - a*sin(x)/(cos(x) + 1) + sqrt(a^2 + b^2))/(b - a*
sin(x)/(cos(x) + 1) - sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2
+ b^2)) - 2*(3*a^2*b + (a^3 + 4*a*b^2)*sin(x)/(cos(x) + 1) + (a^2*b - 2*b^
3)*sin(x)^2/(cos(x) + 1)^2 - (a^3 - 2*a*b^2)*sin(x)^3/(cos(x) + 1)^3)/(a^5
 + 2*a^3*b^2 + a*b^4 + 2*(a^4*b + 2*a^2*b^3 + b^5)*sin(x)/(cos(x) + 1) + 2
*(a^4*b + 2*a^2*b^3 + b^5)*sin(x)^3/(cos(x) + 1)^3 - (a^5 + 2*a^3*b^2 + a*
b^4)*sin(x)^4/(cos(x) + 1)^4)
```

3.285.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.90

$$\int \frac{\cos(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx = -\frac{(a^3-2\,ab^2)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}}$$
$$-\frac{2\left(a^3\tan\left(\frac{1}{2}\,x\right)^3-2\,ab^2\tan\left(\frac{1}{2}\,x\right)^3-a^2b\tan\left(\frac{1}{2}\,x\right)^2+2\,b^3\tan\left(\frac{1}{2}\,x\right)^2-a^3\tan\left(\frac{1}{2}\,x\right)-4\,ab^2\tan\left(\frac{1}{2}\,x\right)-3\right.}{\left(a\tan\left(\frac{1}{2}\,x\right)^4-2\,b\tan\left(\frac{1}{2}\,x\right)^3-2\,b\tan\left(\frac{1}{2}\,x\right)-a\right)(a^4+2\,a^2b^2+b^4)}$$

input
```
integrate(cos(x)*sin(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

output
```
-(a^3 - 2*a*b^2)*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a
*tan(1/2*x) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2
+ b^2)) - 2*(a^3*tan(1/2*x)^3 - 2*a*b^2*tan(1/2*x)^3 - a^2*b*tan(1/2*x)^2
+ 2*b^3*tan(1/2*x)^2 - a^3*tan(1/2*x) - 4*a*b^2*tan(1/2*x) - 3*a^2*b)/((a*
tan(1/2*x)^4 - 2*b*tan(1/2*x)^3 - 2*b*tan(1/2*x) - a)*(a^4 + 2*a^2*b^2 + b
^4))
```

3.285.9 Mupad [B] (verification not implemented)

Time = 23.44 (sec) , antiderivative size = 249, normalized size of antiderivative = 2.26

$$\int \frac{\cos(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx = -\frac{\frac{2\tan\left(\frac{x}{2}\right)\left(a^3+4\,a\,b^2\right)}{a^4+2\,a^2\,b^2+b^4}+\frac{6\,a^2\,b}{a^4+2\,a^2\,b^2+b^4}-\frac{2\,a\tan\left(\frac{x}{2}\right)^3\left(a^2-2\,b^2\right)}{a^4+2\,a^2\,b^2+b^4}+\frac{2\,b\tan\left(\frac{x}{2}\right)^2\left(a^2-2\,b^2\right)}{a^4+2\,a^2\,b^2+b^4}}{-a\tan\left(\frac{x}{2}\right)^4+2\,b\tan\left(\frac{x}{2}\right)^3+2\,b\tan\left(\frac{x}{2}\right)+a} - \frac{a\,\mathrm{atan}\left(\frac{1\mathrm{i}\tan\left(\frac{x}{2}\right)a^5-a^4\,b\,1\mathrm{i}+2\mathrm{i}\tan\left(\frac{x}{2}\right)a^3\,b^2-a^2\,b^3\,2\mathrm{i}+1\mathrm{i}\tan\left(\frac{x}{2}\right)a\,b^4-b^5\,1\mathrm{i}}{\left(a^2+b^2\right)^{5/2}}\right)\left(a^2-2\,b^2\right)2\mathrm{i}}{\left(a^2+b^2\right)^{5/2}}$$

input
```
int((cos(x)*sin(x)^2)/(a*cos(x) + b*sin(x))^2,x)
```

output
```
- ((2*tan(x/2)*(4*a*b^2 + a^3))/(a^4 + b^4 + 2*a^2*b^2) + (6*a^2*b)/(a^4 +
 b^4 + 2*a^2*b^2) - (2*a*tan(x/2)^3*(a^2 - 2*b^2))/(a^4 + b^4 + 2*a^2*b^2)
 + (2*b*tan(x/2)^2*(a^2 - 2*b^2))/(a^4 + b^4 + 2*a^2*b^2))/(a + 2*b*tan(x/
2) - a*tan(x/2)^4 + 2*b*tan(x/2)^3) - (a*atan((a^5*tan(x/2)*1i - a^4*b*1i
- b^5*1i - a^2*b^3*2i + a^3*b^2*tan(x/2)*2i + a*b^4*tan(x/2)*1i)/(a^2 + b^
2)^(5/2))*(a^2 - 2*b^2)*2i)/(a^2 + b^2)^(5/2)
```

3.286 $\int \frac{\cos(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

3.286.1 Optimal result

Integrand size = 18, antiderivative size = 129

$$\int \frac{\cos(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{b(3a^3-ab^2)\,x}{(a^2+b^2)^3} - \frac{a^2(a^2-3b^2)\log(a\cos(x)+b\sin(x))}{(a^2+b^2)^3} - \frac{ab\cos(x)\sin(x)}{(a^2+b^2)^2} - \frac{(a^2-b^2)\sin^2(x)}{2\,(a^2+b^2)^2} - \frac{a^2b\sin(x)}{(a^2+b^2)^2\,(a\cos(x)+b\sin(x))}$$

output
```
b*(3*a^3-a*b^2)*x/(a^2+b^2)^3-a^2*(a^2-3*b^2)*ln(a*cos(x)+b*sin(x))/(a^2+b
^2)^3-a*b*cos(x)*sin(x)/(a^2+b^2)^2-1/2*(a^2-b^2)*sin(x)^2/(a^2+b^2)^2-a^2
*b*sin(x)/(a^2+b^2)^2/(a*cos(x)+b*sin(x))
```

3.286.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.80 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.75

$$\int \frac{\cos(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{4ia^2(a^2-3b^2)\arctan(\tan(x))(a\cos(x)+b\sin(x)) + a\cos(x)\left((a^4-b^4)\cos(2x)+2a(2(ia-b)^3x - a(a^2 \cdots\right.}{}$$

input
```
Integrate[(Cos[x]*Sin[x]^3)/(a*Cos[x] + b*Sin[x])^2,x]
```

output
```
((4*I)*a^2*(a^2 - 3*b^2)*ArcTan[Tan[x]]*(a*Cos[x] + b*Sin[x]) + a*Cos[x]*(
(a^4 - b^4)*Cos[2*x] + 2*a*(2*(I*a - b)^3*x - a*(a^2 - 3*b^2)*Log[(a*Cos[x
] + b*Sin[x])^2] - b*(a^2 + b^2)*Sin[2*x])) - b*Sin[x]*((-a^4 + b^4)*Cos[2
*x] + 2*a*(2*(a^3*(1 + I*x) + a*b^2*(1 - (3*I)*x) - 3*a^2*b*x + b^3*x) + a
*(a^2 - 3*b^2)*Log[(a*Cos[x] + b*Sin[x])^2] + b*(a^2 + b^2)*Sin[2*x])))/(4
*(a^2 + b^2)^3*(a*Cos[x] + b*Sin[x]))
```

3.286.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 295 vs. 2(129) = 258.

Time = 2.24 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.29, number of steps used = 27, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.444, Rules used = {3042, 3590, 3042, 3564, 3042, 3578, 3042, 3115, 24, 3576, 3042, 3588, 3042, 3044, 15, 3115, 24, 3576, 3042, 3612, 3964, 3042, 4014, 25, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(x)\cos(x)}{(a\cos(x)+b\sin(x))^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(x)^3\cos(x)}{(a\cos(x)+b\sin(x))^2}dx$$

$$\Big\downarrow \text{ 3590}$$

$$\frac{a\int \frac{\sin^3(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}-\frac{ab\int \frac{\sin^2(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{b\int \frac{\cos(x)\sin^2(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{ab\int \frac{\sin(x)^2}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{b\int \frac{\cos(x)\sin(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\int \frac{\sin(x)^3}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{ 3564}$$

$$-\frac{ab\int \frac{1}{(b+a\cot(x))^2}dx}{a^2+b^2}+\frac{b\int \frac{\cos(x)\sin(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\int \frac{\sin(x)^3}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{ab\int \frac{1}{\left(b-a\tan\left(x+\frac{\pi}{2}\right)\right)^2}dx}{a^2+b^2}+\frac{b\int \frac{\cos(x)\sin(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\int \frac{\sin(x)^3}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \ 3578$$

$$-\frac{ab\int \frac{1}{\left(b-a\tan\left(x+\frac{\pi}{2}\right)\right)^2}dx}{a^2+b^2}+\frac{a\left(\frac{b\int \sin^2(x)dx}{a^2+b^2}+\frac{a^2\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}-\frac{a\sin^2(x)}{2(a^2+b^2)}\right)}{a^2+b^2}+\frac{b\int \frac{\cos(x)\sin(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \ 3042$$

$$-\frac{ab\int \frac{1}{\left(b-a\tan\left(x+\frac{\pi}{2}\right)\right)^2}dx}{a^2+b^2}+\frac{a\left(\frac{b\int \sin(x)^2dx}{a^2+b^2}+\frac{a^2\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}-\frac{a\sin^2(x)}{2(a^2+b^2)}\right)}{a^2+b^2}+\frac{b\int \frac{\cos(x)\sin(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \ 3115$$

$$-\frac{ab\int \frac{1}{\left(b-a\tan\left(x+\frac{\pi}{2}\right)\right)^2}dx}{a^2+b^2}+\frac{a\left(\frac{a^2\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\left(\frac{\int 1dx}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}-\frac{a\sin^2(x)}{2(a^2+b^2)}\right)}{a^2+b^2}+\frac{b\int \frac{\cos(x)\sin(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \ 24$$

$$-\frac{ab\int \frac{1}{\left(b-a\tan\left(x+\frac{\pi}{2}\right)\right)^2}dx}{a^2+b^2}+\frac{a\left(\frac{a^2\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}-\frac{a\sin^2(x)}{2(a^2+b^2)}+\frac{b\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\int \frac{\cos(x)\sin(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \ 3576$$

$$-\frac{ab\int \frac{1}{\left(b-a\tan\left(x+\frac{\pi}{2}\right)\right)^2}dx}{a^2+b^2}+\frac{a\left(\frac{a^2\left(\frac{bx}{a^2+b^2}-\frac{a\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}-\frac{a\sin^2(x)}{2(a^2+b^2)}+\frac{b\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\int \frac{\cos(x)\sin(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \ 3042$$

3.286. $\int \frac{\cos(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

$$-\frac{ab\int \frac{1}{\left(b-a\tan\left(x+\frac{\pi}{2}\right)\right)^2}dx}{a^2+b^2}+\frac{a\left(\frac{a^2\left(\frac{bx}{a^2+b^2}-\frac{a\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}-\frac{a\sin^2(x)}{2(a^2+b^2)}+\frac{b\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\int \frac{\cos(x)\sin(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \; 3588$$

$$-\frac{ab\int \frac{1}{\left(b-a\tan\left(x+\frac{\pi}{2}\right)\right)^2}dx}{a^2+b^2}+\frac{a\left(\frac{a^2\left(\frac{bx}{a^2+b^2}-\frac{a\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}-\frac{a\sin^2(x)}{2(a^2+b^2)}+\frac{b\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\left(\frac{a\int \sin^2(x)dx}{a^2+b^2}+\frac{b\int \cos(x)\sin(x)dx}{a^2+b^2}-\frac{ab\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{ab\int \frac{1}{\left(b-a\tan\left(x+\frac{\pi}{2}\right)\right)^2}dx}{a^2+b^2}+\frac{a\left(\frac{a^2\left(\frac{bx}{a^2+b^2}-\frac{a\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}-\frac{a\sin^2(x)}{2(a^2+b^2)}+\frac{b\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\left(\frac{a\int \sin(x)^2dx}{a^2+b^2}+\frac{b\int \cos(x)\sin(x)dx}{a^2+b^2}-\frac{ab\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \; 3044$$

$$-\frac{ab\int \frac{1}{\left(b-a\tan\left(x+\frac{\pi}{2}\right)\right)^2}dx}{a^2+b^2}+\frac{a\left(\frac{a^2\left(\frac{bx}{a^2+b^2}-\frac{a\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}-\frac{a\sin^2(x)}{2(a^2+b^2)}+\frac{b\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\left(\frac{a\int \sin(x)^2dx}{a^2+b^2}+\frac{b\int \sin(x)d\sin(x)}{a^2+b^2}-\frac{ab\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \; 15$$

3.286. $\int \frac{\cos(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

$$-\frac{ab\int \frac{1}{\left(b-a\tan\left(x+\frac{\pi}{2}\right)\right)^2}dx}{a^2+b^2}+\frac{b\left(\frac{a\int \sin(x)^2dx}{a^2+b^2}-\frac{ab\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\sin^2(x)}{2(a^2+b^2)}\right)}{a^2+b^2}+\frac{a\left(\frac{a^2\left(\frac{bx}{a^2+b^2}-\frac{a\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}-\frac{a\sin^2(x)}{2(a^2+b^2)}+\frac{b\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{3115}$$

$$-\frac{ab\int \frac{1}{\left(b-a\tan\left(x+\frac{\pi}{2}\right)\right)^2}dx}{a^2+b^2}+\frac{b\left(\frac{a\left(\frac{\int 1dx}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}-\frac{ab\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\sin^2(x)}{2(a^2+b^2)}\right)}{a^2+b^2}+\frac{a\left(\frac{a^2\left(\frac{bx}{a^2+b^2}-\frac{a\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}-\frac{a\sin^2(x)}{2(a^2+b^2)}+\frac{b\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{24}$$

$$-\frac{ab\int \frac{1}{\left(b-a\tan\left(x+\frac{\pi}{2}\right)\right)^2}dx}{a^2+b^2}+\frac{b\left(-\frac{ab\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\sin^2(x)}{2(a^2+b^2)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(\frac{a^2\left(\frac{bx}{a^2+b^2}-\frac{a\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}-\frac{a\sin^2(x)}{2(a^2+b^2)}+\frac{b\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{3576}$$

$$-\frac{ab\int \frac{1}{\left(b-a\tan\left(x+\frac{\pi}{2}\right)\right)^2}dx}{a^2+b^2}+\frac{a\left(\frac{a^2\left(\frac{bx}{a^2+b^2}-\frac{a\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}-\frac{a\sin^2(x)}{2(a^2+b^2)}+\frac{b\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\left(-\frac{ab\left(\frac{bx}{a^2+b^2}-\frac{a\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\sin^2(x)}{2(a^2+b^2)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

3.286. $\displaystyle\int \frac{\cos(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

$$-\frac{ab\int\frac{1}{\left(b-a\tan\left(x+\frac{\pi}{2}\right)\right)^2}dx}{a^2+b^2}+\frac{a\left(\frac{a^2\left(\frac{bx}{a^2+b^2}-\frac{a\int\frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}-\frac{a\sin^2(x)}{2(a^2+b^2)}+\frac{b\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\left(-\frac{ab\left(\frac{bx}{a^2+b^2}-\frac{a\int\frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\sin^2(x)}{2(a^2+b^2)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}$$

$$\downarrow \text{3612}$$

$$-\frac{ab\int\frac{1}{\left(b-a\tan\left(x+\frac{\pi}{2}\right)\right)^2}dx}{a^2+b^2}+\frac{b\left(\frac{b\sin^2(x)}{2(a^2+b^2)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}-\frac{ab\left(\frac{bx}{a^2+b^2}-\frac{a\log(a\cos(x)+b\sin(x))}{a^2+b^2}\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(-\frac{a\sin^2(x)}{2(a^2+b^2)}+\frac{b\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}+\frac{a^2\left(\frac{bx}{a^2+b^2}-\frac{a\log(a\cos(x)+b\sin(x))}{a^2+b^2}\right)}{a^2+b^2}\right)}{a^2+b^2}$$

$$\downarrow \text{3964}$$

$$-\frac{ab\left(\frac{\int\frac{b-a\cot(x)}{b+a\cot(x)}dx}{a^2+b^2}+\frac{a}{(a^2+b^2)(a\cot(x)+b)}\right)}{a^2+b^2}+\frac{b\left(\frac{b\sin^2(x)}{2(a^2+b^2)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}-\frac{ab\left(\frac{bx}{a^2+b^2}-\frac{a\log(a\cos(x)+b\sin(x))}{a^2+b^2}\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(-\frac{a\sin^2(x)}{2(a^2+b^2)}+\frac{b\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}+\frac{a^2\left(\frac{bx}{a^2+b^2}-\frac{a\log(a\cos(x)+b\sin(x))}{a^2+b^2}\right)}{a^2+b^2}\right)}{a^2+b^2}$$

$$\downarrow \text{3042}$$

$$-\frac{ab\left(\frac{\int\frac{b+a\tan\left(x+\frac{\pi}{2}\right)}{b-a\tan\left(x+\frac{\pi}{2}\right)}dx}{a^2+b^2}+\frac{a}{(a^2+b^2)(a\cot(x)+b)}\right)}{a^2+b^2}+\frac{b\left(\frac{b\sin^2(x)}{2(a^2+b^2)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}-\frac{ab\left(\frac{bx}{a^2+b^2}-\frac{a\log(a\cos(x)+b\sin(x))}{a^2+b^2}\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(-\frac{a\sin^2(x)}{2(a^2+b^2)}+\frac{b\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}+\frac{a^2\left(\frac{bx}{a^2+b^2}-\frac{a\log(a\cos(x)+b\sin(x))}{a^2+b^2}\right)}{a^2+b^2}\right)}{a^2+b^2}$$

$$\downarrow \text{4014}$$

3.286. $\int \frac{\cos(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

$$-\frac{ab\left(\frac{-\frac{2ab\int -\frac{a-b\cot(x)}{b+a\cot(x)}dx}{a^2+b^2}-\frac{x\left(a^2-b^2\right)}{a^2+b^2}}{a^2+b^2}+\frac{a}{(a^2+b^2)(a\cot(x)+b)}\right)}{a^2+b^2}+\frac{b\left(\frac{b\sin^2(x)}{2(a^2+b^2)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}-\frac{ab\left(\frac{bx}{a^2+b^2}-\frac{a\log(a\cos(x)+b\sin(x))}{a^2+b^2}\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(-\frac{a\sin^2(x)}{2(a^2+b^2)}+\frac{b\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}+\frac{a^2\left(\frac{bx}{a^2+b^2}-\frac{a\log(a\cos(x)+b\sin(x))}{a^2+b^2}\right)}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{ab\left(\frac{\frac{2ab\int \frac{a-b\cot(x)}{b+a\cot(x)}dx}{a^2+b^2}-\frac{x\left(a^2-b^2\right)}{a^2+b^2}}{a^2+b^2}+\frac{a}{(a^2+b^2)(a\cot(x)+b)}\right)}{a^2+b^2}+\frac{b\left(\frac{b\sin^2(x)}{2(a^2+b^2)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}-\frac{ab\left(\frac{bx}{a^2+b^2}-\frac{a\log(a\cos(x)+b\sin(x))}{a^2+b^2}\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(-\frac{a\sin^2(x)}{2(a^2+b^2)}+\frac{b\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}+\frac{a^2\left(\frac{bx}{a^2+b^2}-\frac{a\log(a\cos(x)+b\sin(x))}{a^2+b^2}\right)}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{ab\left(\frac{\frac{2ab\int \frac{a+b\tan\left(x+\frac{\pi}{2}\right)}{b-a\tan\left(x+\frac{\pi}{2}\right)}dx}{a^2+b^2}-\frac{x\left(a^2-b^2\right)}{a^2+b^2}}{a^2+b^2}+\frac{a}{(a^2+b^2)(a\cot(x)+b)}\right)}{a^2+b^2}+\frac{b\left(\frac{b\sin^2(x)}{2(a^2+b^2)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}-\frac{ab\left(\frac{bx}{a^2+b^2}-\frac{a\log(a\cos(x)+b\sin(x))}{a^2+b^2}\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(-\frac{a\sin^2(x)}{2(a^2+b^2)}+\frac{b\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}+\frac{a^2\left(\frac{bx}{a^2+b^2}-\frac{a\log(a\cos(x)+b\sin(x))}{a^2+b^2}\right)}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{4013}$$

3.286. $\int \frac{\cos(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

$$\frac{b\left(\frac{b\sin^2(x)}{2(a^2+b^2)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}-\frac{ab\left(\frac{bx}{a^2+b^2}-\frac{a\log(a\cos(x)+b\sin(x))}{a^2+b^2}\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(-\frac{a\sin^2(x)}{2(a^2+b^2)}+\frac{b\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}+\frac{a^2\left(\frac{bx}{a^2+b^2}-\frac{a\log(a\cos(x)+b\sin(x))}{a^2+b^2}\right)}{a^2+b^2}\right)}{a^2+b^2}-\frac{ab\left(\frac{a}{(a^2+b^2)(a\cot(x)+b)}+\frac{-\frac{x(a^2-b^2)}{a^2+b^2}-\frac{2ab\log(a\cos(x)+b\sin(x))}{a^2+b^2}}{a^2+b^2}\right)}{a^2+b^2}$$

input
```
Int[(Cos[x]*Sin[x]^3)/(a*Cos[x] + b*Sin[x])^2,x]
```

output
```
-((a*b*(a/((a^2 + b^2)*(b + a*Cot[x])) + (-(((a^2 - b^2)*x)/(a^2 + b^2)) -
 (2*a*b*Log[a*Cos[x] + b*Sin[x]])/(a^2 + b^2))/(a^2 + b^2)))/(a^2 + b^2))
+ (b*(-((a*b*((b*x)/(a^2 + b^2) - (a*Log[a*Cos[x] + b*Sin[x]])/(a^2 + b^2)
))/(a^2 + b^2)) + (b*Sin[x]^2)/(2*(a^2 + b^2)) + (a*(x/2 - (Cos[x]*Sin[x])
/2))/(a^2 + b^2)))/(a^2 + b^2) + (a*((a^2*((b*x)/(a^2 + b^2) - (a*Log[a*Co
s[x] + b*Sin[x]])/(a^2 + b^2)))/(a^2 + b^2) - (a*Sin[x]^2)/(2*(a^2 + b^2))
 + (b*(x/2 - (Cos[x]*Sin[x])/2))/(a^2 + b^2)))/(a^2 + b^2)
```

3.286.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3564
```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(b + a*Cot[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

rule 3576
```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(x/(a^2 + b^2)), x] - Simp[a/(a^2 + b
^2)   Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x
]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3578
```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-a)*(Sin[c + d*x]^(m - 1)/(d*(a^2
 + b^2)*(m - 1))), x] + (Simp[a^2/(a^2 + b^2)   Int[Sin[c + d*x]^(m - 2)/(a
*Cos[c + d*x] + b*Sin[c + d*x]), x], x] + Simp[b/(a^2 + b^2)   Int[Sin[c +
d*x]^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ
[m, 1]
```

rule 3588
```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b
/(a^2 + b^2)   Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Simp[a/(a
^2 + b^2)   Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Simp[a*(b/(a^
2 + b^2))   Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x]
+ b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
&& IGtQ[m, 0] && IGtQ[n, 0]
```

3.286. $\int \frac{\cos(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

rule 3590
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Sim
p[b/(a^2 + b^2)   Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
 b*Sin[c + d*x])^(p + 1), x], x] + (Simp[a/(a^2 + b^2)   Int[Cos[c + d*x]^(
m - 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - S
imp[a*(b/(a^2 + b^2))   Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Co
s[c + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^
2 + b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

rule 3612
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

rule 3964
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)
 Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

3.286. $\int \frac{\cos(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

3.286.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.07

method	result
default	$\frac{\frac{(-a^3b-a\,b^3)\tan(x)+\frac{a^4}{2}-\frac{b^4}{2}}{1+\tan(x)^2}+a\left(\frac{(a^3-3a\,b^2)\ln(1+\tan(x)^2)}{2}+(3a^2b-b^3)\arctan(\tan(x))\right)}{(a^2+b^2)^3}+\frac{a^3}{(a^2+b^2)^2(a+b\tan(x))}-\frac{a^2(a^2\cdots}{\cdots}$
parallelrisch	$\frac{-8a^2(a^2-3b^2)(a\cos(x)+b\sin(x))\ln\left(\frac{-a\cos(x)-b\sin(x)}{\cos(x)+1}\right)+8a^2(a^2-3b^2)(a\cos(x)+b\sin(x))\ln\left(\frac{1}{\cos(x)+1}\right)+a(a^2+b^2)^2\cos(3x)\cdots}{8(a\cos(x)+b\sin(x)\cdots}$
risch	$\frac{iax}{3ib\,a^2-ib^3-a^3+3a\,b^2}+\frac{e^{2ix}}{-16iba+8a^2-8b^2}+\frac{e^{-2ix}}{16iba+8a^2-8b^2}+\frac{2ia^4x}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{6ia^2x\,b^2}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{\cdots}{(ib-\cdots}$
norman	$\frac{-\frac{2a\tan(\frac{x}{2})^8}{a^2+b^2}+\frac{(3a^2-b^2)a^2bx\tan(\frac{x}{2})^{10}}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{2a\tan(\frac{x}{2})^4}{a^2+b^2}-\frac{2a\tan(\frac{x}{2})^6}{a^2+b^2}+\frac{2a\tan(\frac{x}{2})^2}{a^2+b^2}+\frac{4b\,a^2\tan(\frac{x}{2})}{a^4+2a^2b^2+b^4}+\frac{4b\,a^2\tan(\frac{x}{2})^9}{a^4+2a^2b^2+b^4}-\frac{4b(-3a^3+a\,b^2)\cdots}{a(a^4+2a^2b^2\cdots}}{\cdots}$

input
```
int(cos(x)*sin(x)^3/(a*cos(x)+b*sin(x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/(a^2+b^2)^3*(((-a^3*b-a*b^3)*tan(x)+1/2*a^4-1/2*b^4)/(1+tan(x)^2)+a*(1/2
*(a^3-3*a*b^2)*ln(1+tan(x)^2)+(3*a^2*b-b^3)*arctan(tan(x))))+a^3/(a^2+b^2)
^2/(a+b*tan(x))-a^2*(a^2-3*b^2)/(a^2+b^2)^3*ln(a+b*tan(x))
```

3.286.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.83

$$\int \frac{\cos(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{2\,(a^5+2\,a^3b^2+ab^4)\cos{(x)}^3-(a^5+3\,ab^4-4\,(3\,a^4b-a^2b^3)x)\cos{(x)}-2\,((a^5-3\,a^3b^2)\cos{(x)}+(a^4b-\cdots}{4\,((a^7+3\,a^5b^2+3\,a^3b\cdots}$$

input
```
integrate(cos(x)*sin(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

output
```
1/4*(2*(a^5 + 2*a^3*b^2 + a*b^4)*cos(x)^3 - (a^5 + 3*a*b^4 - 4*(3*a^4*b -
a^2*b^3)*x)*cos(x) - 2*((a^5 - 3*a^3*b^2)*cos(x) + (a^4*b - 3*a^2*b^3)*sin
(x))*log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) - (5*a^4*b - b^
5 + 2*(a^4*b + 2*a^2*b^3 + b^5)*cos(x)^2 - 4*(3*a^3*b^2 - a*b^4)*x)*sin(x)
)/((a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cos(x) + (a^6*b + 3*a^4*b^3 + 3*a
^2*b^5 + b^7)*sin(x))
```

3.286.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(x)*sin(x)**3/(a*cos(x)+b*sin(x))**2,x)
```

output
```
Timed out
```

3.286.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(127) = 254$.

Time = 0.30 (sec) , antiderivative size = 259, normalized size of antiderivative = 2.01

$$\int \frac{\cos(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{(3\,a^3b-ab^3)x}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} - \frac{(a^4-3\,a^2b^2)\log\left(b\tan\left(x\right)+a\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{(a^4-3\,a^2b^2)\log\left(\tan\left(x\right)^2+1\right)}{2\,(a^6+3\,a^4b^2+3\,a^2b^4+b^6)} + \frac{3\,a^3-ab^2+2\,(a^3-ab^2)\tan\left(x\right)^2-(a^2b+b^3)\tan\left(x\right)}{2\,\left(a^5+2\,a^3b^2+ab^4+(a^4b+2\,a^2b^3+b^5)\tan\left(x\right)^3+(a^5+2\,a^3b^2+ab^4)\tan\left(x\right)^2+(a^4b+2\,a^2b^3+b^5)\,\text{ta}\right.}$$

input
```
integrate(cos(x)*sin(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

output
```
(3*a^3*b - a*b^3)*x/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (a^4 - 3*a^2*b^2
)*log(b*tan(x) + a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 1/2*(a^4 - 3*a^2
*b^2)*log(tan(x)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 1/2*(3*a^3 -
 a*b^2 + 2*(a^3 - a*b^2)*tan(x)^2 - (a^2*b + b^3)*tan(x))/(a^5 + 2*a^3*b^2
 + a*b^4 + (a^4*b + 2*a^2*b^3 + b^5)*tan(x)^3 + (a^5 + 2*a^3*b^2 + a*b^4)*
tan(x)^2 + (a^4*b + 2*a^2*b^3 + b^5)*tan(x))
```

3.286.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.73

$$\int \frac{\cos(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{(3\,a^3b-ab^3)x}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{(a^4-3\,a^2b^2)\log\left(\tan(x)^2+1\right)}{2\,(a^6+3\,a^4b^2+3\,a^2b^4+b^6)} - \frac{(a^4b-3\,a^2b^3)\log\left(|b\tan(x)+a|\right)}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7} + \frac{2\,a^3\tan(x)^2-2\,ab^2\tan(x)^2-a^2b\tan(x)-b^3\tan(x)+3\,a^3-ab^2}{2\,(a^4+2\,a^2b^2+b^4)\left(b\tan(x)^3+a\tan(x)^2+b\tan(x)+a\right)}$$

input
```
integrate(cos(x)*sin(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

output
```
(3*a^3*b - a*b^3)*x/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 1/2*(a^4 - 3*a^2
*b^2)*log(tan(x)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (a^4*b - 3*a
^2*b^3)*log(abs(b*tan(x) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7) + 1/2
*(2*a^3*tan(x)^2 - 2*a*b^2*tan(x)^2 - a^2*b*tan(x) - b^3*tan(x) + 3*a^3 -
a*b^2)/((a^4 + 2*a^2*b^2 + b^4)*(b*tan(x)^3 + a*tan(x)^2 + b*tan(x) + a))
```

3.286.9 Mupad [B] (verification not implemented)

Time = 30.77 (sec) , antiderivative size = 5431, normalized size of antiderivative = 42.10

$$\int \frac{\cos(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \text{Too large to display}$$

input
```
int((cos(x)*sin(x)^3)/(a*cos(x) + b*sin(x))^2,x)
```

output
```
(log(1/(cos(x) + 1))*(2*a^4 - 6*a^2*b^2))/(2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^
4*b^2)) - (log(a + 2*b*tan(x/2) - a*tan(x/2)^2)*(a^4 - 3*a^2*b^2))/(a^6 +
b^6 + 3*a^2*b^4 + 3*a^4*b^2) - ((2*a*tan(x/2)^2)/(a^2 + b^2) - (2*a*tan(x/
2)^4)/(a^2 + b^2) + (4*a^2*b*tan(x/2))/(a^2 + b^2)^2 + (4*a^2*b*tan(x/2)^5
)/(a^4 + b^4 + 2*a^2*b^2) + (4*b*tan(x/2)^3*(a^2 - b^2))/(a^2 + b^2)^2)/(a
 + 2*b*tan(x/2) + a*tan(x/2)^2 - a*tan(x/2)^4 - a*tan(x/2)^6 + 4*b*tan(x/2
)^3 + 2*b*tan(x/2)^5) + (2*a*b*atan((((((a*b*((32*(3*a^4*b^11 - a^2*b^13 -
 4*a^14*b + 18*a^6*b^9 + 22*a^8*b^7 + 3*a^10*b^5 - 9*a^12*b^3))/(a^12 + b^
12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) - (16
*(2*a^4 - 6*a^2*b^2)*(3*a^16*b + 3*a^2*b^15 + 21*a^4*b^13 + 63*a^6*b^11 +
105*a^8*b^9 + 105*a^10*b^7 + 63*a^12*b^5 + 21*a^14*b^3))/((a^6 + b^6 + 3*a
^2*b^4 + 3*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 +
15*a^8*b^4 + 6*a^10*b^2)))*(3*a^2 - b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b
^2) - (16*a*b*(2*a^4 - 6*a^2*b^2)*(3*a^2 - b^2)*(3*a^16*b + 3*a^2*b^15 + 2
1*a^4*b^13 + 63*a^6*b^11 + 105*a^8*b^9 + 105*a^10*b^7 + 63*a^12*b^5 + 21*a
^14*b^3))/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^2*(a^12 + b^12 + 6*a^2*b^10
 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(2*a^4 - 6*a^2*b^2
))/(2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (a*b*((32*(5*a^4*b^9 - 3*a^12
*b + 12*a^6*b^7 + 6*a^8*b^5 - 4*a^10*b^3))/(a^12 + b^12 + 6*a^2*b^10 + 15*
a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) + (((32*(3*a^4*b^11 - a...
```

3.286. $\int \frac{\cos(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

3.287 $\int \frac{\cos^2(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

3.287.1 Optimal result

Integrand size = 18, antiderivative size = 109

$$\int \frac{\cos^2(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx = -\frac{b(-2a^2+b^2)\,\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} - \frac{(a^2-b^2)\cos(x)}{(a^2+b^2)^2} + \frac{2ab\sin(x)}{(a^2+b^2)^2} + \frac{ab^2}{(a^2+b^2)^2\,(a\cos(x)+b\sin(x))}$$

output
```
-b*(-2*a^2+b^2)*arctanh((b*cos(x)-a*sin(x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/
2)-(a^2-b^2)*cos(x)/(a^2+b^2)^2+2*a*b*sin(x)/(a^2+b^2)^2+a*b^2/(a^2+b^2)^2
/(a*cos(x)+b*sin(x))
```

3.287.2 Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.01

$$\int \frac{\cos^2(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{2b(-2a^2+b^2)\,\text{arctanh}\left(\frac{-b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} - \frac{a^3-5ab^2+a(a^2+b^2)\cos(2x)-b(a^2+b^2)\sin(2x)}{2\,(a^2+b^2)^2\,(a\cos(x)+b\sin(x))}$$

input
```
Integrate[(Cos[x]^2*Sin[x])/(a*Cos[x] + b*Sin[x])^2,x]
```

output
```
(2*b*(-2*a^2 + b^2)*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2
)^(5/2) - (a^3 - 5*a*b^2 + a*(a^2 + b^2)*Cos[2*x] - b*(a^2 + b^2)*Sin[2*x]
)/(2*(a^2 + b^2)^2*(a*Cos[x] + b*Sin[x]))
```

3.287.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 228 vs. $2(109) = 218$.

Time = 1.31 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.09, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 3590, 3042, 3579, 3042, 3117, 3553, 219, 3588, 3042, 3117, 3118, 3553, 219, 3634, 3042, 3553, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)\cos^2(x)}{(a\cos(x)+b\sin(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)\cos(x)^2}{(a\cos(x)+b\sin(x))^2}dx$$

$$\Big\downarrow \text{3590}$$

$$\frac{b\int \frac{\cos^2(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}-\frac{ab\int \frac{\cos(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{a\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{ab\int \frac{\cos(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{b\int \frac{\cos(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3579}$$

$$-\frac{ab\int \frac{\cos(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{a\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\left(\frac{a\int \cos(x)dx}{a^2+b^2}+\frac{b^2\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{ab\int \frac{\cos(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{a\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\left(\frac{a\int \sin\left(x+\frac{\pi}{2}\right)dx}{a^2+b^2}+\frac{b^2\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{3117}$$

$$-\frac{ab\int \frac{\cos(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{b\left(\frac{b^2\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin(x)}{a^2+b^2}+\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{ 3553}$$

$$-\frac{ab\int \frac{\cos(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{a\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\left(-\frac{b^2\int \frac{1}{a^2+b^2-(b\cos(x)-a\sin(x))^2}d(b\cos(x)-a\sin(x))}{a^2+b^2}+\frac{a\sin(x)}{a^2+b^2}+\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{ab\int \frac{\cos(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{a\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\left(-\frac{b^2\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{a\sin(x)}{a^2+b^2}+\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{ 3588}$$

$$-\frac{ab\int \frac{\cos(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{a\left(\frac{a\int \sin(x)dx}{a^2+b^2}+\frac{b\int \cos(x)dx}{a^2+b^2}-\frac{ab\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\left(-\frac{b^2\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{a\sin(x)}{a^2+b^2}+\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{ab\int \frac{\cos(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{a\left(\frac{a\int \sin(x)dx}{a^2+b^2}+\frac{b\int \sin\left(x+\frac{\pi}{2}\right)dx}{a^2+b^2}-\frac{ab\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\left(-\frac{b^2\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{a\sin(x)}{a^2+b^2}+\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{ 3117}$$

$$-\frac{ab\int \frac{\cos(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{a\left(\frac{a\int \sin(x)dx}{a^2+b^2}-\frac{ab\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\sin(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\left(-\frac{b^2\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{a\sin(x)}{a^2+b^2}+\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}$$

3.287. $\int \frac{\cos^2(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

$$\Big\downarrow \text{3118}$$

$$-\frac{ab\int \frac{\cos(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{a\left(-\frac{ab\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\sin(x)}{a^2+b^2}-\frac{a\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\left(-\frac{b^2\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{a\sin(x)}{a^2+b^2}+\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{3553}$$

$$-\frac{ab\int \frac{\cos(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{a\left(\frac{ab\int \frac{1}{a^2+b^2-(b\cos(x)-a\sin(x))^2}d(b\cos(x)-a\sin(x))}{a^2+b^2}+\frac{b\sin(x)}{a^2+b^2}-\frac{a\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\left(-\frac{b^2\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{a\sin(x)}{a^2+b^2}+\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{219}$$

$$-\frac{ab\int \frac{\cos(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{b\left(-\frac{b^2\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{a\sin(x)}{a^2+b^2}+\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(\frac{ab\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{b\sin(x)}{a^2+b^2}-\frac{a\cos(x)}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{3634}$$

$$-\frac{ab\left(\frac{a\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}-\frac{b}{(a^2+b^2)(a\cos(x)+b\sin(x))}\right)}{a^2+b^2}+\frac{b\left(-\frac{b^2\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{a\sin(x)}{a^2+b^2}+\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(\frac{ab\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{b\sin(x)}{a^2+b^2}-\frac{a\cos(x)}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

3.287. $\int \frac{\cos^2(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

$$-\frac{ab\left(\frac{a\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}-\frac{b}{(a^2+b^2)(a\cos(x)+b\sin(x))}\right)}{a^2+b^2}+\frac{b\left(-\frac{b^2\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{a\sin(x)}{a^2+b^2}+\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(\frac{ab\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{b\sin(x)}{a^2+b^2}-\frac{a\cos(x)}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \; 3553$$

$$-\frac{ab\left(-\frac{a\int \frac{1}{a^2+b^2-(b\cos(x)-a\sin(x))^2}d(b\cos(x)-a\sin(x))}{a^2+b^2}-\frac{b}{(a^2+b^2)(a\cos(x)+b\sin(x))}\right)}{a^2+b^2}+\frac{b\left(-\frac{b^2\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{a\sin(x)}{a^2+b^2}+\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(\frac{ab\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{b\sin(x)}{a^2+b^2}-\frac{a\cos(x)}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \; 219$$

$$\frac{b\left(-\frac{b^2\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{a\sin(x)}{a^2+b^2}+\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(\frac{ab\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{b\sin(x)}{a^2+b^2}-\frac{a\cos(x)}{a^2+b^2}\right)}{a^2+b^2}-\frac{ab\left(-\frac{a\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}-\frac{b}{(a^2+b^2)(a\cos(x)+b\sin(x))}\right)}{a^2+b^2}$$

input
```
Int[(Cos[x]^2*Sin[x])/(a*Cos[x] + b*Sin[x])^2,x]
```

3.287. $\int \frac{\cos^2(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

output
```
(b*(-((b^2*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/
2)) + (b*Cos[x])/(a^2 + b^2) + (a*Sin[x])/(a^2 + b^2)))/(a^2 + b^2) + (a*(
(a*b*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2) - (
a*Cos[x])/(a^2 + b^2) + (b*Sin[x])/(a^2 + b^2)))/(a^2 + b^2) - (a*b*(-((a*
ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2)) - b/((a
^2 + b^2)*(a*Cos[x] + b*Sin[x]))))/(a^2 + b^2)
```

3.287.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3579
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(Cos[c + d*x]^(m - 1)/(d*(a^2 +
b^2)*(m - 1))), x] + (Simp[a/(a^2 + b^2)   Int[Cos[c + d*x]^(m - 1), x], x]
 + Simp[b^2/(a^2 + b^2)   Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[
c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m,
 1]
```

3.287. $\int \frac{\cos^2(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

rule 3588
```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b
/(a^2 + b^2)   Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Simp[a/(a
^2 + b^2)   Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Simp[a*(b/(a^
2 + b^2))   Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x]
+ b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
&& IGtQ[m, 0] && IGtQ[n, 0]
```

rule 3590
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Sim
p[b/(a^2 + b^2)   Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
 b*Sin[c + d*x])^(p + 1), x], x] + (Simp[a/(a^2 + b^2)   Int[Cos[c + d*x]^(
m - 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - S
imp[a*(b/(a^2 + b^2))   Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Co
s[c + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^
2 + b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

rule 3634
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[(c*B + c*A*Co
s[d + e*x] + (a*B - b*A)*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])), x] + Simp[(a*A - b*B)/(a^2 - b^2 - c^2)   Int[1/(a
 + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B},
 x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - b*B, 0]
```

3.287.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.31

method	result
default	$\frac{4b\left(\frac{-\frac{b^2\tan\left(\frac{x}{2}\right)}{2}-\frac{ab}{2}}{\tan\left(\frac{x}{2}\right)^2 a-2b\tan\left(\frac{x}{2}\right)-a}-\frac{\left(2a^2-b^2\right)\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{2\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^2}+\frac{4\tan\left(\frac{x}{2}\right)ab-2a^2+2b^2}{\left(a^4+2a^2b^2+b^4\right)\left(1+\tan\left(\frac{x}{2}\right)^2\right)}$
risch	$-\frac{e^{ix}}{2(-2iba+a^2-b^2)}-\frac{e^{-ix}}{2(2iba+a^2-b^2)}+\frac{2ia\,b^2e^{ix}}{(-ia+b)^2(ia+b)^2(b\,e^{2ix}+ia\,e^{2ix}-b+ia)}+\frac{2ib\ln\left(e^{ix}+\frac{ib+a}{\sqrt{-a^2-b^2}}\right)a^2}{\sqrt{-a^2-b^2}\,(a^2+b^2)^2}-\frac{ib^3\ln\left(e^{ix}+\right.}{\sqrt{-a^2-b}}$

input
```
int(cos(x)^2*sin(x)/(a*cos(x)+b*sin(x))^2,x,method=_RETURNVERBOSE)
```

output
```
4*b/(a^2+b^2)^2*((-1/2*b^2*tan(1/2*x)-1/2*a*b)/(tan(1/2*x)^2*a-2*b*tan(1/2
*x)-a)-1/2*(2*a^2-b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*x)-2*b)/(a
^2+b^2)^(1/2)))+4/(a^4+2*a^2*b^2+b^4)*(tan(1/2*x)*a*b-1/2*a^2+1/2*b^2)/(1+
tan(1/2*x)^2)
```

3.287.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(106) = 212$.

Time = 0.27 (sec) , antiderivative size = 252, normalized size of antiderivative = 2.31

$$\int \frac{\cos^2(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{6\,a^3b^2+6\,ab^4-2\,(a^5+2\,a^3b^2+ab^4)\cos(x)^2+2\,(a^4b+2\,a^2b^3+b^5)\cos(x)\sin(x)-\sqrt{a^2+b^2}((2\,a^3b-a}{2\,((a^7+3\,a^5b^2+3\,a^3b^4+ab^6)\cos(x)+(a}$$

input
```
integrate(cos(x)^2*sin(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

output
```
1/2*(6*a^3*b^2 + 6*a*b^4 - 2*(a^5 + 2*a^3*b^2 + a*b^4)*cos(x)^2 + 2*(a^4*b
 + 2*a^2*b^3 + b^5)*cos(x)*sin(x) - sqrt(a^2 + b^2)*((2*a^3*b - a*b^3)*cos
(x) + (2*a^2*b^2 - b^4)*sin(x))*log(-(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*co
s(x)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos
(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2)))/((a^7 + 3*a^5*b^2 + 3*a^3*b^4 +
 a*b^6)*cos(x) + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*sin(x))
```

3.287.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(x)**2*sin(x)/(a*cos(x)+b*sin(x))**2,x)
```

output
```
Timed out
```

3.287.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(106) = 212$.

Time = 0.32 (sec) , antiderivative size = 264, normalized size of antiderivative = 2.42

$$\int \frac{\cos^2(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{(2\,a^2b-b^3)\log\left(\frac{b-\frac{a\sin(x)}{\cos(x)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(x)}{\cos(x)+1}-\sqrt{a^2+b^2}}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}} - \frac{2\left(a^3-2\,ab^2-\frac{3\,b^3\sin(x)}{\cos(x)+1}-\frac{(a^3+4\,ab^2)\sin(x)^2}{(\cos(x)+1)^2}+\frac{(2\,a^2b-b^3)\sin(x)^3}{(\cos(x)+1)^3}\right)}{a^5+2\,a^3b^2+ab^4+\frac{2\,(a^4b+2\,a^2b^3+b^5)\sin(x)}{\cos(x)+1}+\frac{2\,(a^4b+2\,a^2b^3+b^5)\sin(x)^3}{(\cos(x)+1)^3}-\frac{(a^5+2\,a^3b^2+ab^4)\sin(x)^4}{(\cos(x)+1)^4}}$$

input
```
integrate(cos(x)^2*sin(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

output
```
(2*a^2*b - b^3)*log((b - a*sin(x)/(cos(x) + 1) + sqrt(a^2 + b^2))/(b - a*s
in(x)/(cos(x) + 1) - sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 +
 b^2)) - 2*(a^3 - 2*a*b^2 - 3*b^3*sin(x)/(cos(x) + 1) - (a^3 + 4*a*b^2)*si
n(x)^2/(cos(x) + 1)^2 + (2*a^2*b - b^3)*sin(x)^3/(cos(x) + 1)^3)/(a^5 + 2*
a^3*b^2 + a*b^4 + 2*(a^4*b + 2*a^2*b^3 + b^5)*sin(x)/(cos(x) + 1) + 2*(a^4
*b + 2*a^2*b^3 + b^5)*sin(x)^3/(cos(x) + 1)^3 - (a^5 + 2*a^3*b^2 + a*b^4)*
sin(x)^4/(cos(x) + 1)^4)
```

3.287.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.87

$$\int \frac{\cos^2(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{(2\,a^2b-b^3)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}} + \frac{2\left(2\,a^2b\tan\left(\frac{1}{2}\,x\right)^3-b^3\tan\left(\frac{1}{2}\,x\right)^3-a^3\tan\left(\frac{1}{2}\,x\right)^2-4\,ab^2\tan\left(\frac{1}{2}\,x\right)^2-3\,b^3\tan\left(\frac{1}{2}\,x\right)+a^3-2\,ab^2\right)}{\left(a\tan\left(\frac{1}{2}\,x\right)^4-2\,b\tan\left(\frac{1}{2}\,x\right)^3-2\,b\tan\left(\frac{1}{2}\,x\right)-a\right)(a^4+2\,a^2b^2+b^4)}$$

input
```
integrate(cos(x)^2*sin(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

output
```
(2*a^2*b - b^3)*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*
tan(1/2*x) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 +
 b^2)) + 2*(2*a^2*b*tan(1/2*x)^3 - b^3*tan(1/2*x)^3 - a^3*tan(1/2*x)^2 - 4
*a*b^2*tan(1/2*x)^2 - 3*b^3*tan(1/2*x) + a^3 - 2*a*b^2)/((a*tan(1/2*x)^4 -
 2*b*tan(1/2*x)^3 - 2*b*tan(1/2*x) - a)*(a^4 + 2*a^2*b^2 + b^4))
```

3.287.9 Mupad [B] (verification not implemented)

Time = 24.45 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.32

$$\int \frac{\cos^2(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{\frac{2\,(2\,a\,b^2-a^3)}{a^4+2\,a^2\,b^2+b^4}+\frac{6\,b^3\tan\left(\frac{x}{2}\right)}{a^4+2\,a^2\,b^2+b^4}+\frac{2\tan\left(\frac{x}{2}\right)^2\,(a^3+4\,a\,b^2)}{a^4+2\,a^2\,b^2+b^4}-\frac{2\,b\tan\left(\frac{x}{2}\right)^3\,(2\,a^2-b^2)}{a^4+2\,a^2\,b^2+b^4}}{-a\tan\left(\frac{x}{2}\right)^4+2\,b\tan\left(\frac{x}{2}\right)^3+2\,b\tan\left(\frac{x}{2}\right)+a} + \frac{b\,\mathrm{atan}\left(\frac{1\mathrm{i}\tan\left(\frac{x}{2}\right)a^5-a^4\,b\,1\mathrm{i}+2\mathrm{i}\tan\left(\frac{x}{2}\right)a^3\,b^2-a^2\,b^3\,2\mathrm{i}+1\mathrm{i}\tan\left(\frac{x}{2}\right)a\,b^4-b^5\,1\mathrm{i}}{(a^2+b^2)^{5/2}}\right)\,(2\,a^2-b^2)\,2\mathrm{i}}{(a^2+b^2)^{5/2}}$$

input
```
int((cos(x)^2*sin(x))/(a*cos(x) + b*sin(x))^2,x)
```

output
```
((2*(2*a*b^2 - a^3))/(a^4 + b^4 + 2*a^2*b^2) + (6*b^3*tan(x/2))/(a^4 + b^4
 + 2*a^2*b^2) + (2*tan(x/2)^2*(4*a*b^2 + a^3))/(a^4 + b^4 + 2*a^2*b^2) - (
2*b*tan(x/2)^3*(2*a^2 - b^2))/(a^4 + b^4 + 2*a^2*b^2))/(a + 2*b*tan(x/2) -
 a*tan(x/2)^4 + 2*b*tan(x/2)^3) + (b*atan((a^5*tan(x/2)*1i - a^4*b*1i - b^
5*1i - a^2*b^3*2i + a^3*b^2*tan(x/2)*2i + a*b^4*tan(x/2)*1i)/(a^2 + b^2)^(
5/2))*(2*a^2 - b^2)*2i)/(a^2 + b^2)^(5/2)
```

3.288 $\int \frac{\cos^2(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

3.288.1 Optimal result

Integrand size = 20, antiderivative size = 131

$$\int \frac{\cos^2(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{(a^4-6a^2b^2+b^4)\,x}{2\,(a^2+b^2)^3}+\frac{2ab(a^2-b^2)\log(a\cos(x)+b\sin(x))}{(a^2+b^2)^3}+\frac{(-a^2+b^2)\cos(x)\sin(x)}{2\,(a^2+b^2)^2}+\frac{ab\sin^2(x)}{(a^2+b^2)^2}+\frac{ab^2\sin(x)}{(a^2+b^2)^2\,(a\cos(x)+b\sin(x))}$$

output
```
1/2*(a^4-6*a^2*b^2+b^4)*x/(a^2+b^2)^3+2*a*b*(a^2-b^2)*ln(a*cos(x)+b*sin(x)
)/(a^2+b^2)^3+1/2*(-a^2+b^2)*cos(x)*sin(x)/(a^2+b^2)^2+a*b*sin(x)^2/(a^2+b
^2)^2+a*b^2*sin(x)/(a^2+b^2)^2/(a*cos(x)+b*sin(x))
```

3.288.2 Mathematica [A] (verified)

Time = 1.97 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.11

$$\int \frac{\cos^2(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{\sin(x)}{8a(a\cos(x)+b\sin(x))} - \frac{-4(a^4-6a^2b^2+b^4)\,x+4ab(a^2+b^2)\cos(2x)-16ab(a^2-b^2)\log(a\cos(x)+b\sin(x))+\frac{(a^2+b^2)\,(a^4-6a^2b^2}{a(a\cos(x)+b\,\mathrm{s}}}{8\,(a^2+b^2)^3}$$

input
```
Integrate[(Cos[x]^2*Sin[x]^2)/(a*Cos[x] + b*Sin[x])^2,x]
```

output
```
Sin[x]/(8*a*(a*Cos[x] + b*Sin[x])) - (-4*(a^4 - 6*a^2*b^2 + b^4)*x + 4*a*b
*(a^2 + b^2)*Cos[2*x] - 16*a*b*(a^2 - b^2)*Log[a*Cos[x] + b*Sin[x]] + ((a^
2 + b^2)*(a^4 - 6*a^2*b^2 + b^4)*Sin[x])/(a*(a*Cos[x] + b*Sin[x])) + 2*(a^
4 - b^4)*Sin[2*x])/(8*(a^2 + b^2)^3)
```

3.288.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 336 vs. $2(131) = 262$.

Time = 2.43 (sec) , antiderivative size = 336, normalized size of antiderivative = 2.56, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 1.050$, Rules used = {3042, 3590, 3042, 3588, 3042, 3044, 15, 3115, 24, 3576, 3042, 3577, 3042, 3590, 3042, 3554, 3576, 3042, 3577, 3042, 3612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(x)\cos^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)^2\cos(x)^2}{(a\cos(x)+b\sin(x))^2}dx$$

$$\Big\downarrow \text{3590}$$

$$\frac{b\int \frac{\cos^2(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\int \frac{\cos(x)\sin^2(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}-\frac{ab\int \frac{\cos(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{ab\int \frac{\cos(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{b\int \frac{\cos(x)^2\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\int \frac{\cos(x)\sin(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3588}$$

$$\frac{b\left(\frac{b\int \cos^2(x)dx}{a^2+b^2}+\frac{a\int \cos(x)\sin(x)dx}{a^2+b^2}-\frac{ab\int \frac{\cos(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(\frac{a\int \sin^2(x)dx}{a^2+b^2}+\frac{b\int \cos(x)\sin(x)dx}{a^2+b^2}-\frac{ab\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}-\frac{ab\int \frac{\cos(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{ab\int \frac{\cos(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{a\left(\frac{a\int \sin(x)^2dx}{a^2+b^2}+\frac{b\int \cos(x)\sin(x)dx}{a^2+b^2}-\frac{ab\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\left(\frac{b\int \sin\left(x+\frac{\pi}{2}\right)^2dx}{a^2+b^2}+\frac{a\int \cos(x)\sin(x)dx}{a^2+b^2}-\frac{ab\int \frac{\cos(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \; 3044$$

$$-\frac{ab\int \frac{\cos(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{b\left(\frac{b\int \sin\left(x+\frac{\pi}{2}\right)^2dx}{a^2+b^2}+\frac{a\int \sin(x)d\sin(x)}{a^2+b^2}-\frac{ab\int \frac{\cos(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(\frac{a\int \sin(x)^2dx}{a^2+b^2}+\frac{b\int \sin(x)d\sin(x)}{a^2+b^2}-\frac{ab\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \; 15$$

$$\frac{a\left(\frac{a\int \sin(x)^2dx}{a^2+b^2}-\frac{ab\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\sin^2(x)}{2(a^2+b^2)}\right)}{a^2+b^2}+\frac{b\left(\frac{b\int \sin\left(x+\frac{\pi}{2}\right)^2dx}{a^2+b^2}-\frac{ab\int \frac{\cos(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^2(x)}{2(a^2+b^2)}\right)}{a^2+b^2}-\frac{ab\int \frac{\cos(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}$$

$$\Big\downarrow \; 3115$$

$$\frac{b\left(\frac{b\left(\frac{\int 1dx}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}-\frac{ab\int \frac{\cos(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^2(x)}{2(a^2+b^2)}\right)}{a^2+b^2}+\frac{a\left(\frac{a\left(\frac{\int 1dx}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}-\frac{ab\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\sin^2(x)}{2(a^2+b^2)}\right)}{a^2+b^2}-\frac{ab\int \frac{\cos(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}$$

$$\Big\downarrow \; 24$$

$$\frac{b\left(-\frac{ab\int \frac{\cos(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^2(x)}{2(a^2+b^2)}+\frac{b\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(-\frac{ab\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\sin^2(x)}{2(a^2+b^2)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}-\frac{ab\int \frac{\cos(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}$$

$$\Big\downarrow \; 3576$$

3.288. $\int \frac{\cos^2(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

$$\frac{b\left(-\frac{ab\int \frac{\cos(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^2(x)}{2(a^2+b^2)}+\frac{b\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(-\frac{ab\left(\frac{bx}{a^2+b^2}-\frac{a\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\sin^2(x)}{2(a^2+b^2)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}-\frac{ab\int \frac{\cos(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}$$

$\downarrow$ 3042

$$\frac{b\left(-\frac{ab\int \frac{\cos(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^2(x)}{2(a^2+b^2)}+\frac{b\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(-\frac{ab\left(\frac{bx}{a^2+b^2}-\frac{a\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\sin^2(x)}{2(a^2+b^2)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}-\frac{ab\int \frac{\cos(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}$$

$\downarrow$ 3577

$$\frac{a\left(-\frac{ab\left(\frac{bx}{a^2+b^2}-\frac{a\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\sin^2(x)}{2(a^2+b^2)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\left(-\frac{ab\left(\frac{b\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{ax}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^2(x)}{2(a^2+b^2)}+\frac{b\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}-\frac{ab\int \frac{\cos(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}$$

$\downarrow$ 3042

$$\frac{a\left(-\frac{ab\left(\frac{bx}{a^2+b^2}-\frac{a\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\sin^2(x)}{2(a^2+b^2)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\left(-\frac{ab\left(\frac{b\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{ax}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^2(x)}{2(a^2+b^2)}+\frac{b\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}-\frac{ab\int \frac{\cos(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}$$

$\downarrow$ 3590

3.288. $\int \frac{\cos^2(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

$$\frac{a\left(-\frac{ab\left(\frac{bx}{a^2+b^2}-\frac{a\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\sin^2(x)}{2(a^2+b^2)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\left(-\frac{ab\left(\frac{b\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{ax}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^2(x)}{2(a^2+b^2)}+\frac{b\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}-\frac{ab\left(-\frac{ab\int \frac{1}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{b\int \frac{\cos(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{a\left(-\frac{ab\left(\frac{bx}{a^2+b^2}-\frac{a\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\sin^2(x)}{2(a^2+b^2)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\left(-\frac{ab\left(\frac{b\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{ax}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^2(x)}{2(a^2+b^2)}+\frac{b\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}-\frac{ab\left(-\frac{ab\int \frac{1}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{b\int \frac{\cos(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \ 3554$$

$$\frac{a\left(-\frac{ab\left(\frac{bx}{a^2+b^2}-\frac{a\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\sin^2(x)}{2(a^2+b^2)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\left(-\frac{ab\left(\frac{b\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{ax}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^2(x)}{2(a^2+b^2)}+\frac{b\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}-\frac{ab\left(\frac{b\int \frac{\cos(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}-\frac{b\sin(x)}{(a^2+b^2)(a\cos(x)+b\sin(x))}\right)}{a^2+b^2}$$

3.288. $\int \frac{\cos^2(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

$$\Big\downarrow \; 3576$$

$$\frac{a\left(-\frac{ab\left(\frac{bx}{a^2+b^2}-\frac{a\int\frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\sin^2(x)}{2(a^2+b^2)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\left(-\frac{ab\left(\frac{b\int\frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{ax}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^2(x)}{2(a^2+b^2)}+\frac{b\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}-\frac{ab\left(\frac{b\int\frac{\cos(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\left(\frac{bx}{a^2+b^2}-\frac{a\int\frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}-\frac{b\sin(x)}{(a^2+b^2)(a\cos(x)+b\sin(x))}\right)}{a^2+b^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{a\left(-\frac{ab\left(\frac{bx}{a^2+b^2}-\frac{a\int\frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\sin^2(x)}{2(a^2+b^2)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\left(-\frac{ab\left(\frac{b\int\frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{ax}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^2(x)}{2(a^2+b^2)}+\frac{b\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}-\frac{ab\left(\frac{b\int\frac{\cos(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\left(\frac{bx}{a^2+b^2}-\frac{a\int\frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}-\frac{b\sin(x)}{(a^2+b^2)(a\cos(x)+b\sin(x))}\right)}{a^2+b^2}$$

$$\Big\downarrow \; 3577$$

3.288. $\int \frac{\cos^2(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

$$\frac{a\left(-\frac{ab\left(\frac{bx}{a^2+b^2}-\frac{a\int\frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\sin^2(x)}{2(a^2+b^2)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\left(-\frac{ab\left(\frac{b\int\frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{ax}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^2(x)}{2(a^2+b^2)}+\frac{b\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}-\frac{ab\left(\frac{a\left(\frac{bx}{a^2+b^2}-\frac{a\int\frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\left(\frac{b\int\frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{ax}{a^2+b^2}\right)}{a^2+b^2}-\frac{b\sin(x)}{(a^2+b^2)(a\cos(x)+b\sin(x))}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\left(-\frac{ab\left(\frac{bx}{a^2+b^2}-\frac{a\int\frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\sin^2(x)}{2(a^2+b^2)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\left(-\frac{ab\left(\frac{b\int\frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{ax}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^2(x)}{2(a^2+b^2)}+\frac{b\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}-\frac{ab\left(\frac{a\left(\frac{bx}{a^2+b^2}-\frac{a\int\frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\left(\frac{b\int\frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{ax}{a^2+b^2}\right)}{a^2+b^2}-\frac{b\sin(x)}{(a^2+b^2)(a\cos(x)+b\sin(x))}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{3612}$$

$$\frac{a\left(\frac{b\sin^2(x)}{2(a^2+b^2)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}-\frac{ab\left(\frac{bx}{a^2+b^2}-\frac{a\log(a\cos(x)+b\sin(x))}{a^2+b^2}\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\left(\frac{a\sin^2(x)}{2(a^2+b^2)}+\frac{b\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}-\frac{ab\left(\frac{ax}{a^2+b^2}+\frac{b\log(a\cos(x)+b\sin(x))}{a^2+b^2}\right)}{a^2+b^2}\right)}{a^2+b^2}-\frac{ab\left(-\frac{b\sin(x)}{(a^2+b^2)(a\cos(x)+b\sin(x))}+\frac{a\left(\frac{bx}{a^2+b^2}-\frac{a\log(a\cos(x)+b\sin(x))}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\left(\frac{ax}{a^2+b^2}+\frac{b\log(a\cos(x)+b\sin(x))}{a^2+b^2}\right)}{a^2+b^2}\right)}{a^2+b^2}$$

3.288. $\int \frac{\cos^2(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

input
```
Int[(Cos[x]^2*Sin[x]^2)/(a*Cos[x] + b*Sin[x])^2,x]
```

output
```
-((a*b*((a*((b*x)/(a^2 + b^2) - (a*Log[a*Cos[x] + b*Sin[x]])/(a^2 + b^2)))
/(a^2 + b^2) + (b*((a*x)/(a^2 + b^2) + (b*Log[a*Cos[x] + b*Sin[x]])/(a^2 +
 b^2)))/(a^2 + b^2) - (b*Sin[x])/((a^2 + b^2)*(a*Cos[x] + b*Sin[x]))))/(a^
2 + b^2)) + (a*(-((a*b*((b*x)/(a^2 + b^2) - (a*Log[a*Cos[x] + b*Sin[x]])/(
a^2 + b^2)))/(a^2 + b^2)) + (b*Sin[x]^2)/(2*(a^2 + b^2)) + (a*(x/2 - (Cos[
x]*Sin[x])/2))/(a^2 + b^2)))/(a^2 + b^2) + (b*(-((a*b*((a*x)/(a^2 + b^2) +
 (b*Log[a*Cos[x] + b*Sin[x]])/(a^2 + b^2)))/(a^2 + b^2)) + (a*Sin[x]^2)/(2
*(a^2 + b^2)) + (b*(x/2 + (Cos[x]*Sin[x])/2))/(a^2 + b^2)))/(a^2 + b^2)
```

3.288.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3554
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

3.288. $\int \frac{\cos^2(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

rule 3576
```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(x/(a^2 + b^2)), x] - Simp[a/(a^2 + b
^2)   Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x
]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3577
```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[a*(x/(a^2 + b^2)), x] + Simp[b/(a^2 + b
^2)   Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x
]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3588
```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b
/(a^2 + b^2)   Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Simp[a/(a
^2 + b^2)   Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Simp[a*(b/(a^
2 + b^2))   Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x]
+ b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
&& IGtQ[m, 0] && IGtQ[n, 0]
```

rule 3590
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Sim
p[b/(a^2 + b^2)   Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
 b*Sin[c + d*x])^(p + 1), x], x] + (Simp[a/(a^2 + b^2)   Int[Cos[c + d*x]^(
m - 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - S
imp[a*(b/(a^2 + b^2))   Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Co
s[c + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^
2 + b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

rule 3612
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

3.288. $\int \frac{\cos^2(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

3.288.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.09

method	result
default	$\frac{\frac{\left(-\frac{a^4}{2}+\frac{b^4}{2}\right)\tan(x)-a^3b-a\,b^3}{1+\tan(x)^2}+\frac{\left(-4a^3b+4a\,b^3\right)\ln\left(1+\tan(x)^2\right)}{4}+\frac{\left(a^4-6a^2b^2+b^4\right)\arctan(\tan(x))}{2}}{(a^2+b^2)^3}-\frac{a^2b}{(a^2+b^2)^2(a+b\tan(x))}+\frac{2ab(}{}$
parallelrisch	$16\left(\left(a^5b-a\,b^5\right)\cos(2x)+ba(a-b)^2(a+b)^2\right)\ln\left(\frac{-a\cos(x)-b\sin(x)}{\cos(x)+1}\right)+16\left(\left(-a^5b+a\,b^5\right)\cos(2x)-ba(a-b)^2(a+b)^2\right)\ln\left(\frac{1}{\cos(x)+1}\right)$
risch	$-\frac{ixb}{2(3ib\,a^2-ib^3-a^3+3a\,b^2)}-\frac{xa}{2(3ib\,a^2-ib^3-a^3+3a\,b^2)}+\frac{ie^{2ix}}{-16iba+8a^2-8b^2}-\frac{ie^{-2ix}}{8(2iba+a^2-b^2)}-\frac{4ia^3bx}{a^6+3a^4b^2+3a^2b^4+b^6}$
norman	$\frac{2b\tan\left(\frac{x}{2}\right)^8}{a^2+b^2}+\frac{\left(a^4-6a^2b^2+b^4\right)ax\tan\left(\frac{x}{2}\right)^6}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{2b\tan\left(\frac{x}{2}\right)^4}{a^2+b^2}+\frac{2b\tan\left(\frac{x}{2}\right)^6}{a^2+b^2}-\frac{2b\tan\left(\frac{x}{2}\right)^2}{a^2+b^2}-\frac{\left(-a^4+3a^2b^2\right)\tan\left(\frac{x}{2}\right)}{a\left(a^4+2a^2b^2+b^4\right)}-\frac{\left(-a^4+3a^2b^2\right)\tan\left(\frac{x}{2}\right)^9}{a\left(a^4+2a^2b^2+b^4\right)}$

input
```
int(cos(x)^2*sin(x)^2/(a*cos(x)+b*sin(x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/(a^2+b^2)^3*(((-1/2*a^4+1/2*b^4)*tan(x)-a^3*b-a*b^3)/(1+tan(x)^2)+1/4*(-
4*a^3*b+4*a*b^3)*ln(1+tan(x)^2)+1/2*(a^4-6*a^2*b^2+b^4)*arctan(tan(x)))-a^
2*b/(a^2+b^2)^2/(a+b*tan(x))+2*a*b*(a^2-b^2)/(a^2+b^2)^3*ln(a+b*tan(x))
```

3.288.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.86

$$\int \frac{\cos^2(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx = -\frac{(a^4b+2\,a^2b^3+b^5)\cos(x)^3+(a^2b^3-b^5-(a^5-6\,a^3b^2+ab^4)x)\cos(x)-2\,((a^4b-a^2b^3)\cos(x)+(a^3b}{2\,((a^7+3\,a^5b^2+3}$$

input
```
integrate(cos(x)^2*sin(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

output
```
-1/2*((a^4*b + 2*a^2*b^3 + b^5)*cos(x)^3 + (a^2*b^3 - b^5 - (a^5 - 6*a^3*b
^2 + a*b^4)*x)*cos(x) - 2*((a^4*b - a^2*b^3)*cos(x) + (a^3*b^2 - a*b^4)*si
n(x))*log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) - (3*a^3*b^2 +
 a*b^4 - (a^5 + 2*a^3*b^2 + a*b^4)*cos(x)^2 + (a^4*b - 6*a^2*b^3 + b^5)*x)
*sin(x))/((a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cos(x) + (a^6*b + 3*a^4*b^
3 + 3*a^2*b^5 + b^7)*sin(x))
```

3.288.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(x)**2*sin(x)**2/(a*cos(x)+b*sin(x))**2,x)
```

output
```
Timed out
```

3.288.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(127) = 254$.

Time = 0.33 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.96

$$\int \frac{\cos^2(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{(a^4-6\,a^2b^2+b^4)x}{2\,(a^6+3\,a^4b^2+3\,a^2b^4+b^6)} + \frac{2\,(a^3b-ab^3)\log\left(b\tan(x)+a\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} - \frac{(a^3b-ab^3)\log\left(\tan(x)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} - \frac{4\,a^2b+(3\,a^2b-b^3)\tan(x)^2+(a^3+ab^2)\tan(x)}{2\left(a^5+2\,a^3b^2+ab^4+(a^4b+2\,a^2b^3+b^5)\tan(x)^3+(a^5+2\,a^3b^2+ab^4)\tan(x)^2+(a^4b+2\,a^2b^3+b^5)\,\text{ta}\right.}$$

input
```
integrate(cos(x)^2*sin(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

output
```
1/2*(a^4 - 6*a^2*b^2 + b^4)*x/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 2*(a^3
*b - a*b^3)*log(b*tan(x) + a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (a^3*b
 - a*b^3)*log(tan(x)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 1/2*(4*a
^2*b + (3*a^2*b - b^3)*tan(x)^2 + (a^3 + a*b^2)*tan(x))/(a^5 + 2*a^3*b^2 +
 a*b^4 + (a^4*b + 2*a^2*b^3 + b^5)*tan(x)^3 + (a^5 + 2*a^3*b^2 + a*b^4)*ta
n(x)^2 + (a^4*b + 2*a^2*b^3 + b^5)*tan(x))
```

3.288.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.67

$$\int \frac{\cos^2(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{(a^4-6\,a^2b^2+b^4)x}{2\,(a^6+3\,a^4b^2+3\,a^2b^4+b^6)} - \frac{(a^3b-ab^3)\log\left(\tan(x)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{2\,(a^3b^2-ab^4)\log(|b\tan(x)+a|)}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7} - \frac{3\,a^2b\tan(x)^2-b^3\tan(x)^2+a^3\tan(x)+ab^2\tan(x)+4\,a^2b}{2\,(a^4+2\,a^2b^2+b^4)\left(b\tan(x)^3+a\tan(x)^2+b\tan(x)+a\right)}$$

input
```
integrate(cos(x)^2*sin(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

output
```
1/2*(a^4 - 6*a^2*b^2 + b^4)*x/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (a^3*b
 - a*b^3)*log(tan(x)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 2*(a^3*b
^2 - a*b^4)*log(abs(b*tan(x) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7) -
 1/2*(3*a^2*b*tan(x)^2 - b^3*tan(x)^2 + a^3*tan(x) + a*b^2*tan(x) + 4*a^2*
b)/((a^4 + 2*a^2*b^2 + b^4)*(b*tan(x)^3 + a*tan(x)^2 + b*tan(x) + a))
```

3.288.9 Mupad [B] (verification not implemented)

Time = 36.24 (sec) , antiderivative size = 6012, normalized size of antiderivative = 45.89

$$\int \frac{\cos^2(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \text{Too large to display}$$

input
```
int((cos(x)^2*sin(x)^2)/(a*cos(x) + b*sin(x))^2,x)
```

output
```
((tan(x/2)^5*(3*a*b^2 - a^3))/(a^4 + b^4 + 2*a^2*b^2) + (2*b*tan(x/2)^2)/(
a^2 + b^2) - (2*b*tan(x/2)^4)/(a^2 + b^2) + (tan(x/2)*(3*a*b^2 - a^3))/(a^
2 + b^2)^2 + (2*tan(x/2)^3*(5*a*b^2 + a^3))/(a^2 + b^2)^2)/(a + 2*b*tan(x/
2) + a*tan(x/2)^2 - a*tan(x/2)^4 - a*tan(x/2)^6 + 4*b*tan(x/2)^3 + 2*b*tan
(x/2)^5) - (log(a + 2*b*tan(x/2) - a*tan(x/2)^2)*(2*a*b^3 - 2*a^3*b))/(a^6
 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (log(1/(cos(x) + 1))*(16*a*b^3 - 16*a^3*
b))/(2*(4*a^6 + 4*b^6 + 12*a^2*b^4 + 12*a^4*b^2)) - (atan((tan(x/2)*((((((
((8*(4*a^2*b^13 - 20*a^14*b + 48*a^4*b^11 + 132*a^6*b^9 + 128*a^8*b^7 + 12
*a^10*b^5 - 48*a^12*b^3))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*
b^6 + 15*a^8*b^4 + 6*a^10*b^2) - (4*(16*a*b^3 - 16*a^3*b)*(12*a*b^16 + 84*
a^3*b^14 + 252*a^5*b^12 + 420*a^7*b^10 + 420*a^9*b^8 + 252*a^11*b^6 + 84*a
^13*b^4 + 12*a^15*b^2))/((4*a^6 + 4*b^6 + 12*a^2*b^4 + 12*a^4*b^2)*(a^12 +
 b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*
(2*a*b - a^2 + b^2)*(2*a*b + a^2 - b^2))/(2*(a^2 + b^2)*(a^4 + b^4 + 2*a^2
*b^2)) - (2*(16*a*b^3 - 16*a^3*b)*(2*a*b - a^2 + b^2)*(2*a*b + a^2 - b^2)*
(12*a*b^16 + 84*a^3*b^14 + 252*a^5*b^12 + 420*a^7*b^10 + 420*a^9*b^8 + 252
*a^11*b^6 + 84*a^13*b^4 + 12*a^15*b^2))/((a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^
2)*(4*a^6 + 4*b^6 + 12*a^2*b^4 + 12*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 1
5*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(16*a*b^3 - 16*a^3*b))
/(2*(4*a^6 + 4*b^6 + 12*a^2*b^4 + 12*a^4*b^2)) - (((8*(2*a*b^12 + a^13 ...
```

3.288. $\int \frac{\cos^2(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

3.289 $\int \frac{\cos^2(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

3.289.1 Optimal result

Integrand size = 20, antiderivative size = 172

$$\int \frac{\cos^2(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{a^2b(2a^2-3b^2)\operatorname{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}} - \frac{a^2(a^2-3b^2)\cos(x)}{(a^2+b^2)^3} + \frac{(a^2-b^2)\cos^3(x)}{3(a^2+b^2)^2} + \frac{2ab(a^2-b^2)\sin(x)}{(a^2+b^2)^3} + \frac{2ab\sin^3(x)}{3(a^2+b^2)^2} + \frac{a^3b^2}{(a^2+b^2)^3(a\cos(x)+b\sin(x))}$$

output
```
a^2*b*(2*a^2-3*b^2)*arctanh((b*cos(x)-a*sin(x))/(a^2+b^2)^(1/2))/(a^2+b^2)
^(7/2)-a^2*(a^2-3*b^2)*cos(x)/(a^2+b^2)^3+1/3*(a^2-b^2)*cos(x)^3/(a^2+b^2)
^2+2*a*b*(a^2-b^2)*sin(x)/(a^2+b^2)^3+2/3*a*b*sin(x)^3/(a^2+b^2)^2+a^3*b^2
/(a^2+b^2)^3/(a*cos(x)+b*sin(x))
```

3.289.2 Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.16

$$\int \frac{\cos^2(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx = -\frac{2a^2b(2a^2-3b^2)\operatorname{arctanh}\left(\frac{-b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}} + \frac{-9a^5+90a^3b^2-21ab^4+(-8a^5+4a^3b^2+12ab^4)\cos(2x)+a(a^2+b^2)^2\cos(4x)+18a^4b\sin(2x)+16a^{\cdots}}{24(a^2+b^2)^3(a\cos(x)+b\sin(x))}$$

input
```
Integrate[(Cos[x]^2*Sin[x]^3)/(a*Cos[x] + b*Sin[x])^2,x]
```

output
```
(-2*a^2*b*(2*a^2 - 3*b^2)*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/(a^2
 + b^2)^(7/2) + (-9*a^5 + 90*a^3*b^2 - 21*a*b^4 + (-8*a^5 + 4*a^3*b^2 + 12
*a*b^4)*Cos[2*x] + a*(a^2 + b^2)^2*Cos[4*x] + 18*a^4*b*Sin[2*x] + 16*a^2*b
^3*Sin[2*x] - 2*b^5*Sin[2*x] - a^4*b*Sin[4*x] - 2*a^2*b^3*Sin[4*x] - b^5*S
in[4*x])/(24*(a^2 + b^2)^3*(a*Cos[x] + b*Sin[x]))
```

3.289.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(x)\cos^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)^3\cos(x)^2}{(a\cos(x)+b\sin(x))^2}dx$$

$$\Big\downarrow \text{3590}$$

$$\frac{b\int \frac{\cos^2(x)\sin^2(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\int \frac{\cos(x)\sin^3(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}-\frac{ab\int \frac{\cos(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{ab\int \frac{\cos(x)\sin(x)^2}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{b\int \frac{\cos(x)^2\sin(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\int \frac{\cos(x)\sin(x)^3}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3588}$$

$$\frac{b\left(\frac{b\int \cos^2(x)\sin(x)dx}{a^2+b^2}+\frac{a\int \cos(x)\sin^2(x)dx}{a^2+b^2}-\frac{ab\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(\frac{a\int \sin^3(x)dx}{a^2+b^2}+\frac{b\int \cos(x)\sin^2(x)dx}{a^2+b^2}-\frac{ab\int \frac{\sin^2(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}-\frac{ab\int \frac{\cos(x)\sin(x)^2}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{ab\int \frac{\cos(x)\sin(x)^2}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{b\left(\frac{b\int \cos(x)^2\sin(x)dx}{a^2+b^2}+\frac{a\int \cos(x)\sin(x)^2dx}{a^2+b^2}-\frac{ab\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(\frac{a\int \sin(x)^3dx}{a^2+b^2}+\frac{b\int \cos(x)\sin(x)^2dx}{a^2+b^2}-\frac{ab\int \frac{\sin(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \ 3044$$

$$\frac{b\left(\frac{a\int \sin^2(x)d\sin(x)}{a^2+b^2}+\frac{b\int \cos(x)^2\sin(x)dx}{a^2+b^2}-\frac{ab\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(\frac{b\int \sin^2(x)d\sin(x)}{a^2+b^2}+\frac{a\int \sin(x)^3dx}{a^2+b^2}-\frac{ab\int \frac{\sin(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}-\frac{ab\int \frac{\cos(x)\sin(x)^2}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}$$

$$\Big\downarrow \ 15$$

$$\frac{b\left(\frac{b\int \cos(x)^2\sin(x)dx}{a^2+b^2}-\frac{ab\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}+\frac{a\left(\frac{a\int \sin(x)^3dx}{a^2+b^2}-\frac{ab\int \frac{\sin(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\sin^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}-\frac{ab\int \frac{\cos(x)\sin(x)^2}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}$$

$$\Big\downarrow \ 3045$$

$$\frac{b\left(-\frac{b\int \cos^2(x)d\cos(x)}{a^2+b^2}-\frac{ab\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}+\frac{a\left(\frac{a\int \sin(x)^3dx}{a^2+b^2}-\frac{ab\int \frac{\sin(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\sin^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}-\frac{ab\int \frac{\cos(x)\sin(x)^2}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}$$

$$\Big\downarrow \ 15$$

$$\frac{b\left(-\frac{ab\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{b\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}+\frac{a\left(\frac{a\int \sin(x)^3dx}{a^2+b^2}-\frac{ab\int \frac{\sin(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\sin^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}-\frac{ab\int \frac{\cos(x)\sin(x)^2}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}$$

$$\Big\downarrow \ 3113$$

3.289. $\int \frac{\cos^2(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

$$\frac{b\left(-\frac{ab\int\frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{b\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}+\frac{a\left(-\frac{a\int(1-\cos^2(x))\,d\cos(x)}{a^2+b^2}-\frac{ab\int\frac{\sin(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\sin^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}-\frac{ab\int\frac{\cos(x)\sin(x)^2}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{b\left(-\frac{ab\int\frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{b\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}+\frac{a\left(-\frac{ab\int\frac{\sin(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\sin^3(x)}{3(a^2+b^2)}-\frac{a\left(\cos(x)-\frac{\cos^3(x)}{3}\right)}{a^2+b^2}\right)}{a^2+b^2}-\frac{ab\int\frac{\cos(x)\sin(x)^2}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}$$

$$\Big\downarrow \text{ 3578}$$

$$\frac{b\left(-\frac{ab\int\frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{b\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}+\frac{a\left(-\frac{ab\left(\frac{b\int\sin(x)dx}{a^2+b^2}+\frac{a^2\int\frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}-\frac{a\sin(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\sin^3(x)}{3(a^2+b^2)}-\frac{a\left(\cos(x)-\frac{\cos^3(x)}{3}\right)}{a^2+b^2}\right)}{a^2+b^2}-\frac{ab\int\frac{\cos(x)\sin(x)^2}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{a\left(-\frac{ab\left(\frac{b\int\sin(x)dx}{a^2+b^2}+\frac{a^2\int\frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}-\frac{a\sin(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\sin^3(x)}{3(a^2+b^2)}-\frac{a\left(\cos(x)-\frac{\cos^3(x)}{3}\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\left(-\frac{ab\int\frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{b\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}-\frac{ab\int\frac{\cos(x)\sin(x)^2}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}$$

$$\Big\downarrow \text{ 3118}$$

$$\frac{a\left(-\frac{ab\left(\frac{a^2\int\frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}-\frac{a\sin(x)}{a^2+b^2}-\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\sin^3(x)}{3(a^2+b^2)}-\frac{a\left(\cos(x)-\frac{\cos^3(x)}{3}\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\left(-\frac{ab\int\frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{b\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}-\frac{ab\int\frac{\cos(x)\sin(x)^2}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}$$

3.289. $\int\frac{\cos^2(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

$$\Big\downarrow \ 3553$$

$$\frac{b\left(-\frac{ab\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{b\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}+\frac{a\left(-\frac{ab\left(-\frac{a^2\int \frac{1}{a^2+b^2-(b\cos(x)-a\sin(x))^2}d(b\cos(x)-a\sin(x))}{a^2+b^2}-\frac{a\sin(x)}{a^2+b^2}-\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\sin^3(x)}{3(a^2+b^2)}-\frac{a\left(\cos(x)-\frac{\cos^3(x)}{3}\right)}{a^2+b^2}\right)}{a^2+b^2}-\frac{ab\int \frac{\cos(x)\sin(x)^2}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}$$

$$\Big\downarrow \ 219$$

$$\frac{b\left(-\frac{ab\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{b\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}-\frac{ab\int \frac{\cos(x)\sin(x)^2}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{a\left(-\frac{ab\left(-\frac{a^2\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}-\frac{a\sin(x)}{a^2+b^2}-\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\sin^3(x)}{3(a^2+b^2)}-\frac{a\left(\cos(x)-\frac{\cos^3(x)}{3}\right)}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \ 3588$$

$$\frac{b\left(-\frac{ab\left(\frac{a\int \sin(x)dx}{a^2+b^2}+\frac{b\int \cos(x)dx}{a^2+b^2}-\frac{ab\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{b\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}-\frac{ab\int \frac{\cos(x)\sin(x)^2}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{a\left(-\frac{ab\left(-\frac{a^2\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}-\frac{a\sin(x)}{a^2+b^2}-\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\sin^3(x)}{3(a^2+b^2)}-\frac{a\left(\cos(x)-\frac{\cos^3(x)}{3}\right)}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \ 3042$$

3.289. $\int \frac{\cos^2(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

$$\frac{b\left(-\frac{ab\left(\frac{a\int\sin(x)dx}{a^2+b^2}+\frac{b\int\sin\left(x+\frac{\pi}{2}\right)dx}{a^2+b^2}-\frac{ab\int\frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{b\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}-\frac{ab\int\frac{\cos(x)\sin(x)^2}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{a\left(-\frac{ab\left(-\frac{a^2\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}-\frac{a\sin(x)}{a^2+b^2}-\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\sin^3(x)}{3(a^2+b^2)}-\frac{a\left(\cos(x)-\frac{\cos^3(x)}{3}\right)}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{3117}$$

$$\frac{b\left(-\frac{ab\left(\frac{a\int\sin(x)dx}{a^2+b^2}-\frac{ab\int\frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\sin(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{b\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}-\frac{ab\int\frac{\cos(x)\sin(x)^2}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{a\left(-\frac{ab\left(-\frac{a^2\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}-\frac{a\sin(x)}{a^2+b^2}-\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\sin^3(x)}{3(a^2+b^2)}-\frac{a\left(\cos(x)-\frac{\cos^3(x)}{3}\right)}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{3118}$$

$$\frac{b\left(-\frac{ab\left(-\frac{ab\int\frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\sin(x)}{a^2+b^2}-\frac{a\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{b\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}-\frac{ab\int\frac{\cos(x)\sin(x)^2}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{a\left(-\frac{ab\left(-\frac{a^2\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}-\frac{a\sin(x)}{a^2+b^2}-\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\sin^3(x)}{3(a^2+b^2)}-\frac{a\left(\cos(x)-\frac{\cos^3(x)}{3}\right)}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{3553}$$

3.289. $\int \frac{\cos^2(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

$$\frac{b\left(-\frac{ab\left(\frac{ab\int \frac{1}{a^2+b^2-(b\cos(x)-a\sin(x))^2}d(b\cos(x)-a\sin(x))}{a^2+b^2}+\frac{b\sin(x)}{a^2+b^2}-\frac{a\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{b\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}-\frac{ab\int \frac{\cos(x)\sin(x)^2}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{a\left(-\frac{ab\left(-\frac{a^2\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}-\frac{a\sin(x)}{a^2+b^2}-\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\sin^3(x)}{3(a^2+b^2)}-\frac{a\left(\cos(x)-\frac{\cos^3(x)}{3}\right)}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \; 219$$

$$-\frac{ab\int \frac{\cos(x)\sin(x)^2}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{a\left(-\frac{ab\left(-\frac{a^2\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}-\frac{a\sin(x)}{a^2+b^2}-\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\sin^3(x)}{3(a^2+b^2)}-\frac{a\left(\cos(x)-\frac{\cos^3(x)}{3}\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\left(-\frac{ab\left(\frac{ab\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{b\sin(x)}{a^2+b^2}-\frac{a\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{b\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}$$

$$\Big\downarrow \; 3590$$

$$-\frac{ab\left(\frac{a\int \frac{\sin^2(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}-\frac{ab\int \frac{\sin(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{b\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(-\frac{ab\left(-\frac{a^2\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}-\frac{a\sin(x)}{a^2+b^2}-\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\sin^3(x)}{3(a^2+b^2)}-\frac{a\left(\cos(x)-\frac{\cos^3(x)}{3}\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\left(-\frac{ab\left(\frac{ab\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{b\sin(x)}{a^2+b^2}-\frac{a\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{b\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}$$

$$\Big\downarrow \; 3042$$

3.289. $\int \frac{\cos^2(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

$$-\frac{ab\left(-\frac{ab\int\frac{\sin(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{b\int\frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\int\frac{\sin(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(-\frac{ab\left(-\frac{a^2\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}-\frac{a\sin(x)}{a^2+b^2}-\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\sin^3(x)}{3(a^2+b^2)}-\frac{a\left(\cos(x)-\frac{\cos^3(x)}{3}\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\left(-\frac{ab\left(\frac{ab\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{b\sin(x)}{a^2+b^2}-\frac{a\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{b\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}$$

$$\downarrow 3578$$

$$-\frac{ab\left(-\frac{ab\int\frac{\sin(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{b\int\frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\left(\frac{b\int\sin(x)dx}{a^2+b^2}+\frac{a^2\int\frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}-\frac{a\sin(x)}{a^2+b^2}\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(-\frac{ab\left(-\frac{a^2\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}-\frac{a\sin(x)}{a^2+b^2}-\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\sin^3(x)}{3(a^2+b^2)}-\frac{a\left(\cos(x)-\frac{\cos^3(x)}{3}\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\left(-\frac{ab\left(\frac{ab\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{b\sin(x)}{a^2+b^2}-\frac{a\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{b\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}$$

$$\downarrow 3042$$

3.289. $\int\frac{\cos^2(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

$$-\frac{ab\left(-\frac{ab\int \frac{\sin(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{a\left(\frac{b\int \sin(x)dx}{a^2+b^2}+\frac{a^2\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}-\frac{a\sin(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(-\frac{ab\left(-\frac{a^2\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}-\frac{a\sin(x)}{a^2+b^2}-\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\sin^3(x)}{3(a^2+b^2)}-\frac{a\left(\cos(x)-\frac{\cos^3(x)}{3}\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\left(-\frac{ab\left(\frac{ab\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}+\frac{b\sin(x)}{a^2+b^2}-\frac{a\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{b\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}$$

$$\Big\downarrow \; 3118$$

$$-\frac{ab\left(-\frac{ab\int \frac{\sin(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{a\left(\frac{a^2\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}-\frac{a\sin(x)}{a^2+b^2}-\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(-\frac{ab\left(-\frac{a^2\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}-\frac{a\sin(x)}{a^2+b^2}-\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\sin^3(x)}{3(a^2+b^2)}-\frac{a\left(\cos(x)-\frac{\cos^3(x)}{3}\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\left(-\frac{ab\left(\frac{ab\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}+\frac{b\sin(x)}{a^2+b^2}-\frac{a\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{b\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}$$

$$\Big\downarrow \; 3553$$

3.289. $\int \frac{\cos^2(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

$$-\frac{ab\left(-\frac{ab\int \frac{\sin(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{b\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\left(-\frac{a^2\int \frac{1}{a^2+b^2-(b\cos(x)-a\sin(x))^2}d(b\cos(x)-a\sin(x))}{a^2+b^2}-\frac{a\sin(x)}{a^2+b^2}-\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(-\frac{ab\left(-\frac{a^2\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}-\frac{a\sin(x)}{a^2+b^2}-\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\sin^3(x)}{3(a^2+b^2)}-\frac{a\left(\cos(x)-\frac{\cos^3(x)}{3}\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\left(-\frac{ab\left(\frac{ab\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{b\sin(x)}{a^2+b^2}-\frac{a\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{b\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{219}$$

$$-\frac{ab\left(-\frac{ab\int \frac{\sin(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{b\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\left(-\frac{a^2\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}-\frac{a\sin(x)}{a^2+b^2}-\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(-\frac{ab\left(-\frac{a^2\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}-\frac{a\sin(x)}{a^2+b^2}-\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\sin^3(x)}{3(a^2+b^2)}-\frac{a\left(\cos(x)-\frac{\cos^3(x)}{3}\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\left(-\frac{ab\left(\frac{ab\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{b\sin(x)}{a^2+b^2}-\frac{a\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{b\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{3588}$$

3.289. $\int \frac{\cos^2(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

$$-\frac{ab\left(-\frac{ab\int \frac{\sin(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{b\left(\frac{a\int \sin(x)dx}{a^2+b^2}+\frac{b\int \cos(x)dx}{a^2+b^2}-\frac{ab\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(-\frac{a^2\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}-\frac{a\sin(x)}{a^2+b^2}-\right.}{a^2+b^2}}{a^2+b^2}$$

$$\frac{a\left(-\frac{ab\left(-\frac{a^2\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}-\frac{a\sin(x)}{a^2+b^2}-\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\sin^3(x)}{3(a^2+b^2)}-\frac{a\left(\cos(x)-\frac{\cos^3(x)}{3}\right)}{a^2+b^2}\right)}{a^2+b^2}+$$

$$\frac{b\left(-\frac{ab\left(\frac{ab\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{b\sin(x)}{a^2+b^2}-\frac{a\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{b\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}$$

input
```
Int[(Cos[x]^2*Sin[x]^3)/(a*Cos[x] + b*Sin[x])^2,x]
```

output
```
$Aborted
```

3.289.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3578
```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-a)*(Sin[c + d*x]^(m - 1)/(d*(a^2
 + b^2)*(m - 1))), x] + (Simp[a^2/(a^2 + b^2)   Int[Sin[c + d*x]^(m - 2)/(a
*Cos[c + d*x] + b*Sin[c + d*x]), x], x] + Simp[b/(a^2 + b^2)   Int[Sin[c +
d*x]^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ
[m, 1]
```

3.289. $\int \frac{\cos^2(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

rule 3588
```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b
/(a^2 + b^2)   Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Simp[a/(a
^2 + b^2)   Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Simp[a*(b/(a^
2 + b^2))   Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x]
+ b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
&& IGtQ[m, 0] && IGtQ[n, 0]
```

rule 3590
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Sim
p[b/(a^2 + b^2)   Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
 b*Sin[c + d*x])^(p + 1), x], x] + (Simp[a/(a^2 + b^2)   Int[Cos[c + d*x]^(
m - 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - S
imp[a*(b/(a^2 + b^2))   Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Co
s[c + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^
2 + b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

3.289.4 Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.58

method	result
default	$\frac{4a^2b\left(\frac{-\frac{b^2\tan\left(\frac{x}{2}\right)}{2}-\frac{ab}{2}}{\tan\left(\frac{x}{2}\right)^2a-2b\tan\left(\frac{x}{2}\right)-a}-\frac{\left(2a^2-3b^2\right)\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{2\sqrt{a^2+b^2}}\right)}{(a^4+2a^2b^2+b^4)(a^2+b^2)}+\frac{4\left(a^3b-a\,b^3\right)\tan\left(\frac{x}{2}\right)^5+4\left(\frac{3}{2}a^2b^2-\frac{1}{2}b^4\right)\tan\left(\frac{x}{2}\right)^4+4\left(\frac{10}{3}a^3b\right.}{(a^2+b^2}$
risch	$\frac{e^{3ix}}{-48iba+24a^2-24b^2}-\frac{ie^{ix}b}{8(-3ib\,a^2+ib^3+a^3-3a\,b^2)}-\frac{3\,e^{ix}a}{8(-3ib\,a^2+ib^3+a^3-3a\,b^2)}+\frac{ie^{-ix}b}{8(ib+a)^3}-\frac{3\,e^{-ix}a}{8(ib+a)^3}+\frac{e^{-3ix}}{24(ib+a)^2}+\frac{}{(-}$

input
```
int(cos(x)^2*sin(x)^3/(a*cos(x)+b*sin(x))^2,x,method=_RETURNVERBOSE)
```

output
```
4*a^2*b/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*((-1/2*b^2*tan(1/2*x)-1/2*a*b)/(tan(
1/2*x)^2*a-2*b*tan(1/2*x)-a)-1/2*(2*a^2-3*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2
*(2*a*tan(1/2*x)-2*b)/(a^2+b^2)^(1/2)))+4/(a^2+b^2)/(a^4+2*a^2*b^2+b^4)*((
a^3*b-a*b^3)*tan(1/2*x)^5+(3/2*a^2*b^2-1/2*b^4)*tan(1/2*x)^4+(10/3*a^3*b-2
/3*a*b^3)*tan(1/2*x)^3+(-a^4+3*a^2*b^2)*tan(1/2*x)^2+(a^3*b-a*b^3)*tan(1/2
*x)-1/3*a^4+3/2*a^2*b^2-1/6*b^4)/(1+tan(1/2*x)^2)^3
```

3.289.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. 2(164) = 328.

Time = 0.29 (sec) , antiderivative size = 360, normalized size of antiderivative = 2.09

$$\int \frac{\cos^2(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx$$
$$= \frac{22\,a^5b^2+14\,a^3b^4-8\,ab^6+2\,(a^7+3\,a^5b^2+3\,a^3b^4+ab^6)\cos{(x)}^4-2\,(3\,a^7+4\,a^5b^2-a^3b^4-2\,ab^6)\cos{(x)}}{}$$

input
```
integrate(cos(x)^2*sin(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

output
```
1/6*(22*a^5*b^2 + 14*a^3*b^4 - 8*a*b^6 + 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 +
a*b^6)*cos(x)^4 - 2*(3*a^7 + 4*a^5*b^2 - a^3*b^4 - 2*a*b^6)*cos(x)^2 - 3*s
qrt(a^2 + b^2)*((2*a^5*b - 3*a^3*b^3)*cos(x) + (2*a^4*b^2 - 3*a^2*b^4)*sin
(x))*log(-(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 - 2*a^2 - b^2 + 2*sq
rt(a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*co
s(x)^2 + b^2)) - 2*((a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cos(x)^3 - 5*(a^
6*b + 2*a^4*b^3 + a^2*b^5)*cos(x))*sin(x))/((a^9 + 4*a^7*b^2 + 6*a^5*b^4 +
 4*a^3*b^6 + a*b^8)*cos(x) + (a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 +
b^9)*sin(x))
```

3.289.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(x)**2*sin(x)**3/(a*cos(x)+b*sin(x))**2,x)
```

output
```
Timed out
```

3.289.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 611 vs. $2(164) = 328$.

Time = 0.40 (sec) , antiderivative size = 611, normalized size of antiderivative = 3.55

$$\int \frac{\cos^2(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{(2a^2b-3b^3)a^2\log\left(\frac{b-\frac{a\sin(x)}{\cos(x)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(x)}{\cos(x)+1}-\sqrt{a^2+b^2}}\right)}{(a^6+3a^4b^2+3a^2b^4+b^6)\sqrt{a^2+b^2}} - \frac{2\left(2a^5-12a^3b^2+ab^4-\frac{(2a^4b+15a^2b^3-2b^5)\sin(x)}{\cos(x)+1}+\frac{(4a^5-30a^3b^2+11ab^4)\sin(x)^2}{(\cos(x)+1)^2}-\frac{(2a^4b+47\cdots}{(\cos(\cdots}\right.}{3\left(a^7+3a^5b^2+3a^3b^4+ab^6+\frac{2(a^6b+3a^4b^3+3a^2b^5+b^7)\sin(x)}{\cos(x)+1}+\frac{2(a^7+3a^5b^2+3a^3b^4+ab^6)\sin(x)^2}{(\cos(x)+1)^2}+\frac{6(a^6b+3a^4b^3+3a^2b\cdots}{(\cos(x)+1\cdots}\right.}$$

input
```
integrate(cos(x)^2*sin(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

output
```
(2*a^2*b - 3*b^3)*a^2*log((b - a*sin(x)/(cos(x) + 1) + sqrt(a^2 + b^2))/(b
 - a*sin(x)/(cos(x) + 1) - sqrt(a^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4
 + b^6)*sqrt(a^2 + b^2)) - 2/3*(2*a^5 - 12*a^3*b^2 + a*b^4 - (2*a^4*b + 15
*a^2*b^3 - 2*b^5)*sin(x)/(cos(x) + 1) + (4*a^5 - 30*a^3*b^2 + 11*a*b^4)*si
n(x)^2/(cos(x) + 1)^2 - (2*a^4*b + 47*a^2*b^3)*sin(x)^3/(cos(x) + 1)^3 - (
6*a^5 + 40*a^3*b^2 - 11*a*b^4)*sin(x)^4/(cos(x) + 1)^4 + (14*a^4*b - 25*a^
2*b^3 + 6*b^5)*sin(x)^5/(cos(x) + 1)^5 - 3*(2*a^3*b^2 - 3*a*b^4)*sin(x)^6/
(cos(x) + 1)^6 + 3*(2*a^4*b - 3*a^2*b^3)*sin(x)^7/(cos(x) + 1)^7)/(a^7 + 3
*a^5*b^2 + 3*a^3*b^4 + a*b^6 + 2*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*sin
(x)/(cos(x) + 1) + 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*sin(x)^2/(cos(x
) + 1)^2 + 6*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*sin(x)^3/(cos(x) + 1)^3
 + 6*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*sin(x)^5/(cos(x) + 1)^5 - 2*(a^
7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*sin(x)^6/(cos(x) + 1)^6 + 2*(a^6*b + 3*
a^4*b^3 + 3*a^2*b^5 + b^7)*sin(x)^7/(cos(x) + 1)^7 - (a^7 + 3*a^5*b^2 + 3*
a^3*b^4 + a*b^6)*sin(x)^8/(cos(x) + 1)^8)
```

3.289.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 342 vs. $2(164) = 328$.

Time = 0.33 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.99

$$\int \frac{\cos^2(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{(2\,a^4b-3\,a^2b^3)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\sqrt{a^2+b^2}} - \frac{2\left(a^2b^3\tan\left(\frac{1}{2}\,x\right)+a^3b^2\right)}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\left(a\tan\left(\frac{1}{2}\,x\right)^2-2\,b\tan\left(\frac{1}{2}\,x\right)-a\right)} + \frac{2\left(6\,a^3b\tan\left(\frac{1}{2}\,x\right)^5-6\,ab^3\tan\left(\frac{1}{2}\,x\right)^5+9\,a^2b^2\tan\left(\frac{1}{2}\,x\right)^4-3\,b^4\tan\left(\frac{1}{2}\,x\right)^4+20\,a^3b\tan\left(\frac{1}{2}\,x\right)^3-4\,ab^3\tan\right.}{3\,(a^6+3\,a^4b^2+3\,a^2b^4+}$$

input
```
integrate(cos(x)^2*sin(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

output
```
(2*a^4*b - 3*a^2*b^3)*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/ab
s(2*a*tan(1/2*x) - 2*b + 2*sqrt(a^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4
 + b^6)*sqrt(a^2 + b^2)) - 2*(a^2*b^3*tan(1/2*x) + a^3*b^2)/((a^6 + 3*a^4*
b^2 + 3*a^2*b^4 + b^6)*(a*tan(1/2*x)^2 - 2*b*tan(1/2*x) - a)) + 2/3*(6*a^3
*b*tan(1/2*x)^5 - 6*a*b^3*tan(1/2*x)^5 + 9*a^2*b^2*tan(1/2*x)^4 - 3*b^4*ta
n(1/2*x)^4 + 20*a^3*b*tan(1/2*x)^3 - 4*a*b^3*tan(1/2*x)^3 - 6*a^4*tan(1/2*
x)^2 + 18*a^2*b^2*tan(1/2*x)^2 + 6*a^3*b*tan(1/2*x) - 6*a*b^3*tan(1/2*x) -
 2*a^4 + 9*a^2*b^2 - b^4)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*(tan(1/2*x)
^2 + 1)^3)
```

3.289.9 Mupad [B] (verification not implemented)

Time = 25.44 (sec) , antiderivative size = 594, normalized size of antiderivative = 3.45

$$\int \frac{\cos^2(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx = -\frac{\frac{2\tan\left(\frac{x}{2}\right)^6(3\,a\,b^4-2\,a^3\,b^2)}{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6}-\frac{2\tan\left(\frac{x}{2}\right)^4(6\,a^5+40\,a^3\,b^2-11\,a\,b^4)}{3\,(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6)}-\frac{2\tan\left(\frac{x}{2}\right)^3(2\,a^4\,b+47\,a^2\,b^3)}{3\,(a^2+b^2)\,(a^4+2\,a^2\,b^2+b^4)}+\frac{2\,a\,(2\,a^4-12\,a^2\,b^2+b^4)}{3\,(a^2+b^2)\,(a^4+2\,a^2\,b^2+b^4)}+\frac{2\,a\tan}{3\,(}}{-a\tan\left(\frac{x}{2}\right)^8+2\,b\tan\left(\frac{x}{2}\right)^7-2\,a\tan\left(\frac{x}{2}\right)^6+6\,b\tan\left(\frac{}{2}\right.} + \frac{a^2\,b\,\mathrm{atan}\left(\frac{1\mathrm{i}\tan\left(\frac{x}{2}\right)a^7-a^6\,b\,1\mathrm{i}+3\mathrm{i}\tan\left(\frac{x}{2}\right)a^5\,b^2-a^4\,b^3\,3\mathrm{i}+3\mathrm{i}\tan\left(\frac{x}{2}\right)a^3\,b^4-a^2\,b^5\,3\mathrm{i}+1\mathrm{i}\tan\left(\frac{x}{2}\right)a\,b^6-b^7\,1\mathrm{i}}{(a^2+b^2)^{7/2}}\right)(2\,a^2-3\,b^2)\,2\mathrm{i}}{(a^2+b^2)^{7/2}}$$

input
```
int((cos(x)^2*sin(x)^3)/(a*cos(x) + b*sin(x))^2,x)
```

output
```
(a^2*b*atan((a^7*tan(x/2)*1i - a^6*b*1i - b^7*1i - a^2*b^5*3i - a^4*b^3*3i
 + a^3*b^4*tan(x/2)*3i + a^5*b^2*tan(x/2)*3i + a*b^6*tan(x/2)*1i)/(a^2 + b
^2)^(7/2))*(2*a^2 - 3*b^2)*2i)/(a^2 + b^2)^(7/2) - ((2*tan(x/2)^6*(3*a*b^4
 - 2*a^3*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (2*tan(x/2)^4*(6*a^5
- 11*a*b^4 + 40*a^3*b^2))/(3*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (2*tan
(x/2)^3*(2*a^4*b + 47*a^2*b^3))/(3*(a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2)) +
(2*a*(2*a^4 + b^4 - 12*a^2*b^2))/(3*(a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2)) +
 (2*a*tan(x/2)^2*(4*a^4 + 11*b^4 - 30*a^2*b^2))/(3*(a^2 + b^2)*(a^4 + b^4
+ 2*a^2*b^2)) + (2*b*tan(x/2)^5*(14*a^4 + 6*b^4 - 25*a^2*b^2))/(3*(a^2 + b
^2)*(a^4 + b^4 + 2*a^2*b^2)) + (2*a^2*b*tan(x/2)^7*(2*a^2 - 3*b^2))/(a^6 +
 b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (2*b*tan(x/2)*(2*a^4 - 2*b^4 + 15*a^2*b^2)
)/(3*(a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2)))/(a + 2*b*tan(x/2) + 2*a*tan(x/2
)^2 - 2*a*tan(x/2)^6 - a*tan(x/2)^8 + 6*b*tan(x/2)^3 + 6*b*tan(x/2)^5 + 2*
b*tan(x/2)^7)
```

3.289. $\int \frac{\cos^2(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

3.290 $\int \frac{\cos^3(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

3.290.1 Optimal result

Integrand size = 18, antiderivative size = 128

$$\int \frac{\cos^3(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx = -\frac{ab(a^2-3b^2)\,x}{(a^2+b^2)^3} - \frac{b^2(3a^2-b^2)\log(a\cos(x)+b\sin(x))}{(a^2+b^2)^3} + \frac{ab\cos(x)\sin(x)}{(a^2+b^2)^2} + \frac{(a^2-b^2)\sin^2(x)}{2\,(a^2+b^2)^2} + \frac{ab^2\cos(x)}{(a^2+b^2)^2\,(a\cos(x)+b\sin(x))}$$

output
```
-a*b*(a^2-3*b^2)*x/(a^2+b^2)^3-b^2*(3*a^2-b^2)*ln(a*cos(x)+b*sin(x))/(a^2+
b^2)^3+a*b*cos(x)*sin(x)/(a^2+b^2)^2+1/2*(a^2-b^2)*sin(x)^2/(a^2+b^2)^2+a*
b^2*cos(x)/(a^2+b^2)^2/(a*cos(x)+b*sin(x))
```

3.290.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.63 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.73

$$\int \frac{\cos^3(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{-4ib^2(-3a^2+b^2)\arctan(\tan(x))(a\cos(x)+b\sin(x)) - a\cos(x)\left((a^4-b^4)\cos(2x)+2b(2(a+ib)^3x-b(\cdots\right.}{}$$

input
```
Integrate[(Cos[x]^3*Sin[x])/(a*Cos[x] + b*Sin[x])^2,x]
```

output
```
((-4*I)*b^2*(-3*a^2 + b^2)*ArcTan[Tan[x]]*(a*Cos[x] + b*Sin[x]) - a*Cos[x]
*((a^4 - b^4)*Cos[2*x] + 2*b*(2*(a + I*b)^3*x - b*(-3*a^2 + b^2)*Log[(a*Co
s[x] + b*Sin[x])^2] - a*(a^2 + b^2)*Sin[2*x])) + b*Sin[x]*((-a^4 + b^4)*Co
s[2*x] + 2*b*(-2*(a + I*b)*(a^2*x - b^2*(I + x) + a*(b + (2*I)*b*x)) + (-3
*a^2*b + b^3)*Log[(a*Cos[x] + b*Sin[x])^2] + a*(a^2 + b^2)*Sin[2*x])))/(4*
(a^2 + b^2)^3*(a*Cos[x] + b*Sin[x]))
```

3.290.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 293 vs. 2(128) = 256.

Time = 2.19 (sec) , antiderivative size = 293, normalized size of antiderivative = 2.29, number of steps used = 26, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 1.389$, Rules used = {3042, 3590, 3042, 3565, 3042, 3579, 3042, 3115, 24, 3577, 3042, 3588, 3042, 3044, 15, 3115, 24, 3577, 3042, 3612, 3964, 3042, 4014, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)\cos^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)\cos(x)^3}{(a\cos(x)+b\sin(x))^2}dx$$

$$\Big\downarrow \text{3590}$$

$$\frac{b\int \frac{\cos^3(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}-\frac{ab\int \frac{\cos^2(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{a\int \frac{\cos^2(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{ab\int \frac{\cos(x)^2}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{b\int \frac{\cos(x)^3}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\int \frac{\cos(x)^2\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3565}$$

$$-\frac{ab\int \frac{1}{(a+b\tan(x))^2}dx}{a^2+b^2}+\frac{b\int \frac{\cos(x)^3}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\int \frac{\cos(x)^2\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{ab\int \frac{1}{(a+b\tan(x))^2}dx}{a^2+b^2}+\frac{b\int \frac{\cos(x)^3}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\int \frac{\cos(x)^2\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3579}$$

$$-\frac{ab\int \frac{1}{(a+b\tan(x))^2}dx}{a^2+b^2}+\frac{b\left(\frac{a\int \cos^2(x)dx}{a^2+b^2}+\frac{b^2\int \frac{\cos(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\cos^2(x)}{2(a^2+b^2)}\right)}{a^2+b^2}+\frac{a\int \frac{\cos(x)^2\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{ab\int \frac{1}{(a+b\tan(x))^2}dx}{a^2+b^2}+\frac{b\left(\frac{a\int \sin\left(x+\frac{\pi}{2}\right)^2dx}{a^2+b^2}+\frac{b^2\int \frac{\cos(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\cos^2(x)}{2(a^2+b^2)}\right)}{a^2+b^2}+\frac{a\int \frac{\cos(x)^2\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3115}$$

$$-\frac{ab\int \frac{1}{(a+b\tan(x))^2}dx}{a^2+b^2}+\frac{b\left(\frac{b^2\int \frac{\cos(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\left(\frac{\int 1dx}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}+\frac{b\cos^2(x)}{2(a^2+b^2)}\right)}{a^2+b^2}+\frac{a\int \frac{\cos(x)^2\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{24}$$

$$-\frac{ab\int \frac{1}{(a+b\tan(x))^2}dx}{a^2+b^2}+\frac{b\left(\frac{b^2\int \frac{\cos(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\cos^2(x)}{2(a^2+b^2)}+\frac{a\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\int \frac{\cos(x)^2\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3577}$$

$$-\frac{ab\int \frac{1}{(a+b\tan(x))^2}dx}{a^2+b^2}+\frac{b\left(\frac{b^2\left(\frac{b\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{ax}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\cos^2(x)}{2(a^2+b^2)}+\frac{a\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\int \frac{\cos(x)^2\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{ab\int \frac{1}{(a+b\tan(x))^2}dx}{a^2+b^2}+\frac{b\left(\frac{b^2\left(\frac{b\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{ax}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\cos^2(x)}{2(a^2+b^2)}+\frac{a\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\int \frac{\cos(x)^2\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

3.290. $\int \frac{\cos^3(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

$$\Big\downarrow \; 3588$$

$$-\frac{ab\int \frac{1}{(a+b\tan(x))^2}dx}{a^2+b^2}+\frac{b\left(\frac{b^2\left(\frac{b\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{ax}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\cos^2(x)}{2(a^2+b^2)}+\frac{a\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(\frac{b\int \cos^2(x)dx}{a^2+b^2}+\frac{a\int \cos(x)\sin(x)dx}{a^2+b^2}-\frac{ab\int \frac{\cos(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{ab\int \frac{1}{(a+b\tan(x))^2}dx}{a^2+b^2}+\frac{b\left(\frac{b^2\left(\frac{b\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{ax}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\cos^2(x)}{2(a^2+b^2)}+\frac{a\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(\frac{b\int \sin\left(x+\frac{\pi}{2}\right)^2dx}{a^2+b^2}+\frac{a\int \cos(x)\sin(x)dx}{a^2+b^2}-\frac{ab\int \frac{\cos(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \; 3044$$

$$-\frac{ab\int \frac{1}{(a+b\tan(x))^2}dx}{a^2+b^2}+\frac{b\left(\frac{b^2\left(\frac{b\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{ax}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\cos^2(x)}{2(a^2+b^2)}+\frac{a\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(\frac{b\int \sin\left(x+\frac{\pi}{2}\right)^2dx}{a^2+b^2}+\frac{a\int \sin(x)d\sin(x)}{a^2+b^2}-\frac{ab\int \frac{\cos(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \; 15$$

$$-\frac{ab\int \frac{1}{(a+b\tan(x))^2}dx}{a^2+b^2}+\frac{b\left(\frac{b^2\left(\frac{b\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{ax}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\cos^2(x)}{2(a^2+b^2)}+\frac{a\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(\frac{b\int \sin\left(x+\frac{\pi}{2}\right)^2dx}{a^2+b^2}-\frac{ab\int \frac{\cos(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^2(x)}{2(a^2+b^2)}\right)}{a^2+b^2}$$

$$\Big\downarrow \; 3115$$

3.290. $\int \frac{\cos^3(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

$$-\frac{ab\int \frac{1}{(a+b\tan(x))^2}dx}{a^2+b^2}+\frac{b\left(\frac{b^2\left(\frac{b\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{ax}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\cos^2(x)}{2(a^2+b^2)}+\frac{a\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(\frac{b\left(\frac{\int 1dx}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}-\frac{ab\int \frac{\cos(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^2(x)}{2(a^2+b^2)}\right)}{a^2+b^2}$$

$$\Big\downarrow \; 24$$

$$-\frac{ab\int \frac{1}{(a+b\tan(x))^2}dx}{a^2+b^2}+\frac{b\left(\frac{b^2\left(\frac{b\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{ax}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\cos^2(x)}{2(a^2+b^2)}+\frac{a\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(-\frac{ab\int \frac{\cos(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^2(x)}{2(a^2+b^2)}+\frac{b\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \; 3577$$

$$-\frac{ab\int \frac{1}{(a+b\tan(x))^2}dx}{a^2+b^2}+\frac{b\left(\frac{b^2\left(\frac{b\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{ax}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\cos^2(x)}{2(a^2+b^2)}+\frac{a\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(-\frac{ab\left(\frac{b\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{ax}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^2(x)}{2(a^2+b^2)}+\frac{b\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{ab\int \frac{1}{(a+b\tan(x))^2}dx}{a^2+b^2}+\frac{b\left(\frac{b^2\left(\frac{b\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{ax}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\cos^2(x)}{2(a^2+b^2)}+\frac{a\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(-\frac{ab\left(\frac{b\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{ax}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^2(x)}{2(a^2+b^2)}+\frac{b\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \; 3612$$

3.290. $\int \frac{\cos^3(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

$$-\frac{ab\int \frac{1}{(a+b\tan(x))^2}dx}{a^2+b^2}+\frac{b\left(\frac{b\cos^2(x)}{2(a^2+b^2)}+\frac{a\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}+\frac{b^2\left(\frac{ax}{a^2+b^2}+\frac{b\log(a\cos(x)+b\sin(x))}{a^2+b^2}\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(\frac{a\sin^2(x)}{2(a^2+b^2)}+\frac{b\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}-\frac{ab\left(\frac{ax}{a^2+b^2}+\frac{b\log(a\cos(x)+b\sin(x))}{a^2+b^2}\right)}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{3964}$$

$$-\frac{ab\left(\frac{\int \frac{a-b\tan(x)}{a+b\tan(x)}dx}{a^2+b^2}-\frac{b}{(a^2+b^2)(a+b\tan(x))}\right)}{a^2+b^2}+\frac{b\left(\frac{b\cos^2(x)}{2(a^2+b^2)}+\frac{a\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}+\frac{b^2\left(\frac{ax}{a^2+b^2}+\frac{b\log(a\cos(x)+b\sin(x))}{a^2+b^2}\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(\frac{a\sin^2(x)}{2(a^2+b^2)}+\frac{b\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}-\frac{ab\left(\frac{ax}{a^2+b^2}+\frac{b\log(a\cos(x)+b\sin(x))}{a^2+b^2}\right)}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{ab\left(\frac{\int \frac{a-b\tan(x)}{a+b\tan(x)}dx}{a^2+b^2}-\frac{b}{(a^2+b^2)(a+b\tan(x))}\right)}{a^2+b^2}+\frac{b\left(\frac{b\cos^2(x)}{2(a^2+b^2)}+\frac{a\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}+\frac{b^2\left(\frac{ax}{a^2+b^2}+\frac{b\log(a\cos(x)+b\sin(x))}{a^2+b^2}\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(\frac{a\sin^2(x)}{2(a^2+b^2)}+\frac{b\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}-\frac{ab\left(\frac{ax}{a^2+b^2}+\frac{b\log(a\cos(x)+b\sin(x))}{a^2+b^2}\right)}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{4014}$$

$$-\frac{ab\left(\frac{\frac{2ab\int \frac{b-a\tan(x)}{a+b\tan(x)}dx}{a^2+b^2}+\frac{x(a^2-b^2)}{a^2+b^2}}{a^2+b^2}-\frac{b}{(a^2+b^2)(a+b\tan(x))}\right)}{a^2+b^2}+\frac{b\left(\frac{b\cos^2(x)}{2(a^2+b^2)}+\frac{a\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}+\frac{b^2\left(\frac{ax}{a^2+b^2}+\frac{b\log(a\cos(x)+b\sin(x))}{a^2+b^2}\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(\frac{a\sin^2(x)}{2(a^2+b^2)}+\frac{b\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}-\frac{ab\left(\frac{ax}{a^2+b^2}+\frac{b\log(a\cos(x)+b\sin(x))}{a^2+b^2}\right)}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

3.290. $\int \frac{\cos^3(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}dx$

$$-\frac{ab\left(\frac{\frac{2ab\int \frac{b-a\tan(x)}{a+b\tan(x)}dx}{a^2+b^2}+\frac{x\left(a^2-b^2\right)}{a^2+b^2}}{a^2+b^2}-\frac{b}{(a^2+b^2)(a+b\tan(x))}\right)}{a^2+b^2}+\frac{b\left(\frac{b\cos^2(x)}{2(a^2+b^2)}+\frac{a\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}+\frac{b^2\left(\frac{ax}{a^2+b^2}+\frac{b\log(a\cos(x)+b\sin(x))}{a^2+b^2}\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(\frac{a\sin^2(x)}{2(a^2+b^2)}+\frac{b\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}-\frac{ab\left(\frac{ax}{a^2+b^2}+\frac{b\log(a\cos(x)+b\sin(x))}{a^2+b^2}\right)}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{ 4013}$$

$$\frac{b\left(\frac{b\cos^2(x)}{2(a^2+b^2)}+\frac{a\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}+\frac{b^2\left(\frac{ax}{a^2+b^2}+\frac{b\log(a\cos(x)+b\sin(x))}{a^2+b^2}\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(\frac{a\sin^2(x)}{2(a^2+b^2)}+\frac{b\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}-\frac{ab\left(\frac{ax}{a^2+b^2}+\frac{b\log(a\cos(x)+b\sin(x))}{a^2+b^2}\right)}{a^2+b^2}\right)}{a^2+b^2}-\frac{ab\left(\frac{\frac{x\left(a^2-b^2\right)}{a^2+b^2}+\frac{2ab\log(a\cos(x)+b\sin(x))}{a^2+b^2}}{a^2+b^2}-\frac{b}{(a^2+b^2)(a+b\tan(x))}\right)}{a^2+b^2}$$

input
```
Int[(Cos[x]^3*Sin[x])/(a*Cos[x] + b*Sin[x])^2,x]
```

output
```
(b*((b*Cos[x]^2)/(2*(a^2 + b^2)) + (b^2*((a*x)/(a^2 + b^2) + (b*Log[a*Cos[
x] + b*Sin[x]])/(a^2 + b^2)))/(a^2 + b^2) + (a*(x/2 + (Cos[x]*Sin[x])/2))/
(a^2 + b^2)))/(a^2 + b^2) + (a*(-((a*b*((a*x)/(a^2 + b^2) + (b*Log[a*Cos[x
] + b*Sin[x]])/(a^2 + b^2)))/(a^2 + b^2)) + (a*Sin[x]^2)/(2*(a^2 + b^2)) +
 (b*(x/2 + (Cos[x]*Sin[x])/2))/(a^2 + b^2)))/(a^2 + b^2) - (a*b*((((a^2 -
b^2)*x)/(a^2 + b^2) + (2*a*b*Log[a*Cos[x] + b*Sin[x]])/(a^2 + b^2))/(a^2 +
 b^2) - b/((a^2 + b^2)*(a + b*Tan[x]))))/(a^2 + b^2)
```

3.290. $\int \frac{\cos^3(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

3.290.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3565
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(a + b*Tan[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

rule 3577
```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[a*(x/(a^2 + b^2)), x] + Simp[b/(a^2 + b
^2)   Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x
]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3579
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(Cos[c + d*x]^(m - 1)/(d*(a^2 +
b^2)*(m - 1))), x] + (Simp[a/(a^2 + b^2)   Int[Cos[c + d*x]^(m - 1), x], x]
 + Simp[b^2/(a^2 + b^2)   Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[
c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m,
 1]
```

3.290. $\int \frac{\cos^3(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

rule 3588
```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b
/(a^2 + b^2)   Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Simp[a/(a
^2 + b^2)   Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Simp[a*(b/(a^
2 + b^2))   Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x]
+ b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
&& IGtQ[m, 0] && IGtQ[n, 0]
```

rule 3590
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Sim
p[b/(a^2 + b^2)   Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
 b*Sin[c + d*x])^(p + 1), x], x] + (Simp[a/(a^2 + b^2)   Int[Cos[c + d*x]^(
m - 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - S
imp[a*(b/(a^2 + b^2))   Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Co
s[c + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^
2 + b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

rule 3612
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

rule 3964
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)
 Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

3.290. $\int \frac{\cos^3(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

3.290.4 Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.10

method	result
default	$\frac{a\,b^2}{(a^2+b^2)^2(a+b\tan(x))}-\frac{b^2(3a^2-b^2)\ln(a+b\tan(x))}{(a^2+b^2)^3}+\frac{\frac{\left(a^3b+a\,b^3\right)\tan(x)-\frac{a^4}{2}+\frac{b^4}{2}}{1+\tan(x)^2}+b\left(\frac{\left(3a^2b-b^3\right)\ln\left(1+\tan(x)^2\right)}{2}+(-a^3+3a\right.}{(a^2+b^2)^3}$
parallelrisch	$\frac{-24b^2\left(a^2-\frac{b^2}{3}\right)(a\cos(x)+b\sin(x))\ln\left(\frac{-a\cos(x)-b\sin(x)}{\cos(x)+1}\right)+24b^2\left(a^2-\frac{b^2}{3}\right)(a\cos(x)+b\sin(x))\ln\left(\frac{1}{\cos(x)+1}\right)-a\left(a^2+b^2\right)^2\cos(3}{8(a\cos(x)+b\sin(x)}$
risch	$-\frac{ixb}{ia^3-3ia\,b^2+3a^2b-b^3}-\frac{e^{2ix}}{8(-2iba+a^2-b^2)}-\frac{e^{-2ix}}{8(2iba+a^2-b^2)}+\frac{6ia^2x\,b^2}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{2ib^4x}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{}{(-}$
norman	$\frac{\frac{2a\tan\left(\frac{x}{2}\right)^8}{a^2+b^2}+\frac{b\,a^2\left(a^2-3b^2\right)x}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{2a\tan\left(\frac{x}{2}\right)^4}{a^2+b^2}+\frac{2a\tan\left(\frac{x}{2}\right)^6}{a^2+b^2}-\frac{2a\tan\left(\frac{x}{2}\right)^2}{a^2+b^2}-\frac{2b\left(a^3-a\,b^2\right)\tan\left(\frac{x}{2}\right)}{a\left(a^4+2a^2b^2+b^4\right)}-\frac{2b\left(a^3-a\,b^2\right)\tan\left(\frac{x}{2}\right)^9}{a\left(a^4+2a^2b^2+b^4\right)}-\frac{4(a^3-}{a(a}}{}$

input
```
int(cos(x)^3*sin(x)/(a*cos(x)+b*sin(x))^2,x,method=_RETURNVERBOSE)
```

output
```
a*b^2/(a^2+b^2)^2/(a+b*tan(x))-b^2*(3*a^2-b^2)/(a^2+b^2)^3*ln(a+b*tan(x))+
1/(a^2+b^2)^3*(((a^3*b+a*b^3)*tan(x)-1/2*a^4+1/2*b^4)/(1+tan(x)^2)+b*(1/2*
(3*a^2*b-b^3)*ln(1+tan(x)^2)+(-a^3+3*a*b^2)*arctan(tan(x))))
```

3.290.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.97

$$\int \frac{\cos^3(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx =$$

$$-\frac{2\left(a^5+2\,a^3b^2+ab^4\right)\cos\left(x\right)^3-\left(a^5+4\,a^3b^2+7\,ab^4-4\left(a^4b-3\,a^2b^3\right)x\right)\cos\left(x\right)+2\left(\left(3\,a^3b^2-ab^4\right)\cos\left(\right.\right.}{4\left(\left(a^7+3\,a^5\right.\right.}$$

input
```
integrate(cos(x)^3*sin(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

output
```
-1/4*(2*(a^5 + 2*a^3*b^2 + a*b^4)*cos(x)^3 - (a^5 + 4*a^3*b^2 + 7*a*b^4 -
4*(a^4*b - 3*a^2*b^3)*x)*cos(x) + 2*((3*a^3*b^2 - a*b^4)*cos(x) + (3*a^2*b
^3 - b^5)*sin(x))*log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) -
(a^4*b - 4*a^2*b^3 - b^5 + 2*(a^4*b + 2*a^2*b^3 + b^5)*cos(x)^2 - 4*(a^3*b
^2 - 3*a*b^4)*x)*sin(x))/((a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cos(x) + (
a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*sin(x))
```

3.290.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(x)**3*sin(x)/(a*cos(x)+b*sin(x))**2,x)
```

output
```
Timed out
```

3.290.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(126) = 252$.

Time = 0.33 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.00

$$\begin{aligned}\int \frac{\cos^3(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx = &-\frac{(a^3b-3\,ab^3)x}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} \\ &-\frac{(3\,a^2b^2-b^4)\log\left(b\tan\left(x\right)+a\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{(3\,a^2b^2-b^4)\log\left(\tan\left(x\right)^2+1\right)}{2\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)} \\ &+\frac{4\,ab^2\tan\left(x\right)^2-a^3+3\,ab^2+(a^2b+b^3)\tan\left(x\right)}{2\left(a^5+2\,a^3b^2+ab^4+(a^4b+2\,a^2b^3+b^5)\tan\left(x\right)^3+(a^5+2\,a^3b^2+ab^4)\tan\left(x\right)^2+(a^4b+2\,a^2b^3+b^5)\,\mathrm{ta}\right.}\end{aligned}$$

input
```
integrate(cos(x)^3*sin(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

output
```
-(a^3*b - 3*a*b^3)*x/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (3*a^2*b^2 - b^
4)*log(b*tan(x) + a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 1/2*(3*a^2*b^2
- b^4)*log(tan(x)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 1/2*(4*a*b^
2*tan(x)^2 - a^3 + 3*a*b^2 + (a^2*b + b^3)*tan(x))/(a^5 + 2*a^3*b^2 + a*b^
4 + (a^4*b + 2*a^2*b^3 + b^5)*tan(x)^3 + (a^5 + 2*a^3*b^2 + a*b^4)*tan(x)^
2 + (a^4*b + 2*a^2*b^3 + b^5)*tan(x))
```

3.290.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.67

$$\int \frac{\cos^3(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx = -\frac{(a^3b-3\,ab^3)x}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{(3\,a^2b^2-b^4)\log\left(\tan(x)^2+1\right)}{2\,(a^6+3\,a^4b^2+3\,a^2b^4+b^6)} - \frac{(3\,a^2b^3-b^5)\log\left(|b\tan(x)+a|\right)}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7} + \frac{4\,ab^2\tan(x)^2+a^2b\tan(x)+b^3\tan(x)-a^3+3\,ab^2}{2\,(a^4+2\,a^2b^2+b^4)\left(b\tan(x)^3+a\tan(x)^2+b\tan(x)+a\right)}$$

input
```
integrate(cos(x)^3*sin(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

output
```
-(a^3*b - 3*a*b^3)*x/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 1/2*(3*a^2*b^2
- b^4)*log(tan(x)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (3*a^2*b^3
- b^5)*log(abs(b*tan(x) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7) + 1/2*
(4*a*b^2*tan(x)^2 + a^2*b*tan(x) + b^3*tan(x) - a^3 + 3*a*b^2)/((a^4 + 2*a
^2*b^2 + b^4)*(b*tan(x)^3 + a*tan(x)^2 + b*tan(x) + a))
```

3.290.9 Mupad [B] (verification not implemented)

Time = 30.83 (sec) , antiderivative size = 5428, normalized size of antiderivative = 42.41

$$\int \frac{\cos^3(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \text{Too large to display}$$

input
```
int((cos(x)^3*sin(x))/(a*cos(x) + b*sin(x))^2,x)
```

output

```
((2*a*tan(x/2)^2)/(a^2 + b^2) - (8*b^3*tan(x/2)^3)/(a^2 + b^2)^2 - (2*a*ta
n(x/2)^4)/(a^2 + b^2) + (2*b*tan(x/2)*(a^2 - b^2))/(a^2 + b^2)^2 + (2*b*ta
n(x/2)^5*(a^2 - b^2))/(a^4 + b^4 + 2*a^2*b^2))/(a + 2*b*tan(x/2) + a*tan(x
/2)^2 - a*tan(x/2)^4 - a*tan(x/2)^6 + 4*b*tan(x/2)^3 + 2*b*tan(x/2)^5) + (
log(a + 2*b*tan(x/2) - a*tan(x/2)^2)*(b^4 - 3*a^2*b^2))/(a^6 + b^6 + 3*a^2
*b^4 + 3*a^4*b^2) - (log(1/(cos(x) + 1))*(2*b^4 - 6*a^2*b^2))/(2*(a^6 + b^
6 + 3*a^2*b^4 + 3*a^4*b^2)) - (2*a*b*atan((tan(x/2)*((((((a*b*((32*(a*b^14
 + 9*a^3*b^12 + 18*a^5*b^10 + 2*a^7*b^8 - 27*a^9*b^6 - 27*a^11*b^4 - 8*a^1
3*b^2))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 +
 6*a^10*b^2) - (16*(2*b^4 - 6*a^2*b^2)*(3*a*b^16 + 21*a^3*b^14 + 63*a^5*b^
12 + 105*a^7*b^10 + 105*a^9*b^8 + 63*a^11*b^6 + 21*a^13*b^4 + 3*a^15*b^2))
/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b
^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(a^2 - 3*b^2))/(a^6 + b^6 + 3
*a^2*b^4 + 3*a^4*b^2) - (16*a*b*(a^2 - 3*b^2)*(2*b^4 - 6*a^2*b^2)*(3*a*b^1
6 + 21*a^3*b^14 + 63*a^5*b^12 + 105*a^7*b^10 + 105*a^9*b^8 + 63*a^11*b^6 +
 21*a^13*b^4 + 3*a^15*b^2))/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^2*(a^12 +
 b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*
(2*b^4 - 6*a^2*b^2))/(2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (a*b*(a^2 -
 3*b^2)*((32*(3*a*b^12 - 21*a^3*b^10 - 34*a^5*b^8 + 6*a^7*b^6 + 15*a^9*b^4
 - a^11*b^2))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*...
```

3.290. $\int \frac{\cos^3(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

3.291 $\int \frac{\cos^3(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

3.291.1 Optimal result

Integrand size = 20, antiderivative size = 176

$$\int \frac{\cos^3(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx = -\frac{ab^2(3a^2-2b^2)\,\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}} + \frac{2ab(a^2-b^2)\cos(x)}{(a^2+b^2)^3} - \frac{2ab\cos^3(x)}{3(a^2+b^2)^2} - \frac{b^2(3a^2-b^2)\sin(x)}{(a^2+b^2)^3} + \frac{(a^2-b^2)\sin^3(x)}{3(a^2+b^2)^2} - \frac{a^2b^3}{(a^2+b^2)^3(a\cos(x)+b\sin(x))}$$

output
```
-a*b^2*(3*a^2-2*b^2)*arctanh((b*cos(x)-a*sin(x))/(a^2+b^2)^(1/2))/(a^2+b^2
)^(7/2)+2*a*b*(a^2-b^2)*cos(x)/(a^2+b^2)^3-2/3*a*b*cos(x)^3/(a^2+b^2)^2-b^
2*(3*a^2-b^2)*sin(x)/(a^2+b^2)^3+1/3*(a^2-b^2)*sin(x)^3/(a^2+b^2)^2-a^2*b^
3/(a^2+b^2)^3/(a*cos(x)+b*sin(x))
```

3.291.2 Mathematica [A] (verified)

Time = 1.49 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.12

$$\int \frac{\cos^3(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{2ab^2(3a^2-2b^2)\,\text{arctanh}\left(\frac{-b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}} - \frac{-21a^4b+90a^2b^3-9b^5-4b(3a^4+a^2b^2-2b^4)\cos(2x)+b(a^2+b^2)^2\cos(4x)-2a^5\sin(2x)+16a^3b^2\sin}{24(a^2+b^2)^3(a\cos(x)+b\sin(x))}$$

input
```
Integrate[(Cos[x]^3*Sin[x]^2)/(a*Cos[x] + b*Sin[x])^2,x]
```

output
```
(2*a*b^2*(3*a^2 - 2*b^2)*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/(a^2
+ b^2)^(7/2) - (-21*a^4*b + 90*a^2*b^3 - 9*b^5 - 4*b*(3*a^4 + a^2*b^2 - 2*
b^4)*Cos[2*x] + b*(a^2 + b^2)^2*Cos[4*x] - 2*a^5*Sin[2*x] + 16*a^3*b^2*Sin
[2*x] + 18*a*b^4*Sin[2*x] + a^5*Sin[4*x] + 2*a^3*b^2*Sin[4*x] + a*b^4*Sin[
4*x])/(24*(a^2 + b^2)^3*(a*Cos[x] + b*Sin[x]))
```

3.291.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(x)\cos^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sin(x)^2\cos(x)^3}{(a\cos(x)+b\sin(x))^2}dx$$

$$\Big\downarrow \ 3590$$

$$\frac{b\int \frac{\cos^3(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\int \frac{\cos^2(x)\sin^2(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}-\frac{ab\int \frac{\cos^2(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}$$

$$\Big\downarrow \ 3042$$

$$-\frac{ab\int \frac{\cos(x)^2\sin(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{b\int \frac{\cos(x)^3\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\int \frac{\cos(x)^2\sin(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \ 3588$$

$$\frac{a\left(\frac{b\int \cos^2(x)\sin(x)dx}{a^2+b^2}+\frac{a\int \cos(x)\sin^2(x)dx}{a^2+b^2}-\frac{ab\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\left(\frac{b\int \cos^3(x)dx}{a^2+b^2}+\frac{a\int \cos^2(x)\sin(x)dx}{a^2+b^2}-\frac{ab\int \frac{\cos^2(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}-\frac{ab\int \frac{\cos(x)^2\sin(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}$$

$$\Big\downarrow \ 3042$$

$$-\frac{ab\int \frac{\cos(x)^2\sin(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{a\left(\frac{b\int \cos(x)^2\sin(x)dx}{a^2+b^2}+\frac{a\int \cos(x)\sin(x)^2dx}{a^2+b^2}-\frac{ab\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\left(\frac{b\int \sin\left(x+\frac{\pi}{2}\right)^3dx}{a^2+b^2}+\frac{a\int \cos(x)^2\sin(x)dx}{a^2+b^2}-\frac{ab\int \frac{\cos(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{3044}$$

$$\frac{a\left(\frac{a\int \sin^2(x)d\sin(x)}{a^2+b^2}+\frac{b\int \cos(x)^2\sin(x)dx}{a^2+b^2}-\frac{ab\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}-\frac{ab\int \frac{\cos(x)^2\sin(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{b\left(\frac{b\int \sin\left(x+\frac{\pi}{2}\right)^3dx}{a^2+b^2}+\frac{a\int \cos(x)^2\sin(x)dx}{a^2+b^2}-\frac{ab\int \frac{\cos(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{15}$$

$$\frac{a\left(\frac{b\int \cos(x)^2\sin(x)dx}{a^2+b^2}-\frac{ab\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}-\frac{ab\int \frac{\cos(x)^2\sin(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{b\left(\frac{b\int \sin\left(x+\frac{\pi}{2}\right)^3dx}{a^2+b^2}+\frac{a\int \cos(x)^2\sin(x)dx}{a^2+b^2}-\frac{ab\int \frac{\cos(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{3045}$$

$$\frac{a\left(-\frac{b\int \cos^2(x)d\cos(x)}{a^2+b^2}-\frac{ab\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}+\frac{b\left(\frac{b\int \sin\left(x+\frac{\pi}{2}\right)^3dx}{a^2+b^2}-\frac{a\int \cos^2(x)d\cos(x)}{a^2+b^2}-\frac{ab\int \frac{\cos(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}-\frac{ab\int \frac{\cos(x)^2\sin(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}$$

$$\Big\downarrow \text{15}$$

$$\frac{a\left(-\frac{ab\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{b\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}+\frac{b\left(\frac{b\int \sin\left(x+\frac{\pi}{2}\right)^3dx}{a^2+b^2}-\frac{ab\int \frac{\cos(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}-\frac{a\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}-\frac{ab\int \frac{\cos(x)^2\sin(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3113}$$

3.291. $\int \frac{\cos^3(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

$$\frac{a\left(-\frac{ab\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{b\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}+\frac{b\left(-\frac{b\int(1-\sin^2(x))d(-\sin(x))}{a^2+b^2}-\frac{ab\int \frac{\cos(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}-\frac{a\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}-\frac{ab\int \frac{\cos(x)^2\sin(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{b\left(-\frac{ab\int \frac{\cos(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}-\frac{b\left(\frac{\sin^3(x)}{3}-\sin(x)\right)}{a^2+b^2}-\frac{a\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}+\frac{a\left(-\frac{ab\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{b\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}-\frac{ab\int \frac{\cos(x)^2\sin(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}$$

$$\Big\downarrow \text{ 3579}$$

$$\frac{a\left(-\frac{ab\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{b\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}+\frac{b\left(-\frac{ab\left(\frac{a\int\cos(x)dx}{a^2+b^2}+\frac{b^2\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}-\frac{b\left(\frac{\sin^3(x)}{3}-\sin(x)\right)}{a^2+b^2}-\frac{a\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}-\frac{ab\int \frac{\cos(x)^2\sin(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{a\left(-\frac{ab\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{b\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}+\frac{b\left(-\frac{ab\left(\frac{a\int\sin\left(x+\frac{\pi}{2}\right)dx}{a^2+b^2}+\frac{b^2\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}-\frac{b\left(\frac{\sin^3(x)}{3}-\sin(x)\right)}{a^2+b^2}-\frac{a\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}-\frac{ab\int \frac{\cos(x)^2\sin(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}$$

$$\Big\downarrow \text{ 3117}$$

3.291. $\int \frac{\cos^3(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

$$\frac{b\left(-\frac{ab\left(\frac{b^2\int\frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin(x)}{a^2+b^2}+\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}-\frac{b\left(\frac{\sin^3(x)}{3}-\sin(x)\right)}{a^2+b^2}-\frac{a\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}+\frac{a\left(-\frac{ab\int\frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{b\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}-\frac{ab\int\frac{\cos(x)^2\sin(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}$$

$$\Big\downarrow \; 3553$$

$$\frac{a\left(-\frac{ab\int\frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{b\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}+\frac{b\left(-\frac{ab\left(-\frac{b^2\int\frac{1}{a^2+b^2-(b\cos(x)-a\sin(x))^2}d(b\cos(x)-a\sin(x))}{a^2+b^2}+\frac{a\sin(x)}{a^2+b^2}+\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}-\frac{b\left(\frac{\sin^3(x)}{3}-\sin(x)\right)}{a^2+b^2}-\frac{a\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}-\frac{ab\int\frac{\cos(x)^2\sin(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}$$

$$\Big\downarrow \; 219$$

$$\frac{a\left(-\frac{ab\int\frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{b\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}-\frac{ab\int\frac{\cos(x)^2\sin(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{b\left(-\frac{ab\left(-\frac{b^2\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{a\sin(x)}{a^2+b^2}+\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}-\frac{b\left(\frac{\sin^3(x)}{3}-\sin(x)\right)}{a^2+b^2}-\frac{a\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}$$

$$\Big\downarrow \; 3588$$

$$\frac{a\left(-\frac{ab\left(\frac{a\int\sin(x)dx}{a^2+b^2}+\frac{b\int\cos(x)dx}{a^2+b^2}-\frac{ab\int\frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{b\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}-\frac{ab\int\frac{\cos(x)^2\sin(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{b\left(-\frac{ab\left(-\frac{b^2\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{a\sin(x)}{a^2+b^2}+\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}-\frac{b\left(\frac{\sin^3(x)}{3}-\sin(x)\right)}{a^2+b^2}-\frac{a\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}$$

$$\Big\downarrow \; 3042$$

3.291. $\int \frac{\cos^3(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

$$\frac{a\left(-\frac{ab\left(\frac{a\int \sin(x)dx}{a^2+b^2}+\frac{b\int \sin\left(x+\frac{\pi}{2}\right)dx}{a^2+b^2}-\frac{ab\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{b\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}-\frac{ab\int \frac{\cos(x)^2\sin(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{b\left(-\frac{ab\left(-\frac{b^2\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}+\frac{a\sin(x)}{a^2+b^2}+\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}-\frac{b\left(\frac{\sin^3(x)}{3}-\sin(x)\right)}{a^2+b^2}-\frac{a\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{3117}$$

$$\frac{a\left(-\frac{ab\left(\frac{a\int \sin(x)dx}{a^2+b^2}-\frac{ab\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\sin(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{b\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}-\frac{ab\int \frac{\cos(x)^2\sin(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{b\left(-\frac{ab\left(-\frac{b^2\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}+\frac{a\sin(x)}{a^2+b^2}+\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}-\frac{b\left(\frac{\sin^3(x)}{3}-\sin(x)\right)}{a^2+b^2}-\frac{a\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{3118}$$

$$\frac{a\left(-\frac{ab\left(-\frac{ab\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\sin(x)}{a^2+b^2}-\frac{a\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{b\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}-\frac{ab\int \frac{\cos(x)^2\sin(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{b\left(-\frac{ab\left(-\frac{b^2\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}+\frac{a\sin(x)}{a^2+b^2}+\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}-\frac{b\left(\frac{\sin^3(x)}{3}-\sin(x)\right)}{a^2+b^2}-\frac{a\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{3553}$$

3.291. $\int \frac{\cos^3(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

$$\frac{a\left(-\frac{ab\left(\frac{ab\int \frac{1}{a^2+b^2-(b\cos(x)-a\sin(x))^2}d(b\cos(x)-a\sin(x))}{a^2+b^2}+\frac{b\sin(x)}{a^2+b^2}-\frac{a\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{b\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}-\frac{ab\int \frac{\cos(x)^2\sin(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{b\left(-\frac{ab\left(-\frac{b^2\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{a\sin(x)}{a^2+b^2}+\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}-\frac{b\left(\frac{\sin^3(x)}{3}-\sin(x)\right)}{a^2+b^2}-\frac{a\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}$$

$$\Big\downarrow \; 219$$

$$-\frac{ab\int \frac{\cos(x)^2\sin(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{a\left(-\frac{ab\left(\frac{ab\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{b\sin(x)}{a^2+b^2}-\frac{a\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{b\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}+\frac{b\left(-\frac{ab\left(-\frac{b^2\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{a\sin(x)}{a^2+b^2}+\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}-\frac{b\left(\frac{\sin^3(x)}{3}-\sin(x)\right)}{a^2+b^2}-\frac{a\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}$$

$$\Big\downarrow \; 3590$$

$$-\frac{ab\left(\frac{b\int \frac{\cos^2(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}-\frac{ab\int \frac{\cos(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{a\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(-\frac{ab\left(\frac{ab\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{b\sin(x)}{a^2+b^2}-\frac{a\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{b\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}+\frac{b\left(-\frac{ab\left(-\frac{b^2\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{a\sin(x)}{a^2+b^2}+\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}-\frac{b\left(\frac{\sin^3(x)}{3}-\sin(x)\right)}{a^2+b^2}-\frac{a\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}$$

$$\Big\downarrow \; 3042$$

3.291. $\int \frac{\cos^3(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

$$-\frac{ab\left(-\frac{ab\int \frac{\cos(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{b\int \frac{\cos(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(-\frac{ab\left(\frac{ab\operatorname{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{b\sin(x)}{a^2+b^2}-\frac{a\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{b\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}+\frac{b\left(-\frac{ab\left(-\frac{b^2\operatorname{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{a\sin(x)}{a^2+b^2}+\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}-\frac{b\left(\frac{\sin^3(x)}{3}-\sin(x)\right)}{a^2+b^2}-\frac{a\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}$$

$$\Big\downarrow \; 3579$$

$$-\frac{ab\left(-\frac{ab\int \frac{\cos(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{a\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\left(\frac{a\int \cos(x)dx}{a^2+b^2}+\frac{b^2\int \frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(-\frac{ab\left(\frac{ab\operatorname{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{b\sin(x)}{a^2+b^2}-\frac{a\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{b\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}+\frac{b\left(-\frac{ab\left(-\frac{b^2\operatorname{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{a\sin(x)}{a^2+b^2}+\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}-\frac{b\left(\frac{\sin^3(x)}{3}-\sin(x)\right)}{a^2+b^2}-\frac{a\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}$$

$$\Big\downarrow \; 3042$$

3.291. $\int \frac{\cos^3(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

$$-\frac{ab\left(-\frac{ab\int\frac{\cos(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{a\int\frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\left(\frac{a\int\sin\left(x+\frac{\pi}{2}\right)dx}{a^2+b^2}+\frac{b^2\int\frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(-\frac{ab\left(\frac{ab\operatorname{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{b\sin(x)}{a^2+b^2}-\frac{a\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{b\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}+\frac{b\left(-\frac{ab\left(-\frac{b^2\operatorname{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{a\sin(x)}{a^2+b^2}+\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}-\frac{b\left(\frac{\sin^3(x)}{3}-\sin(x)\right)}{a^2+b^2}-\frac{a\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}$$

$$\Big\downarrow \; 3117$$

$$-\frac{ab\left(-\frac{ab\int\frac{\cos(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{b\left(\frac{b^2\int\frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin(x)}{a^2+b^2}+\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\int\frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(-\frac{ab\left(\frac{ab\operatorname{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{b\sin(x)}{a^2+b^2}-\frac{a\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{b\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}+\frac{b\left(-\frac{ab\left(-\frac{b^2\operatorname{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{a\sin(x)}{a^2+b^2}+\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}-\frac{b\left(\frac{\sin^3(x)}{3}-\sin(x)\right)}{a^2+b^2}-\frac{a\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}$$

$$\Big\downarrow \; 3553$$

3.291. $\int\frac{\cos^3(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

$$-\frac{ab\left(-\frac{ab\int \frac{\cos(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{a\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\left(-\frac{b^2\int \frac{1}{a^2+b^2-(b\cos(x)-a\sin(x))^2}d(b\cos(x)-a\sin(x))}{a^2+b^2}+\frac{a\sin(x)}{a^2+b^2}+\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(-\frac{ab\left(\frac{ab\,\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{b\sin(x)}{a^2+b^2}-\frac{a\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{b\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}+\frac{b\left(-\frac{ab\left(-\frac{b^2\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{a\sin(x)}{a^2+b^2}+\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}-\frac{b\left(\frac{\sin^3(x)}{3}-\sin(x)\right)}{a^2+b^2}-\frac{a\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}$$

$$\Big\downarrow \; 219$$

$$-\frac{ab\left(-\frac{ab\int \frac{\cos(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{a\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\left(-\frac{b^2\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{a\sin(x)}{a^2+b^2}+\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(-\frac{ab\left(\frac{ab\,\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{b\sin(x)}{a^2+b^2}-\frac{a\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{b\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}+\frac{b\left(-\frac{ab\left(-\frac{b^2\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{a\sin(x)}{a^2+b^2}+\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}-\frac{b\left(\frac{\sin^3(x)}{3}-\sin(x)\right)}{a^2+b^2}-\frac{a\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}$$

$$\Big\downarrow \; 3588$$

3.291. $\int \frac{\cos^3(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

$$-\frac{ab\left(-\frac{ab\int\frac{\cos(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{a\left(\frac{a\int\sin(x)dx}{a^2+b^2}+\frac{b\int\cos(x)dx}{a^2+b^2}-\frac{ab\int\frac{1}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\left(-\frac{b^2\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{a\sin(x)}{a^2+b^2}+\right.}{a^2+b^2}}{a^2+b^2}$$

$$\frac{a\left(-\frac{ab\left(\frac{ab\,\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{b\sin(x)}{a^2+b^2}-\frac{a\cos(x)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{b\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}+$$

$$\frac{b\left(-\frac{ab\left(-\frac{b^2\text{arctanh}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{a\sin(x)}{a^2+b^2}+\frac{b\cos(x)}{a^2+b^2}\right)}{a^2+b^2}-\frac{b\left(\frac{\sin^3(x)}{3}-\sin(x)\right)}{a^2+b^2}-\frac{a\cos^3(x)}{3(a^2+b^2)}\right)}{a^2+b^2}$$

input
```
Int[(Cos[x]^3*Sin[x]^2)/(a*Cos[x] + b*Sin[x])^2,x]
```

output
```
$Aborted
```

3.291.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3579
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(Cos[c + d*x]^(m - 1)/(d*(a^2 +
b^2)*(m - 1))), x] + (Simp[a/(a^2 + b^2)   Int[Cos[c + d*x]^(m - 1), x], x]
 + Simp[b^2/(a^2 + b^2)   Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[
c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m,
 1]
```

3.291. $\int \frac{\cos^3(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

rule 3588
```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b
/(a^2 + b^2)   Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Simp[a/(a
^2 + b^2)   Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Simp[a*(b/(a^
2 + b^2))   Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x]
+ b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
&& IGtQ[m, 0] && IGtQ[n, 0]
```

rule 3590
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Sim
p[b/(a^2 + b^2)   Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
 b*Sin[c + d*x])^(p + 1), x], x] + (Simp[a/(a^2 + b^2)   Int[Cos[c + d*x]^(
m - 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - S
imp[a*(b/(a^2 + b^2))   Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Co
s[c + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^
2 + b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

3.291.4 Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.51

method	result
default	$-\frac{2\left(\left(3a^2b^2-b^4\right)\tan\left(\frac{x}{2}\right)^5+4a\,b^3\tan\left(\frac{x}{2}\right)^4+\left(-\frac{4}{3}a^4+6a^2b^2-\frac{2}{3}b^4\right)\tan\left(\frac{x}{2}\right)^3+\left(-4a^3b+4a\,b^3\right)\tan\left(\frac{x}{2}\right)^2+\left(3a^2b^2-b^4\right)\tan\left(\frac{x}{2}\right)-\frac{4a^3b}{3}+\frac{8a}{}\right.}{\left(a^2+b^2\right)\left(a^4+2a^2b^2+b^4\right)\left(1+\tan\left(\frac{x}{2}\right)^2\right)^3}$
risch	$\frac{i\mathrm{e}^{3ix}}{-48iba+24a^2-24b^2}+\frac{3\,\mathrm{e}^{ix}b}{8(-3ib\,a^2+ib^3+a^3-3a\,b^2)}-\frac{i\mathrm{e}^{ix}a}{8(-3ib\,a^2+ib^3+a^3-3a\,b^2)}+\frac{3\,\mathrm{e}^{-ix}b}{8(ib+a)^3}+\frac{i\mathrm{e}^{-ix}a}{8(ib+a)^3}-\frac{i\mathrm{e}^{-3ix}}{24(ib+a)^2}-\frac{}{(ib}$

input
```
int(cos(x)^3*sin(x)^2/(a*cos(x)+b*sin(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/(a^2+b^2)/(a^4+2*a^2*b^2+b^4)*((3*a^2*b^2-b^4)*tan(1/2*x)^5+4*a*b^3*tan
(1/2*x)^4+(-4/3*a^4+6*a^2*b^2-2/3*b^4)*tan(1/2*x)^3+(-4*a^3*b+4*a*b^3)*tan
(1/2*x)^2+(3*a^2*b^2-b^4)*tan(1/2*x)-4/3*a^3*b+8/3*a*b^3)/(1+tan(1/2*x)^2)
^3-2*a*b^2/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*((-b^2*tan(1/2*x)-a*b)/(tan(1/2*x
)^2*a-2*b*tan(1/2*x)-a)-(3*a^2-2*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan
(1/2*x)-2*b)/(a^2+b^2)^(1/2)))
```

3.291.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 369 vs. 2(168) = 336.

Time = 0.29 (sec) , antiderivative size = 369, normalized size of antiderivative = 2.10

$$\int \frac{\cos^3(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx$$
$$= \frac{2\,a^6b-22\,a^4b^3-20\,a^2b^5+4\,b^7-2\left(a^6b+3\,a^4b^3+3\,a^2b^5+b^7\right)\cos\left(x\right)^4+2\left(4\,a^6b+7\,a^4b^3+2\,a^2b^5-b^7\right)\cos}{}$$

input
```
integrate(cos(x)^3*sin(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

output
```
1/6*(2*a^6*b - 22*a^4*b^3 - 20*a^2*b^5 + 4*b^7 - 2*(a^6*b + 3*a^4*b^3 + 3*
a^2*b^5 + b^7)*cos(x)^4 + 2*(4*a^6*b + 7*a^4*b^3 + 2*a^2*b^5 - b^7)*cos(x)
^2 - 3*sqrt(a^2 + b^2)*((3*a^4*b^2 - 2*a^2*b^4)*cos(x) + (3*a^3*b^3 - 2*a*
b^5)*sin(x))*log((2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 - 2*a^2 - b^2
 - 2*sqrt(a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos(x)*sin(x) + (a^2 -
b^2)*cos(x)^2 + b^2)) - 2*((a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cos(x)^3
- (a^7 - 2*a^5*b^2 - 7*a^3*b^4 - 4*a*b^6)*cos(x))*sin(x))/((a^9 + 4*a^7*b^
2 + 6*a^5*b^4 + 4*a^3*b^6 + a*b^8)*cos(x) + (a^8*b + 4*a^6*b^3 + 6*a^4*b^5
 + 4*a^2*b^7 + b^9)*sin(x))
```

3.291.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(x)**3*sin(x)**2/(a*cos(x)+b*sin(x))**2,x)
```

output
```
Timed out
```

3.291.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 606 vs. $2(168) = 336$.

Time = 0.33 (sec) , antiderivative size = 606, normalized size of antiderivative = 3.44

$$\int \frac{\cos^3(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx = -\frac{(3\,a^2b^2-2\,b^4)a\log\left(\frac{b-\frac{a\sin(x)}{\cos(x)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(x)}{\cos(x)+1}-\sqrt{a^2+b^2}}\right)}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\sqrt{a^2+b^2}} + \frac{2\left(4\,a^4b-11\,a^2b^3-\frac{(a^3b^2+16\,ab^4)\sin(x)}{\cos(x)+1}+\frac{(8\,a^4b-31\,a^2b^3+6\,b^5)\sin(x)^2}{(\cos(x)+1)^2}+\frac{(4\,a^5+15\,a^3b^2-34\,a \ldots}{(\cos(x)+1) \ldots}\right.}{3\left(a^7+3\,a^5b^2+3\,a^3b^4+ab^6+\frac{2\,(a^6b+3\,a^4b^3+3\,a^2b^5+b^7)\sin(x)}{\cos(x)+1}+\frac{2\,(a^7+3\,a^5b^2+3\,a^3b^4+ab^6)\sin(x)^2}{(\cos(x)+1)^2}+\frac{6\,(a^6b+3\,a^4b^3+3\,a^2b \ldots}{(\cos(x)+1 \ldots}\right.}$$

input
```
integrate(cos(x)^3*sin(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

output
```
-(3*a^2*b^2 - 2*b^4)*a*log((b - a*sin(x)/(cos(x) + 1) + sqrt(a^2 + b^2))/(
b - a*sin(x)/(cos(x) + 1) - sqrt(a^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^
4 + b^6)*sqrt(a^2 + b^2)) + 2/3*(4*a^4*b - 11*a^2*b^3 - (a^3*b^2 + 16*a*b^
4)*sin(x)/(cos(x) + 1) + (8*a^4*b - 31*a^2*b^3 + 6*b^5)*sin(x)^2/(cos(x) +
 1)^2 + (4*a^5 + 15*a^3*b^2 - 34*a*b^4)*sin(x)^3/(cos(x) + 1)^3 - (4*a^4*b
 + 45*a^2*b^3 - 4*b^5)*sin(x)^4/(cos(x) + 1)^4 - (4*a^5 - 9*a^3*b^2 + 32*a
*b^4)*sin(x)^5/(cos(x) + 1)^5 - 3*(3*a^2*b^3 - 2*b^5)*sin(x)^6/(cos(x) + 1
)^6 + 3*(3*a^3*b^2 - 2*a*b^4)*sin(x)^7/(cos(x) + 1)^7)/(a^7 + 3*a^5*b^2 +
3*a^3*b^4 + a*b^6 + 2*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*sin(x)/(cos(x)
 + 1) + 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*sin(x)^2/(cos(x) + 1)^2 +
6*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*sin(x)^3/(cos(x) + 1)^3 + 6*(a^6*b
 + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*sin(x)^5/(cos(x) + 1)^5 - 2*(a^7 + 3*a^5*b
^2 + 3*a^3*b^4 + a*b^6)*sin(x)^6/(cos(x) + 1)^6 + 2*(a^6*b + 3*a^4*b^3 + 3
*a^2*b^5 + b^7)*sin(x)^7/(cos(x) + 1)^7 - (a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a
*b^6)*sin(x)^8/(cos(x) + 1)^8)
```

3.291.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.90

$$\int \frac{\cos^3(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx = -\frac{(3\,a^3b^2-2\,ab^4)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\sqrt{a^2+b^2}} + \frac{2\left(ab^4\tan\left(\frac{1}{2}\,x\right)+a^2b^3\right)}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\left(a\tan\left(\frac{1}{2}\,x\right)^2-2\,b\tan\left(\frac{1}{2}\,x\right)-a\right)} - \frac{2\left(9\,a^2b^2\tan\left(\frac{1}{2}\,x\right)^5-3\,b^4\tan\left(\frac{1}{2}\,x\right)^5+12\,ab^3\tan\left(\frac{1}{2}\,x\right)^4-4\,a^4\tan\left(\frac{1}{2}\,x\right)^3+18\,a^2b^2\tan\left(\frac{1}{2}\,x\right)^3-2\,b^4\tan\cdots\right.}{3\,(a^6+3\,a^4b^2+3\,a^2b^4+b\cdots}$$

input
```
integrate(cos(x)^3*sin(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

output
```
-(3*a^3*b^2 - 2*a*b^4)*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/a
bs(2*a*tan(1/2*x) - 2*b + 2*sqrt(a^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^
4 + b^6)*sqrt(a^2 + b^2)) + 2*(a*b^4*tan(1/2*x) + a^2*b^3)/((a^6 + 3*a^4*b
^2 + 3*a^2*b^4 + b^6)*(a*tan(1/2*x)^2 - 2*b*tan(1/2*x) - a)) - 2/3*(9*a^2*
b^2*tan(1/2*x)^5 - 3*b^4*tan(1/2*x)^5 + 12*a*b^3*tan(1/2*x)^4 - 4*a^4*tan(
1/2*x)^3 + 18*a^2*b^2*tan(1/2*x)^3 - 2*b^4*tan(1/2*x)^3 - 12*a^3*b*tan(1/2
*x)^2 + 12*a*b^3*tan(1/2*x)^2 + 9*a^2*b^2*tan(1/2*x) - 3*b^4*tan(1/2*x) -
4*a^3*b + 8*a*b^3)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*(tan(1/2*x)^2 + 1)
^3)
```

3.291.9 Mupad [B] (verification not implemented)

Time = 24.91 (sec) , antiderivative size = 586, normalized size of antiderivative = 3.33

$$\int \frac{\cos^3(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx = -\frac{\frac{2\tan\left(\frac{x}{2}\right)^4\left(4\,a^4\,b+45\,a^2\,b^3-4\,b^5\right)}{3\,(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6)}-\frac{2\tan\left(\frac{x}{2}\right)^6\left(2\,b^5-3\,a^2\,b^3\right)}{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6}+\frac{2\tan\left(\frac{x}{2}\right)^5\left(4\,a^5-9\,a^3\,b^2+32\,a\,b^4\right)}{3\,(a^2+b^2)\,(a^4+2\,a^2\,b^2+b^4)}-\frac{2\tan\left(\frac{x}{2}\right)^3\left(4\,a^5+15\,a^3\,b^2-34\,a\,b^4\right)}{3\,(a^2+b^2)\,(a^4+2\,a^2\,b^2+b^4)}\cdots}{-a\tan\left(\frac{x}{2}\right)^8+2\,b\tan\left(\frac{x}{2}\right)^7-2\,a\tan\left(\frac{x}{2}\right)^6+6\,b\tan\left(\frac{x}{2}\right)\cdots} - \frac{a\,b^2\,\mathrm{atan}\left(\frac{1\mathrm{i}\tan\left(\frac{x}{2}\right)a^7-a^6\,b\,1\mathrm{i}+3\mathrm{i}\tan\left(\frac{x}{2}\right)a^5\,b^2-a^4\,b^3\,3\mathrm{i}+3\mathrm{i}\tan\left(\frac{x}{2}\right)a^3\,b^4-a^2\,b^5\,3\mathrm{i}+1\mathrm{i}\tan\left(\frac{x}{2}\right)a\,b^6-b^7\,1\mathrm{i}}{(a^2+b^2)^{7/2}}\right)\left(3\,a^2-2\,b^2\right)2\mathrm{i}}{(a^2+b^2)^{7/2}}$$

input
```
int((cos(x)^3*sin(x)^2)/(a*cos(x) + b*sin(x))^2,x)
```

output
```
- ((2*tan(x/2)^4*(4*a^4*b - 4*b^5 + 45*a^2*b^3))/(3*(a^6 + b^6 + 3*a^2*b^4
 + 3*a^4*b^2)) - (2*tan(x/2)^6*(2*b^5 - 3*a^2*b^3))/(a^6 + b^6 + 3*a^2*b^4
 + 3*a^4*b^2) + (2*tan(x/2)^5*(32*a*b^4 + 4*a^5 - 9*a^3*b^2))/(3*(a^2 + b^
2)*(a^4 + b^4 + 2*a^2*b^2)) - (2*tan(x/2)^3*(4*a^5 - 34*a*b^4 + 15*a^3*b^2
))/(3*(a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2)) - (2*tan(x/2)^2*(8*a^4*b + 6*b^
5 - 31*a^2*b^3))/(3*(a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2)) + (2*a*(11*a*b^3
- 4*a^3*b))/(3*(a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2)) + (2*b*tan(x/2)^7*(2*a
*b^3 - 3*a^3*b))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (2*b*tan(x/2)*(16*a
*b^3 + a^3*b))/(3*(a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2)))/(a + 2*b*tan(x/2)
+ 2*a*tan(x/2)^2 - 2*a*tan(x/2)^6 - a*tan(x/2)^8 + 6*b*tan(x/2)^3 + 6*b*ta
n(x/2)^5 + 2*b*tan(x/2)^7) - (a*b^2*atan((a^7*tan(x/2)*1i - a^6*b*1i - b^7
*1i - a^2*b^5*3i - a^4*b^3*3i + a^3*b^4*tan(x/2)*3i + a^5*b^2*tan(x/2)*3i
+ a*b^6*tan(x/2)*1i)/(a^2 + b^2)^(7/2))*(3*a^2 - 2*b^2)*2i)/(a^2 + b^2)^(7
/2)
```

3.291. $\int \frac{\cos^3(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

3.292 $\int \frac{\cos^3(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

3.292.1 Optimal result

Integrand size = 20, antiderivative size = 210

$$\int \frac{\cos^3(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx = -\frac{3ab(a^4-6a^2b^2+b^4)\,x}{4\,(a^2+b^2)^4} - \frac{b^2\cos^4(x)}{4\,(a^2+b^2)^2} - \frac{3a^2b^2(a^2-b^2)\log(a\cos(x)+b\sin(x))}{(a^2+b^2)^4} + \frac{ab(5a^2-3b^2)\cos(x)\sin(x)}{4\,(a^2+b^2)^3} - \frac{ab\cos^3(x)\sin(x)}{2\,(a^2+b^2)^2} - \frac{2a^2b^2\sin^2(x)}{(a^2+b^2)^3} + \frac{a^2\sin^4(x)}{4\,(a^2+b^2)^2} - \frac{a^2b^3\sin(x)}{(a^2+b^2)^3\,(a\cos(x)+b\sin(x))}$$

output
```
-3/4*a*b*(a^4-6*a^2*b^2+b^4)*x/(a^2+b^2)^4-1/4*b^2*cos(x)^4/(a^2+b^2)^2-3*
a^2*b^2*(a^2-b^2)*ln(a*cos(x)+b*sin(x))/(a^2+b^2)^4+1/4*a*b*(5*a^2-3*b^2)*
cos(x)*sin(x)/(a^2+b^2)^3-1/2*a*b*cos(x)^3*sin(x)/(a^2+b^2)^2-2*a^2*b^2*si
n(x)^2/(a^2+b^2)^3+1/4*a^2*sin(x)^4/(a^2+b^2)^2-a^2*b^3*sin(x)/(a^2+b^2)^3
/(a*cos(x)+b*sin(x))
```

3.292.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.44 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.95

$$\int \frac{\cos^3(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx$$
$$= \frac{-12ab(a^2-3b^2)(3a^2-b^2)x+6i(a^6-15a^4b^2+15a^2b^4-b^6)x-6i(a^6-15a^4b^2+15a^2b^4-b^6)\arctan(\mathrm{t}}{}$$

input
```
Integrate[(Cos[x]^3*Sin[x]^3)/(a*Cos[x] + b*Sin[x])^2,x]
```

output
```
(-12*a*b*(a^2 - 3*b^2)*(3*a^2 - b^2)*x + (6*I)*(a^6 - 15*a^4*b^2 + 15*a^2*
b^4 - b^6)*x - (6*I)*(a^6 - 15*a^4*b^2 + 15*a^2*b^4 - b^6)*ArcTan[Tan[x]]
- 4*(a^2 + b^2)*(a^4 - 6*a^2*b^2 + b^4)*Cos[2*x] + (a^2 - b^2)*(a^2 + b^2)
^2*Cos[4*x] + 3*(a^6 - 15*a^4*b^2 + 15*a^2*b^4 - b^6)*Log[(a*Cos[x] + b*Si
n[x])^2] + (2*b*(a^2 + b^2)*(3*a^4 - 10*a^2*b^2 + 3*b^4)*Sin[x])/(a*Cos[x]
 + b*Sin[x]) + (3*(a^2 + b^2)^2*(a*Cos[x]*((-2*I)*(a + I*b)^2*x + (-a^2 +
b^2)*Log[(a*Cos[x] + b*Sin[x])^2]) + b*(2*(a + I*b)*(a*(-1 - I*x) + b*(I +
 x)) + (-a^2 + b^2)*Log[(a*Cos[x] + b*Sin[x])^2])*Sin[x] + (2*I)*(a^2 - b^
2)*ArcTan[Tan[x]]*(a*Cos[x] + b*Sin[x])))/(a*Cos[x] + b*Sin[x]) + 16*a*b*(
a^4 - b^4)*Sin[2*x] - 2*a*b*(a^2 + b^2)^2*Sin[4*x])/(32*(a^2 + b^2)^4)
```

3.292.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(x)\cos^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin(x)^3\cos(x)^3}{(a\cos(x)+b\sin(x))^2}dx$$
$$\Big\downarrow \text{3590}$$
$$\frac{b\int \frac{\cos^3(x)\sin^2(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\int \frac{\cos^2(x)\sin^3(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}-\frac{ab\int \frac{\cos^2(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}$$
$$\Big\downarrow \text{3042}$$

$$-\frac{ab\int\frac{\cos(x)^2\sin(x)^2}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{b\int\frac{\cos(x)^3\sin(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\int\frac{\cos(x)^2\sin(x)^3}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}$$

$$\Big\downarrow \text{ 3588}$$

$$\frac{a\left(\frac{b\int\cos^2(x)\sin^2(x)dx}{a^2+b^2}+\frac{a\int\cos(x)\sin^3(x)dx}{a^2+b^2}-\frac{ab\int\frac{\cos(x)\sin^2(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\left(\frac{b\int\cos^3(x)\sin(x)dx}{a^2+b^2}+\frac{a\int\cos^2(x)\sin^2(x)dx}{a^2+b^2}-\frac{ab\int\frac{\cos^2(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}-\frac{ab\int\frac{\cos(x)^2\sin(x)^2}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{ab\int\frac{\cos(x)^2\sin(x)^2}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{b\left(\frac{b\int\cos(x)^3\sin(x)dx}{a^2+b^2}+\frac{a\int\cos(x)^2\sin(x)^2dx}{a^2+b^2}-\frac{ab\int\frac{\cos(x)^2\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(\frac{b\int\cos(x)^2\sin(x)^2dx}{a^2+b^2}+\frac{a\int\cos(x)\sin(x)^3dx}{a^2+b^2}-\frac{ab\int\frac{\cos(x)\sin(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{ 3044}$$

$$\frac{a\left(\frac{a\int\sin^3(x)d\sin(x)}{a^2+b^2}+\frac{b\int\cos(x)^2\sin(x)^2dx}{a^2+b^2}-\frac{ab\int\frac{\cos(x)\sin(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}-\frac{ab\int\frac{\cos(x)^2\sin(x)^2}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{b\left(\frac{b\int\cos(x)^3\sin(x)dx}{a^2+b^2}+\frac{a\int\cos(x)^2\sin(x)^2dx}{a^2+b^2}-\frac{ab\int\frac{\cos(x)^2\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{a\left(\frac{b\int\cos(x)^2\sin(x)^2dx}{a^2+b^2}-\frac{ab\int\frac{\cos(x)\sin(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^4(x)}{4(a^2+b^2)}\right)}{a^2+b^2}-\frac{ab\int\frac{\cos(x)^2\sin(x)^2}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{b\left(\frac{b\int\cos(x)^3\sin(x)dx}{a^2+b^2}+\frac{a\int\cos(x)^2\sin(x)^2dx}{a^2+b^2}-\frac{ab\int\frac{\cos(x)^2\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \text{ 3045}$$

3.292. $\int\frac{\cos^3(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}dx$

$$\frac{b\left(-\frac{b\int\cos^3(x)d\cos(x)}{a^2+b^2}+\frac{a\int\cos(x)^2\sin(x)^2dx}{a^2+b^2}-\frac{ab\int\frac{\cos(x)^2\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(\frac{b\int\cos(x)^2\sin(x)^2dx}{a^2+b^2}-\frac{ab\int\frac{\cos(x)\sin(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^4(x)}{4(a^2+b^2)}\right)}{a^2+b^2}-\frac{ab\int\frac{\cos(x)^2\sin(x)^2}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}$$

$$\Big\downarrow \text{15}$$

$$\frac{b\left(\frac{a\int\cos(x)^2\sin(x)^2dx}{a^2+b^2}-\frac{ab\int\frac{\cos(x)^2\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}-\frac{b\cos^4(x)}{4(a^2+b^2)}\right)}{a^2+b^2}+\frac{a\left(\frac{b\int\cos(x)^2\sin(x)^2dx}{a^2+b^2}-\frac{ab\int\frac{\cos(x)\sin(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^4(x)}{4(a^2+b^2)}\right)}{a^2+b^2}-\frac{ab\int\frac{\cos(x)^2\sin(x)^2}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3048}$$

$$\frac{a\left(\frac{b\left(\frac{1}{4}\int\cos^2(x)dx-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}-\frac{ab\int\frac{\cos(x)\sin(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^4(x)}{4(a^2+b^2)}\right)}{a^2+b^2}+\frac{b\left(\frac{a\left(\frac{1}{4}\int\cos^2(x)dx-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}-\frac{ab\int\frac{\cos(x)^2\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}-\frac{b\cos^4(x)}{4(a^2+b^2)}\right)}{a^2+b^2}-\frac{ab\int\frac{\cos(x)^2\sin(x)^2}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\left(\frac{b\left(\frac{1}{4}\int\sin\left(x+\frac{\pi}{2}\right)^2dx-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}-\frac{ab\int\frac{\cos(x)\sin(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^4(x)}{4(a^2+b^2)}\right)}{a^2+b^2}+\frac{b\left(\frac{a\left(\frac{1}{4}\int\sin\left(x+\frac{\pi}{2}\right)^2dx-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}-\frac{ab\int\frac{\cos(x)^2\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}-\frac{b\cos^4(x)}{4(a^2+b^2)}\right)}{a^2+b^2}-\frac{ab\int\frac{\cos(x)^2\sin(x)^2}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3115}$$

$$\frac{a\left(\frac{b\left(\frac{1}{4}\left(\frac{\int 1dx}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}-\frac{ab\int\frac{\cos(x)\sin(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^4(x)}{4(a^2+b^2)}\right)}{a^2+b^2}+\frac{b\left(\frac{a\left(\frac{1}{4}\left(\frac{\int 1dx}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}-\frac{ab\int\frac{\cos(x)^2\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}-\frac{b\cos^4(x)}{4(a^2+b^2)}\right)}{a^2+b^2}-\frac{ab\int\frac{\cos(x)^2\sin(x)^2}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}$$

3.292. $\int\frac{\cos^3(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

$$\downarrow 24$$

$$\frac{a\left(-\frac{ab\int\frac{\cos(x)\sin(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^4(x)}{4(a^2+b^2)}+\frac{b\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\left(-\frac{ab\int\frac{\cos(x)^2\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}-\frac{b\cos^4(x)}{4(a^2+b^2)}+\frac{a\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}\right)}{a^2+b^2}-\frac{ab\int\frac{\cos(x)^2\sin(x)^2}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}$$

$$\downarrow 3588$$

$$\frac{a\left(-\frac{ab\left(\frac{a\int\sin^2(x)dx}{a^2+b^2}+\frac{b\int\cos(x)\sin(x)dx}{a^2+b^2}-\frac{ab\int\frac{\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^4(x)}{4(a^2+b^2)}+\frac{b\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\left(-\frac{ab\left(\frac{b\int\cos^2(x)dx}{a^2+b^2}+\frac{a\int\cos(x)\sin(x)dx}{a^2+b^2}-\frac{ab\int\frac{\cos(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}-\frac{b\cos^4(x)}{4(a^2+b^2)}+\frac{a\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}\right)}{a^2+b^2}-\frac{ab\int\frac{\cos(x)^2\sin(x)^2}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}$$

$$\downarrow 3042$$

$$\frac{a\left(-\frac{ab\left(\frac{a\int\sin(x)^2dx}{a^2+b^2}+\frac{b\int\cos(x)\sin(x)dx}{a^2+b^2}-\frac{ab\int\frac{\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^4(x)}{4(a^2+b^2)}+\frac{b\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\left(-\frac{ab\left(\frac{b\int\sin\left(x+\frac{\pi}{2}\right)^2dx}{a^2+b^2}+\frac{a\int\cos(x)\sin(x)dx}{a^2+b^2}-\frac{ab\int\frac{\cos(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}-\frac{b\cos^4(x)}{4(a^2+b^2)}+\frac{a\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}\right)}{a^2+b^2}-\frac{ab\int\frac{\cos(x)^2\sin(x)^2}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}$$

$$\downarrow 3044$$

3.292. $\int\frac{\cos^3(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}dx$

$$\frac{a\left(-\frac{ab\left(\frac{a\int\sin(x)^2dx}{a^2+b^2}+\frac{b\int\sin(x)d\sin(x)}{a^2+b^2}-\frac{ab\int\frac{\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^4(x)}{4(a^2+b^2)}+\frac{b\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\left(-\frac{ab\left(\frac{b\int\sin\left(x+\frac{\pi}{2}\right)^2dx}{a^2+b^2}+\frac{a\int\sin(x)d\sin(x)}{a^2+b^2}-\frac{ab\int\frac{\cos(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}-\frac{b\cos^4(x)}{4(a^2+b^2)}+\frac{a\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}\right)}{a^2+b^2}-\frac{ab\int\frac{\cos(x)^2\sin(x)^2}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}$$

$$\Big\downarrow \ 15$$

$$\frac{a\left(-\frac{ab\left(\frac{a\int\sin(x)^2dx}{a^2+b^2}-\frac{ab\int\frac{\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\sin^2(x)}{2\left(a^2+b^2\right)}\right)}{a^2+b^2}+\frac{a\sin^4(x)}{4(a^2+b^2)}+\frac{b\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\left(-\frac{ab\left(\frac{b\int\sin\left(x+\frac{\pi}{2}\right)^2dx}{a^2+b^2}-\frac{ab\int\frac{\cos(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^2(x)}{2\left(a^2+b^2\right)}\right)}{a^2+b^2}-\frac{b\cos^4(x)}{4(a^2+b^2)}+\frac{a\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}\right)}{a^2+b^2}-\frac{ab\int\frac{\cos(x)^2\sin(x)^2}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}$$

$$\Big\downarrow \ 3115$$

$$\frac{a\left(-\frac{ab\left(\frac{a\left(\frac{\int 1dx}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}-\frac{ab\int\frac{\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\sin^2(x)}{2\left(a^2+b^2\right)}\right)}{a^2+b^2}+\frac{a\sin^4(x)}{4(a^2+b^2)}+\frac{b\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\left(-\frac{ab\left(\frac{b\left(\frac{\int 1dx}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}-\frac{ab\int\frac{\cos(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^2(x)}{2\left(a^2+b^2\right)}\right)}{a^2+b^2}-\frac{b\cos^4(x)}{4(a^2+b^2)}+\frac{a\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}\right)}{a^2+b^2}-\frac{ab\int\frac{\cos(x)^2\sin(x)^2}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}$$

3.292. $\int\frac{\cos^3(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}dx$

$$\Big\downarrow \text{24}$$

$$\frac{a\left(-\frac{ab\left(-\frac{ab\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\sin^2(x)}{2\left(a^2+b^2\right)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^4(x)}{4(a^2+b^2)}+\frac{b\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\left(-\frac{ab\left(-\frac{ab\int \frac{\cos(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^2(x)}{2\left(a^2+b^2\right)}+\frac{b\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}-\frac{b\cos^4(x)}{4(a^2+b^2)}+\frac{a\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}\right)}{a^2+b^2}-\frac{ab\int \frac{\cos(x)^2\sin(x)^2}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3576}$$

$$\frac{a\left(-\frac{ab\left(-\frac{ab\left(\frac{bx}{a^2+b^2}-\frac{a\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\sin^2(x)}{2\left(a^2+b^2\right)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^4(x)}{4(a^2+b^2)}+\frac{b\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\left(-\frac{ab\left(-\frac{ab\int \frac{\cos(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^2(x)}{2\left(a^2+b^2\right)}+\frac{b\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}-\frac{b\cos^4(x)}{4(a^2+b^2)}+\frac{a\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}\right)}{a^2+b^2}-\frac{ab\int \frac{\cos(x)^2\sin(x)^2}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

3.292. $\int \frac{\cos^3(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

$$\frac{a\left(-\frac{ab\left(-\frac{ab\left(\frac{bx}{a^2+b^2}-\frac{a\int\frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\sin^2(x)}{2\left(a^2+b^2\right)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^4(x)}{4(a^2+b^2)}+\frac{b\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}\right)}{a^2+b^2}$$

$$\frac{b\left(-\frac{ab\left(-\frac{ab\int\frac{\cos(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^2(x)}{2\left(a^2+b^2\right)}+\frac{b\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}-\frac{b\cos^4(x)}{4(a^2+b^2)}+\frac{a\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}\right)}{a^2+b^2}-\frac{ab\int\frac{\cos(x)^2\sin(x)^2}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}$$

$$\Big\downarrow \; 3577$$

$$\frac{a\left(-\frac{ab\left(-\frac{ab\left(\frac{bx}{a^2+b^2}-\frac{a\int\frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\sin^2(x)}{2\left(a^2+b^2\right)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^4(x)}{4(a^2+b^2)}+\frac{b\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}\right)}{a^2+b^2}$$

$$\frac{b\left(-\frac{ab\left(-\frac{ab\left(\frac{b\int\frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{ax}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^2(x)}{2\left(a^2+b^2\right)}+\frac{b\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}-\frac{b\cos^4(x)}{4(a^2+b^2)}+\frac{a\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}\right)}{a^2+b^2}$$

$$\frac{ab\int\frac{\cos(x)^2\sin(x)^2}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}$$

$$\Big\downarrow \; 3042$$

3.292. $\displaystyle\int \frac{\cos^3(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

$$\frac{a\left(-\frac{ab\left(-\frac{ab\left(\frac{bx}{a^2+b^2}-\frac{a\int\frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\sin^2(x)}{2\left(a^2+b^2\right)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^4(x)}{4(a^2+b^2)}+\frac{b\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}\right)}{a^2+b^2}$$

$$\frac{b\left(-\frac{ab\left(-\frac{ab\left(\frac{b\int\frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{ax}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^2(x)}{2\left(a^2+b^2\right)}+\frac{b\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}-\frac{b\cos^4(x)}{4(a^2+b^2)}+\frac{a\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}\right)}{a^2+b^2}$$

$$\frac{ab\int\frac{\cos(x)^2\sin(x)^2}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}$$

$$\Big\downarrow \; 3590$$

$$\frac{a\left(-\frac{ab\left(-\frac{ab\left(\frac{bx}{a^2+b^2}-\frac{a\int\frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\sin^2(x)}{2\left(a^2+b^2\right)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^4(x)}{4(a^2+b^2)}+\frac{b\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}\right)}{a^2+b^2}$$

$$\frac{ab\left(\frac{b\int\frac{\cos^2(x)\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\int\frac{\cos(x)\sin^2(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}-\frac{ab\int\frac{\cos(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}\right)}{a^2+b^2}+$$

$$\frac{b\left(-\frac{ab\left(-\frac{ab\left(\frac{b\int\frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{ax}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^2(x)}{2\left(a^2+b^2\right)}+\frac{b\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}-\frac{b\cos^4(x)}{4(a^2+b^2)}+\frac{a\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \; 3042$$

3.292. $\int \frac{\cos^3(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

$$\frac{a\left(-\frac{ab\left(-\frac{ab\left(\frac{bx}{a^2+b^2}-\frac{a\int\frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\sin^2(x)}{2\left(a^2+b^2\right)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^4(x)}{4(a^2+b^2)}+\frac{b\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}\right.}{a^2+b^2}$$

$$\frac{b\left(-\frac{ab\left(-\frac{ab\left(\frac{b\int\frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{ax}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^2(x)}{2\left(a^2+b^2\right)}+\frac{b\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}-\frac{b\cos^4(x)}{4(a^2+b^2)}+\frac{a\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}\right.}{a^2+b^2}$$

$$\frac{ab\left(-\frac{ab\int\frac{\cos(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{b\int\frac{\cos(x)^2\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\int\frac{\cos(x)\sin(x)^2}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}$$

$$\Big\downarrow \; 3588$$

$$\frac{a\left(-\frac{ab\left(-\frac{ab\left(\frac{bx}{a^2+b^2}-\frac{a\int\frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\sin^2(x)}{2\left(a^2+b^2\right)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^4(x)}{4(a^2+b^2)}+\frac{b\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}\right.}{a^2+b^2}$$

$$\frac{ab\left(\frac{b\left(\frac{b\int\cos^2(x)dx}{a^2+b^2}+\frac{a\int\cos(x)\sin(x)dx}{a^2+b^2}-\frac{ab\int\frac{\cos(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(\frac{a\int\sin^2(x)dx}{a^2+b^2}+\frac{b\int\cos(x)\sin(x)dx}{a^2+b^2}-\frac{ab\int\frac{\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}-\frac{ab\int\ldots}{(a\cos\ldots}\right.}{a^2+b^2}$$

$$\frac{b\left(-\frac{ab\left(-\frac{ab\left(\frac{b\int\frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{ax}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^2(x)}{2\left(a^2+b^2\right)}+\frac{b\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}-\frac{b\cos^4(x)}{4(a^2+b^2)}+\frac{a\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}\right.}{a^2+b^2}$$

3.292. $\displaystyle\int\frac{\cos^3(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

$\downarrow$ 3042

$$\frac{a\Bigg(-\frac{ab\Big(-\frac{ab\big(\frac{bx}{a^2+b^2}-\frac{a\int\frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\big)}{a^2+b^2}+\frac{b\sin^2(x)}{2(a^2+b^2)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\Big)}{a^2+b^2}+\frac{a\sin^4(x)}{4(a^2+b^2)}+\frac{b\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}}{a^2+b^2}$$

$$\frac{b\Bigg(-\frac{ab\Big(-\frac{ab\big(\frac{b\int\frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{ax}{a^2+b^2}\big)}{a^2+b^2}+\frac{a\sin^2(x)}{2(a^2+b^2)}+\frac{b\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\Big)}{a^2+b^2}-\frac{b\cos^4(x)}{4(a^2+b^2)}+\frac{a\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}}{a^2+b^2}$$

$$\frac{ab\Bigg(-\frac{ab\int\frac{\cos(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{a\Big(\frac{a\int\sin(x)^2dx}{a^2+b^2}+\frac{b\int\cos(x)\sin(x)dx}{a^2+b^2}-\frac{ab\int\frac{\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\Big)}{a^2+b^2}+\frac{b\Big(\frac{b\int\sin\left(x+\frac{\pi}{2}\right)^2dx}{a^2+b^2}+\frac{a\int\cos(x)\sin(x)dx}{a^2+b^2}-\frac{ab}{}}{a^2+b^2}}{a^2+b^2}$$

$\downarrow$ 3044

3.292. $\displaystyle\int\frac{\cos^3(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

$$\frac{a\left(-\frac{ab\left(-\frac{ab\left(\frac{bx}{a^2+b^2}-\frac{a\int\frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\sin^2(x)}{2\left(a^2+b^2\right)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^4(x)}{4(a^2+b^2)}+\frac{b(\frac{1}{4}(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x))-\frac{1}{4}\sin(x)\cos^3(x))}{a^2+b^2}\right.}{a^2+b^2}$$

$$\frac{b\left(-\frac{ab\left(-\frac{ab\left(\frac{b\int\frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{ax}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^2(x)}{2\left(a^2+b^2\right)}+\frac{b\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}-\frac{b\cos^4(x)}{4(a^2+b^2)}+\frac{a(\frac{1}{4}(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x))-\frac{1}{4}\sin(x)\cos^3(x))}{a^2+b^2}\right.}{a^2+b^2}$$

$$\frac{ab\left(-\frac{ab\int\frac{\cos(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}dx}{a^2+b^2}+\frac{b\left(\frac{b\int\sin\left(x+\frac{\pi}{2}\right)^2dx}{a^2+b^2}+\frac{a\int\sin(x)d\sin(x)}{a^2+b^2}-\frac{ab\int\frac{\cos(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\left(\frac{a\int\sin(x)^2dx}{a^2+b^2}+\frac{b\int\sin(x)d\sin(x)}{a^2+b^2}-\frac{ab\int\right.}{a^2+b^2}\right.}{a^2+b^2}$$

$$\Big\downarrow \; 15$$

3.292. $\int \frac{\cos^3(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

$$\frac{a\left(-\frac{ab\left(-\frac{ab\left(\frac{bx}{a^2+b^2}-\frac{a\int\frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\sin^2(x)}{2(a^2+b^2)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^4(x)}{4(a^2+b^2)}+\frac{b\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}\right.}{a^2+b^2}$$

$$\frac{b\left(-\frac{ab\left(-\frac{ab\left(\frac{b\int\frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{ax}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^2(x)}{2(a^2+b^2)}+\frac{b\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}-\frac{b\cos^4(x)}{4(a^2+b^2)}+\frac{a\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}\right.}{a^2+b^2}$$

$$\frac{ab\left(\frac{a\left(\frac{a\int\sin(x)^2dx}{a^2+b^2}-\frac{ab\int\frac{\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\sin^2(x)}{2(a^2+b^2)}\right)}{a^2+b^2}+\frac{b\left(\frac{b\int\sin\left(x+\frac{\pi}{2}\right)^2dx}{a^2+b^2}-\frac{ab\int\frac{\cos(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^2(x)}{2(a^2+b^2)}\right)}{a^2+b^2}-\frac{ab\int\frac{\cos(x)\sin(x)}{(a\cos(x)+b\sin(x}}{a^2+b^2}\right.}{a^2+b^2}$$

$\downarrow$ 3115

3.292. $\int \frac{\cos^3(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

$$\frac{a\left(-\frac{ab\left(-\frac{ab\left(\frac{bx}{a^2+b^2}-\frac{a\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}\right)}{a^2+b^2}+\frac{b\sin^2(x)}{2\left(a^2+b^2\right)}+\frac{a\left(\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^4(x)}{4(a^2+b^2)}+\frac{b\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}\right.}{a^2+b^2}$$

$$\frac{b\left(-\frac{ab\left(-\frac{ab\left(\frac{b\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{ax}{a^2+b^2}\right)}{a^2+b^2}+\frac{a\sin^2(x)}{2\left(a^2+b^2\right)}+\frac{b\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}\right)}{a^2+b^2}-\frac{b\cos^4(x)}{4(a^2+b^2)}+\frac{a\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)}{a^2+b^2}\right.}{a^2+b^2}$$

$$\frac{ab\left(\frac{b\left(\frac{b\left(\frac{\int 1dx}{2}+\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}-\frac{ab\int \frac{\cos(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{a\sin^2(x)}{2\left(a^2+b^2\right)}\right)}{a^2+b^2}+\frac{a\left(\frac{a\left(\frac{\int 1dx}{2}-\frac{1}{2}\sin(x)\cos(x)\right)}{a^2+b^2}-\frac{ab\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\sin^2(x)}{2\left(a^2+b^2\right)}\right)}{a^2+b^2}-\right.}{a^2+b^2}$$

input
```
Int[(Cos[x]^3*Sin[x]^3)/(a*Cos[x] + b*Sin[x])^2,x]
```

output
```
$Aborted
```

3.292.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.292. $\int \frac{\cos^3(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3576
```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(x/(a^2 + b^2)), x] - Simp[a/(a^2 + b
^2)   Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x
]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3577
```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[a*(x/(a^2 + b^2)), x] + Simp[b/(a^2 + b
^2)   Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x
]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

3.292. $\int \frac{\cos^3(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

rule 3588

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b
/(a^2 + b^2)   Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Simp[a/(a
^2 + b^2)   Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Simp[a*(b/(a^
2 + b^2))   Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x]
+ b*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
&& IGtQ[m, 0] && IGtQ[n, 0]
```

rule 3590

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Sim
p[b/(a^2 + b^2)   Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
 b*Sin[c + d*x])^(p + 1), x], x] + (Simp[a/(a^2 + b^2)   Int[Cos[c + d*x]^(
m - 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - S
imp[a*(b/(a^2 + b^2))   Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Co
s[c + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^
2 + b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

3.292.4 Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.10

method	result
default	$\frac{a^{3} b^{2}}{\left(a^{2}+b^{2}\right)^{3} \left(a+b \tan \left(x \right)\right)}-\frac{3 a^{2} b^{2} \left(a^{2}-b^{2}\right) \ln \left(a+b \tan \left(x \right)\right)}{\left(a^{2}+b^{2}\right)^{4}}+\frac{\frac{\left(\frac{1}{2} a^{3} b^{3}-\frac{3}{4} a \,b^{5}+\frac{5}{4} a^{5} b \right) \tan \left(x \right)^{3}+\left(-\frac{1}{2} a^{6}+a^{4} b^{2}+\frac{3}{2} a^{2} b^{4}\right) \tan \left(x \right)^{2}+\left(\frac{3}{4} a \cdots\right.}{\left(1+\tan \left(x \right)^{2}\right)^{2}}}{\cdots}$
parallelrisch	$\frac{-192 b^{2} \left(a^{2}-b^{2}+\cos \left(2 x \right) \left(a^{2}+b^{2}\right)\right) \left(a -b \right) a^{2} \left(a +b \right) \ln \left(\frac{-a \cos \left(x \right)-b \sin \left(x \right)}{\cos \left(x \right)+1}\right)+192 b^{2} \left(a^{2}-b^{2}+\cos \left(2 x \right) \left(a^{2}+b^{2}\right)\right) \left(a -b \right) a^{2} \left(a +b \right) \ln \left(\frac{\cdots}{\cos \cdots}\right.}{\cdots}$
risch	$\frac{3 x a b}{4 \left(4 i a^{3} b -4 i a \,b^{3}-a^{4}+6 a^{2} b^{2}-b^{4}\right)}+\frac{{\mathrm e}^{4 i x}}{-128 i b a +64 a^{2}-64 b^{2}}-\frac{i {\mathrm e}^{2 i x} b}{16 \left(-3 i b \,a^{2}+i b^{3}+a^{3}-3 a \,b^{2}\right)}-\frac{{\mathrm e}^{2 i x} a}{16 \left(-3 i b \,a^{2}+i b^{3}+a^{3}-3 a \,b^{2}\right)}+\cdots$
norman	Expression too large to display

input

```
int(cos(x)^3*sin(x)^3/(a*cos(x)+b*sin(x))^2,x,method=_RETURNVERBOSE)
```

output

```
a^3*b^2/(a^2+b^2)^3/(a+b*tan(x))-3*a^2*b^2*(a^2-b^2)/(a^2+b^2)^4*ln(a+b*ta
n(x))+1/(a^2+b^2)^4*(((1/2*a^3*b^3-3/4*a*b^5+5/4*a^5*b)*tan(x)^3+(-1/2*a^6
+a^4*b^2+3/2*a^2*b^4)*tan(x)^2+(3/4*a^5*b-1/2*a^3*b^3-5/4*a*b^5)*tan(x)-1/
4*a^6+5/4*a^4*b^2+5/4*a^2*b^4-1/4*b^6)/(1+tan(x)^2)^2+3/4*a*b*(1/2*(4*a^3*
b-4*a*b^3)*ln(1+tan(x)^2)+(-a^4+6*a^2*b^2-b^4)*arctan(tan(x))))
```

3.292.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.77

$$\int \frac{\cos^3(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{8\,(a^7+3\,a^5b^2+3\,a^3b^4+ab^6)\cos{(x)}^5-8\,(2\,a^7+3\,a^5b^2-ab^6)\cos{(x)}^3+(5\,a^7+21\,a^5b^2+27\,a^3b^4-21\,ab}{}$$

input
```
integrate(cos(x)^3*sin(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

output
```
1/32*(8*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cos(x)^5 - 8*(2*a^7 + 3*a^5*
b^2 - a*b^6)*cos(x)^3 + (5*a^7 + 21*a^5*b^2 + 27*a^3*b^4 - 21*a*b^6 - 24*(
a^6*b - 6*a^4*b^3 + a^2*b^5)*x)*cos(x) - 48*((a^5*b^2 - a^3*b^4)*cos(x) +
(a^4*b^3 - a^2*b^5)*sin(x))*log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2
 + b^2) + (5*a^6*b - 51*a^4*b^3 - 21*a^2*b^5 + 3*b^7 - 8*(a^6*b + 3*a^4*b^
3 + 3*a^2*b^5 + b^7)*cos(x)^4 + 24*(a^6*b + 2*a^4*b^3 + a^2*b^5)*cos(x)^2
- 24*(a^5*b^2 - 6*a^3*b^4 + a*b^6)*x)*sin(x))/((a^9 + 4*a^7*b^2 + 6*a^5*b^
4 + 4*a^3*b^6 + a*b^8)*cos(x) + (a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7
 + b^9)*sin(x))
```

3.292.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(x)**3*sin(x)**3/(a*cos(x)+b*sin(x))**2,x)
```

output
```
Timed out
```

3.292.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 456 vs. $2(200) = 400$.

Time = 0.31 (sec) , antiderivative size = 456, normalized size of antiderivative = 2.17

$$\int \frac{\cos^3(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx = -\frac{3\,(a^5b-6\,a^3b^3+ab^5)x}{4\,(a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8)} - \frac{3\,(a^4b^2-a^2b^4)\log\left(b\tan(x)+a\right)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8} + \frac{3\,(a^4b^2-a^2b^4)\log\left(\tan(x)^2+1\right)}{2\,(a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8)} - \frac{a^5-10\,a^3b^2+ab^4-3\,(3\,a^3b^2-ab^4)\tan(x)^4-3\,(a^4b+a}{4\,(a^7+3\,a^5b^2+3\,a^3b^4+ab^6+(a^6b+3\,a^4b^3+3\,a^2b^5+b^7)\tan(x)^5+(a^7+3\,a^5b^2+3\,a^3b^4+ab^6)\tan(x}$$

input
```
integrate(cos(x)^3*sin(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

output
```
-3/4*(a^5*b - 6*a^3*b^3 + a*b^5)*x/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^
6 + b^8) - 3*(a^4*b^2 - a^2*b^4)*log(b*tan(x) + a)/(a^8 + 4*a^6*b^2 + 6*a^
4*b^4 + 4*a^2*b^6 + b^8) + 3/2*(a^4*b^2 - a^2*b^4)*log(tan(x)^2 + 1)/(a^8
+ 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - 1/4*(a^5 - 10*a^3*b^2 + a*b^4
 - 3*(3*a^3*b^2 - a*b^4)*tan(x)^4 - 3*(a^4*b + a^2*b^3)*tan(x)^3 + (2*a^5
- 17*a^3*b^2 + 5*a*b^4)*tan(x)^2 - (2*a^4*b + a^2*b^3 - b^5)*tan(x))/(a^7
+ 3*a^5*b^2 + 3*a^3*b^4 + a*b^6 + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*ta
n(x)^5 + (a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*tan(x)^4 + 2*(a^6*b + 3*a^4
*b^3 + 3*a^2*b^5 + b^7)*tan(x)^3 + 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)
*tan(x)^2 + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*tan(x))
```

3.292.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 435 vs. $2(200) = 400$.

Time = 0.29 (sec) , antiderivative size = 435, normalized size of antiderivative = 2.07

$$\int \frac{\cos^3(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx = -\frac{3\,(a^5b-6\,a^3b^3+ab^5)x}{4\,(a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8)} + \frac{3\,(a^4b^2-a^2b^4)\log\left(\tan(x)^2+1\right)}{2\,(a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8)} - \frac{3\,(a^4b^3-a^2b^5)\log\left(|b\tan(x)+a|\right)}{a^8b+4\,a^6b^3+6\,a^4b^5+4\,a^2b^7+b^9} + \frac{3\,a^4b^3\tan(x)-3\,a^2b^5\tan(x)+4\,a^5b^2-2\,a^3b^4}{(a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8)(b\tan(x)+a)} - \frac{9\,a^4b^2\tan(x)^4-9\,a^2b^4\tan(x)^4-5\,a^5b\tan(x)^3-2\,a^3b^3\tan(x)^3+3\,ab^5\tan(x)^3+2\,a^6\tan(x)^2+14\,a^4}{4\,(a^8+4\,a^6b^2+6\,a^4b^4+4}$$

input
```
integrate(cos(x)^3*sin(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

output
```
-3/4*(a^5*b - 6*a^3*b^3 + a*b^5)*x/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^
6 + b^8) + 3/2*(a^4*b^2 - a^2*b^4)*log(tan(x)^2 + 1)/(a^8 + 4*a^6*b^2 + 6*
a^4*b^4 + 4*a^2*b^6 + b^8) - 3*(a^4*b^3 - a^2*b^5)*log(abs(b*tan(x) + a))/
(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9) + (3*a^4*b^3*tan(x) - 3*
a^2*b^5*tan(x) + 4*a^5*b^2 - 2*a^3*b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*
a^2*b^6 + b^8)*(b*tan(x) + a)) - 1/4*(9*a^4*b^2*tan(x)^4 - 9*a^2*b^4*tan(x
)^4 - 5*a^5*b*tan(x)^3 - 2*a^3*b^3*tan(x)^3 + 3*a*b^5*tan(x)^3 + 2*a^6*tan
(x)^2 + 14*a^4*b^2*tan(x)^2 - 24*a^2*b^4*tan(x)^2 - 3*a^5*b*tan(x) + 2*a^3
*b^3*tan(x) + 5*a*b^5*tan(x) + a^6 + 4*a^4*b^2 - 14*a^2*b^4 + b^6)/((a^8 +
 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*(tan(x)^2 + 1)^2)
```

3.292.9 Mupad [B] (verification not implemented)

Time = 39.73 (sec) , antiderivative size = 8198, normalized size of antiderivative = 39.04

$$\int \frac{\cos^3(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \text{Too large to display}$$

input
```
int((cos(x)^3*sin(x)^3)/(a*cos(x) + b*sin(x))^2,x)
```

output
```
((tan(x/2)^4*(a*b^2 + 4*a^3))/(a^4 + b^4 + 2*a^2*b^2) - (tan(x/2)^6*(a*b^2
 + 4*a^3))/(a^4 + b^4 + 2*a^2*b^2) - (3*a*b^2*tan(x/2)^2)/(a^4 + b^4 + 2*a
^2*b^2) + (3*a*b^2*tan(x/2)^8)/(a^4 + b^4 + 2*a^2*b^2) + (3*b*tan(x/2)^9*(
a^4 - 3*a^2*b^2))/(2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (3*b*tan(x/2)*
(a^4 - 3*a^2*b^2))/(2*(a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2)) + (4*b*tan(x/2)
^3*(a^4 + b^4 - 4*a^2*b^2))/((a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2)) + (4*b*t
an(x/2)^7*(a^4 + b^4 - 4*a^2*b^2))/((a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2)) -
 (3*b*tan(x/2)^5*(a^4 + 13*a^2*b^2))/((a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2))
)/(a + 2*b*tan(x/2) + 3*a*tan(x/2)^2 + 2*a*tan(x/2)^4 - 2*a*tan(x/2)^6 - 3
*a*tan(x/2)^8 - a*tan(x/2)^10 + 8*b*tan(x/2)^3 + 12*b*tan(x/2)^5 + 8*b*tan
(x/2)^7 + 2*b*tan(x/2)^9) + (log(a + 2*b*tan(x/2) - a*tan(x/2)^2)*(3*a^2*b
^4 - 3*a^4*b^2))/(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2) - (log(1/
(cos(x) + 1))*(96*a^2*b^4 - 96*a^4*b^2))/(2*(16*a^8 + 16*b^8 + 64*a^2*b^6
+ 96*a^4*b^4 + 64*a^6*b^2)) + (3*a*b*atan((tan(x/2)*((((6*(45*a^7*b^10 - 1
8*a^5*b^12 - 135*a^9*b^8 + 99*a^11*b^6 + 9*a^13*b^4))/(a^18 + b^18 + 9*a^2
*b^16 + 36*a^4*b^14 + 84*a^6*b^12 + 126*a^8*b^10 + 126*a^10*b^8 + 84*a^12*
b^6 + 36*a^14*b^4 + 9*a^16*b^2) - (((6*(6*a^3*b^16 - 153*a^5*b^14 - 180*a^
7*b^12 + 357*a^9*b^10 + 534*a^11*b^8 + 81*a^13*b^6 - 72*a^15*b^4 + 3*a^17*
b^2))/(a^18 + b^18 + 9*a^2*b^16 + 36*a^4*b^14 + 84*a^6*b^12 + 126*a^8*b^10
 + 126*a^10*b^8 + 84*a^12*b^6 + 36*a^14*b^4 + 9*a^16*b^2) - ((96*a^2*b^...
```

3.292. $\int \frac{\cos^3(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

3.293 $\int \frac{\tan(x)}{b\cos(x)+a\sin(x)}\,dx$

3.293.1 Optimal result

Integrand size = 14, antiderivative size = 47

$$\int \frac{\tan(x)}{b\cos(x)+a\sin(x)}\,dx = \frac{\operatorname{arctanh}(\sin(x))}{a} + \frac{b\operatorname{arctanh}\left(\frac{a\cos(x)-b\sin(x)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}$$

output
```
arctanh(sin(x))/a+b*arctanh((a*cos(x)-b*sin(x))/(a^2+b^2)^(1/2))/a/(a^2+b^
2)^(1/2)
```

3.293.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.62

$$\int \frac{\tan(x)}{b\cos(x)+a\sin(x)}\,dx = \frac{-\frac{2b\operatorname{arctanh}\left(\frac{-a+b\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} - \log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right) + \log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)}{a}$$

input
```
Integrate[Tan[x]/(b*Cos[x] + a*Sin[x]),x]
```

output
```
((-2*b*ArcTanh[(-a + b*Tan[x/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2] - Log[C
os[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]])/a
```

3.293.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 3589, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(x)}{a\sin(x)+b\cos(x)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin(x)}{\cos(x)(a\sin(x)+b\cos(x))}dx$$
$$\Big\downarrow \text{3589}$$
$$\int \left(\frac{\sec(x)}{a}-\frac{b}{a(a\sin(x)+b\cos(x))}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{b\text{arctanh}\left(\frac{a\cos(x)-b\sin(x)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}+\frac{\text{arctanh}(\sin(x))}{a}$$

input
```
Int[Tan[x]/(b*Cos[x] + a*Sin[x]),x]
```

output
```
ArcTanh[Sin[x]]/a + (b*ArcTanh[(a*Cos[x] - b*Sin[x])/Sqrt[a^2 + b^2]])/(a*
Sqrt[a^2 + b^2])
```

3.293.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3589
```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[Ex
pandTrig[cos[c + d*x]^m*(sin[c + d*x]^n/(a*cos[c + d*x] + b*sin[c + d*x])),
 x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]
```

3.293.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.34

method	result	size
default	$-\frac{\ln\left(\tan\left(\frac{x}{2}\right)-1\right)}{a}+\frac{2b\,\mathrm{arctanh}\left(\frac{-2b\tan\left(\frac{x}{2}\right)+2a}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)+1\right)}{a}$	63
risch	$\frac{ib\ln\left(\mathrm{e}^{ix}+\frac{ia+b}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}\,a}-\frac{ib\ln\left(\mathrm{e}^{ix}-\frac{ia+b}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}\,a}+\frac{\ln\left(i+\mathrm{e}^{ix}\right)}{a}-\frac{\ln\left(\mathrm{e}^{ix}-i\right)}{a}$	124

input
```
int(tan(x)/(b*cos(x)+a*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/a*ln(tan(1/2*x)-1)+2*b/a/(a^2+b^2)^(1/2)*arctanh(1/2*(-2*b*tan(1/2*x)+2
*a)/(a^2+b^2)^(1/2))+1/a*ln(tan(1/2*x)+1)
```

3.293.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(43) = 86.

Time = 0.29 (sec) , antiderivative size = 140, normalized size of antiderivative = 2.98

$$\int \frac{\tan(x)}{b\cos(x)+a\sin(x)}\,dx$$
$$=\frac{\sqrt{a^2+b^2}b\log\left(\frac{2\,ab\cos(x)\sin(x)-(a^2-b^2)\cos(x)^2-a^2-2\,b^2-2\,\sqrt{a^2+b^2}(a\cos(x)-b\sin(x))}{2\,ab\cos(x)\sin(x)-(a^2-b^2)\cos(x)^2+a^2}\right)+(a^2+b^2)\log\left(\sin\left(x\right)+1\right)-(a}{2\left(a^3+ab^2\right)}$$

input
```
integrate(tan(x)/(b*cos(x)+a*sin(x)),x, algorithm="fricas")
```

output
```
1/2*(sqrt(a^2 + b^2)*b*log((2*a*b*cos(x)*sin(x) - (a^2 - b^2)*cos(x)^2 - a
^2 - 2*b^2 - 2*sqrt(a^2 + b^2)*(a*cos(x) - b*sin(x)))/(2*a*b*cos(x)*sin(x)
 - (a^2 - b^2)*cos(x)^2 + a^2)) + (a^2 + b^2)*log(sin(x) + 1) - (a^2 + b^2
)*log(-sin(x) + 1))/(a^3 + a*b^2)
```

3.293.6 Sympy [F]

$$\int \frac{\tan(x)}{b\cos(x)+a\sin(x)}\,dx = \int \frac{\tan(x)}{a\sin(x)+b\cos(x)}\,dx$$

input
```
integrate(tan(x)/(b*cos(x)+a*sin(x)),x)
```

output
```
Integral(tan(x)/(a*sin(x) + b*cos(x)), x)
```

3.293.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(43) = 86$.

Time = 0.33 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.09

$$\int \frac{\tan(x)}{b\cos(x)+a\sin(x)}\,dx = \frac{b\log\left(\frac{a-\frac{b\sin(x)}{\cos(x)+1}+\sqrt{a^2+b^2}}{a-\frac{b\sin(x)}{\cos(x)+1}-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}a}+\frac{\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(x)}{\cos(x)+1}-1\right)}{a}$$

input
```
integrate(tan(x)/(b*cos(x)+a*sin(x)),x, algorithm="maxima")
```

output
```
b*log((a - b*sin(x)/(cos(x) + 1) + sqrt(a^2 + b^2))/(a - b*sin(x)/(cos(x)
+ 1) - sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a) + log(sin(x)/(cos(x) + 1) + 1
)/a - log(sin(x)/(cos(x) + 1) - 1)/a
```

3.293.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(43) = 86$.

Time = 0.32 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.91

$$\int \frac{\tan(x)}{b\cos(x)+a\sin(x)}\,dx = \frac{b\log\left(\frac{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}a} + \frac{\log\left(\left|\tan\left(\frac{1}{2}\,x\right)+1\right|\right)}{a}-\frac{\log\left(\left|\tan\left(\frac{1}{2}\,x\right)-1\right|\right)}{a}$$

input
```
integrate(tan(x)/(b*cos(x)+a*sin(x)),x, algorithm="giac")
```

output
```
b*log(abs(2*b*tan(1/2*x) - 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*tan(1/2*x) - 2
*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a) + log(abs(tan(1/2*x) + 1))/a
- log(abs(tan(1/2*x) - 1))/a
```

3.293.9 Mupad [B] (verification not implemented)

Time = 23.29 (sec) , antiderivative size = 408, normalized size of antiderivative = 8.68

$$\int \frac{\tan(x)}{b\cos(x) + a\sin(x)}\,dx = \frac{2\,\mathrm{atanh}\left(\tan\left(\frac{x}{2}\right)\right)}{a} - \frac{2\,b\,\mathrm{atanh}\left(\frac{64\,b^3}{\sqrt{a^2+b^2}\left(128\,b^2\tan\left(\frac{x}{2}\right) - \frac{128\,b^4\tan\left(\frac{x}{2}\right)}{a^2+b^2} + \frac{64\,a\,b^3}{a^2+b^2}\right)} - \frac{64\,b^5}{(a^2+b^2)^{3/2}\left(128\,b^2\tan\left(\frac{x}{2}\right) - \frac{128\,b^4\tan\left(\frac{x}{2}\right)}{a^2+b^2} + \frac{64\,a\,b^3}{a^2+b^2}\right)} + \frac{64}{\sqrt{a^2+b^2}\left(\frac{64}{a^2}\right.}\right.}{}$$

input
```
int(tan(x)/(b*cos(x) + a*sin(x)),x)
```

output
```
(2*atanh(tan(x/2)))/a - (2*b*atanh((64*b^3)/((a^2 + b^2)^(1/2)*(128*b^2*ta
n(x/2) - (128*b^4*tan(x/2))/(a^2 + b^2) + (64*a*b^3)/(a^2 + b^2))) - (64*b
^5)/((a^2 + b^2)^(3/2)*(128*b^2*tan(x/2) - (128*b^4*tan(x/2))/(a^2 + b^2)
+ (64*a*b^3)/(a^2 + b^2))) + (128*b^4*tan(x/2))/((a^2 + b^2)^(1/2)*((64*a^
2*b^3)/(a^2 + b^2) + 128*a*b^2*tan(x/2) - (128*a*b^4*tan(x/2))/(a^2 + b^2)
)) - (128*b^6*tan(x/2))/((a^2 + b^2)^(3/2)*((64*a^2*b^3)/(a^2 + b^2) + 128
*a*b^2*tan(x/2) - (128*a*b^4*tan(x/2))/(a^2 + b^2))) + (128*a*b^2*tan(x/2)
)/((a^2 + b^2)^(1/2)*(128*b^2*tan(x/2) - (128*b^4*tan(x/2))/(a^2 + b^2) +
(64*a*b^3)/(a^2 + b^2))) - (192*a*b^4*tan(x/2))/((a^2 + b^2)^(3/2)*(128*b^
2*tan(x/2) - (128*b^4*tan(x/2))/(a^2 + b^2) + (64*a*b^3)/(a^2 + b^2)))))/(
a*(a^2 + b^2)^(1/2))
```

3.294 $\int \frac{\cot(x)}{b\cos(x)+a\sin(x)}\,dx$

3.294.1 Optimal result

Integrand size = 14, antiderivative size = 48

$$\int \frac{\cot(x)}{b\cos(x)+a\sin(x)}\,dx = -\frac{\operatorname{arctanh}(\cos(x))}{b}+\frac{a\operatorname{arctanh}\left(\frac{a\cos(x)-b\sin(x)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}$$

output
```
-arctanh(cos(x))/b+a*arctanh((a*cos(x)-b*sin(x))/(a^2+b^2)^(1/2))/b/(a^2+b
^2)^(1/2)
```

3.294.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.25

$$\int \frac{\cot(x)}{b\cos(x)+a\sin(x)}\,dx = \frac{-\frac{2a\operatorname{arctanh}\left(\frac{-a+b\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}-\log\left(\cos\left(\frac{x}{2}\right)\right)+\log\left(\sin\left(\frac{x}{2}\right)\right)}{b}$$

input
```
Integrate[Cot[x]/(b*Cos[x] + a*Sin[x]),x]
```

output
```
((-2*a*ArcTanh[(-a + b*Tan[x/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2] - Log[C
os[x/2]] + Log[Sin[x/2]])/b
```

3.294.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 3589, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(x)}{a\sin(x)+b\cos(x)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\cos(x)}{\sin(x)(a\sin(x)+b\cos(x))}dx$$

$$\Big\downarrow \ 3589$$

$$\int \left(\frac{\csc(x)}{b}-\frac{a}{b(a\sin(x)+b\cos(x))}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{a\text{arctanh}\left(\frac{a\cos(x)-b\sin(x)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}-\frac{\text{arctanh}(\cos(x))}{b}$$

input
```
Int[Cot[x]/(b*Cos[x] + a*Sin[x]),x]
```

output
```
-(ArcTanh[Cos[x]]/b) + (a*ArcTanh[(a*Cos[x] - b*Sin[x])/Sqrt[a^2 + b^2]])/
(b*Sqrt[a^2 + b^2])
```

3.294.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3589
```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[Ex
pandTrig[cos[c + d*x]^m*(sin[c + d*x]^n/(a*cos[c + d*x] + b*sin[c + d*x])),
 x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]
```

3.294.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02

method	result	size
default	$\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{b}+\frac{2a\,\operatorname{arctanh}\left(\frac{-2b\tan\left(\frac{x}{2}\right)+2a}{2\sqrt{a^{2}+b^{2}}}\right)}{b\sqrt{a^{2}+b^{2}}}$	49
risch	$-\frac{ia\ln\left(\mathrm{e}^{ix}-\frac{ia+b}{\sqrt{-a^{2}-b^{2}}}\right)}{\sqrt{-a^{2}-b^{2}}\,b}+\frac{ia\ln\left(\mathrm{e}^{ix}+\frac{ia+b}{\sqrt{-a^{2}-b^{2}}}\right)}{\sqrt{-a^{2}-b^{2}}\,b}+\frac{\ln\left(\mathrm{e}^{ix}-1\right)}{b}-\frac{\ln\left(\mathrm{e}^{ix}+1\right)}{b}$	122

input
```
int(cot(x)/(b*cos(x)+a*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
1/b*ln(tan(1/2*x))+2*a/b/(a^2+b^2)^(1/2)*arctanh(1/2*(-2*b*tan(1/2*x)+2*a)
/(a^2+b^2)^(1/2))
```

3.294.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. 2(44) = 88.

Time = 0.29 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.96

$$\int \frac{\cot(x)}{b\cos(x)+a\sin(x)}\,dx$$
$$=\frac{\sqrt{a^{2}+b^{2}}a\log\left(\frac{2\,ab\cos(x)\sin(x)-(a^{2}-b^{2})\cos(x)^{2}-a^{2}-2\,b^{2}-2\,\sqrt{a^{2}+b^{2}}(a\cos(x)-b\sin(x))}{2\,ab\cos(x)\sin(x)-(a^{2}-b^{2})\cos(x)^{2}+a^{2}}\right)-(a^{2}+b^{2})\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+}{2\,(a^{2}b+b^{3})}$$

input
```
integrate(cot(x)/(b*cos(x)+a*sin(x)),x, algorithm="fricas")
```

output
```
1/2*(sqrt(a^2 + b^2)*a*log((2*a*b*cos(x)*sin(x) - (a^2 - b^2)*cos(x)^2 - a
^2 - 2*b^2 - 2*sqrt(a^2 + b^2)*(a*cos(x) - b*sin(x)))/(2*a*b*cos(x)*sin(x)
 - (a^2 - b^2)*cos(x)^2 + a^2)) - (a^2 + b^2)*log(1/2*cos(x) + 1/2) + (a^2
 + b^2)*log(-1/2*cos(x) + 1/2))/(a^2*b + b^3)
```

3.294.6 Sympy [F]

$$\int \frac{\cot(x)}{b\cos(x) + a\sin(x)}\,dx = \int \frac{\cot(x)}{a\sin(x) + b\cos(x)}\,dx$$

input
```
integrate(cot(x)/(b*cos(x)+a*sin(x)),x)
```

output
```
Integral(cot(x)/(a*sin(x) + b*cos(x)), x)
```

3.294.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.65

$$\int \frac{\cot(x)}{b\cos(x) + a\sin(x)}\,dx = \frac{a\log\left(\frac{a - \frac{b\sin(x)}{\cos(x)+1} + \sqrt{a^2+b^2}}{a - \frac{b\sin(x)}{\cos(x)+1} - \sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,b} + \frac{\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{b}$$

input
```
integrate(cot(x)/(b*cos(x)+a*sin(x)),x, algorithm="maxima")
```

output
```
a*log((a - b*sin(x)/(cos(x) + 1) + sqrt(a^2 + b^2))/(a - b*sin(x)/(cos(x)
+ 1) - sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b) + log(sin(x)/(cos(x) + 1))/b
```

3.294.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.56

$$\int \frac{\cot(x)}{b\cos(x) + a\sin(x)}\,dx = \frac{a\log\left(\frac{\left|2\,b\tan\left(\frac{1}{2}\,x\right) - 2\,a - 2\,\sqrt{a^2+b^2}\right|}{\left|2\,b\tan\left(\frac{1}{2}\,x\right) - 2\,a + 2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,b} + \frac{\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)}{b}$$

input
```
integrate(cot(x)/(b*cos(x)+a*sin(x)),x, algorithm="giac")
```

output
```
a*log(abs(2*b*tan(1/2*x) - 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*tan(1/2*x) - 2
*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b) + log(abs(tan(1/2*x)))/b
```

3.294.9 Mupad [B] (verification not implemented)

Time = 23.21 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.56

$$\int \frac{\cot(x)}{b\cos(x)+a\sin(x)}\,dx = \frac{\ln\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{b} - \frac{2\,a\,\mathrm{atanh}\left(\frac{\sqrt{a^2+b^2}\,\left(4\mathrm{i}\,\sin\left(\frac{x}{2}\right)\,a^2+2\mathrm{i}\,\cos\left(\frac{x}{2}\right)\,a\,b+1\mathrm{i}\,\sin\left(\frac{x}{2}\right)\,b^2\right)}{a^3\,\sin\left(\frac{x}{2}\right)\,4\mathrm{i}+a^2\,b\,\cos\left(\frac{x}{2}\right)\,1\mathrm{i}+a\,b^2\,\sin\left(\frac{x}{2}\right)\,3\mathrm{i}+b\,\cos\left(\frac{x}{2}\right)\,\left(a^2+b^2\right)\,1\mathrm{i}}\right)}{b\,\sqrt{a^2+b^2}}$$

input
```
int(cot(x)/(b*cos(x) + a*sin(x)),x)
```

output
```
log(sin(x/2)/cos(x/2))/b - (2*a*atanh(((a^2 + b^2)^(1/2)*(a^2*sin(x/2)*4i
+ b^2*sin(x/2)*1i + a*b*cos(x/2)*2i))/(a^3*sin(x/2)*4i + a^2*b*cos(x/2)*1i
 + a*b^2*sin(x/2)*3i + b*cos(x/2)*(a^2 + b^2)*1i)))/(b*(a^2 + b^2)^(1/2))
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                               convert(leaf_count_result,string)," vs. $2 (",
                               convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
            return False
        else:
            return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
            'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
            'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
            'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
            'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```